

Probability

* Probability of occurrence of event E is denoted by $P(E)$ and

$$P(E) = \frac{\text{no. of cases favourable to } E}{\text{total no. of cases}} = \frac{a}{a+b}$$

$$\Rightarrow 0 \leq P(E) \leq 1$$

↑
Impossible

↑ Sure event

where, $a \Rightarrow$ favourable no. of cases

$b \Rightarrow$ non-favourable " " " "

\Rightarrow Here $a : b =$ odd. in favour of event E
 $b : a =$ or against favour of event E .

$$P(E') = \frac{b}{a+b}$$

$$P(E) + P(E') = 1$$

Q.1 From a pack of 52 cards 3 cards are drawn at random. Find the probability that drawn is a King, a Q, and a J.

$$P(E) = \frac{{}^4C_1 \cdot {}^4C_1 \cdot {}^4C_1}{{}^{52}C_3}$$

Q. From a pack of cards are drawn. Find the chance are both are King.

$$P(E) = \frac{{}^{52}C_2}{{}^{52}C_2}$$

Q. 2 dice through 6 times

(i) What is the chance that sum of score is 7.

(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)

$$P(E) = \frac{6}{36} = \frac{a}{a+b} \quad a=1 \quad b=5$$

odd in favour of $E = 1:5$

odd against favour of $E = 5:1$

(ii) Chance that selected no. are Relatively Prime (Co-prime)

(1,2) (1,3) (1,5) (2,3)

(1,2) (2,1) (2,3) (3,2)
(1,3) (3,1) (2,5) (5,2)
(1,4) (4,1) (3,5) (5,3)
(1,5) (5,1) (3,4) (4,3)
(4,6) (6,4) (4,5) (5,4)
(6,5) (5,6)

$$P(E) = \frac{22}{6 \cdot 6}$$

(iii) getting the score is less than 12.

(1,6) (6,1) (2,6) (6,2) (3,6) (6,3) (4,6) (6,4) (5,6) (6,5)
(1,5) (5,1) (2,5) (5,2) (3,5) (5,3) (4,5) (5,4) (5,5) (5,5)

E — Sum < 12

E' = Sum $= 12$

$$P(E') = \frac{1}{6 \cdot 6} = \frac{1}{36}$$

$P(E) = 1 - P(E')$

$$= 1 - \frac{1}{36}$$

Q. 10 shoes (5 Pairs) are lying in a almirah rack
 of ~~10~~ Chhanna reaches there and randomly picks 4 shoes
 find the probability that they form exactly one pair
 of shoes

L_1, L_2, L_3, L_4, L_5
 R_1, R_2, R_3, R_4, R_5

$P(E) = \frac{10 \cdot 8 \cdot 6 \cdot 4}{10 \cdot 9 \cdot 8 \cdot 7}$

$$P(E) = \frac{5C_1 \cdot 4C_2 \cdot 2C_1 \cdot 2C_2}{10C_4}$$

Q. A matrix of 2nd order is made with the element 0 or 1
 A matrix is chosen from above set then what is the chance
 that it is singular.

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ Element $\in [0, 1]$

$|A| = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \rightarrow$ If $|A| \neq 0$ then A is non-singular
 If $|A| = 0$ then A is singular

$\{ \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \}$

$$P(E) = \frac{6}{16}$$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P(E) = \frac{10}{2 \cdot 2 \cdot 2 \cdot 2} \quad A$$

Q. 4 A's and 3 O's are randomly placed in a line.
find the chance that the 2 extreme fruits are both oranges

$$P(E) = \frac{{}^3C_2 \cdot {}^5C_1}{{}^7C_2} \text{ Ans} = \frac{3 \cdot 5}{21}$$

~~10/10000~~

Q. $N = \{1, 2, 3, \dots, 20\}$

first 20 Natural No.

2 Natural no. are randomly selected

(i) what is the chance that the sum is odd.

$\{1, 2\}, \{1, 4\}, \{1, 6\}, \{1, 10\}, \{1, 12\}, \{1, 16\}, \{1, 18\}$

odd - $\{1, 3, \dots, 19\}$

Even - $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$$P(E) = \frac{(\text{odd} + \text{Even})}{{}^{20}C_2} = P(E) = \frac{{}^{10}C_1 \cdot {}^{10}C_1}{{}^{20}C_2}$$

(ii) what is the P that sum even

$$P(E) = \frac{{}^{10}C_2 \cdot {}^{10}C_2}{{}^{20}C_2} \text{ Ans}$$

$$E + O =$$

$$E + E = E$$

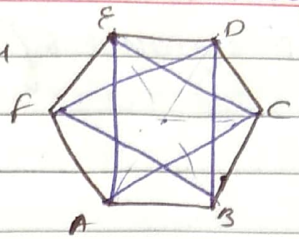
$$O + O = E$$

(iii) if 3 no. are selected then what is the chance that the no. arrange A.P.

$(1, 2, 3)$

$$P(E) = \frac{1 \cdot {}^{10}C_2 + {}^{10}C_2 \cdot 1}{{}^{20}C_3}$$

Q. A regular hexagon 3 vertices of a Regular hexagon are joined randomly. find the chance that an equilateral triangle is formed.

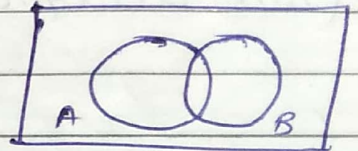


$$P(E) = \frac{6C_3}{6C_3}$$

* Addition theorem of Probability

If A and B are two events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$\frac{n(A \cup B)}{n(S)} = \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

all divide by $n(S)$

(i) $(A \cup B)$ = occurrence of at least 1



(ii) $(A \cap B)$ = simultaneous occurrence of both



- \bar{A} Joint occurrence of both

(iii) \bar{A} = non occurrence of A

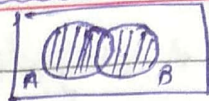


(iv) $A \cap B'$ = occurrence of A but not B
= $A - B$



$$P(A \cap B') = P(A) - P(A \cap B)$$

* $P(\text{Occurance of exactly one}) = P(A \cup B) - P(A \cap B)$



$$= P(A) + P(B) - 2P(A \cap B)$$

$$P(A \cap B) = P(A \cap B)$$

* $(A \cup B)' = A' \cap B'$
 $(A \cap B)' = A' \cup B'$ → De Morgan Law

→ $P(\text{non occurrence of both}) = P(A' \cap B') =$



$$P(A' \cap B') = 1 - P(A \cup B)$$

$$P(A' \cup B') = 1 - P(A \cap B)$$

⇒ $P(\text{occurrence of at least one or both}) = P(A \cup B) =$

$$P(A \cup B) = 1 - P(\text{non-occurrence of } A \text{ \& } B)$$

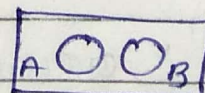
$$P(A \cup B) = 1 - P(\text{neither } A \text{ nor } B)$$

Occurance of either of them = 1 - neither A nor B

no common on both

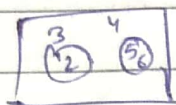
* If A and B are mutually Exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



disjoint →

* Ex! $S = \{1, 2, 3, 4, 5, 6\}$ $A = \{1, 2\}$ $B = \{5, 6\}$
 disjoint. (nothing common).



