

Handwritten Notes
On
Wave Optics



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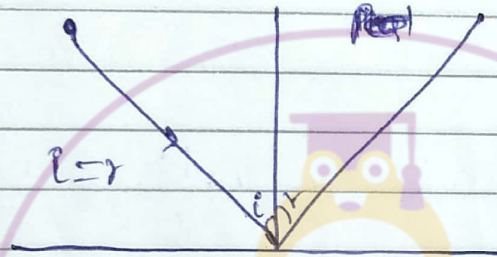
Wave optics

* Nature of light:

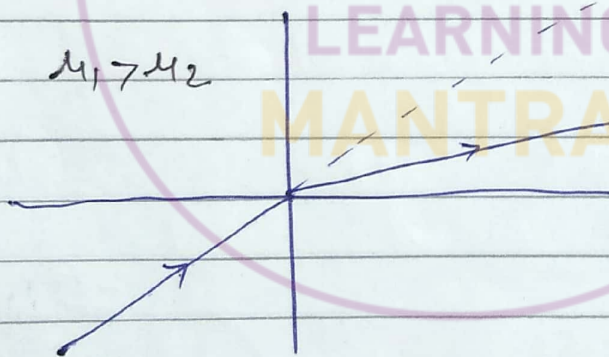
(i) Newton's Corpuscles theory: (Particles)

Light wave consist of very small particles called Corpuscles

In Reflection corpuscles is strike a surface perfectly elastically



In Refraction Corpuscles are ^{more} attracted by denser medium relative to rarer medium

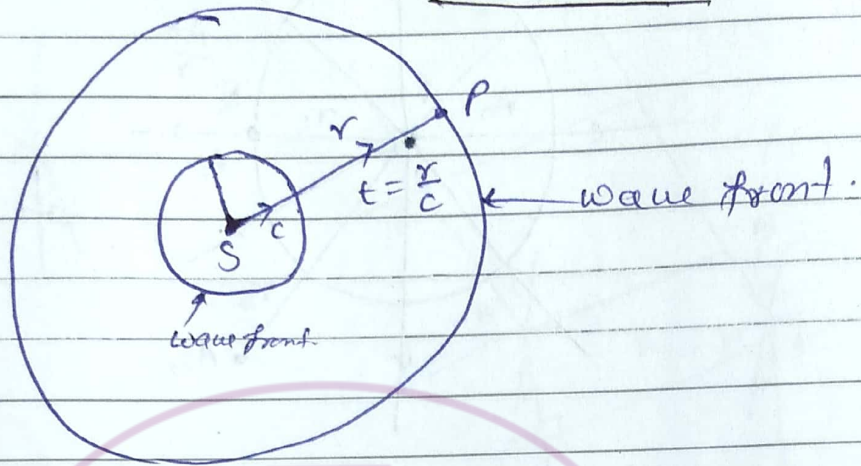


(ii) Huygen's Principle:

(1) There a universally present medium called ether

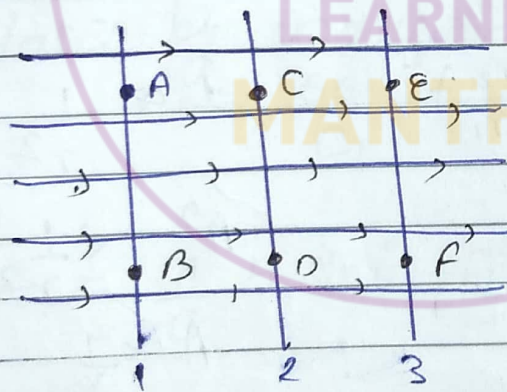
(2) Light wave emitting from a source these medium particles and medium particles all start oscillate. Oscillating medium particles becomes new source of light producing light wave in all directions called secondary wave or secondary wavelets.

* A Surface at which all the particles are vibrating in same phase called a wave front.



* Wave front:

- 1) It is the locus of all the particles vibrating in same phase
- 2) It is \perp to the direction of propagation of wave

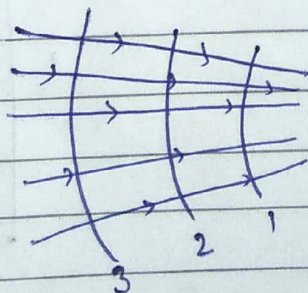


Planes

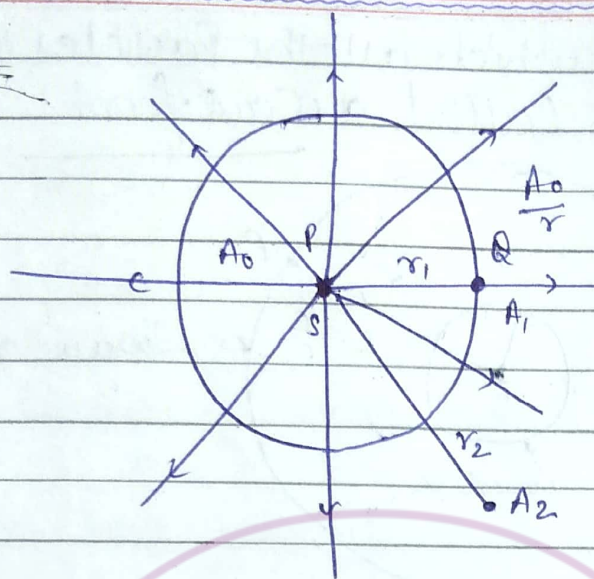
$$\phi_A = \phi_B$$

$$\phi_C = \phi_D$$

$$\phi_E = \phi_F$$



* Point Source



$$I \propto A^2$$

$$y = A \sin(kx - \omega t + \theta)$$

Intensity of point source

$$I = \frac{\text{Energy}}{\text{time} \times \text{Area}} = \frac{P}{4\pi r^2}$$

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2 \quad I \propto \frac{1}{r^2}$$

$$\frac{A_1}{A_2} = \frac{r_2}{r_1}$$

$$A^2 \propto \frac{1}{r^2}$$

$$A \propto \frac{1}{r}$$

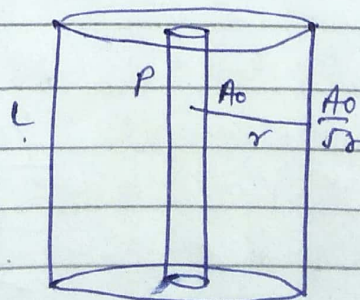
* Cylindrical source or extended source

or Line source!

$$I = \frac{P}{2\pi r l}$$

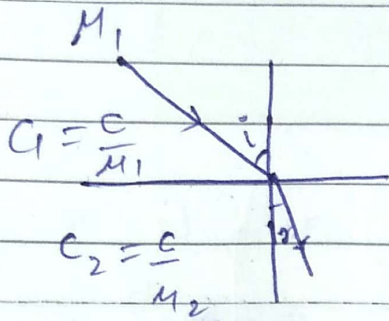
$$I \propto \frac{1}{r} \propto A^2$$

$$A \propto \frac{1}{\sqrt{r}}$$

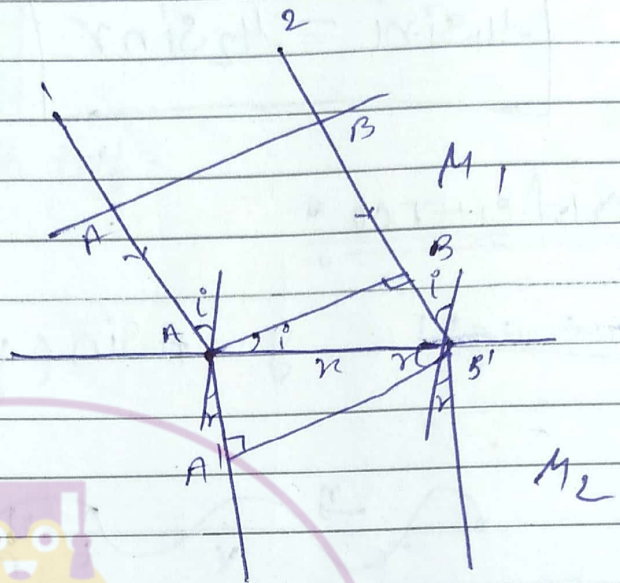


Important for Board

Refraction by Huygen's principle!



