Handwritten Notes

on

Optics

Optics

(i) Wave Optics (ii) Ray optics

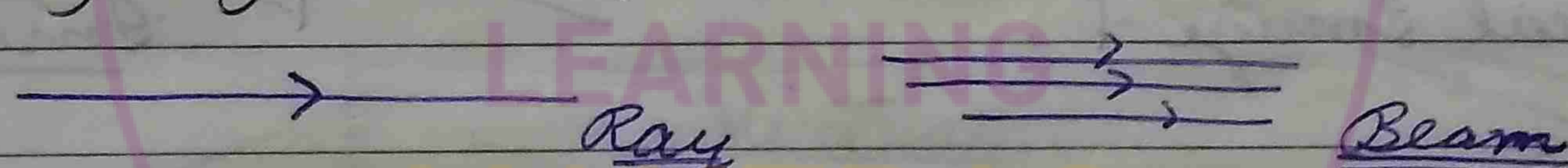
- light is an electromagnetic wave

Visible light: 400nm $\leq \lambda \leq$ 700nm

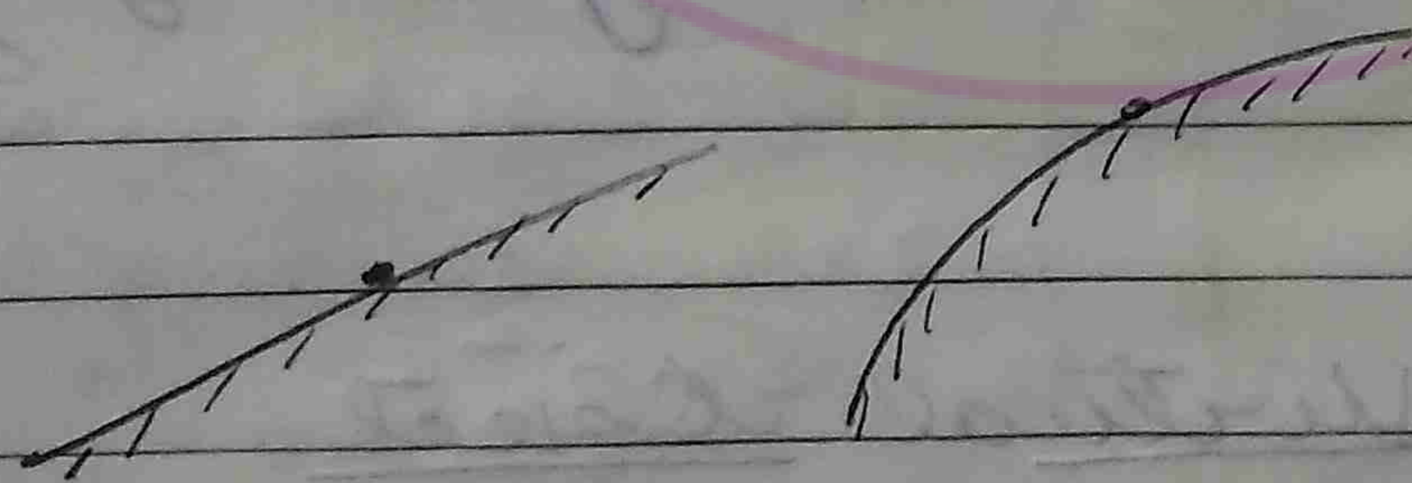
400nm ⇓ Blue, 700nm ⇓ Red

- If λ <<< size of obstacle then light travels in a straight line.

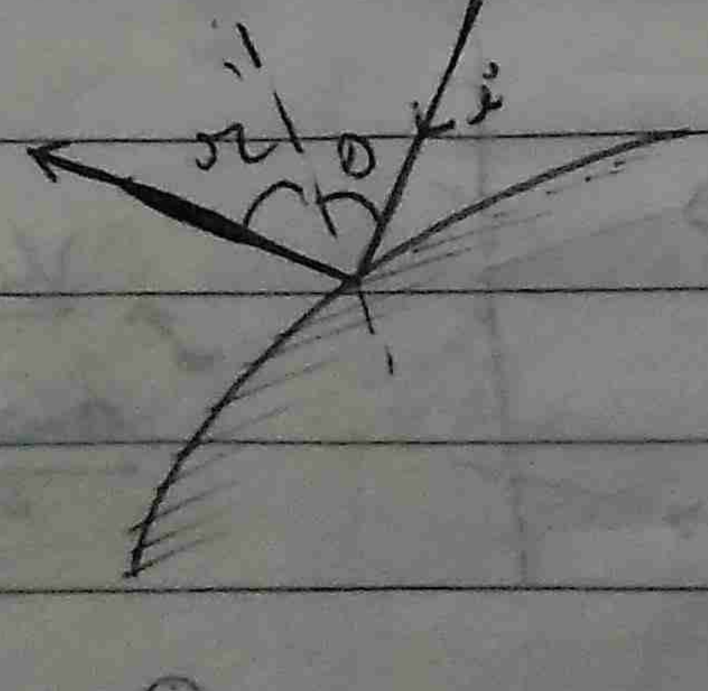
- Propagation of light

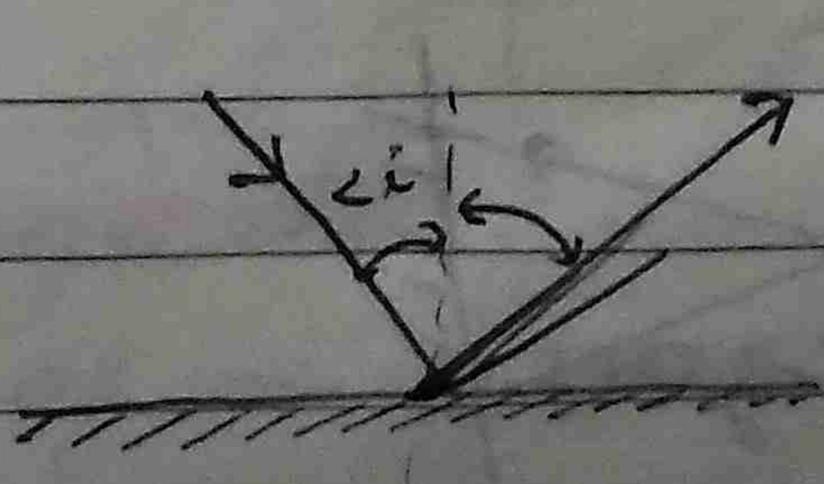


laws of reflection



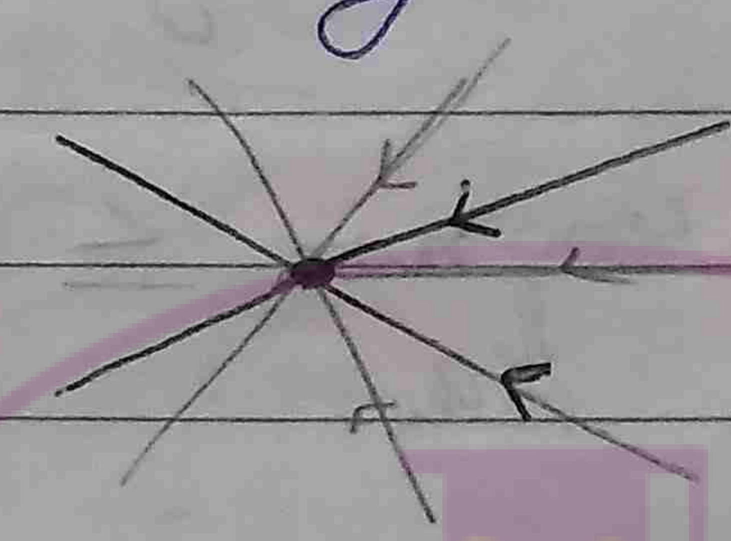
(i) $\angle i = \angle r$

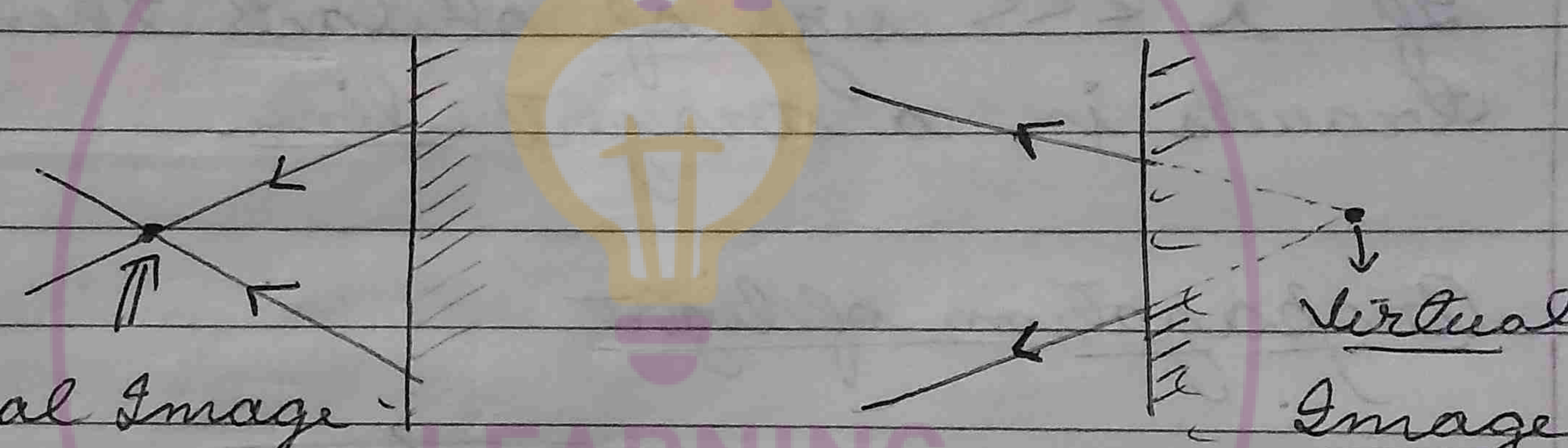




2 Incident Ray, Reflected ray and Normal are coplanar.

- Image → The intersection of reflected ray.

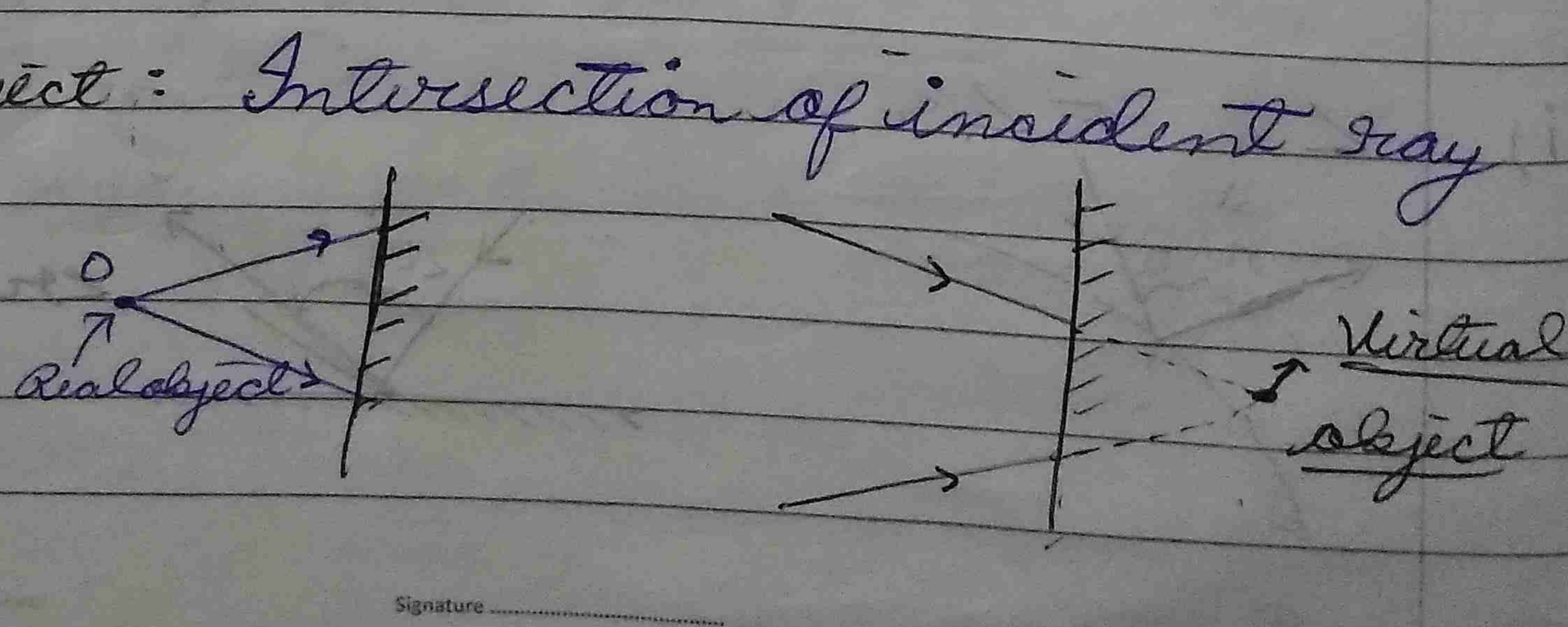




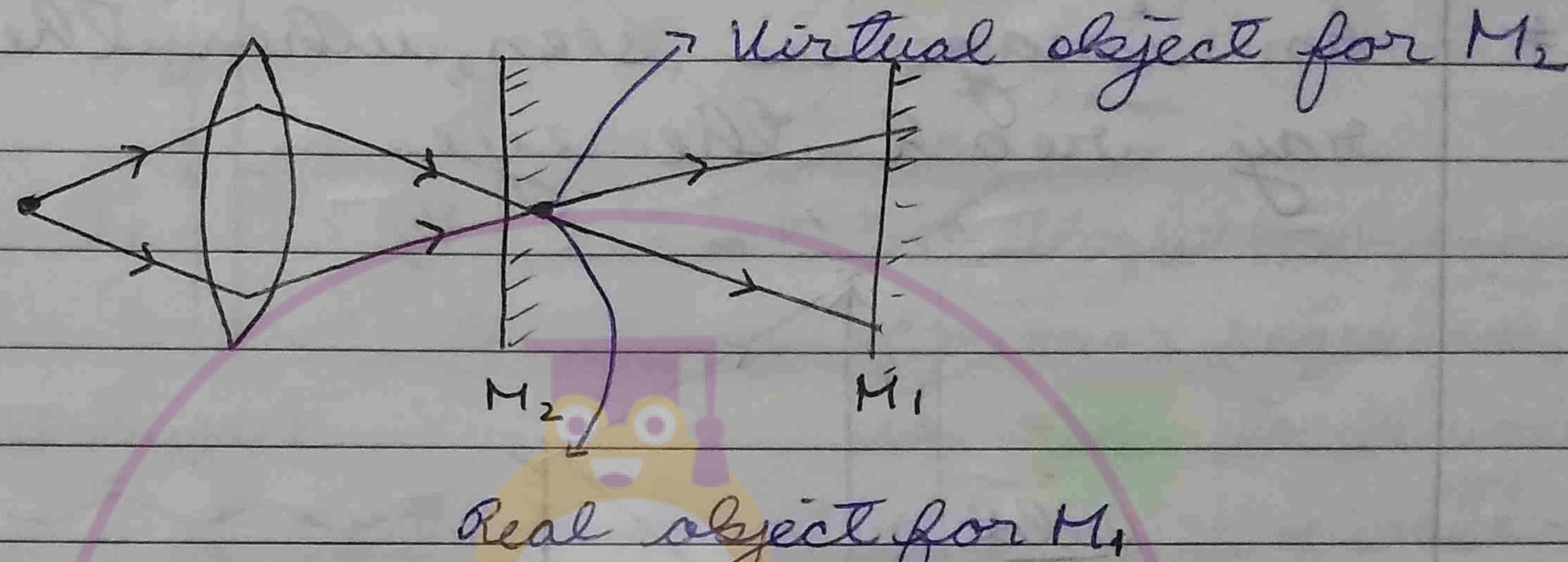
- If after reflection the rays converge → Real Image
- If after reflection the rays diverge → Virtual Image

- Real and Virtual Object

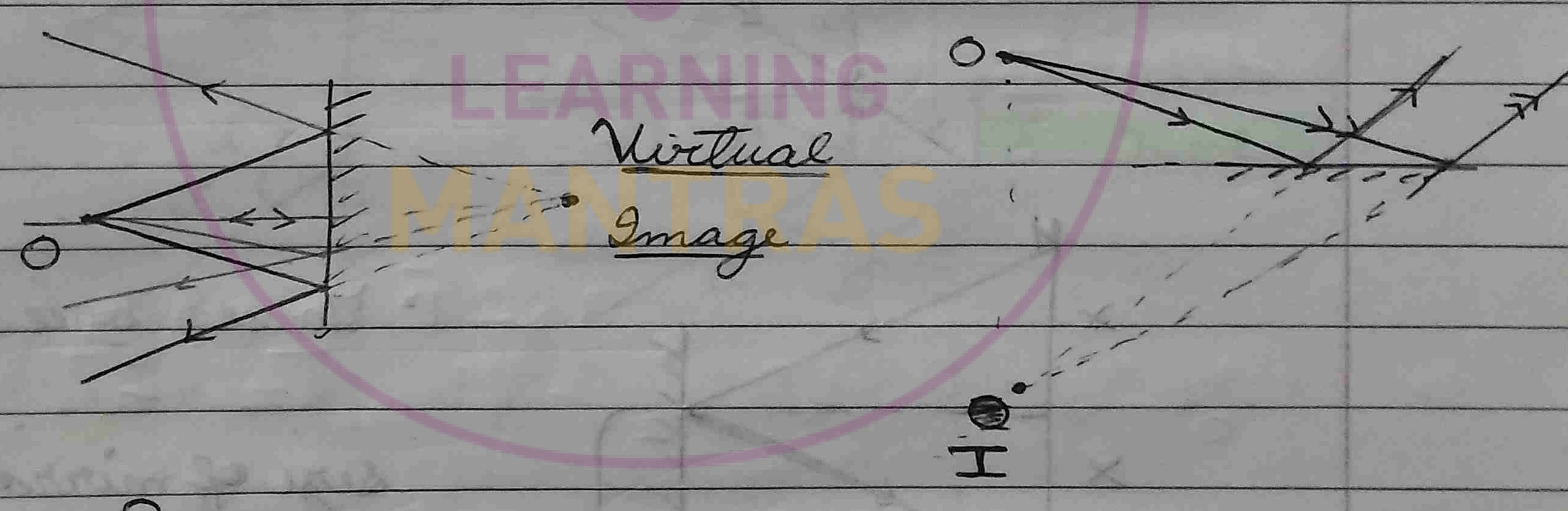
- Object : Intersection of incident ray

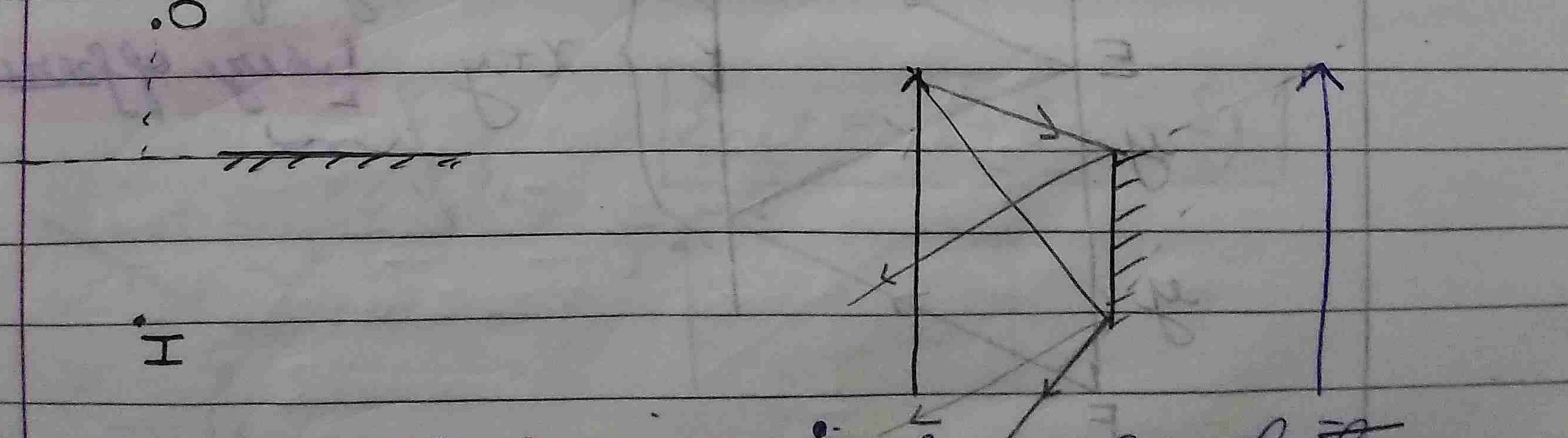


- If converging set of incident beam is blocked by an optical element then virtual object is formed.



- Image formation in Plane Mirror

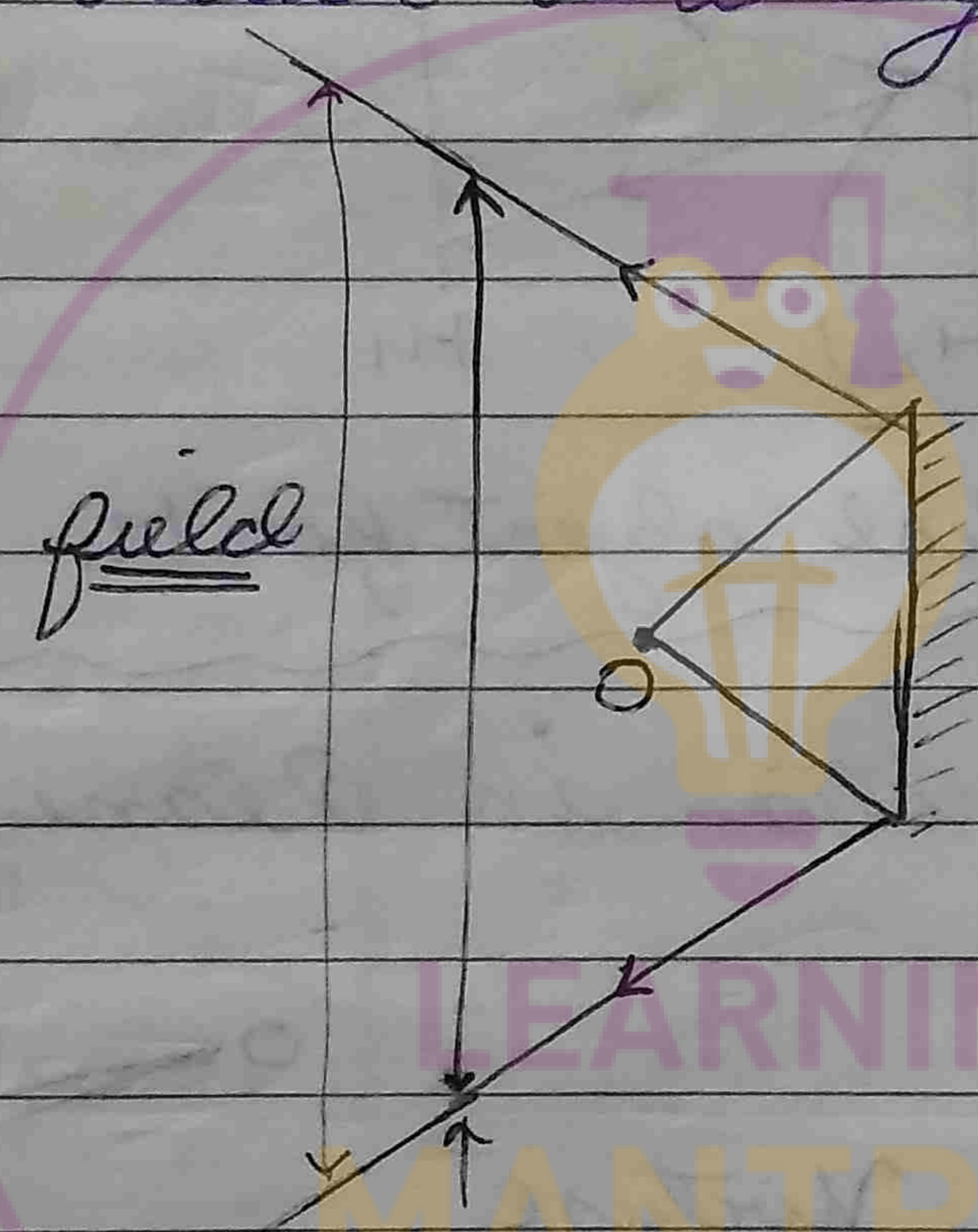


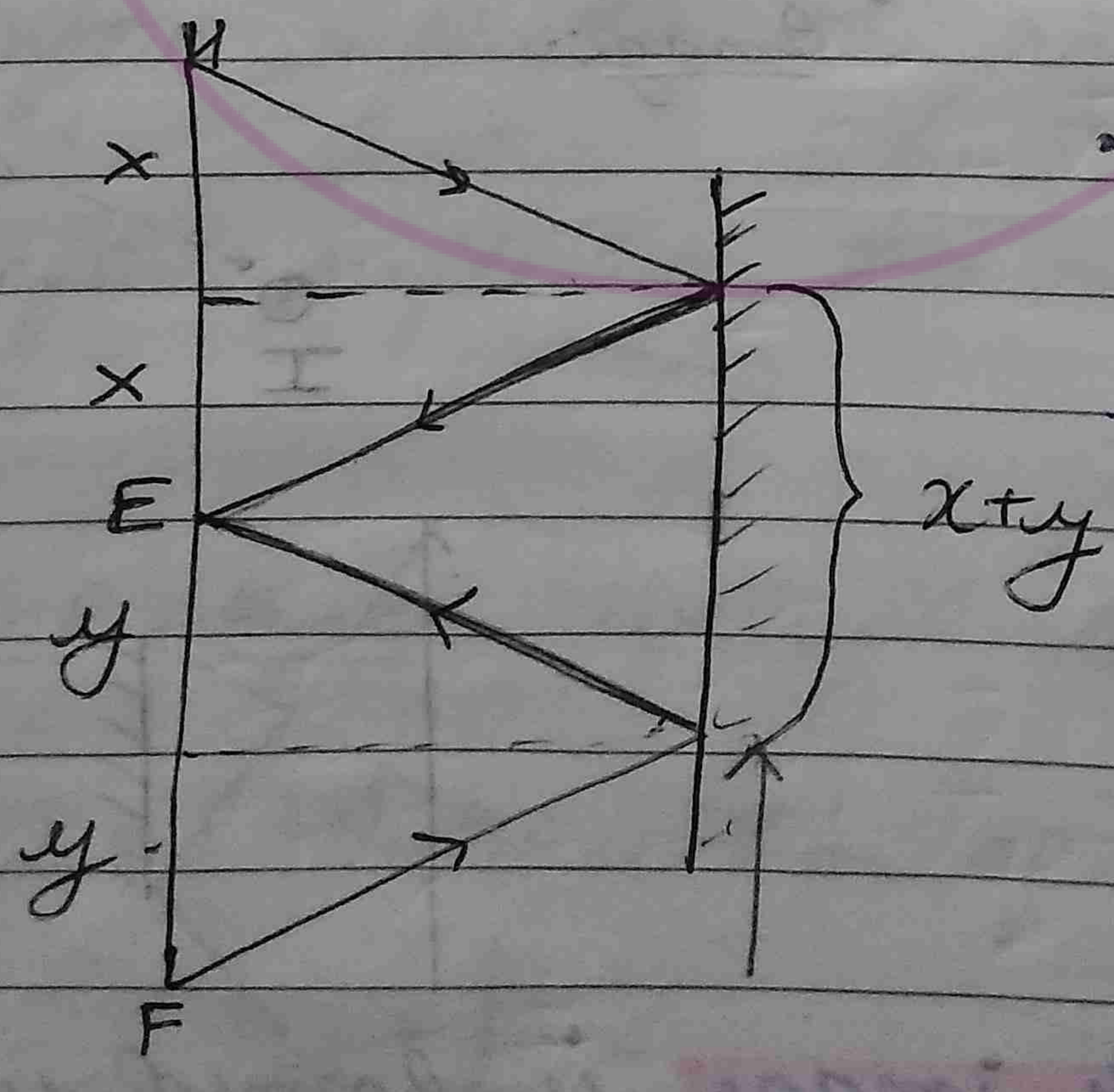


#* A complete image is formed whatever be the size of mirror.

★# If size of mirror is increased the brightness of image increases.

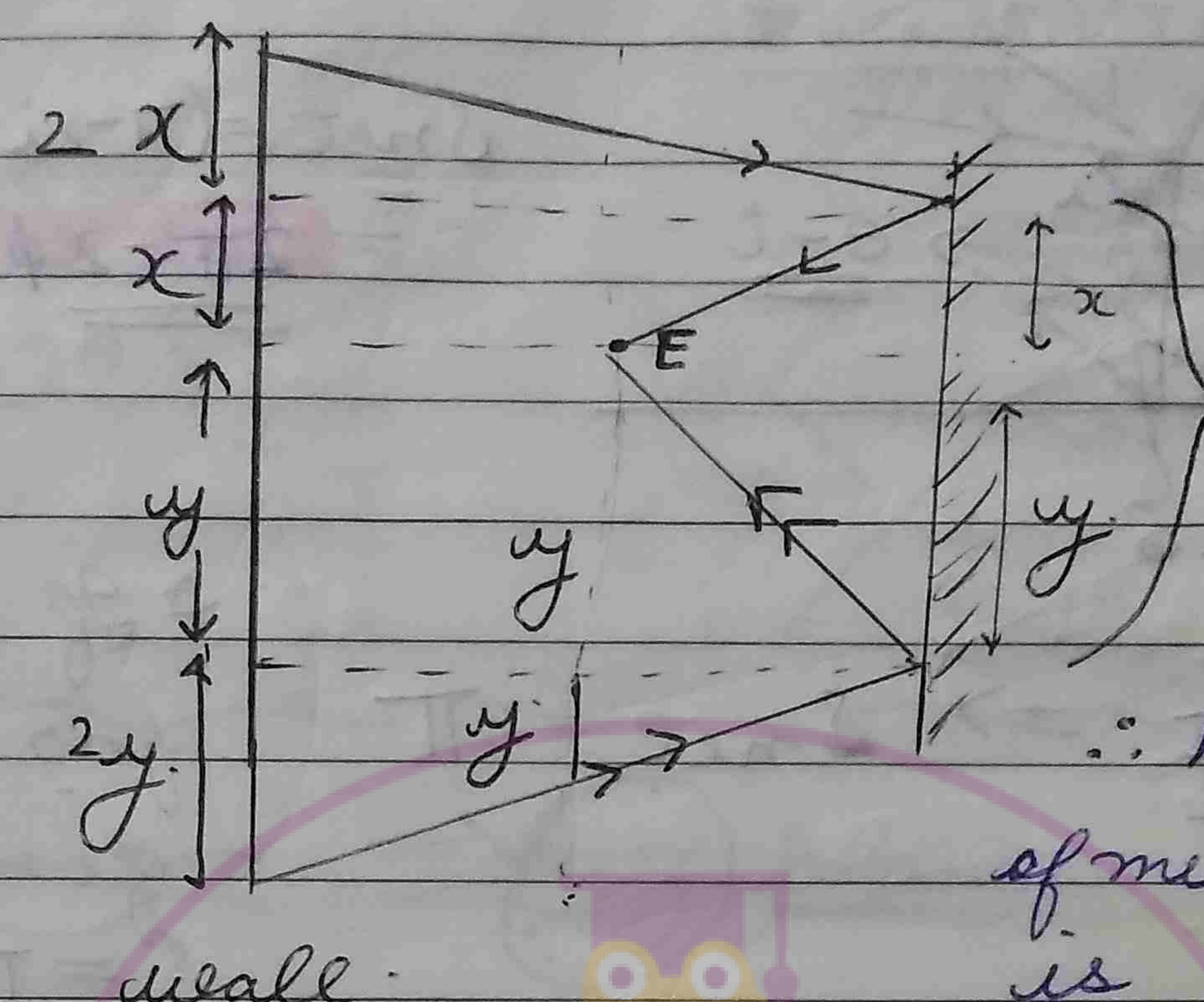
An image is seen, when the reflected ray reach the eye.





- To see ones image

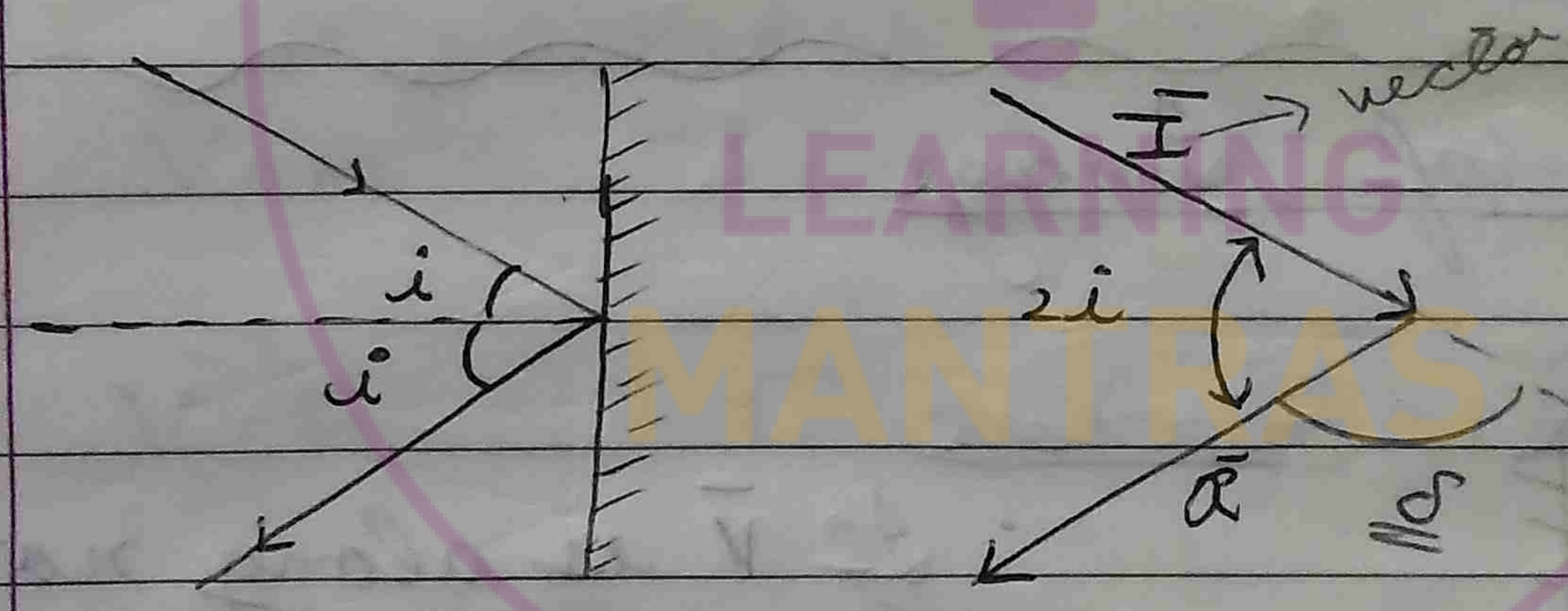
size of mirror ≥ $\frac{1}{2}$ size of person.



- If the person stands in the middle of the room.

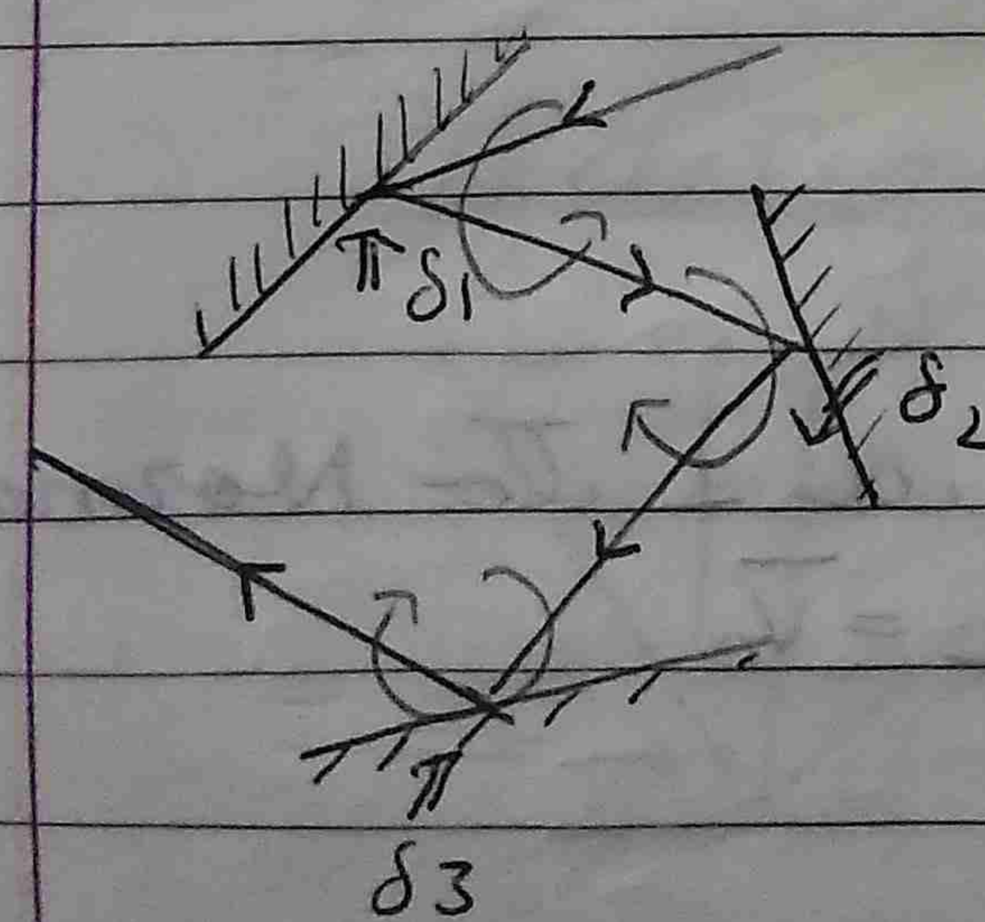
∴ Height (minimum) of mirror required is $= \frac{H}{3}$ to see wall.
Here H is size of wall.