It is the disjoint

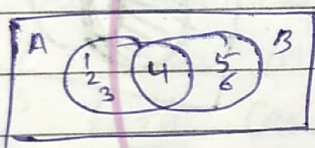
$n(A) = 2$ $n(B) = 2$

\Rightarrow if A and B are Exhaustive event
 Ans: \emptyset A disjoint

$S = \{1, 2, 3, 4, 5, 6\}$



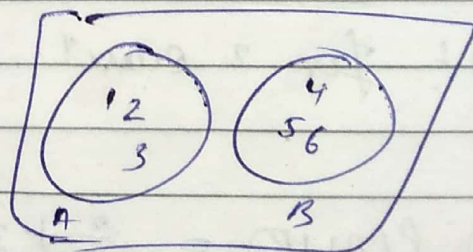
$A = \{1, 2, 3, 4\}$ A and B
 $B = \{4, 5, 6\}$ are not exhaustive.



$P(A \cup B) = 1 = P(A) + P(B) - P(A \cap B)$

Here A and B are not any mutual Exclusive.

$\Rightarrow S = \{1, 2, 3, 4, 5, 6\}$
 $A = \{1, 2, 3\}$
 $B = \{4, 5, 6\}$



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $1 = P(A) + P(B)$

Both Mutually Exclusive and Exhaustive
 = $A \cap B = \emptyset$

Mutually Excl. = no Common. $P(A \cap B) = 0$
 Exhaustive = A हो या तो B होना जरूरी है

Q. For two events A and B

Let $P(A \cup B) = \frac{7}{8}$

$P(A \cap B) = \frac{1}{4}$

$P(A) = \frac{5}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

find $P(A)$

$P(B)$

$P(A \cap B')$

~~$P(A) = \frac{5}{8}$~~

$\frac{7}{8} = \frac{3}{8} + P(B) - \frac{1}{4}$

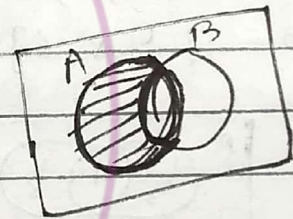
~~$P(A) =$~~

$P(B) = \frac{3}{4}$

~~$\frac{7}{8} = P(A) +$~~

$1 - P(A) = \frac{5}{8}$

$P(A) = 1 - \frac{5}{8} = \frac{3}{8}$



at least

$P(A \cap B') = P(A) - P(A \cap B)$

$= \frac{3}{8} - \frac{1}{4} = \frac{1}{4}$

Ques! For 2 event Let $P(A) = \frac{3}{8}$

$P(B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$

(i) $P(A \cup B) = \frac{3}{8} + \frac{1}{2} - \frac{1}{4} =$

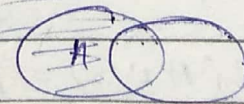
(ii) $P(\bar{A}) = 1 - P(A) = \frac{5}{8}$

(iii) $P(\bar{B}) = 1 - P(B) = \frac{1}{2}$

(iv) $P(A' \cap B') = P(\overline{A \cup B}) = P(A \cup B)' = 1 - P(A \cup B)$

(v) $P(A' \cup B') = P(\overline{A \cap B}) = P(A \cap B)' = 1 - P(A \cap B)$

(vi) $P(A' \cap B) = P(A) \cup P(B) = P(B) - P(A \cap B)$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Auto = no. of common.

A and B - Exh.

$$200 \frac{282}{1326}$$

$A \cup B \Rightarrow$ occurrence of at least one

$$\frac{52}{52} \frac{53}{53}$$

Q. 2 cards are drawn from a pack of 52 cards. Find the probability that both cards are of red colour or they are queen.

A \rightarrow get red
B \rightarrow get king

$$\frac{26}{52} = \frac{1}{2}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^{26}C_2}{{}^{52}C_2} - \frac{{}^{26}C_2}{{}^{52}C_2}$$

$$\frac{12 \times 11}{12 \times 11} = \frac{12 \times 23}{26 \times 51} = \frac{12 \times 23}{26 \times 51} = \frac{282}{26 \times 51}$$

Q. Prob. that franchi will get plumbing contract is $\frac{2}{3}$ and he will get an electric contract $\frac{5}{9}$. If the prob. that franchi will get at least one contract is $\frac{4}{5}$ then what is the prob. that franchi will get both contracts?

(AUB)

$P(A) = \frac{2}{3}$ and $P(B) = \frac{5}{9}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{4}{5} = \frac{2}{3} + \frac{5}{9} - P(A \cap B)$$

Q. 2 for a prob. that horse A winning race is $\frac{1}{3}$ and that of horse B is winning race is $\frac{1}{5}$. And Prob. that either of them will win the race (ii) none of them will win the race

E = at least A win race

$$P(E) = \frac{1}{3}$$

F = A B

$$P(F) = \frac{1}{5}$$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(ii) $P(A \cup B) = P(A \cup B)$

(iii) $P(E \cup F) = P(E \cup F)$

1ES

P(E)

Event E and F are mutually and not exhaustive

$$(i) P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{1}{3} + \frac{1}{5}$$

$$(ii) P(E' \cap F') = P(E \cup F)' = 1 - P(E \cup F)$$

Q. For a class ~~bus~~ in India railway french and change appear in ~~change~~ interview. the probabilities that for being sel. is 3 that of ~~change~~. what is the individual prob. of each.