$\mu_2 (> \mu_1)$



$\Delta ABB'$

$$\sin i = \frac{BB'}{\mu}$$

$$BB' = \mu \sin i$$

time taken by wave from B to B'

$$t_1 = \frac{BB'}{c_1} = \frac{\mu \sin i}{\frac{c}{\mu_1}}$$

$$t_1 = \frac{\mu}{c} \sin i \quad \text{--- (1)}$$

$\Delta ABA'$

$$\sin r = \frac{AA'}{\mu}$$

$$AA' = \mu \sin r$$

time taken to reach from A to A'

$$t_2 = \frac{AA'}{c_2} = \frac{\mu \sin r}{\frac{c}{\mu_2}}$$

$$= \frac{\mu}{c} \mu_2 \sin r$$

$$t_1 = t_2$$

$$\mu_1 \sin i = \mu_2 \sin r$$

★★★ Interference

Wave eqⁿ: $y = A \sin(kx - \omega t + \theta)$ - sinusoidal wave



$$\omega - \text{Ang. freq.} = 2\pi f = \frac{2\pi}{T}$$

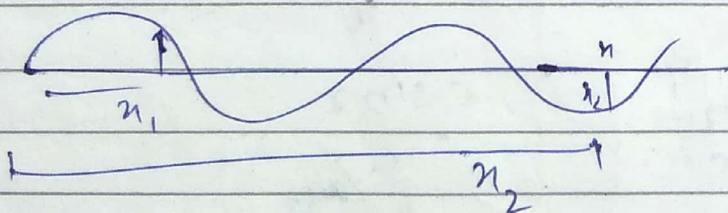
$$k - \text{Propagation Const.} = \frac{2\pi}{\lambda}$$

Ang. wave No.

$$\text{phase} = kx - \omega t + \theta$$

$$\phi = \underbrace{kx - \omega t}_{\text{phase angle}} + \underbrace{\theta}_{\text{phase const.}}$$

If t is fixed



$$\phi_1 = kx_1 - \omega t + \theta$$

$$\phi_2 = kx_2 - \omega t + \theta$$

$$\Delta\phi = k\Delta x$$

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x$$

Path diff.

if x is fixed

$$\phi_1 = kx - \omega t + \phi$$

$$\phi_2 = kx - \omega t + \phi$$

$$\Delta\phi = \omega\Delta t$$

$$\Delta\phi = \frac{2\pi}{T} \Delta t$$

* Superposition Principle:

Two or more waves can travel in a medium simultaneously without affecting each other when these waves reach a medium Particle simultaneously - Only net displacement of particle is given by sum of the displacements produce by each wave.

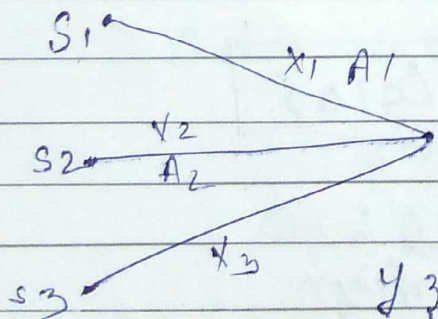
$$y_1 = A_1 \sin(kx_1 - \omega t) \quad I_1 \propto A_1^2$$

$$y_2 = A_2 \sin(kx_2 - \omega t) \quad I_2 \propto A_2^2$$

$$y_3 = A_3 \sin(kx_3 - \omega t) \quad I_3 \propto A_3^2$$

$$y = y_1 + y_2 + y_3$$

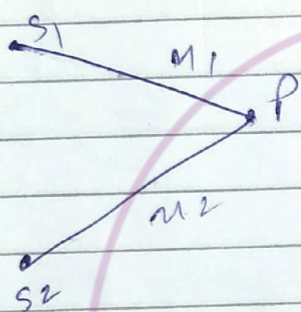
$$I \propto A^2$$



Interference:

It is the phenomena of redistribution of energy in a medium due to superposition of two or more waves.

At some Δ amplitude and intensity are maximum called constructive interference, and where intensity and amplitude are minimum called destructive interference.



$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \Delta\phi}$$

$$y_1 = A_1 \sin(kx_1 - \omega t) \quad I_1 \propto A_1^2$$

$$y_2 = A_2 \sin(kx_2 - \omega t) \quad I_2 \propto A_2^2$$

Phase diff $\Delta\phi = \frac{2\pi \Delta x}{\lambda}$ (Path diff.)

$$I \propto A^2$$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \Delta\phi$$

$$\frac{I}{\propto} = \frac{I_1}{\propto} + \frac{I_2}{\propto} + 2 \frac{I_1 I_2}{\propto} \cos \Delta\phi$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta\phi$$

(1) for constructive interference:
or
maxima or bright fringe

$$A \ \& \ I = \text{max}$$

$$\cos \Delta\phi = \text{max}$$

$$\cos \Delta\phi = 1$$

$$\Delta\phi = 2n\pi$$

$\Delta\phi = 0$ - zero order max

$\Delta\phi = 2\pi$ - 1st order max

$$\Delta\phi = \phi\pi = 2nd$$

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x = 2n\pi$$

$\Delta x = n\lambda$

$$\Delta n = d \quad \text{- 1st}$$

$$|n_1 - n_2| = m$$

$$A_{max} = A_1 + A_2$$

$$I_{max} = \left[\sqrt{I_1} + \sqrt{I_2} \right]^2$$

② for Destructive Interference
or minima or Dark fringe!

$$A \& I = \text{min}$$

$$\cos \Delta\phi = \text{min} = -1$$

$$\phi = (2n \pm 1)\pi$$

$$\Delta\phi = \pi, 3\pi, 5\pi, 7\pi$$

1st order 2nd order

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x = (2n \pm 1)\pi$$

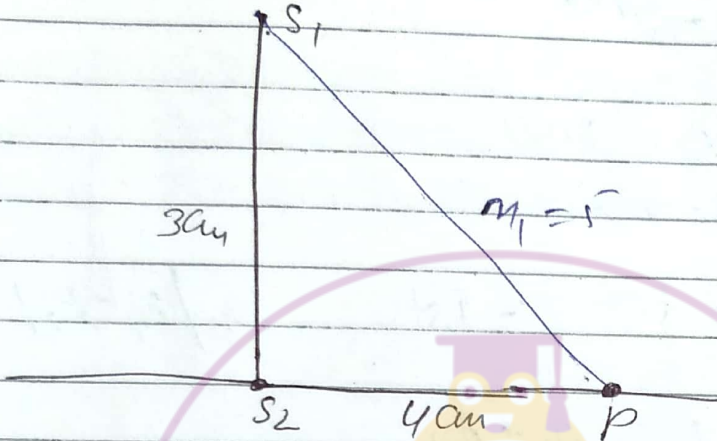
$$\Delta x = (2n \pm 1) \frac{\lambda}{2}$$

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}$$

$$A_{\min} = |A_1 - A_2|$$

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

Ex



monochromatic

Possible values of source.

