Handwritten Notes

On

Simple Harmonic Motion

(SHM)

'SIMPLE HARMONIC MOTION [S·H·M]'

|1| → **Aperiodic motion** → A particle not Repet path in a same time interval.
* also called Non-periodic motion.

Eg → * A Bus or vehicle move in st. line
* Motion of gas molecule.

|2| → **Periodic Motion** → If particle Repet its path in a same time interval.

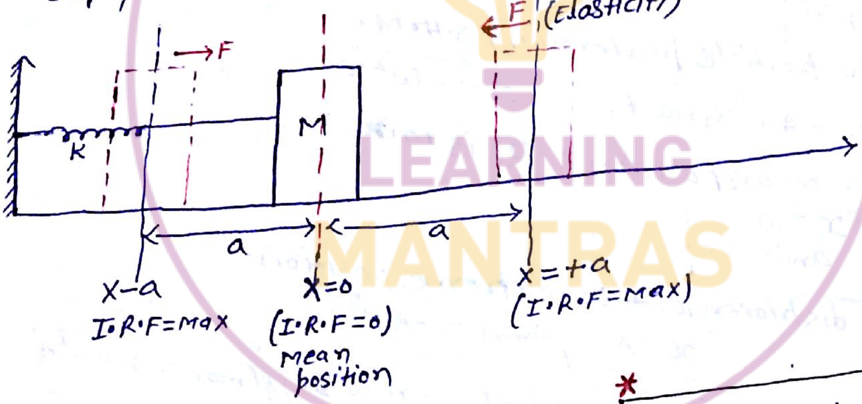
EX → * Motion of pendulum
* Motion around the nucleus
* Motion of planet around the sun.
* Motion of Earth w.r.t self axis.
* Motion of swift.
* Motion of liq in 'U' tube.
* Motion of Wooden plank at surface of liq.

|3| → **Osscilatory/Vibrational Motion** → If particle repet its path w.r.t same point & complete throw & flow Motion in same time interval.

Eg → * Motion of pendulum
* Motion of swimmer
* Motion of liq in 'U' or 'V' tube.
* Motion of Medium particle when wave propagate in a medium.

AIIMS
NOTE → All oscillating motion is periodic but all periodic motion is not oscilatory

|4| → **simple harmonic motion** → If a osscilatory Motion is directelly propersnal to the disp. & always towards mean position.

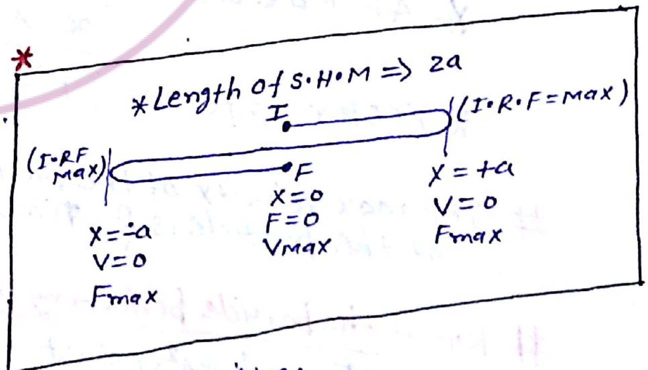


$\boxed{F \propto -X}$

* $\boxed{F = -KX}$

Disp. from mean position.
R.F coffizient
offosite direction of force.

I.R.F
⇓
Elasticity



Exam
* Disp. and distance in one osscilation Resp. zero & 4a.

Amplitude → Max disp. of medium particle from mean position.

NOTE → * Elasticity Inertia is necessory for oscillatory motion.
* S.H.M occur in stable eqm.
* General vibration of polyatomic gas molecule about its eqm position → S.H.M

Condition for S.H.M

$\boxed{F = -Kx}$ $\boxed{A = -\omega^2 x}$ $\boxed{\frac{K}{m} = \omega^2}$ (ω = angular freq.)

$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$ $\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$ (Differential eqn of S.H.M)

Velocity of the particle performing S.H.M →

$\boxed{V = A\omega\sqrt{\frac{A^2 - x^2}{A^2}}}$ $\boxed{V = \omega\sqrt{A^2 - x^2}}$ $\boxed{V = \frac{2\pi}{T}\sqrt{A^2 - x^2}}$

$\boxed{V = 2\pi n\sqrt{A^2 - x^2}}$

x = disp. of the particle from mean position.
T = Time period
A = Max disp. or, Amplitude.
ω = Angular freq.
n = Linear freq.

Imp
* Velocity of the particle at extreme position or at max disp. is equal to zero or min.

$\boxed{x = A = 90° \quad V = \omega\sqrt{A^2 - x^2} \quad V_{min} = 0}$

* At mean position or, at an equilibrium position the velocity of the particle will be maximum.

$x = 0$
$V = \omega\sqrt{A^2 - 0^2}$

$\boxed{V_{max} = A\omega = A(2\pi n) = A \times \left(\frac{2\pi}{T}\right)}$

* Averge velocity or, mean velocity

$\boxed{\vec{V}_{avg} = 0}$

* Accln of the particle performing S.H.M.

