Learning Mantras

LEARNING MANTRAS

Handwritten Notes

On

Vector

* Vector *

* ~~Physical Quantity~~:
Physical Quantity having both magnitude and direction and which can be menicualted by certain specific rule. vector addition and substraction.

Scalar:
Physical Quantity such as mass, length have magnitude only.

* Null vector:
vector having magnitude zero and direction Random. i.e is initial and terminal point consided.

* Null vector or zero vector has many properties similar to no. zero.

* ~~Unique Vector~~:
unit vector A vector of ~~unique~~ unit magnitud in the direction of vector $\vec{a}$ is denoted by symbol $\hat{a}$

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

$$\bar{a} = a_1 i + a_2 j + a_3 k$$

$$|\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

* Equal vectors!

Two vectors are said to be equal if they have same magnitude, same direction, and represent same physical Quantities.

free vector

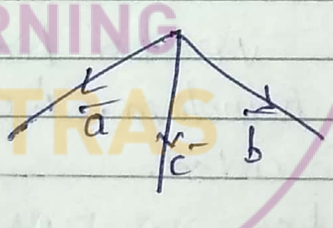
* All such vectors which can be transformed in space from one point to another point. Wiout affecting there magnitude and direction

Locdasied vector

* It initial position is fixed
*

* co-initial vector!

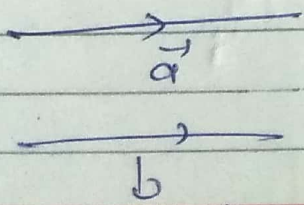
Initial and terminal point consided



** * Co-lineary vectors!- (Parallel vectors)

Two vectors are said to be co-lineay if there directed line segment to their dir. line this regard to their direction.

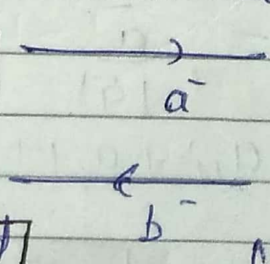
They are also called Parallel vectors.



like vectors

$\bar{a} = \lambda \bar{b}$

Here a and b are linearly (L.D) dependent and relative



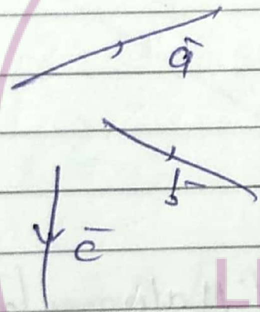
un like vecter.

system

$\boxed{\text{Colinear formula} \rightarrow \vec{a} = \lambda \vec{b}}$

non colinear vector

* **Co-Planar vectors:** Given no. of vectors are coplanar vector if their line segment are all parallel to same.
* Vector line same of parallel plane.



$\bar{n}$ is normal vector to the plane

then $\bar{n} \cdot \bar{a} = 0$

$\bar{n} \cdot \bar{b} = 0$

$\bar{n} \cdot \bar{c} = 0$

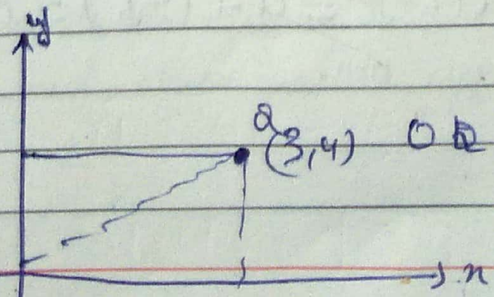
* Two non colinear are always co-planar.

* **Position vector:** To specify the position of an object in 3D space position vector is used. Let $\vec{OB}$ the fixed $\vec{OB}$ then position vector is point P is OP where

$P(x, y, z)$.

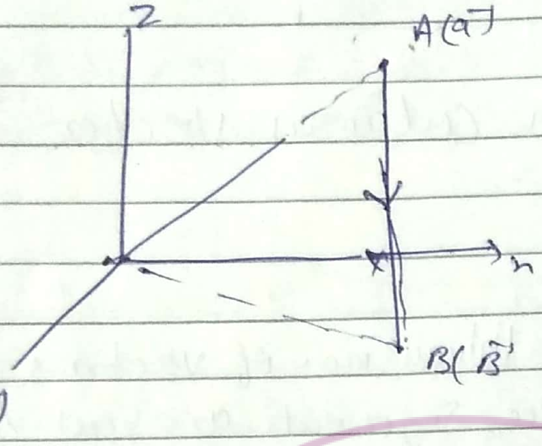
$OP = x\hat{i} + y\hat{j} + z\hat{k}$.

*



$OQ = 3\hat{i} + 4\hat{j}$

* if two point in space. then $\vec{AB} = \vec{B} - \vec{A}$



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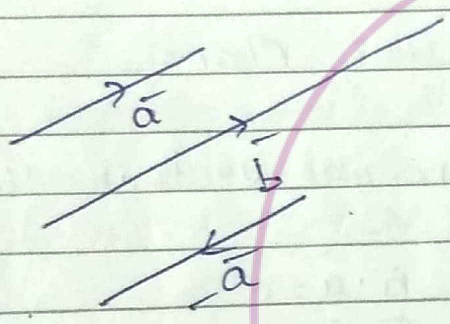
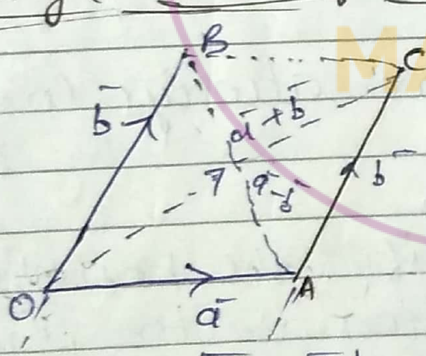
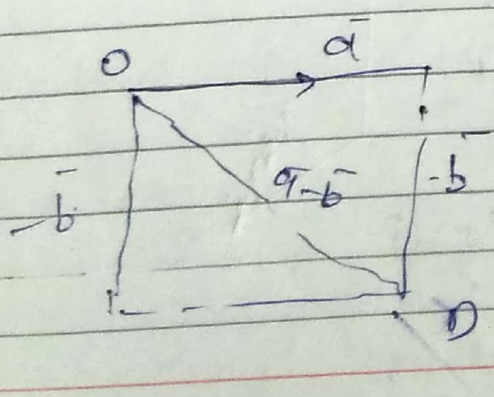


* Triangle law of addition : (||alogram law of addition).



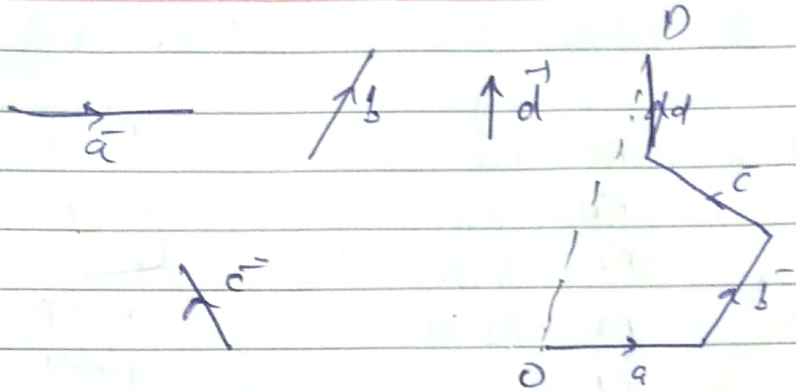
$$\bar{OA} + \bar{OB} = \bar{OA} + \bar{AC}$$
$$= \bar{OC}$$
$$= \bar{a} + \bar{b}$$

* $|\bar{a} + \bar{b}|$ = Dist. b/w Point O and C.



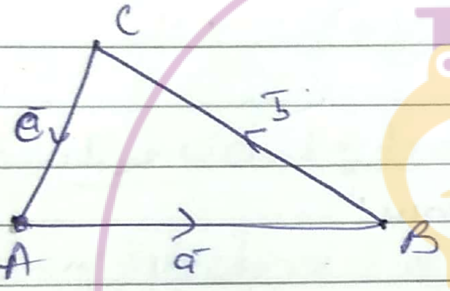
$$\bar{a} - \bar{b} = \bar{a} + (-\bar{b}) = OD$$

*



$\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{OD}$ Polygon law of addition.

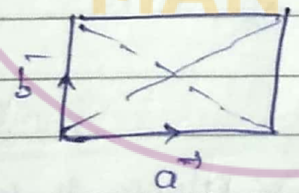
*



$\vec{AB} + \vec{BC} + \vec{CA} = \vec{a} + \vec{b} + \vec{c} = 0.$

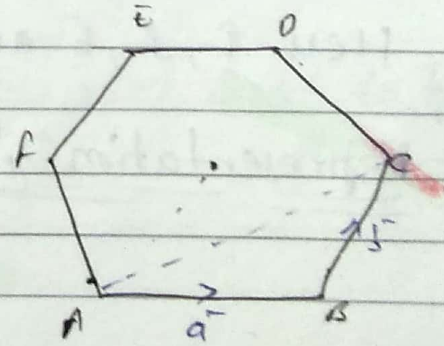
* If $\bar{a}$ and $\bar{b}$ are two vector such that $|\bar{a}+\bar{b}| = |\bar{a}-\bar{b}|$ then

$\Rightarrow$

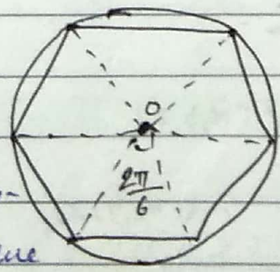


$\bar{a} \cdot \bar{b} = 0$

Que: ABCDEF is Regular Hexagon



→ hexagonal has six equilateral triangle and all equilateral triangle have same angle and same radius $\angle\left(\frac{2\pi}{6}\right)$.



$\bar{AB} = \bar{a}$

$\bar{BC} = \bar{b}$

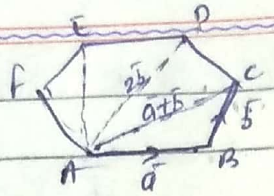
then find $\bar{AC} = \bar{a}+\bar{b}$

$\bar{AD} = 2\bar{b}$

$\bar{AF} = \bar{b} - \bar{A}$

$\bar{AE} = 2\bar{b} - \bar{a}$

$\bar{CF} =$



$\bar{AB} + \bar{BC} = \bar{AC} = \bar{a} + \bar{b}$

$\bar{AC} + \bar{CD} = \bar{AD} = 2\bar{b}$

$\bar{AE} + \bar{ED} = \bar{AE} = \bar{b} - \bar{A}$

~~AE+E~~

$\bar{AF} + \bar{FE} = \bar{AE}$

$\bar{AE} + \bar{ED} = \bar{AD}$

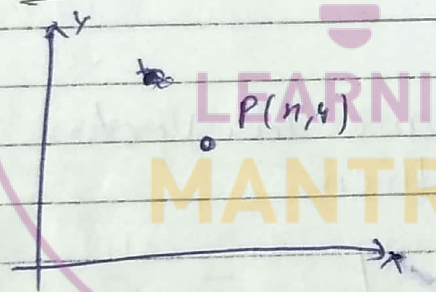
$\bar{AC} + \bar{CD} = \bar{AD}$

$\bar{a}+\bar{b} + \bar{CD} = 2\bar{b}$

$\bar{CD} = \bar{b} - \bar{a}$

$AE = 2\bar{b} - \bar{a}$

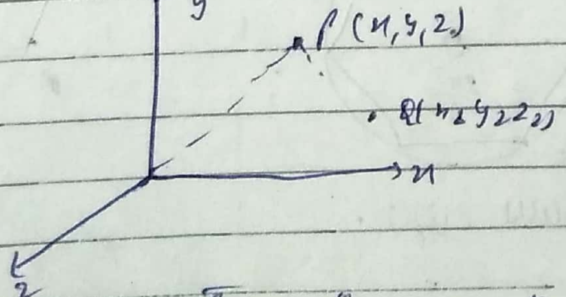
Representation of vector in plane:



$\bar{OP} = x\hat{i} + y\hat{j}$

Here $\hat{i}, \hat{j}, \hat{k}$ are unit vector along $x, y,$ and z axis.

Representation in 3 Dim.:



$\bar{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\bar{OP}| = \sqrt{x^2 + y^2 + z^2}$ Dist. b/w Point O and P

If two point representate in space. $P(x, y, z), Q(x_2, y_2, z_2)$

$$\overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}.$$

* **Linear Combination of Vectors:**

If $\bar{a}, \bar{b}, \bar{c}$ ---- are n vectors and x, y, z --- are n scalars then $x\bar{a} + y\bar{b} + z\bar{c} + \text{------}$ is called linear combination of given vectors.

$$\overline{OP} = x\hat{i} + y\hat{j}$$

$$x\hat{i} + y\hat{j} + (-\overline{OP}) = 0$$

* **Linear dependen Vectors:**

Set of vectors $\bar{a}, \bar{b}, \bar{c}$ ----- are said to be (L.D) if their exist n scalars x, y, z ---- (not all of them equal to zero)

such that $x\bar{a} + y\bar{b} + z\bar{c} \text{--} = 0$ $\quad x = y = z \neq 0$

* **Linear independent vector:**

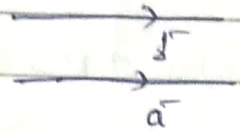
Set of vectors $\bar{a}, \bar{b}, \bar{c}$ ----- are said to be L.I. if $x\bar{a} + y\bar{b} + z\bar{c} \text{---} = 0$

$$\Rightarrow x = y = z = \text{------} = 0.$$

Note: Any two Co-linear vector (∥el vectors) are L.D

(2) Any two non-colinear vectors are L.I

L.D



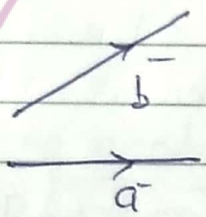
Collinear (||el)

$\bar{a} = \lambda \bar{b}$

$\Rightarrow \quad \bar{a} = \left(-\frac{n}{m}\right)\bar{b}$

$\Rightarrow \quad m\bar{a} + n\bar{b} = 0$

Here m & n are non-zero constant.



L.D

$\bar{a} = \lambda \bar{b}$

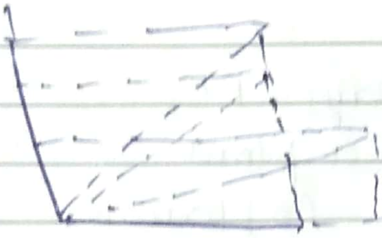
$\bar{a} = \left(-\frac{n}{m}\right)\bar{b}, \quad m\bar{a} + n\bar{b} = 0$

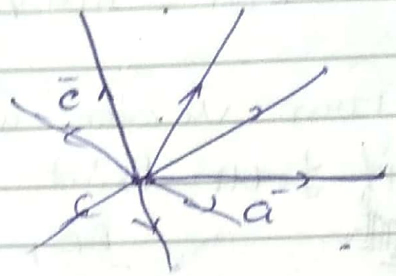
$m = 0$

$n = 0$

(3)

(4) Three coplanar vectors are ~~necessary~~ LD
(5) Three non-coplanar vector are LI





$x_1\bar{a}+y_1\bar{c}$

$x\bar{a}+1\bar{c}$

$\bar{b}=x\bar{a}+y\bar{c}$

$=x\bar{a}+(-1)\bar{b}+z\bar{c}=0$

$\bar{b}=x\bar{a}+y\cdot\bar{b}$

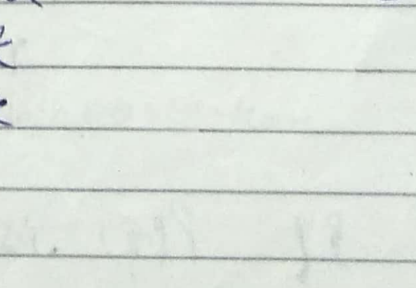
collinear $=\bar{a}=\lambda\bar{b}$

coplanar

Imp: $\boxed{\lambda\vec{a}+\mu\vec{b}}$ where $\vec{a}$ and $\bar{b}$ are coplanar.