Find wave length so that there is maxing at point P

$$m_2 = 4$$

Path diff

$$\Delta x = 10\text{ cm}$$

$$r = h \lambda$$

$$x = \frac{1}{n}$$

$$\frac{4 \times 3 = 12}{4 \times 4 = 16}$$

$$\left(\frac{3}{4} = \frac{\text{max } 12}{16}\right)$$

Enq

Q. ~~Same frequency~~ Interference pattern ratio of maxⁿ to minⁿ intensity is $\frac{42}{16} = \frac{I_{\max}}{I_{\min}}$

find the Ratio of Amplitudes of Interfering wave

$$\frac{11}{3}$$

~~$$\frac{11}{3}$$~~

$$\frac{I_{max}}{I_{min}} = \frac{49}{16}$$

$$\left(\frac{A_{max}}{A_{min}} \right)^2 = \frac{49}{16}$$

$$\frac{A_{max}}{A_{min}} = \frac{7}{4}$$

$$\frac{A_1 + A_2}{A_1 - A_2} = \frac{7}{4}$$

* Condition for good Interference:
 (1) Amplitude and Intensity of Interfering waves must be nearly equal

$$A_1 = 100$$

$$A_2 = 2$$

$$A_1 = 100$$

$$A_2 = 98$$

$$A_1 = 100$$

$$A_2 = 100$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \Delta\phi}$$

$$I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \Delta\phi$$

$$A_1 = A_2 = A_0$$

$$I_1 = I_2 = I_0$$

$$= A_0 \sqrt{2(1 + \cos \Delta\phi)}$$

$$= A_0 \sqrt{2 \times 2 \cos^2 \frac{\Delta\phi}{2}}$$

$$A = 2A_0 \cos \frac{\Delta\phi}{2}$$

$$I = 4I_0 \cos^2 \frac{\Delta\phi}{2}$$

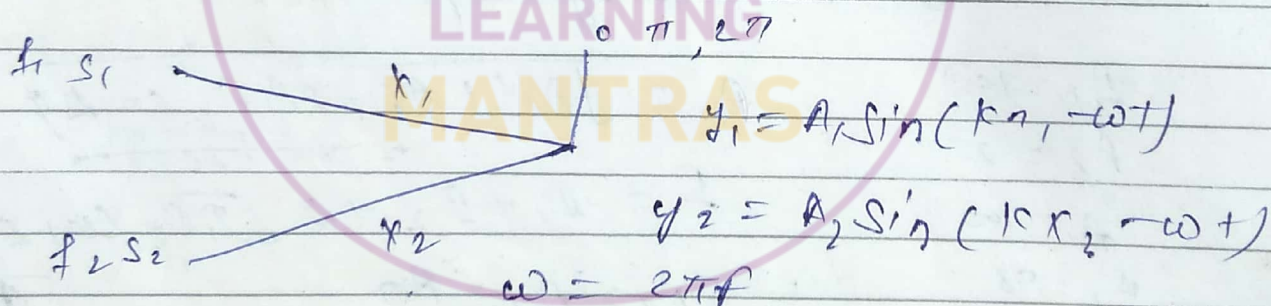
(1) Frequencies of both the wave must be equal

(2) Phase diff b/w the waves must be independent of time

(3) Sources producing wave must be ~~totally~~ coherent

* Coherent source: All the sources producing waves of same frequency called coherent source

Phase difference b/w the coherent source does not change with time or independent of time



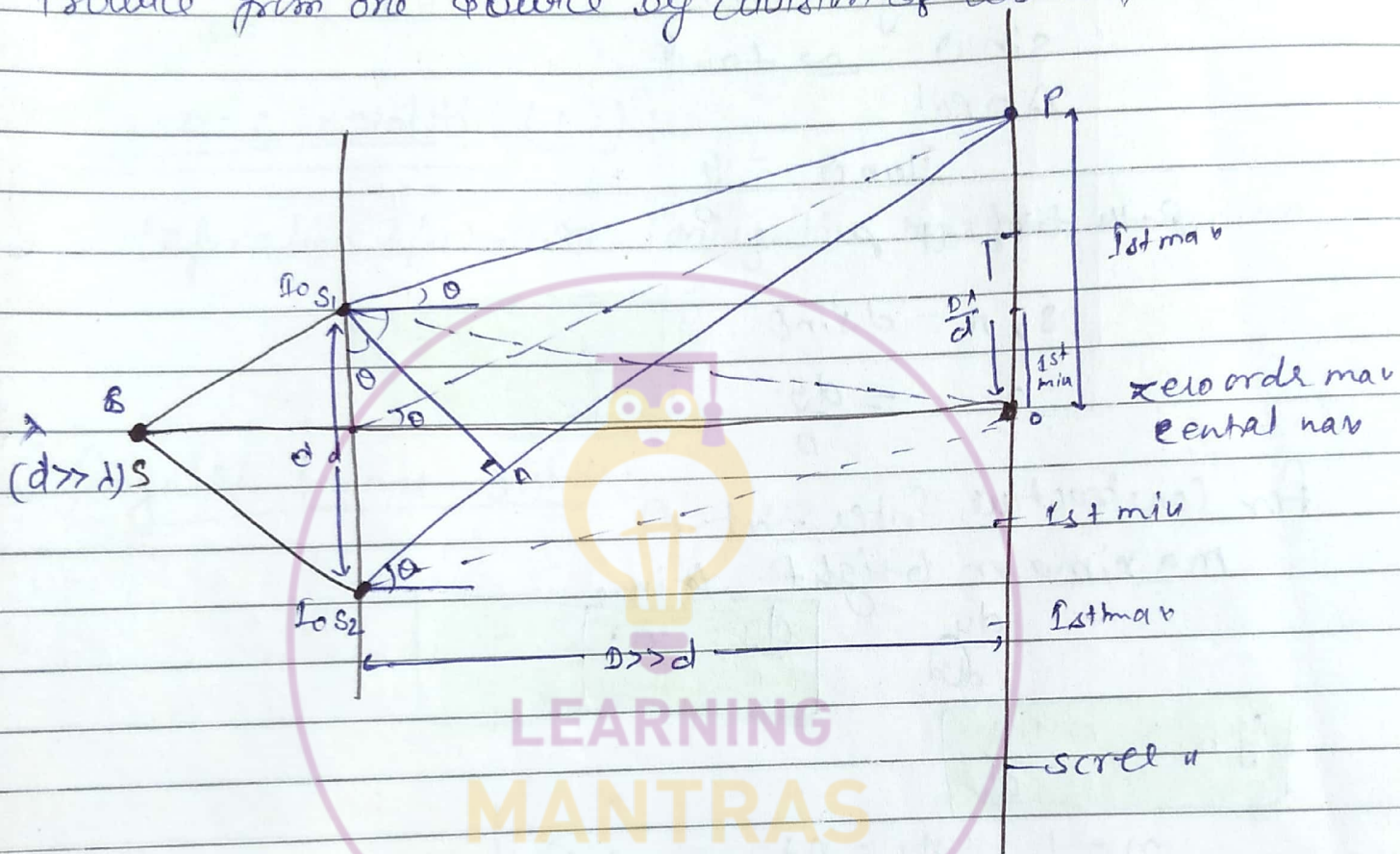
Due to coherent source interference pattern does not change with time