$\boxed{a = -A\omega^2 \sin\omega t}$ $\boxed{a = -\omega^2 x}$

* At eqm or, mean position Accln is min.

$\boxed{x = 0 \quad a_{min} = 0}$

* At max displacement or, extreme position.

$\boxed{x = A}$ $\boxed{a_{max} = -A\omega^2}$

* Restoring force $\boxed{F = ma}$, $\boxed{F = -m\omega^2 x}$, $\boxed{F_{max} = -m\omega^2 A}$

The max velocity of the particle performing S.H.M is 'α' & max accln of the particle is β. Time period of oscillation → $\boxed{T = 2\pi\left(\frac{\alpha}{\beta}\right)}$

K.E of the particle performing S.H.M →

$\boxed{E_K = \frac{1}{2}KA^2\cos^2\omega t}$

$\boxed{E_K = E_{Kmax} \cdot \cos^2\omega t}$

$\boxed{K.E_{max} = \frac{1}{2}KA^2}$

Relation b/w K.E & disp →

$\boxed{E_K = \frac{1}{2}K(A^2 - x^2)}$

* K.E of the particle is max at mean position or, equilibrium position → $\boxed{E_{Kmax} = \frac{1}{2}KA^2}$
* At max disp or, at extreme position

$\boxed{x = A}$ $\boxed{E_{Kmin} = 0}$

Avg. K.E or, Mean K.E →

$$\langle K.E \rangle = \frac{E_{K\max} + E_{K\min}}{2}$$

$$\langle K.E \rangle = \frac{\frac{1}{2}KA^2 + 0}{2}$$

$$\langle K.E \rangle = \frac{1}{4}KA^2$$

$$\langle K.E \rangle = \frac{1}{2} \times \underbrace{\frac{1}{2}KA^2}$$

$$\langle K.E \rangle = \frac{1}{2}E_{K\max}$$

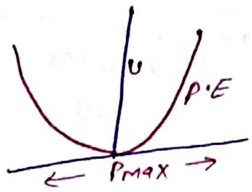
$$\langle K.E \rangle = \frac{K.E_{\max} + K.E_{\min}}{2}$$

$$\langle K.E \rangle = \frac{1}{}KA^2$$

$$\boxed{\langle K.E \rangle = \frac{1}{2}K.E_{\max}}$$

P.E of Particle performing S.H.M →

$\boxed{U = \frac{1}{2}Kx^2}$ $\boxed{U = \frac{1}{2}Kx^2}$



Avg. P.E or, Mean K.E ⇒

$$\langle P.E \rangle = \frac{U_{K\max} + U_{K\min}}{2}$$

$$\langle P.E \rangle = \frac{1}{2}U_{K\max}$$

Relation b/w P.E & time →

$\boxed{U = \frac{1}{2}Kx^2}$

$\boxed{U = \frac{1}{2}KA^2 \sin^2 \omega t}$

Total energy of the particle performing S.H.M → Total E in S.H.M remain Conserved it is equal to sum of K.E & P.E

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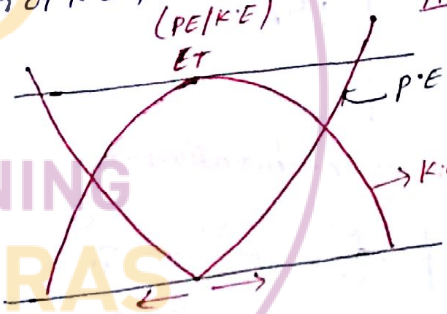
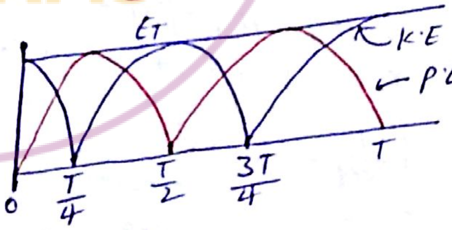
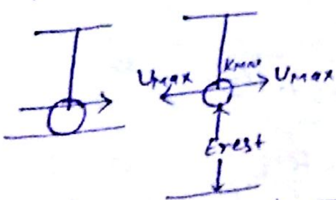
$$\boxed{\begin{array}{l} E_T = \frac{1}{2}KA^2 \\ E_T = \frac{1}{2}m\omega^2A^2 \\ E_T \propto m \\ E_T \propto \omega^2 \propto n^2 \propto \frac{1}{T^2} \propto A^2 \\ E_T \propto A^2 \end{array}}$$





** NOTE → If Energy at rest is given in question.



* $\boxed{E_T = E_{rest} + U_{\max}}$

* Fraction of P.E in T.E → $\frac{\frac{1}{2}Kx^2}{\frac{1}{2}KA^2}$

* A particle having mass 'm' moving with a velo. 'v' strike from spring & compress it. cal. compression in spring → $\boxed{x = v\sqrt{\frac{m}{k}}}$

* A block is placed at a spring. If block is sepreaked from the plateform which is performing S.H.M then the condn. → $\boxed{\begin{array}{l} \omega^2 A = g \\ \frac{K}{m} A = g \end{array}}$

$$\boxed{E_T = K_{max} = U_{max} = \frac{1}{2} K A^2}$$

Spring pendulum

Spring is used to store elastic P.E.