* If three points are collinear

$\boxed{\overline{AB}=\lambda\overline{AC}}$



* Three vectors $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$c=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

are coplanar then

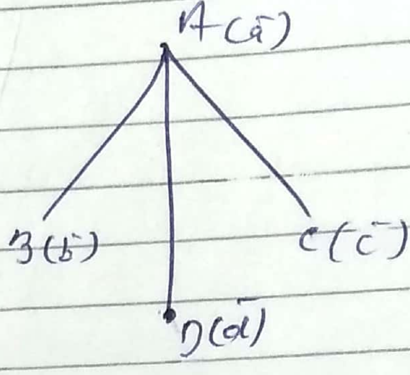
(i) $\bar{c}=\lambda\bar{a}+\mu\vec{b}$ OR.

(ii)
$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$$

* $\hat{i}, \hat{j}, \hat{k}$ are LI

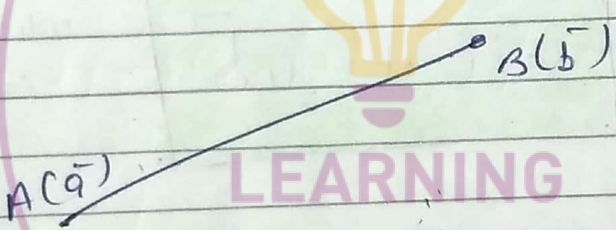
* $\overline{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

* four vector are always LD.
* if four Point lie in a plane
  $\overline{AB}, \overline{AC}, \overline{AD}$ coplanar
  $\overline{AB} = \lambda \overline{AC}$

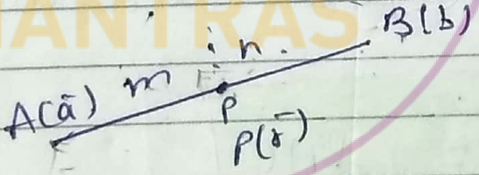


formula:

(1) if $A(\bar{a})$ the $\overline{AB}$ = final − initial
and dist. b/w ~~A~~B $|\bar{a}-\bar{b}|$ and $(\bar{b}-\bar{a})$



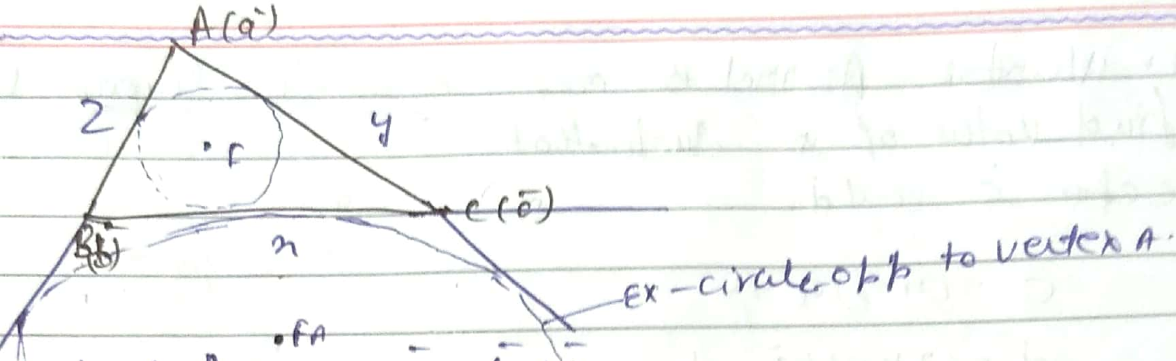
(2) Section formula:



$$\frac{AP}{PB} = \frac{m}{n}$$

$$\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}.$$

If $P(\bar{r})$ is mid Point then $\bar{r} = \dfrac{\bar{a}+\bar{b}}{2}$



(i) Centriod of $\Delta = \dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

(ii) Incentre of $\Delta = \dfrac{x\bar{a}+y\bar{b}+z\bar{c}}{x+y+z}$

$$E_A = \frac{-x\bar{a}+y\bar{b}+z\bar{c}}{-x+y+z}$$

Ex circle opp to vertex B $\quad y \longrightarrow -y$

(iii) ortho centre of $\Delta = o(\bar{c}) = \dfrac{\bar{a}\tan A+\bar{b}\tan B+\bar{c}\tan C}{\tan A+\tan B+\tan C}$

Curcum centre $= c(\bar{c}) = \dfrac{\bar{a}\sin 2A+\bar{b}\sin 2B+\bar{c}\sin 2C}{\sin 2A+\sin 2B+\sin 2C}$

Que: if Vectors $a\hat{i}+\hat{j}+k$

$\hat{i}+b\hat{j}+k$

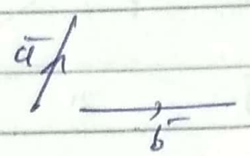
$\hat{i}+\hat{j}+ck$ are Coplanar then find Condition

Ans. $a\hat{i}+\hat{j}-k$
$\hat{i}+b\hat{j}+k$
$\hat{i}-\hat{j}+ck$ $= \begin{vmatrix} a & 1 & -1 \\ 1 & b & 1 \\ 1 & -1 & c \end{vmatrix} = 0$

QP formula use.

O-1 = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
S-1 = 1, 2, 3

Q. Vector $\bar{A}$ and $\bar{B}$ are non - Collinear Vectors find value of x such that vector $\bar{c}$ and $\bar{d}$



$\bar{c} = (x-2)\bar{a} + \bar{b}$

$\bar{d} = (2x+1)\bar{a} - \bar{b}$

are Collinear

$= \bar{C} = \lambda \bar{d}$

$(x-2)\bar{a} + \bar{b} - \lambda(2x+1)\bar{a} + \lambda b = 0$

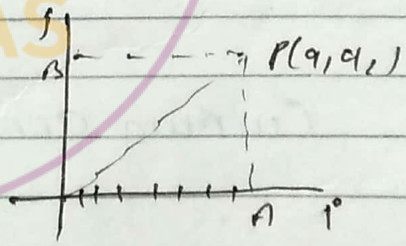
$\bar{a}\left[(x-2) - \lambda(2x+1)\right] + \bar{b}\left[1+\lambda\right] = 0$

$(x-2) - \lambda(2x+1) = 0$ & $1+\lambda = 0$

$\lambda = -1$

O-1 (8)

* $\bar{a} = a_1 i + a_2 j$



$OA = a_1$

Proj of $\bar{OP}$ upon unit vector $i = a_1 i$

$a = 2pi + 1j$

$a' = (P+1)i + 1j$

$|\bar{a}'| = |\bar{a}|$

$4p^2 + 1 = (P+1)^2 + 1$

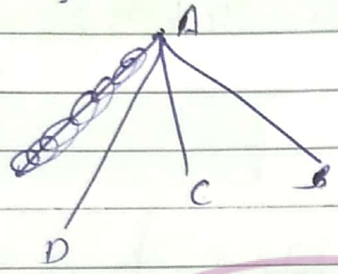
L1RU

Coplanar

Q. P.V of four point A, B, C, D are A(3, −2, −1) B(2, 3, −4) C(−1, 1, 2) D(4, 5, λ) then find λ.

Ans: $AB = \frac{(2-3)(3+2)(-4-1)}{7}$ $BC = (1+2)(1+2)$

$$\begin{vmatrix} 5 & -2 & + \\ 2 & 3 & -4 \\ \frac{2+\lambda}{2} = \lambda = 0 \end{vmatrix}$$



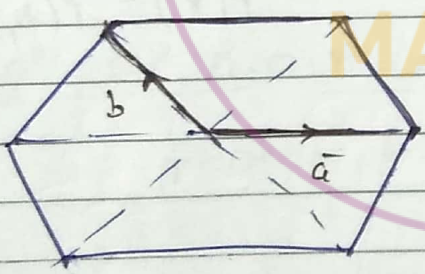
$\bar{AB} = -\hat{i} + 5\hat{j} - 3k$

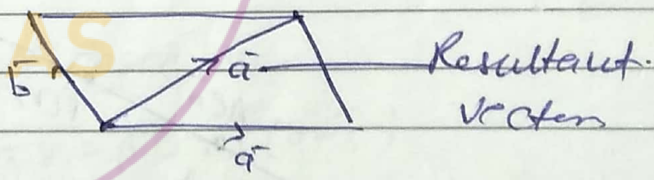
$AC = -4i + 3j + 3k$

$AD = i + 7j + (\lambda+1)k$

$$\begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda+1 \end{vmatrix} = 0$$

*





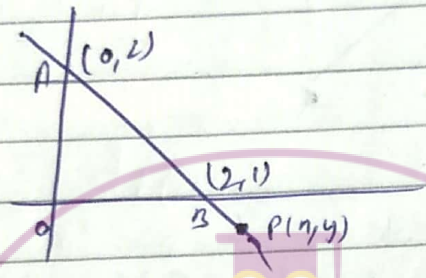
* If sum of two unit vector is another unit vector then angle b/w two given vectors will be 120°

i.e $\bar{a} \wedge \bar{b} = 120°$

* Vector eqⁿ of line : (Perametric form):

(1) Line Passes throw **Two Points**

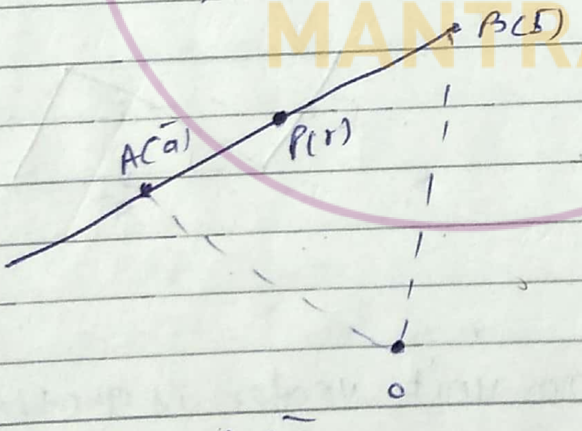
$A(\bar{a})$ & $B(\bar{b})$ $\quad$ $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$



$y - 2 = \frac{1-2}{2-0}(x-0)$

or $\quad \Delta_{PAB} = 0$

$$\begin{vmatrix} x & y & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 0$$



$P(\bar{r}) = P(x, y, z)$

$\bar{AP} = \lambda \bar{AB}$

$\bar{b} - \bar{a} = \lambda(\bar{b} - \bar{a})$

Position vector of on line $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$ ← shows dir

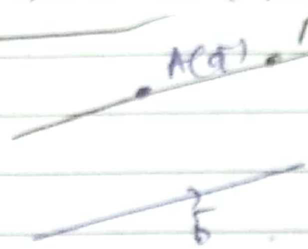
Perameter.

Passes throw a.

for (dif. λ diff. Point on the line)

Dis

(2) Line passes throw $A(\vec{a})$ and parallel ~~throw~~ to $\vec{b}$



$$\boxed{\vec{r} = \vec{a} + \lambda \vec{b}}$$

* $\vec{r} = i - 2j + \lambda(i + j - 3k)$

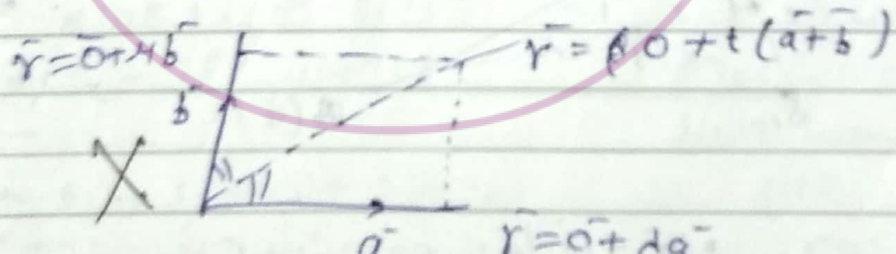
Passes throw $(1, -2, 0)$ & in the dir^n of $\hat{i} + \hat{j} - 3k$.

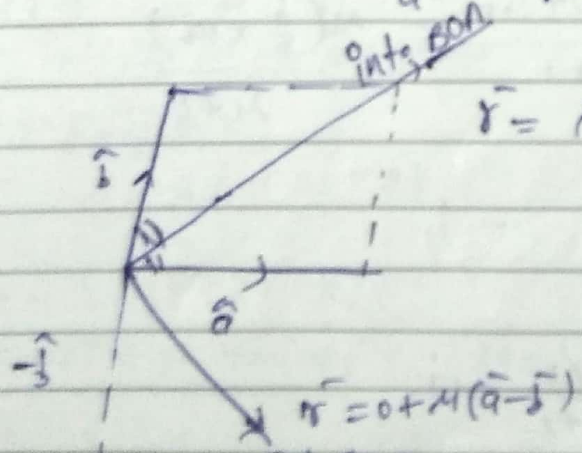
Any point on the line $(1+\lambda, -2+\lambda, -3\lambda)$

* $\vec{r} = \lambda(i + j - 3k)$

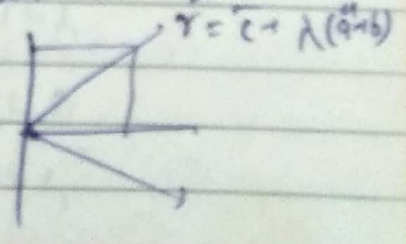
Line Passes throw origin.

* eq^n of B.O.A (Bisector of Angle) :-





if ref would be $\vec{c}$



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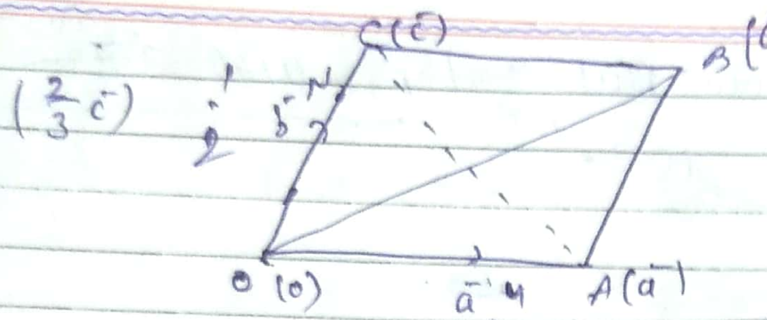
* non Parallel non-Intersecting line are Called skew lines (not come in a plane)



OABC is ||gram.