Two independent source can never be coherent

To produce coherent source two or more they have sources are produce from one source

* Young's double slits Exp. | YDSE:

This exp. Prove wave nature of light. Two sources are produce from one source by division of wave front



at 0

$$\text{path diff} = 0$$

$$\text{phase diff} = \frac{2\pi}{\lambda} \text{ path diff}$$

$$= 0$$

$$I = 4 I_0 \cos^2 \frac{\Delta d}{2}$$

at point P path diff = $s_2P - s_1P$
 $= s_2P$

$$\Delta S_1 S_2 A$$

$$\sin \theta = \frac{s_2P}{d}$$

Path diff at P

$$s_2 r = d \sin \theta$$

$$\sin \theta \quad \theta \gg d$$

θ - very small

$$\sin \theta \approx \tan \theta$$

$$\Delta \theta \approx \theta$$

$$\tan \theta = \frac{y}{D}$$

Path diff at P

$$s_2 r = d \sin \theta$$

$$= \frac{dy}{D}$$

For Constructive Inter. at P

maxima or bright fringe

$$\frac{dy}{dn}$$

$$\frac{dy}{D} = n \lambda$$

$$y_n = \frac{n D \lambda}{d}$$

$$n=1 \quad y_1 = \frac{D \lambda}{d} \quad \text{--- 1st order}$$

$$n=2 \quad y_2 = \frac{2 D \lambda}{d} \quad \text{--- 2nd order}$$

$$n=3 \quad y_3 = \frac{3 D \lambda}{d}$$

For Destructive Int. minima or dark fringe

$$\frac{dy}{D} = \frac{(2n-1) \lambda}{2}$$

$$y_n = \frac{(2n-1) D \lambda}{2d}$$

$$y_1 = \frac{D \lambda}{2d} \quad \text{1st order min}$$

$$y_2 = \frac{3D\lambda}{2d}$$

$$y_3 = \frac{5D\lambda}{2d}$$

* Fringe width (w) :

separation b/w two consecutive maxima or minima.

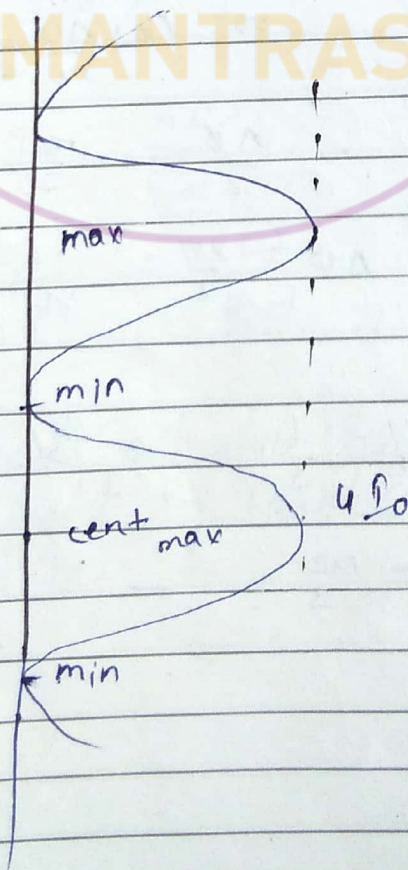
$$w = \frac{D\lambda}{d}$$

* Angular fringe width :

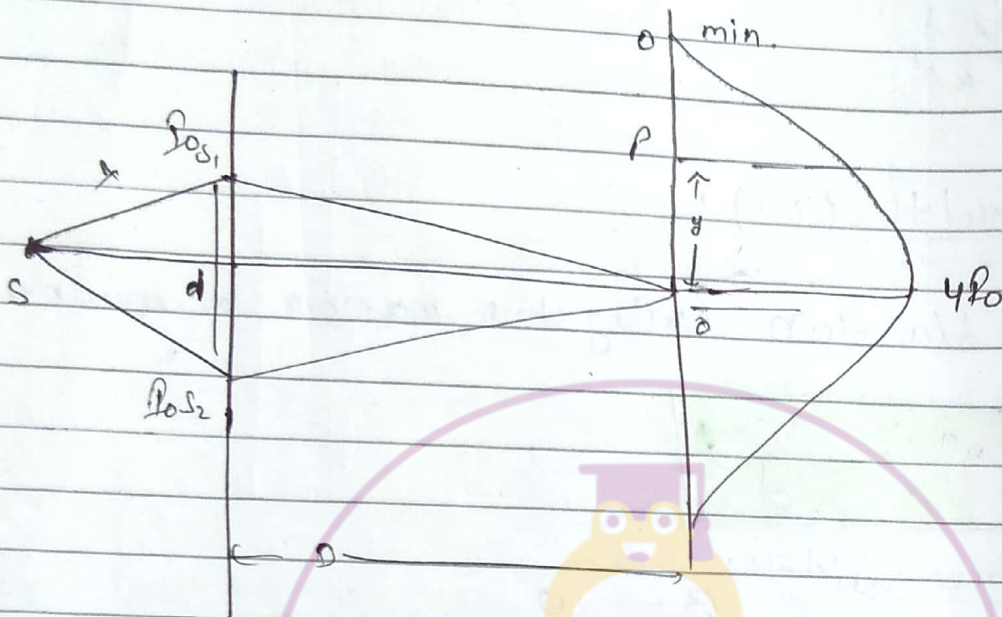
$$\beta = \frac{w}{D}$$

$$\beta = \frac{\lambda}{d}$$

LEARNING
MANTRAS



Ques: Find y so that Intensity at P is 25% of Intensity at central maxima



$$I = 4 I_0 \cos^2 \frac{\Delta \phi}{2}$$

$$I_0 = 4 I_0 \cos^2 \frac{\Delta \phi}{2}$$

$$\cos^2 \frac{\Delta \phi}{2} = \frac{1}{4} \Rightarrow \cos \frac{\Delta \phi}{2} = \pm \frac{1}{2}$$

$$\cos \frac{\Delta \phi}{2} = \frac{1}{2} \Rightarrow \frac{\Delta \phi}{2} = \frac{\pi}{3}$$

$$\Delta \phi = \frac{2\pi}{3}$$

$$\frac{2\pi}{\lambda} \text{ path} = \frac{2\pi}{3}$$

$$\text{path diff} = \frac{\lambda}{3} \Rightarrow \frac{dy}{D} = \frac{\lambda}{3}$$