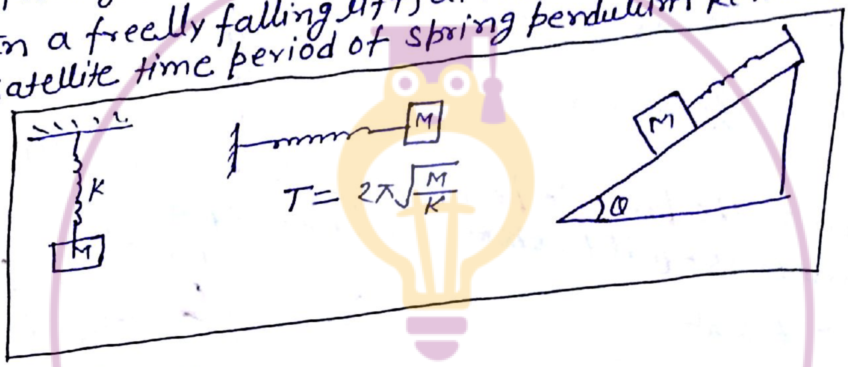
* When spring is compressed or extended then work is done & this work is stored in the form elastic P.E.
* The force const. of a spring is inversally propersnal to the natural length.
* If a block of mass 'M' is suspended from a spring & it is slightly pulled & released it will perform S.H.M in this case restoring force is ∝ to the disp.
* Time period of spring pendulum only depend on the mass of suspended body & on force const. & it is independent from gravitation accln. & extension in spring.
* In a freelly falling lift, at the centre of Earth & in artificial satellite time period of spring pendulum remain unchange.

*$K = M\omega^2$

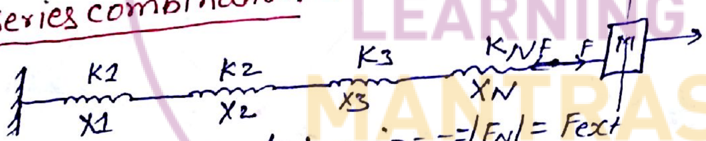
$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{x}}$

*$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{K}}$

$= 2\pi\sqrt{\frac{x}{g}}$



Series combination →



$|F_N| = F_{ext}$

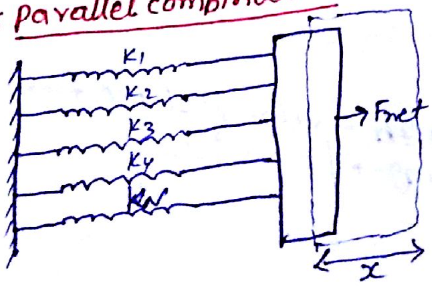
* $|F_1| = |F_2| = |F_3| = \text{-----} = |F_N| = F_{ext}$
* $X = x_1 + x_2 + \text{-----} x_N$

$$\boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} \text{----} \frac{1}{K_N}}$$ (spring const. of combination)

$$T = 2\pi\sqrt{\frac{M}{K_{eq}}}$$

$$\boxed{T = 2\pi\sqrt{M\left(\frac{1}{K_1} + \frac{1}{K_2} + \text{---} \frac{1}{K_N}\right)}}$$ Time period of combination.

Parallel combination



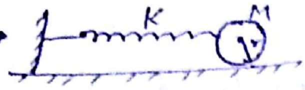
$$\boxed{K_{eq} = K_1 + K_2 + K_3 \text{---} K_N}$$

$$T = 2\pi\sqrt{\frac{M}{K_{eq}}}$$

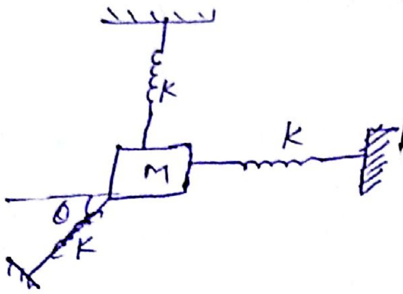
$$\boxed{T = 2\pi\sqrt{\frac{M}{K_1 + K_2 + \text{---} + K_N}}}$$

Standard Result for Spring :→

|a|→  $\boxed{T = 2\pi\sqrt{\frac{\mu}{k}}}$ $\boxed{\mu = \frac{M_1 M_2}{M_1 + M_2}}$ μ = Reduce Mass.