- **Deviation:** It is the angle between incident and reflected ray.



$$\delta = \pi - 2i$$

Ex



$$\delta_{net} = \delta_2 + \delta_3 - \delta_1$$

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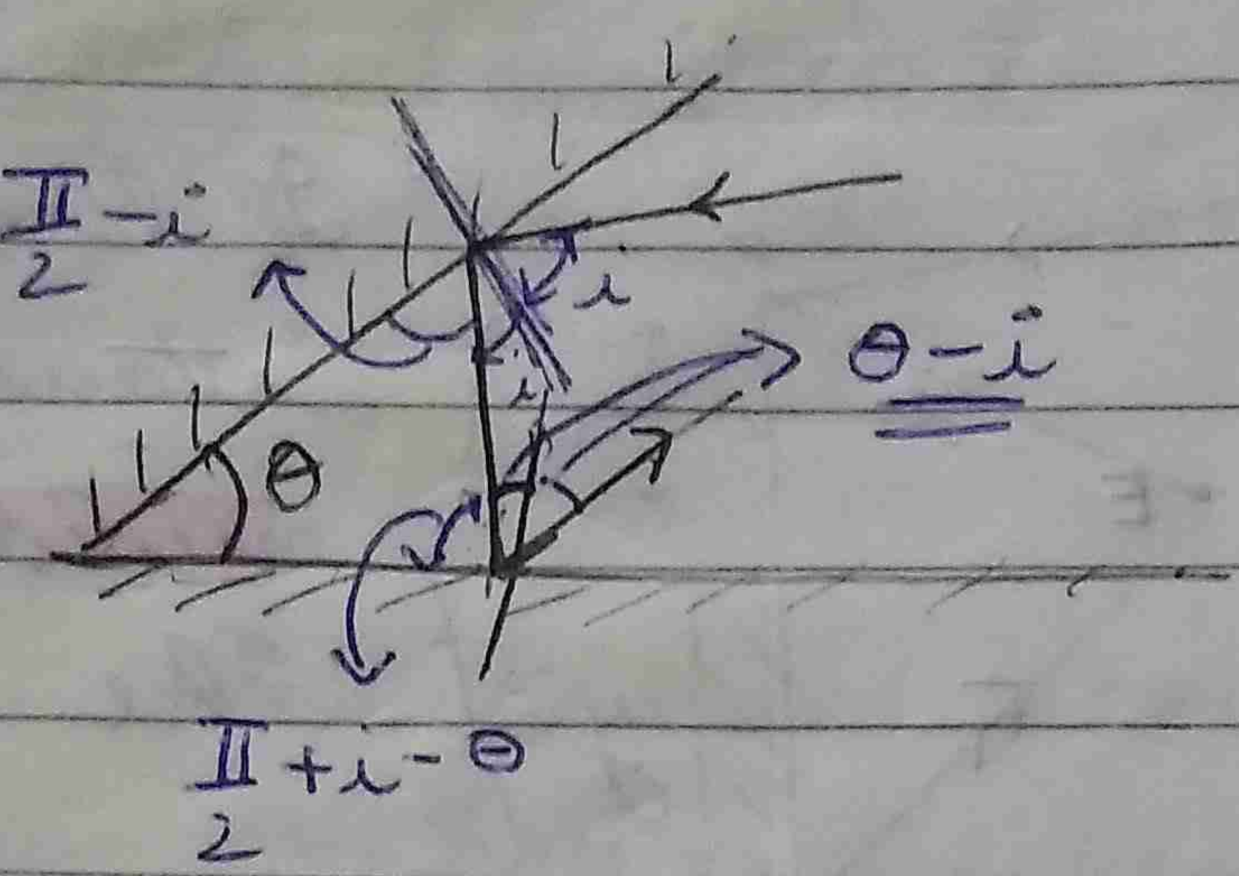
Ex

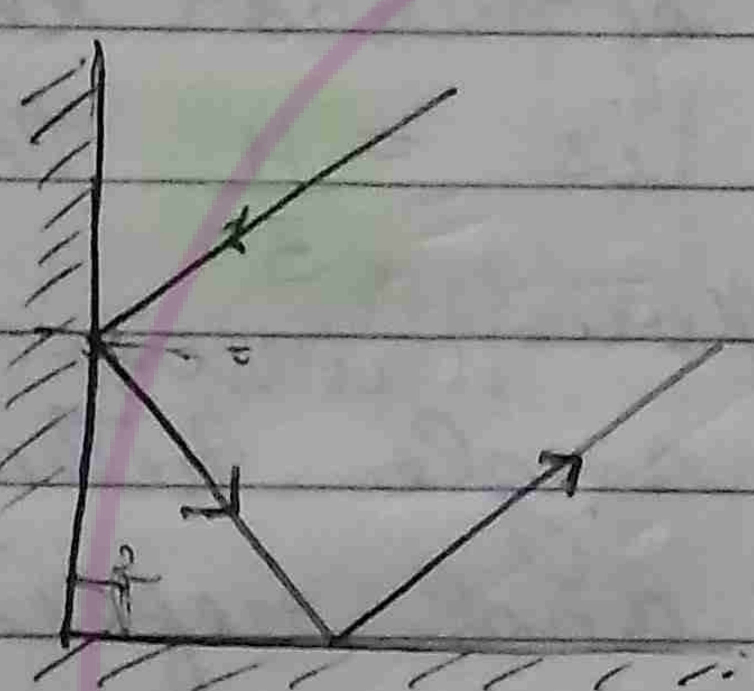


$\delta_{net} = (\pi - 2i) + (\pi - 2(\theta - i))$

$= 2\pi - 2\phi$

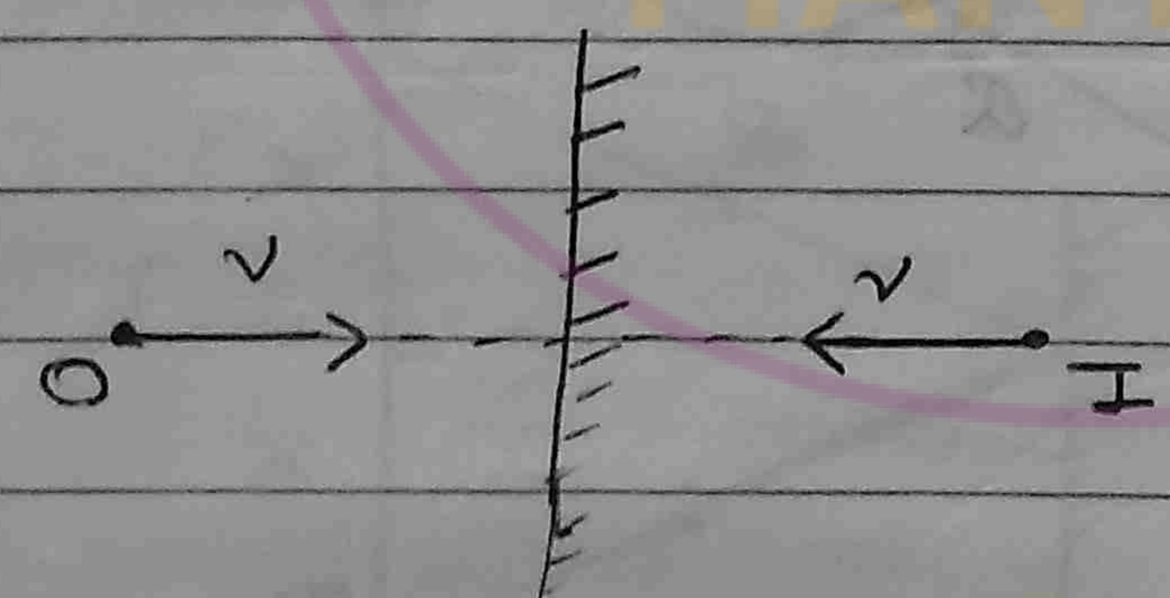
- If $\phi = \frac{\pi}{2} \Rightarrow \delta_{net} = \pi$

☆ If $\theta = \frac{\pi}{2}$ then for any angle of incidence $\delta = \pi$

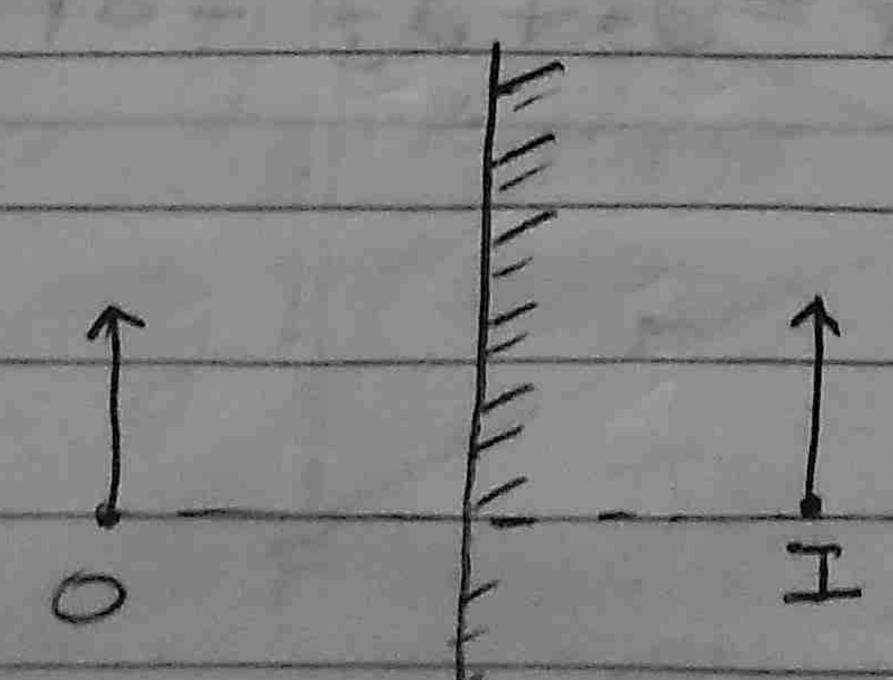


$\delta_{net} = \pi$

- **Velocity of Image**

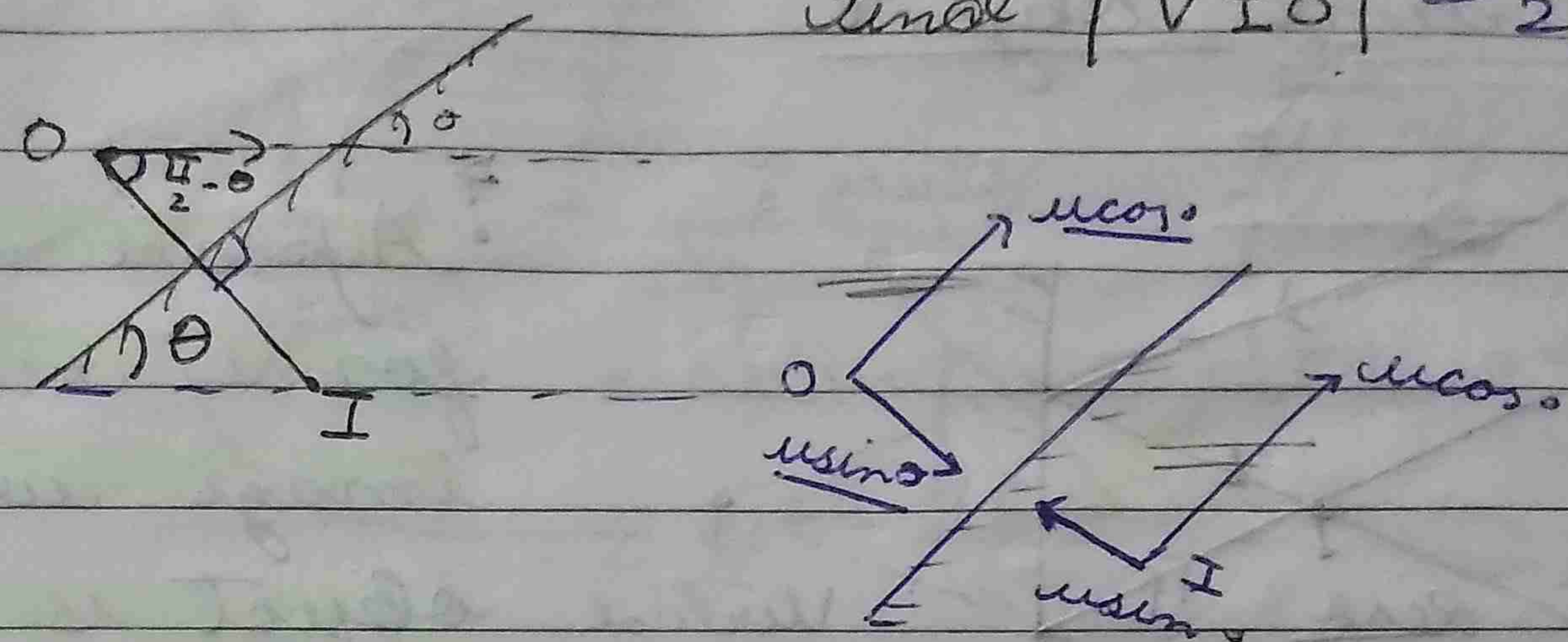


- If $\bar{V}$ is along normal then $\bar{V}_I = -\bar{V}_o$



- If $\bar{V}$ is ⊥ to Normal $\bar{V}_I = \bar{V}_o$

Final $|\vec{V}_{IO}| = 2u\sin\theta$



Mirror ⇒ yz plane

$\bar{V}_m = 5\hat{i} - 2\hat{j}$; $\bar{V}_o = 2\hat{i} - 3\hat{j} + \hat{k}$

find V_I

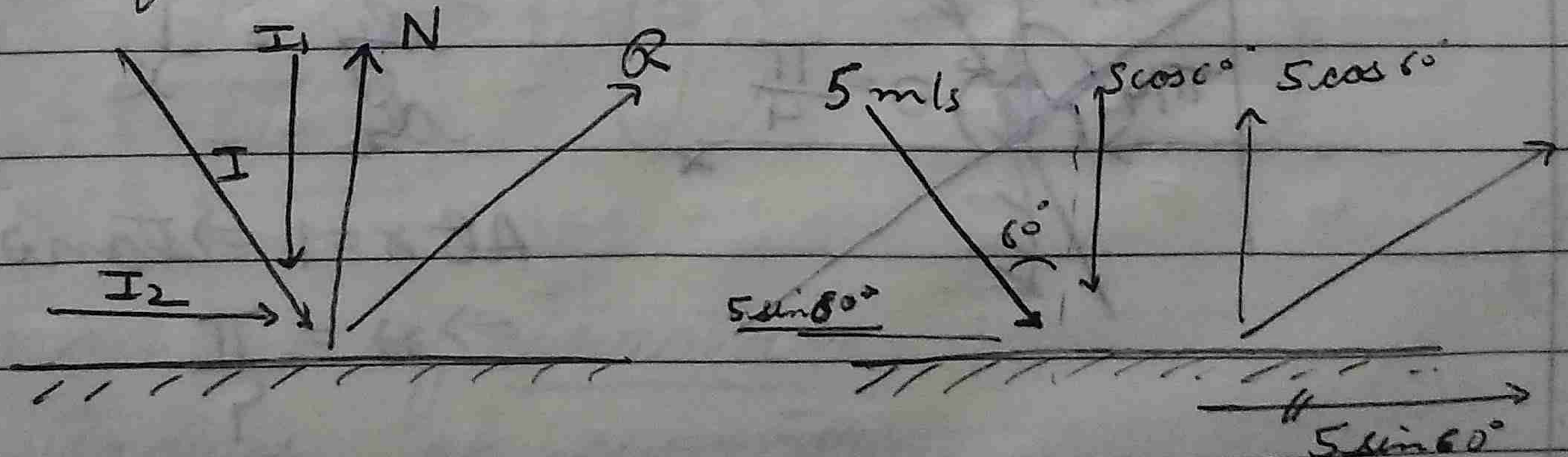
Normal ⇒ x axis

$\bar{V}_{OM} = -3\hat{i} - \hat{j} + \hat{k} \Rightarrow -3\hat{i} + (-1\hat{j} + 1\hat{k})$

$\vec{V}_{IM} = +3\hat{i} + (-1\hat{j} + 1\hat{k})$

$\bar{V}_I = \bar{V}_{IM} + \bar{V}_M = (3\hat{i} - \hat{j} + \hat{k}) + (5\hat{i} - 2\hat{j})$

- Reflection Vectorially



$\bar{I} = \bar{I}_1 + \bar{I}_2 \Rightarrow \bar{I}_1 = (\bar{I}.\hat{N})\hat{N}$; $\bar{I}_2 = \bar{I} - \bar{I}_1$

$\bar{R} = -\bar{I}_1 + \bar{I}_2$

• Object distance and image distance are measured perpendicularly from mirror.

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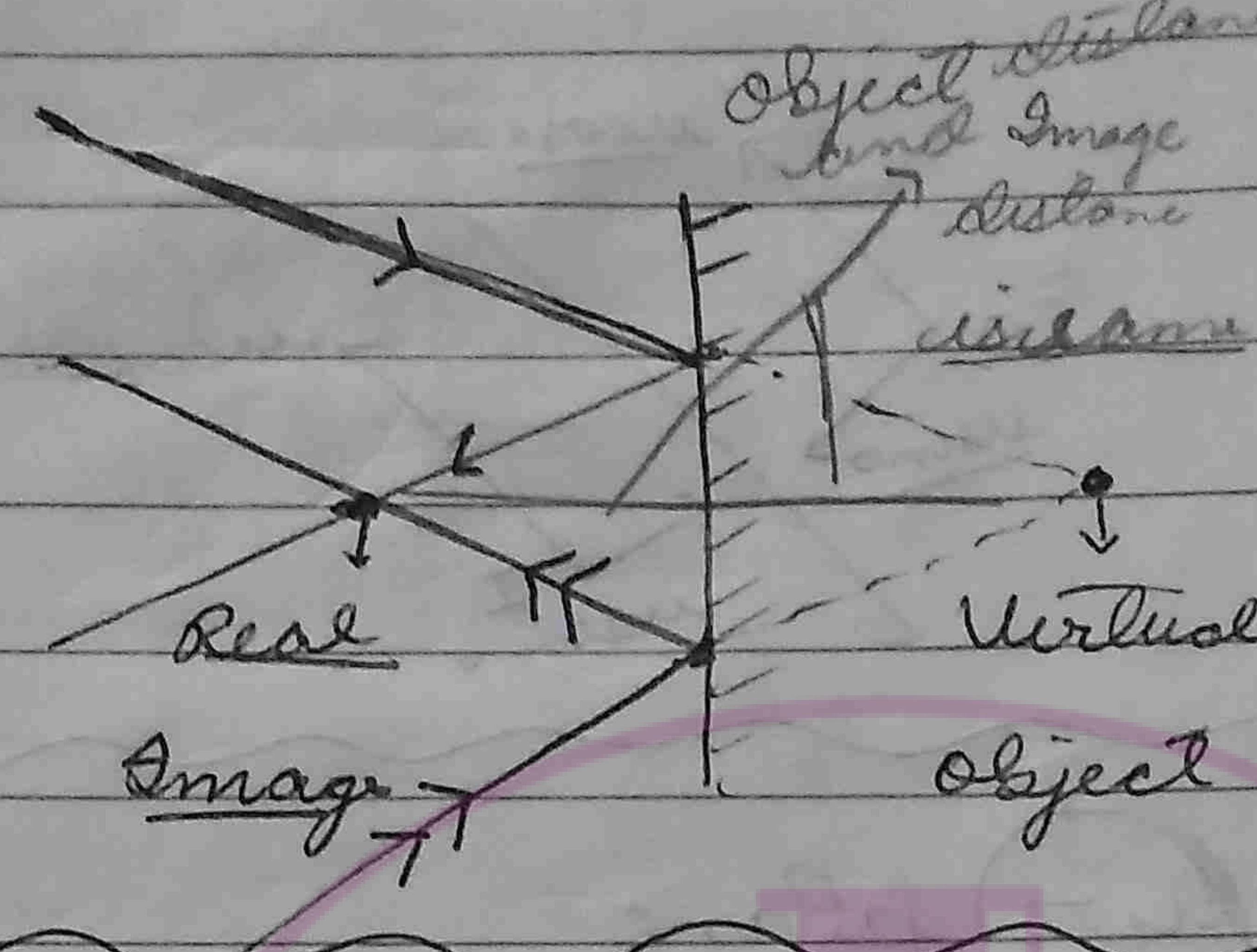
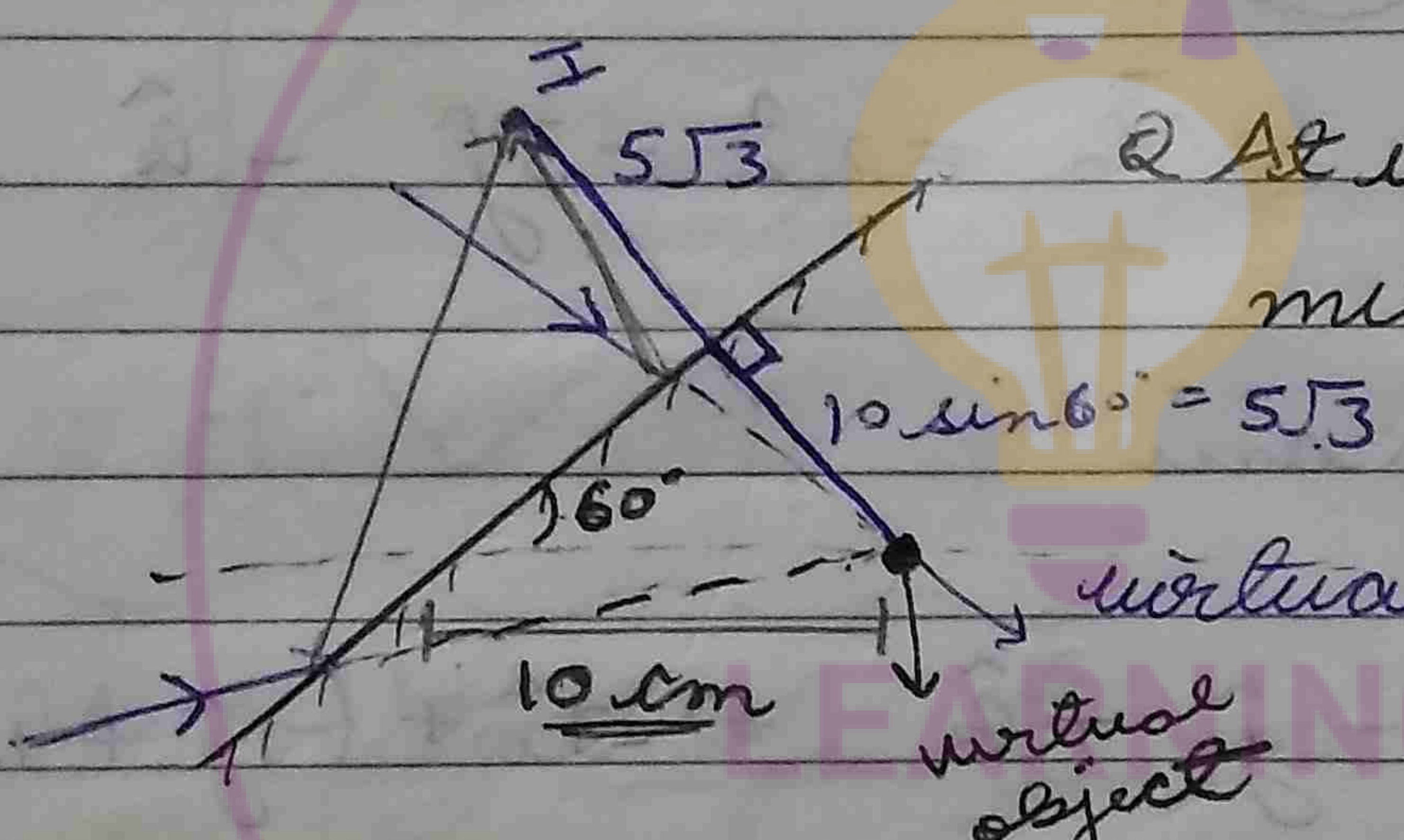
• ## Virtual Object



☆ • A plane mirror forms real image when object is virtual.

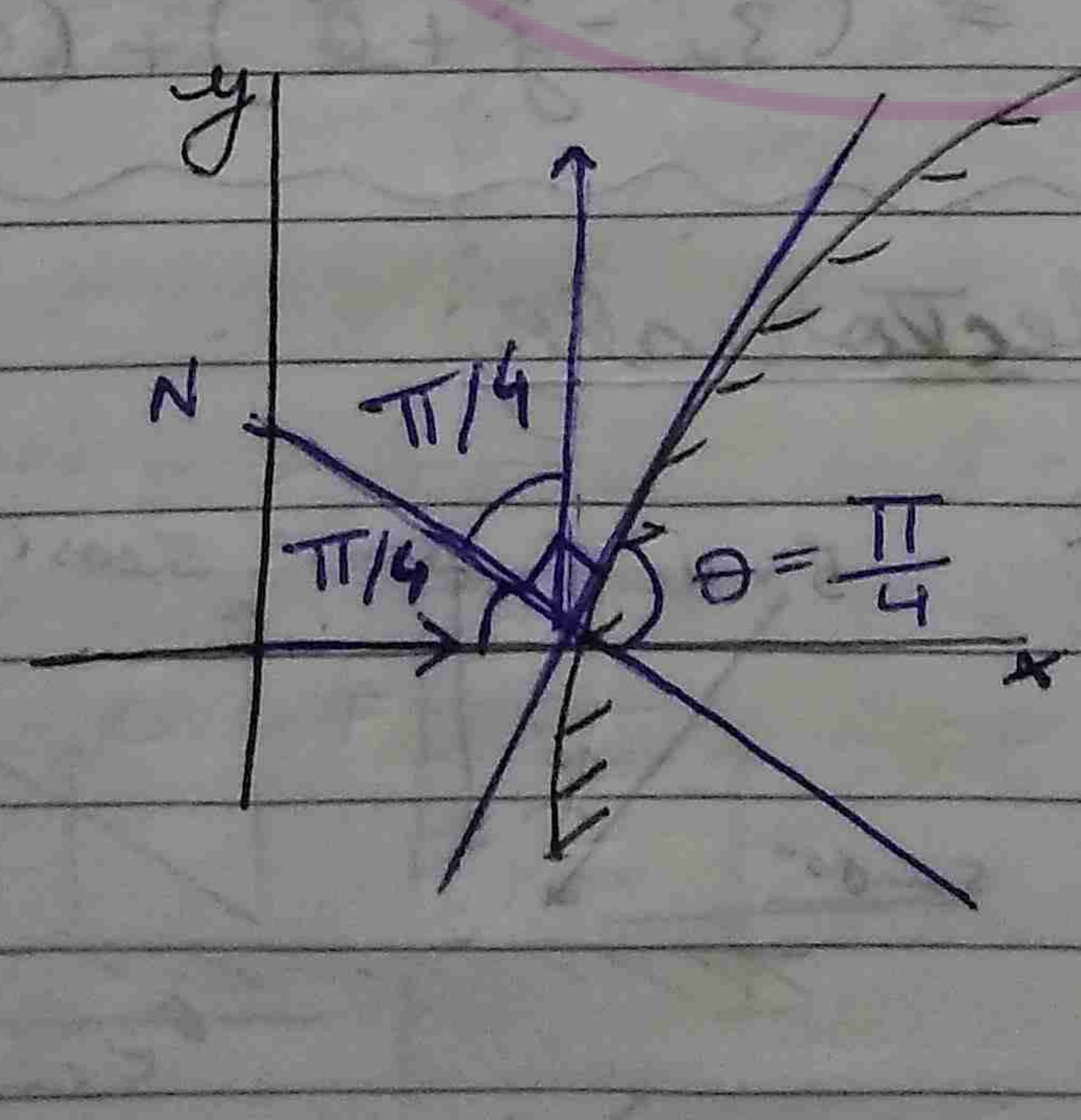
Ex



Q At what distance from the mirror is the image formed

• ## Curved Mirror

\#



$y = \ln x$ angle of reflection = ?

$y = \ln x$

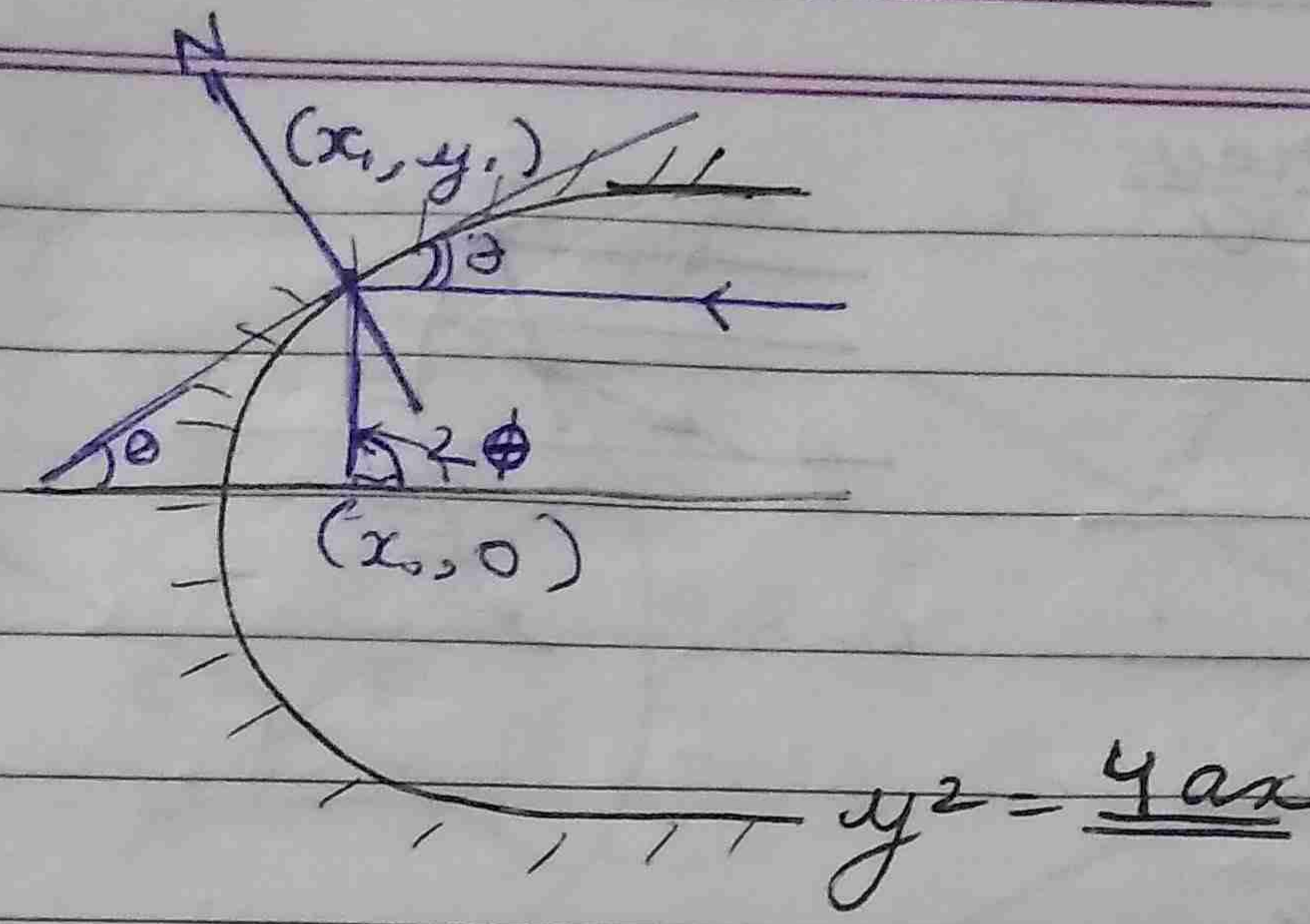
$\frac{dy}{dx} = \frac{1}{x}$

At $x = 1 \Rightarrow \tan\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

• Remember principle of reversibility!

#



$y^2 = 4ax$

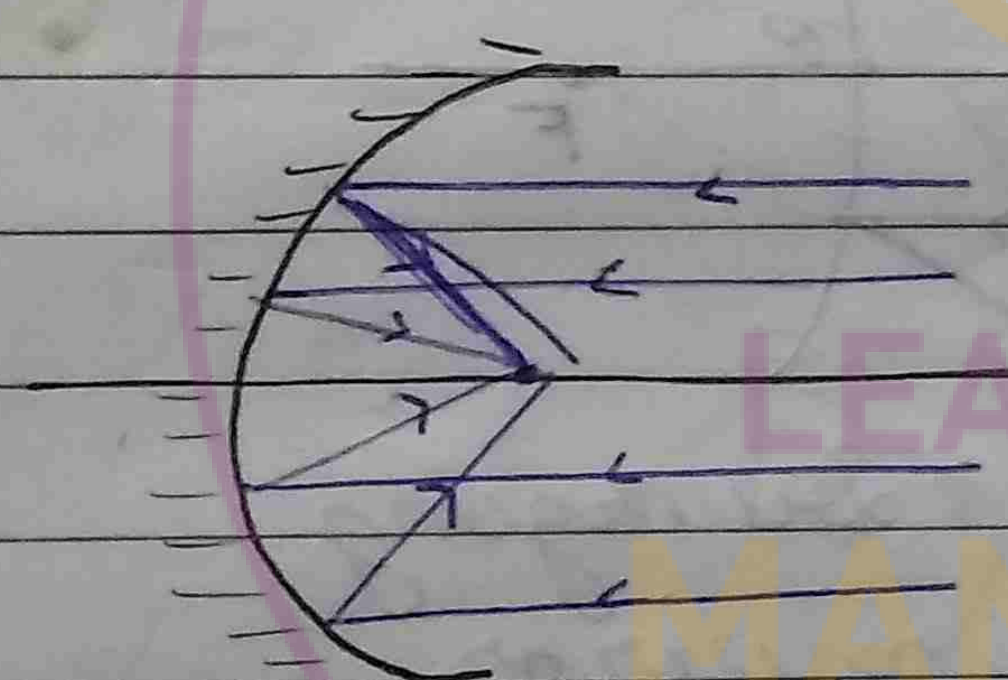
$y\dfrac{dy}{dx} = 2a$

$\tan\theta = \dfrac{2a}{y_1}$ - - (i)

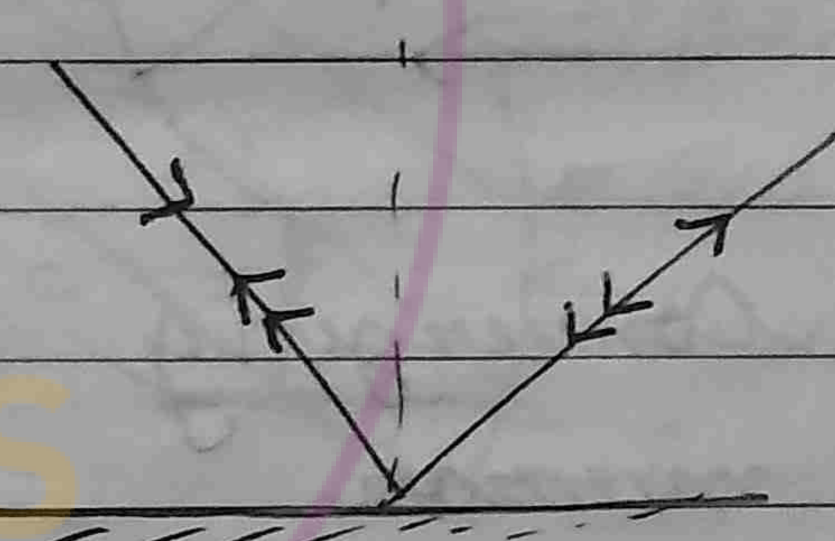
equation of reflected line $(y - y_1) = m(x - x_1)$

$0 - y_1 = \tan 2\theta\,(x_0 - x_1)$ - - (ii) $\quad y_1 = 4ax_1$ - - (iii)

$\Rightarrow \underline{x_0 = a}$

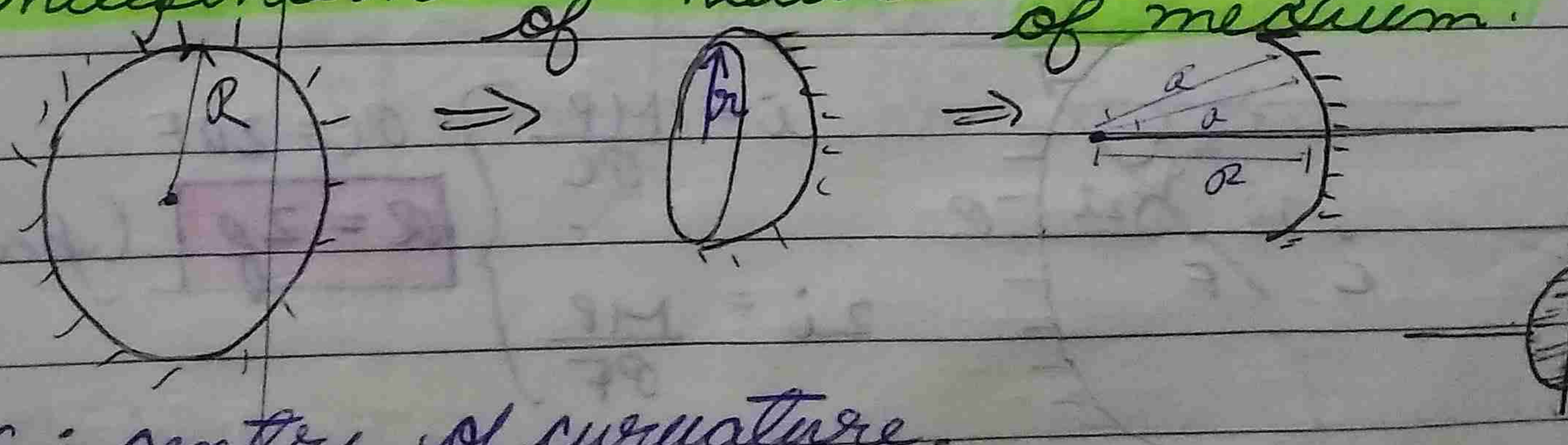


#



• Spherical Mirrors

☆ Focal length of spherical mirror is independent of nature of medium.



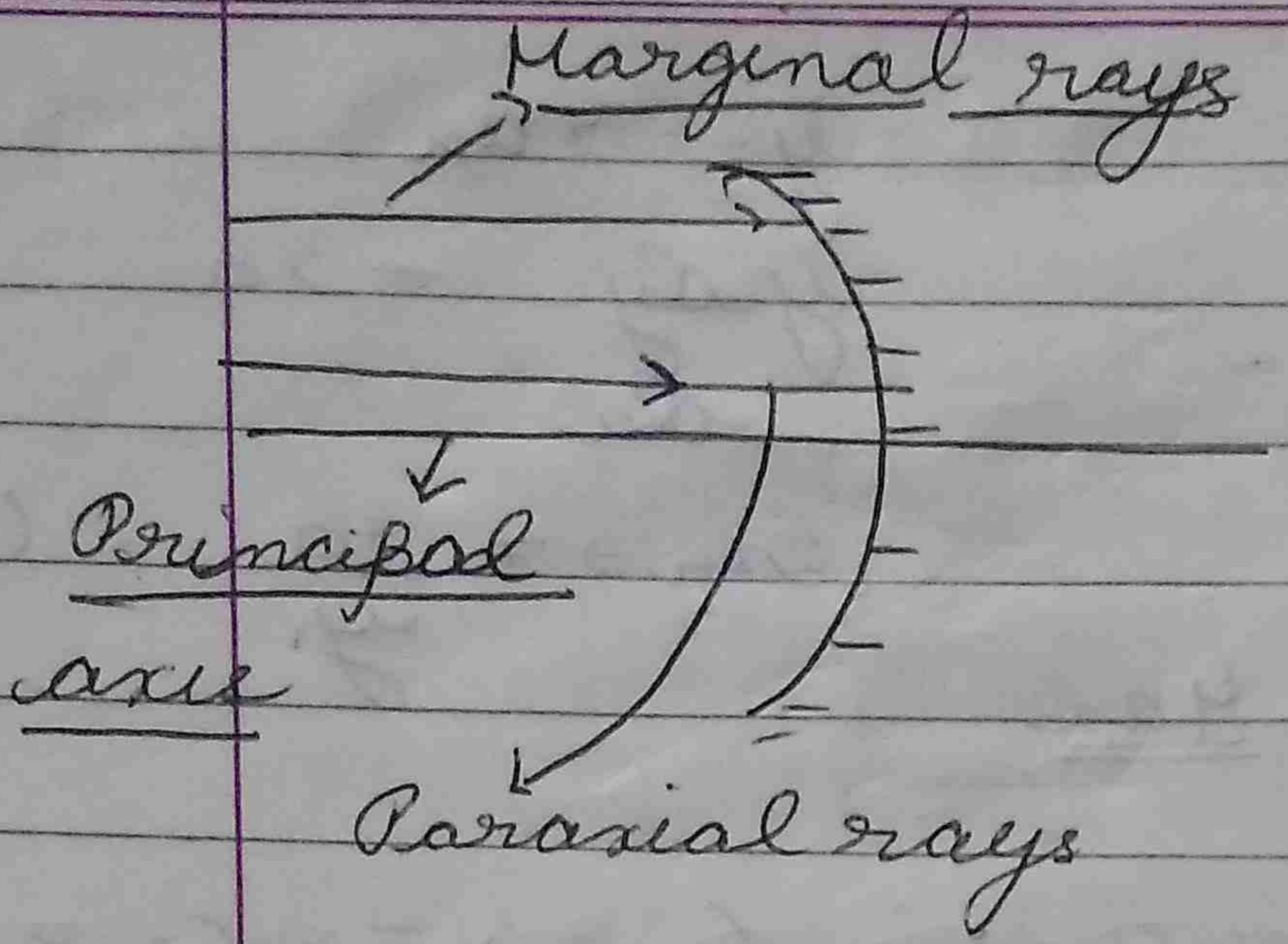
C : centre of curvature.

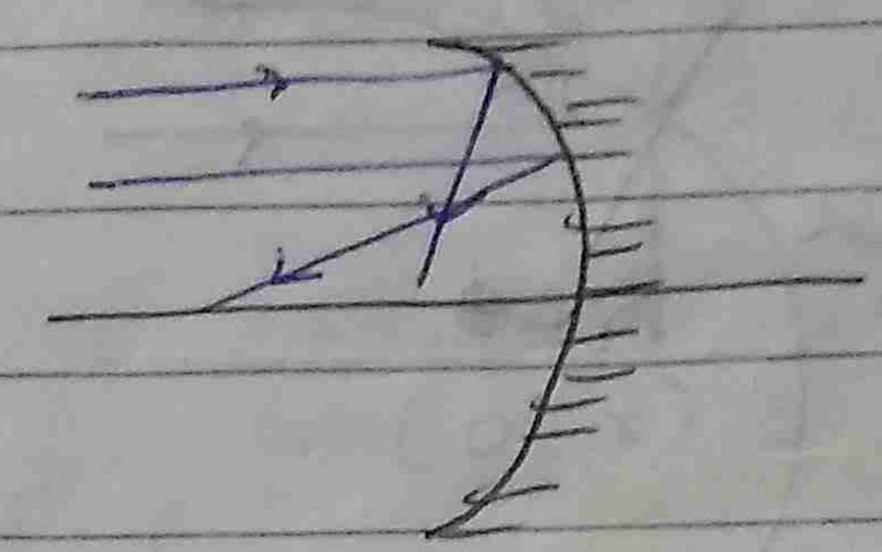
R : Radius of curvature

P - - - pole.