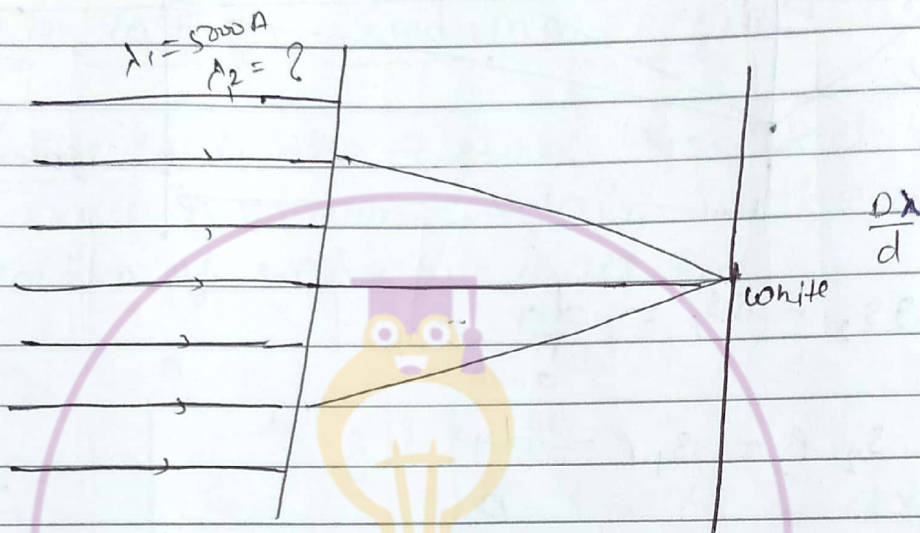
$$y = \frac{D\lambda}{3d} = \frac{4\lambda}{3}$$

Q

Case-1 If monochromatic light is not used

If white light is used in YDSE instead of monochromatic light then central maxima is white and rest of the screen is coloured.

Q x:



Find d_2 if 3rd Maxima of λ_1 coincides to the 5th min of λ_2

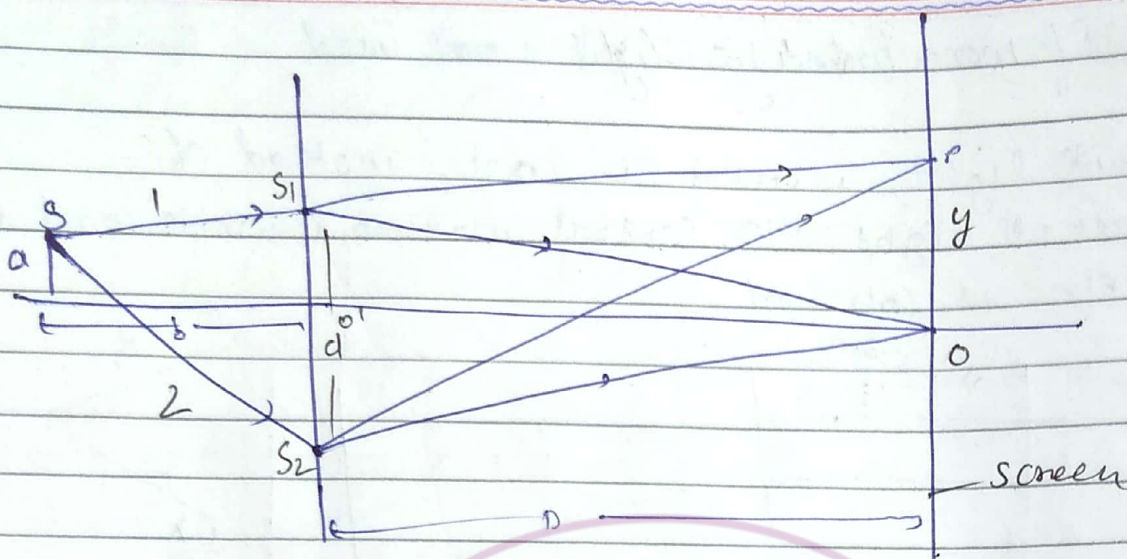
$\lambda_1 = 5000 \text{ \AA}$
 $\lambda_2 = ?$

max $y_n = \frac{nD\lambda}{d}$

min $y_n = (2n-1) \frac{D\lambda}{2d}$

$$\frac{3D\lambda_1}{d} = \frac{5D\lambda_2}{2d}$$

Case: II: If source is not placed symmetrically to the slits i.e. the light ray has path diff. before reaching the slits.



$$SS_2 - SS_1 = \frac{da}{b}$$

$$S_2P - S_1P = \frac{dy}{D}$$

net path diff at point P

$$\text{path diff} = \frac{da}{b} + \frac{dy}{D}$$

$$\text{fringe width} = \frac{D\lambda}{d}$$

for maxima

$$\text{path diff} = m\lambda$$

$$\frac{da}{b} + \frac{dy}{D} = m\lambda \Rightarrow \frac{dy}{D} = m\lambda - \frac{da}{b}$$

$$y = \frac{bD\lambda}{d} - \frac{Dd}{b}$$

central maxima $= m=0$

$$y_0 = -\frac{Dd}{b}$$

$$y_1 = \frac{D\lambda}{d} - \frac{Dd}{b}$$

$$y_2 = \frac{2D\lambda}{d} - \frac{Dd}{b}$$

Case ¹²
 Ex 5 + 2-6
 2, 3, 6-1, 1-14

$$\frac{dy}{\phi} = h d - \frac{da}{b}$$

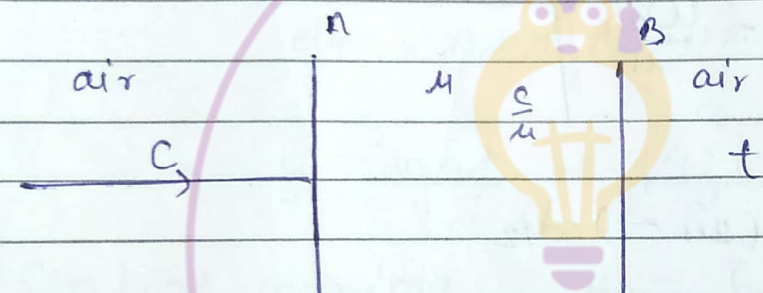
$$y = \frac{h \phi d}{d} - \frac{da}{b}$$