A = french being selected

B = H.P being selected

$$(A+B) = 1$$

$$P(A) = 3P(B)$$

$$P(E \cup F) = 1$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$1 = 3n + n$$

$$n = \frac{1}{4}$$

* Conditional Prob.

let A and B are 2 events such that $P(A) > 0$ then $P(B|A)$ denotes conditional prob. of B

given that A has occurred. Since A is known to occurred so it become new sample space and

$$P\left(\frac{B}{A}\right) \text{ is } = \frac{P(A \cap B)}{P(A)}$$

* multiplication theorem

$$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right)$$

$$P(A \cap B \cap C) = P(A) \cdot P\left(\frac{B}{A}\right) \cdot P\left(\frac{C}{A \cap B}\right)$$

Note:

$$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right) = P(B) P\left(\frac{A}{B}\right)$$

Ex 1 $S = \{1, 2, 3, 4, 5, 6\}$ $P(\text{odd}) = \frac{1}{2}$

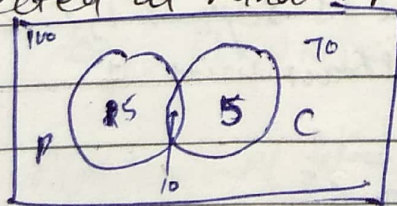
$\{2, 4, 6\}$ $P(\text{even}) = \frac{1}{3}$

Ex 2 $80B + 20G$ $P = \frac{1}{100}$

T & O 14 is selected

$$P(\text{selected}) = \frac{1}{20}$$

Q. In a certain college 25% student fail in physics, 15% fail in Chem. and 10% fail in both. A student is selected at randomly



$$P(A) = 10$$

(i) If he is fail in Chem. then what is the prob. that he failed in Phy

$$P(A) = 25 \quad P(B) = 15$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P\left(\frac{\text{fail in Physics}}{\text{fail in Chem}}\right) = \frac{10}{15} = \frac{2}{3}$$

$$\frac{P(E \cap F)}{P(F)} = \frac{10/100}{15/100} = \frac{10}{15} = \frac{2}{3}$$

(ii) what is failed in phy and chem or both

$$P(\overbrace{P(E \cup F)}^{\text{failed in phy or chem}}) = \frac{30}{100} = \frac{3}{10}$$

$$P(E \cup F) = \frac{30}{100} = \frac{3}{10}$$

Q Event

$$P(E) = 0.6$$

$$P(F) = 0.3$$

$$\& P(E \cap F) = 0.2$$

find $P(\frac{E}{F})$

(ii) $P(\frac{F}{E})$

(i) $P(\frac{E}{F}) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3}$

(ii) $P(\frac{F}{E}) = \frac{P(F \cap E)}{P(E)} = \frac{0.2}{0.6}$

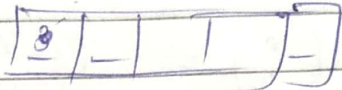
Q. in a experiment we roll a fair die five times
define two event $E \rightarrow$ outcome is sequ. (1, 2, 3, 4, 5)

$F \rightarrow$ sequenc start with 1

(i) $P(E) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{6^5}$

(ii) $P(F) = \frac{1}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} = \frac{1}{6}$

(iii) What is Prob. that no. 2 appears exactly twice

n_3 $\frac{1}{6}$ P(appears exactly twice) 

$${}^5C_2 \cdot \frac{5 \cdot 1 \cdot 5 \cdot 5 \cdot 1}{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}$$

(iv) find $P\left(\frac{F}{E}\right) = 1$, $\frac{P(F \cap E)}{P(E)} = \frac{1}{10 \cdot 6^5}$

(v) $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{1/6^5}{1/6} = \frac{1}{6^4}$

(vi) A ship is fitted with 3 engine E_1, E_2, E_3 the engine fu independently each other w. respective P.

$$E_1 = \frac{1}{2}$$

$$E_2 = \frac{1}{4}$$

$$E_3 = \frac{1}{4}$$

For the ship to be operational at least 2 engine functionally.

Let $X \rightarrow$ ship is operational

$X_1, X_2, X_3 \rightarrow$ engine E_1 functional

$X_2 \rightarrow$ engine E_2 "

$X_3 \rightarrow$ " engine E_3 "

$$P(X_1) = \frac{1}{2}$$

$$P(X_2) = \frac{1}{4}$$

$$P(X_3) = \frac{1}{4}$$

then find value of $P(X)$ -

$${}^3C_2 \cdot P(X_1 \cup X_2) =$$

$$P(n) = P(\text{at least } 2 \text{ engine } E_i) + P(\text{at least } 2 \text{ engine } E_j)$$

$$= P(n_1, n_2, n_3) + P(n_1, n_2', n_3) + P(n_1', n_2, n_3) + P(n_1, n_2, n_3')$$

$$= \frac{1}{2} \cdot \frac{1}{4} \cdot (1 - \frac{1}{4}) + \frac{1}{2} \cdot (1 - \frac{1}{4}) \cdot \frac{1}{4} + (1 - \frac{1}{2}) \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4}$$

$$Q \quad P\left(\frac{x}{x_2}\right) =$$

$$P\left(\frac{x}{x_2}\right) = \frac{P(n \cap n_2)}{P(n_2)} = \frac{P(x_1, y_2, x_3) + P(x_1', y_2, x_3) + P(n_1, n_2, x_3)}{1/4}$$

$$Q. \quad P\left(\frac{n_1'}{n}\right) = \frac{P(n_1' \cap n)}{P(n)} = \frac{P(x_1', n_2, n_3) + P(n_1, n_2, n_3') + P(n_1, n_2, n_3)}{P(n)}$$

* Independent Events

Two events A and B are said to be independent if occurrence or non-occurrence of one does not affect the probability of occurrence or non-occurrence of other

if E and F independent

Ex!

E = strs str IIT/ee 4G/2E E'
 F = Trump President USA

Both are independent

$$P(E \cap F) = P(E) \cdot P(F)$$

E & F are independent then

- E & F' are I
- E' & F are II
- E' & F' are III

$$P(E \cap F) = P(E) \cdot P(F)$$

* If E and F is independent

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= P(E) + P(F) - P(E)P(F)$$

or
 $P(E) + P(F)(1 - P(E))$

* If E and F is independent then

P(neither A occur nor B)

$$P(\text{Occurrence of at least one}) = 1 - P(E')P(F')$$

Note, if E, F and G are independent

$$P(E \cup F \cup G) = 1 - P(E')P(F')P(G')$$

In the prob. that a man live to more than 10 years $\left(\frac{1}{4}\right)$
 prob. of his wife will live to more than 10 years is $\frac{1}{3}$
 $E \Rightarrow$ man live to more than 10 years $= P(E) = \frac{1}{4}$

Find the prob. that $F =$ his wife to more than 10 years $= P(F) = \frac{1}{3}$

(i) Both will alive in next 10 years

$$P(E \cap F) = P(E)P(F) = \frac{1}{4} \cdot \frac{1}{3}$$

(ii) at least one will alive? $P(A \cup B) = P(E) + P(F) - P(E \cap F)$
 $P(A) = 1 - P(E')P(F') \Rightarrow \frac{1}{4} + \frac{1}{3} - \frac{1}{4} \cdot \frac{1}{3}$