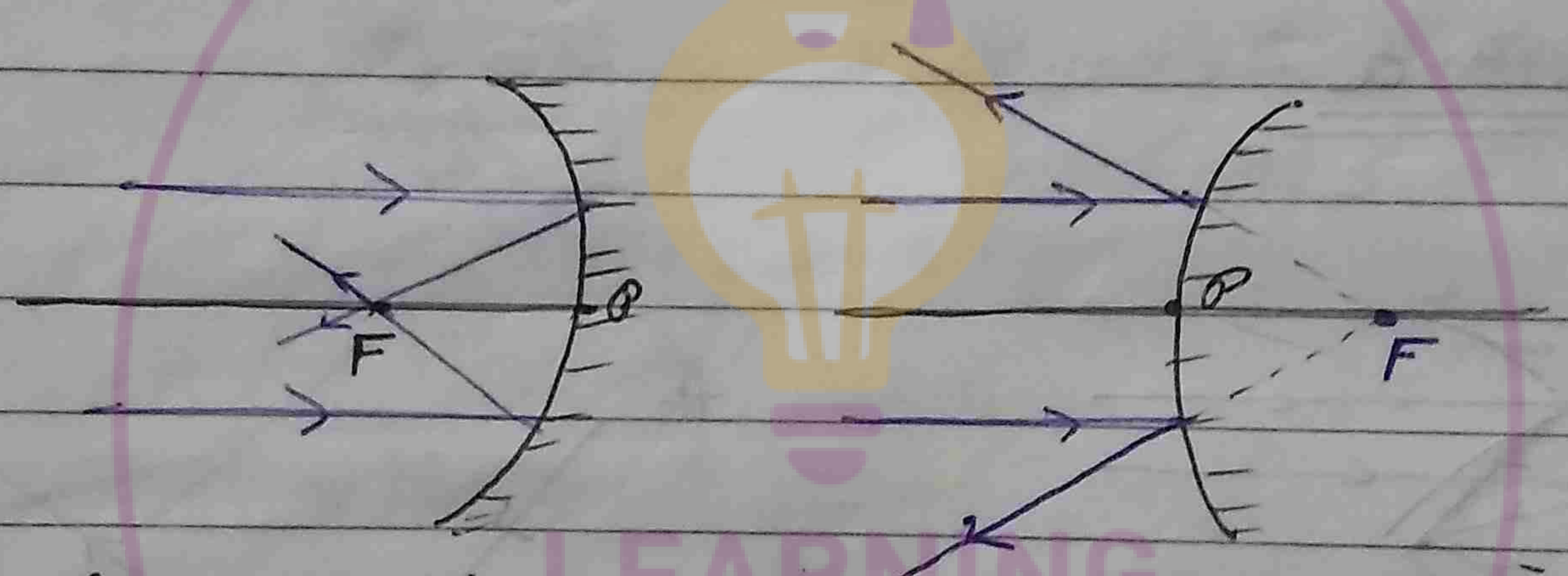
r → radius of aperture

- For a sphere, normal to the surface is radius.



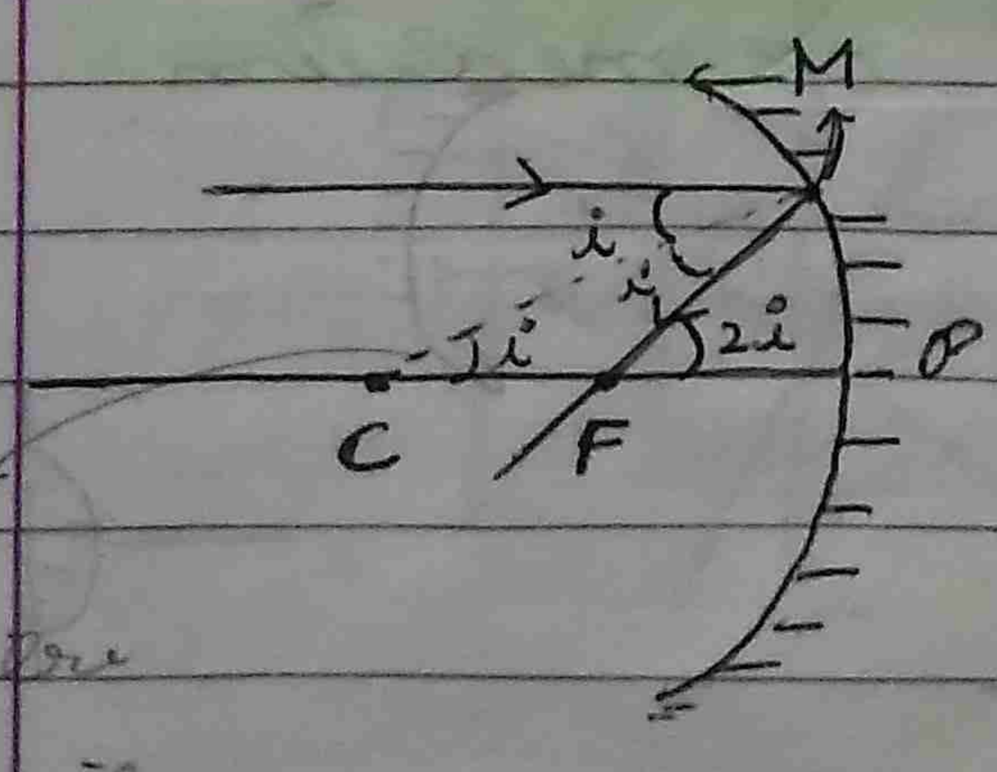


- Focus



Converging mirror   Diverging mirror

$PF = f$ --- focal length



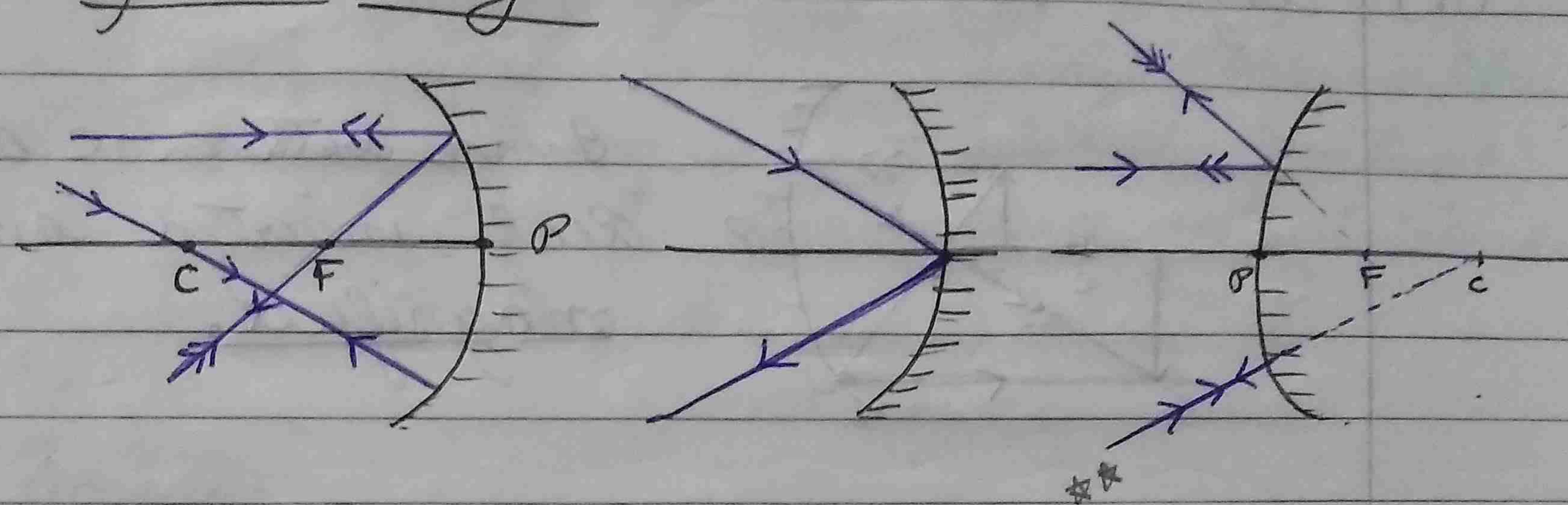
$$i = \frac{MP}{PC}$$
$$2i = \frac{MP}{PF}$$

$PC = 2PF$

$R = 2f$ (paraxial)

any line from centre of curvature to the mirror acts as normal

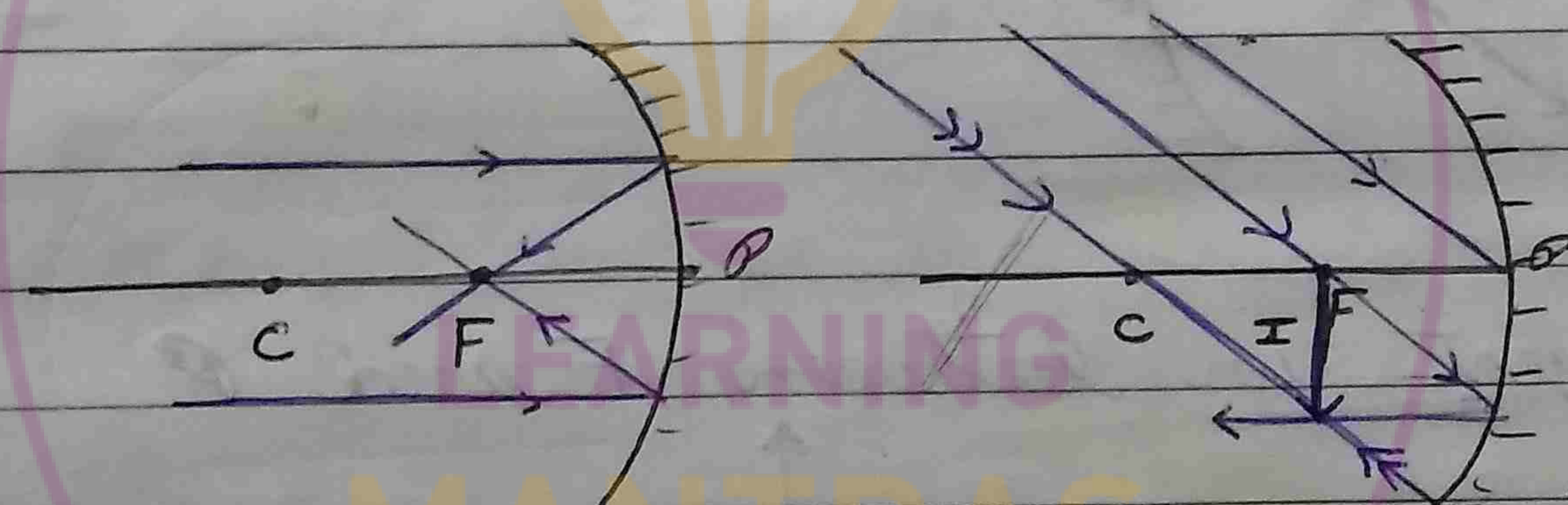
- Special Rays



- Ray Tracing

(a) Concave Mirror

(i) Object at Infinity



- Image at F
Real,
Inverted and
Diminished.

(ii) Object between ∞ and C

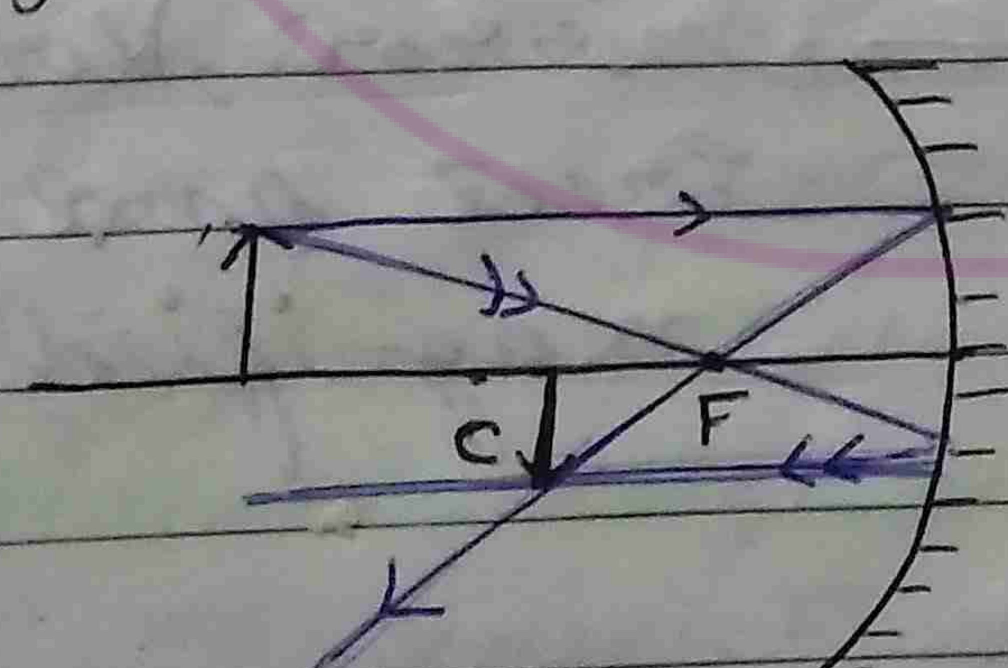
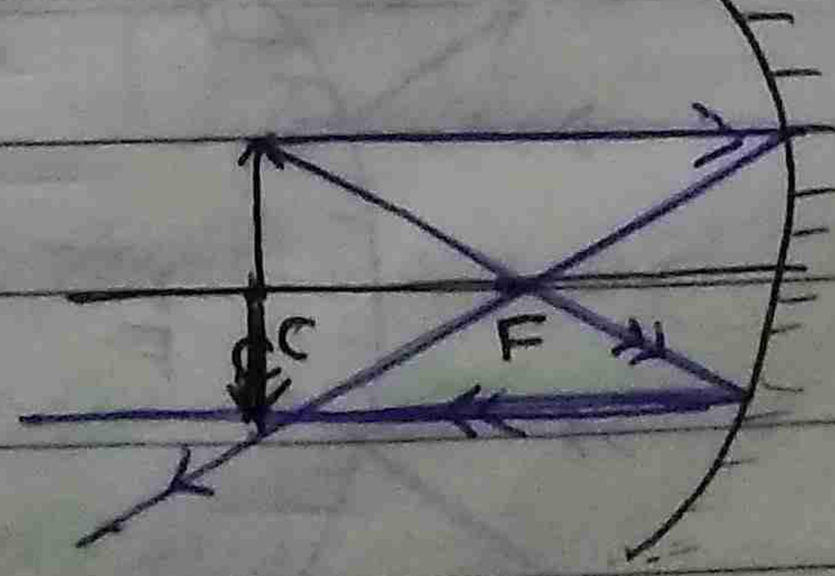


Image b/w F and C
Real, Inverted and
Diminished.

(iii) Object at C



- Image at C,
Real, Inverted and
of same size.

(iv) Object between C and F

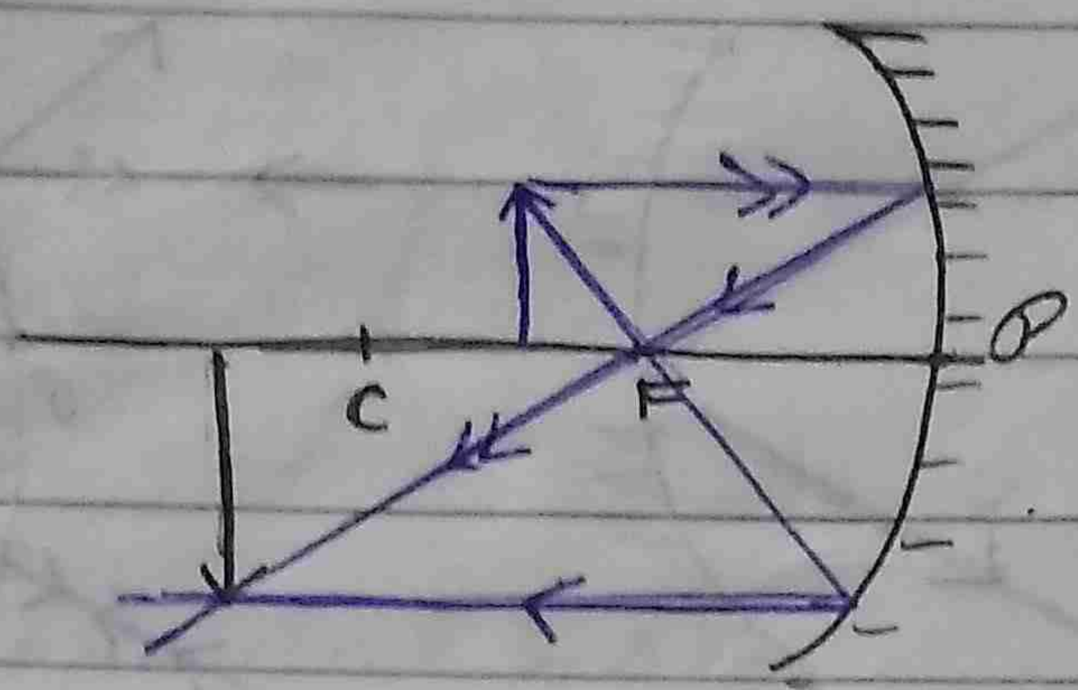


Image between C and ∞
Real, inverted and *magnified*.

(v) Object at F

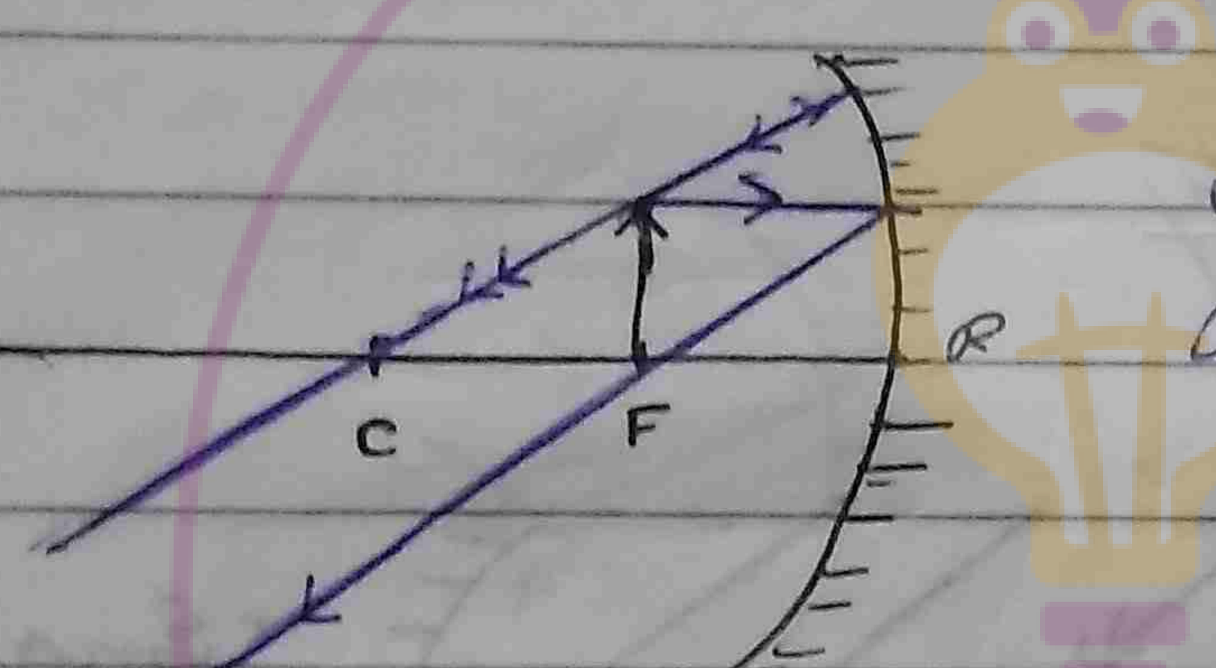
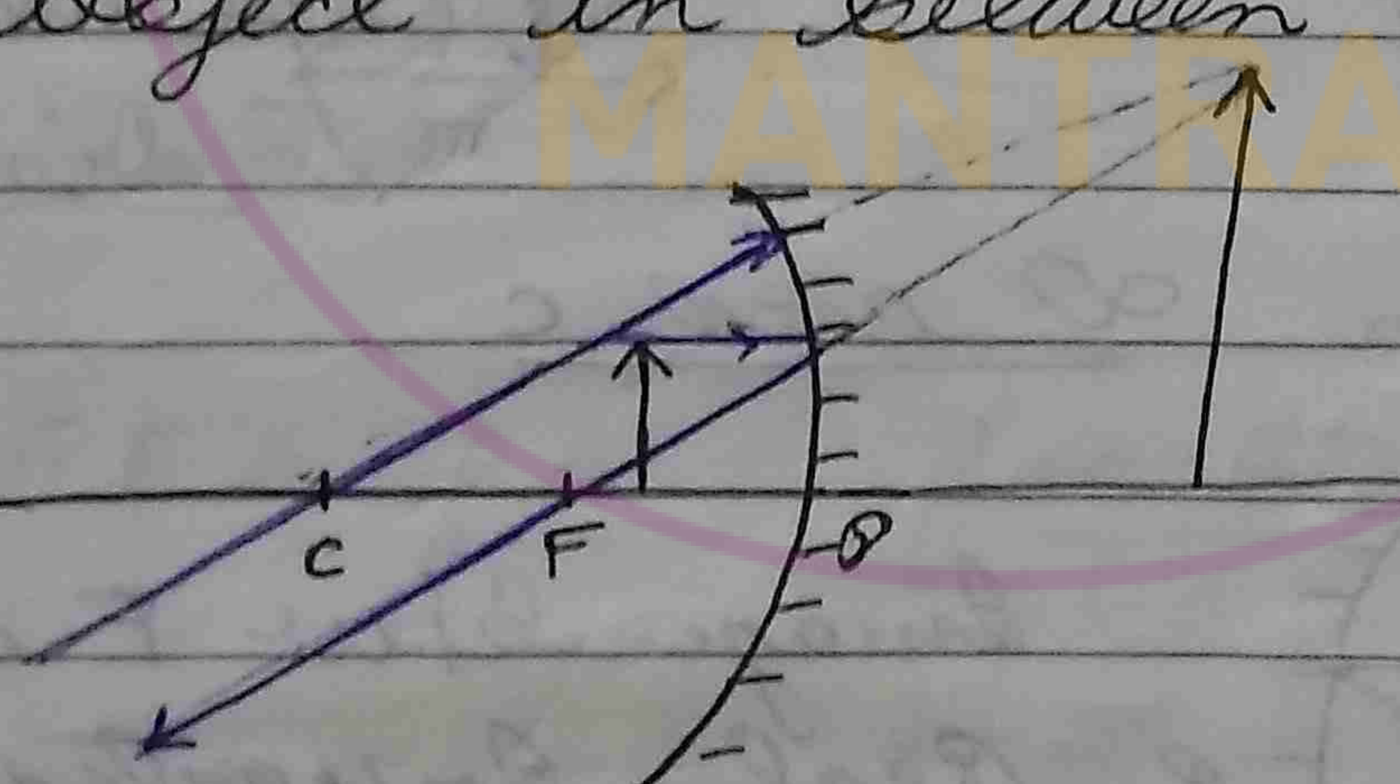


Image at ∞
Real, Inverted and magnified

(vi) Object in between F and P



Image, behind the mirror, Virtual, Erect and magnified.

(b) Convex Mirror

(i) Object is at infinity

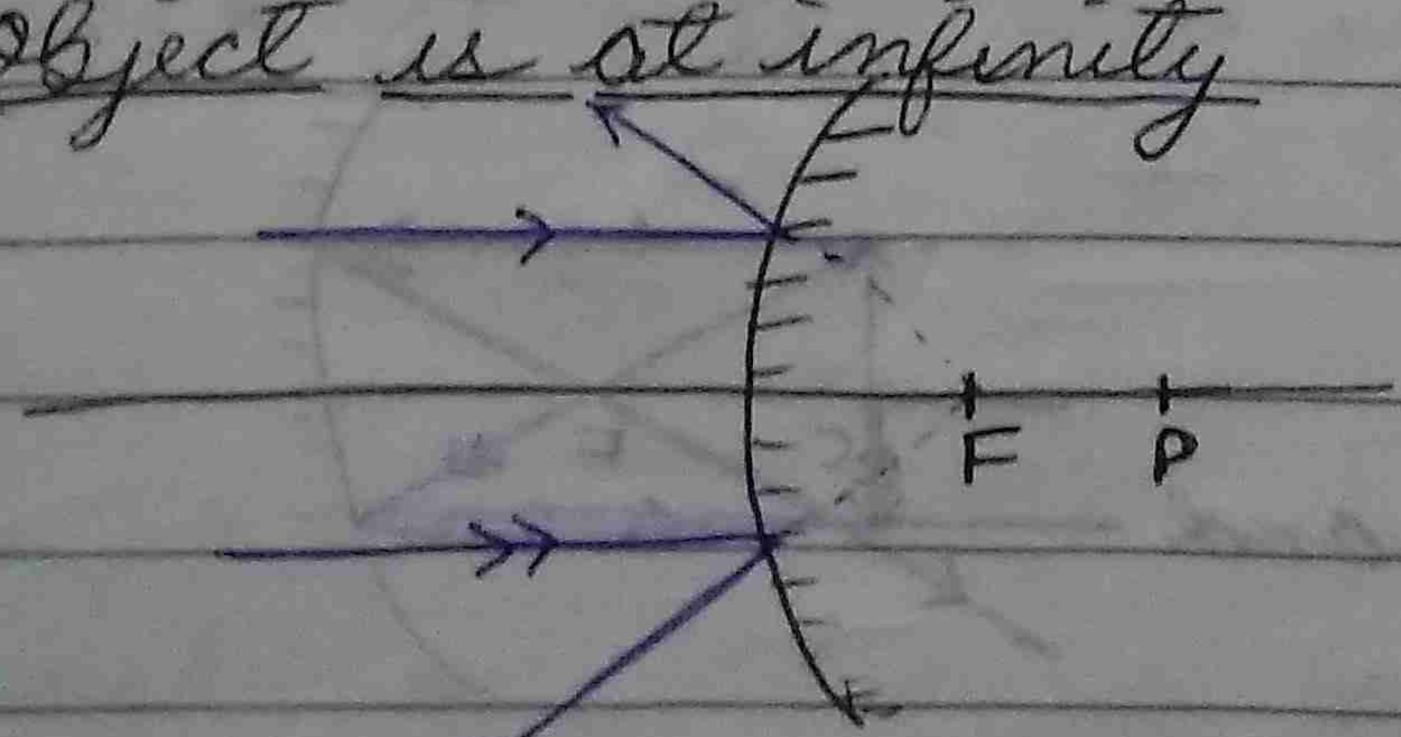


Image at F, virtual, erect and diminished

(ii) Object placed anywhere.

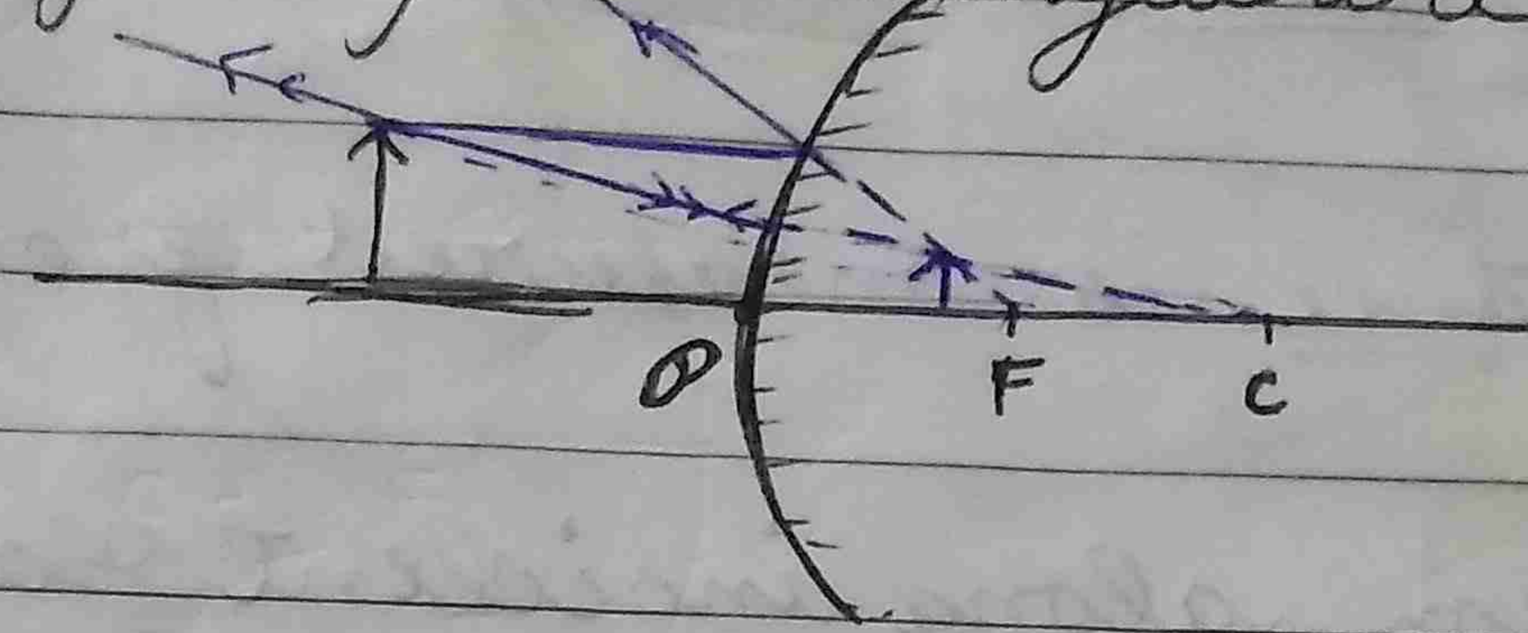


Image behind the mirror, Virtual, Erect, diminished.

• Concave

Object position	Image position	Nature
At ∞	At F	R, I, D
b/w C and ∞	b/w C and F	R, I, D
C	C	R, I, of same size
b/w C and F	b/w C and ∞	R, I, M
F	∞	R, I, M
b/w F and O	behind the mirror	V, E, M

• Convex

Object position	Image position	Nature
At ∞	F (behind)	V, E, D
Anywhere	behind	V, E, D

• Real image is ~~erect~~ inverted
• Virtual image is erect

⇒ if the object is real and it is one optical system. (valid for concave → ∞ ⇄ F, C ⇄ C (till image is real))

★ Characteristics of virtual image:

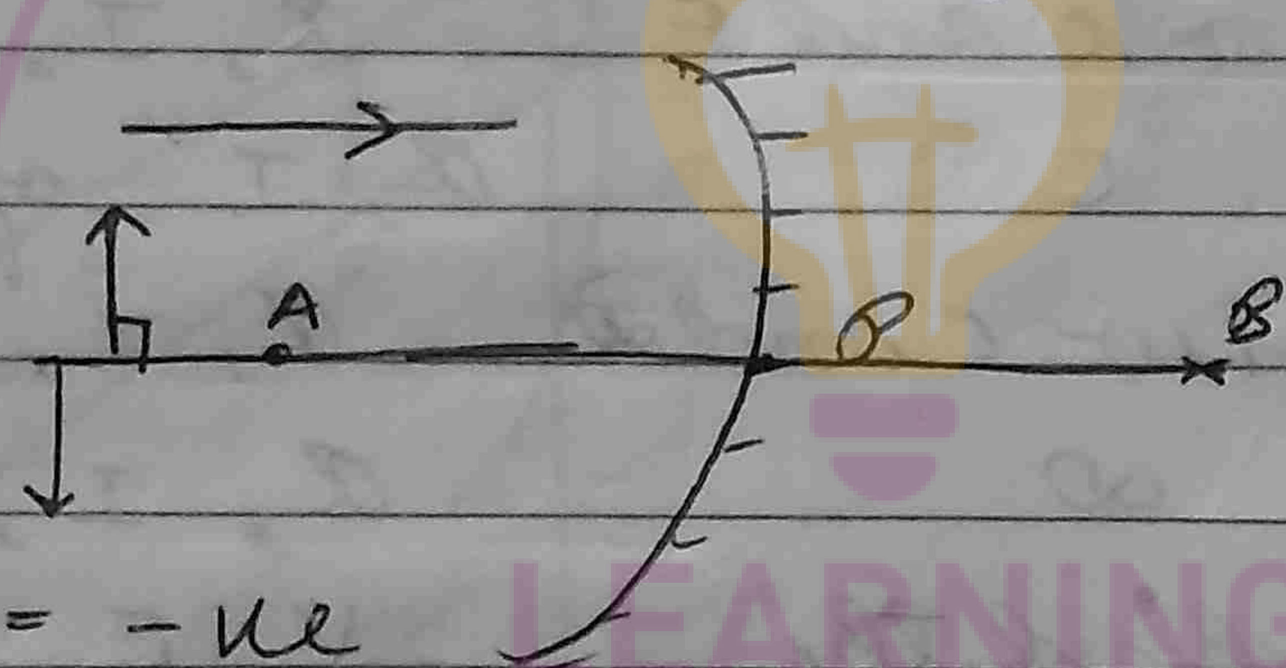
→ Concave → magnified | Plane → of same size
→ Convex → diminished

- <u>Sign Convection</u>

1 Every distance is measured from the pole.

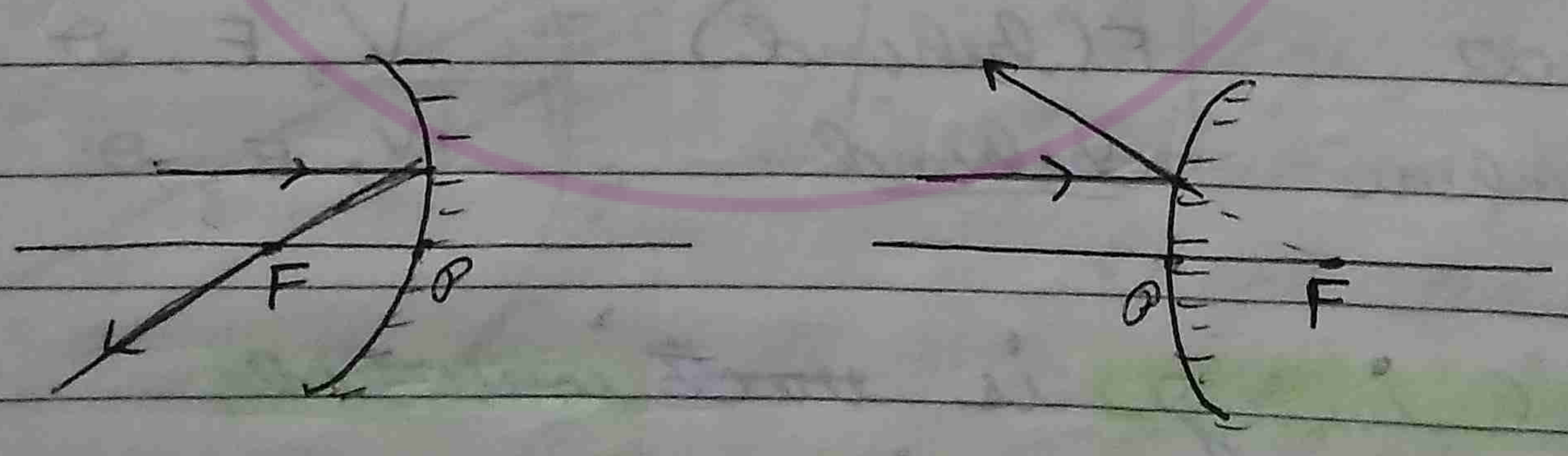
2 The direction along incident ray is taken positive.

3 upright height is taken as positive and inverted height is taken -ve.