$$w = \frac{\lambda}{\phi}$$

No. of fringes
 shift $h = \frac{y_0}{w}$

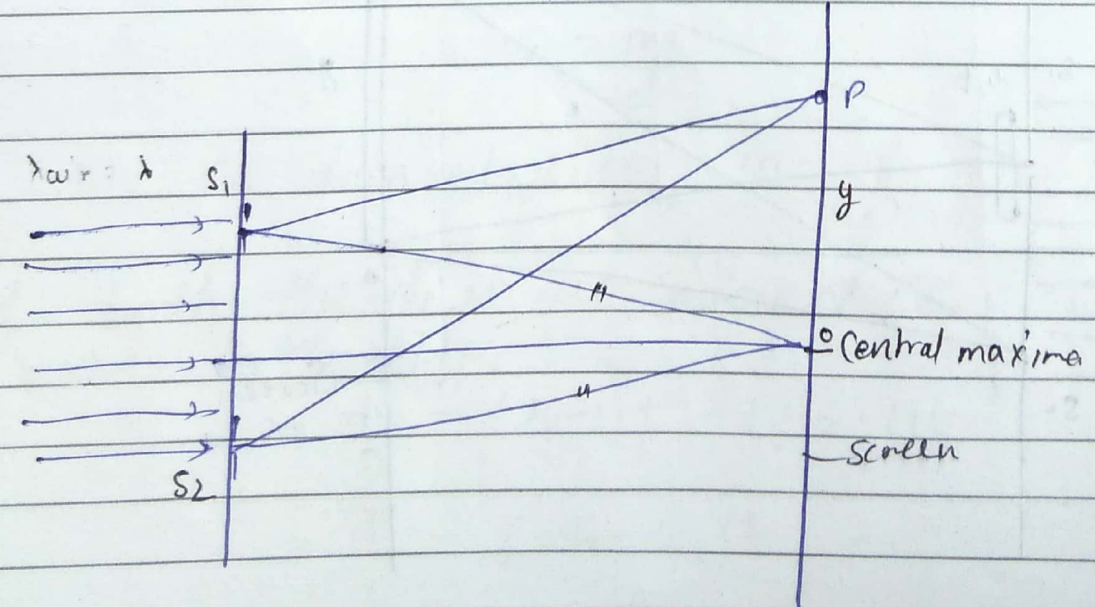
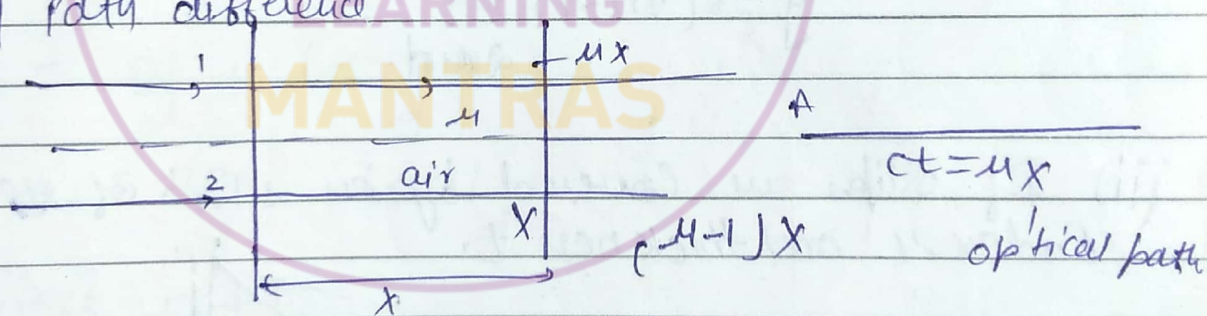
* Case-III \Rightarrow YDSE in a medium of refractive index (μ)?

Optical Path - It is the distance travelled by the light wave in vacuum or air in the same time in a medium of refractive index μ .



$$t = \frac{x}{c/\mu} = \frac{\mu x}{c}$$

Optical path difference



Geometrical Path diff.

$$S_2P - S_1P = \frac{dy}{D}$$

Optical Path diff.

$$\mu(S_2P - S_1P) = \frac{\mu dy}{D}$$

For maxima $\frac{\mu dy}{D} = n\lambda \Rightarrow y_n = \frac{nD\lambda}{\mu d}$

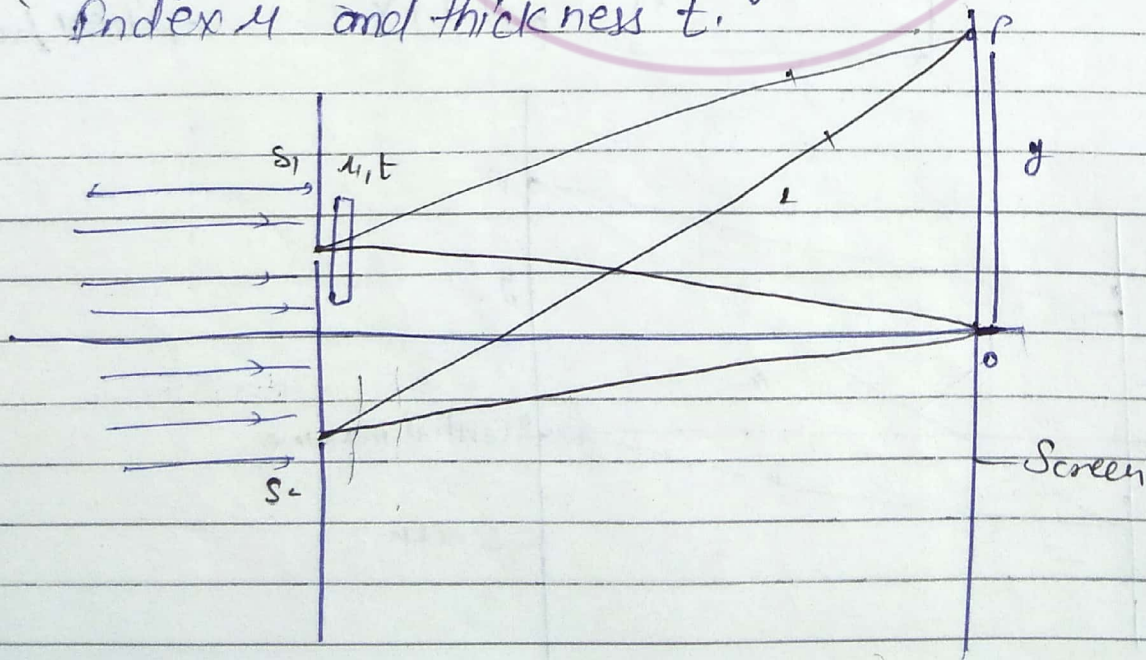
$$y_1 = \frac{D\lambda}{\mu d} \quad y_2 = \frac{2D\lambda}{\mu d}$$

$$\boxed{\omega_{\text{medium}} = \frac{D\lambda}{\mu d} = \frac{\omega_{\text{air}}}{\mu}}$$

minima $\frac{\mu dy}{D} = (2n-1)\lambda/2$

$$y = (2n-1) \frac{D\lambda}{2\mu d}$$

(iii) If slits are covered by a slab of refractive index μ and thickness t .



Geometrical path diff.

$$\Delta p - S, p = \frac{dy}{D}$$

optical path diff.
 $= (\mu - 1)t$

Net path diff.

$$= \frac{dy}{D} - (\mu - 1)t$$

For maxima

$$\frac{dy}{D} - (\mu - 1)t = n\lambda$$

$$\frac{dy}{D} = n\lambda + (\mu - 1)t$$

$$y = \frac{n\lambda D}{d} + \frac{(\mu - 1)t D}{d}$$

Central maxima $y_0 = \frac{(\mu - 1)t D}{d}$

$$y_1 = \frac{D\lambda}{d} + \frac{(\mu - 1)t D}{d}$$

$$y_2 = \frac{2D\lambda}{d} + \frac{(\mu - 1)t D}{d}$$

fringe width

$$w = \frac{D\lambda}{d}$$

No. of fringes shift $n = \frac{y}{w}$

if both the slits are covered
net path diff.