(iii) neither will alive $= P(E') \cdot P(F')$

$$= \left(1 - \frac{1}{4}\right) \cdot \left(1 - \frac{1}{3}\right)$$

Q. E and F are two ind. Events. the prob. that both E and F happen is $\left(\frac{1}{12}\right)$. Prob. the neither E nor F is $\frac{1}{2}$

$\frac{1}{2}$

Find $P(E)$ and $P(F)$.

$$P(E \cap F) = P(A) \quad P(E) = x$$

$$P(F) = y$$

$$= P(\bar{E} \cap \bar{F}) = P(E')P(F') \quad P(E')P(F') = \frac{1}{2}$$

face

$$P(E \cap F) = P(E) \cdot P(F)$$

$$\frac{1}{2} \Rightarrow n_y =$$

$$P(E' \cap F') = P(E') \cdot P(F')$$

$$= \frac{1}{2} = (1 - P(E))(1 - P(F))$$

Ques! 4 Person are asked the same Ques by an interviewer if each has independent probability $\frac{1}{8}$ of Ans Ques independently. then find the prob⁶ that atleast one Ans correctly

Ans $P(A \cap B) = P(A) \cdot P(B)$
 $= \frac{1}{6}$

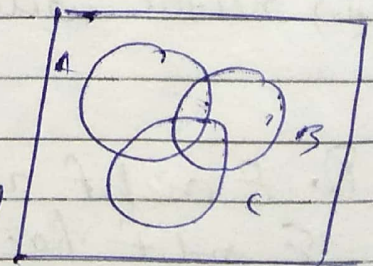
$$P(E \cup F \cup G \cup H) = 1 - P(E')P(F')P(G')P(H')$$
$$= 1 - \left(1 - \frac{1}{8}\right)^4$$
$$= 1 - \left(\frac{7}{8}\right)^4$$

* 3 three events!

if A, B, and C are three events then addition theorem

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(BC) - P(CA) + P(A \cap B \cap C)$$

P(A)

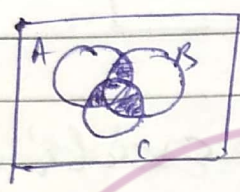


occurrence of at least one

* $(A \cup B \cup C) \rightarrow$ occurrence of at least one

$A \cap B \cap C \rightarrow$ Joint occurrence of all

$(A \cap B \bar{C}) \cup (A \bar{B} \cap C) \cup (\bar{A} \cap B \cap C) =$ occurrence of exactly two

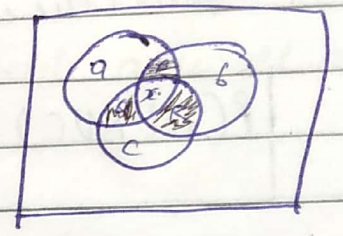


OR.

$$\Rightarrow P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$$

$$P + q + r + 2 + x - 3x$$

$$P + q + r$$



* Occurrence of at least two

$$P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$$

$(A \cap B \bar{C}) \cup (A \bar{B} \cap C) \cup (\bar{A} \cap B \cap C) \cup (A \cap B \cap C)$

* Occurrence of exactly one

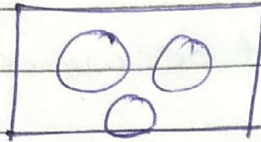
$$P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A) + 3P(A \cap B \cap C)$$

OR

$(A \bar{B} \bar{C}) \cup (\bar{A} \bar{B} \cap C) \cup (\bar{A} \bar{B} \cap C)$

*

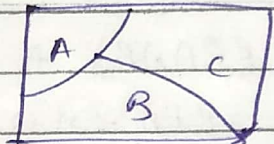
* If A, B, C are mutually Exclusive events



$$P(A \cup B \cup C) = P(A) + P(B) + P(C) \leq 1$$

* If A, B, and C are Mutually Exclusive and exhaustive event

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$$

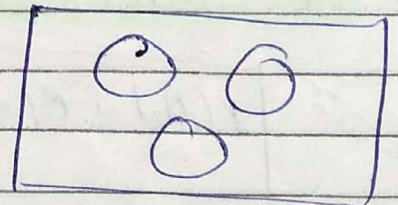


* Occurrence of at least one event \bar{P}

$$P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$$

* If A, B and C are Three Pair wise Mut. Exclusive then they are mutually Exclusive

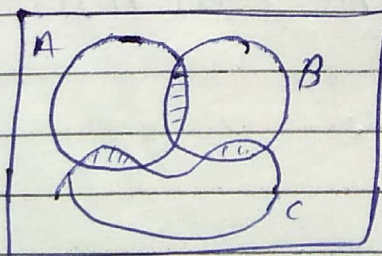
• However if A, B and C are M.E this does not



\Rightarrow they are Pair M.E

Exact

$$A \cap B \cap C = \phi$$



* * * * * If A, B and C are independent events
* then

$$P(A \cap B) = P(A) \cdot P(B)$$

→ Here A & B, A' & B, A & B', A' & B' are independent.
addition theorem holds

* If A, B and C are independent
then

(i) they are pair wise independent as well as

(ii) m. Independent taken all
i.e. if A, B and C are independent

- (i) A & B are independent i.e. $P(A \cap B) = P(A) \cdot P(B)$
- (ii) A & C are independent i.e. $P(A \cap C) = P(A) \cdot P(C)$
- (iii) B & C " " i.e. $P(B \cap C) = P(B) \cdot P(C)$
- (iv) A & B, C all " " i.e. $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

[all three must be satisfied in all the points.]

Note for n independent events, the total no. of
condition required =

$$2^n - n - 1$$

n = 2 the cond = $2^2 - 2 - 1$
= 1

V.I.P

V.I.P

$n=3$ then condition
 $= 2^3 - 3 - 1 = 4$

* If A, B and C are incl. Events then occurrence of at least one event

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= P(A) + P(B) + P(C) - P(A)P(B) - P(B)P(C) - P(C)P(A) + P(A)P(B)P(C)$$

OR:

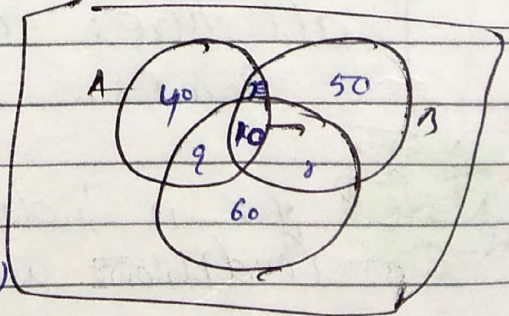
$$= 1 - P(A') P(B') P(C')$$

Q

There are 3 clubs A, B, C in big city with
 A — 40
 B — 50
 C — 60 members respectively.