($\frac{3}{4}\bar{a}$)

eq^n of line $\bar{OA} = \bar{r} = \lambda \bar{a}$

$\bar{OB}$, $\bar{r} = \lambda(\bar{a}+\bar{c})$

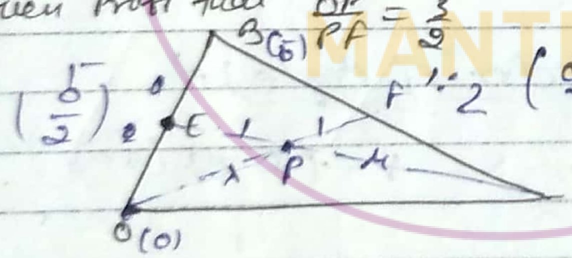
$\bar{AC} \Rightarrow \bar{r} = \bar{a} + \mu(\bar{a}-\bar{c})$

$\bar{r} = \frac{2}{3}\bar{a} + t\left(\frac{3}{4}\bar{a} - \frac{2}{3}\bar{c}\right)$ $\quad \frac{1.0 + 3\bar{a}}{1+3}$

M: $\bar{r} = 3\bar{a}$

N: $\bar{r} = 2\lambda + (1/3 - 2)$

Que: In a Δ AOB E is mid point of OB and F divide BA in ratio 1:2. If line joining OF and OA intercept at P then Prove that $\frac{OP}{PF} = \frac{3}{2}$



$\left(\frac{\bar{a}+2\bar{b}}{3}\right)$ OP

$= \frac{\lambda\left(\frac{\bar{a}+2\bar{b}}{3}\right) + \frac{0}{3}}{\lambda+1}$

$= \frac{\mu\left(\frac{\bar{b}}{2}\right) + \bar{a}}{\mu+1}$

Ans P.V of P is $\frac{\lambda\left(\frac{\bar{a}+2\bar{b}}{3}\right) + 1.\bar{0}}{\lambda+1} = \frac{\mu\left(\frac{\bar{b}}{2}\right) + 1\bar{a}}{\mu+1}$

$\frac{\frac{\lambda}{3}}{\lambda+1} = \frac{1}{\mu+1}$ — i)

$\frac{2\lambda/3}{\lambda+1} = \frac{\mu/2}{\mu+1}$ — ii)

non-parallel
non-intersecting ar

(10-11): 11, 12, 13, 15, 20,
→ (5.1) 4, 10, 11

Intersecting species are coplanar line

$\bar{a}[\ -\ ] + \bar{b}[\quad] = 0 \quad \therefore \bar{a}$ and $\bar{b}$ are L.I.

M.2: eq^n of line OF = $\bar{r} = 0 + t\left(\frac{\bar{a}+2\bar{b}}{3}\right)$

eq^n of line $AF = \bar{a} + s\left(\bar{a} - \frac{\bar{b}}{2}\right)$

If intercept then $\bar{r} = \bar{r}$

$$t\left(\frac{\bar{a}+2\bar{b}}{3}\right) = \bar{a} + s\left(\bar{a} - \frac{\bar{b}}{2}\right) \Rightarrow$$

$$\frac{t}{3} = 1 + s \quad -(i)$$

$$\frac{2}{3}t = -\frac{s}{2} \quad -(ii)$$

$$\frac{\bar{OP}}{PF} = \frac{\ }{\ } = \frac{3}{2}$$

Que: Find position vector point of intersection of line

(1) $\bar{r} = \hat{i} - j - 10k + \lambda(2i - 3j + 8k)$
(ii) $\bar{r} = 4i - 3j - k + \mu(i - 4j + 7k)$

Non-Intersecting
Non-~~planar~~ parall
are
Non coplanar

(2) $\bar{r} = -3i + 6j + \lambda(-4i + 3j + 2k)$
$\bar{r} = -2i + 7k + \mu(-4i + j + k)$

Ans: Non-Intersecting species.

(3) $\bar{r} = t(3i - j + k)$
$r = 2t + s(-6i + 2j - 2k)$

parall - and
non-Intersecting
are
coplanar

(4) $\bar{r} = 2k + \lambda(3i + 2j + k)$
$\bar{r} = 3i + 2j + 3k + \mu(8i + 4j + 2k)$

Intersecting lines
are co-planar

* Shortest dist. b/w line are ~~prep~~ perpendicular. Skew lines.

Ans (17) $r = r'$

$= 1 + 2\lambda = 4 + \mu \longrightarrow \lambda = 2$

$-1 - 3\lambda = -3 - 4\mu \longrightarrow \mu = 1$

$-10 + 8\lambda = -1 + 7\mu$

Solve any two, get the value of λ and μ. If it satisfy 3rd eqn. then lines will be intersecting otherwise not.

$-10 + 8 \times 2 = -1 + 7 \times 1$

$-10 + 16 = 6 = 6 = 6$

(1) and (2) solve and satisfy 3rd eqn.

Intersecting.

* Hence line intersecting species

⇒ Now find ~~two~~ point of intersecting, ~~to~~

Put $\mu = 1$ ~~5 -7~~ $(5, -7, 6)$ Ans.

Ans: (2)

$= 2\hat{j} + s(3\hat{i} - \hat{j} + \hat{k})$

$(0, 0, 0)$, $(2, 0, 0)$

Non intersecting and parallel lines.

(9) $\bar{r} = 2k + \lambda(3i + 2j + k)$ A(0, 0, 2) — (3, 2, 1)

$\bar{r} = 2\hat{i} + 2j + 3k + 2\mu(3i + 2j + k)$. B(3, 2, 3) — (3, 2, 1)

Non Intersecting and Parallel.

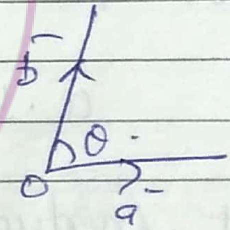
Both lines are coinsided lines

$$\bar{AB} = \underline{3i + 2j + k}.$$

* Dot Product :

If $\bar{a}$ and $\bar{b}$ are two vectors then their dot product as $\bar{a}.\bar{b}$ where

$$\boxed{\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta}$$



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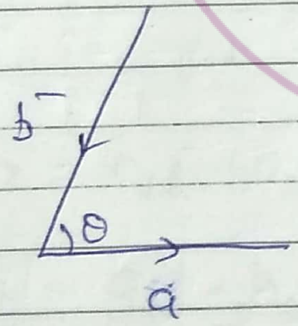
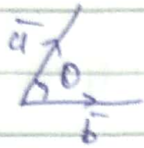


$$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos(\pi - \theta)$$

* Properties/ $\bar{a}$ and $\bar{b}$ are non zero Vectors

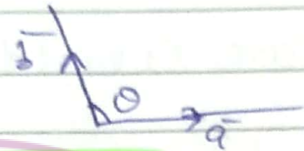
(1)

(i) $\bar{a}\cdot\bar{b} > 0 \Rightarrow$



then θ is acute angle

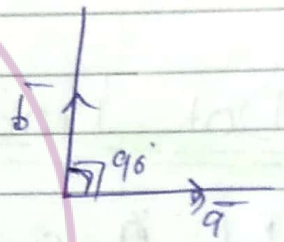
(ii) $\bar{a}\cdot\bar{b} < 0 \Rightarrow$



then $\angle$ — — optus angle

(iii)☆ $\bar{a}\cdot\bar{b} = 0$

then — — — then $90°$



(2) Dot Product is commucative.

$$\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a}$$

(3) Dot Product is distributed

$$\bar{a}\cdot(\bar{b}+\bar{c}) = \bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c}$$

(4)☆ $\bar{a}\cdot\bar{a} = |\bar{a}|^2 \Rightarrow (\bar{a})^2$

$$(\bar{a}+\bar{b})\cdot(\bar{a}+\bar{b}) = |\bar{a}+\bar{b}|^2 = \bar{a}\cdot\bar{a} + \bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{a} + \bar{b}\cdot\bar{b}$$

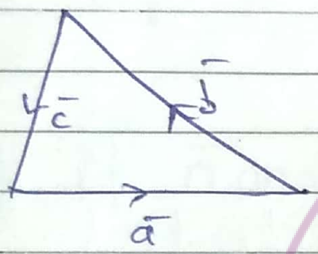
$$= |\bar{a}|^2 + 2\bar{a}\cdot\bar{b} + |\bar{b}|^2.$$

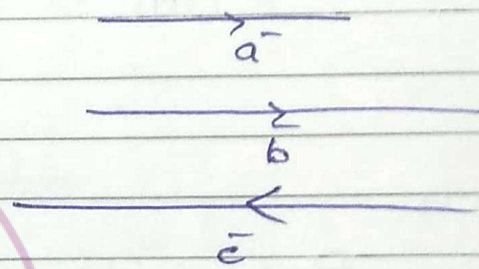
* $(\bar{a}+\bar{b})\cdot(\bar{a}-\bar{b}) = |\bar{a}|^2 - |\bar{b}|^2$

* $(\bar{a}+\bar{b})^2 = (\bar{a}-\bar{b})^2 + 4\bar{a}\cdot\bar{b}$

A $(\bar{a}+\bar{b}+\bar{c})^2 = |\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a})$

$\bar{a}+\bar{b}+\bar{c} = 0$ in two situation.





* $\bar{OP} = x\hat{i} + y\hat{j} + z\hat{k}$.

$|\bar{OP}| = \sqrt{x^2+y^2+z^2}$

(5) $\hat{i}\cdot\hat{i} = 1 = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k}$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\hat{j} = 0$

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3k$.

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3k$

$\bar{a}\cdot\bar{b} = a_1b_1 + a_2b_2 + a_3$

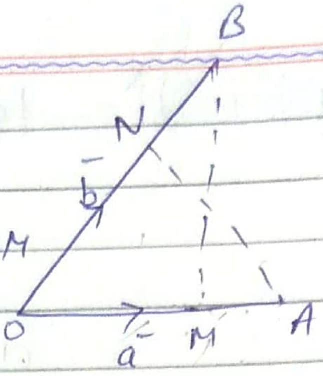
$|\bar{a}||\bar{b}|\cos\theta =$

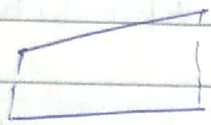
$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$$

Scalar Quantity

* Projection of Vector:

P. of vector $\frac{\bar{b}}{\bar{a}}$ = $\bar{b}$ upon $\bar{a}$ = OM





$$OM = |\bar{b}| \cos\theta$$

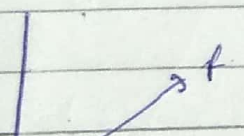
$$= \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|}$$

Projection of $\bar{a}$ upon $\bar{b}$ = ON

$$= |\bar{a}| \cos\theta$$

$$= \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$$

* $\frac{\text{Proj. of } \bar{a} \text{ upon } \bar{b}}{\text{Proj. of } \bar{b} \text{ upon } \bar{a}} = \frac{\frac{\bar{a}\cdot\bar{b}}{|\bar{b}|}}{\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|}} = \frac{|\bar{a}|}{|\bar{b}|}$



f_y = Proj. of f along y axis.

$\bar{f} = f_x\hat{i} + f_y\hat{j}$ → Component of f along y axis.

= Component of f along x axis =

f_x ⇒ Proj. of f along x-axis.

* Component of $\bar{b}$ upon $\bar{a}$ = $\overrightarrow{OM}$

$$= \left(\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|}\right)\hat{a}$$

$\bar{OM} + \bar{MB} = \bar{b}$

$$\boxed{\bar{MB} = \bar{b} - \left(\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|}\right)\hat{a}}$$

Comp. of vector $\bar{b}$ $\perp$cular to the Dir^n of $\bar{a}$

* If $\bar{a}$ and $\bar{b}$ are any two vectors

$(\bar{a}\cdot\bar{b}) \rightarrow$ Its max. value $\Rightarrow |\bar{a}||\bar{b}|$ $\theta = 0°$

$(\bar{a}\cdot\bar{b}) \rightarrow -|\bar{a}||\bar{b}|$ $(\theta = \pi)$

* Any Vector $\bar{b}$ can be expresed as

$$\bar{a} = (\bar{a}\cdot\hat{i})\hat{i} + (\bar{a}\hat{j})\hat{j} + (\bar{a}\cdot\hat{k})k.$$

$$= \bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

* $\bar{r} = \bar{a} + \lambda\bar{b}$

$\bar{r} = \bar{b} + \mu\bar{q}$ $\qquad \cos\theta = \dfrac{\bar{p}\cdot\bar{q}}{|\bar{p}||\bar{q}|}$

*** If ~~$\bar{a}$ and~~ any vector Coplanar with $\bar{a}$ and $\bar{b}$ is $x\bar{a} + y\bar{b}$

H.W. O-1 ⇒ 16, 19, 21, 23, 25, 26, 27,
S-1 : 8 V. Impt ~~ter~~ 9, 12, 13,

J.M ⇒ 1, 6, 7, 9,
J.A : 3 (a)(b)(c),

* if $\bar{a}$, $\bar{b}$ and $\bar{c}$ are any three Unit vector.

$$|\bar{a}+\bar{b}+\bar{c}| \geq 0$$

$$|\bar{a}+\bar{b}+\bar{c}|^2 \geq 0$$

$$|\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}) \geq 0$$

$$\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a} \geq -\frac{3}{2}.$$

Que:

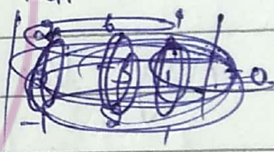
$\bar{a} = 2i+2j-k$

$\bar{b} = 6i-3j+2k$

find angle $(\bar{a}\wedge\bar{b})$ b/w $\bar{a}$ and $\bar{b}$.

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$$

$$= \frac{12-6-2}{3\cdot\sqrt{36+9+4}}.$$



Q. $\bar{a} = 2i - j - 2k$

$\bar{b} = i - 3j + 2k$.

(i) find projection of Vector $\bar{b}$ upon $\bar{a}$

$$\frac{a\cdot b}{|a|} = \frac{2+3-4}{3} = \frac{1}{3}$$

$$\frac{b\cdot a}{|a|}$$

(ii) find component of $\bar{b}$ $= \frac{a\cdot b}{|a|}\times\hat{a}$.