$$= \frac{dy}{D} - (\mu_1 - 1)t + (\mu_2 - 1)t$$

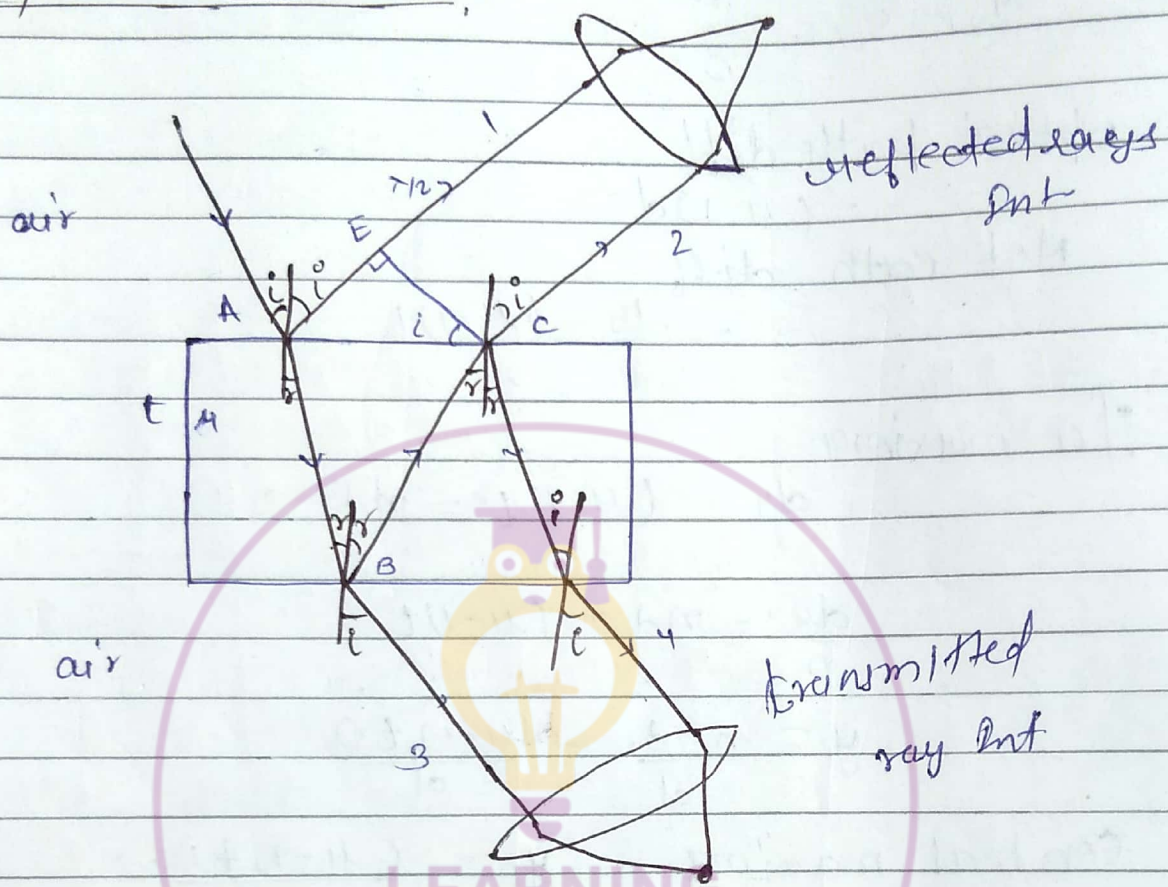
$$= \frac{dy}{D} - (\mu_2 - \mu_1)t$$

$$\mu = 1.0 \quad = 5$$
$$w = 20 \mu$$

$$\frac{y}{w} = 5.5$$

both thickness same

* Thin film Interference



* → when light wave reflect from denser medium ~~for~~ phase of the wave changes by 180°

$$\Delta\phi = \frac{2\pi}{\lambda} \text{ Path diff.}$$

$$\pi = \frac{2\pi}{\lambda} \text{ Path diff.}$$

$$\text{Path diff.} = \frac{\lambda}{2}$$

* when light ray reflect from rarer medium it phase does not change

* During refraction phase of the wave does not change

Optical path travelled by ray 2 in the slab

$$= \mu (AB + BC)$$

$$= 2 \mu AB$$

$$= \frac{2 \mu t}{\cos r}$$

ΔABD

$$\cos r = \frac{t}{AB}$$

$$AB = \frac{t}{\cos r}$$

ΔACE

$$\sin i = \frac{AE}{AC}$$

$$AE = 2AD \sin i$$

$$AE = 2AD \mu \sin r$$

$$= 2 \mu t \tan r \sin r$$

$$= \frac{2 \mu t \sin^2 r}{\cos r}$$

ΔABD $\tan r = \frac{AD}{t}$ $AD = t \tan r$

$$\Delta \phi = \frac{2\pi}{\lambda} \text{ Path diff.}$$

$$\text{Path diff.} = \frac{2 \mu t}{\cos r} (1 - \sin^2 r)$$

$$= 2 \mu t \cos r$$

$$\text{Let Path diff} = 2 \mu t \cos r - \frac{\lambda}{2}$$

* for Constructive inter. or maxima.

$$2 \mu t \cos r - \frac{\lambda}{2}$$

for Constructive int or maxima

$$2 \mu t \cos r - \frac{\lambda}{2} = n \lambda$$

$$2 \mu t \cos r = (n + \frac{1}{2}) \lambda \quad - (1)$$

+ = -

Destructive Int-

$$2\mu + \cos r = \frac{\lambda}{2} = (2n-1) \frac{\lambda}{2}$$

$$2\mu + \cos r = n\lambda \quad (ii)$$

Transmitted ray

path diff. $2\mu + \cos r$

for maxima

$$2\mu + \cos r = n\lambda$$

for minima

$$2\mu + \cos r = (2n-1) \frac{\lambda}{2}$$

film

In thin Int. the wavelengths producing maxima in Reflected Rays produce minima in transmitted rays.

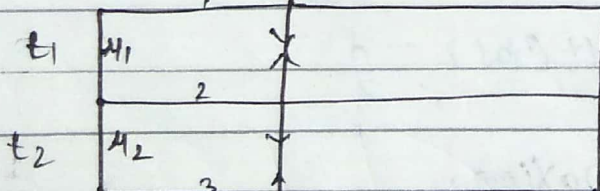
for normal incidence

$$r = 0$$

$$\cos r = 1$$

Que!

air



air

(Q) if $\mu_1 > \mu_2$ find t_1 show that light rays reflected from 1 and 2 makes constructive inter

for constructive int

$$2\mu t \cos r = m\lambda$$

Ans: $2\mu_1 t_1 - \frac{\lambda}{2} = n\lambda$

$2\mu_1 t_1 = (n + \frac{1}{2})\lambda$

$t_1 = \frac{(n + \frac{1}{2})\lambda}{2\mu_1}$

(ii) if $\mu_1 < \mu_2$ find t_1 show that light waves reflected at ~~one~~ 1 and 2 interfere constructively.