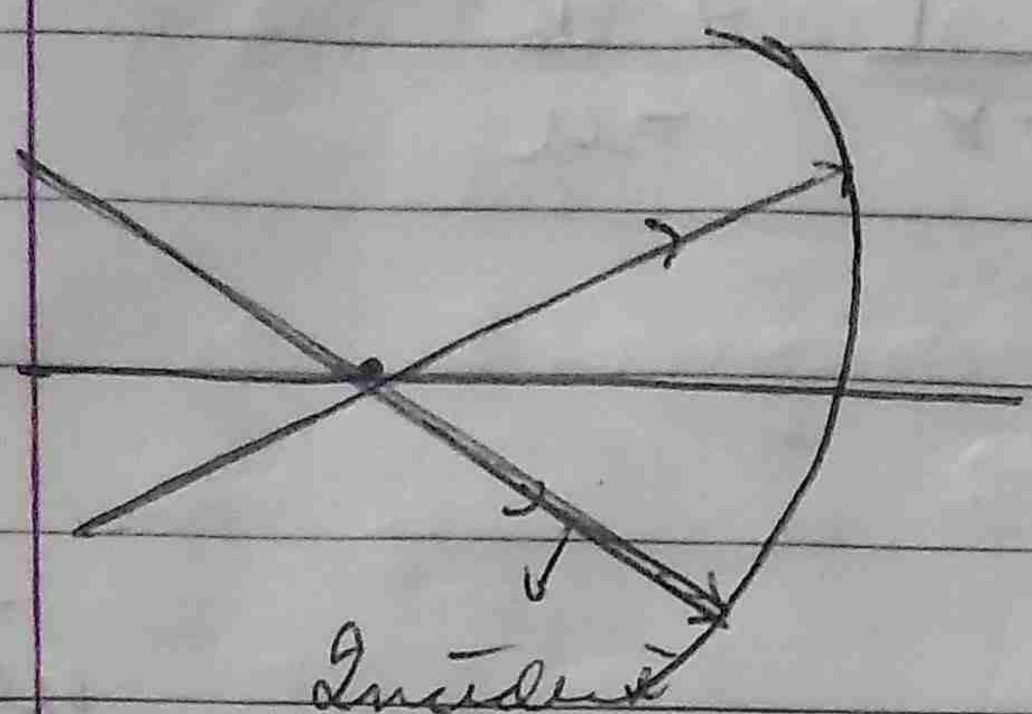
PA = -ve
PB = +ve

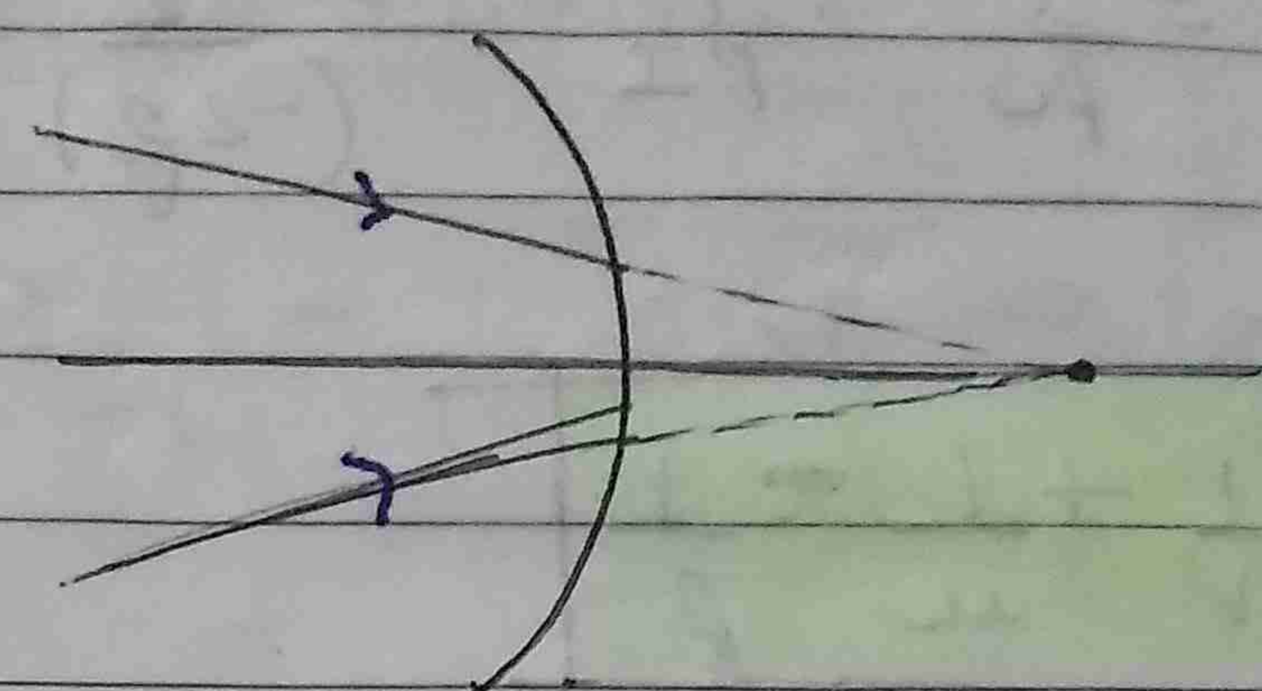
\#



concave => f is -ve
Convex => f is +ve

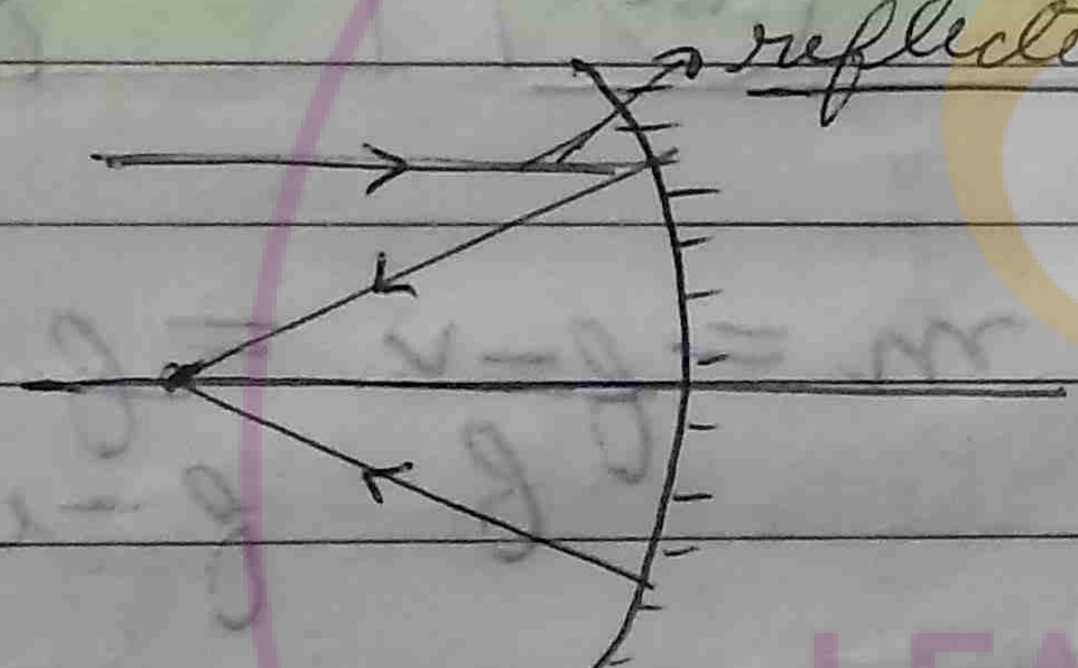
#

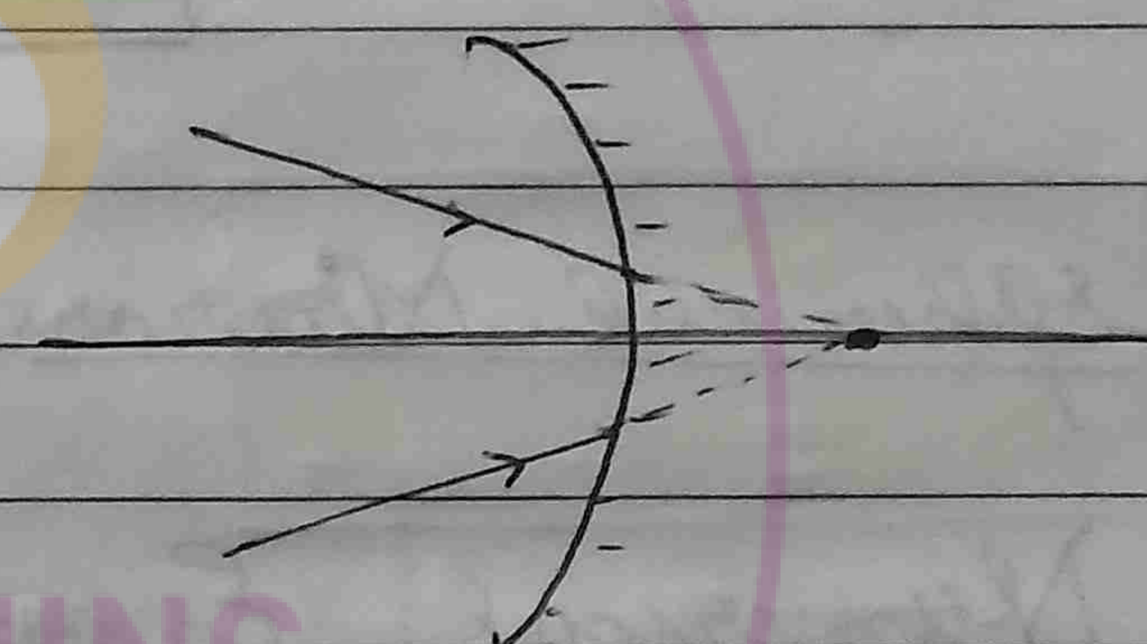




Real object => $u = -ve$.
Virtual object => $u = +ve$

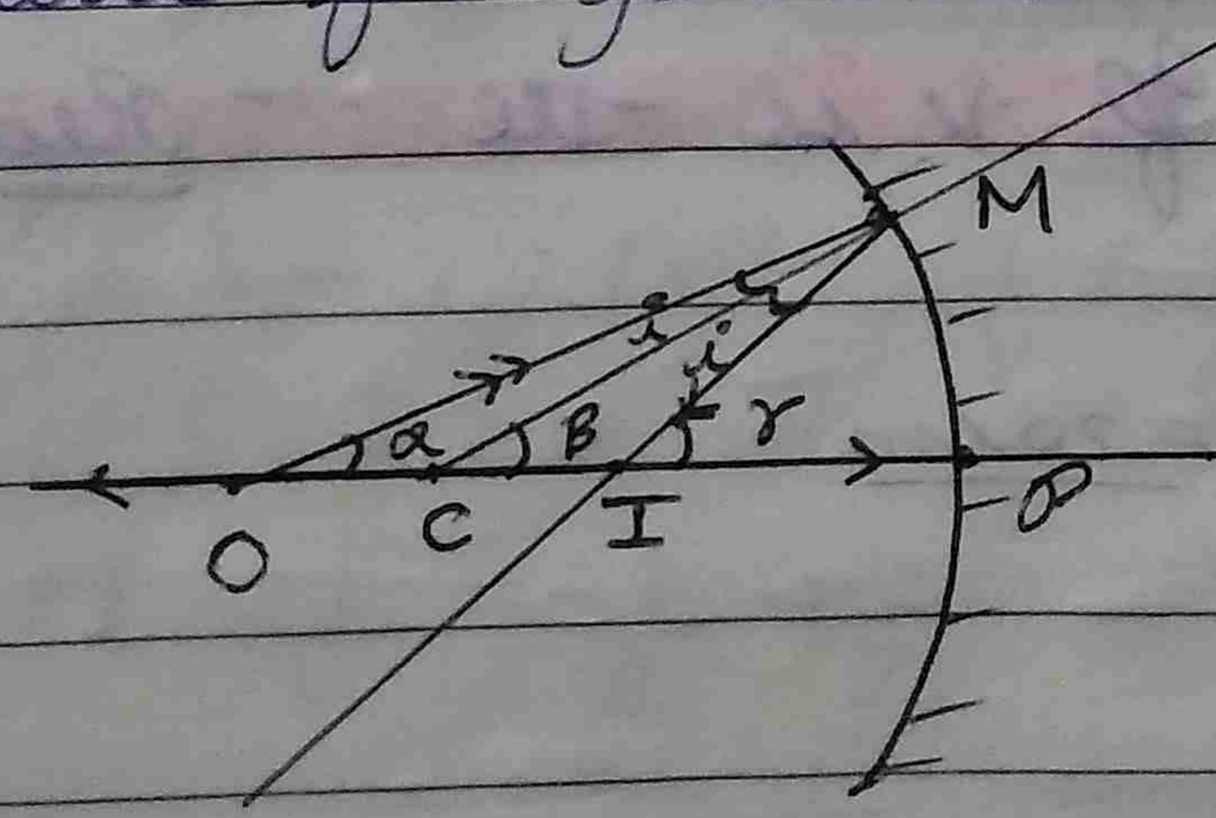
#





Real image $v = -ve$
Virtual image => $v = +ve$

- Mirror formula
- valid for paraxial ray only.



$\beta + \pi - (\alpha + i) = \pi$

$\beta = \alpha + i$ - - (i)

$\gamma = \beta + i$ - - (ii)

$\beta - \alpha = \gamma - \beta$

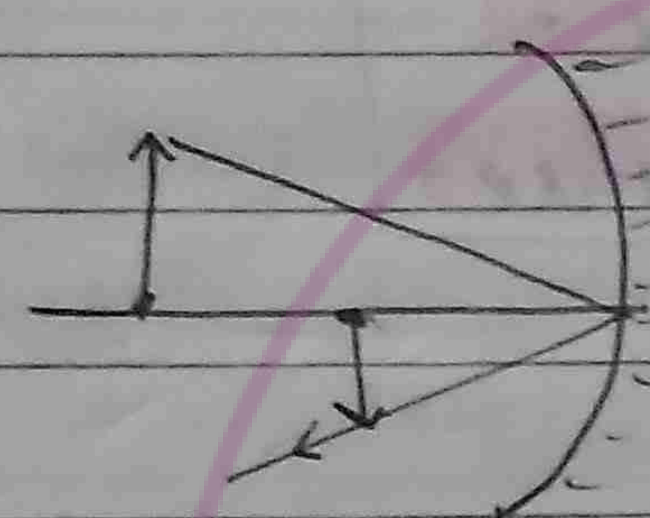
$2\beta = \alpha + \gamma$

$$2\frac{MP}{PC} = \frac{MP}{PO} + \frac{MP}{PI}$$

$$\frac{2}{PC} = \frac{1}{PO} + \frac{1}{PI} \Rightarrow \frac{2}{(-2f)} = \frac{1}{-v} + \frac{1}{-u}$$

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$

Magnification



only valid when object is perpendicular to the axis.

$$\boxed{m = \frac{I}{O} = \frac{-v}{u}} \qquad \boxed{P = -\frac{1}{f}}$$

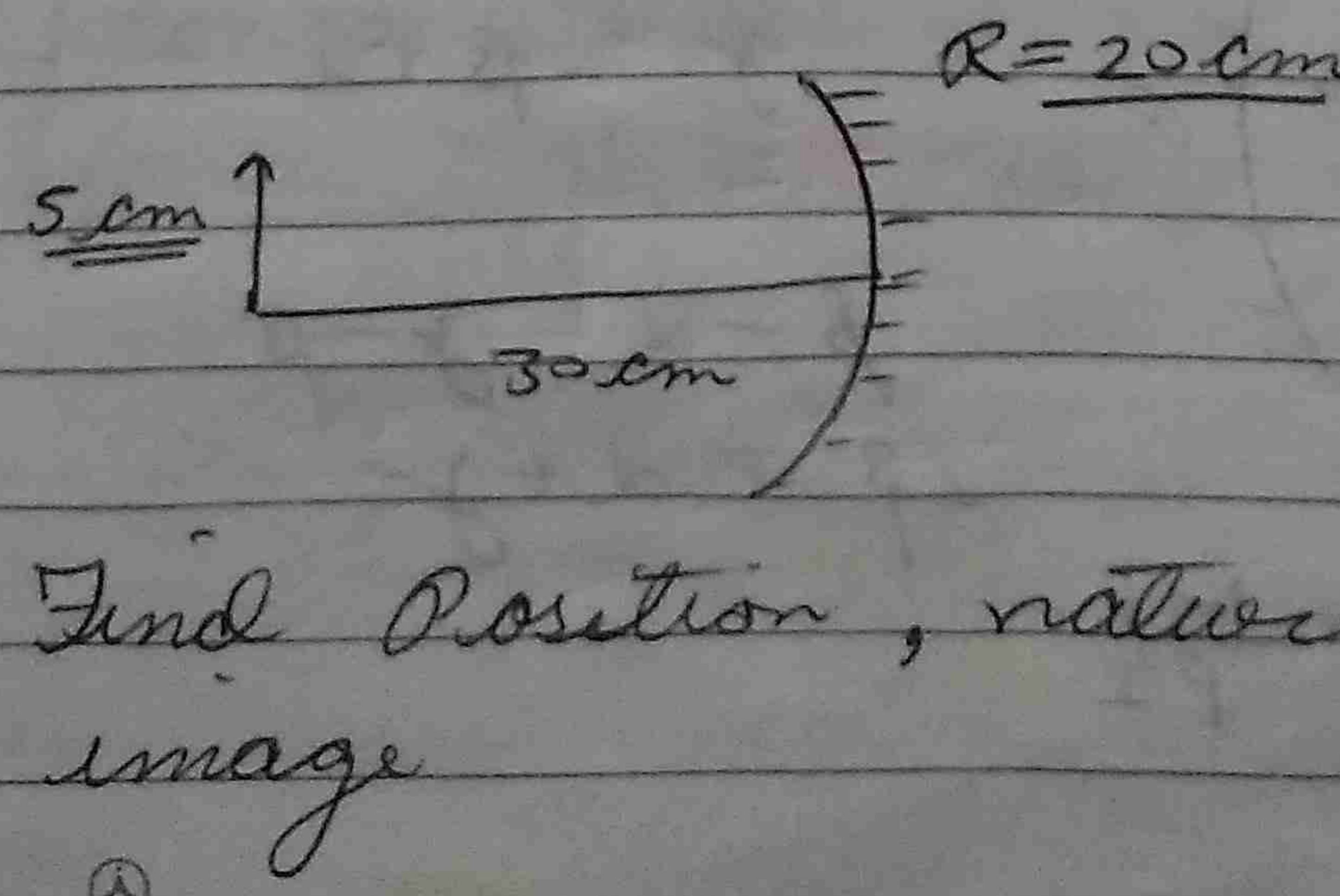
- Spherical Mirrors $m = \frac{f-v}{f} = \frac{f}{f-u}$

Numerical Examples

$$\boxed{\begin{aligned} &\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \\ &m = \frac{I}{O} = \frac{-v}{u} \\ &P = -\frac{1}{f} = -\frac{2}{R} \end{aligned}}$$

- If sign of m is $-ve \Rightarrow$ Inverted
- If $|m| > 1$ ... Magnified
- If v is $-ve$ ... Real ✓

#



Find Position, nature and size of image

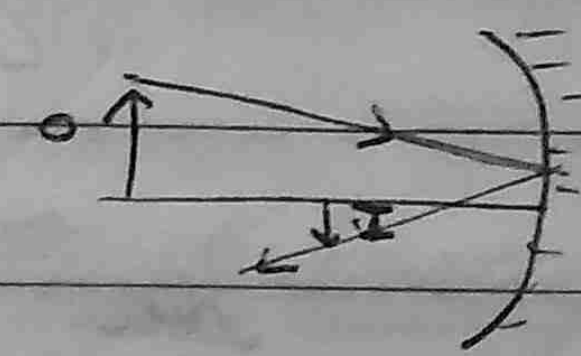
(A)

$f = -10\,cm \qquad u = -30\,cm$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{30} = \frac{-1}{10} \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{10}$

$\Rightarrow v = -15$

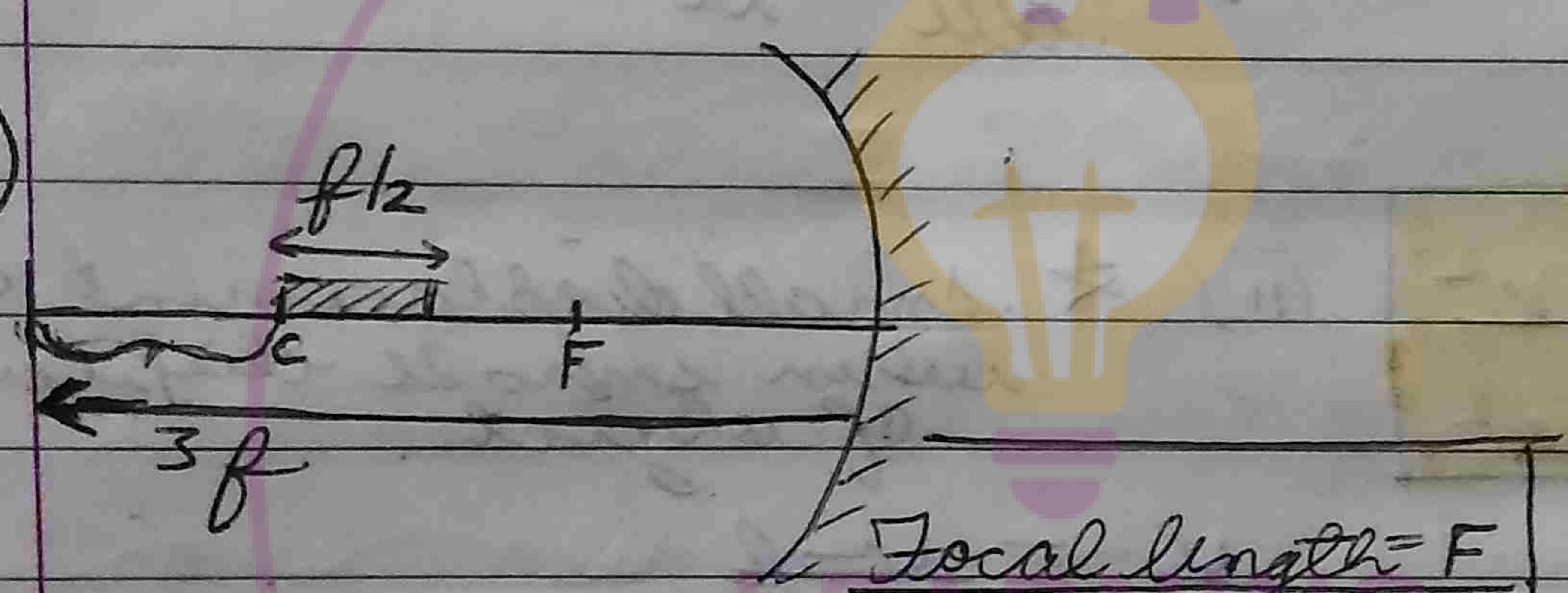
Real



$m = \frac{-v}{u} = \frac{I}{O} \Rightarrow -\left(\frac{-15}{-30}\right) = \frac{I}{+5} \Rightarrow I = -2.5$

inverted

★★★ **Ex**



- The object is kept along the principal axis such that one end of the object touches one end of image. If length of object is f/2 and is magnified then find the magnification.

$\Rightarrow u = -3f/2 \qquad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-f} + \frac{1}{3f/2}$

$\Rightarrow \frac{1}{v} = \frac{-1}{f} + \frac{2}{3f} \Rightarrow v = -3f$

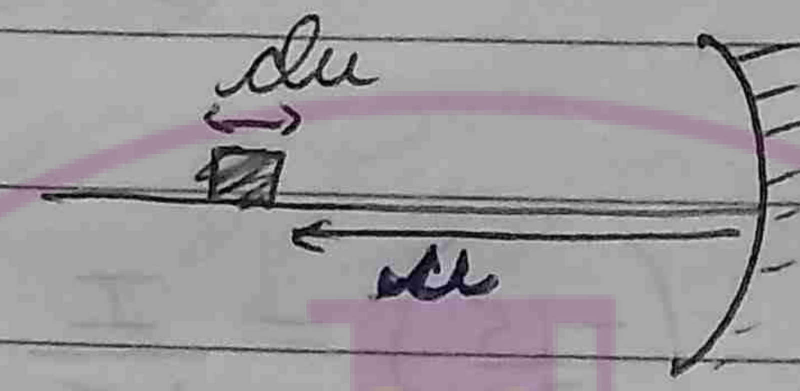
- length of image = f
  length of objects = $f/2$

$m = \frac{I}{O} = \frac{f}{f/2} = 2$ → valid.

~~$m = \frac{-v}{u}$~~ → not valid because here the object and image are not ⊥

• Lateral magnification for concave mirror ⇒ $\frac{-v^2}{u^2}$ for convex ⇒ $\frac{v^2}{u^2}$ of a very small object

Q. A short linear object of size 'a' is kept along principal axis at a distance 'u' from a concave mirror of focal length f. Find size of image.



$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ (i) $\Rightarrow -\frac{1}{v^2}\frac{dv}{du}-\frac{1}{u^2}=0$

$\delta v = -m^2 \delta u$ ← $dv = -\frac{du\, v^2}{u^2}$ (ii) ★ small displacement of image when small displacement of object.

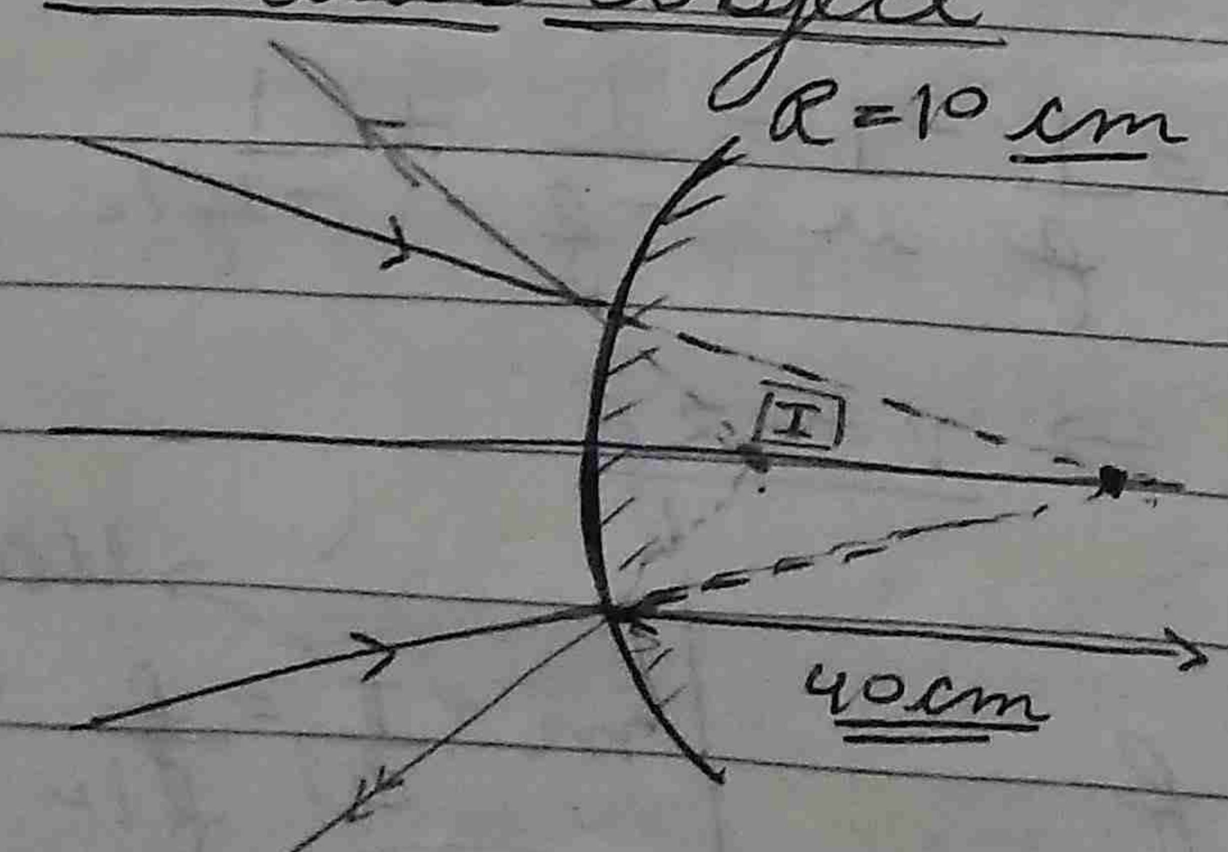
$\frac{u}{v}+1=\frac{u}{f}$ $\quad \frac{u}{v}=\frac{u-f}{f}$

Putting in equation (iii)

$dv = -du\left(\frac{f}{u-f}\right)^2 = -a\left(\frac{-f}{-u+f}\right)^2$

size of image.

• **Virtual Object**



Find position and nature of image.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{+5} - \frac{1}{+40} \Rightarrow \frac{1}{5} - \frac{1}{40} = \frac{7}{40}$$

$v = +\frac{40}{7}$ ---- virtual image

$m = \frac{-v}{u} = -\left[\frac{40}{7} \times \frac{1}{40}\right] = -\frac{1}{7}$ → inverted, diminished

★ **When object is real**

concave mirror ⟶ real and virtual image

convex mirror ⟶ virtual image

★ **When object is virtual** : $u = +ve$

• **Concave mirror**

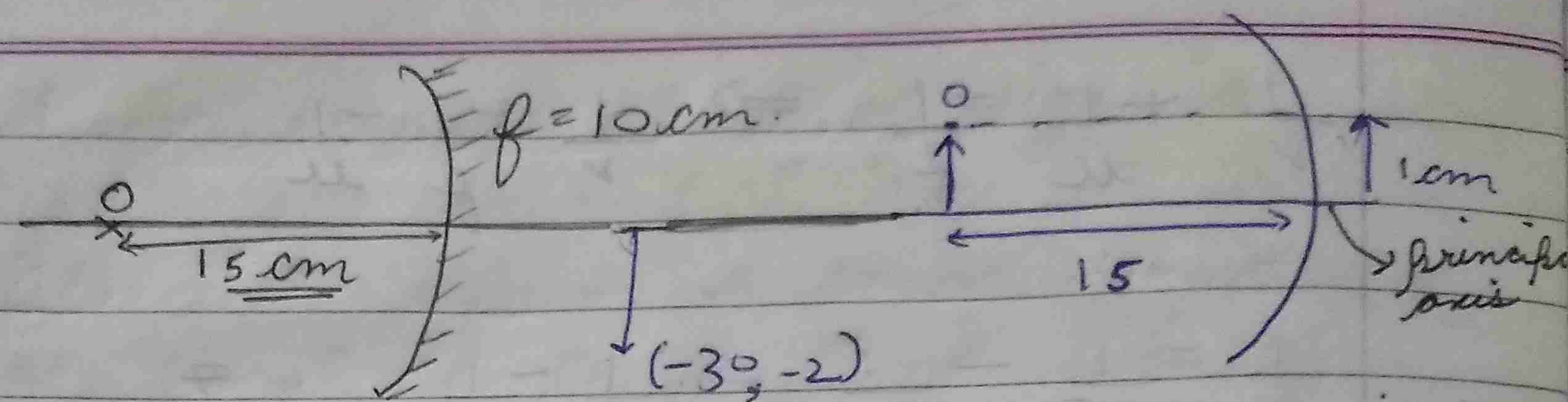
★ $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-f} - \frac{1}{+u} = -ve$

⇒ when object is virtual point object then the image is real, erect and smaller.

• **Convex**

★ $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{+f} - \frac{1}{+u}$: Image could be real or virtual.

e.g



If mirror is shifted down by 1 cm, find the co-ordinate of image, taking new position of pole as origin.

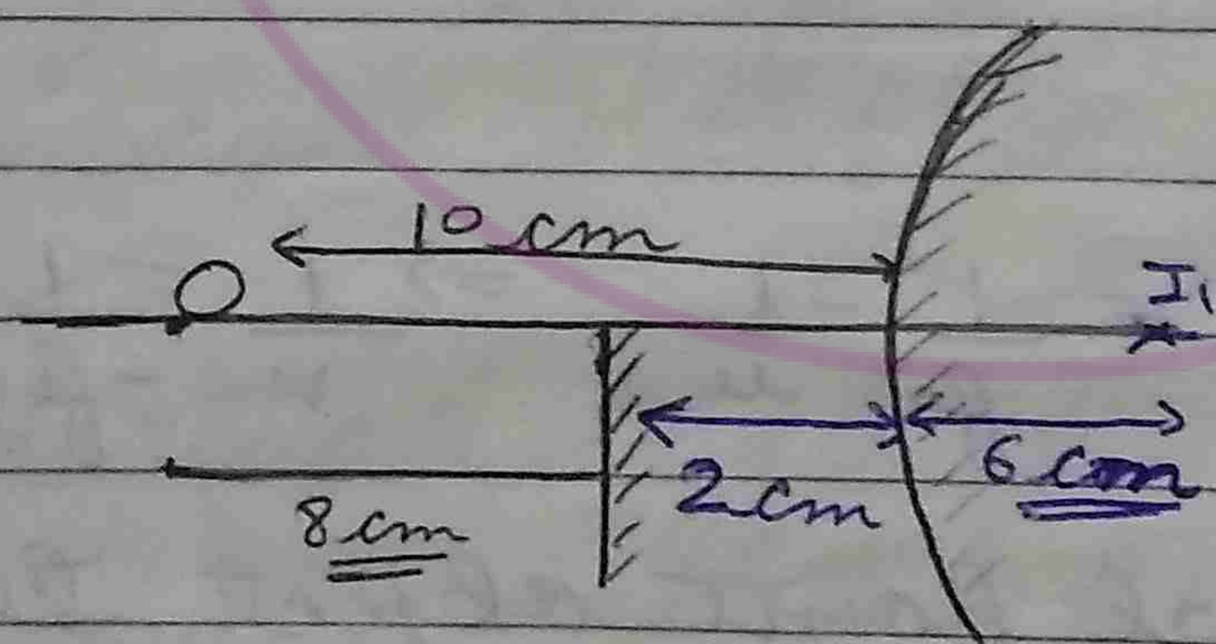
$u = -15$ $f = -10 \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-10} + \frac{1}{+15} \Rightarrow \frac{15-10}{-150}$

$m = \frac{-v}{u} = \frac{I}{O} \Rightarrow \frac{-(-30)}{(-15)} = \frac{I}{1}$ $\Rightarrow v = -30$

$\Rightarrow I = -2\,cm$

Multiple Mirror

Q Find the R of convex mirror so that both the images coincide



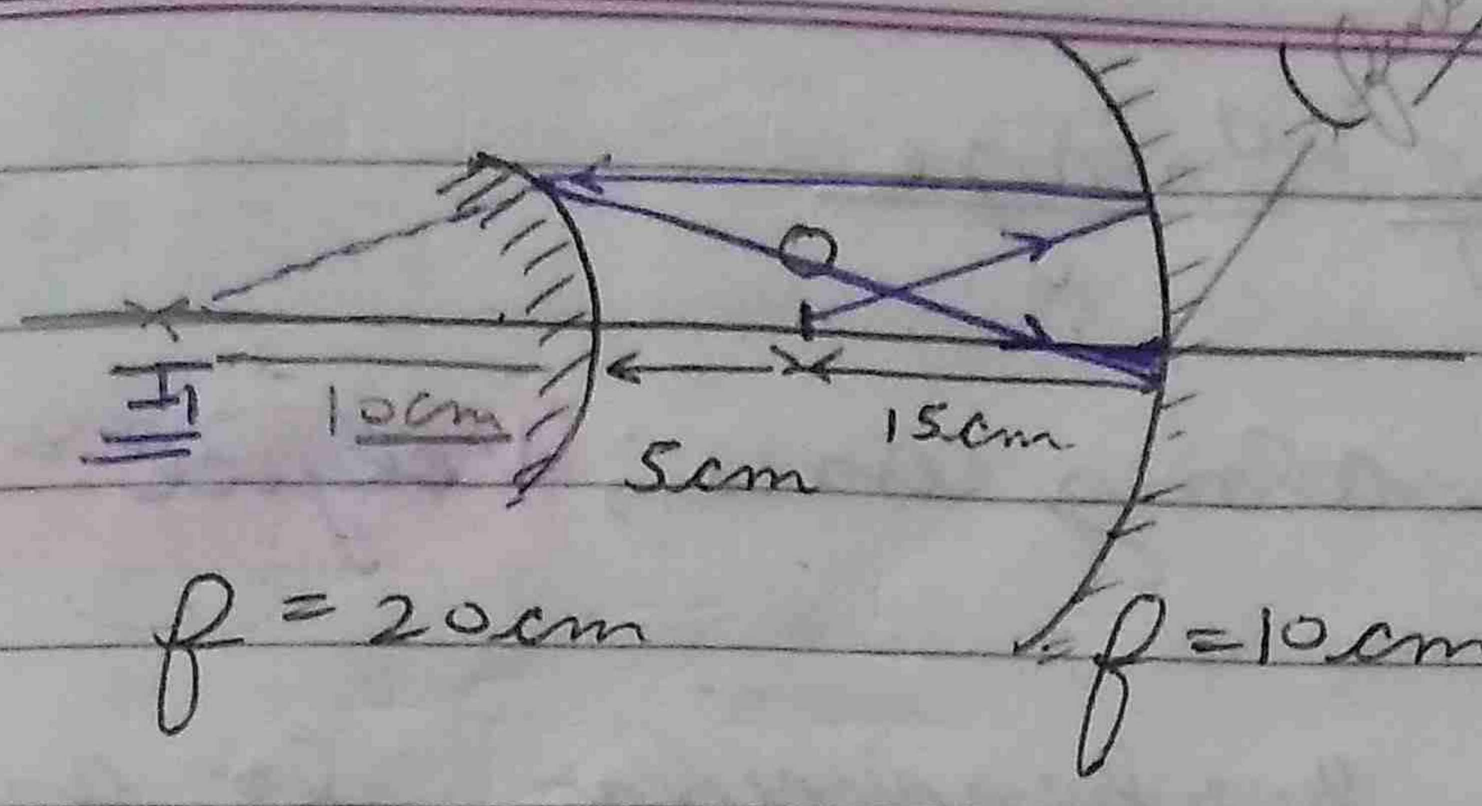
Convex $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{+6} + \frac{1}{-10} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{10-6}{60}$

$\Rightarrow \frac{1}{f} = \frac{4}{60}$

$f = 15\,cm$ $R = 2f = 30\,cm$

$f = 15$

Q



Considering reflection first from concave. Then convex find the final location of image.

Concave

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-15} \Rightarrow v = -30$

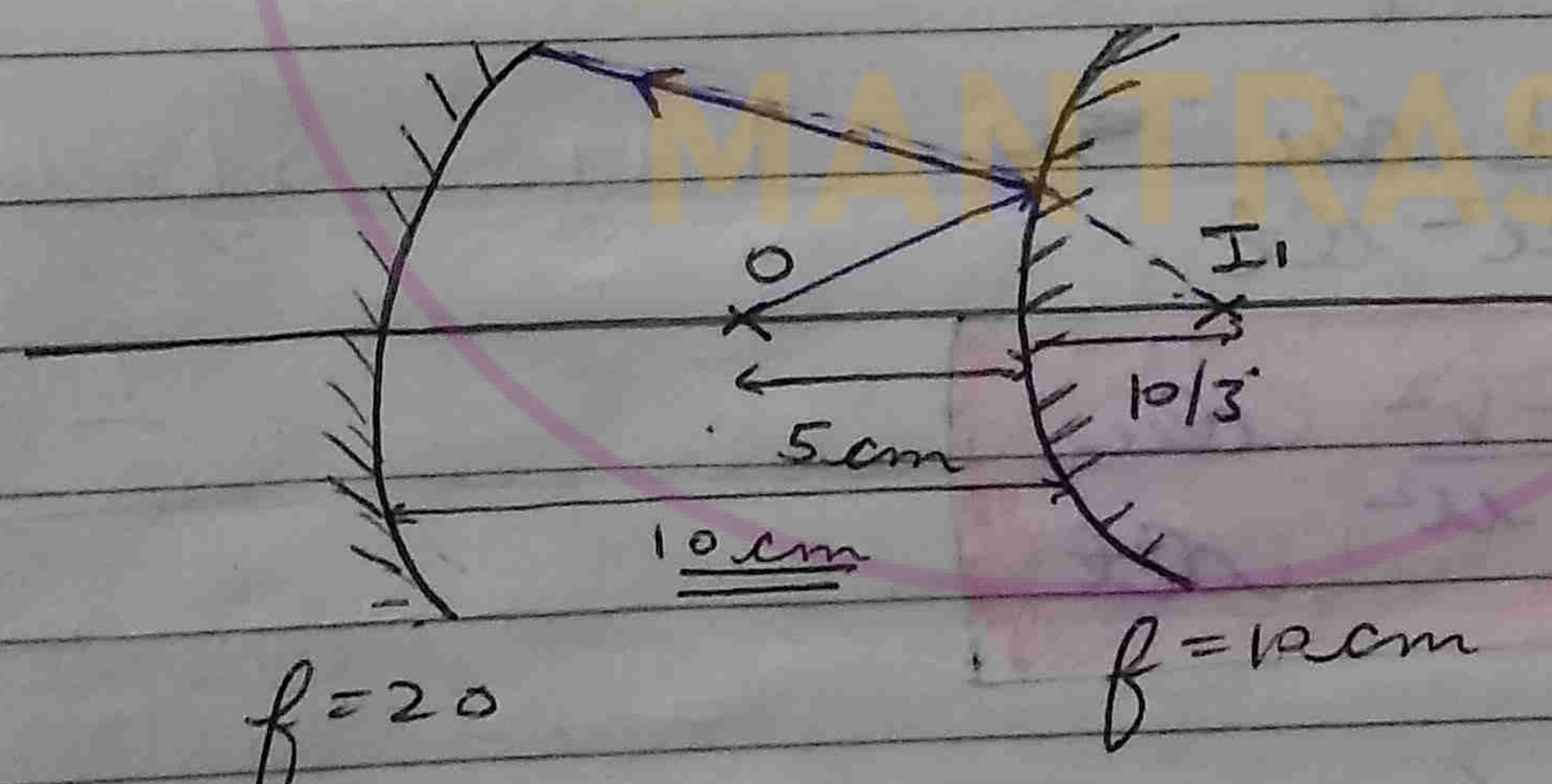
Convex

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{10} = \frac{1}{20}$

[Image of first is object of second]

$\Rightarrow \frac{1}{v} = \frac{-1}{10} + \frac{1}{20} \Rightarrow v = -20$

Q



Considering reflection from convex to concave find position of final image

Convex

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{10} + \frac{1}{5} \Rightarrow v = \frac{10}{3}$

Concave = $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{-20} - \frac{1}{-40/3} \Rightarrow v =$

$\Rightarrow \frac{1}{v} \Rightarrow \frac{-2+3}{40} \Rightarrow v \Rightarrow 40\text{ cm}$

★ $V_{image} = -m^2 V_o$ (velocity of object)
velocity of

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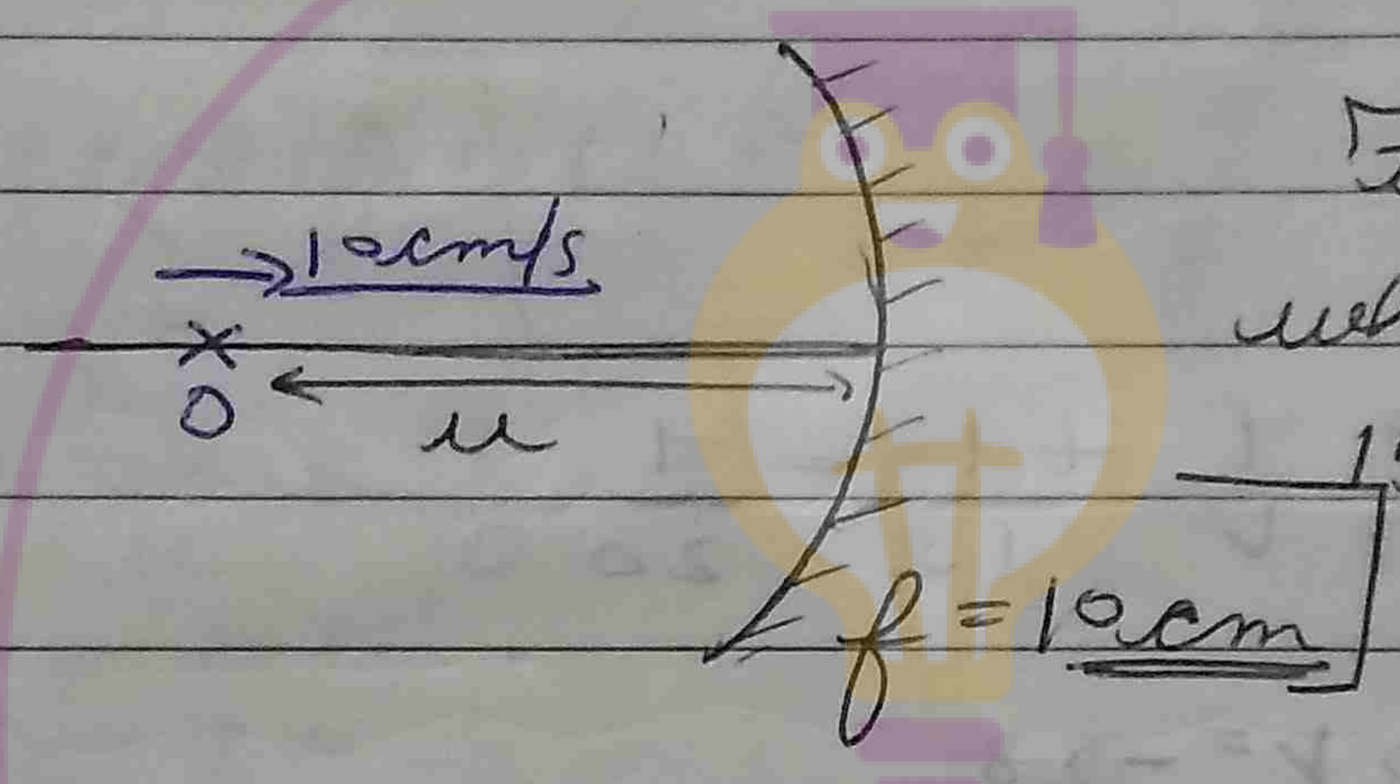
Velocity of Image

(object distance)

- If object moves along axis; $V_{object} = \frac{du}{dt}$
- If object moves perpendicular to axis:
  $V_{object} = \frac{do}{dt}$ → object height

Q



Find image velocity when object is at 15 cm from pole.