10 people members of all 3 clubs
 70 are members only one club

A person is selected randomly, find the Prob. that he had membership of 2 clubs



$$P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$$

$$= n + n + n - 3(10)$$

$$= 3n - 30$$

the value of n

$$a + b + c = 70 \text{ (i)}$$

$$40 = a + 10 + 10 + 10$$

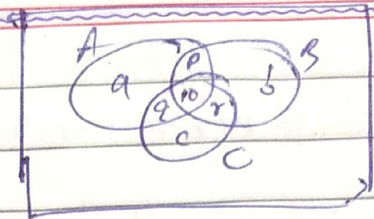
$$a + 10 + 10 = 30 \text{ (ii)}$$

$$b + 10 + 10 = 40 \text{ (iii)}$$

$$p+q+r=50 \quad \text{--- (iv)}$$

$$a+b+c+2(p+q+r)=120$$

$$p+q+r=25 \quad \text{--- (v)}$$



$$P(E) = \frac{25}{a+b+c+p+q+r+10}$$

$$= \frac{25}{70+25+10} = \frac{25}{105} \quad n$$

Q. 3 persons A, B, C fired at a target independently. Suppose $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{3}$ denote the prob. the readings the target.

(i) find the prob. that exactly one of them will target.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A) + 3P(A \cap B \cap C)$$

$$= \frac{1}{6} + \frac{1}{4} + \frac{1}{3}$$

$$= P(A) + \text{OR}$$

$$P(A)P(B')P(C') + P(A')P(B)P(C') + P(A')P(B')P(C)$$

(ii) find prob. that target by hit exact two

$$= P(A)P(B)P(C') + P(A)P(B')P(C) + P(A')P(B)P(C)$$

OR

$$2P(AB) + 2P(BC) + 2P(CA) - 3P(ABC)$$

∴ at least one of them hit the target

$$P(\bar{A}\bar{B}\bar{C}) = 1 - P(A)P(B)P(C)$$

OR

$$= P(A) + P(B) + P(C) - P(A)P(B) - P(B)P(C) - P(C)P(A) + P(A)P(B)P(C)$$

Q. if target is hit only once then find the prob. that it was the man A.

$$P\left(\frac{B}{A}\right) = \frac{P(B|A)}{P(A)}$$

A → man A
B → not hit

$$P\left(\frac{\text{Bis A}}{\text{Exp. I (A)}}\right) = \frac{1}{P(A)P(B)P(C) + P(A)P(C)P(B) + P(A)P(B)P(C)}$$

$$P(A) = \frac{1}{2} \quad P(B) = \frac{1}{3} \quad P(A \cap B) = \frac{1}{4}$$

$$(i) \text{ find } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$$

$$(ii) P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{1/4}{1/2}$$

$$(iii) P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$$

$$(4) P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{P(1 - A \cap B)}{P(B')} = \frac{P(A \cup B)'}{1 - P(B)}$$

$$= \frac{1 - P(A \cup B)}{1 - 1/3}$$

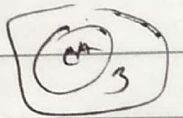
$$(5) P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{P(B \cap A')}{P(A')} = \frac{P(B) - P(A \cap B)}{1 - P(A)}$$

Note!

M. Exclusive events can be used when events are taken from same experiment and independence can be used when the events are taken from diff.

Q. find $P\left(\frac{B}{A}\right)$ if $e = \frac{P(A \cap B)}{P(A)}$



(i) A is subset of B $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$

(ii) A and B are disjoint $= \frac{0}{P(A)} = 0$

Q. A Bag contain 1 Red and 2 Black Ball. Two people A & B in order draw one Ball from the Bag and put it Bag after noting its Colour then cont. doing it indef. unless one who draw the first @uce the game. Compute their. of Red win

$\boxed{1R+2B}$

$$P(A \text{ win}) = R + BBR + BBABR + BBABBR + \dots$$
$$R + B^2R + B^4R + B^6R + \dots$$
$$= \frac{1}{3} + \left(\frac{2}{3}\right)^2 \frac{1}{3} + \left(\frac{2}{3}\right)^4 \frac{1}{3} + \left(\frac{2}{3}\right)^6 \frac{1}{3} + \dots$$

$$a = \frac{1}{3}$$

$$r = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{a}{1-r}\right)$$

$$\frac{1}{3} \left(1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^4 + \dots\right)$$

$$= \frac{\frac{1}{3}}{1 - \frac{4}{9}} = \frac{\frac{1}{3}}{\frac{5}{9}} = \frac{3}{5}$$

$$\textcircled{2} P(B \text{ win}) = B + RBB + RRRB + RRRRB + \dots$$

$$= \cancel{B} + \cancel{R}B + \cancel{R}R\cancel{B} + \cancel{R}R\cancel{R}B + \dots$$

$$= BR + B^3R + B^5R + \dots$$

$$= \frac{2}{3} \frac{1}{3} + \left(\frac{2}{3}\right)^3 \frac{1}{3} + \dots$$

$$\frac{\frac{2}{3} \cdot \frac{1}{3}}{1 - \frac{4}{9}}$$

$$\text{or } 1 - P(A)$$

$$= 1 - \frac{3}{5} = \frac{2}{5}$$

$\textcircled{3}$

$\boxed{1R+2B+3Y}$

$$P(A \text{ win}) = R + BBYR + BBBBYR + \dots$$

$$\frac{1}{6} + \frac{2}{6} \left(\frac{3}{6}\right) \left(\frac{1}{6}\right) + \left(\frac{2}{6}\right)^2 \left(\frac{3}{6}\right)^2 \left(\frac{1}{6}\right) + \dots$$

$$P(A) = w + LLLw + L^6w + \dots + \infty$$

$$P(w) = \frac{1}{6} \quad P(L) = \frac{5}{6}$$

$$P(B \text{ win}) = Lw + LLLLw + L^7w + \dots + \infty$$

* Binomial Prob. distribution :-

for independent trials an experiment

↓
n independent trials (Bernoulli trials).

↓
Each of these trials has 2 possible outcomes

Success
p

$$p + q = 1$$

Failure
q

$$P(\text{Success}) = p$$

$$P(\text{Failure}) = q = 1 - p$$

then $P(n=r) \equiv P(r \text{ successes}) = {}^n C_r p^r q^{n-r}$.