(i) along vector $\bar{a}$

(ii) ⊥ to $\bar{a}$ =

150 | 120

150

(i) OM $= \frac{2+3-4}{3} = \frac{1}{3}\cdot\hat{a}$ $\frac{a\cdot b}{|a|}\cdot\hat{a}$

$= \frac{1}{3} 2i - j - 2k$

(ii) $=$

$0 - \frac{1}{3}\hat{a}$

$MB = \bar{b} - \bar{OM}$

Q. if $|\bar{a}| = 2$ $|\bar{b}| = 6$ $|\bar{a}-\bar{b}| = 3$

find $|\bar{a}+\bar{b}|^2$

$|\bar{a}+\bar{b}| = |\bar{a}| + |\bar{b}|$
$= 6+2 = 8.$

$= \cancel{(\bar{a}+\bar{b})(\bar{a}-\bar{b}) = (\bar{a}-\bar{b})^2}$

$= \cancel{(\bar{a}+\bar{b})} = \cancel{\frac{9}{3}}\ 3$

$$\boxed{|\bar{a}+\bar{b}|^2 + |\bar{a}-\bar{b}|^2 = 2|\bar{a}|^2 + 2|\bar{b}|^2}$$

$+3^2 = 2\cdot 2^2 + 2\cdot 6^2$

$= 8 + 72$

$= 80$

$|\bar{a}+\bar{b}|^2 = 71$

$|\bar{a}+\bar{b}| = \sqrt{71}$

M-2 $|a-b|^2 = 3^2$

$|\bar{a}|^2 + |\bar{b}|^2 - 2a\cdot b = 9$

$2\bar{a}\cdot b = -$

$|\bar{a}+\bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}\cdot\bar{b}$

$= 4 + 36 - 2a\cdot b = 9$

$-2a\cdot b = 9 - 4 - 36$

$a\cdot b = \frac{31}{2}$

$4 + 36 + 2a\cdot b =$

$40 + 2\cdot\frac{31}{2}$

$\sqrt{71}$

Q. If $\bar{a}+\bar{b}+\bar{c} = 0$ & $|\bar{a}| = 3$, $|\bar{b}| = 1$, $|\bar{c}| = 4$.

find $\Sigma \bar{a}\cdot\bar{b}$

$=$ $|\bar{a}+\bar{b}+\bar{c}|^2 = 0.$

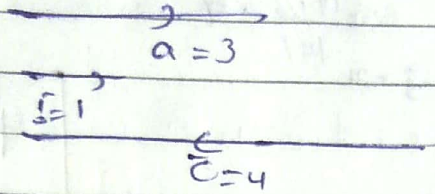
$|a|^2 + |b|^2 + |c|^2 + 2(a\cdot b + b\cdot c + c\cdot a) =$

$= 0 = 9 + 1 + 16 + 2\cdot ?$

$? = -13$

Ch.

M-2,



$$\sum \bar{a}\cdot\bar{b} = \bar{a}\cdot\bar{b} + a\cdot c + b\cdot c$$
$$= 3\cdot 1\cos 0 + 3\cdot 4\cos\pi + 1\cdot 4\cos\pi$$
$$= 3 - 12 - 4$$
$$= -13$$

Q. $\bar{a}+\bar{b}+\bar{c}=0$ $|\bar{a}|=3$, $|\bar{b}|=2$ $|\bar{c}|=4$. find $\bar{b}\wedge\bar{c}$

$\bar{b}+\bar{c} = -\bar{a}$

angle find.

$|\bar{b}+\bar{c}|^2 = |-a|^2$

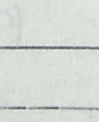
$|b\wedge c|^2 =$ find Ans $|a|^2$

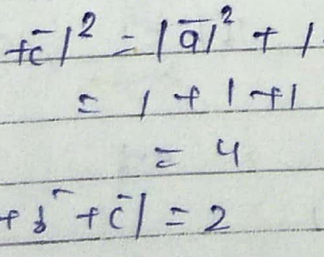
$|\bar{b}|^2 + |c|^2 + 2\bar{b}\bar{c} = |\bar{a}|$

or

$\bar{a}+2b-3c=0$

Q. $\hat{a}$ ~~an~~ $\hat{b}$ and $\hat{c}$ are ~~and~~ unit vector such that $\bar{b}\wedge\bar{c} = \frac{\pi}{3}$ and ~~and~~ ~~angle~~ ~~and~~ $\bar{a}$ is $\perp$ular to the Plane of $\bar{b}$ and $\bar{c}$ then find $|\bar{a}+\bar{b}+\bar{c}|$.





$\hat{b}\wedge\hat{c} = \frac{\pi}{3}$

$\bar{a} = \bar{b}+\bar{c}$

$|a|^2 = |\bar{b}+\bar{c}|^2$

$$|\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\cancel{\bar{a}\cdot\bar{b}} + \bar{b}\cdot\bar{c} + \cancel{\bar{c}\cdot\bar{a}})$$
$$= 1+1+1+2\cdot 1\cdot 1\cos\frac{\pi}{3}$$
$$= 4$$

$|\bar{a}+\bar{b}+\bar{c}| = 2$

Que: $\bar{a} = x\hat{i} - 3\hat{j} - \hat{k}$, $\bar{b} = 2x\hat{i} + x\hat{j} - \hat{k}$. Find set of value of x for which $\bar{a} \wedge \bar{b}$ is acute.

$\bar{a}\cdot\bar{b} = 2x^2 - 3x + 1 > 0$

$= (x-1)(2x-1) > 0$

$x \in \left(-\infty, \frac{1}{2}\right) \cup (1, \infty)$

Que: $\bar{a} = (c\log_2 x)\hat{i} - 6\hat{j} + 3\hat{k}$

$\bar{b} = (\log_2 x)\hat{i} + 2\hat{j} + (2c\log_2 x)\hat{k}$ $\quad x \in (0, \infty)$

makes an obtuse angle then find Range of c.

$\boxed{a\cdot b < 0}$

$\bar{a}\cdot\bar{b} < 0$

$(c\log_2 x)(\log_2 x) - 12 + 3(2c\log_2 x) < 0$

$= c\cdot(\log_2 x)^2 - 12 + 3(2c\log_2 x) < 0$

$c\log_2 x(\log_2 x - 12 + 6) < 0$

$c(\log_2 x)^2 + 6c\log_2 x - 12) < 0$ $\quad c x^2 - 12 + 6c = 0$

$c < 0$ -(i) & $D < 0$

i.e $36c^2 - 4c(-12) < 0$ - -(ii)

$c(9c + 4) < 0$

$-\frac{4}{9} < c < 0$ An.

Que: find unit vector ($\hat{c}$) coplanar with vector $\bar{a}$ and $\bar{b}$ and making an angle 60° with z axis.

$\bar{a} = i + j \quad , b = 2\hat{i} - \hat{j} + k$

Any vector Coplanar with $\bar{a}$ and $\bar{b}$

$\Longrightarrow \boxed{\bar{C} = \lambda\bar{a} + \mu\bar{b}}$

$= \lambda(\hat{i} + \hat{j}) + \mu(2\hat{i} - \hat{j} + k)$

$= \hat{i}(\lambda + 2\mu) + \hat{j}(\lambda - \mu) + \mu k.$

$|\hat{C}| = 1 \Rightarrow$

$(\lambda + 2\mu)^2 + (\lambda - \mu)^2 + \mu^2 = 1 \quad -(i)$

$\bar{C} \cdot \hat{k} = |\bar{C}|\,|\hat{k}| \cos\frac{\pi}{3}$

$\mu = 1 \cdot 1 \cdot \frac{1}{2} \qquad \mu = \frac{1}{2}$

$\cos 60° = |a| \cdot |b|$

$= \frac{1}{2}|a||b|$

$2|\bar{a}||\bar{b}| = 1$

$\hat{a} = \frac{a}{|a|} = \frac{a\hat{j} + bk}{|a||b|}$

Que: $\bar{a} = 5i - 2j + 5k$

$\bar{b} = 3i + k.$

Expressed $\bar{a}$ as a sum of Two vectors such that one is $\parallel$ to $\bar{b}$ and other is $\perp$cular to b.

Parallel $\bar{a} \cdot \bar{b} = 0 \quad -(i)$

$\perp$cular $\bar{a} \cdot \bar{b} = 15 + 5 = 20$

$a + b = 0, \quad , a + b = 1$

Sol^n: $\bar{a} = \bar{h} + \bar{v}$

$\bar{a} = \lambda\hat{b} + (u_1\hat{i} + u_2\hat{j} + v_3 k)$

$(5i - 2j + 5k) = \lambda(3i + k) + (u_1\hat{i} + u_2\hat{j} + u_3 k)$

$5 = 3\lambda + u_1$

$-2 = u_2$

$5 = \lambda - 3u_1$

$\bar{v} \cdot \bar{b} = 0$

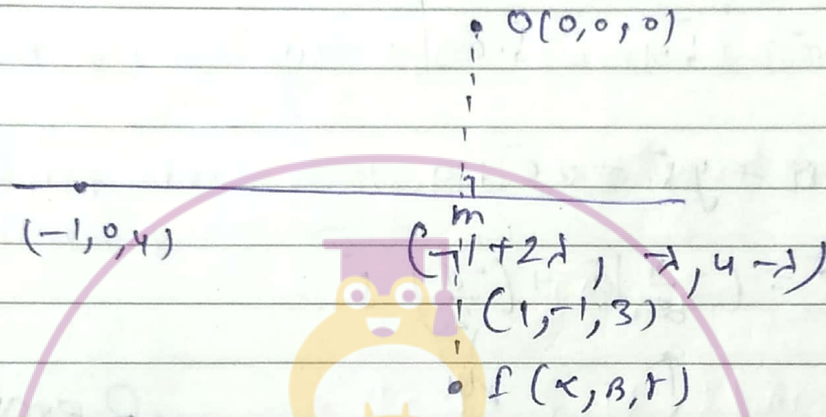
$3u_1 + v_3 = 0$

RK TECK

Q. Find foot of ⊥ar drawn from origin to the line.

$$\bar{r} = -\hat{i} + 4\hat{k} + \lambda(2\hat{i} - \hat{j} - \hat{k})$$

also find its image.



$\bar{OM} =$

$\bar{OM} \perp (2\hat{i} - \hat{j} - \hat{k})$

$[(2\lambda-1)\hat{i} - \lambda\hat{j} + (4-\lambda)\hat{k}] \cdot [2\hat{i} - \hat{j} - \hat{k}] = 0$

$2(2\lambda-1) + \lambda - 4 + \lambda = 0$

$4\lambda - 2 + \lambda - 4 + \lambda = 0$

$\lambda = 1$

$OM = \sqrt{(1-0)^2 + (-1-0)^2 + (3-0)^2}$

$= \sqrt{11}$

$= \frac{\alpha+0}{2} = 0 \qquad \frac{\beta+0}{2} = -1, \quad \frac{\gamma+0}{2} = 3.$

H.W: [O-1, J.M, J.A]

O-1 ⇒ 28, 29, 30, 31, 32, 33, 34 Parsi

O-1: 14, 15, 16 (write down), 24

J-A: 2(a), 4(b), 7(a) Linked.

Remember

Remember

Q. If $\bar{a}, \bar{b}$ and $\bar{c}$ are coplanar vector then prove that

$$\begin{vmatrix} \bar{a} & \bar{b} & \bar{c} \\ \bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c} \\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c} \end{vmatrix} = 0$$

$$x\bar{a} + y\bar{b} + z\bar{c} = 0$$

$$\bar{c} = \left(-\frac{x}{z}\right)\bar{a} + \left(-\frac{y}{z}\right)\bar{b}$$

Remember

$$x\bar{a}\cdot\bar{a} + y\bar{a}\cdot\bar{b} + z\bar{a}\cdot\bar{c} = 0$$

$$x\bar{a}\cdot\bar{b} + y\bar{b}\cdot\bar{b} + z\bar{b}\cdot\bar{c} = 0$$

$$\frac{x}{\xi} = \frac{y}{\theta} = \frac{z}{\partial} \Rightarrow \lambda$$

Eliminate x, y and z.

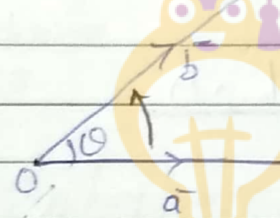
$$\begin{vmatrix} a & b & \bar{c} \\ \bar{a}\cdot\bar{a} & \bar{b}\cdot\bar{c} & \bar{a}\cdot\bar{c} \\ \bar{a}\cdot\bar{b} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c} \end{vmatrix} = 0$$

* Vector Product : (Cross Product) :

If $\bar{a}$ and $\bar{b}$ are two vectors, then their cross product is defined as $\bar{a} \times \bar{b}$ and given by

$$\bar{a} \times \bar{b} = (|\bar{a}||\bar{b}| \sin\theta)\hat{n}$$

where $\hat{n}$ is unit vector ⊥ular to the plane formed by ~~vector~~ $\bar{a}$ and $\bar{b}$



clockwise ⇒ in of paper

Anticlock wise = out

$\bar{a} \times \bar{b}$ is ⊥ular to $\bar{a}$ as well as $\bar{b}$

$$\bar{a} = a_1 i + a_2 j + a_3 k$$

$$\bar{b} = b_1 i + b_2 j + b_3 k$$

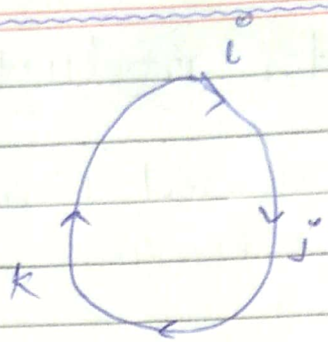
* Properties :

(1) $\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$$= i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

(2) $i \times i = 0 = j \times j = k \times k$.

(3) $\bar{i} \times \bar{j} = k,\ \bar{j} \times \hat{k} = \bar{i},\ \bar{k} \times \bar{i} = \bar{j}$



$\hat{k} \times \hat{j} = -\hat{i}$

If $\bar{a} \times \bar{b} = 0$ and $\bar{a}, \bar{b}$ are not null vectors.

$\bar{a} \times \bar{b} = 0 \Rightarrow \bar{a}$ and $\bar{b}$ are Parallel

(Parallel)(Collinear)

$\bar{a} = \lambda \bar{b}$

i.e $\bar{a}$ and $\bar{b}$ are L.D

* If $\bar{a} \times \bar{b} = \bar{a} \times \bar{c}$

then $\bar{a} \times \bar{b} - \bar{a} \times \bar{c} = 0$ $\rightarrow$ either $\bar{b} = \bar{c}$

$a \times (\bar{b} - \bar{c}) = 0$ $\rightarrow$ $\bar{a}$ is null vector

$\rightarrow$ $\bar{a}$ & $(\bar{b} - \bar{c})$ are Parallel vector.