|b|→  $\boxed{T = 2\pi\sqrt{\frac{M}{K}\left(1 + \frac{K_0^2}{r^2}\right)}}$ (K = spring const, K' = Radius of gyration)

|c|→  $\boxed{T = 2\pi\sqrt{\frac{M + m/3}{K}}}$

|d|→  $\boxed{T = \sqrt{\frac{M}{K(1 - \cos^2\theta)}}}$

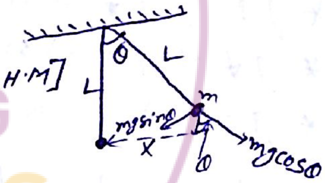
* Astronot use spring pendulum bcoz do not depend on G & L

Two different spring osscilate with same mass. It's time period Resp. T_1 & T_2 calculate time period of combination when—
|i|→ It will connect in series & osscilate ē same mean.
|ii|→ " " " " ||

|i|→ $\boxed{T_{series} = \sqrt{T_1^2 + T_2^2}}$

|ii|→ $\boxed{T_{Parallel} = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}} = \frac{T_1 T_2}{T_S} \Rightarrow T_P T_S = T_1 T_2}$

SIMPLE PENDULUM



$F = -mg\sin\theta$

* $\theta \le 5°$ (small) ⇒ SHM [$\theta > 5$ ⇒ osscilation not S.H.M]

$\boxed{F_R = -\left(\frac{mg}{L}\right)x}$

$\boxed{F = \frac{mg}{L} \Rightarrow \frac{m}{K} = \frac{L}{g}}$

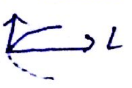
$\boxed{T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{L}{g}}}$

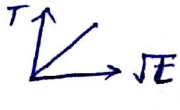
NOTE → Time period of simple pendulum depend on length of pendulum & Gravitation Accln it is independent from mass of ossicilating body.
L = Length of pendulum ×
* If point mass osscilate = length of spring.
* If physical object is suspended = distance b/w point of suspension & centre of mass of osscilating body.

Effecting Factor :→

|a|→ Effect of Length → $T = 2\pi\sqrt{\frac{L}{g}}$

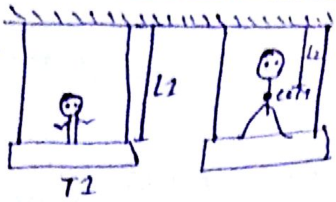
$g = \text{const} \Rightarrow T \propto \sqrt{L}$
* $L\uparrow \Rightarrow T\uparrow \Rightarrow n\downarrow$ (slow)
* $L\downarrow \Rightarrow T\downarrow \Rightarrow n\uparrow$ (Fast)

Graph T v/s L → 

* T v/s $\sqrt{L}$ 

Ex →

|i| →



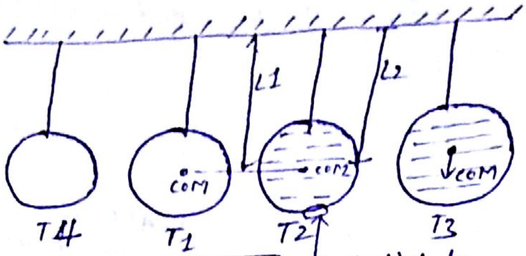
$L_1 > L_2$

$T_1 > T_2$

$\boxed{n_1 > n_2}$

** Exemplar

|ii| →



$\boxed{\begin{matrix} L_1 = L_2 \\ T_1 = T_2 \end{matrix}}$ If a drill/hole.

* L ~~Fast~~ First ↑ & then ↓
* T ~~First~~ First ↑ then ↓

$\boxed{T_3 > T_1 = T_2 = T_4}$

$\boxed{\begin{array}{l} * \text{change in 1 sec} = \frac{1}{2}\alpha\Delta T \\ * \text{change in 't' sec} = \left(\frac{1}{2}\alpha\Delta T\right) t\ \text{sec} \\ * \text{change per day} = \left(\frac{1}{2}\alpha\Delta T\right) 86400\ \frac{\text{sec}}{\text{day}} \\ * 1\ \text{yr} = \pi \times 10^7\ \text{sec} \end{array}}$

|b| → Effect of Gravity :→ $T = 2\pi\sqrt{\frac{L}{g}}$

* $L \Rightarrow$ same $\Rightarrow T = \frac{1}{\sqrt{g}}$
* $g\uparrow \Rightarrow T\downarrow \Rightarrow n\uparrow$ (Fast)
* $g\downarrow \Rightarrow T\uparrow \Rightarrow n\downarrow$ (slow)
* $g = 0 \Rightarrow T = \infty$ →
  - At the centre of Earth.
  - In a freely falling lift.
  - In a Artificial satellite.

Ex → |i| → * At height above 'h' from Earth surface.

✓ $\boxed{T_s = 2\pi\sqrt{\frac{L}{g_s}}}$

$T_h = 2\pi\sqrt{\frac{L}{g_s/(1+h/R)^2}}$

✓ $\boxed{T_h = T_s\,(1 + h/R)}$

✓ $\boxed{\%\ \text{change} = \frac{T_h - T_s}{T_s} = \frac{h}{R} \times 100\%}$

|ii| → * At depth 'd' from earth surface →

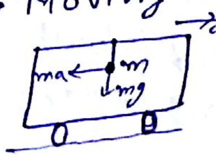
$\boxed{g_d = g_s\left(1 - \frac{d}{R}\right)}$

$\boxed{T_d = \frac{T_s}{\sqrt{1 - d/R}}}$

* $\boxed{d\uparrow \Rightarrow g_d \Rightarrow T_d\uparrow \Rightarrow n\downarrow}$

|iii| → * In a Moving ~~vehicle~~ vehicle in Horizontal plane.

|a| → Moving in a Horizontal plane.