Ex: A pair of dice is thrown is

Q. A pair of dice is thrown 6 times and getting a doublet is considered as success. Compute the prob. of

(i) no success $P(S) = p = \frac{1}{6}$

(ii) exactly two success $P(E) = q = \frac{5}{6}$

(iii) at most two success ()

(iv) atleast ~~two~~ ^{one} success

(v) atleast three success

$$(q + p)^6 = {}^6C_0 q^6 + {}^6C_1 q^5 p^1 + {}^6C_2 q^4 p^2 + {}^6C_3 q^3 p^3 + {}^6C_4 q^2 p^4 + {}^6C_5 q^1 p^5 + {}^6C_6 p^6$$

\downarrow $P(x=0)$ \downarrow $P(x=1)$ \downarrow $P(x=2)$ \downarrow $P(x=3)$ \downarrow $P(x=4)$ \downarrow $P(x=5)$ \downarrow $P(x=6)$

Exactly three success Ex 4 Succ.

$$P(X=r) = P(r \text{ Success}) = {}^n C_r p^r q^{n-r}$$

$\left. \begin{array}{l} \text{All} \\ \text{arrangement} \end{array} \right\} \text{Success on Arrangement or} \\ \text{arrangement}$

① ${}^6C_0 q^6$

② ${}^6C_4 q^2 p^4$

③ $P(x=0) + P(x=1) + P(x=2)$
 $= {}^6C_0 q^6 + {}^6C_1 q^5 p^1 + {}^6C_2 q^4 p^2$

$$P(x=1) + P(x=2) + \dots + P(x=6) = 1 - q^6$$

~~$${}^6C_1 q^5 p^1 + {}^6C_2 q^4 p^2 + {}^6C_3 q^3 p^3 + {}^6C_4 q^2 p^4 + {}^6C_5 q^1 p^5 +$$~~

or

$$P(x=0)$$

$$P(x=3) + P(x=4) + P(x=5) + P(x=6)$$

* In Binomial Prob. distribution mean of BPD = np

$$\text{variance of BPD} = npq$$

$$\text{variance} = \sigma^2$$

$$\text{Standard Dev.} = \sqrt{npq}$$

d. * $n=10$ $P(x=7) = {}^{10}C_7 p^7 q^3$

$n=50$ $P(x=36) = {}^{50}C_{36} p^{36} q^{14}$

Q A fair coin is tossed 10 times (It is called Bernoulli trials), then find the Prob. of getting $p = \frac{1}{2}$ $q = \frac{1}{2}$

(i) exactly 6 Head = ${}^{10}C_6 p^6 q^4$

(ii) atleast 8 head = ${}^{10}C_8 p^8 q^2 + {}^{10}C_9 p^9 q^1 + {}^{10}C_{10} p^{10} q^0$

(iii) atmost 3 head = ${}^{10}C_1 q^1 + {}^{10}C_2 q^2 p^1 + {}^{10}C_3 q^3 p^2 + {}^{10}C_4 q^4 p^3$
 $P(n=0) + P(n=1) + P(n=2) + P(n=3)$

$$(q+p)^{10} =$$

Q. A coin is tossed as likely to head as tail in a sequence of 5 independent trial find the probab. that

$$P(H) + P(T) = 1$$

$$2n + n = 1$$

$$n = \frac{1}{3}$$

$$P(H) = \frac{2}{3} =$$

$$P(T) = \frac{1}{3}$$

(i) Exactly two head occur. $= {}^5C_2 p^2 q^3$ $\frac{5}{43} = 10x$

(ii) Third head occurs on the fifth tossed

$$= {}^4C_2 p^2 q^2 \cdot p$$


Q. 100 identical pts. each following ^{head} coin with prob. p ($0 < p < 1$) are tossed once. If the prob. of 50 coins showing of the heads is = the prob. of 51 coins showing of the heads then find value of p .

$${}^{50}C_0 p^0 q^{50} = {}^{100}C_{50} p^{50} q^{50} = {}^{100}C_{51} p^{51} q^{49}$$

$$p \cdot \frac{100}{50} \cdot q = \frac{100}{51} \cdot p$$

$$= \frac{51}{50} \cdot (1-p) = p$$

$$p(1 + \frac{51}{50}) = \frac{51}{50}$$

$$p = \frac{51}{101}$$

Q. A tracks take a step forward or backward. The prob. that the step forward is 0.4.
 find the prob. that at the end of 11 step he is one step away from the starting point

$$P(\text{forward step}) = 0.4$$

$$\frac{H}{C} = \frac{0.4}{0.4}$$

variate

$$P(X=6) + P(X=5)$$

$${}^{11}C_5 p^5 q^6 + {}^{11}C_5 p^6 q^5$$

$$\text{mean} = 9$$

$$\text{SD} = 3/2$$

Q. If mean and standard deviation of binomial variate X are 9 and $3/2$ respectively find the prob. that n takes the value > 1

(i.e. $P(X > 1)$)

mean = np

$$\text{SD} = 3/2 \quad \text{mean} = 9$$

$$n = 9$$

$$q = np$$

$$p = 9$$

$$\frac{3}{2} = \sqrt{npq}$$

$$P = \frac{q}{n} = \frac{3}{9} = \frac{1}{3} = \sqrt{npq}$$

$$q = \frac{1}{3} \quad p = \frac{2}{3}$$

$$\text{SD}^2 = \left(\frac{3}{2}\right)^2 = npq$$

$$\text{mean} = 9 = np$$

$$\frac{q}{p} = \frac{1/3}{2/3} = \frac{1}{2}$$

$$q = \frac{1}{4}, \quad p = \frac{3}{4}, \quad n = 12$$

$$P(X > 1) = P(X=2) + P(X=3) + \dots + P(X=11) + P(X=12)$$

$$= 1 - \{P(X=0) + P(X=1)\}$$

A of sum of mean and variance for 6 trial
is ~~2.16~~ 2.16

find the distribution

variance = npq

mean = np

npq + np = 2.16

n = 6

6p²

np(q+1) = 2.16

p = $\frac{2.16}{6(q+1)}$

np = $\frac{2.16}{q+1}$

p(q+1) = $\frac{2.16}{6}$

p(q+1) = 0.36

(1-q)(1+q) = 0.36 = $\frac{36}{100} = \frac{9}{25}$

1-q² = $\frac{9}{25}$

q = $\frac{4}{5}$ p = $\frac{1}{5}$ n = 6

p = 1-q

p+q = 1
p = 1-q

Q. A fair die is tossed repeatedly until 6 is obtained. Let X denote the no. of tosses required then

(i) find Prob that P(X=3)

p = success = $\frac{1}{6}$

q = failure = $\frac{5}{6}$

(p+q)⁶ = ~~3C₁q³ + 3C₂q²p¹ + 3C₃p³q³~~

~~P(n=3)~~ P(n=3) = qq²p

P(n=3) + P(n=4) + P(n=5)

(ii) P(n ≥ 3) = qq²p + qq³p + qq⁴p

= 1 - (p+q)

$me = \beta$
 $exh =$

⑤ the Conditional Prob. that $(n \geq 6)$ given that $n > 3$.