Imp.

$$\begin{cases} \bar{a} = \lambda(\bar{b} - \bar{c}) \\ \bar{b} - \bar{c} = \lambda \bar{a} \\ \bar{b} = \bar{c} + \lambda \bar{a} \end{cases}$$

$\bar{a} \times \bar{b} = -\bar{b} \times \bar{c}$

* $\bar{a} \times (\bar{b} + \bar{c}) = \bar{a} \times \bar{b} + \bar{a} \times \bar{c}$

not associative $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$

* $|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin\theta (\hat{n})$

$\bar{a} \times \bar{b}$ = 1 column

$\bar{a} \times \bar{b} \Rightarrow |\bar{a} \times \bar{b}|$ = ? vector ⇒ Parallel make = sub ways.
result = unit: also Scalar ⇒ Area
$\hat{e}$ = ⊥ular Plane

(a) $|\bar{a} \times \bar{b}| = |a||b| \sin(\hat{n})$

$$|\bar{a} \times \bar{b}|^2 = |\bar{a}|^2 |\bar{b}|^2 (1 - \cos^2\theta)$$

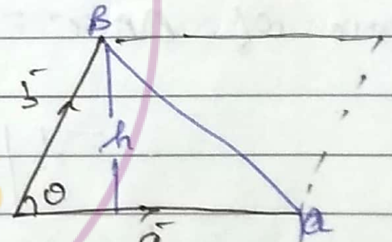
$$= |\bar{a}|^2 |\bar{b}|^2 - |\bar{a}|^2 |\bar{b}|^2 \cos^2\theta$$

$$(\bar{a} \times \bar{b})^2 = |\bar{a}|^2 |\bar{b}|^2 - (\bar{a} \cdot \bar{b})^2$$

Imp: $$\boxed{(\bar{a} \times \bar{b})^2 = |\bar{a} \times \bar{b}|^2 = \begin{vmatrix} \bar{a} \cdot \bar{a} & \bar{a} \cdot \bar{b} \\ \bar{a} \cdot \bar{b} & \bar{b} \cdot \bar{b} \end{vmatrix}}$$ Imp

Langranges formula

* Geometrical Interpretation!



$$|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin\theta$$

$$= 2\left(\tfrac{1}{2} |\bar{a}||\bar{b}| \sin\theta\right)$$

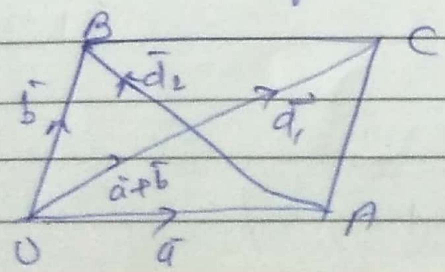
$$\left[\begin{aligned} \Delta OAB &= \tfrac{1}{2} OA \cdot h \\ &= \tfrac{1}{2} |\bar{a}||\bar{b}| \sin\theta \end{aligned}\right.$$

in this fig. unit vector normal to the plane shown $= \boxed{\pm \dfrac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|}}$

$$= 2\,(\Delta OAB)$$

= Area of ||gm OACB.

*



OACB is ||gm

Area of ||gm $= \frac{1}{2} |\bar{d}_1 \times \bar{d}_2|$.

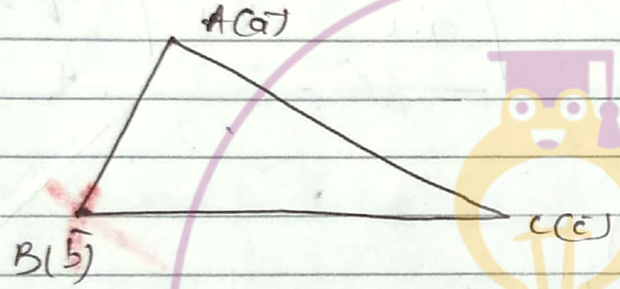
* ~~In~~ above figure.

$$\bar{d_1} = \bar{a} + \bar{b}$$

$$\cancel{d_2 = a} \quad b = \bar{a} + \bar{d_2}$$

$$\bar{d_2} = \bar{b} - \bar{c}$$

$$\bar{b} = \frac{\bar{d_1} + \bar{d_2}}{2} \qquad a = \frac{\bar{d_1} - \bar{d_2}}{2}$$



Area of $\Delta ABC = \frac{1}{2}|\bar{CA} \times \bar{CB}|$

$$= \frac{1}{2}|(\bar{a} - \bar{c}) \times (\bar{b} - \bar{c})|$$

$$= \frac{1}{2}|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$$

* Write unit normal vector ⊥ular to the plane of Δ shown

~~OA~~ + OB + OC

$\frac{AB + AC}{|\bar{a} + \bar{b}|}$

$$= \pm \frac{\bar{CA} \times \bar{CB}}{|\bar{CA} \times \bar{CB}|}$$

R R

$$= \frac{\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}}{|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|}$$

Ex Rrr

Que: find Area of triangle whose vector along side $2i - j - k$ & $i + 3j + k$.

$$= \frac{1}{2} \begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 1 & 3 & 1 \end{vmatrix}$$

$$= i(-1+3) - j(2+1) + k(6+1)$$

$$= \frac{1}{2} |\cancel{4}i - 3j + 7k|$$

$$= \frac{1}{2} \sqrt{16 + 9 + 49}$$

Que: find Area of ||gm whose diagonals are $3i + j - 2k$ & $i - 3j + 4k$

$$= \frac{1}{2} \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$i(4-6) - j(12+2) + k(-9-1)$$

$$= \frac{1}{2} |-2i - 14j - 10k|$$

$$= \frac{1}{2} \left| \sqrt{\cancel{4} - (14)^2 + 10} \right|$$

$$|-i - 7j - 5k| = \sqrt{1 + 49 + 25}$$

$$= \sqrt{75}$$

Que: show that $(\bar{a} - \bar{d}) \times (\bar{b} - \bar{c}) + (\bar{b} - \bar{d}) \times (\bar{c} - \bar{a}) + (\bar{c} - \bar{d}) \times (\bar{a} - \bar{b})$ is independent of vector $\bar{b}$

R

$$= \cancel{a \times b} + \cancel{b^2} - \bar{c}\bar{a} + \cancel{ab} + \bar{b}\bar{c} - \bar{d}\bar{c} - \cancel{ab} + \bar{d}\bar{a} + \cancel{ab} - \bar{a}\bar{d} - \cancel{\bar{c}} + \cancel{bd}$$

$$\cancel{bc} - dc \cancel{+da} \quad \cancel{bc} \cancel{dc} \; 2(b \times c - d \times c)$$

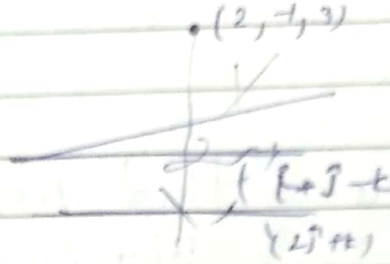
$$= \bar{a} \times b - \bar{a} \times c - \bar{d} \times b + \bar{d} \times \bar{c} + \bar{b} \times c - \bar{b} \times \bar{a} - \bar{d} \times c + \bar{d} \times \bar{a} \; \cancel{+ c \times a}$$

$$- \bar{c} \times b - \cancel{\bar{d} \times \bar{a}} + \cancel{\bar{d} \times \bar{c}}$$

$$2(\bar{c} \times \bar{a} + \bar{a} \times \bar{b} + \bar{b} \times \bar{c})$$

Q. find eq^n of straight line which passes to $A(2,-1,3)$ and $\perp$cular to line

$$\vec{r} = (\hat{i}+\hat{j}-\hat{k}) + \lambda(2\hat{i}+\hat{j}+\hat{k})$$
$$\& \ \vec{r} = (2\hat{j}+\hat{k}) + \mu(\hat{i}-3\hat{i}+2\hat{k}).$$



$$(1+2\lambda)\hat{i} + (1+\lambda)\hat{j} + (-1+\lambda)$$
$$(2+\mu)\hat{i} + (-3\mu)\hat{j} + 1+2\mu$$

$1+2\lambda = 2+\mu$ —(i)

$1+\lambda = -3\mu$ —(ii)

$-1+\lambda = 1+2\mu$ —(iii)

Sol^n dir^n of required line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{vmatrix}$

$= 5\hat{i} - 3\hat{j} - 7\hat{k}.$

$\Rightarrow$

$$\vec{r} = (2\hat{i}-\hat{j}+3\hat{k}) + s(5\hat{i}-3\hat{j}-7\hat{k})$$

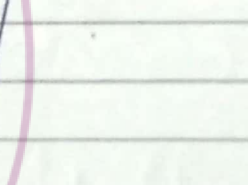
Que:

$\vec{a} = \hat{i}+4\hat{j}+2\hat{k}$

$\vec{b} = 3\hat{i}-2\hat{j}+7\hat{k}$

$\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$

find vec $\vec{d}$ which is $\perp$culer to both $\vec{a}$ and $\vec{b}$ and satisfied $\vec{c}\cdot\vec{d} = 15$



~~da~~ =

$= \vec{c}\cdot\vec{d} = \cancel{6+2+28=15}$

$A\times B = 32\hat{i} - \hat{j} - 14\hat{k}.$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$

$= (28+4)\hat{i} - (1)\hat{j} + (-14)\hat{k}$

$32\hat{i} - \hat{j} - 14\hat{k}$.

Sol^n d is ⊥cular to $\bar{a}$ & $\bar{b}$

$$d = \lambda(\bar{a} + \bar{b})$$
$$= \lambda(32i - j - 14k)$$

$$\lambda(64 + 1 - 56) = 15$$

$$\lambda = \frac{15}{9} = \frac{5}{3}$$

Q. find unknown vector $\bar{R}$ satisfy $\bar{R} \times \bar{B} = \bar{C} \times \bar{B}$

&

$$\bar{R} \cdot \bar{A} = 0$$
$$\bar{A} = 2i + k$$
$$\bar{B} = i + j + k$$
$$\bar{c} = 4i - 3j + 7k$$

$\bar{R} \times \bar{B} = \lambda$

$\bar{R} = \bar{c}$

$= R = 4i - 3j + 7k$

$$R \times B = \lambda(C \times B)$$

$$\bar{R} \times \bar{B} = \left(\begin{vmatrix} i & j & k \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix} \right)$$

$$= \lambda(-10i - 11j + 7k)$$

$\bar{R} \cdot \bar{A} = 0$

$R(2i + k) = 0$

$R = 0$

R

$R \times B - (C \times B) = 0$

$R \times B - C \times B =$

$B(R - C)$

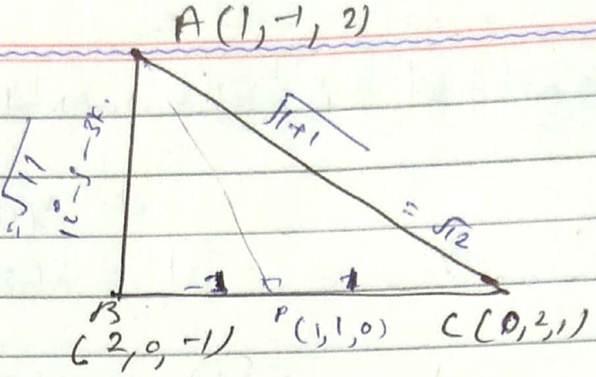
$= 4i + 3i + 7k$

$$\bar{R} \times \bar{B} - \bar{C} \times B = 0$$
$$(\bar{R} - \bar{C}) \times \bar{B} = 0 \qquad = \overline{R - C} = \lambda \bar{B}$$

~~$R = C + \lambda$~~ $\quad R = \bar{C} + \lambda \bar{B}$

$$= (4i - 3j + k) + \lambda(i + j + k)$$
$$R = (4 + \lambda)i + j(\lambda - 3) + k(\lambda + 1)$$
$$2(4 + \lambda) + \lambda + 1 = 0$$
$$\lambda = -3$$
$$R = i - 6j - 2k.$$

Q.(i) find Area of triangle ABC



$\frac{AB}{AC} =$

$1i - j\ 3k$

$\frac{1-k-3}{-1+3-1} = \frac{-3}{1} = \frac{\sqrt{11}}{\sqrt{11}} = \frac{1}{1}$

$(2j - 2k)$

$= \frac{1}{2}|\bar{BA} \times \bar{BC}|$

$\frac{1}{2}\begin{vmatrix} i & j & k \\ -1 & -1 & 3 \\ -2 & 2 & 2 \end{vmatrix}$

$= \frac{1}{2}|-8i - 4j - 4k|$

$= |4i + 2j + 2k|$

$= 4i + 2j + 2k$

Q.2 find unit vector normal to the Plane shown.

$\frac{4i+2j+2k}{\sqrt{4^2+2^2+2^2}} = \frac{4i+2j+2k}{\sqrt{16+4+4}} = \pm\frac{4i+2j+2k}{\sqrt{24}} = \frac{(2i+j+k)}{\sqrt{6}}$

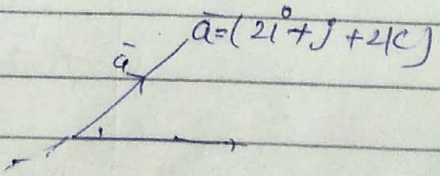
Q. find length of Altitude drawn from vertex A of side BC.

$\sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ $= (2+3)i + -3k$

Que (i) Projection of $\bar{a}$ on x, y and z axis are (2, 1, 2) respective then find angle at which $\bar{a}$ is inclined with z axis

Sol

$\frac{2i+j+4k}{|2|}$



$\bar{a} = a_1 i + a_2 j + a_3 k$

$\left\{ \begin{array}{l} a_1 i \text{ is component of } \bar{a} \text{ along } x\text{-axis} \\ a_1 \text{ " projection " "} \end{array} \right.$

$\begin{cases} a_2 \hat{i} & \text{is Component of } \bar{a} \text{ along y axis} \\ a_2 & \text{" Proj " " " y axis.} \end{cases}$

$\begin{cases} a_3 \hat{i} & \text{" Comp. of } \bar{a} \\ a_3 & \end{cases}$

& z-axis $\rightarrow \hat{k}$

$\vec{a} \cdot k = |\bar{a}||\bar{k}| \cos\theta$

$z = 3 \cdot 1 \cos\theta$

$2 = 3 \cdot 1 \cos\theta$

$\cos\theta = \frac{2}{3}$

$\Rightarrow$ In above Question write $\bar{b}$ or magnitude 9 in the dir of $\bar{a}$

$= \cancel{\vec{b} \cdot \hat{j} = \hat{j} \times \hat{a}}$

$= \bar{b} = \frac{(2\hat{i} + \hat{j} + 3\hat{k})}{3} \times 9$

$= 3(2\hat{i} + \hat{j} + 2\hat{k})$.