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ differentiating wrt to time

$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$\Rightarrow \frac{dv}{dt} = \frac{-v^2}{u^2}\left(\frac{du}{dt}\right)$

$\frac{1}{-10} + \frac{1}{+15}$, $\frac{(15-10)}{-150}$, $\frac{5}{-15}$, $\frac{-150}{5}$

$\Rightarrow \frac{dv}{dt} = -\left(\frac{30}{-15}\right)^2 (+10) \Rightarrow \frac{dv}{dt} = -40$

→ Suppose $\frac{du}{dt}$ is constant

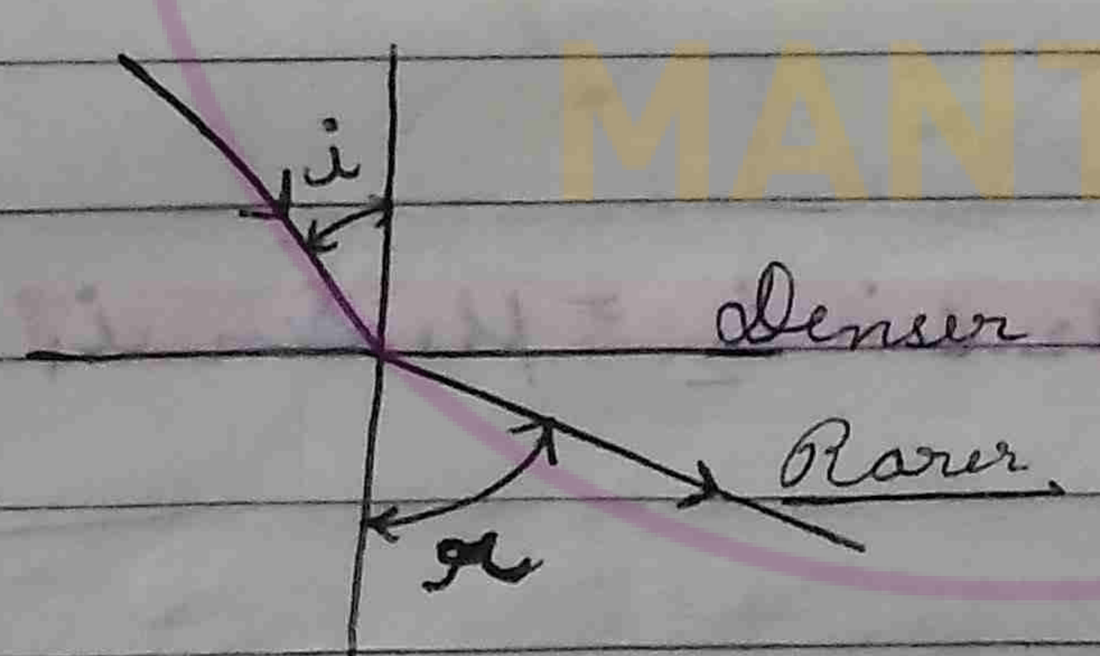
- When object is between ∞ to c then object distance would be greater than image distance ~~and~~ therefore image speed will be less.

- If the object would be at c then object distance would be equal to image distance Therefore image velocity will be equal to image (→ object) velocity.

- When object is between C and F image distance would be greater than ~~im~~ object distance hence speed of image would be greater the object speed.

- <u>Refraction</u>

- It is bending of light when it travels to next medium.



<u>Deviation</u> :

$$\delta = |r - i|$$

- Refractive Index (μ) → <u>Tensor quantity</u>

μ of a medium $= \frac{c}{v}$

v -- speed of light in that medium

μ -- vaccum $= 1$

$\Rightarrow \mu > 1$ for any other medium

→ μ depends on

(a) Nature of medium

(b) Wavelength of light used.

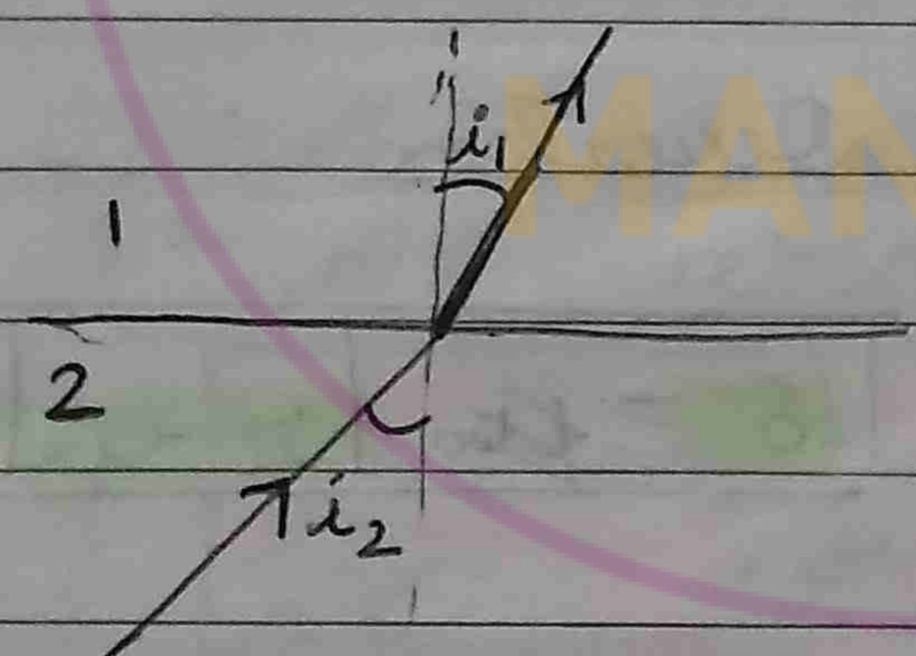
$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \ldots$$

⇒ μ_{red} ——— min

⇒ μ_{violet} ——— max

- Higher the value of μ denser is the medium.

- Snell's law



$$\mu_2 \sin i_2 = \mu_1 \sin i_1$$

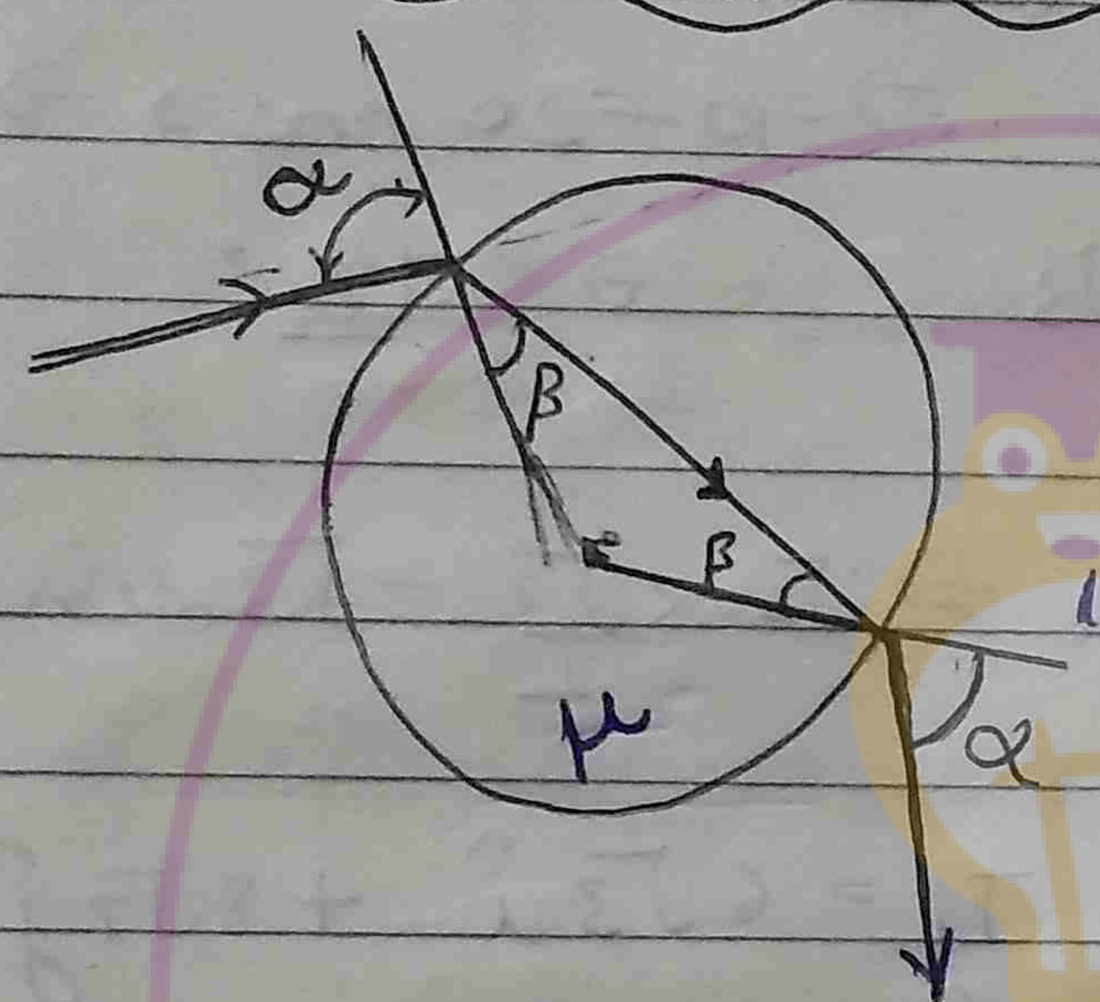
$$\mu \sin i = \text{constant}$$

- If $\mu_1 < \mu_2 \Rightarrow i_1 > i_2$
- If $\mu_1 > \mu_2 \Rightarrow i_1 < i_2$
- If $\mu_1 = \mu_2 \Rightarrow i_1 = i_2$

☆ Light does not bend →

- If the two medium are same.
- If angle of incidence is zero.

Q

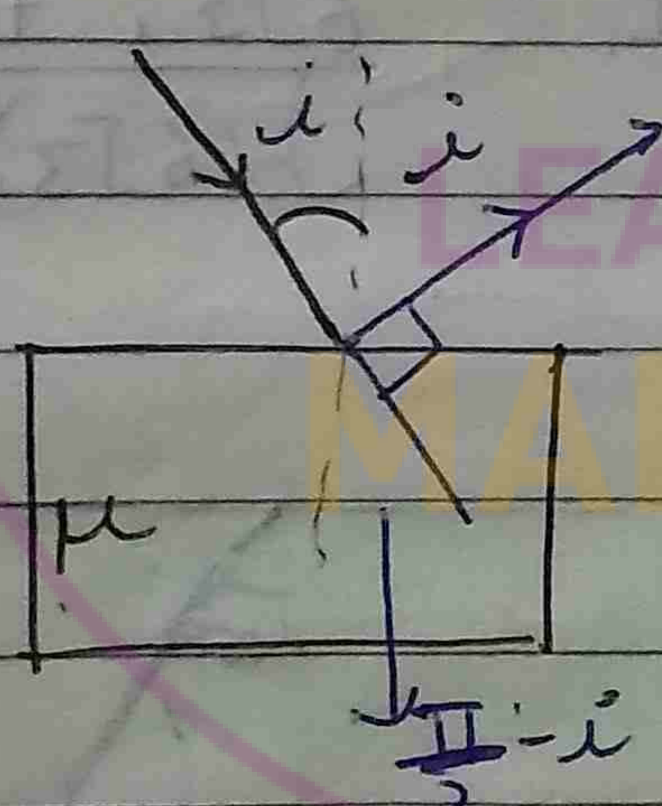


Find deviation

$1 \sin\alpha = \mu \sin\beta$

$\delta_{net} = 2(\alpha - \beta)$

Q



(find angle of incidence so that reflected and refracted rays are perpendicular)

$1 \sin(i) = \mu \sin(\frac{\pi}{2} - i)$

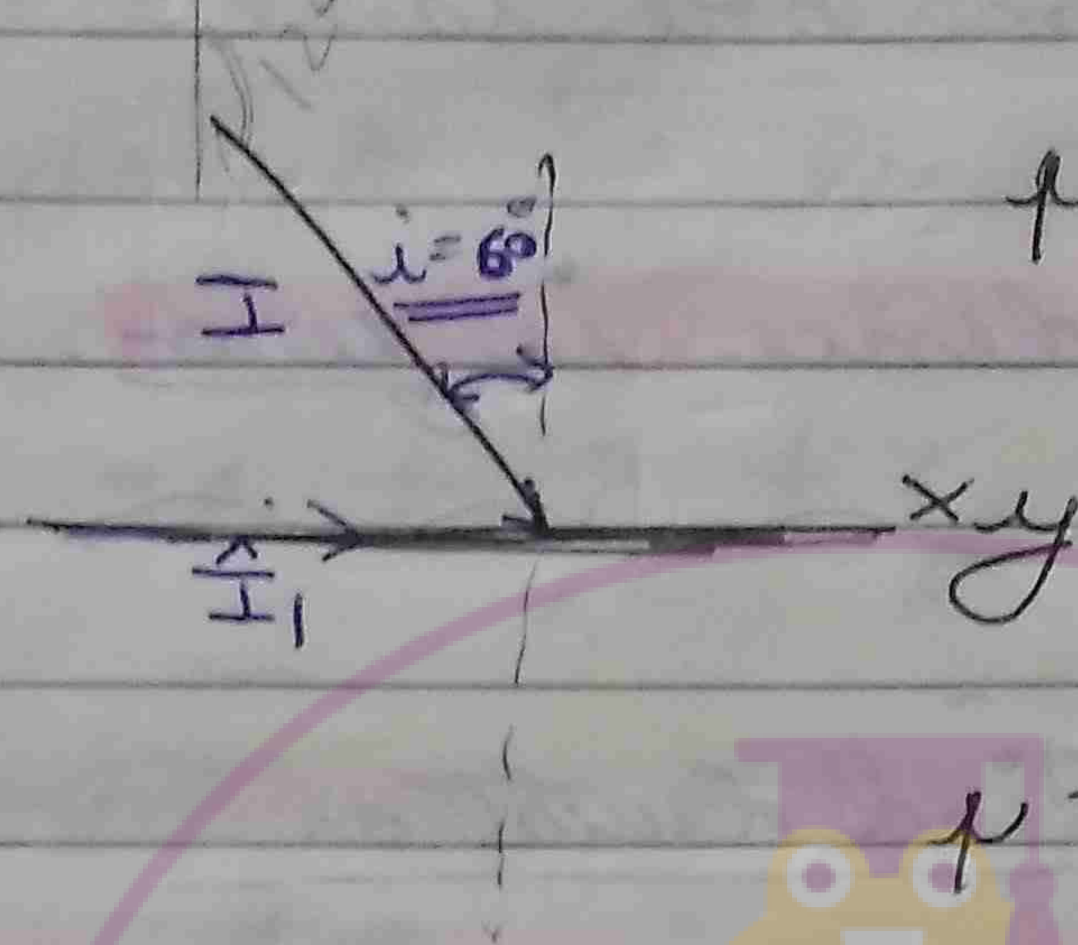
$\Rightarrow$ $\boxed{\tan i = \mu}$ → Brewsters condition

- Refraction Vectorially

med. $z > 0$ $\mu = \sqrt{2}$

$z < 0$ $\mu = \sqrt{3}$

$\vec{I} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ in medium 1.
Find $\hat{R}$?



$\mu = \sqrt{2}$

$\mu = \sqrt{3}$

$\Rightarrow \vec{I}\cdot\hat{k} = I\cos\theta$

$-10 = \sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + 10^2}\,\cos\theta$

$\Rightarrow -10 = 20\cos\theta \Rightarrow$

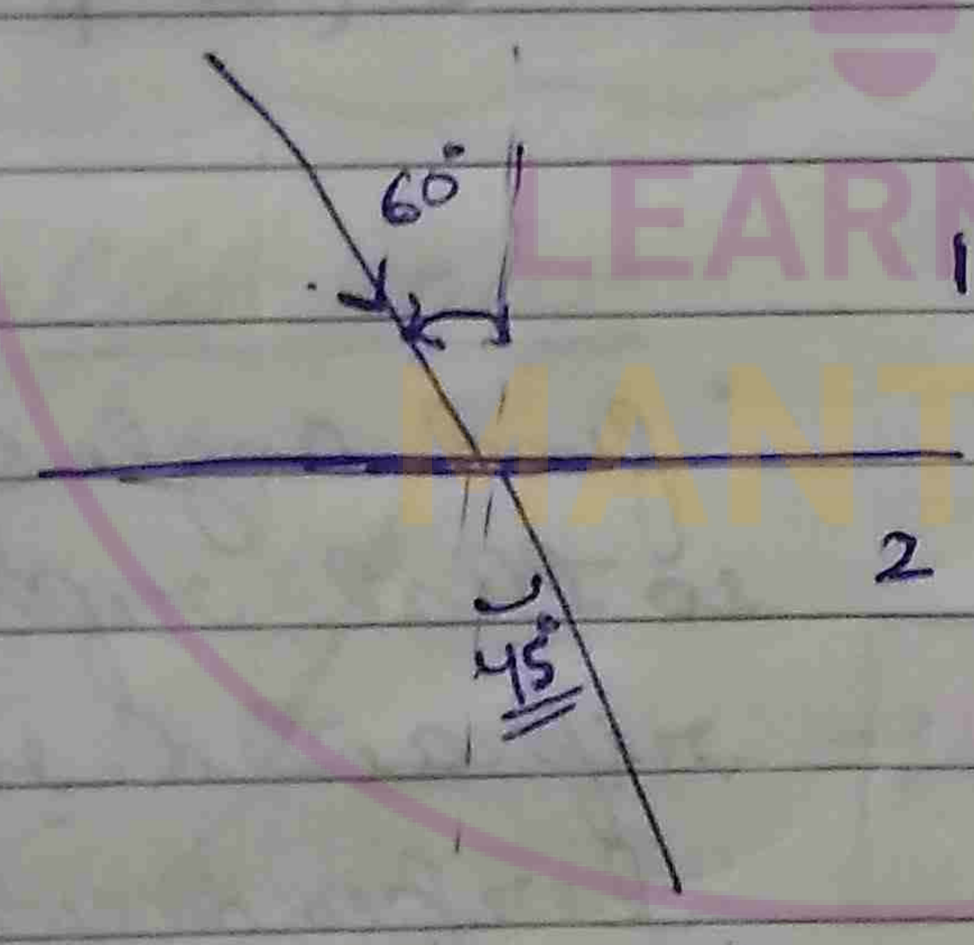
$\theta = 120°$

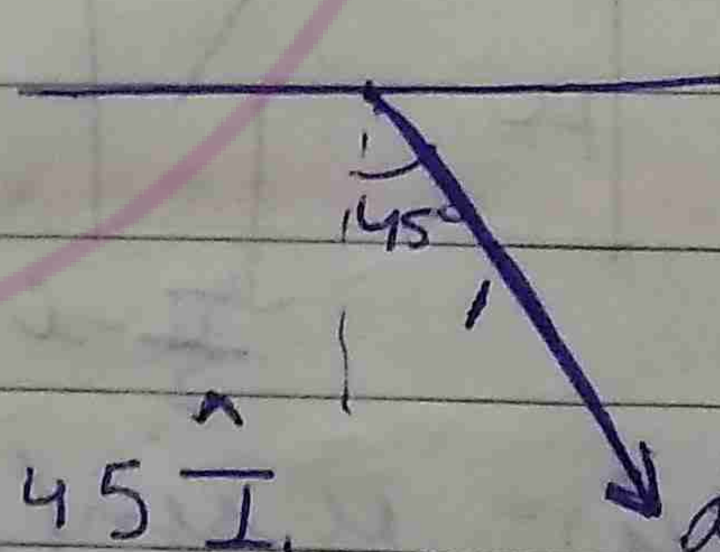
$\sqrt{2}\sin 60 = \sqrt{3}\sin r \Rightarrow \sqrt{2} \times \frac{\sqrt{3}}{2} = \sqrt{3}\sin r$.

$\Rightarrow r = 45°$

$I_1 = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j}$

$\hat{I}_1 = \frac{6\sqrt{3}\,i + 8\sqrt{3}\,j}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}}$



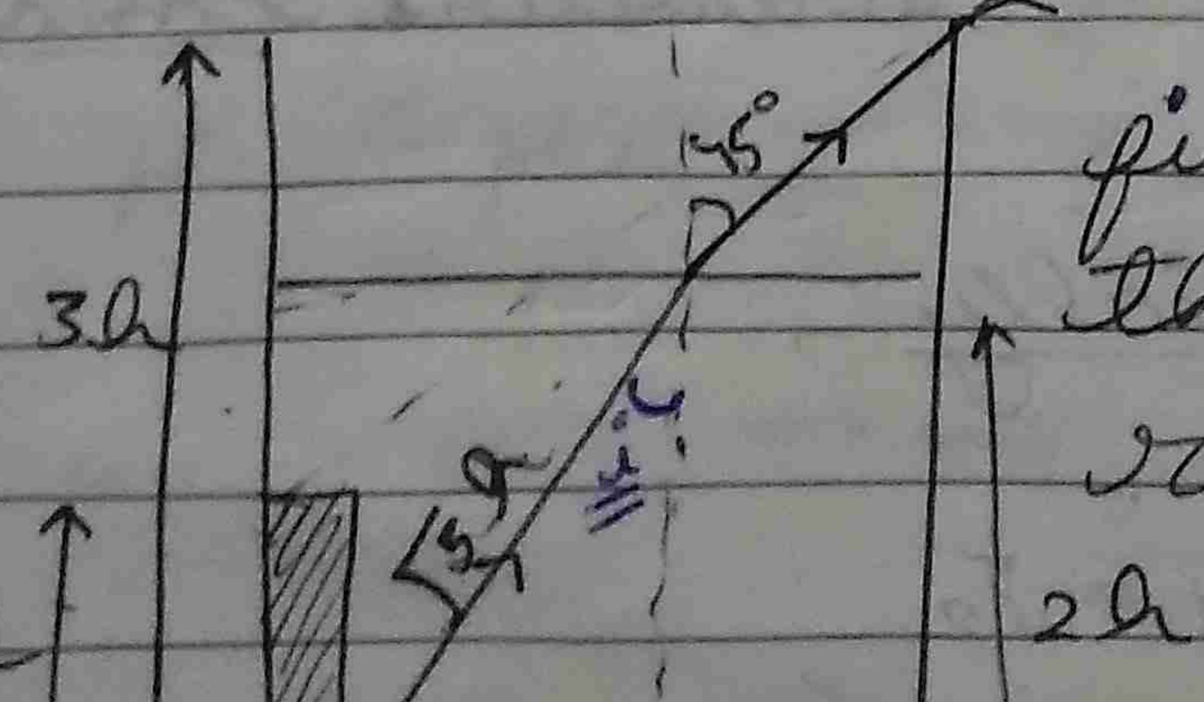


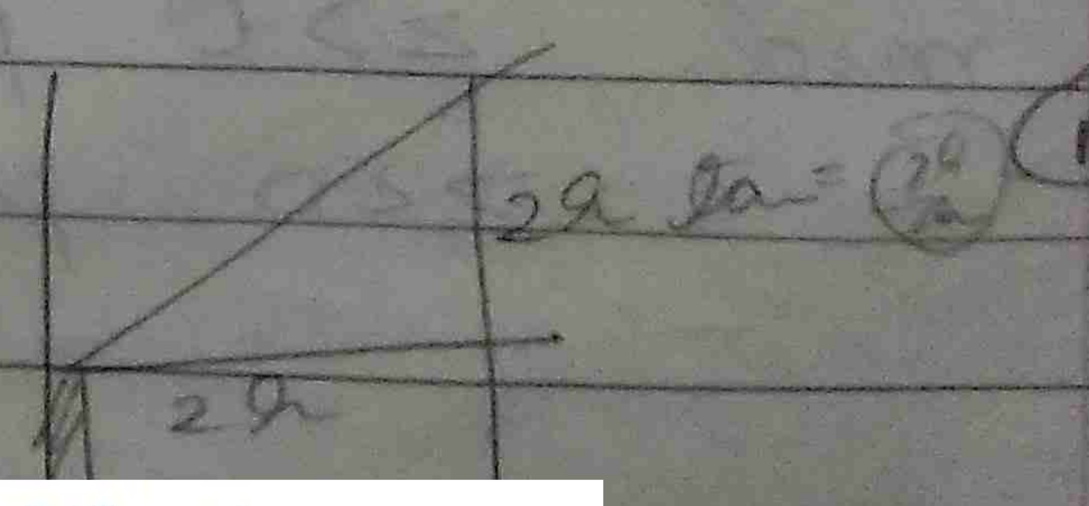
$\hat{R} = 1\cos 45(-\hat{k}) + 1\sin 45\,\hat{I}_1$

---

e.g.

If a liquid is filled upto height 2h, the bottom of the rod is visible. Find μ

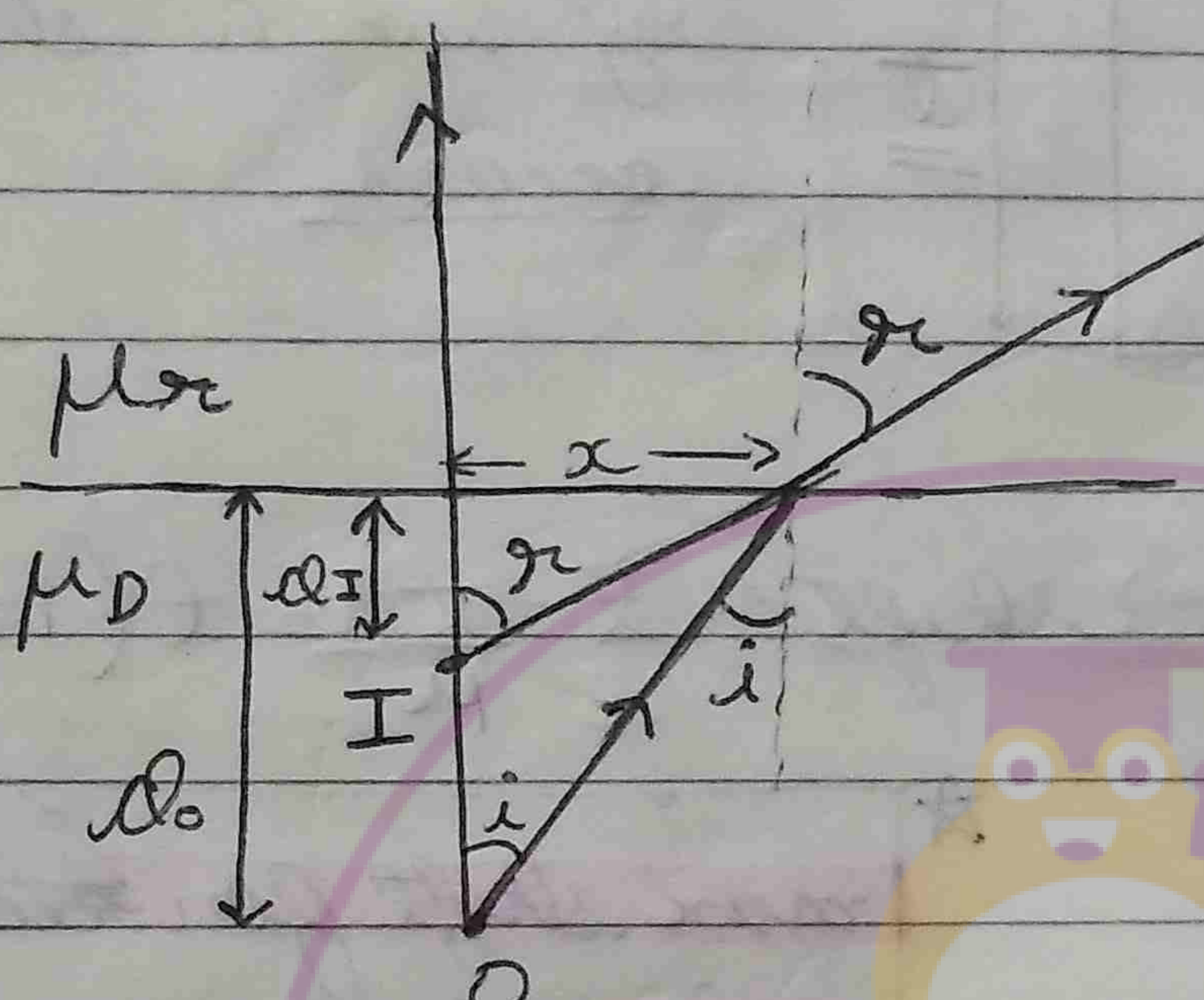




$\mu \sin i = 1 \sin 45 \Rightarrow \mu \times \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{2}}$

- Real and Apparent Depth.

---- paraxial rays



$x = d_I \tan r = d_o \tan i$

here $i \to 0$ $r \to 0$

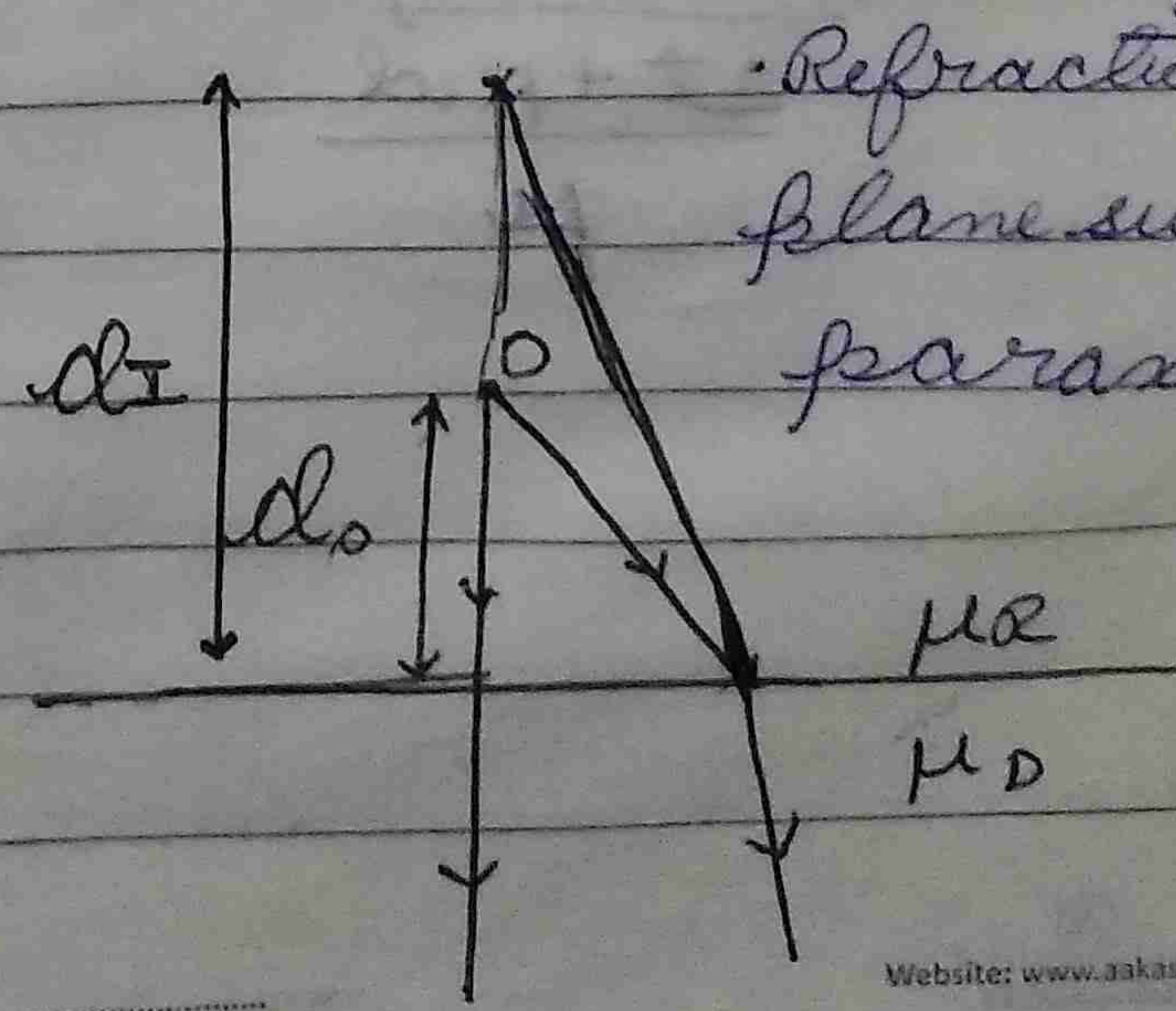
$\Rightarrow d_I \sin r = d_o \sin i$

Using Snells law $\mu_D \sin i = \mu_R \sin r$

$$\frac{\mu_D \sin i = \mu_R \sin r}{d_o \sin i = d_I \sin r} \Rightarrow \frac{d_I}{d_o} = \frac{\mu_R}{\mu_D}$$

$\Rightarrow$ $\boxed{d_I = \frac{d_o}{\mu}}$: when object is at denser and viewed from rarer

- When object is at ~~denser~~ rarer and viewed from denser.



· Refraction at plane surface for paraxial rays.