$$P(n \geq 6) = P(n=6) + P(n=7) + \dots$$

$$\Rightarrow P(n=2) + P(n=3) + P(n=4) + \dots$$

$$= q^2 p + q^3 p + q^4 p + \dots$$

$$\frac{P(n \geq 6)}{P(n > 3)} = \frac{P(A \cap B)}{P(B)} = \frac{P(n \geq 6)}{P(n > 3)}$$

$$= \frac{q^5 p + q^6 p + q^7 p + \dots}{q^3 p + q^4 p + \dots}$$

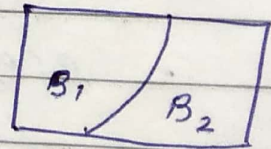
Total Probability and Bayes Theorem

Let an event A be an experiment occur with n mutually exclusive and exhaustive events B_1, B_2, \dots, B_n , then total probability of occurrence of event A is

Two event

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \dots + P(A \cap B_n)$$

$$P(B_1) + P(B_2) = 1$$



$$P(A) = P(A \cap B_1) + P(A \cap B_2)$$

$$P(A) = P(B_1) \cdot P\left(\frac{A}{B_1}\right) + P(B_2) \cdot P\left(\frac{A}{B_2}\right)$$

* Bayes theorem:

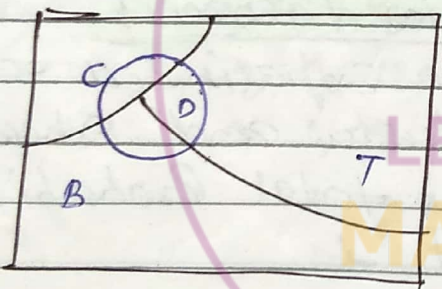
$$P\left(\frac{B_1}{A}\right) = \frac{P(B_1) \cdot P\left(\frac{A}{B_1}\right)}{P(B_2) \cdot P\left(\frac{A}{B_2}\right) + P(B_1) \cdot P\left(\frac{A}{B_1}\right)}$$

$$= \frac{P(B_1) \cdot P\left(\frac{A}{B_1}\right)}{P(B_1) \cdot P\left(\frac{A}{B_1}\right) + P(B_2) \cdot P\left(\frac{A}{B_2}\right)}$$

$$= \frac{P(B_1) \cdot P\left(\frac{A}{B_1}\right)}{P(B_1) \cdot P\left(\frac{A}{B_1}\right) + P(B_2) \cdot P\left(\frac{A}{B_2}\right)}$$

Three event total Probability:

* Lok Sabha 2019 election.



Three event total Probability

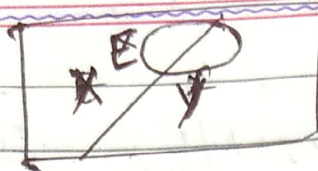
$$P(D) = P(B \cap D) + P(C \cap D) + P(T \cap D)$$

$$= P(B) \cdot P\left(\frac{D}{B}\right) + P(C) \cdot P\left(\frac{D}{C}\right) + P(T) \cdot P\left(\frac{D}{T}\right)$$

Bayes theorem:

$$\Rightarrow P\left(\frac{B}{D}\right) = \frac{P(B) \cdot P\left(\frac{D}{B}\right)}{P(B) \cdot P\left(\frac{D}{B}\right) + P(C) \cdot P\left(\frac{D}{C}\right) + P(T) \cdot P\left(\frac{D}{T}\right)}$$

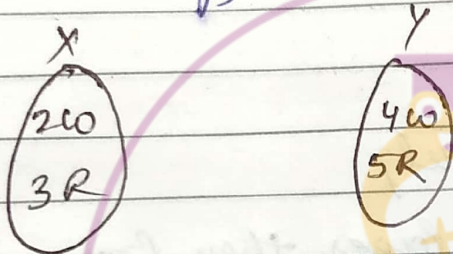
Que,



$$(1) P(E) = P(X) P(E|X) + P(Y) P(E|Y) = P(X) \cdot P\left(\frac{E}{X}\right) + P(Y) P\left(\frac{E}{Y}\right)$$

$$(2) P\left(\frac{Y}{E}\right) = \frac{P(Y) P\left(\frac{E}{Y}\right)}{P(Y) P\left(\frac{E}{Y}\right) + P(X) P\left(\frac{E}{X}\right)}$$

Q.



& Bags are shown in the fig. 1 ball is drawn from the Bag at random then what is the Prob that it is Red.

$$P(E) = P(A \cap E) + P(B \cap E)$$

$$= P(A) P\left(\frac{E}{A}\right) + P(B) P\left(\frac{E}{B}\right)$$

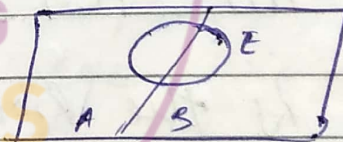
$$= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{5}{9} \text{ Ans}$$

E = event of getting white ball

A = X Bag select

B = Y Bag select

f = white colour ball



Q. In above situation 1 Ball is drawn from a Bag at random and found to be ~~Red~~ white then find the Prob. that it was drawn from Bag Y.

$$P(E) = P\left(\frac{W}{Y}\right) = \frac{P(Y) P\left(\frac{W}{Y}\right)}{P(Y) P\left(\frac{W}{Y}\right) + P(X) P\left(\frac{W}{X}\right)}$$

$$P(Y) P\left(\frac{W}{Y}\right) + P(X) P\left(\frac{W}{X}\right)$$

$$= \frac{\frac{1}{2} \cdot \frac{4}{9}}{\frac{1}{2} \cdot \frac{4}{9} + \frac{1}{2} \cdot \frac{2}{5}}$$

$$\frac{1}{2} \cdot \frac{4}{9} + \frac{1}{2} \cdot \frac{2}{5}$$

H.W
2

Part 5 = 8, 9

See Adv. = 4-5, 12-13

Solve in
fair copy.

$$P\left(\frac{B}{F}\right) = \frac{P(B) P\left(\frac{F}{B}\right)}{P(B) \cdot P\left(\frac{F}{B}\right) + P(A) \cdot P\left(\frac{F}{A}\right)} = \frac{\frac{1}{2} \cdot \frac{4}{9}}{\frac{1}{2} \cdot \frac{4}{9} + \frac{1}{2} \cdot \frac{2}{5}}$$

Q. 3 Political Parties C, B, T are contending for
loksabha election with development as a core
issue. The Prob. C, B, T of C, B, T Parties
meaning R election

$$\begin{aligned} C &= \frac{1}{3} & 0.3 \\ B &= \frac{4}{9} & 0.7 \\ T &= \frac{2}{9} & \text{respectively} \end{aligned}$$

If C comes to the power then probability of
during development in the country is
0.3. Corresponding probabilities for B and
T are 0.7 and 0.1 respectively.