Q. Two sides of ΔABC are $\hat{i} + 2\hat{j}$ & $2\hat{i} - k$. then find length of 3rd side.

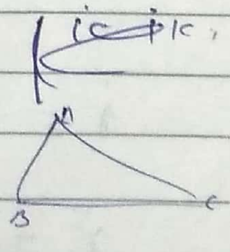
$\bar{A} - (\hat{i} + 2\hat{j}) \quad \bar{B}(2\hat{i} - k)$

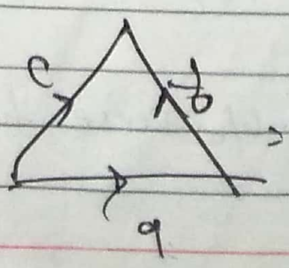
$\cancel{\bar{C} = -(A + B)}$ $\quad c =$

$a + b + c = 0 \quad c = -(3\hat{i} + 2\hat{j} - k)$

$|\bar{c}| = \sqrt{9 + 4 + 1} = \sqrt{14}$

$\bar{a}_4 = 1\hat{q}$





$\vec{a} + \vec{b} + \vec{c} = 0$

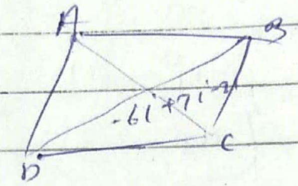
$c = -(\vec{a} + \vec{b})$

$c = |(\vec{a} + \vec{b})|$

Q. In a ||gram ABCD $AC = \cancel{2i} + 3$

$AC = 2i + 3j + 4k$

$BD = -6i + 7j - 2k$



i) find Area of ||gram

(ii) find $\overline{AB} \hat{} \overline{AD}$

$= \frac{1}{2}(d_1 \times d_2)$

(i) $\frac{1}{2}(\overline{AC} \times \overline{BD})$

$\bar{a} + \bar{b} = d_1 = \overline{AC} =$

$\bar{b} - \bar{c} = d_2 = \overline{BD} =$

$= \frac{1}{2} \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -6 & 7 & -2 \end{vmatrix}$

$= \frac{1}{2} [\;$

$= \frac{1}{2}(-4i + 10j + 2k)$

$= -2i + 5j + k$

(ii) $\bar{b} = \frac{d_1 + d_2}{2}$

$\bar{c} = \frac{d_1 - d_2}{2}$

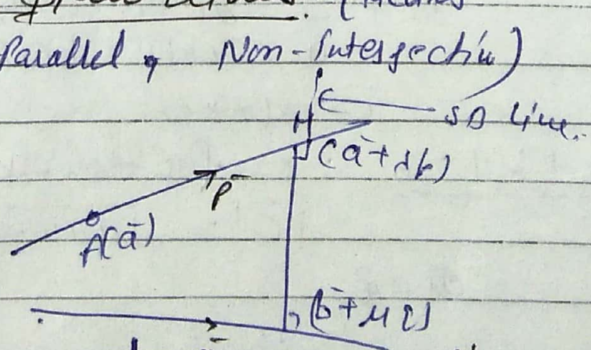
$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|}$

Q. $\left.\begin{array}{l} \bar{a} = 7i - 4j - 4k \\ \bar{b} = 2i - j + 2k \end{array}\right] \times$

* Shortest dist. b/w two skew lines L (Means (Non-Parallel & Non-Intersecting)

$\Rightarrow$ $\bar{r} = \bar{a} + \lambda \bar{p}$

$\bar{r} = b + \mu \bar{q}$



Shortest dist. b/w these two lines

if SD = 0 then given lines are intersect each other

Shortes dis · M and N Quest.

$$\boxed{S.D = \left| \frac{(\bar{b}-\bar{a})\cdot(\bar{p}\times\bar{q})}{|\bar{p}\times\bar{q}|} \right|}$$ — Learn

Method to ^Evaluate^ P.V of Point M & N

[We have to find λ and μ whose ~~respect~~ respited to be M an λ get Value.]

$$\bar{MN} = \bar{a} - \bar{b} + \lambda\bar{p} - \mu\bar{q}$$

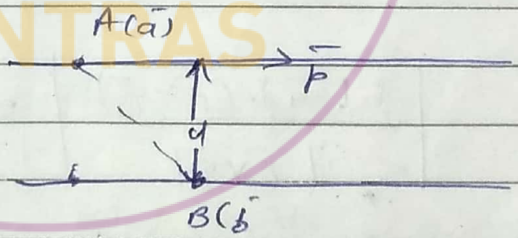
(1) $\bar{MN}\cdot\bar{p} = 0$ —(i)

& $\bar{MN}\cdot q = 0$ —(ii) Solve and get value of.

* Dist. b/w Parallel lines!

$\bar{r} = \bar{a} + \lambda\bar{p}$

$\bar{r} = \bar{b} + \mu\bar{p}$



$$S.D = d = \left| \frac{(\bar{a}-\bar{b})\times\bar{p}}{|\bar{p}|} \right|$$

Que: $\bar{r} = i + j + \lambda(2i - j + k)$ $\quad a(1,1,0)$

$\bar{r} = 2i + j - k + \mu(3i - 5j + 2k)$ $\quad b(2,1,0)$

find S.D b/w

$D = \cancel{\frac{(-1+0-1)\times P}{}}$

$= \frac{(-2) \times P}{|P|\;|P|} = \frac{+2 \times k - 7(i + j + k)}{\sqrt{4+1+1}\;\sqrt{9+25+4}}$

$A(1,1,0) \qquad \bar{p} = 2i - j + k$

$B(2,1,-1) \qquad q = 3i - 5j + 2k$

$\bar{AB} = (i - k)$

$\bar{p} \times \bar{q} = -7i - 7j - 7k$

$= -7(i + j + k)$

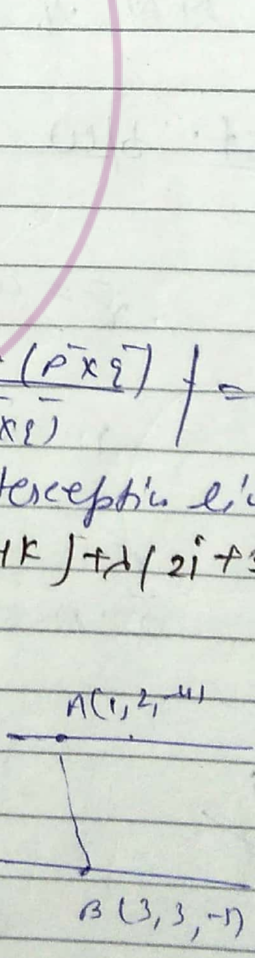
$$\left| \frac{(\bar{b} - \bar{a}) \cdot (\bar{p} \times \bar{q})}{|\bar{p} \times \bar{q}|} \right| = \left| \frac{\bar{AB} \cdot (\bar{p} \times \bar{q})}{(\bar{p} \times \bar{q})} \right| = 0$$

Hence interception line

Q. find dist. b/w two line $\bar{r} = (i - 2j + 4k) + \lambda(2i + 3j + 6k)$

$\bar{r} = (3i + 3j - 5k) + \mu(2i + 3j + 6k)$

$D = \frac{(2i + j - k) \cdot (2i + 3j + 6k)}{|\sqrt{4+9+36}|}$



$= \frac{1}{\sqrt{49}} \frac{(2i + j - k) \times (2i + 3j + 6k)}{\sqrt{49} = 7} = \frac{1}{\sqrt{49}} \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = \frac{\sqrt{293}}{7}$

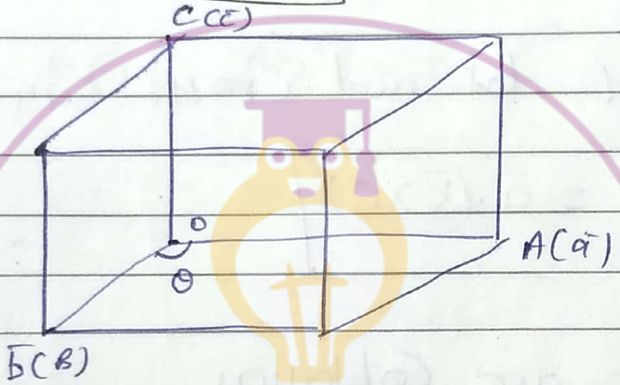
$|A \times B| \Rightarrow$ Area of $\parallel$ gram   H.W $\begin{cases} \text{O-1} & 35, 36, 37, 38, 39, 40, 42, \\ \text{S-1} & \Rightarrow 5, 6, 7, 17(a)(b), 19 \end{cases}$

* S.T.P (Scalar Triple Product) :-

if $\bar{a}, \bar{b}, \bar{c}$ are free vectors then their STP is given by

$$(\bar{a} \times \bar{b}) \cdot \bar{c}$$
$$= [\bar{a}\,\bar{b}\,\bar{c}]$$

* Geometrical Interpretation :



$\theta = \bar{a} \wedge \bar{b}$
$\phi = \hat{n} \wedge \vec{c}$

$$(\bar{a} \times \bar{b}) \cdot \bar{c} = (|\bar{a}||\bar{b}| \sin\theta)\, \hat{n} \cdot \bar{c}$$
$$= |\bar{a}||\bar{b}| \sin\theta\, |\hat{n}||\bar{c}| \cos\phi$$
$$= |\bar{a}||\bar{b}||\bar{c}||\hat{n}| \sin\theta \cos\phi$$

= Volume of $\parallel$elo pipe whose coterminus edges are given by $\bar{a}, \bar{b}, \bar{c}$

* Properties :

$$\bar{a} = a_1 i + a_2 j + a_3 k$$
$$\bar{b} = b_1 i + b_2 j + b_3 k$$
$$\bar{c} = c_1 i + c_2 j + c_3 k.$$

$$[\bar{a}\ \bar{b}\ \bar{c}] = (\bar{a}\times\bar{b})\cdot\bar{c} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \cdot (c_1 i + c_2 j + c_3 k)$$

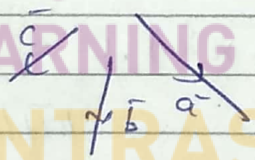
$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(1) Position of dot and cross can be interchanged

$$\boxed{(\bar{a}\times\bar{b})\cdot c = \bar{a}\cdot(\bar{b}\times c)}$$

(2) if $\bar{a}, \bar{b}, c$ are coplanar

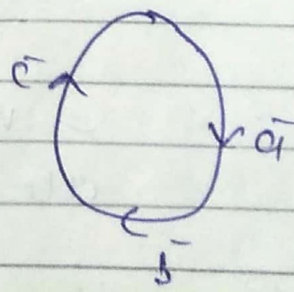
$[\bar{a}\ \bar{b}\ \bar{c}] = 0$



then $\bar{a}\ \bar{b}\ \bar{c}$ are coplanar. i.e L.D.

(3) $[\bar{a}\ \bar{b}\ \bar{c}] = [\bar{b}\ \bar{c}\ \bar{a}] = [\bar{c}\ \bar{a}\ \bar{b}]$

$[\bar{a}\ \bar{b}\ \bar{c}] = -[\bar{b}\ \bar{a}\ \bar{c}]$

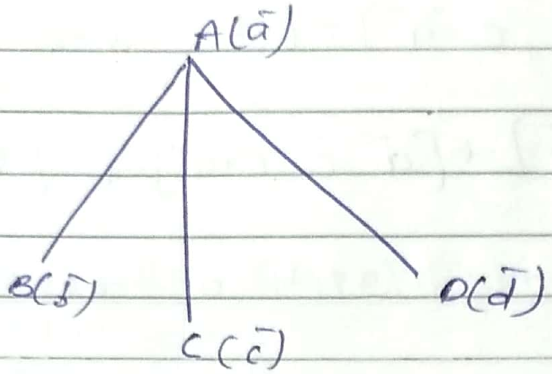


(4) $[k\bar{a}\ \bar{b}\ \bar{c}] = k[\bar{a}\ \bar{b}\ \bar{c}]$

$[\bar{a}+\bar{b}\ \ \bar{c}\ \ \bar{d}] = [\bar{a}\ \bar{c}\ \bar{d}] + [\bar{b}\ \bar{c}\ \bar{d}]$

$[\bar{a}\ \ \bar{a}\ \ \bar{b}] = 0$

5) if four points are coplanar



$[\bar{AB} \quad \bar{AC} \quad \bar{AD}] = 0$

Q 6) if $[\bar{a} \ \bar{b} \ \bar{c}]$

> 0 — $\bar{a}, \bar{b}, \bar{c}$ form R.H. system

< 0 — L.H system.

Q. $[\bar{a}+\bar{b} \quad \bar{b}+\bar{c} \quad \bar{c}+\bar{a}] = 2[\bar{a} \ \bar{b} \ \bar{c}]$
Prove that.

$R_1 \to R_1 + R_2 + R_3$

$[\bar{a} \ \bar{b} \ \bar{c}] + [\bar{b} \ \bar{c}$

$((\bar{a}+\bar{b}) \times (\bar{b} \times \bar{c})) \cdot (\bar{c}+\bar{a})$

$= (\bar{a}\times\bar{b} + \bar{a}\times\bar{c} + \bar{b}\times\bar{c}) \cdot (\bar{c}+\bar{a})$

$= [\bar{a} \ \bar{b} \ \bar{c}] + \cancel{[\bar{a} \ \bar{b} \ \bar{a}]} + \cancel{[\bar{a} \ \bar{c} \ \bar{c}]} + \cancel{[\bar{a} \ \bar{c} \ \bar{a}]} + \cancel{[\bar{b} \ \bar{c} \ \bar{c}]} + [\bar{b} \ \bar{c} \ \bar{a}]$

$= 2[\bar{a} \ \bar{b} \ \bar{c}]$

$[\bar{a}\ \bar{b}\ \bar{c}]$ - volume

M-2

Q. $[\bar{a}+\bar{b} \quad \bar{b}+\bar{c} \quad \bar{c}+\bar{a}]$

$[\bar{a} \quad \bar{b}+\bar{c} \quad \bar{c}+\bar{a}] + [\bar{b} \quad \bar{b}+\bar{c} \quad \bar{c}+\bar{a}]$

$= [\bar{a}\ \bar{b}\ \bar{c}+\bar{a}] + [\bar{a}\ \bar{c}\ \bar{c}+\bar{a}] + [\bar{b}\ \bar{b}\ \bar{c}+\bar{a}] + [\bar{b}\ \bar{c}\ \bar{c}+\bar{a}]$

$= [\bar{a}\bar{b}\bar{c}] + 0 + 0 + 0 + 0 + [\bar{b}\ \bar{c}\ \bar{a}]$

M-3

$[\bar{a}+\bar{b} \quad \bar{b}+\bar{c} \quad \bar{c}+\bar{a}] =$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} [\bar{a}\ \bar{b}\ \bar{c}]$$

$= 2[\bar{a}\ \bar{b}\ \bar{c}]$

Provided $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors.

* $[\bar{a}\times\bar{b} \quad \bar{b}\times\bar{c} \quad \bar{c}\times\bar{a}] = [\bar{a}\ \bar{b}\ \bar{c}]^2$.

$$[\bar{l}\ \ \bar{m}\ \ \bar{n}][\bar{a}\ \ \bar{b}\ \ \bar{c}] = \begin{vmatrix} \bar{a}\cdot\bar{l} & \bar{a}\cdot\bar{m} & \bar{a}\cdot\bar{n} \\ \bar{b}\cdot\bar{l} & \bar{b}\cdot\bar{m} & \bar{b}\cdot\bar{n} \\ \bar{c}\cdot\bar{l} & \bar{c}\cdot\bar{m} & \bar{c}\cdot\bar{n} \end{vmatrix}$$

$$\boxed{[\bar{a}\ \bar{b}\ \bar{c}][\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} \bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c} \\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c} \\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c} \end{vmatrix}}$$

Q. Find Volume of Parallogram formed by Vectors

$\bar{a} = 2i - j - 3k$
$\bar{b} = i + 2j + k$
$\bar{c} = i - 3j + 2k$

$[\bar{a}\ \bar{b}\ \bar{c}] = (a \times b) \cdot c$

$$\begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} (i$$

$$= \begin{vmatrix} i & j & k \\ 2 & -1 & -3 \\ 1 & 2 & 1 \end{vmatrix} (i - 3i + 2k)$$

$$V = \left| \begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} \right| = 33$$

Q. $A(1, 0, 3)$, $B(-1, 3, 4)$ $C(1, 2, 1)$ $D(\lambda, 2, 5)$
if four point are coplane then find λ

~~AB(~~

~~$AB = (-2i + 3j + k)$~~
~~$BC = (2i + j - 3k)$~~
~~$CD = ((\lambda-1)i + 4k)$~~

$$= \begin{vmatrix} -2 & 3 & 1 \\ 2 & -1 & -3 \\ \lambda-1 & 0 & 4 \end{vmatrix} = 0$$

$\lambda =$

$= -2\cancel{(-4)} \cancel{- 3(\ldots)} + 3(\lambda-1)$

$[AB + AC + AD]$

$$\begin{vmatrix} -2 & 3 & 1 \\ 0 & -2 & -2 \\ \lambda-1 & 2 & 2 \end{vmatrix} = 0.$$

$\lambda = 4$

Q. $(p+1)\hat{i} - 3\hat{j} + p\hat{k}$

$p\hat{i} + (p+1)\hat{j} - 3\hat{k}$

$-3\hat{i} + p\hat{j} + (p+1)\hat{k}$

Find value of p for which are linearly dependent

Very useful method

$= (p+p+p) - 3(p+1)$

$$\begin{vmatrix} p+1 & -3 & p \\ p & p+1 & -3 \\ -3 & p & p+1 \end{vmatrix}$$

Q. P. that $[\bar{a}-\bar{b} \quad \bar{b}-\bar{c} \quad \bar{c}-\bar{a}] = 0$

$[\bar{a} \quad \bar{b}-\bar{c} \quad \bar{c}-\bar{a}] - [\bar{b} \quad \bar{b}-\bar{c} \quad \bar{c}-\bar{a}]$

$[\bar{a} \quad \bar{b}$

$$= \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} [a \; b \; c]$$

$= 1 - 1$

$= 0$

Q. Show that lines

$\bar{r} = \bar{p} + \lambda \bar{a}$

& $\bar{r} = \bar{q} + \mu \bar{b}$

intersect each other if

$(\bar{p}-\bar{q}) \cdot (\bar{a} \times \bar{b}) = 0$

$= \bar{p} - \bar{q}$

$|\bar{p} + \bar{r}| = 0$

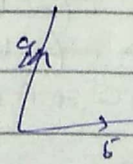
a

$p + a + q + b = 0$

$(\bar{p}-\bar{q}) \cdot \cancel{(\bar{a} \times \bar{b})} = 0$

$(\bar{p}-\bar{q}) \cdot$

$= \dfrac{\bar{a} \; \bar{b}}{|\bar{p}| \; |\bar{q}|}$



if intersect

then $[\bar{p}-\bar{q} \quad \bar{a} \quad \bar{b}] = 0$

Vector Rotate ⇒ Not Change Magnitude.