$\boxed{g_{eff} = \sqrt{g^2 + a^2}}$

$\boxed{T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + a^2}}}}$

|b|→ Move in a circular path

$$\boxed{g_{eff} = \sqrt{g + \frac{V^4}{R^2}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \frac{V^2}{R^2}}}}}$$

|c|→ Move in a incline plane

$$\boxed{T = 2\pi\sqrt{\frac{L}{g\cos\theta}}}$$

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Ex→ In Moving Lift→

|a|→ Moving upward $\boxed{g_{eff} = g+a}$ $\boxed{T_{eff} = 2\pi\sqrt{\frac{L}{g+a}}}$

|b|→ Moving downward $\boxed{g_{eff} = g-a}$ $\boxed{T_{eff} = 2\pi\sqrt{\frac{L}{g-a}}}$

* If V↑ (Accⁿ) ⇒ a = ⊕ve
* If V↓ (Retard) ⇒ a = ⊖ve

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|c|→ Freely falling condⁿ

$a = g \Rightarrow g_{eff} = g - g = 0$

$\boxed{T_{eff} = \infty}$

|d|→ If chain is break → Weightlessness condⁿ $\boxed{a = g}$

|e|→ If lift move with const Accⁿ upward or, downward.

$\boxed{V = const}$ $\boxed{a = 0}$

$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$

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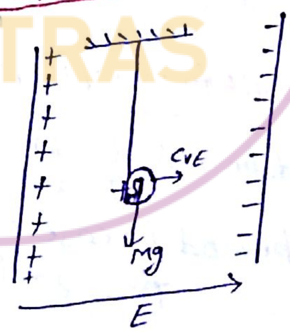
Ex→ osscilate in presence of Electric field→

|a|→ Horizontal electric field

$$\boxed{g_{eff} = m\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}$$

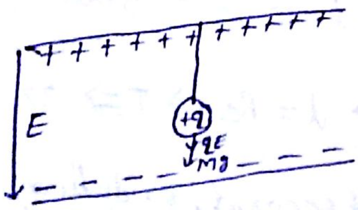
$$\boxed{T_{eff} = 2\pi\sqrt{\frac{L}{g^2 + \left(\frac{qE}{m}\right)^2}}}$$



|b|→ Downward eletric field

$$\boxed{g_{eff} = g + \frac{qE}{m}}$$

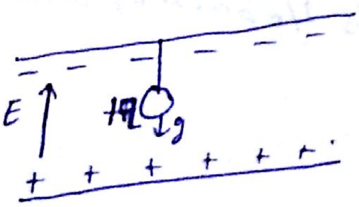
$$\boxed{T_{eff} = 2\pi\sqrt{\frac{L}{g + \frac{qE}{m}}}}$$



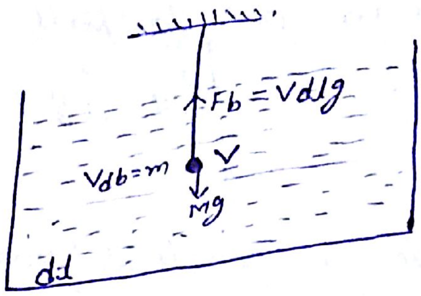
|c|→ upward Electric field.

$$\boxed{T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}}$$

$$\boxed{g_{eff} = g - \frac{qE}{m}}$$



Ex → In a non-viscous liq →



$$W_{eff} = mg\left(1 - \frac{d_l}{d_b}\right)$$

$$\boxed{g_{eff} = g(1 - d_l/d_b)}$$

$$T_{eff} = 2\pi\sqrt{\frac{L}{g(1 - d_l/d_b)}}$$

$$\boxed{T_L = \frac{T_{air}}{\sqrt{1 - \frac{d_l}{d_b}}}}$$

d_l → density of liq
d_b → density of body

* $\boxed{\text{Specific density} = \frac{d_{body}}{d_{H_2O}} = 6}$

$$\boxed{T_{H_2O} = \frac{T_{air}}{\sqrt{1 - \frac{1}{6}}}}$$

Two simple pendulum of time period T_1 & T_2 start vibrating at same phase then min time after which they are again in same phase $(T_1 > T_2)$ → $\boxed{t = \frac{T_1 T_2}{T_1 - T_2}}$

* If $(n+1)$ → No. of osscilation completed by smaller pendulum then, (n) → will be no. of oscillation completed by Larger pendulum. $\boxed{t = (n+1)T_2 = nT_1}$

Standard Result for simple pendulum :→

(a) → second pendulum ⇒ $L = 1m$ at surface of earth
$T = 2sec$ = const.