$d_I = \frac{\mu_D}{\mu_R} d_o$

$\Rightarrow$ $\boxed{d_I = \mu d_o}$

$\mu \to \frac{\mu_D}{\mu_R}$

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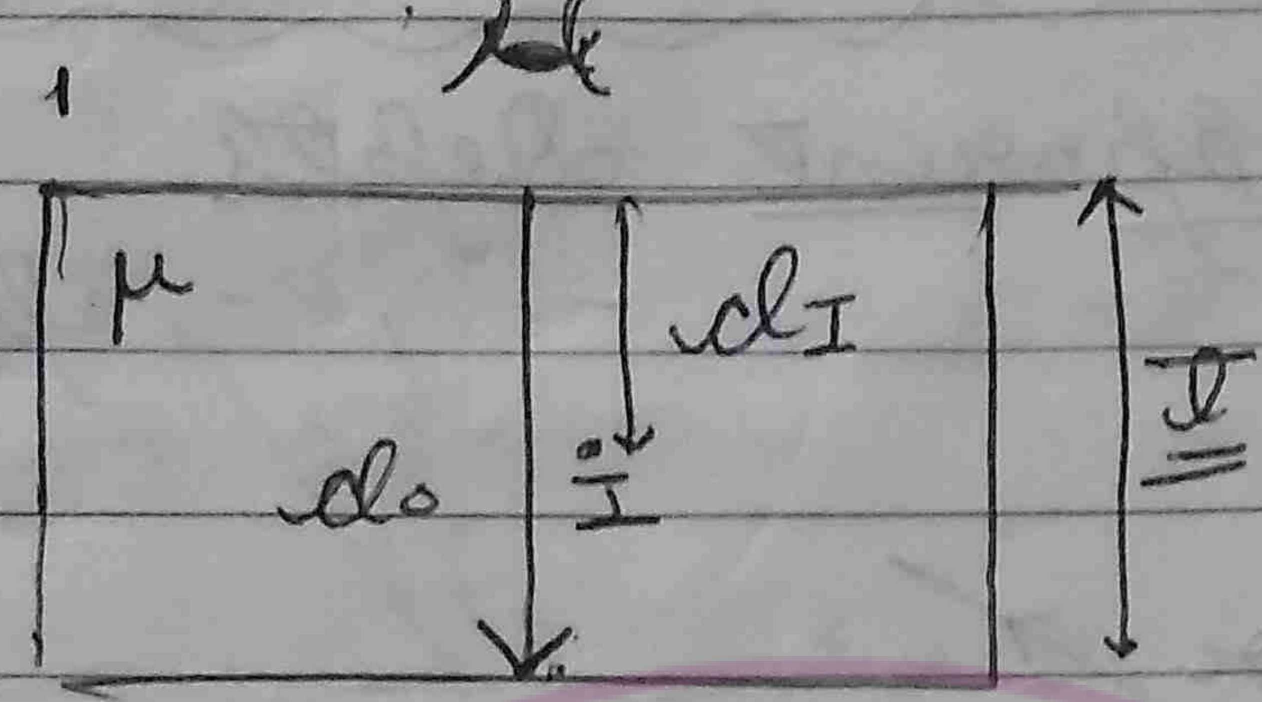
• **Numericals**



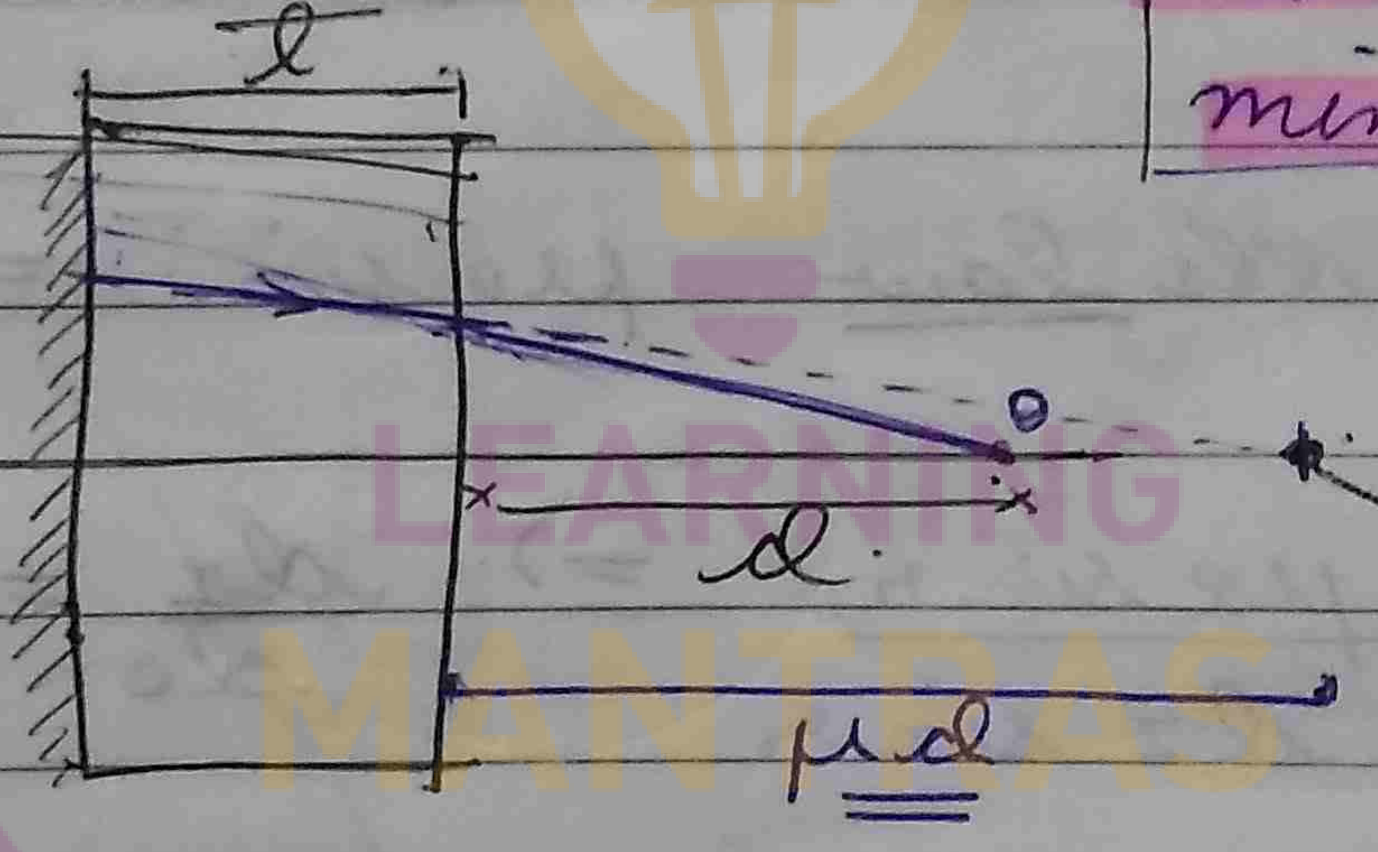
here $\mu = \frac{\mu_D}{\mu_R}$

• Find the distance by which shift occurs.

$$d_I = \frac{d_o}{\mu} = \frac{t}{\mu} \Rightarrow \text{shift} = t - \frac{t}{\mu} = t\left(1 - \frac{1}{\mu}\right)$$

☆ max shift for ~~red~~ blue
min shift for red

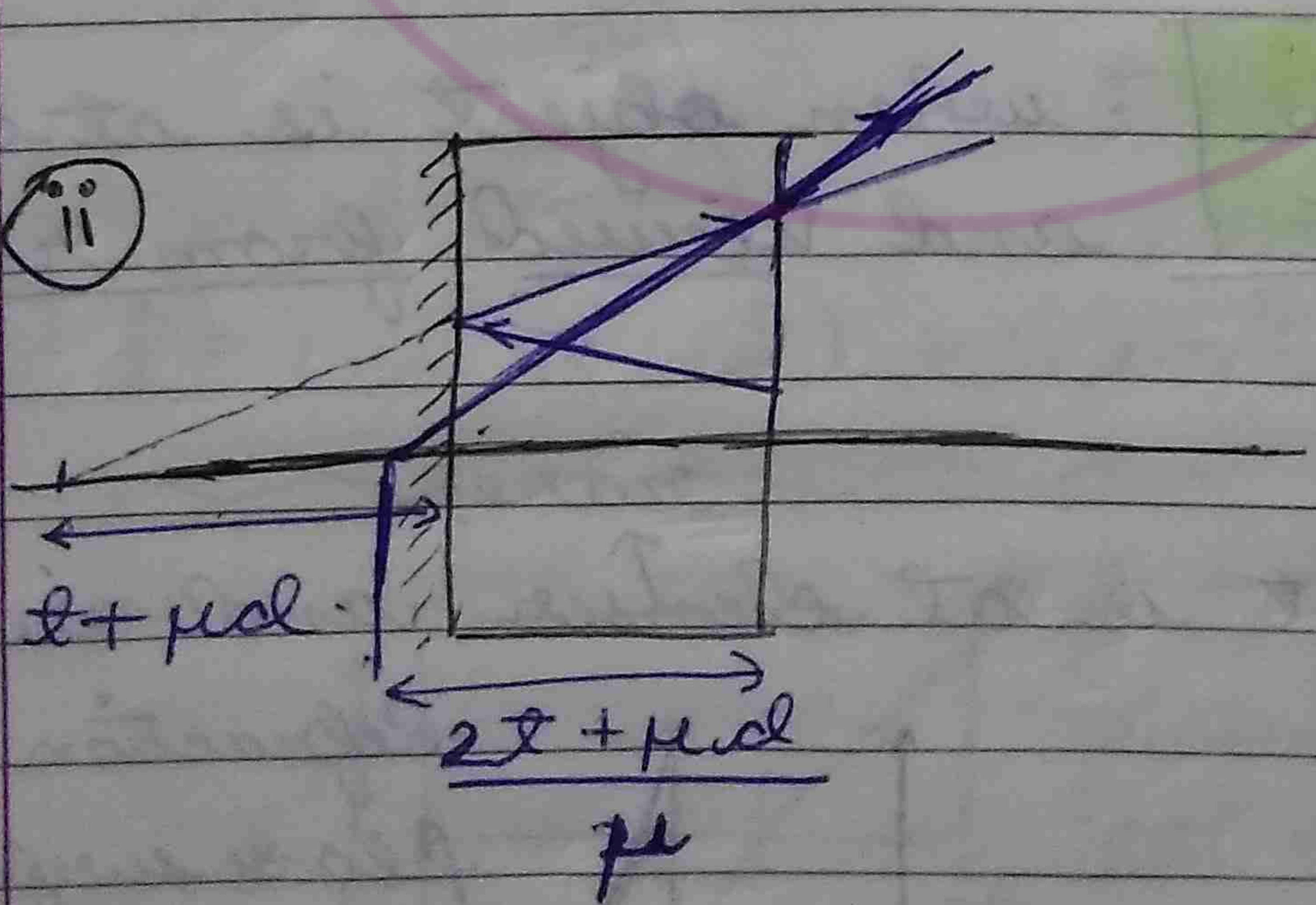
(i)



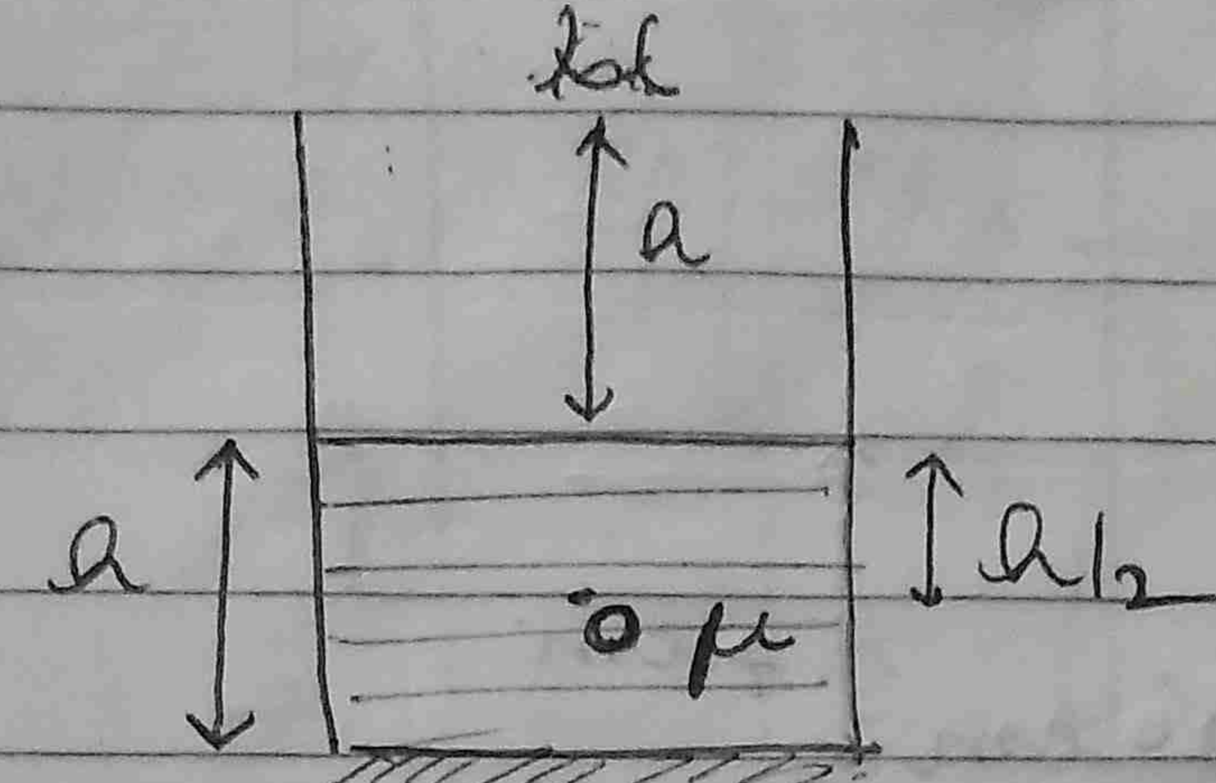
all changes are measured from surface of slab.

mirror thinks this is the object distance

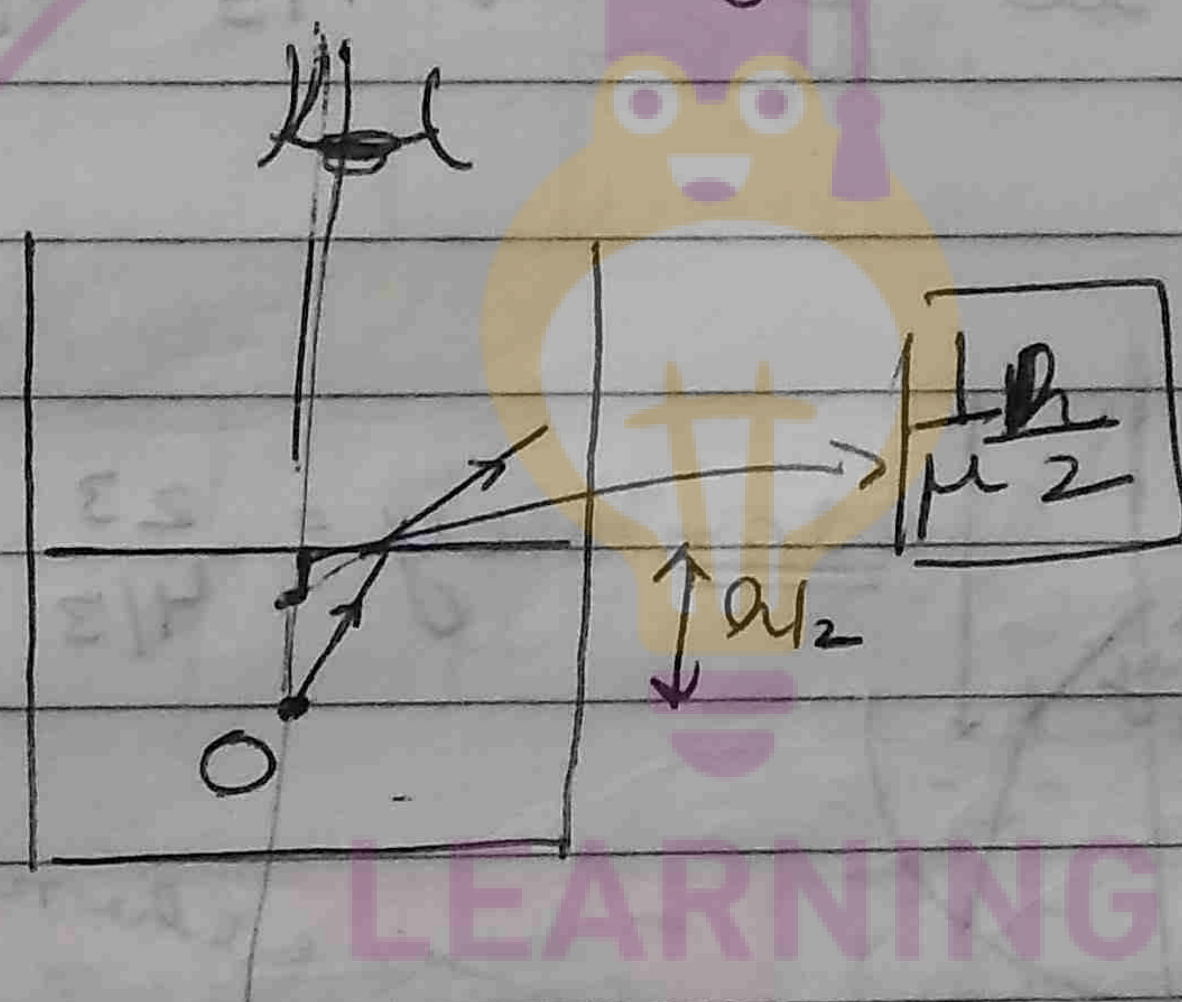
(ii)



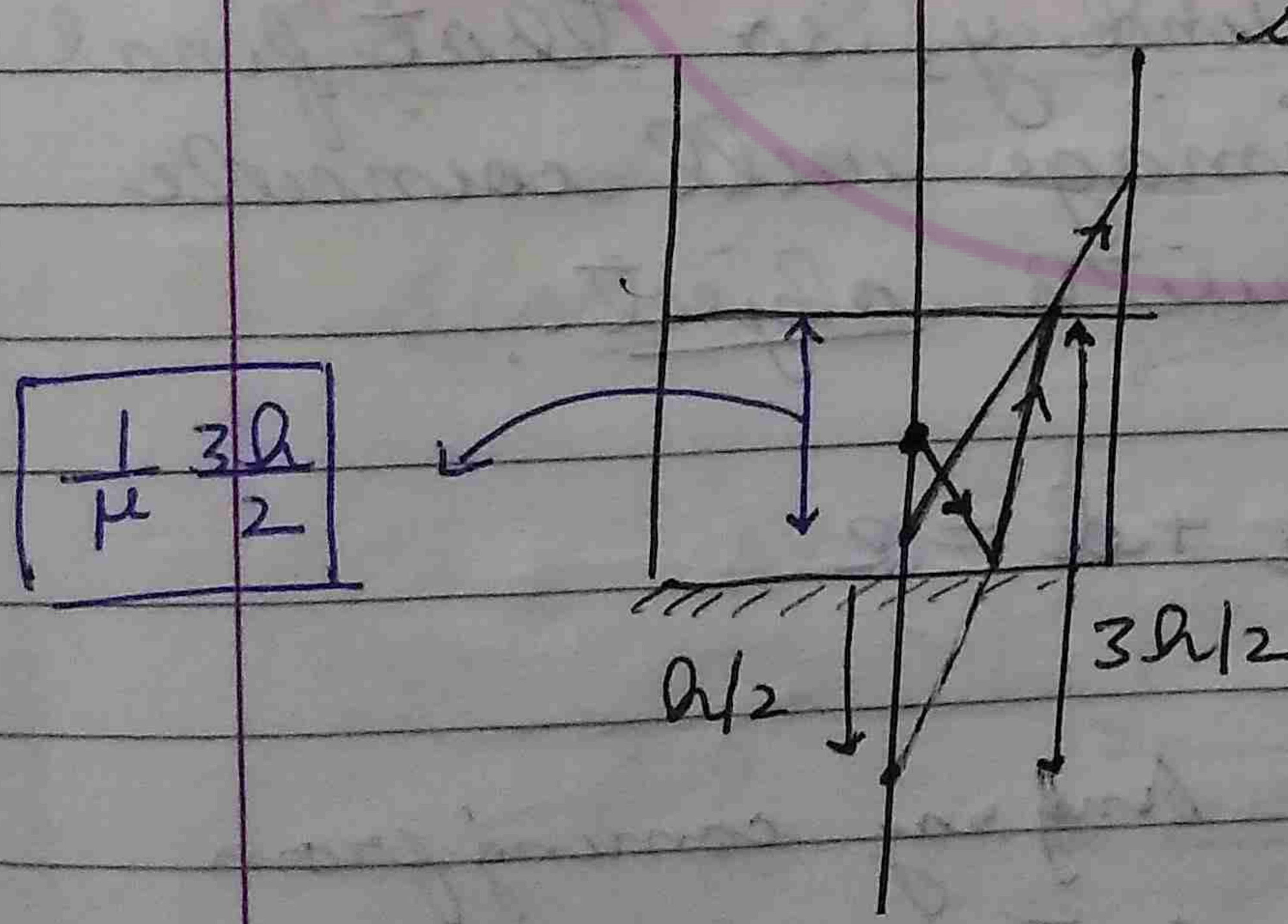
Numericals



- Case A : When object is seen directly.



- Case B : When image of the object is seen.



☆ By principle of reversibility

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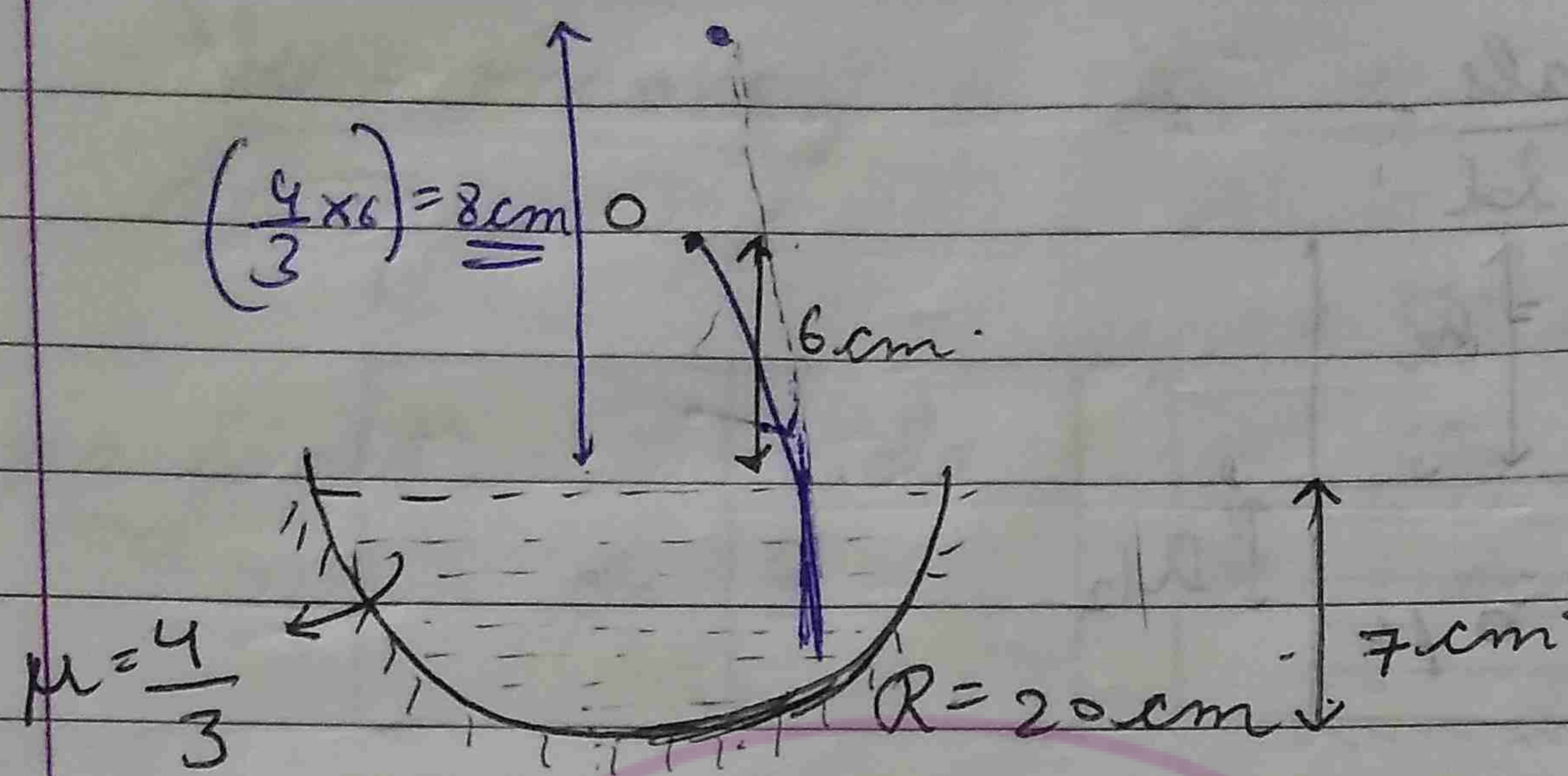
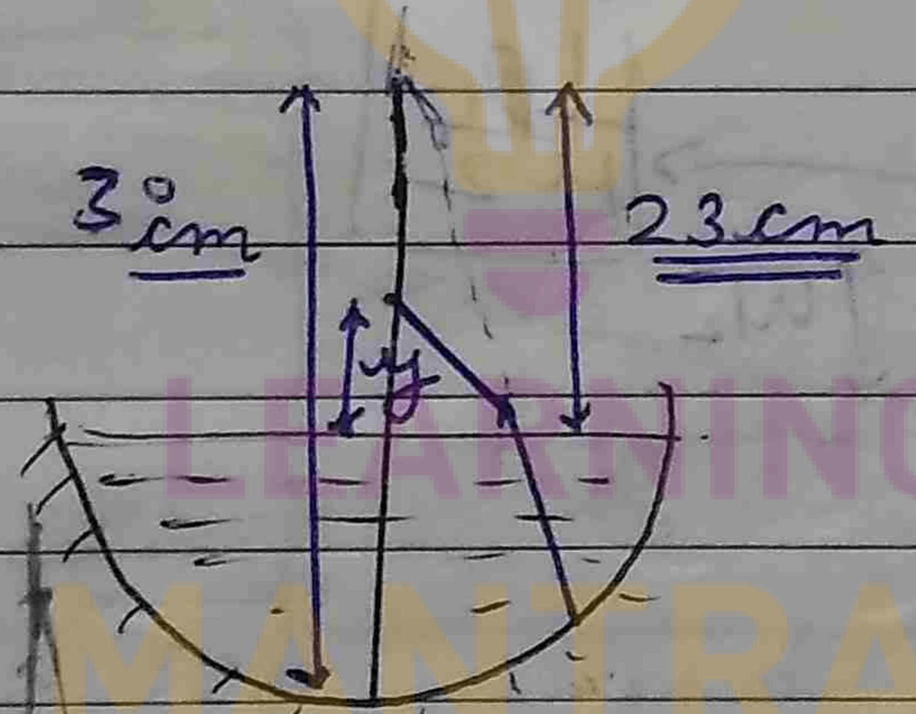
Q



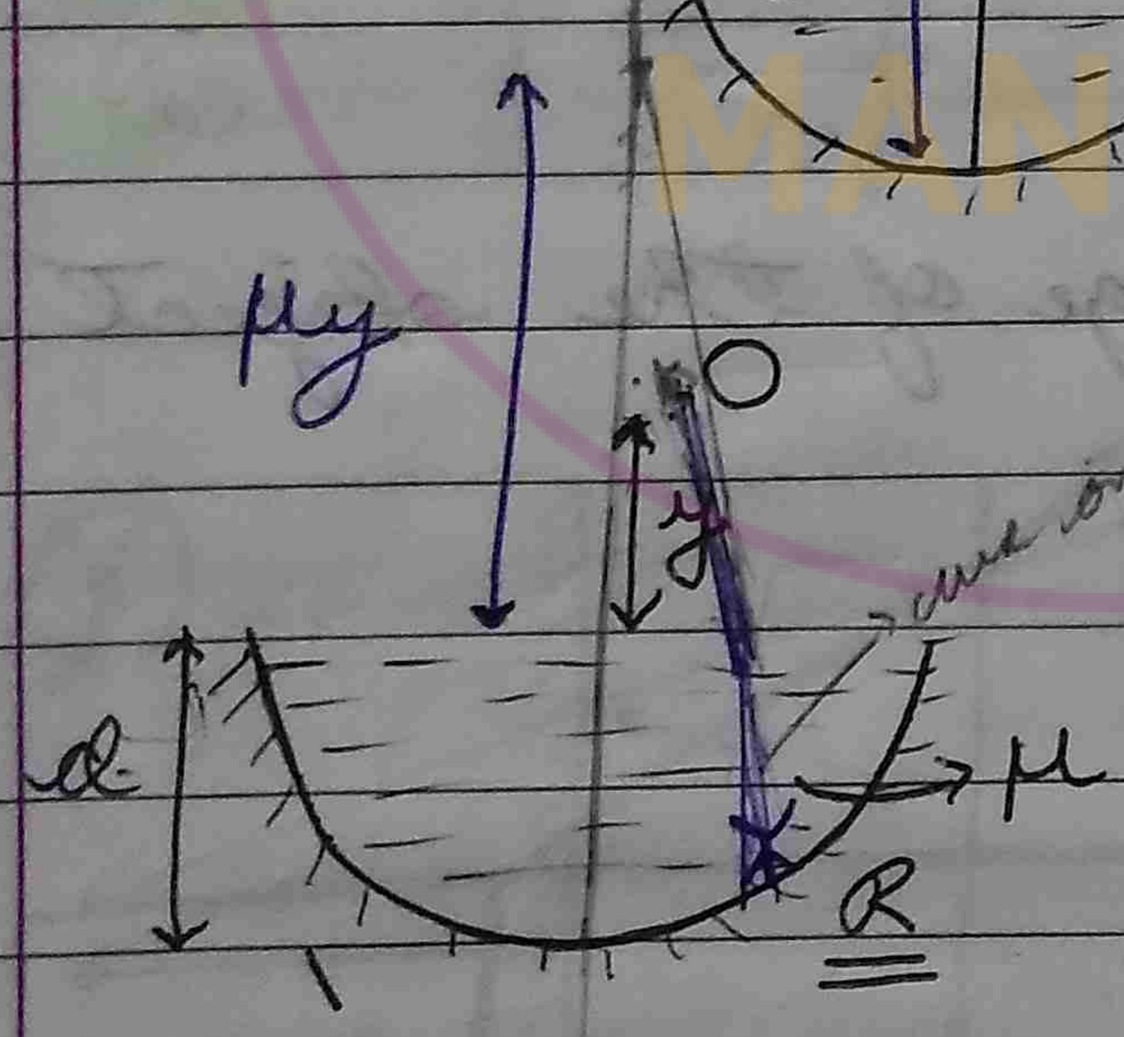
mirror $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{15} = \frac{1}{-10}$

$\Rightarrow v = -30$



$y = \frac{23}{4/3}$

☆Q



• Find y, so that final image will coincide with object.

we only have to retrace the ray ☆

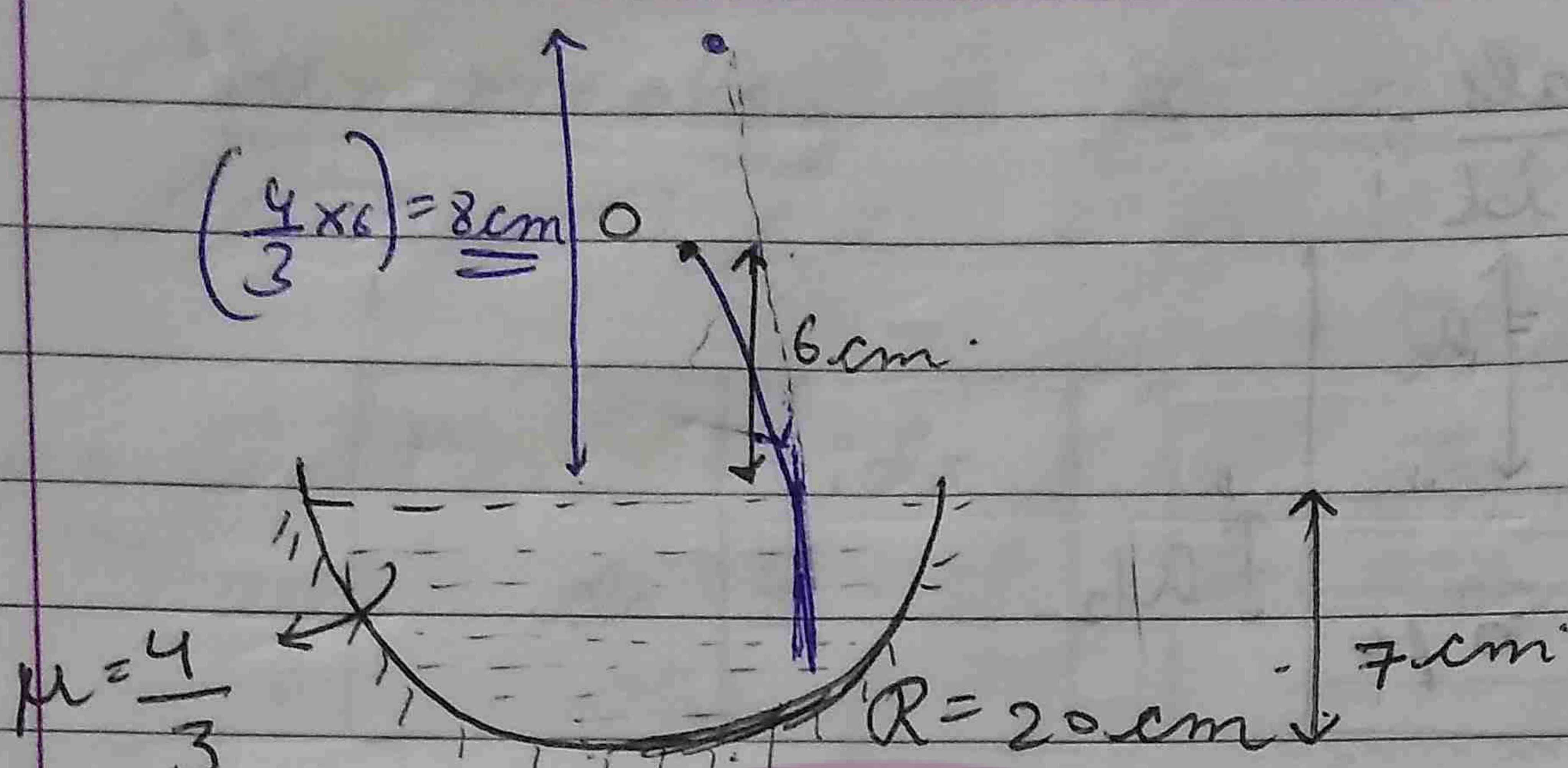
$\mu y + d = R$

$\Rightarrow y = \frac{R-d}{\mu}$

Any ray coming from centre retraces its path.

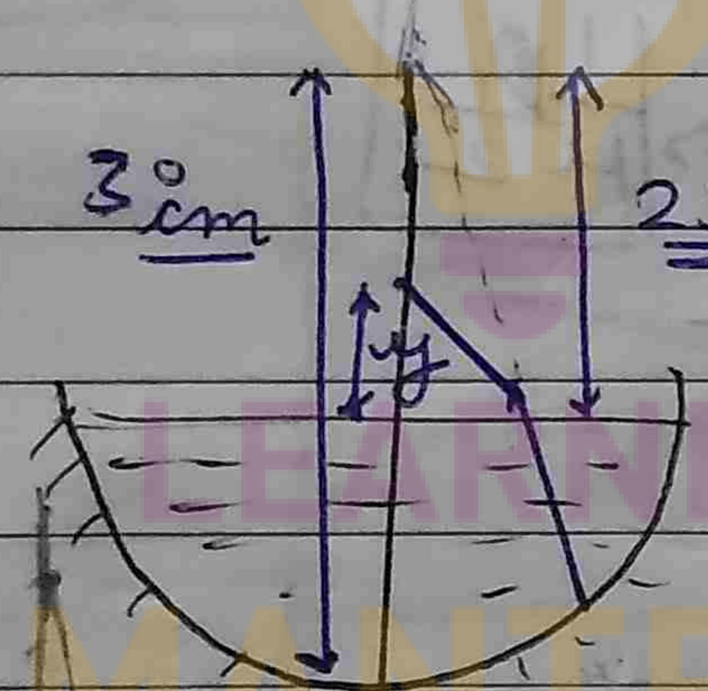
☆ By principle of reversibility

Q



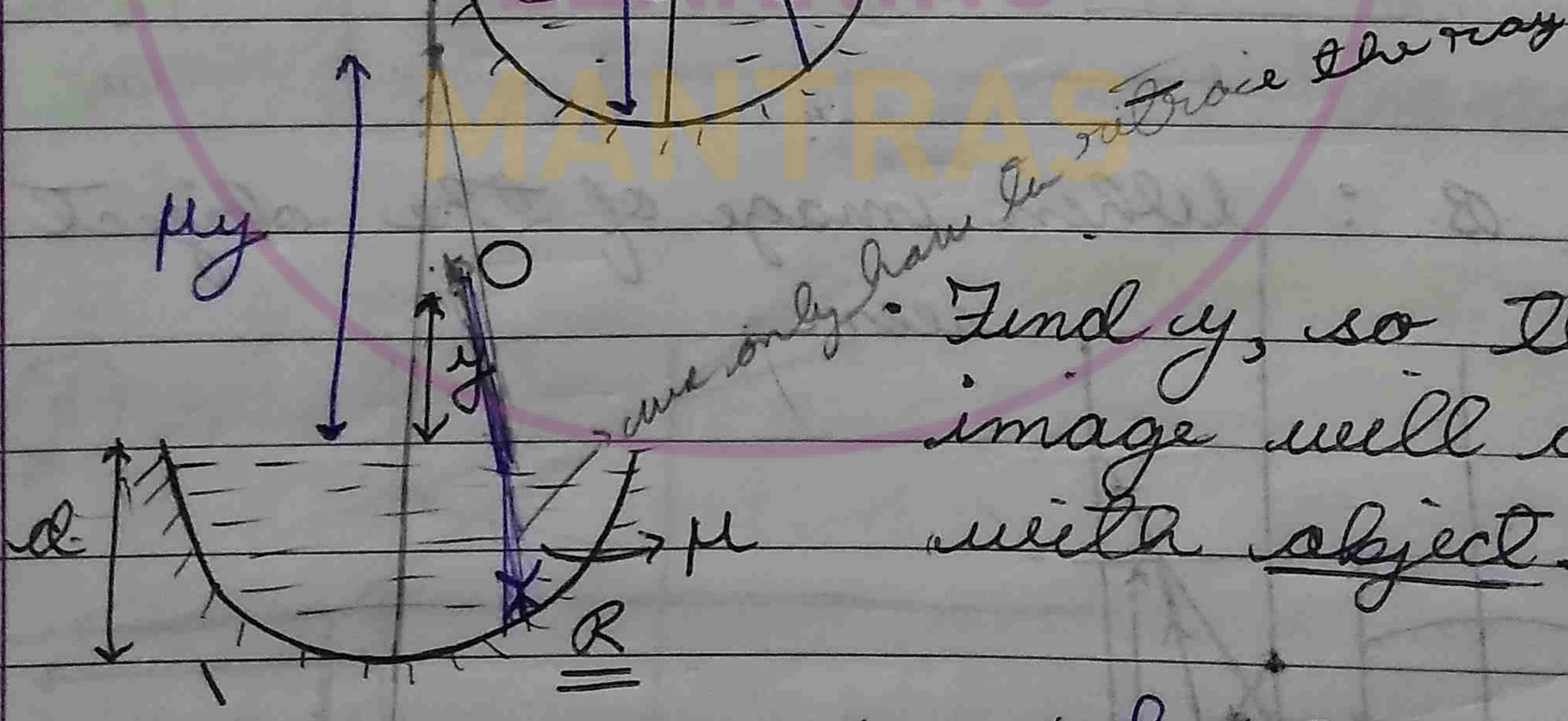
mirror $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{15} = \frac{1}{-10}$

$\Rightarrow v = -30$



$y = \frac{23}{4/3}$

☆ Q



Find y, so that final image will coincide with object.