P No: 34 ⇒ 44, 45, 47, 48, 49, 51, 53, 55, 56,

J.A: 4, 6(a), 7(a), 8(a,b), 9

Q. P be the Point not on the Plane that Passes from P, Q, R, S then show that dist of Point P from Plane

$$= \frac{[\bar{a}\ \bar{b}\ \bar{c}]}{|\bar{a} \times \bar{b}|}$$ where $\bar{QR} = \bar{a}$, $\bar{QS} = \bar{b}$, $\bar{QP} = \bar{c}$

$$= \frac{[\bar{a}\ \bar{b}\ \bar{c}]}{|\bar{a} \times \bar{b}|}$$

Q. $\bar{a} = i + 2j + 2k$ turn throw a Right angle Passing throw Positive x-axis on the way. find eq^n of vector in new Position.

Let New Vector $\bar{b} = xi + yj + zk$.

$\bar{c} = i$

$[\bar{a}\ \bar{b}\ \bar{c}] = 0$

$$\begin{vmatrix} 1 & 2 & 2 \\ 1 & 0 & 0 \\ x & y & z \end{vmatrix} = 0$$

$1(2z - 2y) = 0$

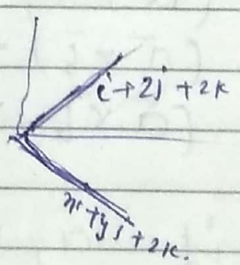
$y = z$ ——(i)

$|\bar{a}| = |\bar{b}|$

$9 = x^2 + y^2 + z^2$ ——(ii)

$\bar{a} \cdot \bar{b} = 0$

$x + 2y + 2z = 0$ ——(iii)



$(\bar{a} \times \bar{b}) = |a||b| \sin\theta$

$= |a||b|$.

$= (1+x)i + (2+y)j + (2+z)k =$

$= \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ x & y & z \end{vmatrix}$

$= (2z - 2y)i - (z - 2x)j + (y - 2x)$

$= (2z - 2y)i - (z - 2x)j + (y - 2x)$

=

$X - \perp$

Non zero Vector $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ such that

$\bar{a}\cdot\bar{d} = 0$, $\bar{b}\cdot\bar{d} = 0$, $\bar{c}\cdot\bar{d} = 0$

P. that $[a\ b\ c] = 0$

Coplanar

3 are $\perp$

Vector Product of three vectors :

Q. which of the following are meaning ful

$(\bar{a}\cdot\bar{b})\,\bar{c}$ ✓ $(\bar{a}\cdot\bar{b}) =$ No.

$(\bar{a}\cdot\bar{b})\cdot\bar{c}$ ✗

$(\bar{a}\cdot\bar{b})\times\bar{c}$ ✗

$(\bar{a}\times\bar{b})\,\bar{c}\,\cdot$ ✗

STP $(\bar{a}\times\bar{b})\cdot\bar{c}$ ✓ STP

VTP $(\bar{a}\times\bar{b})\times\bar{c}$ ✓ VTP

if a, b, c are three vectors then their V.T.P are

with C orthogonal

$(\bar{a}\times\bar{b})\times\bar{c}$

$(\bar{a}\times\bar{b})\times c$ is coplanar with $\bar{a}$ and $\bar{b}$ and $\perp$cular to $\bar{c}$

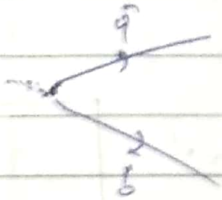
$$\boxed{\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}\cdot\bar{c})\,\bar{b} - (\bar{a}\cdot\bar{b})\,\vec{c}}$$

$(\bar{a}\times\bar{b}) =$ vector

$(\bar{a}\times\bar{b})\times\bar{c}$ = resultant

$\bar{a}\times\bar{b}\times\bar{c}$ coplanar $\times\bar{c}$ $\perp$

Note! $(\bar{a}\times\bar{b})$ is $\perp$ cular to $\bar{a}$ and $\perp$ cular to $\bar{b}$



$$\bar{r} = (\bar{a}\times\bar{b})\times\bar{c}$$

$\bar{r}$ is $\perp^r$ $\bar{a}\times\bar{b}$ $\Rightarrow$ $\bar{r}$ is coplanar with ($\bar{a}$ & $\bar{b}$)
& $\bar{r}$ is $\perp^r$ to $\bar{c}$

[r is coplanar with $\bar{a}$ & $\bar{b}$
then r is orthogonal ($\perp$) with $\bar{c}$]

* ~~Proper~~ Note!

(1) unit vector coplanar with $\bar{a}$ and $\bar{b}$ and orthogonal to $\bar{c}$

$$\pm\frac{(\bar{a}\times\bar{b})\times\bar{c}}{|(\bar{a}\times\bar{b})\times\bar{c}|}$$

(2) $[\bar{a}\ \bar{b}\ \bar{c}]^2 = [\bar{a}\times\bar{b}\ \ \bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}]$

$$= \begin{vmatrix} \bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c} \\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c} \\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c} \end{vmatrix}$$

$\left((a \cdot c) b - (a \cdot b) c \right) \left(\frac{(a \cdot b)}{(a \cdot c)} \right.$ — used VDT

VPT

(3)

$$a \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c}) \bar{b} - (\bar{a} \cdot \bar{b}) \bar{c}$$

Note: $\boxed{\bar{a} \times (\bar{a} \times \bar{b}) = (\bar{a} \cdot \bar{b}) \bar{a} - (\bar{a} \cdot \bar{a}) \bar{b}}$

Que: find unit vector which is orthogonal to

$3i + 2j + 6k$

& coplanar with $2i + j + k$ & $i - j + k$.

$$\frac{(\bar{a} \times \bar{b}) \times \bar{c}}{|(\bar{a} \times \bar{b}) \times \bar{c}|}$$

$$\begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$i(2) - j + k(-3)$

$2i - j - 3k$

$2i - j - 3k \times c$

$$\begin{vmatrix} i & j & k \\ 2 & -1 & -3 \\ 3 & 2 & 6 \end{vmatrix}$$

$= i(-6$

$0 - j(21) + k(7)$

$= -21j + 7k.$

$$\frac{2i - j - 3k \times -21j + 7k}{\sqrt{4+1+9} \; \sqrt{4+49}}$$

$$= \pm \frac{7(-3j + k)}{7\sqrt{10}}$$

$$\begin{vmatrix} i & j & k \\ 2 & -1 & -3 \\ 12 & -2 & 7 \end{vmatrix}$$

Q. Pr. that

$$\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$$

$: (\bar{a} \cdot \bar{c}) \bar{b} - (\bar{a} \cdot \bar{b}) \bar{c} + (\bar{b} \cdot \bar{a}) \bar{c} - (\bar{b} \cdot \bar{c}) \bar{a} + (\bar{c} \cdot \bar{b}) \bar{a} - (\bar{c} \cdot \bar{a}) \bar{b} =$

$= 0$

$V_1 + V_2 + V_3 = 0$

Q. if $a = i + j + k$

$\bar{a} \cdot \bar{b} = 1$

$\bar{a} \times \bar{b} = j - k$

find $\bar{b} = ?$

$\bar{a} \cdot \bar{b} = 1$

$\bar{a} \times (\bar{a} \times \bar{b}) = (\bar{a} \cdot \bar{b}) \bar{a} - (\bar{a} \cdot \bar{a}) \bar{b}$

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \bar{a} - |\bar{a}|^2 \bar{b}$$

$-2i + j + k = i + j + k - 3\bar{b}$

$\bar{b} = i$

$\bar{b} = xi + yj + zk$.

$a \times b = j - k$.

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$$

$= (z-y) i - (z-x) j + (y-x) k$

$(z-y) i - (z-x) j + (y-x) k$.

$x - y = 0 \quad x = y$.

$z - x = 1$

$y - x = -1$

$z = 1 + x$.

Q. if $\bar{v}_1 = \bar{a} \times (\bar{b} \times \bar{c})$

$v_2 = \bar{b} \times (\bar{c} \times \bar{a})$

$v_3 = \bar{c} \times (\bar{a} \times \bar{b})$

$=$ | $a \cdot a$ $\quad a \cdot 1$

then chose correct option

(i) v_1, v_2, v_3 are coplanar

(ii) v_1, v_2, v_3 form side of a Δ

(iii) $\bar{v}_1 + v_2 + v_3 =$ Null vector.

(iv) $\bar{v}_1, \bar{v}_2, v_3$ are linearly dependent

$\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$

All option are correct.]

Q. If $[\bar{a}\ \bar{b}\ \bar{c}] = 3$

then find $[\bar{a} \times (\bar{b} + \bar{c}) \quad \bar{b} \times (\bar{c} - 2\bar{a}) \quad \bar{c} \times (\bar{a} + 3\bar{b})]$

$[(\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c} \quad (\bar{b}\cdot 2\bar{a})\bar{c} - (\bar{b}\cdot\bar{c})2\bar{a} \quad \cancel{(\bar{c}\cdot 3\bar{b})\bar{a} - (\bar{c}\cdot\bar{a})3\bar{b}}]$

$$\begin{vmatrix} 0 & \bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{b} \\ (\bar{b}\cdot\bar{c})2 & 0 & (\bar{b}\cdot 2\bar{a}) \\ (\bar{c}\cdot 3\bar{b}) & 3(\bar{c}\cdot\bar{a}) & 0 \end{vmatrix} [$$

$[\bar{a}\times\bar{b} + \bar{a}\times\bar{c} \quad \bar{b}\times\bar{c} - 2\bar{b}\times\bar{a} \quad \bar{c}\times\bar{a} + 3\bar{c}\times\bar{b}]$

$\bar{a}\times\bar{b} = l$

$\bar{b}\times\bar{c} = m$

$\bar{c}\times\bar{a} = n$

$[l - n \quad m + 2l \quad n - 3m]$

$$= \begin{vmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & -3 & 1 \end{vmatrix} [l\ m\ n] = 5[\bar{a}\ \bar{b}\ \bar{c}]^2$$

$= 5 \cdot 9 = 45$

$1(1-0) - 1(-4+0)$

J.A :- 6(b), 7(b), 8(b), 11, 12, 13, 14,

E-1 : 18, 19, 21 N.B., 23, 26, 27,

Q. $\bar{a}, \bar{b}, \bar{c}$ be three non parall unit vectors

$$\bar{a} \times (\bar{b} \times \bar{c}) = \frac{1}{2}\bar{b}$$

then find ~~which~~ angle which

$\bar{a}$ makes with $\bar{b}$ & $\bar{c}$

$$(\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c} = \frac{1}{2}\bar{b}$$

$$1\bar{b} - \bar{c} = \frac{1}{2}$$

$a\cdot c = \frac{1}{2}\times 1$

$= b$ $\quad a\cdot c = \frac{1}{2}$.

$$\bar{a} \wedge \bar{c} = 60^\circ$$

$$\hat{a} \wedge \bar{b} = 90^\circ$$

$a\cdot b = 0$

$\bar{a} \wedge \bar{b} = 90^\circ$

* Scalar and Vector Product of four vectors:

Scalar Product: if $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are four vectors then their scalar Product is defined as

$$(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})$$

* Vector Product: $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})$

scalar $\underbrace{(\bar{a}\times\bar{b})}_{u}\cdot(\bar{c}\times\bar{d})$ — Remember

$$u\cdot(\bar{c}\times\bar{d})$$

$$(\bar{u}\times\bar{c})\cdot\bar{d}$$

$$= ((\bar{a}\times\bar{b})\times\bar{c})\cdot\bar{d}$$

$$= ((\bar{a}\cdot\bar{c})\bar{b} - (\bar{c}\cdot\bar{b})\bar{a})\cdot\bar{d}$$

$= (\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{d}) - (\bar{b}\cdot\bar{c})(\bar{a}\cdot\bar{d})$

$$= \begin{vmatrix} a\cdot\bar{c} & b\cdot\bar{c} \\ a\cdot d & b\cdot d \end{vmatrix}$$ Remember

Vector:

$\underbrace{(\bar{a}\times\bar{b})}_{u}\times(\bar{c}\times\bar{d})$

$u \times (\bar{c}\times\bar{d})$

$(u\cdot\bar{c})\times\bar{d}$

$\cancel{((\bar{a}\times\bar{b})\cdot\bar{c})\times d}$

$(u\cdot c)\bar{c} - (u\cdot\bar{c})d$

$= ((\bar{a}\times\bar{b})\cdot\bar{d})\bar{c} - ((\bar{a}\times\bar{b})\cdot\bar{c})d$

$= [\bar{a}\,\bar{b}\,\bar{d}]\bar{c} - (\bar{a}\,\bar{b}\,\bar{c}]d$

* Geometrical significance:

$\bar{P} = (\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})$

$\bar{P} \perp$ ~~to~~ $(\bar{a}\times\bar{b})$ & $\bar{P}\perp(\bar{c}\times\bar{d})$

P is coplanar with a & $\bar{b}$) &

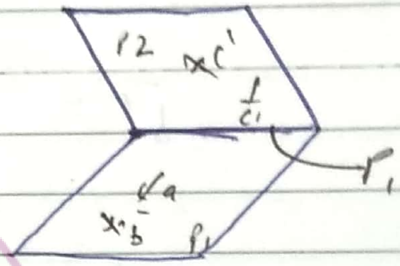
($\bar{P}$ is coplanar with $(\bar{c}$ & $\bar{d})$

If $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=0$ 	P_1 contains $\bar{a}$ & $\bar{b}$
P_2 contains $\bar{c}$ & $\bar{d}$

$\Rightarrow P_1$ & P_2 are Parallel
$\Rightarrow \bar{a},\bar{b},\bar{c},\bar{d}$ are coplanar

If $(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})=0$

$\Rightarrow P_1$ & P_2 are $\perp$ cular.