(b) → Time period in case of Long length pendulum.

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R_e}\right)}}$$

(i) → $l = \infty \Rightarrow T \Rightarrow \infty = 2\pi\sqrt{\frac{R_e}{g}}$ = 84.6 mit
= 1.414 hr
= $\sqrt{2}$ hr.

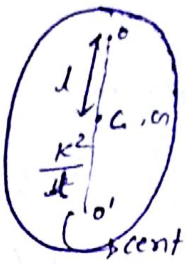
(ii) → $l = R_e \Rightarrow T \Rightarrow 2\pi\sqrt{\frac{R_e}{2g}} \Rightarrow \frac{84.6}{\sqrt{2}}$ mint $= 0.707 \times 84.6$ mint.

* Length of second pendulum at surface of Moon. Where gravitation Accln is 1/6 part of surface of Earth → $\boxed{L_m = \frac{L_e}{6} = 1/6\ m}$

Compound pendulum

l = distance of suspension from centre of gyration (C.G)

r = Radius of gyration about C.G



$$\boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l + \frac{k^2}{l}}{g}}}$$

If 'l' is Replaced by $\frac{k^2}{l}$

$\therefore$ then ($T \rightarrow$ same)

$\therefore$ T is same at distance $l + \frac{k^2}{l}$ from C.G

Linear S.H.M

(i) $\rightarrow$ $F = -kx$

(ii) $\rightarrow$ $k = m\omega^2$

$T = 2\pi\sqrt{\frac{m}{k}}$

(iii) $\rightarrow$ $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Angular S.H.M

(i) $\rightarrow$ $\tau = -C\theta$

(ii) $\rightarrow$ $C = I\omega^2$

$T = 2\pi\sqrt{\frac{I}{C}}$

(iii) $\rightarrow$ $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

For small angle disp.

$\sin\theta \approx \theta$

$$\boxed{\tau = -(mgl)\theta}$$

$$\boxed{\tau = -C\theta}$$

$$\boxed{C = mgl}$$

I = M.O.I

m = mass of body

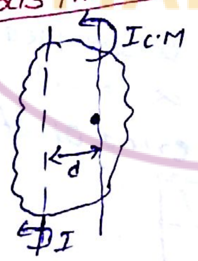
g = gravity

l = effective length.

Parallel axis theorum $\rightarrow$ $\boxed{I = I_{CM} + Md^2}$

$$\boxed{I_{CM} = MK'^2}$$

K' = Radius of gyration



(a) $\rightarrow$ Ring = $I_{C.M} = MR^2 = MK^2 = \boxed{K' = R}$

(b) $\rightarrow$ Disc = $I_{C.M} = \frac{MR^2}{2} = MK'^2 \Rightarrow \boxed{K' = \frac{R}{\sqrt{2}}}$

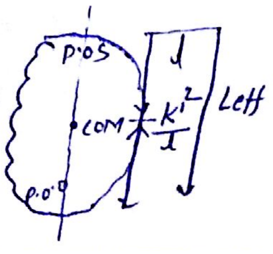
(c) $\rightarrow$ Solid sphere $I_{C.M} = \frac{2}{5}MR^2 = MK'^2 \Rightarrow \boxed{K' = \sqrt{\frac{2}{5}}R}$

(d) $\rightarrow$ Hollow Sphere $I_{C.M} = \frac{2}{3}MR^2 - MK'^2 = \boxed{K' = \sqrt{\frac{2}{3}}R}$

(e) $\rightarrow$ Rod = $I_{C.M} = \frac{ML^2}{12} = MK'^2 \Rightarrow \boxed{K' = \frac{L}{2\sqrt{3}}}$

$$\boxed{T = 2\pi\sqrt{\frac{L_{eff}}{g}}} \qquad \boxed{L_{eff} = \frac{K'^2}{l} + l}$$

$T_{P.O.O} = T_{O.S}$



K'^2 = Radius of gyration.

Disc of Mass 'm' Radius 'R' suspended in a verticle w.r.t horizontal axis. calculate time period of its osscilation. when it is suspended From →

|i| → circumference.

$* \; K' = \frac{R}{\sqrt{2}} \Rightarrow \frac{K'^2}{l} = \frac{R^2/2}{R} = R/2$

$* \; L_{eff} = \frac{K'^2}{l} + l = \frac{3R}{2}$

$* \; T_{eff} = 2\pi\sqrt{\frac{L_{eff}}{g}} = 2\pi\sqrt{\frac{3R}{2g}}$

|ii| → At distance R/2 from COM

$* \; K' = \frac{R}{\sqrt{2}} = \frac{K'^2}{l} = \frac{(R/\sqrt{2})^2}{R/2} = R$

$* \; L_{eff} = \frac{K'^2}{l} + l = \frac{3R}{2}$

$* \; T = 2\pi\sqrt{\frac{3R}{2g}}.$

|iii| → At distance 3R/2 from circumference.