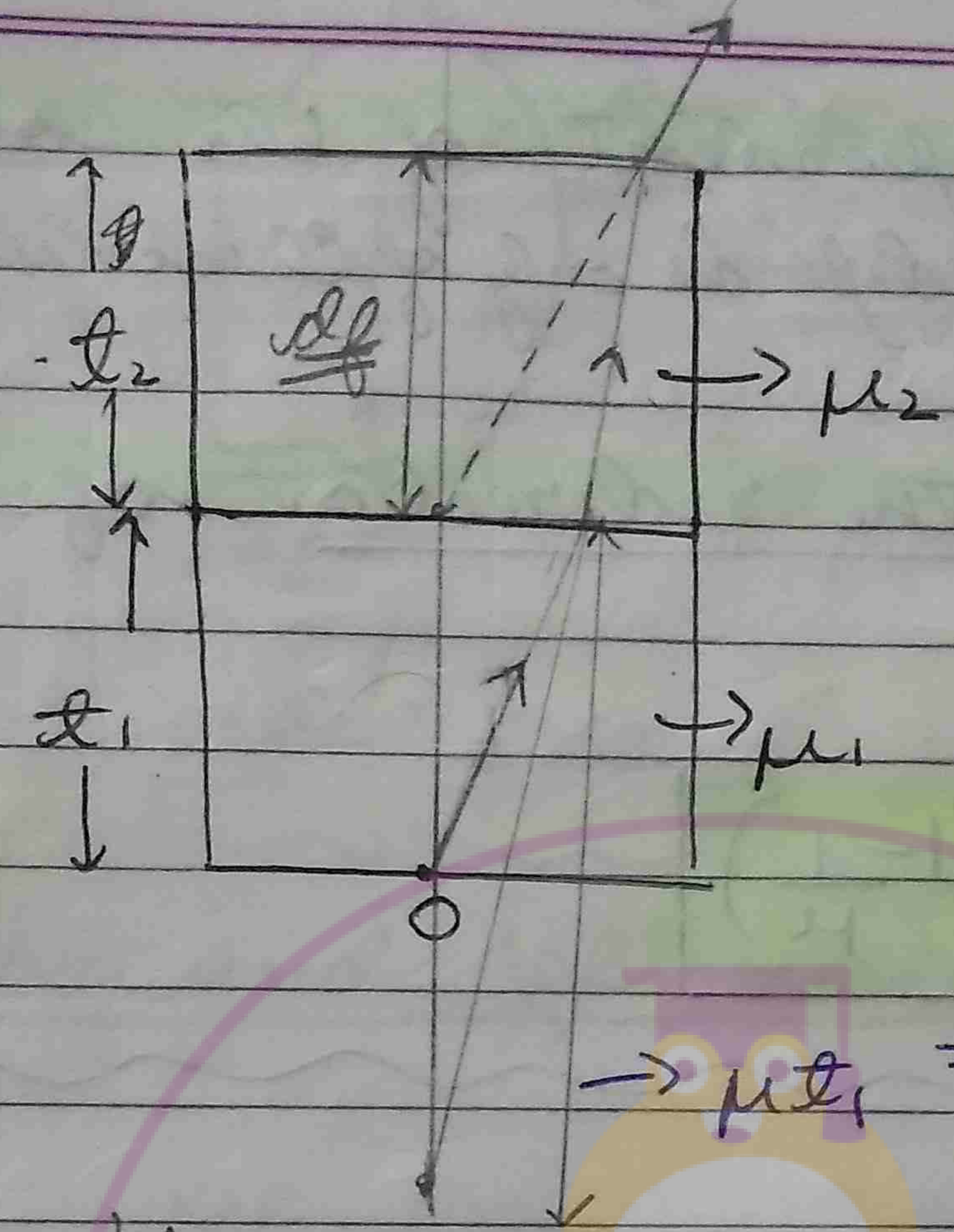
$\mu y + d = R$

$\Rightarrow \boxed{y = \frac{R - d}{\mu}}$

Any ray coming from centre retraces its path.

$$\mu = \frac{\text{real depth}}{\text{app. depth}}$$

e.g



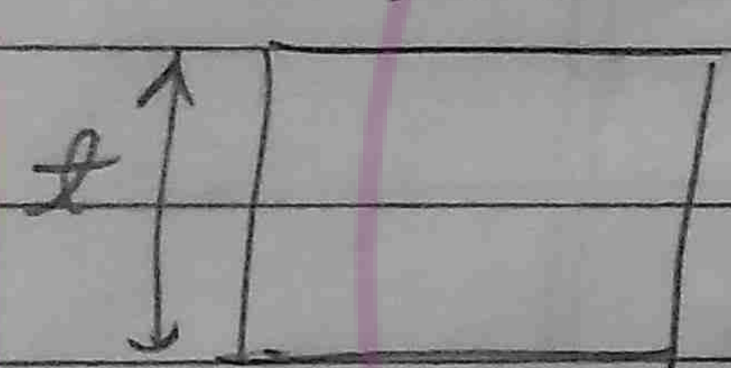
let $\mu_2 > \mu_1$

$$df = \left(t_2 + \frac{\mu_2 t_1}{\mu_1}\right) \times \frac{1}{\mu_{2/1}}$$

$$df = \frac{t_2}{\mu_2} + \frac{t_1}{\mu_1}$$

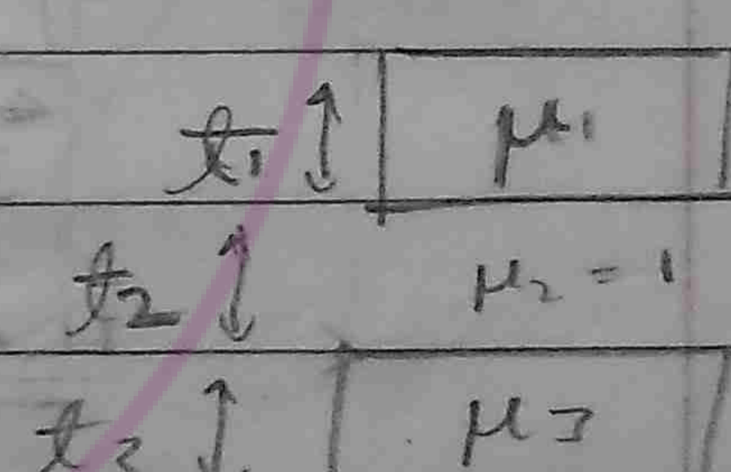
$$\rightarrow \mu t_1 = \frac{\mu_2 t_1}{\mu_1}$$

let



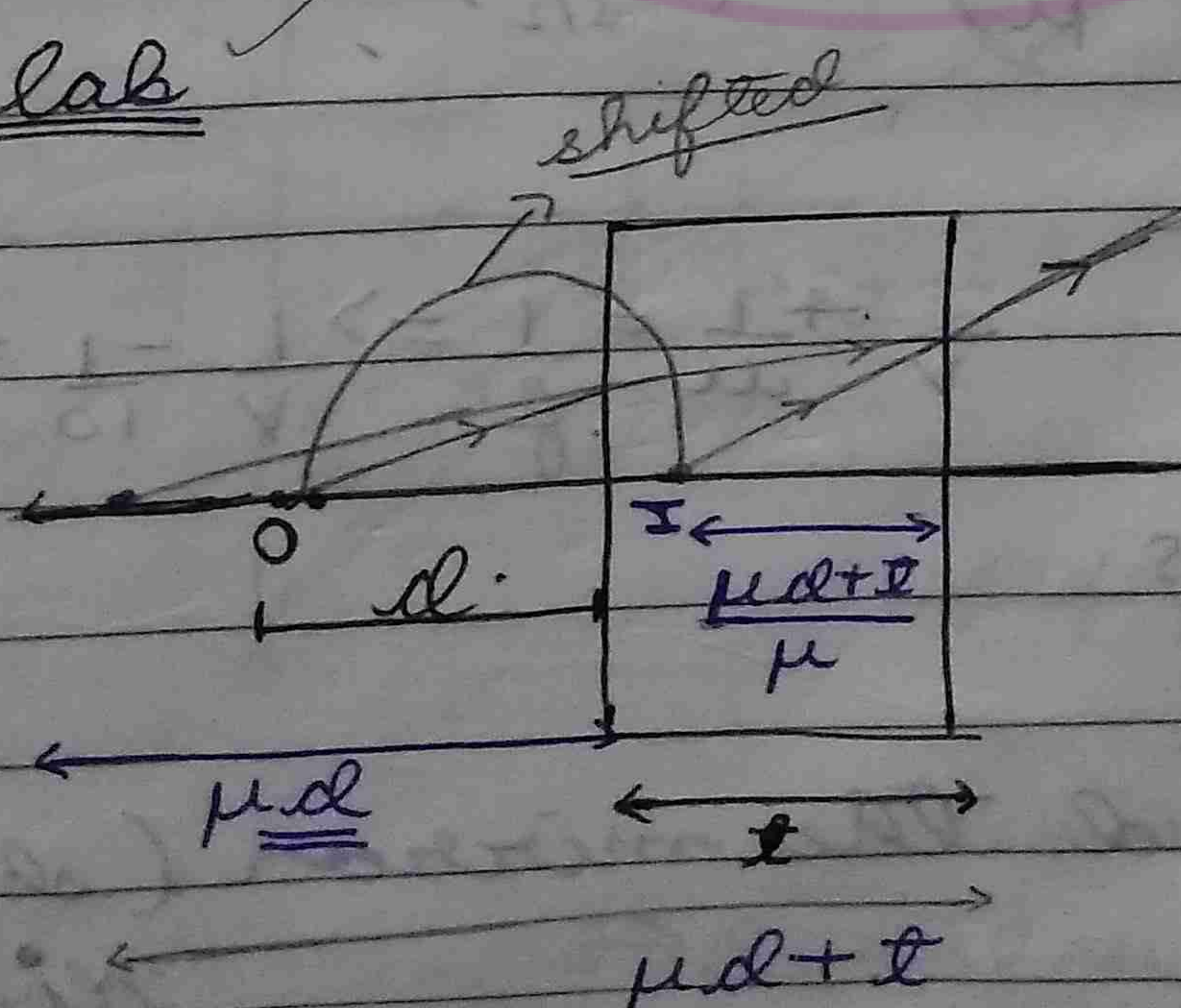
$$\frac{d_{real}}{\mu} = d_{app} \Rightarrow \mu = \frac{d_{real}}{d_{app}}$$

$$\mu_{eq} = \frac{t_1 + t_2}{\frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}} \quad \frac{(d_{real})}{(d_{app})}$$



$$= \frac{t_1 + t_2 + t_3}{\frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \frac{t_3}{\mu_3}}$$

- **Slab**



$$\text{shift} = (t + d) - \left(d + \frac{t}{\mu}\right)$$

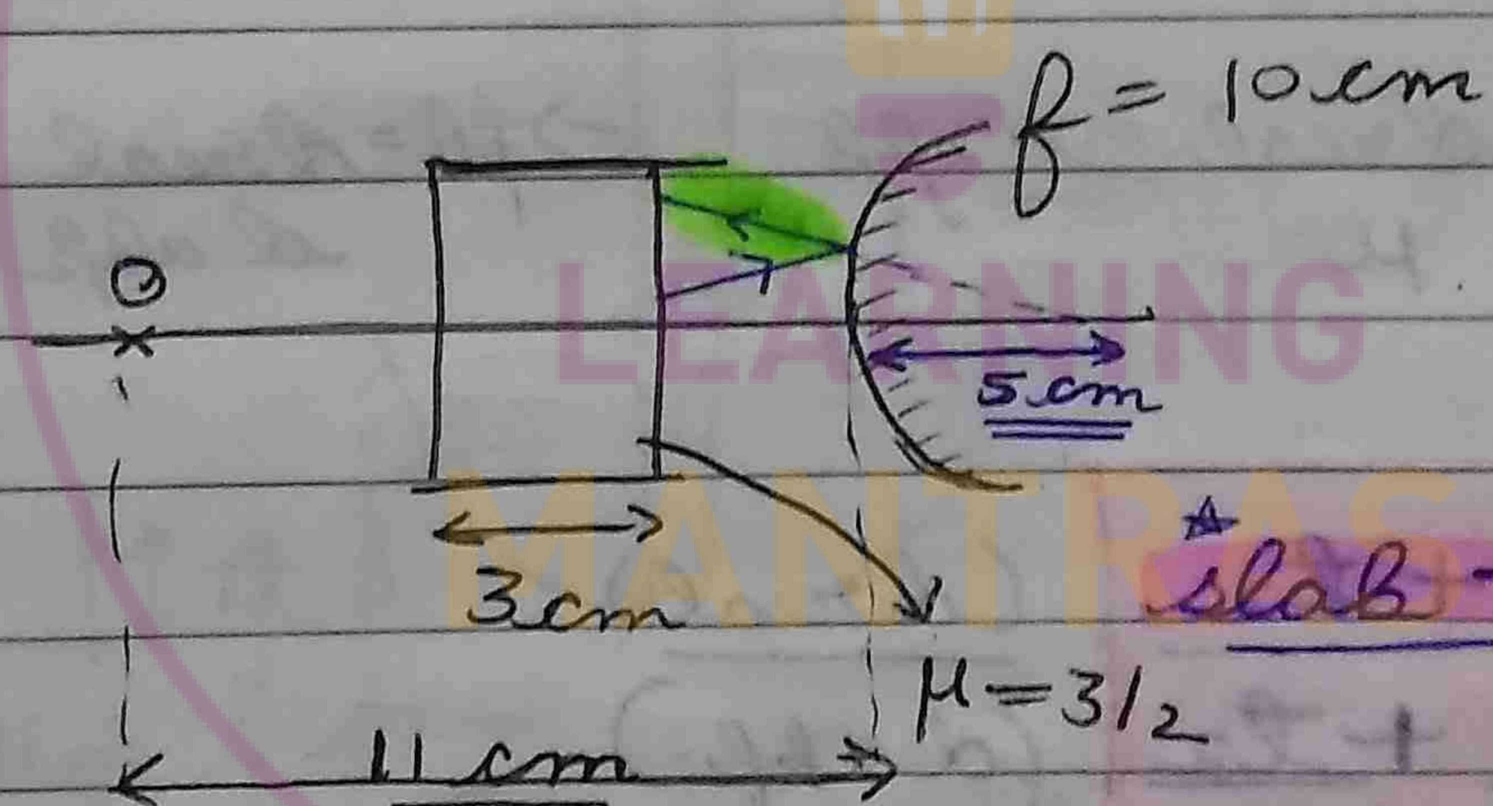
$$= t\left(1 - \frac{1}{\mu}\right)$$

(i) Shift is independent of the object distance and depends only of thickness of slab.

(ii) Shift is in the direction of incident ray.

$$\text{shift} = t\left(1 - \frac{1}{\mu}\right)$$

Numericals



★ slab → reflection → slab

$$\text{shift} = t\left(1 - \frac{1}{\mu}\right) = 3\left(1 - \frac{1}{3/2}\right) = 1\,\text{cm}$$

• Convex Mirror

$u = -10$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{10} = \frac{1}{+10}$$

$\Rightarrow v = +5$

★ 4 cm behind the mirror (shift in the direction of reflected ray)

• In case of curved mirror image coincides with object, if it is kept at centre of curvature. Because if object is kept at centre, then angle of incidence = 0

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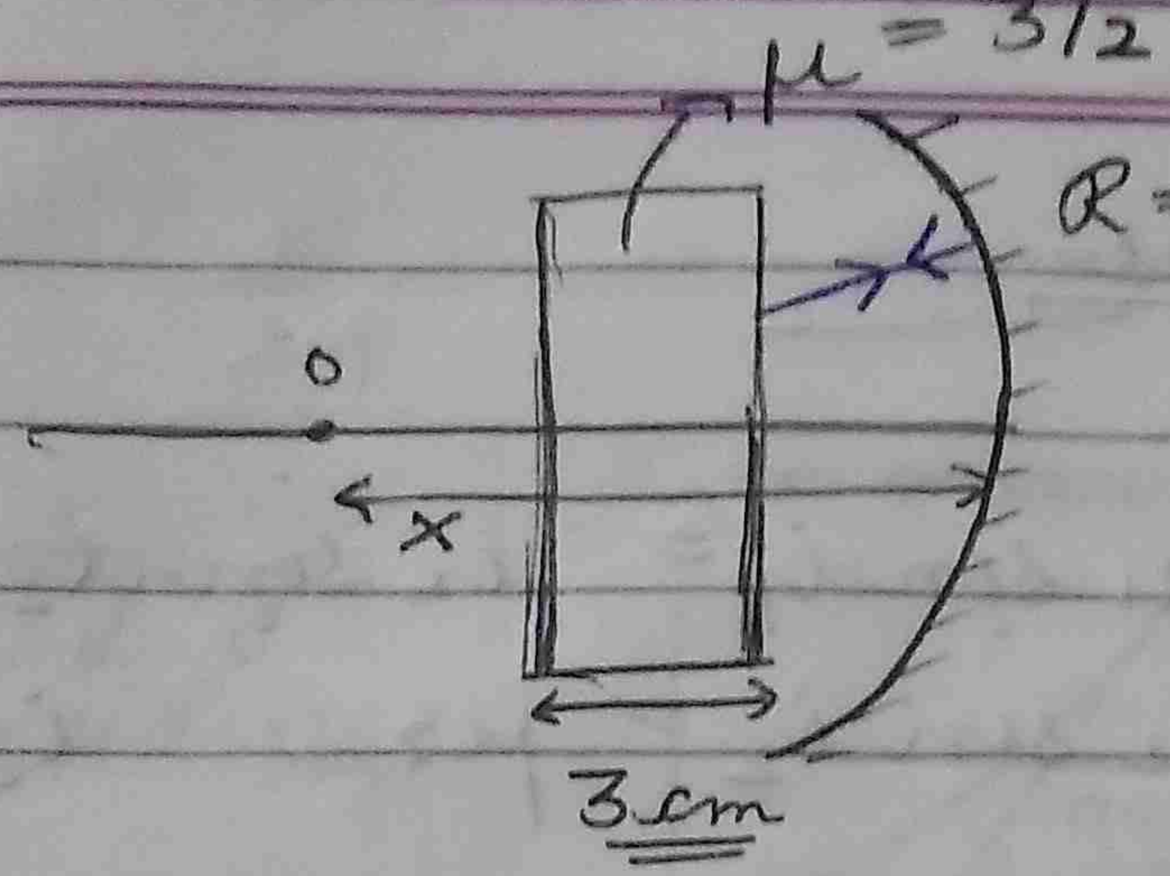
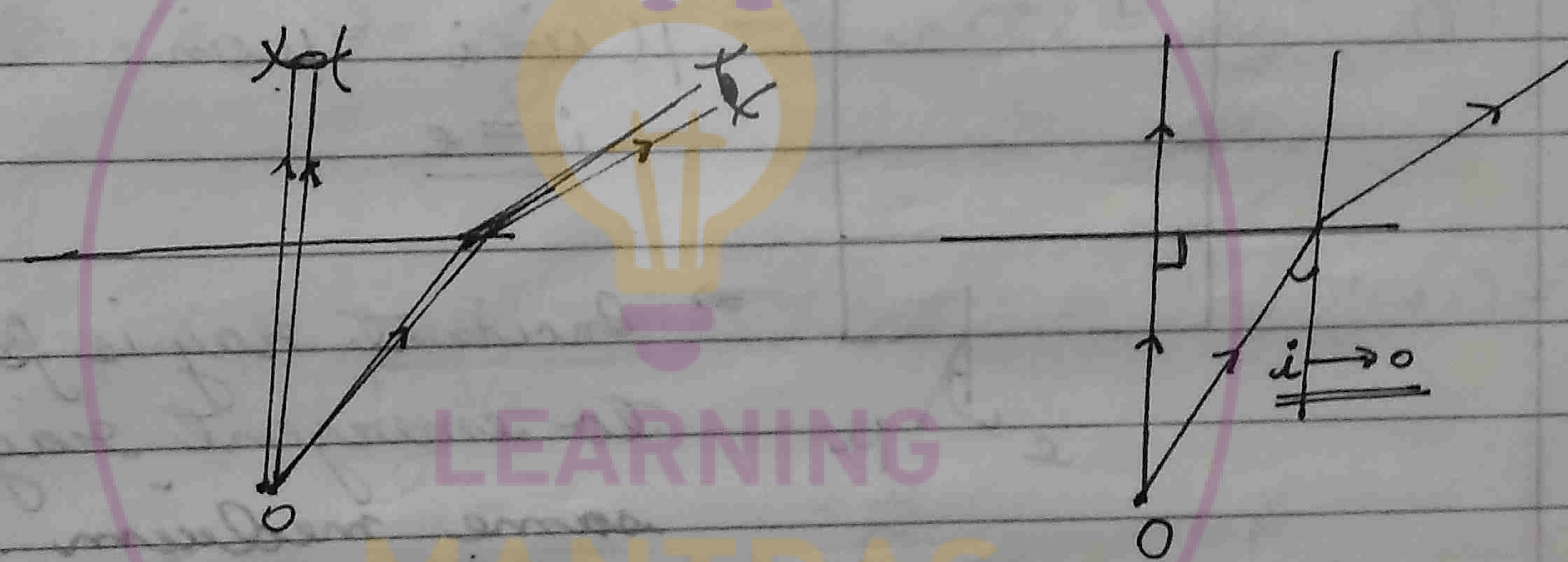
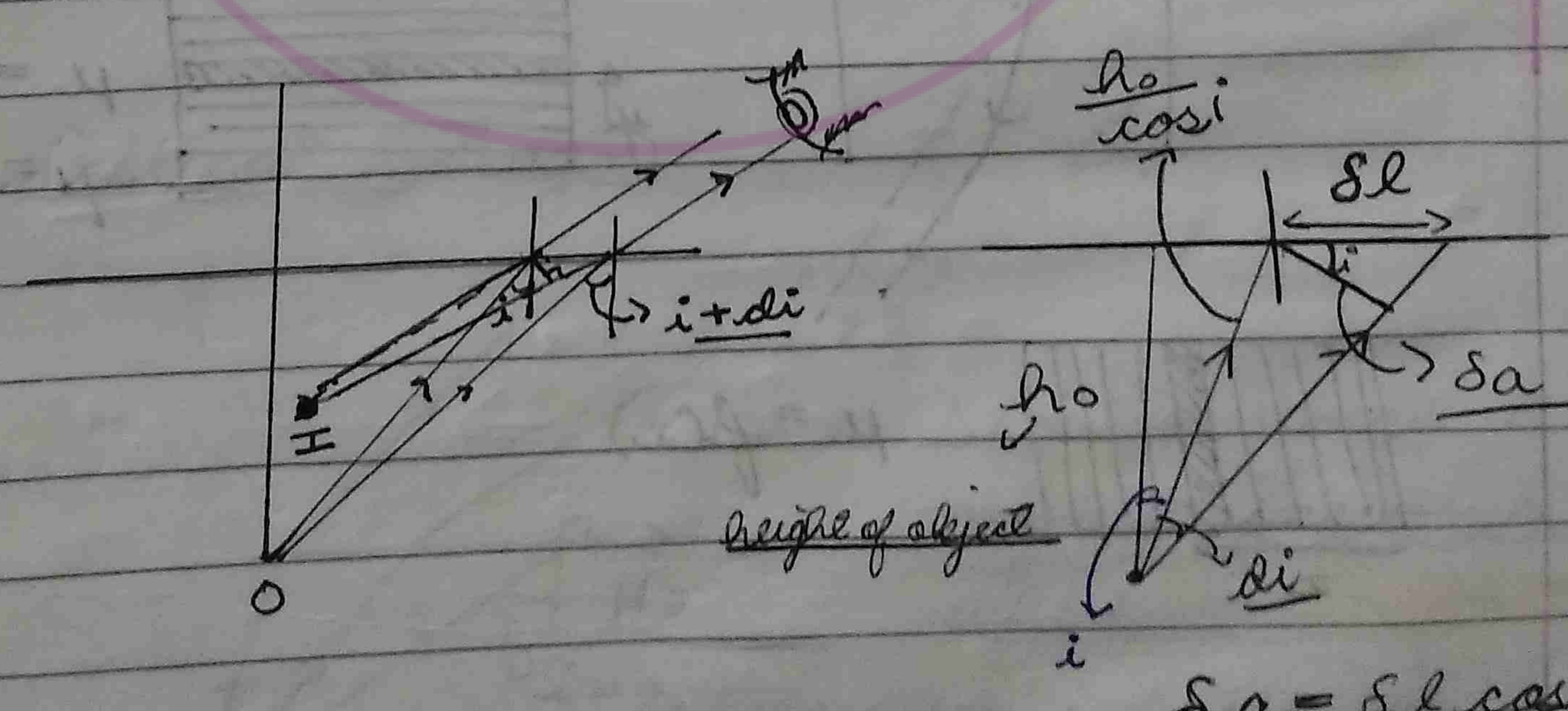
Q 
$R = 10\,cm$ • Find x so that final image will coincide with object.

shift = 1 cm

$x - 1 = R = 10\,cm \Rightarrow x = 11\,cm$

• Real and Apparent depth for oblique vision

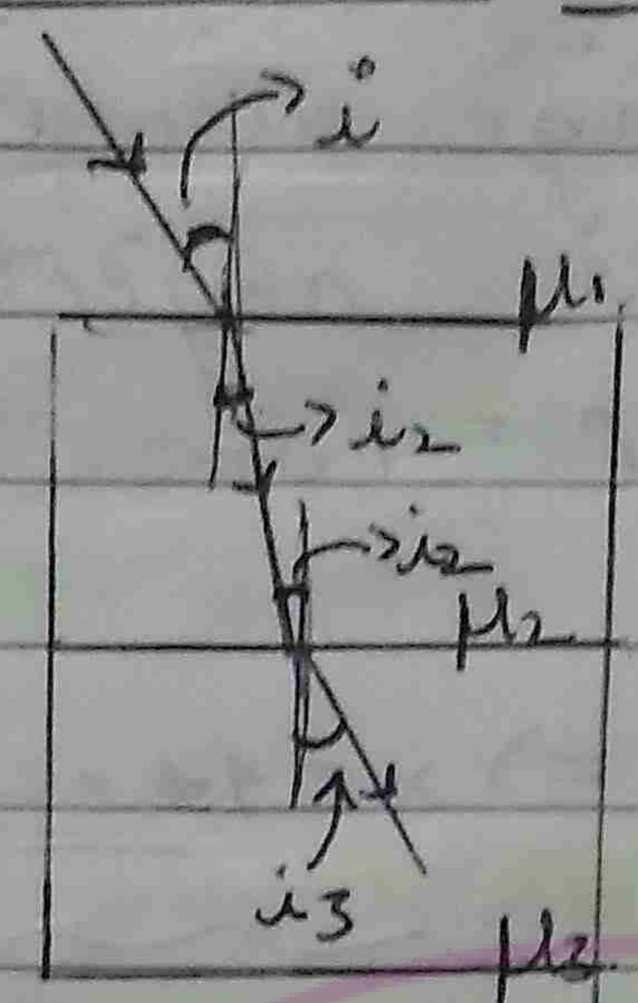




$\delta a = \delta l \cos i$

$\Rightarrow \delta a = \delta l \cos i = \frac{h_0\, di}{\cos i}$

Parallel Slabs



$\mu_1 \sin i_1 = \mu_2 \sin i_2$ (i)

$\mu_2 \sin i_2 = \mu_3 \sin i_3$ (ii)

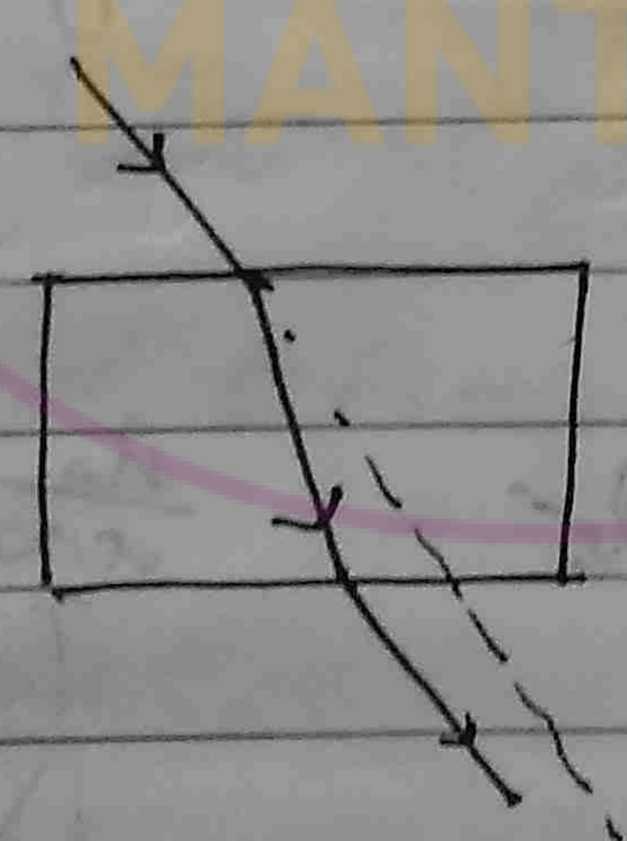
☆ $\mu_1 \sin i = \mu_3 \sin i_3$

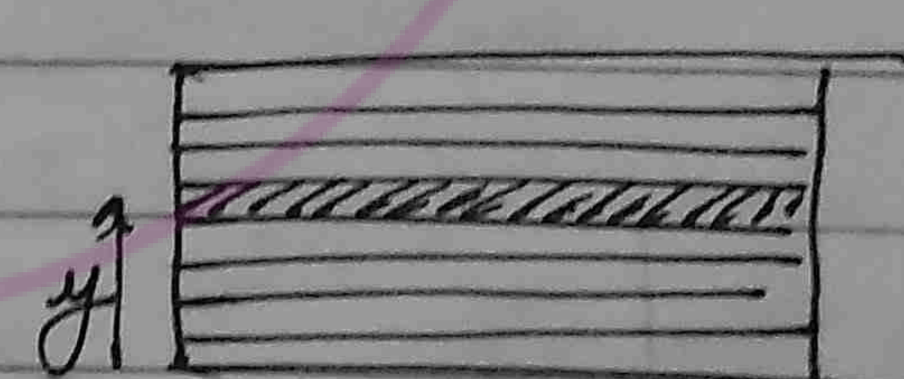
$\mu \sin i = \mu \sin e$

$i = e$

⇒ Incident ray is parallel to emergent ray of same medium.

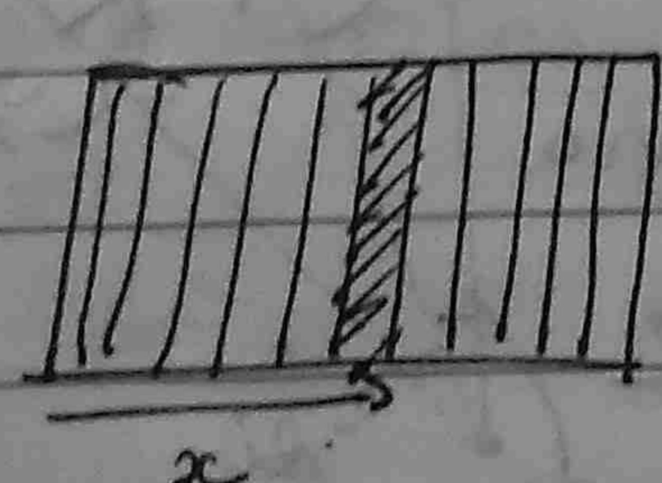
Variable μ



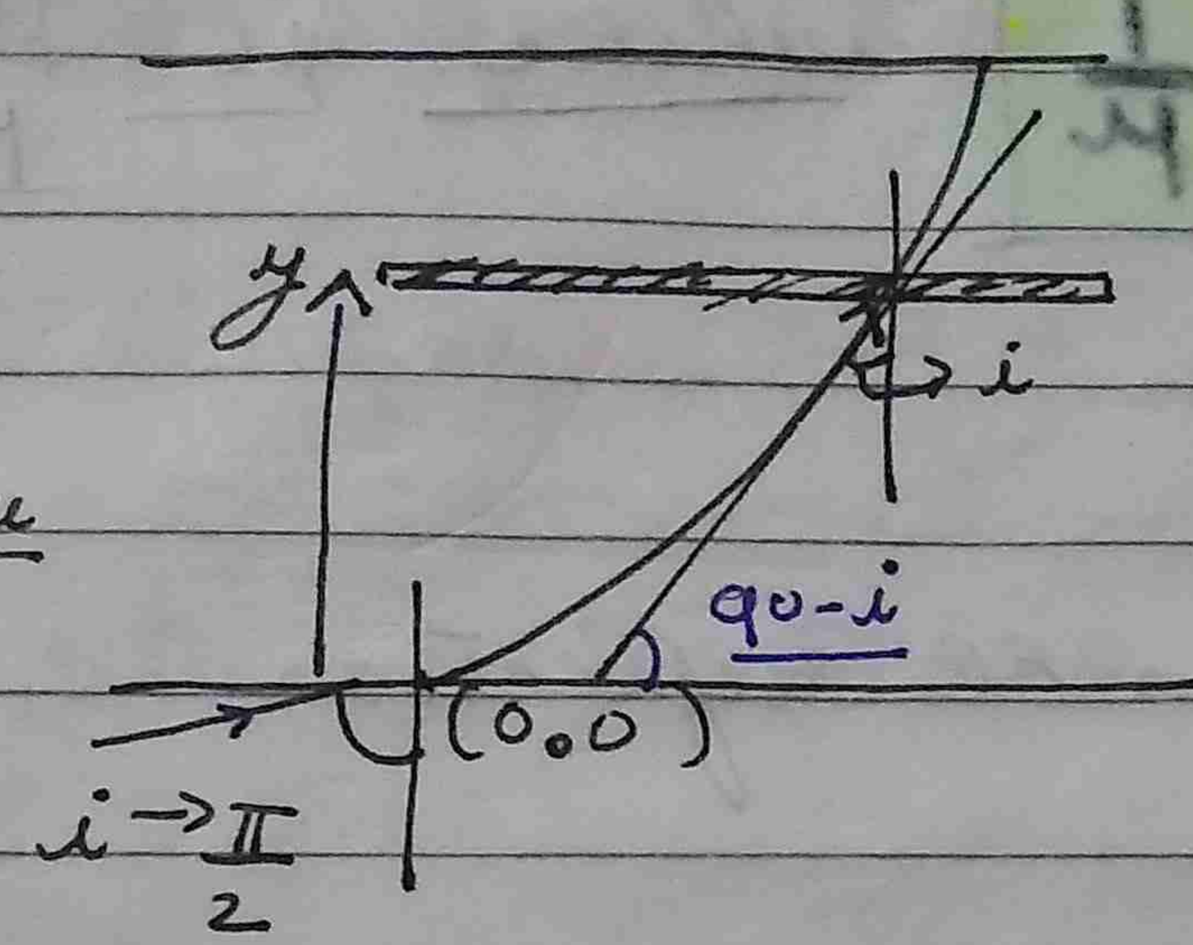


$\mu = f(y)$

$\mu = k(y)$



$\mu = f(x)$



grazing incidence

$\mu = [y^{3/2}+1]^{1/2}$

- Find the trajectory of light.

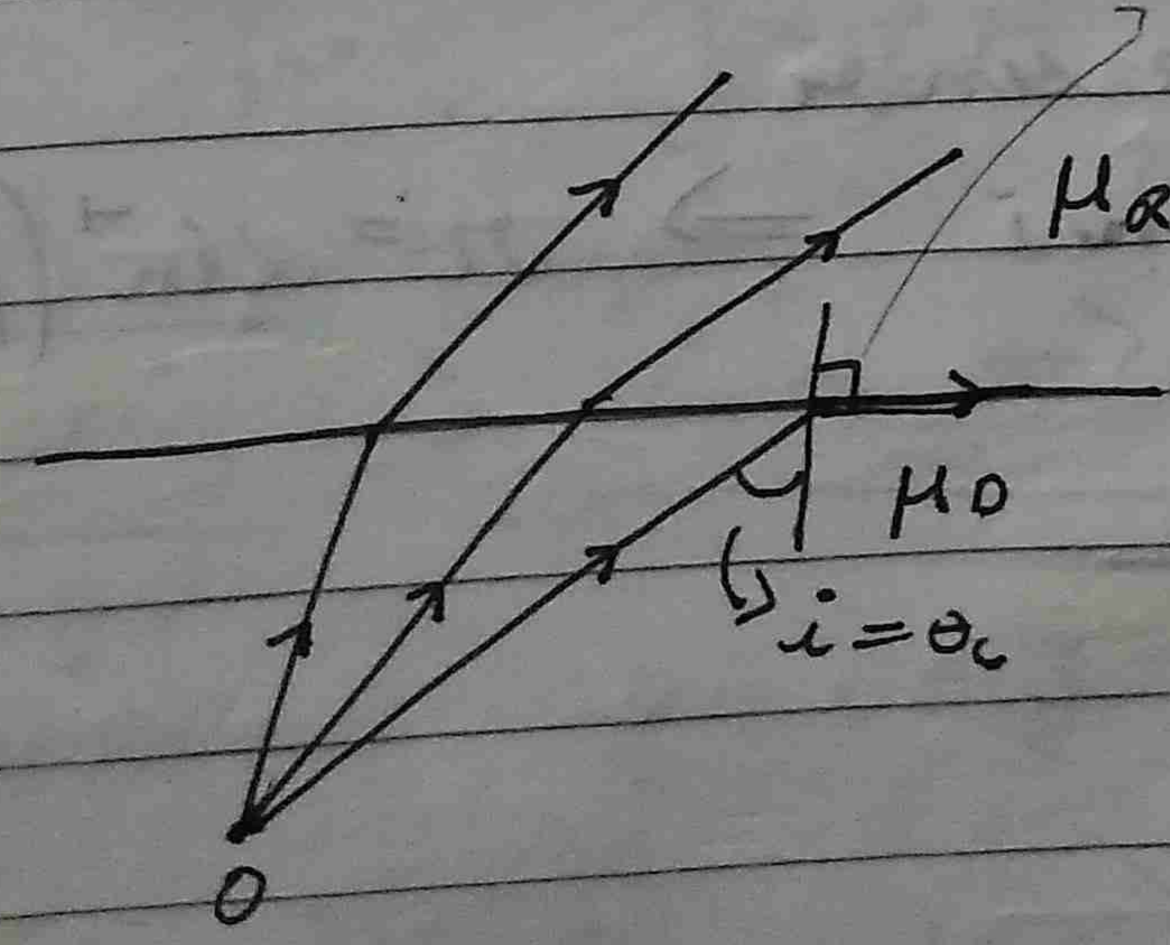
$1 \sin \pi/2 = [y^{3/2}+1]^{1/2} \sin i$ --- (i)

$\tan(90-i) = \frac{dy}{dx} \Rightarrow \cot i = \frac{dy}{dx}$ (ii)

$\sin i = \frac{1}{\sqrt{y^{3/2}+1}}$ $\cot i = \sqrt{\frac{(y^{3/2}+1)-1}{1}} = y^{3/4}$

$\frac{dy}{dx} = y^{3/4} \Rightarrow \int_0^y \frac{dy}{y^{3/4}} = \int_0^x dx$

- Critical Angle



- Critical angle is that angle of incidence for which $\angle r = \pi/2$

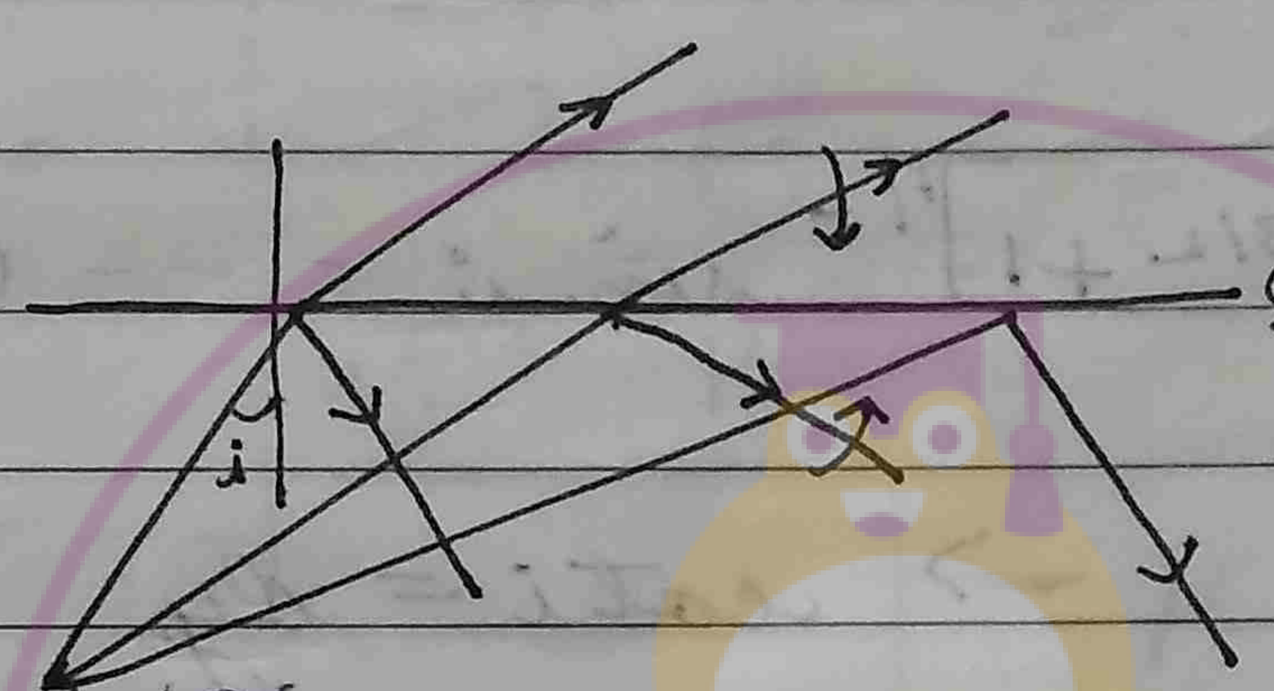
$\mu_D \sin i = \mu_R \sin r$

$\mu_D \sin(\theta_c) = \mu_R \sin \pi/2$

$\sin \theta_c = \frac{\mu_R}{\mu_D} = \frac{1}{\mu}$.