(1) what is the probability that development will occur
C → Congress

$$P(D) = \frac{P(C) P\left(\frac{D}{C}\right)}{P(C) P\left(\frac{D}{C}\right) + P(B) P\left(\frac{D}{B}\right) + P(T) P\left(\frac{D}{T}\right)}$$

B → Congress - BJP etc

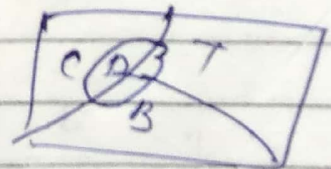
P(C) = Congress power

P(D) = Development by the country

A, T =

$$= \frac{1}{3}$$

$$\begin{aligned} P(D) &= P(B) P\left(\frac{D}{B}\right) + P(C) P\left(\frac{D}{C}\right) + P(T) \cdot P\left(\frac{D}{T}\right) \\ &= \frac{4}{9} \cdot (0.7) + \frac{1}{3} (0.3) + \frac{2}{9} (0.1) \end{aligned}$$



Q (i) if development occur through out the country then what is P that it is done by 3rd

$$P\left(\frac{T}{D}\right) = \frac{P(T) P\left(\frac{D}{T}\right)}{P(T) P\left(\frac{D}{T}\right) + P(B) P\left(\frac{D}{B}\right) + P(C) P\left(\frac{D}{C}\right)}$$

$$= \frac{\frac{2}{9} \times (0.1)}{\frac{4}{9} (0.7) + \frac{1}{3} (0.3) + \frac{2}{9} (0.1)}$$

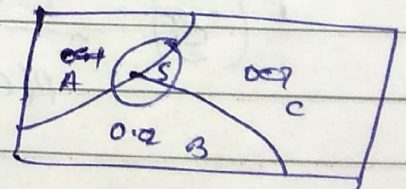
Q. A student stat that he will success in Tel exam with 80% chance if he study 10h per day, with 60% if he study 7h per day and 40% chance if he study 4h per day. Further believe that he will study 10h, 7h, 4h P/ra with P 0.1, 0.2, 0.7 res

(i) what is the chance that you is success

A = 10h study

B = 7h study

C = 4h study



$$P(S) = P(A) P\left(\frac{S}{A}\right) + P(B) P\left(\frac{S}{B}\right) + P(C) P\left(\frac{S}{C}\right)$$

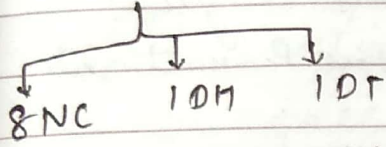
$$= (0.1)(0.8) + (0.2)(0.6) + (0.7)(0.4)$$

(ii) Given that A see what is the chance he study 7h.

$$P\left(\frac{B}{S}\right) = \frac{P(B) P\left(\frac{S}{B}\right)}{P(A) P\left(\frac{S}{A}\right) + P(B) P\left(\frac{S}{B}\right) + P(C) P\left(\frac{S}{C}\right)}$$

$$= \frac{(0.2)(0.6)}{(0.1)(0.8) + (0.2)(0.6) + (0.7)(0.4)}$$

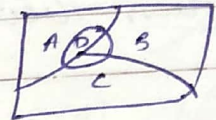
2. A & B lady have to put in help course, & are there are normal coin one 1 DM and 1 DT. She randomly draw a coin and tossed it 5 times. The coin was found was for head for all the time. find the prob. that coin was double head.



A → Normal coin

B → DM coin

C → DT coin



D = all Head 31111

$$P(A|D) = \frac{P(A) \cdot P(D|A)}{P(D)}$$

$$P\left(\frac{DM}{\text{all Head 31111}}\right) = \frac{P(DM) \cdot P\left(\frac{\text{all Head 31111}}{DM}\right)}{P(DM) \cdot P\left(\frac{\text{all Head 31111}}{DM}\right) + P(DT) \cdot P\left(\frac{\text{all Head 31111}}{DT}\right) + P(NC) \cdot P\left(\frac{\text{all Head 31111}}{NC}\right)}$$

$$= \frac{\frac{1}{10} \cdot \left(\frac{1}{10}\right)^5}{\frac{1}{10} \cdot \left(\frac{1}{10}\right)^5 + \frac{1}{10} \cdot \left(\frac{1}{10}\right)^5 + \frac{8}{10} \cdot \left(\frac{1}{2}\right)^5}$$

$$= \frac{1}{10} \cdot (1)$$

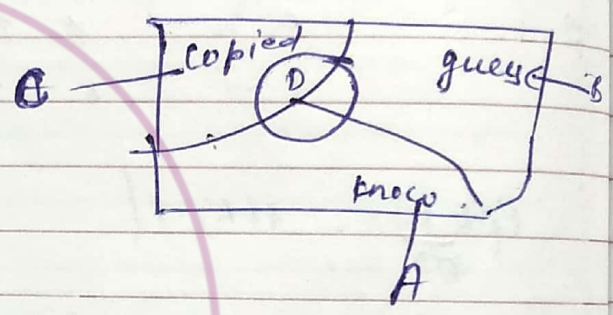
$$\frac{1}{10} \cdot 1 + \frac{1}{10} \cdot 0 + \frac{8}{10} \cdot \left(\frac{1}{2}\right)^5$$

Ans.

Q. In a test An Examinee either ^{guesses} ~~knows~~ or Copies or ~~knows~~ know to the mul. Q. with 4 choices and only one correct. The P. that he makes a guess is $\frac{1}{3}$ $P(\text{Copies the Ans}) = \frac{1}{6}$. The P. that his

Ans is correct given that he copied it $= \frac{1}{8}$.

Find the P. that he knows the Ans to the Q. given that he correctly answered.



$P(\text{he gives})$

$$P\left(\frac{\text{know the correct Ans}}{\text{Ans is correct}}\right) =$$

$$= P(A) \cdot P\left(\frac{\text{correct Ans}}{A = \text{know the Ans}}\right)$$

know

$$P(A) \left[P\left(\frac{\text{correct}}{A}\right) \right] + P(B) \left[P\left(\frac{\text{correct}}{B}\right) \right] + P(C) \left[P\left(\frac{\text{correct}}{C}\right) \right]$$

$$= \frac{\frac{1}{2}(1)}{\frac{1}{2}(1) + \frac{1}{3}\left(\frac{1}{4}\right) + \frac{1}{6}\left(\frac{1}{8}\right)}$$

$$P(A) = 1 - \left(\frac{1}{3} + \frac{1}{6}\right) = \frac{1}{2}$$