$l^2 - \left(\frac{T^2 g}{4\pi^2}\right) l + K'^2 = 0 \qquad \frac{T^2 g}{4\pi^2} = \frac{K'^2 + l^2}{l}$

* crossponding to same time period 4-points is possible.

|iv| → w.r.t COM

$l = 0 \qquad T = 2\pi\sqrt{\frac{L}{g}} = \infty$

$\frac{K'^2}{l} = \infty \qquad L_{eff} = \frac{K'^2}{l} + l = \infty$

NOTE → * If compound pendulum suspended from its C.O.M time period of pendulum is ∞ (max)

* When $K' = l$, time period of of pendulum is min.

$$\boxed{T = 2\pi\sqrt{\frac{K'^2 + l}{g}}}$$

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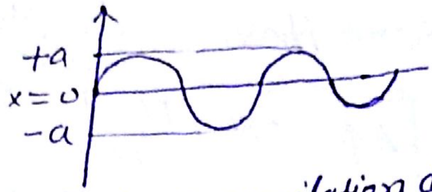
Types of osscilation

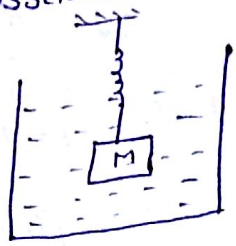
|a| → Free osscilation → If amplitude, energy, time period of osscilation remain same.

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$



BCECE 2015

|b| → Damped osscilation → If Amplitude of osscilation decrese with time.



$F_R = -K_1 x$

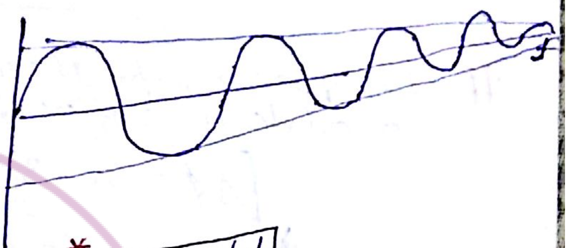
$$\boxed{F_v = -6\pi\eta r v = -K_2 v}$$

viscous force

terminal velo.

$F_{net} = -K_1 x - K_2 v$

$$\boxed{\frac{d^2x}{dt^2} + \left(\frac{K_2}{M}\right)\frac{dx}{dt} + \left(\frac{K_1}{M}\right)x = 0}$$

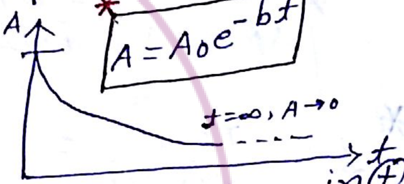


A_0 = Amplitude at $t=0$

b = damping coffcient

t = time

$$\boxed{A = A_0 e^{-bt}}$$



NOTE → In damping osscilation Amplitude ↓ expontially w.r.t time.

|c| → Forced osscilation → If free osscilation perform in ⊕nce of external periodic force.

$$\boxed{\frac{d^2x}{dt^2} + \left(\frac{K_2}{M}\right)\frac{dx}{dt} + \left(\frac{K_1}{M}\right)x = \frac{F_{ex}}{M}}$$

	ωt	$t\ (2\pi = T)$ AIPMT
# * $x=0$ to $x = A/2 = \frac{1}{2}$ or $30°$	$\pi/6 = 30°$	$T/12$
* $x=0$ to $x = \frac{A}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}\right)$ or, $45°$	$\pi/4$	$T/8$
* $x=0$ to $x = \sqrt{3}/2 = 60°$	$\pi/3$	$T/6$
* $x=0$ to $x = A = 90°$	$\pi/2$	$T/4$

A particle excicute S.H.M its velocity at position x_1 & x_2 Resp. v_1 & v_2 then Amplitude & time period →

$$\boxed{a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}}$$

AIPMT 2015

$$\boxed{T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}}$$

particle excicute S.H.M with amplitude 'a' calculate position of particle where,

|i| → $V = \frac{V_{max}}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}}a$

|ii| → $V = \frac{\sqrt{3}}{2}V_{max} \Rightarrow x = \pm\frac{a}{2}$

Two particle exicute S.H.M With time period resp. T_1 & T_2 ($T_1 > T_2$). If Initially it will start With same phase calculate time after Wich again its come in a same phase.

$$\boxed{\text{Same phase} = \text{even Multiple of } \pi}$$

$$\boxed{\Delta\phi = 2\pi\left(\frac{T_1 - T_2}{T_1 T_2}\right)t} = 0, 2\pi, 4\pi$$

$$\Delta\phi = 0, 2\pi, 4\pi, 6\pi \text{ ---- } 2n\pi$$

$$\boxed{t = 0,\ \frac{T_1 T_2}{T_1 - T_2},\ \frac{2T_1 T_2}{T_1 - T_2} \text{ ---- } \frac{nT_1 T_2}{T_1 - T_2}}$$

$$\boxed{T_{min} = \frac{T_1 T_2}{T_1 - T_2}}$$

In upper concept If initially particle ~~ent~~ in same phase in opposite phase. calculate time after Wich it come in opposite phase.