J.A
Q5(a)

$(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})=1$ 	$(\bar{a}\cdot\bar{c})=\frac{1}{2}$

$\bar{u}\cdot(\bar{c}\times\bar{d})=1$

$(\bar{u}\times\bar{c})\cdot\bar{d}=1$

$((\bar{a}\times\bar{b})\times\bar{c})\times\bar{d}=$

$((\bar{a}\cdot\bar{c})\bar{b}-(\bar{c}\cdot\bar{b})\bar{a})\cdot\bar{d}$

$\frac{1}{2}\bar{b}-c$

$(|\bar{a}||\bar{b}|\sin\theta)(\bar{n}_1\cdot\bar{n}_2)\quad(|\bar{c}||\bar{d}|\sin\phi)=1$

$\sin\theta\,\sin\phi\cdot(|\hat{n}_1||\hat{n}_2|\cos\psi=1$

$\theta=90^\circ$
$\phi=90^\circ$
$\psi=0$

JA = 2(6)

* Resiprocal system of vector:-

if $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors then their Resiprocal system $\bar{a}', \bar{b}', \bar{c}'$

Where $\bar{a}' = \dfrac{\bar{b} \times \bar{c}}{[\bar{a}\bar{b}\bar{c}]}$

$$\bar{b}' = \frac{\bar{c} \times \bar{a}}{[\bar{a}\bar{b}\bar{c}]}$$

$$c' = \frac{\bar{a} \times \bar{b}}{[\bar{a}\bar{b}\bar{c}]}$$

$$* \; [a' \; b' \; c'] = \frac{\bar{b} \times \bar{c}}{[\bar{a}\bar{b}\bar{c}]} \quad \frac{\bar{c} \times \bar{a}}{[\bar{a}\bar{b}\bar{c}]} \quad \frac{\bar{a} \times \bar{b}}{[\bar{a}\bar{b}\bar{c}]}$$

$$= \frac{1}{[\bar{a}\bar{b}\bar{c}]^3} [\bar{b} \times \bar{c} \quad \bar{c} \times \bar{a} \quad \bar{a} \times \bar{b}]$$

$$= \frac{1}{[\bar{a}\bar{b}\bar{c}]} [a \; b \; c]^2$$

$$= \frac{1}{[\bar{a}\bar{b}\bar{c}]}$$

$$* \; \bar{a} \cdot \bar{a}' = \bar{a} \cdot \frac{\bar{b} \times \bar{c}}{[\bar{a}\bar{b}\bar{c}]} = \frac{\bar{a}\bar{b}\bar{c}}{[\bar{a}\bar{b}\bar{c}]} = 1$$

$$\bar{b} \cdot \bar{b}' = 1 = \bar{c} \cdot \bar{c}'$$

* $\bar{a}' \cdot \bar{b} = 0 = \bar{a}' \cdot \bar{c} = \bar{b}' \cdot \bar{c} = \bar{b}' \cdot \bar{a}$

$$a' \cdot b = \frac{\bar{b} \times \bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]} \cdot b$$

Note:

i) $\bar{a} \times \bar{a}' + \bar{b} \times \bar{b}' + \bar{c} \times \bar{c}' = 0$

(ii) $(\bar{a} + \bar{b} + \bar{c})(\bar{a}' + \bar{b}' + \bar{c}') = 3$

(iii) $[\bar{a}'\ \bar{b}'\ \bar{c}'] = \dfrac{1}{[\bar{a}\ \bar{b}\ \bar{c}]}$

(iv) $a' \times b' + b' \times c' + c' \times a' = \dfrac{\bar{a} + \bar{b} + \bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]}$

Que:

if $\bar{x} \cdot \bar{a} = c$

$\bar{a} \times \bar{x} = b$

where c is non-zero scalar / constant and $\bar{a}$ and $\bar{b}$ are non zero given vectors. Then Unknown $\bar{x}$

$\cancel{\bar{a} \times (\bar{b} \times \bar{x})} = \cancel{(\bar{a} \cdot \bar{x})\bar{b}}$

$\bar{a} \times (\bar{a} \times \bar{x}) = \bar{a} \times b$

$(a \cdot x)\bar{a} - (\bar{a} \cdot \bar{a})\bar{x} = \bar{a} \times b$

$c\bar{a} - |a|^2 \bar{x} = a \times c$

$$\bar{x} = \frac{c\bar{a} - a \times b}{|\bar{a}|^2}$$

* Ques. solve following system of eq^n for x & y

$\bar{x}+\bar{y}=\bar{a}$

$\bar{x}\times\bar{y}=\bar{b}$ $\qquad$ $\bar{y}=\bar{a}-\bar{x}$

$\bar{x}\cdot\bar{a}=1$

$\bar{x}\cdot\bar{y}=1$

$\bar{x}+\bar{y}=0$

$\bar{x}=1+\bar{a}$

$(1+\bar{a})\cdot\bar{a}=1$

$x\times(\bar{a}-\bar{x})=\bar{b}$

$\bar{x}\times\bar{a}-\bar{x}\times\bar{x}=\bar{b}$

$\bar{x}\times\bar{a}=\bar{b}$ $\qquad$ $\bar{x}\cdot\bar{a}=1$

$\bar{a}\times(\bar{x}\times\bar{a})=\bar{a}\times\bar{b}$

$(\bar{a}\cdot\bar{a})\bar{x}-(\bar{a}\cdot\bar{x})\bar{a}=\bar{a}\times\bar{b}$

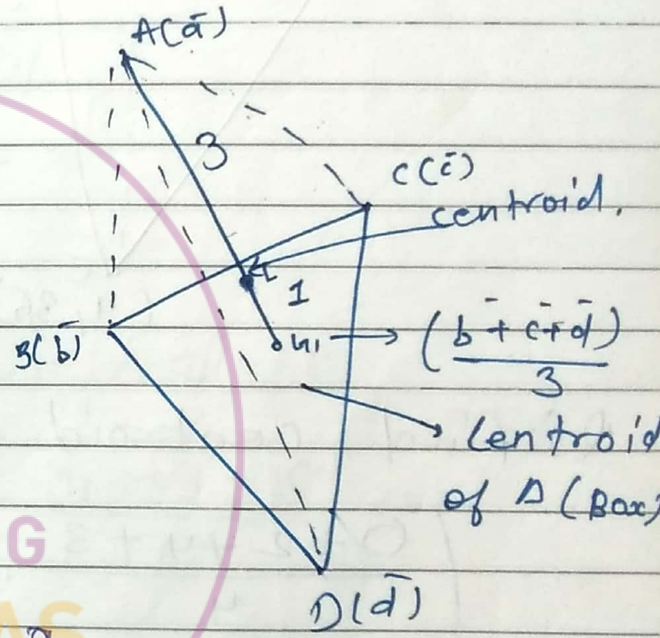
$|\bar{a}|^2\,\bar{x}=\bar{a}+\bar{a}\times\bar{b}$

$\therefore\ \bar{x}=\dfrac{\bar{a}+\bar{a}\times\bar{b}}{|\bar{a}|^2}$ $\qquad$ $\bar{y}=\bar{a}-\bar{x}$

* Tetrahedral :
four faces, six edges, four vertices, ab & cd, ac & bd, ad & bc, are opposite edges

*** Line joining mid point of edges are concurrent & called centroid of tetrahedra

$$G = \frac{\bar{a}+\bar{b}+\bar{c}+\bar{d}}{n}$$
(centroid)

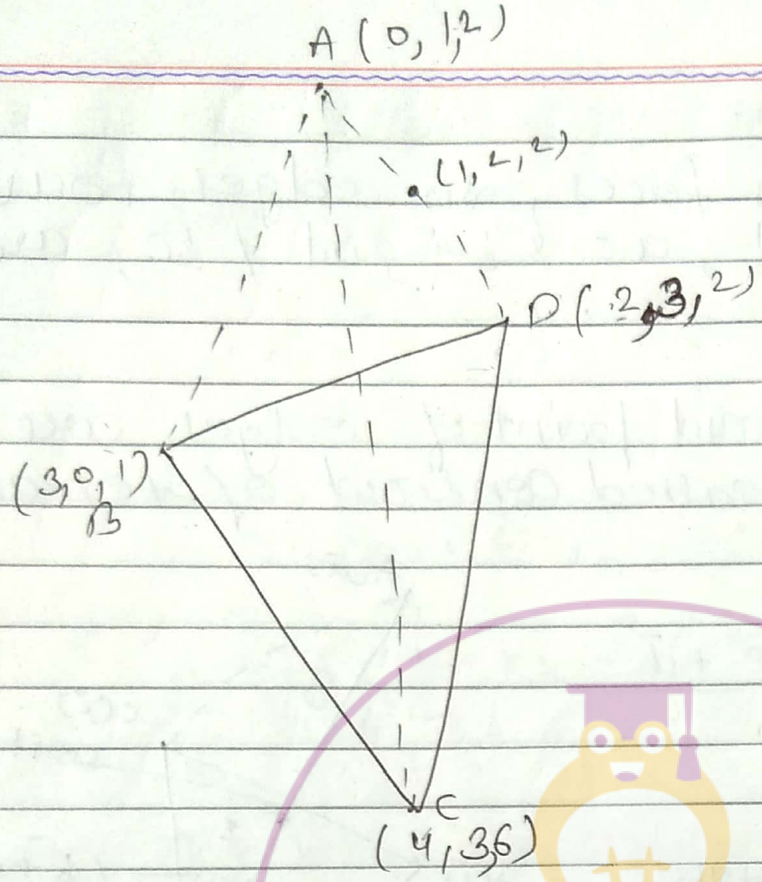


☆ G से A तक की लम्बाई में 3 : 1 मे बटी हुई Line वाला Point ही Tetrahedral का centroid होगा

$$V = \frac{1}{6}\{\vec{AB}\ \ \vec{AC}\ \ \vec{AD}\}$$
or
$$V = \frac{1}{3} A H$$

* Angle b/w two opposite sides = Angle b/w their direction.

* Angle b/w two planes = Angle b/w their Normals



Q(i) find centroid of T ?

$$\left(\frac{0+2+4+3}{4}, \frac{1+0+3+3}{4}, \frac{2+1+6+2}{4} \right)$$

$$\left(\frac{9}{4}, \frac{7}{4}, \frac{11}{4} \right)$$

Q(ii) find Area of ΔBCD

$$= \frac{1}{2} \left| BC \times BD \right|$$

$$= \frac{1}{2} \left| (\hat{i} + 3\hat{j} + 5\hat{k}) \times (-\hat{i} + 3\hat{j} + \hat{k}) \right|$$

$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 5 \\ -1 & 3 & 1 \end{vmatrix} = \frac{1}{2} \left| -12\hat{i} - 6\hat{j} + 6\hat{k} \right|$$

$= 3\,(2i + j - k)$

$= 3\sqrt{4+1+1} = 3\sqrt{6}\,A$

Q. find unit vector Normal to the Plance BCD.

$$\hat{n}_1 = \pm \frac{\vec{BC} \times \overline{BD}}{|\overline{BC} \times \overline{BD}|}$$

$$n_1 = \pm \frac{2i + j - k}{\sqrt{6}}$$

(iv) find volume of Tetrahedral.

$$V = \frac{1}{3}(j + 2k)\left(\pm \frac{2i + j - k}{\sqrt{6}}\right)$$

$$V = \frac{1}{3}\left|(3i - j - k)\,(4i + 2j + 4k)\,(2i + 2j)\right|$$

$$\frac{1}{3}\begin{vmatrix} 3 & -1 & -1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix}$$

$= -6 + 24$

$= 6$

(v) find length of Attitude draw from vertex A to BCD.

$V = \frac{1}{3} A H = \frac{1}{3}\, 3\sqrt{6}\, H = \sqrt{6}$

then $H = \sqrt{6}$

Q. find Agle b/w opp. edges AD & BD.

$\bar{AD} = 2\hat{i} + 2\hat{j}$ $\quad \bar{BC} = \hat{i} + 3\hat{j} + 5\hat{k}$

$$\cos\theta = \frac{\bar{BC} \cdot \bar{AD}}{|\bar{BC}|\,|\bar{AD}|}$$

$$= \frac{2+6}{\sqrt{8}\sqrt{35}}$$

Q. find Acute ~~Ag~~ Angle b/w Plane ABC & BCD

AD (1, 2)

$$= \vec{AB} \times \bar{AC} = (3\hat{i} - \hat{j} - k) \times (4\hat{i} + 2\hat{j} + 4k)$$

$$= \begin{vmatrix} i & j & k \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix}$$

$$\hat{n}_2 = -2\hat{i} - 16\hat{j} + 10k$$

$$\hat{n}_2 = -2(\hat{i} + 8\hat{j} - 5k)$$

$$\cos\theta = \frac{\hat{n}_1 \cdot \hat{n}_2}{|\hat{n}_1|\,|\hat{n}_2|}$$

Q. find Shortest dist. b/w Skew lines. BC & AD

$\bar{BC} = \hat{i} + 3\hat{j} + 5K \quad \bar{AD} = 2\hat{i} + 2\hat{j}$

~~Line~~

Line AD is $\bar{r} = \underbrace{j + 2k}_{\bar{a}} \times \lambda \underbrace{(i + j)}_{\bar{b}}$

Line BC

$$\bar{r} = \underbrace{(3i + k)}_{\bar{p}} + \mu \underbrace{(i + 3j + 5k)}_{\bar{q}}$$

$$SD = \left| \frac{(\bar{a} - \bar{b})(\bar{p} \times \bar{q})}{|\bar{p} \times \bar{q}|} \right|$$

Q. find posit vector of foot N to ~~from A to~~ the plane BCD eq^n of altitude of AN.

$$\bar{r} = j + 2k \cancel{+} \lambda (2i + j - k)$$

co-ordinate of N

$$(2\lambda, 1 + \lambda, 2 - \lambda)$$

$$\bar{BN} = (2\lambda - 3)i + (1 + \lambda)j + (2 - \lambda - 1)k$$

$$(\bar{BN} \ \bar{BC} \ \bar{BD}) = 0$$

* If $\bar{A}$ & $\bar{B}$ are two non co-linear vectors then:

(i) $\bar{r} = \bar{a}x + \bar{b}y$ (Any vector in Plane)

(ii) Any vector in space is given by

$$\bar{r} = \bar{a}x + \bar{b}y + (\bar{a} + \bar{b})z$$

(iii) If a, b & c are non-coplanar vectors then:

Any vector is given by

(a) $\bar{r} = x\bar{a} + y\bar{b} + z\bar{c}$

(b) $\bar{o} = x(\bar{b} \times \bar{c}) + y(\bar{c} \times \bar{a}) + z(\bar{a} \times \bar{b})$