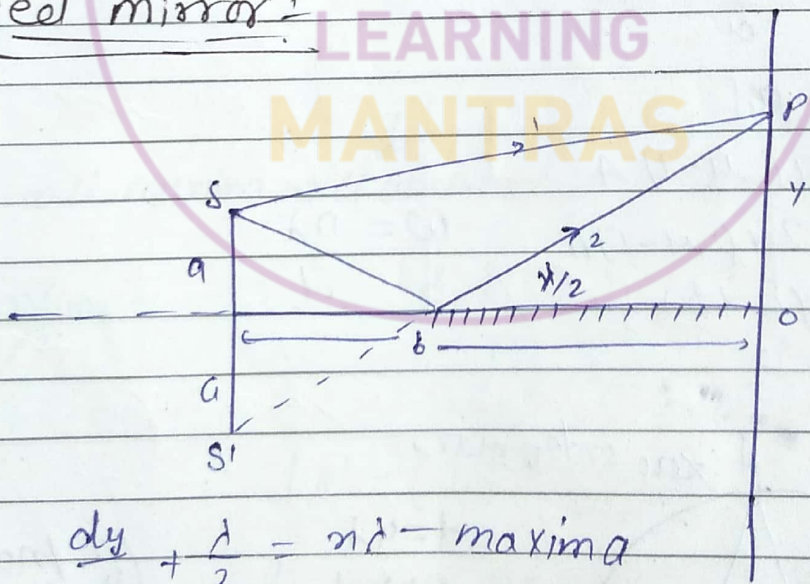
$2\mu_1 t_1 = (n - \frac{1}{2})\lambda$

Path diff = $2\mu_1 t_1$

$2\mu_1 t_1 = n\lambda$

$t_1 = \frac{n\lambda}{2\mu_1}$

* Lloyd mirror:



$d = 2a$

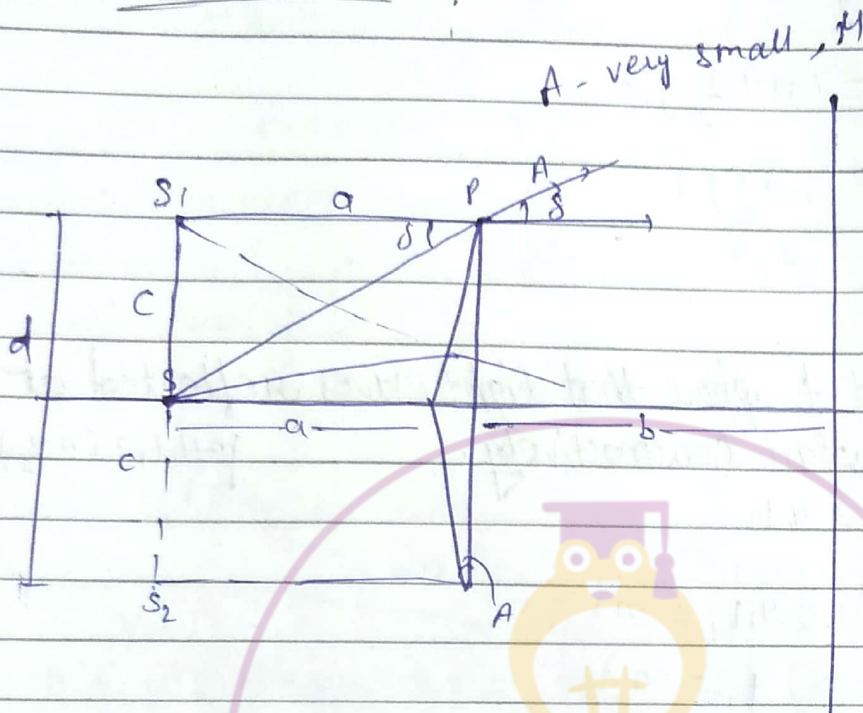
$D = b$

$\frac{dy}{D} + \frac{\lambda}{2} = n\lambda$ — maxima

$= (2n + 1)\frac{\lambda}{2}$ minima

$\omega = \frac{D\lambda}{d}$

* Fresnel's Biprism:



$$\delta = (\mu - 1)A$$

$$\tan \delta = \frac{c}{a}$$

$$c = a\delta$$

$$c = a(\mu - 1)A$$

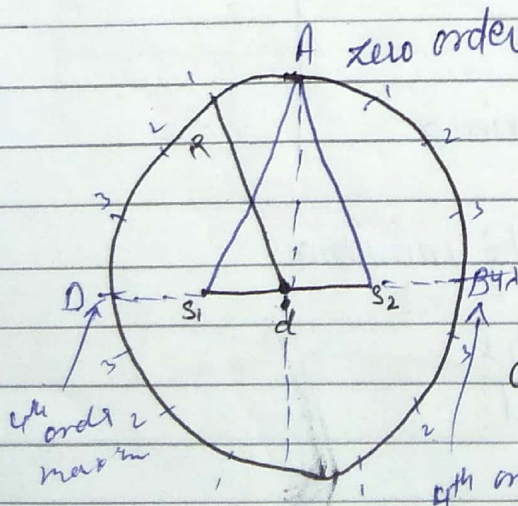
$$d = 2a(\mu - 1)A$$

$$D = (a + b)$$

$$= \lambda$$

$$w = \frac{D\lambda}{d}$$

Ex:



$$d = 4\lambda$$

$$R \gg d$$

16 maxima A

How many maxima are observed on the circumference of

$$d \sin \theta = \frac{n\lambda}{d}$$

$$\theta = 0$$

Op-wave:
 0-1 : 42 to 51

$$\sin \theta = \frac{1}{4} \quad \text{1st order}$$

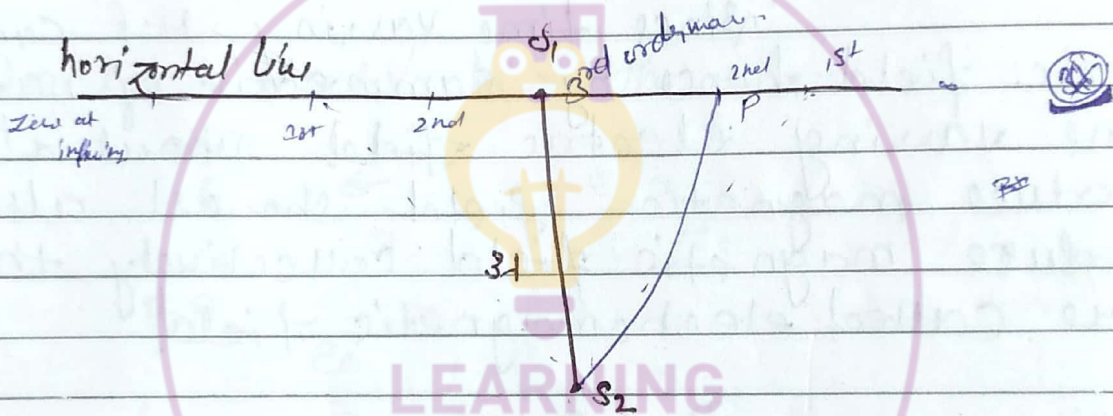
$$\sin \theta = \frac{1}{2} \quad \text{2nd order}$$

$$\sin \theta = \frac{3}{4} \quad \text{3rd order}$$

$$\theta = 90^\circ \quad \text{4th}$$



Ex:



How will maxima will observed on horizontal line.

maxima =

$$S_2P - S_1P$$

$$\sqrt{D^2 + x^2} - x = n\lambda \quad \text{--- maxima}$$

$$\sqrt{x^2 + (D)^2} = (n\lambda + x)$$

$$x^2 + D^2 = (n\lambda + x)^2 + D^2$$

$$x = \frac{D^2 - n^2 \lambda^2}{2n\lambda}$$

for minima
 $n\lambda \rightarrow (n + \frac{1}{2})\lambda$