$$\boxed{\Delta\phi = \pi, 3\pi, 5\pi \text{ ----- } (2n-1)}$$

$n = 1, 2, 3$ ---

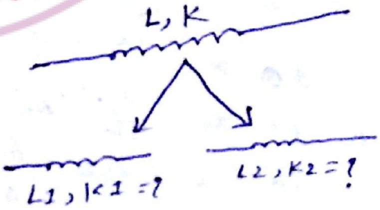
$$* \boxed{\Delta\phi = 2\pi\left(\frac{T_1 - T_2}{T_1 T_2}\right)t} = \pi, 3\pi, 5\pi \text{ --- } (2n-1)\pi$$

$$* \boxed{t = \frac{T_1 T_2}{2(T_1 - T_2)} = \frac{3T_1 T_2}{2(T_1 - T_2)} = \frac{5T_1 T_2}{2(T_1 - T_2)} \text{ --- } \frac{(2n-1)T_1 T_2}{2(T_1 - T_2)}}$$

NOTE → * Time or position is not define Avg of K.E & P.E is $\frac{1}{4}ka^2$

* If freq of S.H.M is 'n' then freq of K.E & P.E is '2n'
* If freq. of SHM is 'n' then conversion of freq (kinetic to potential or, pot. to kinetic) is '4n'

A spring of spring const. 'k' devide into unequal part ratio of length is 1:n. spring const. of indivisual part.



$$\boxed{L_2 = nL_1}$$

$* \ L_1 = \frac{L}{n+1} \Rightarrow K \propto \frac{1}{l}$

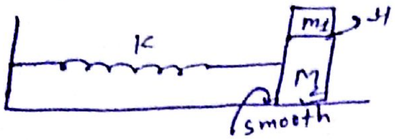
$* \ \frac{K_1}{K} = \frac{L}{l_1} = \frac{L}{\left(\frac{L}{n+1}\right)}$

$$\boxed{K_1 = (n+1)K}$$

$* \ L_2 = nL_1 = \frac{nL}{n+1} \Rightarrow K \propto \frac{1}{K} \Rightarrow \frac{K_2}{K} = \frac{L}{l_2} = \frac{L}{\frac{nL}{n+1}}$

$$\boxed{K_2 = \frac{(n+1)}{n}K}$$

Find max. amplitude of osscilation so that m_1 does no leave on M_2.



$a\left(\frac{K}{m_1+m_2}\right) = \mu g$

$$\boxed{a = \frac{\mu(M_1+m_2)g}{K}}$$

Max amplitude of osscilation so that block remain in contact.

during upward Accl^n

$N_1 = m(g+a)$

during down ward accl^n $N_2 = m(g-a)$

If $N_2 > 0 \Rightarrow$ block Remain in contact.

$\boxed{g > a}$ → Accl^n

$\boxed{g > a\omega^2}$ → Amplitude

$$\boxed{a_{max} = \frac{mg}{K}}$$

2016 JIPMER

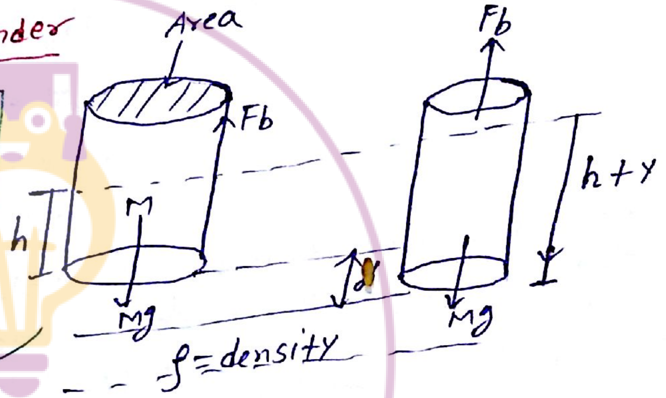
Time period of floating of cyllinder

$$\boxed{T = 2\pi\sqrt{\frac{h}{g}}} \quad (l = h)$$

Time period of liq in U-Tube.

JIPMER 2016**

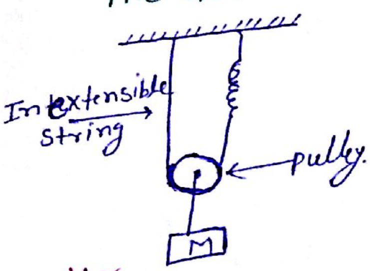
$$\boxed{T = 2\pi\sqrt{\frac{m}{A\rho g}}}$$



Motion of an oscillating liquid in u-tube → simple harmonic & time period Independent of density of liq.

**exemplar

Time period of Mass 'M' when displaced from equilibrium position & then Released for the system shown in fig?



$$* \boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4K}{M}}} = 2\pi\sqrt{\frac{M}{4K}}}$$

Exemplar

The length of second pendulum on the surface of Earth is 1m. then of second pendulum on the Moon? → $\boxed{1/6\ m}$