$\sin\theta_c = \frac{1}{\mu}$ where $\mu = \frac{\mu_D}{\mu_R}$

- Total internal reflection

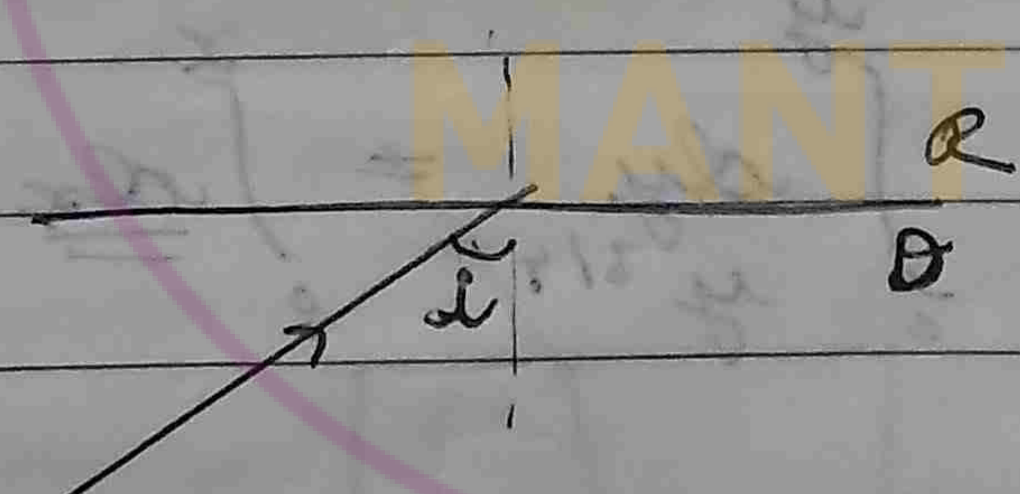


Conditions

(i) $i > \theta_c$

(ii) light should travel from denser to rarer

e.g



$\sin\theta_c = \frac{1}{\mu}$

- when $i < \theta_c$ --- Refraction

$\mu_D \sin i = \mu_R \sin r$

$\sin r = \frac{\mu_D}{\mu_R} \sin i \Rightarrow r = \sin^{-1}(\mu \sin i)$

$\delta = r - i$

☆ when $i = \theta_c$ then light is refracted but critically. i.e $r = \frac{\pi}{2}$

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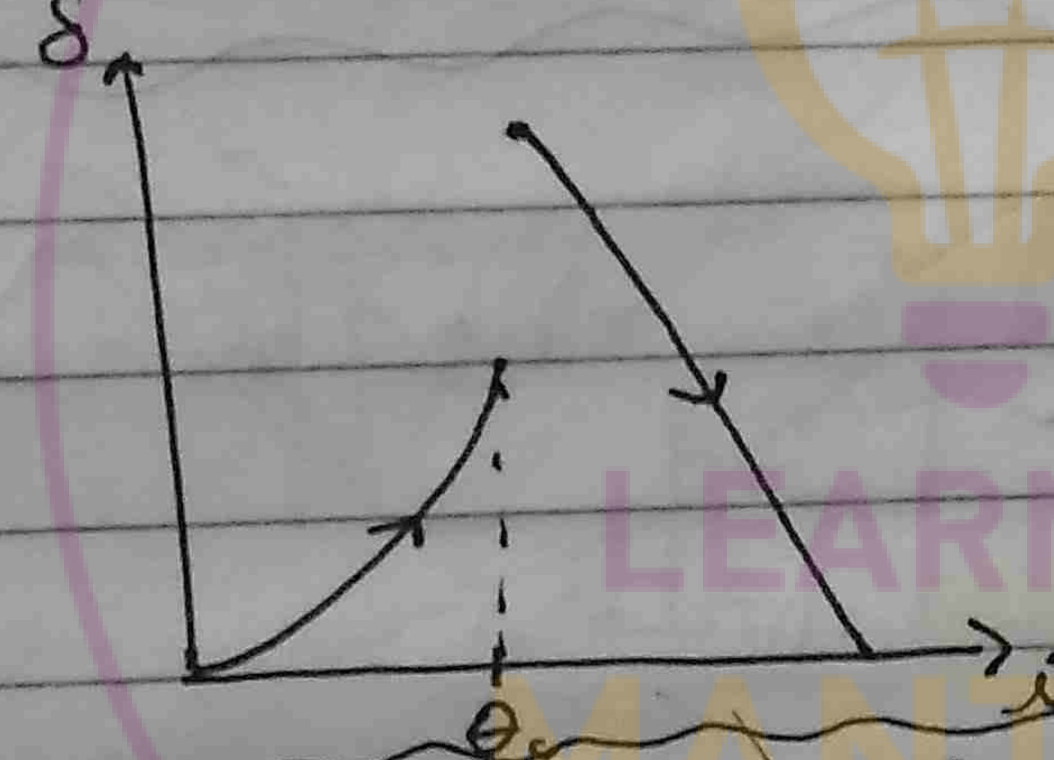
case I — when $i > \theta_c$ $\quad \delta = \pi - 2i$ (TIR) (reflection)

case II — when $i < \theta_c$ $\quad \delta = r - i = \sin^{-1}(\mu \sin i) - i$ (refract)

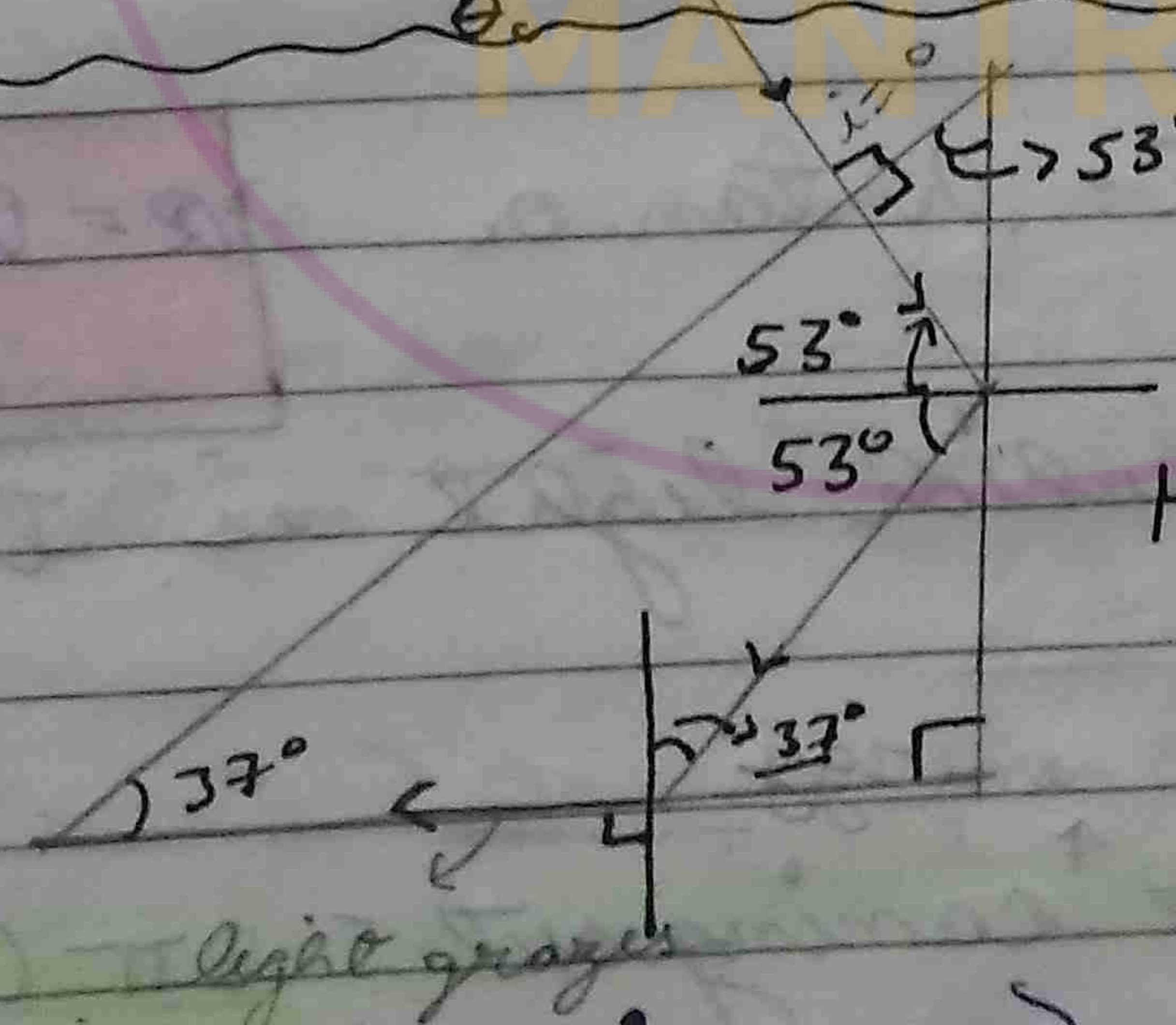
for $i = \theta_c^+$ $\quad \delta = \pi - 2\theta_c$

for $i = \theta_c^-$ $\quad \delta = \sin^{-1}(\mu \sin\theta_c) - \theta_c = \frac{\pi}{2} - \theta_c$

$\Rightarrow \frac{\pi - 2\theta_c}{2}$



e.g



Calculate the net deviation :

$\mu = \frac{5}{3}$

$i \quad \theta_c$

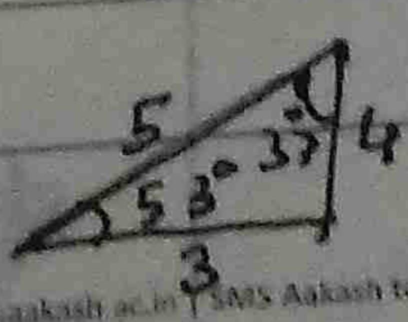
$\sin i = 4/5 > \sin r = \frac{1}{\mu} = \frac{1}{5/3} = \frac{3}{5}$

$\because \sin i > \sin\theta_c \Rightarrow i > \theta_c \Rightarrow$ TIR

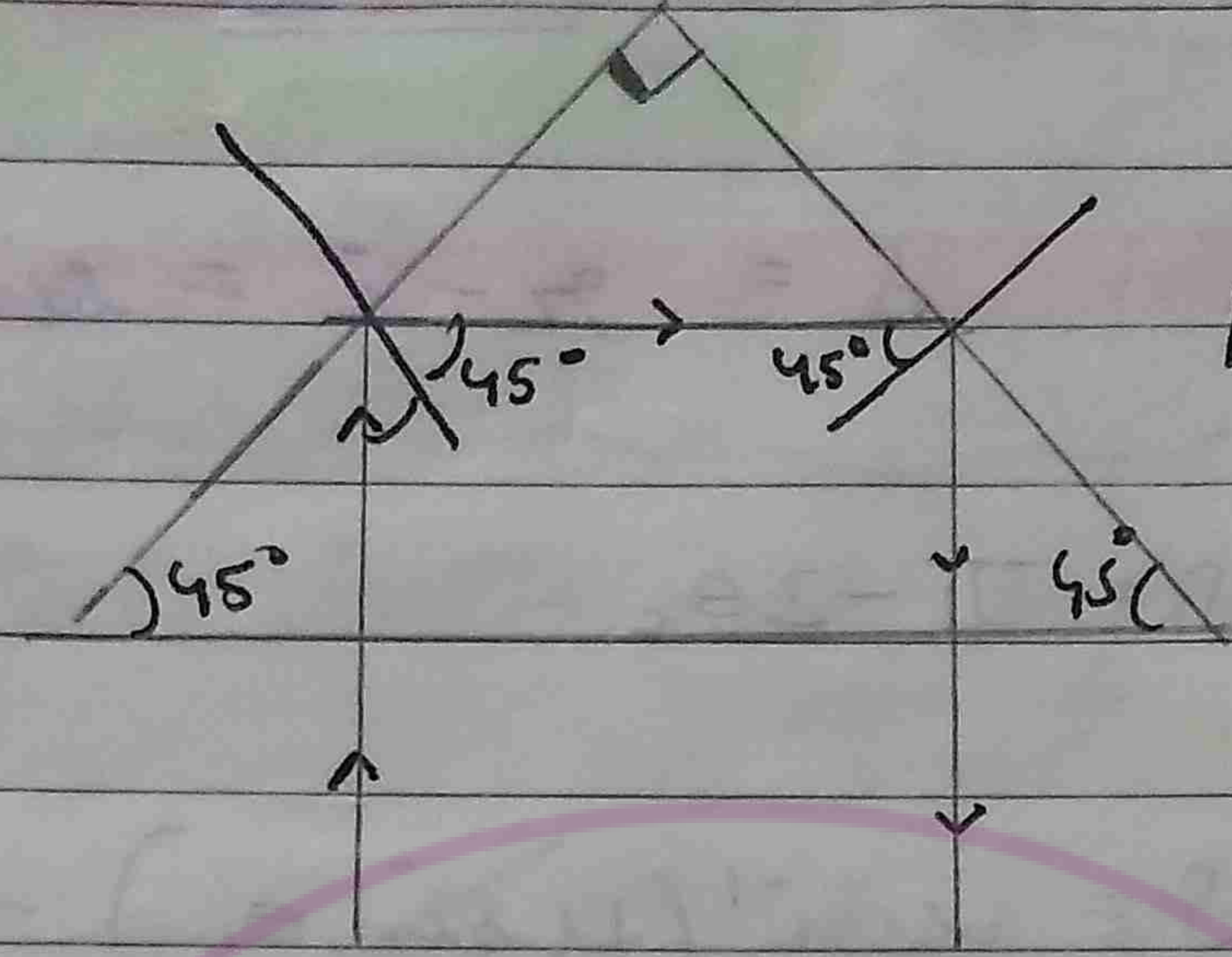
II incidence $\sin i = \frac{3}{5}$ $\quad \sin\theta_c = \frac{3}{5}$ $\Rightarrow i = \theta_c \Rightarrow r = \frac{\pi}{2}$

$\Rightarrow \delta_{net} = (180 - 106) + (90 - 37)$

$(180 - 2i) \quad (r - i)$ refraction



• Use of prism for reflection is better than a mirror because of TIR there is negligible loss of energy.

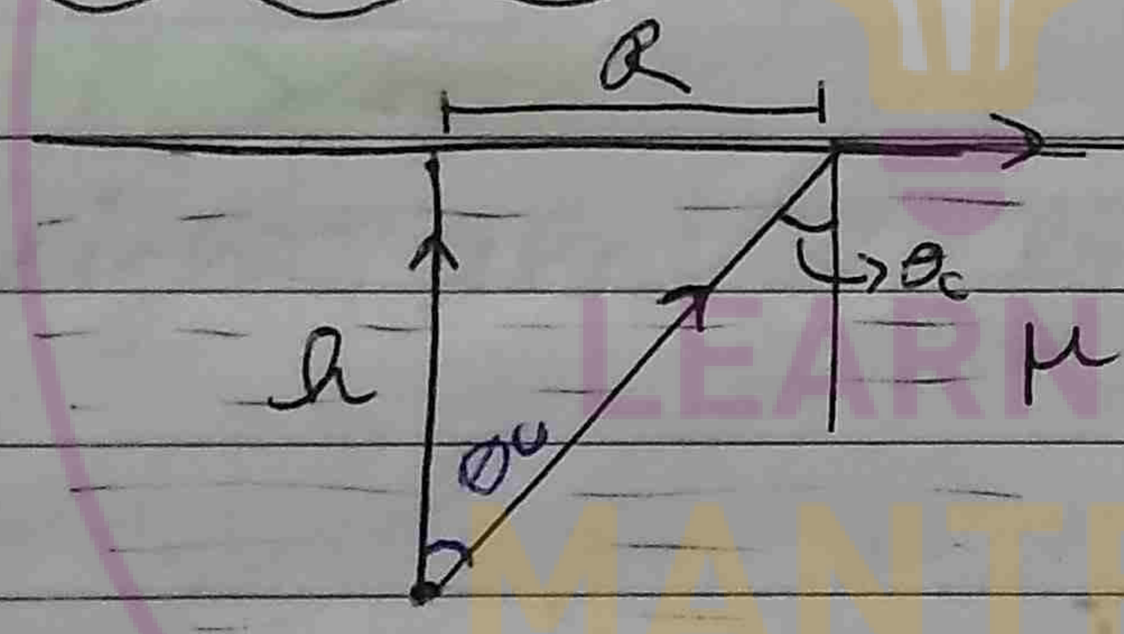


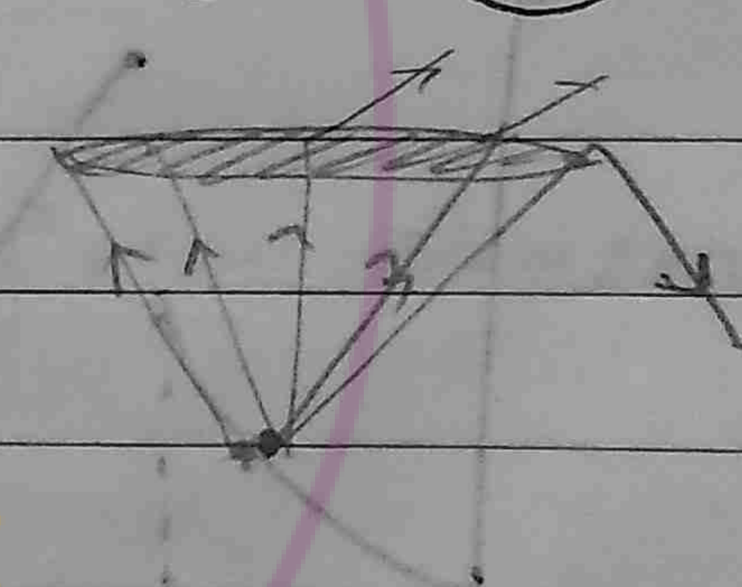
$\mu = 3/2$

$\sin i = \frac{1}{\sqrt{2}} = 0.707 \quad \sin\theta_c = \frac{2}{3} = 0.67$

$\Rightarrow$ TIR

e.g



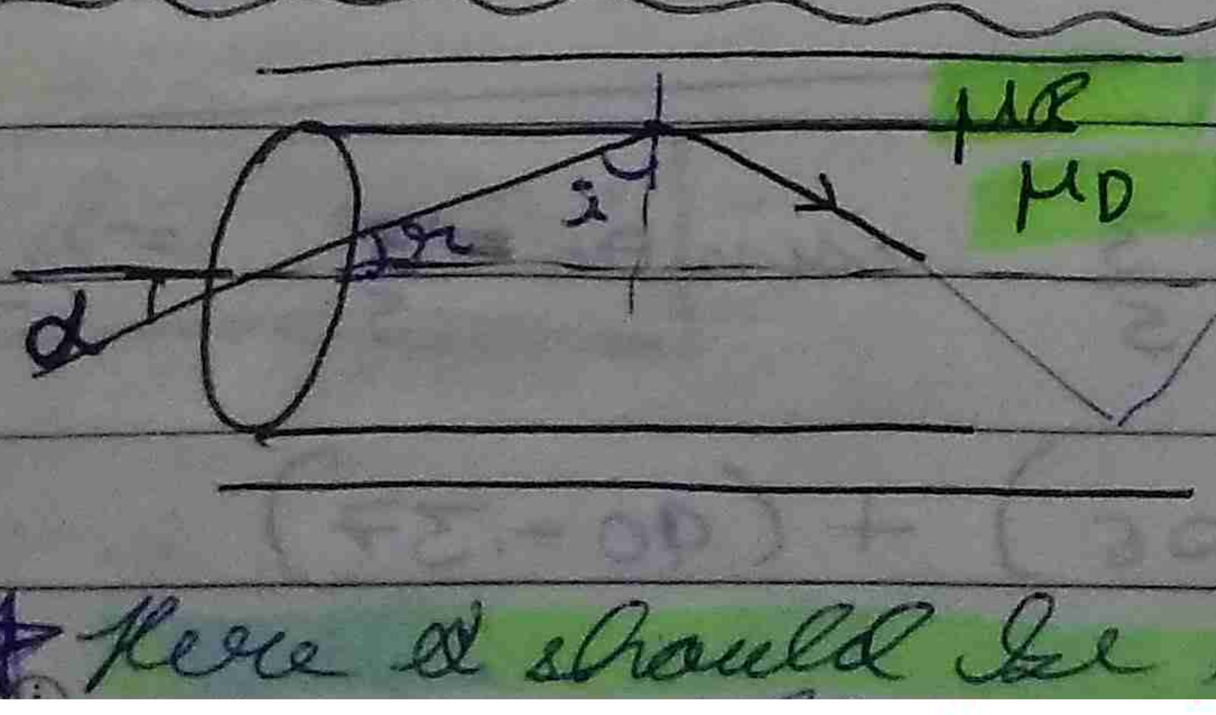


$\sin\theta_c = \frac{1}{\mu} \quad R = h \cdot \tan\theta_c \quad \Rightarrow R = h \times \frac{1}{\sqrt{\mu^2 - 1}}$

• Area through which light is transmitted is πR^2

• fraction of light (energy) coming out $= \frac{2\pi(1-\cos\theta_c)}{4\pi}$

#



• Find max value of α so that light always suffer TIR

★ Here α should be as low as possible so TIR

$i = \frac{\pi}{2} - r > \theta_c \Rightarrow \cos r > \sin\theta_c$

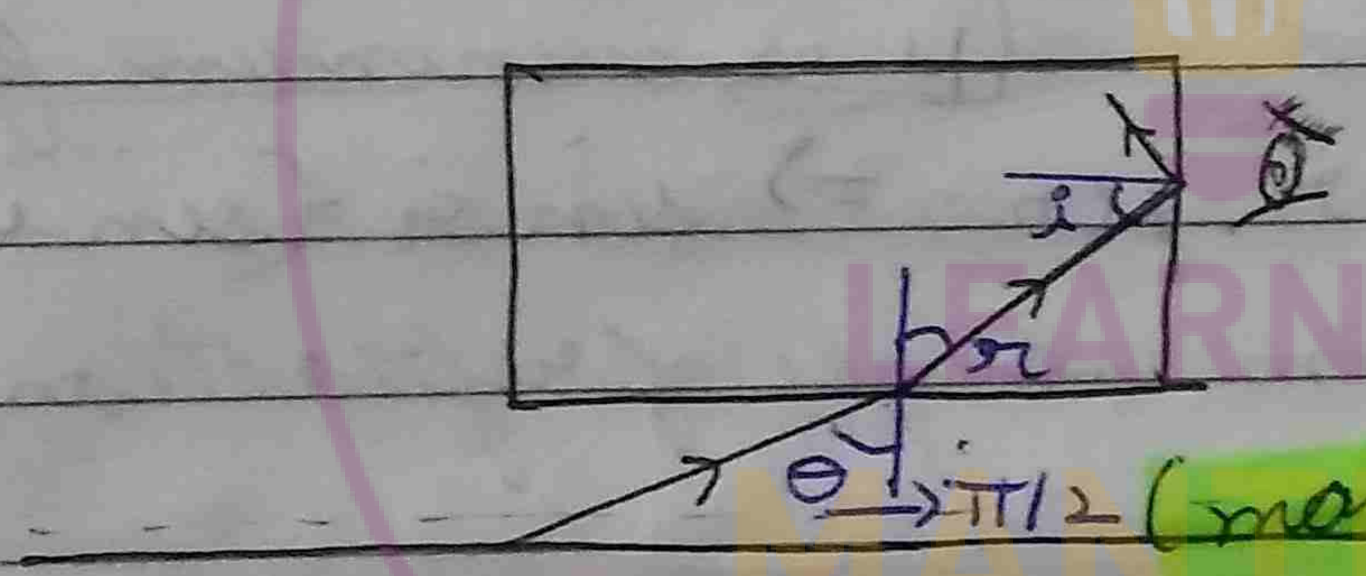
$\Rightarrow \cos r > \frac{1}{\mu_c}$ --- (i)

$1 \sin\alpha = \mu_D \sin r \Rightarrow \sin r = \frac{\sin\alpha}{\mu_D}$

$\Rightarrow \cos r = \sqrt{1 - \frac{\sin^2\alpha}{\mu_D^2}} > \frac{1}{\mu_D/\mu_c}$

$\Rightarrow 1 - \frac{\sin^2\alpha}{\mu_D^2} > \frac{\mu_c^2}{\mu_D^2} \Rightarrow \boxed{\sin\alpha = \sqrt{\mu_D^2 - \mu_c^2}}$

Q



Find the value of μ so that the person can't see the objects

Soln $i > \theta_c$ always so that person will not see

Mini ⇒ r would be max

condition ⇒ $\theta = \pi/2$ $r = \theta_c$ (i) and $i > \theta_c$

(by snells law)

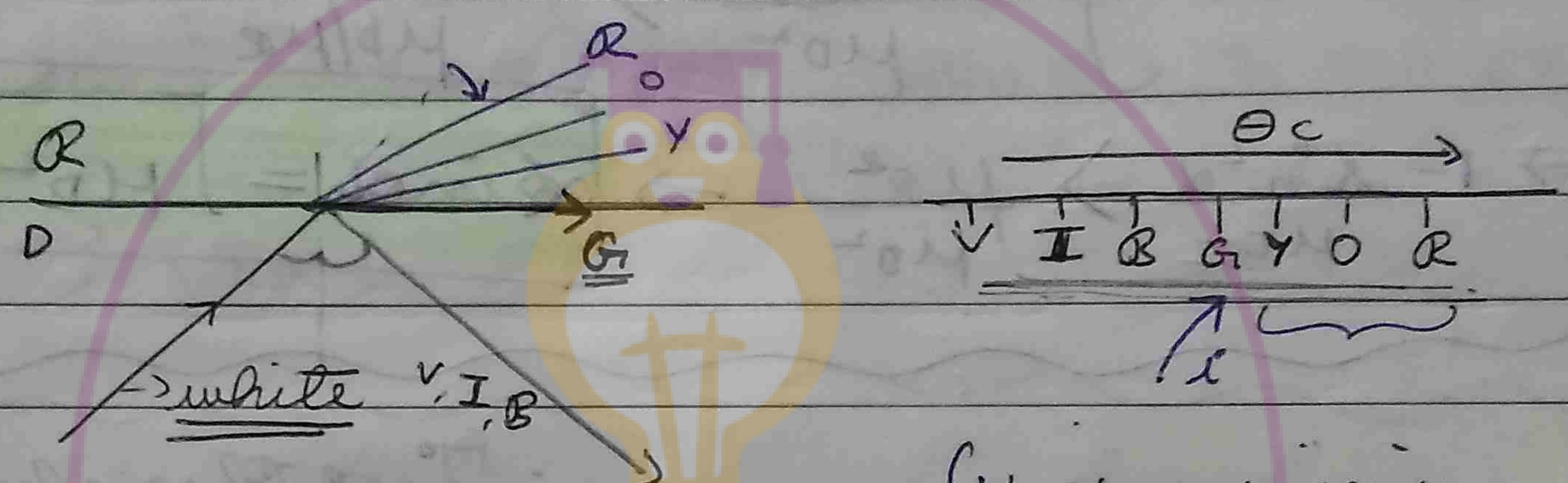
$\pi/2 - r > \theta_c \Rightarrow \frac{\pi}{2} - \theta_c > \theta_c \Rightarrow \frac{\pi}{2} > 2\theta_c \Rightarrow \theta_c < \frac{\pi}{4}$

$\sin\theta_c < \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{\mu} < \frac{1}{\sqrt{2}} \Rightarrow \boxed{\mu > \sqrt{2}}$

- TIR for different λ

$\sin\theta_c = \frac{1}{\mu}$. $\mu \propto \frac{1}{\lambda}$ $\sin\theta \propto \lambda$

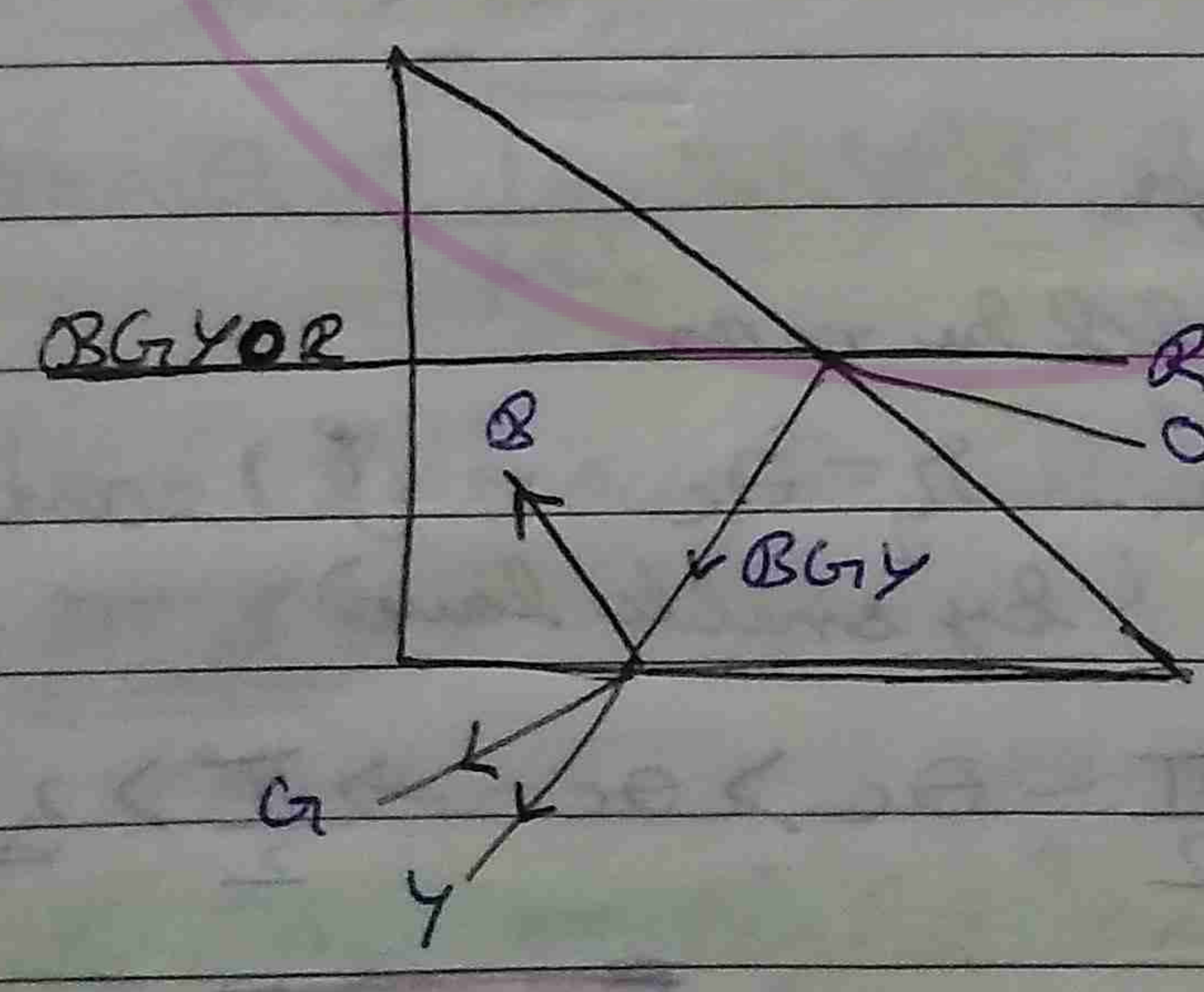
θ_c - - - maximum for red.
θ_c minimum for violet

Q 

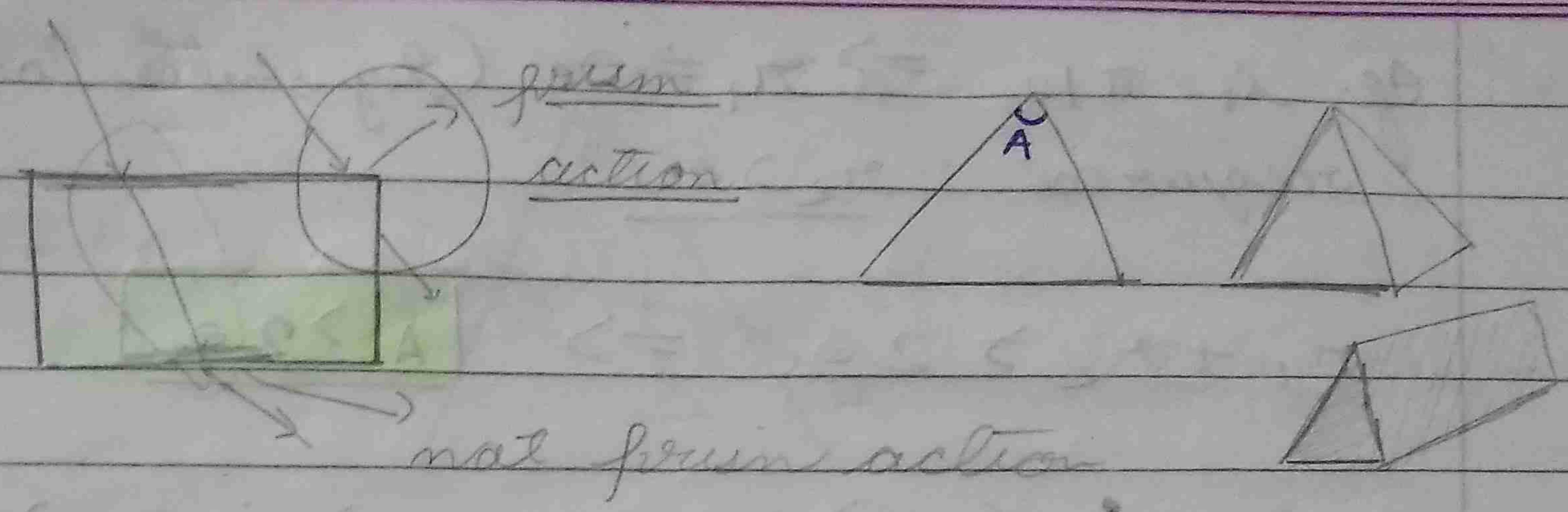
(μ is minimum for red)

$\sin i(\mu_0) = \sin r \ \mu_r \Rightarrow \sin r = (\sin i)(\mu)$

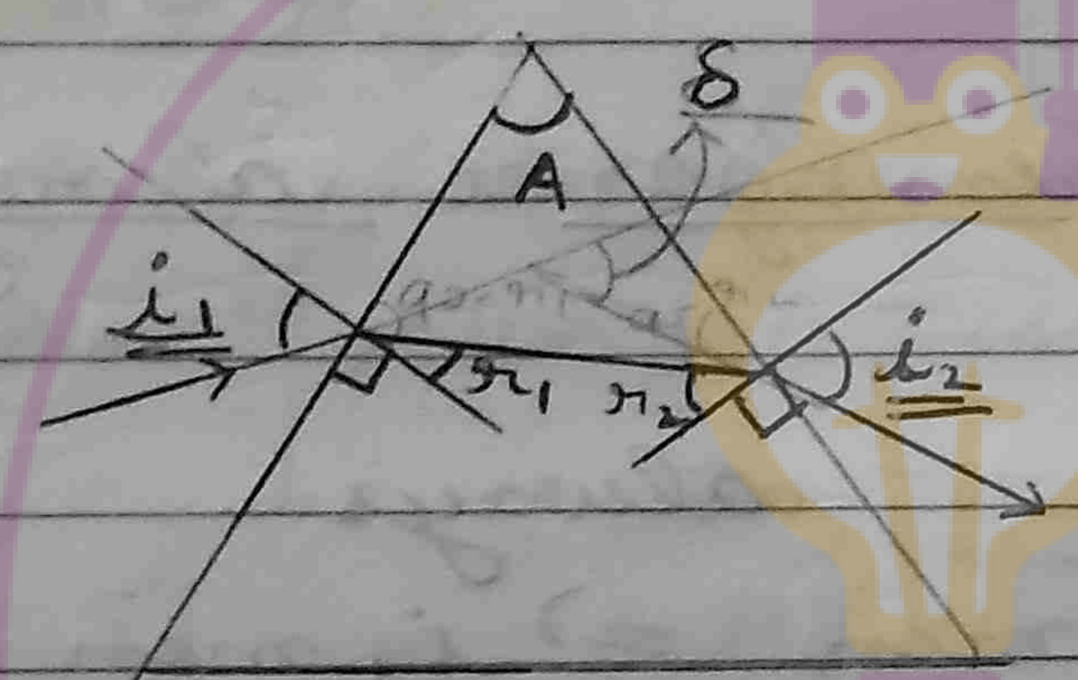
- red has minimum angle of refraction.

Q 

- Prism : It is an arrangement where incident and refracting surface are not parallel.



- Angle of prism is the angle between incident and emergent faces.

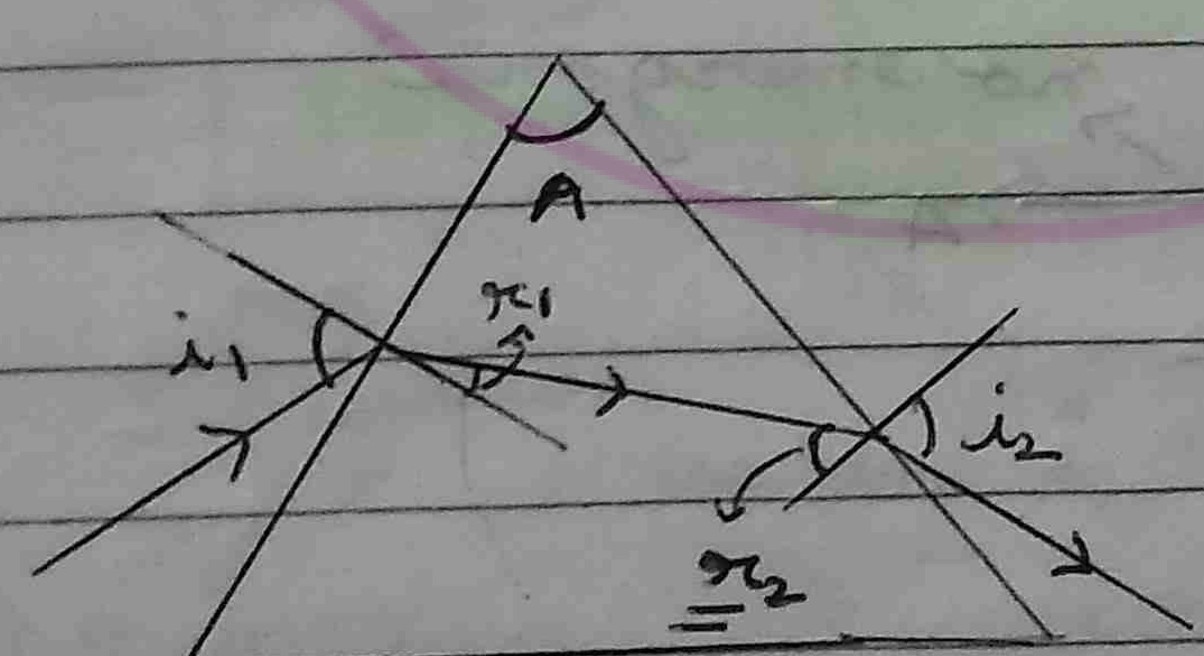


$$A + (90 - r_1) + (90 - r_2) = 180^\circ$$

$$r_1 + r_2 = A$$

$$\delta_{net} = (i_1 - r_1) + (i_2 - r_2) = (i + i_2) - (r_1 + r_2)$$

$$\Rightarrow \delta_{net} = i_1 + i_2 - A$$



$$r_1 + r_2 = 0$$
↑ ↓

- Condition for no emergence

$r_2 > \theta_c$ - - - always

min r_2

min $r_2 \Rightarrow$ max $r_1 \Rightarrow$ max $i_1 = \pi/2$

- Monochromatic light undergoes deviation but not dispersion.

If $i_1 = \pi/2 \Rightarrow r_1 = \theta_c$ (by snell's law)

required $r_2 > \theta_c$

$r_1 + r_2 > 2\theta_c \Rightarrow \boxed{A > 2\theta_c}$

or $\sin(A/2) > \sin\theta_c \Rightarrow \sin(A/2) > \frac{1}{\mu}$

$\boxed{\mu > \operatorname{cosec}(A/2)}$

- Condition for emergance always

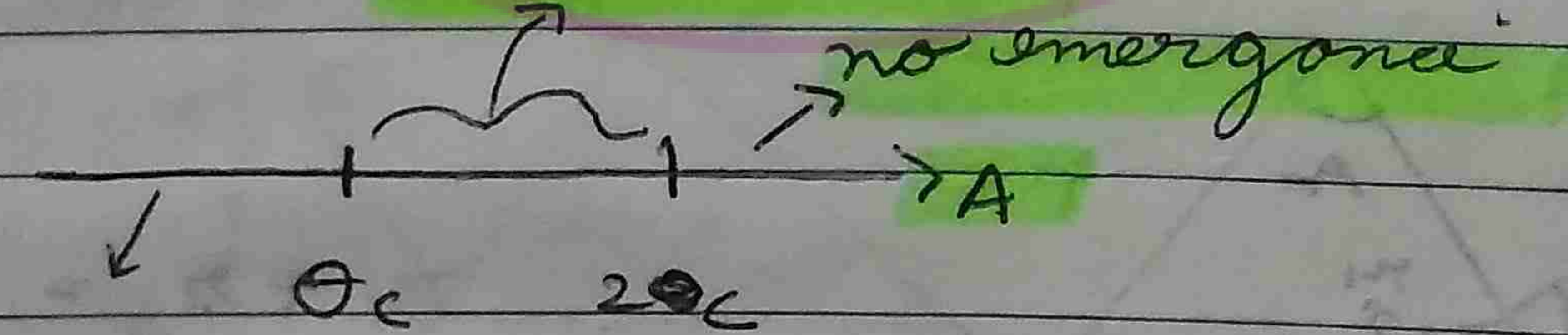
$r_2 < \theta_c$ - - - always.

max $r_2 \Rightarrow r_1$ min $\Rightarrow i_1$ min

$i_1 = 0 \Rightarrow r_1 = 0 \Rightarrow \boxed{r_2 = A}$

$\Rightarrow \boxed{A < \theta_c}$

☆ $\theta_c \le A \le 2\theta_c$



emergance always

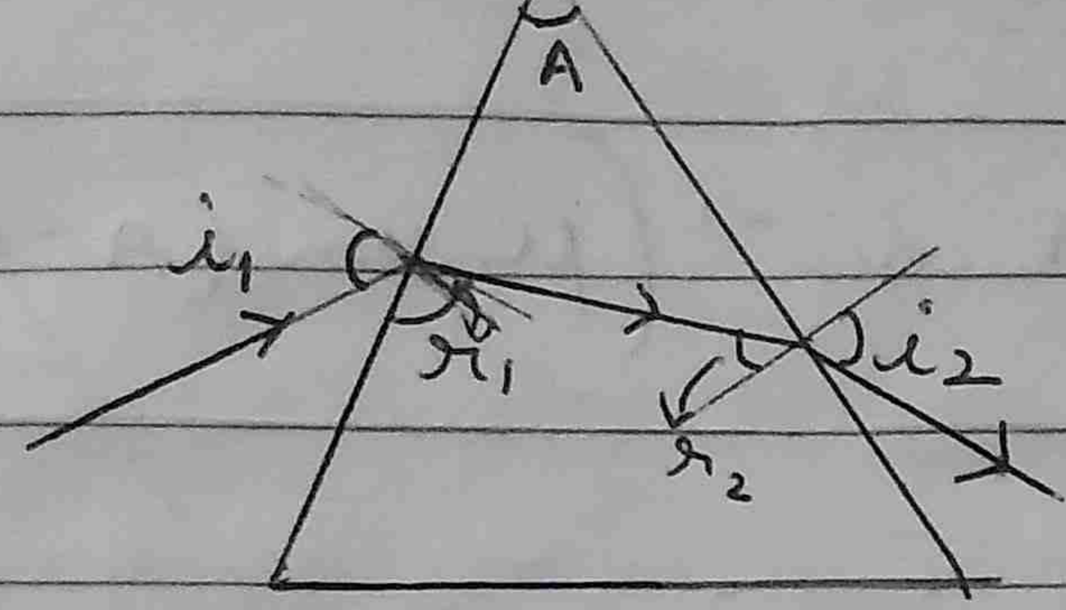
- Deviation by Prism

- Prism action
  - → Deviation
  - → Dispersion

- Only deviation is possible with monochromatic light.

minimum deviation: $i_1 = i_2$

Name of the Chapter max deviation $i_1 = \frac{\pi}{2}$ Date ______ Page No.: ______



$\delta = i_1 + i_2 - A$

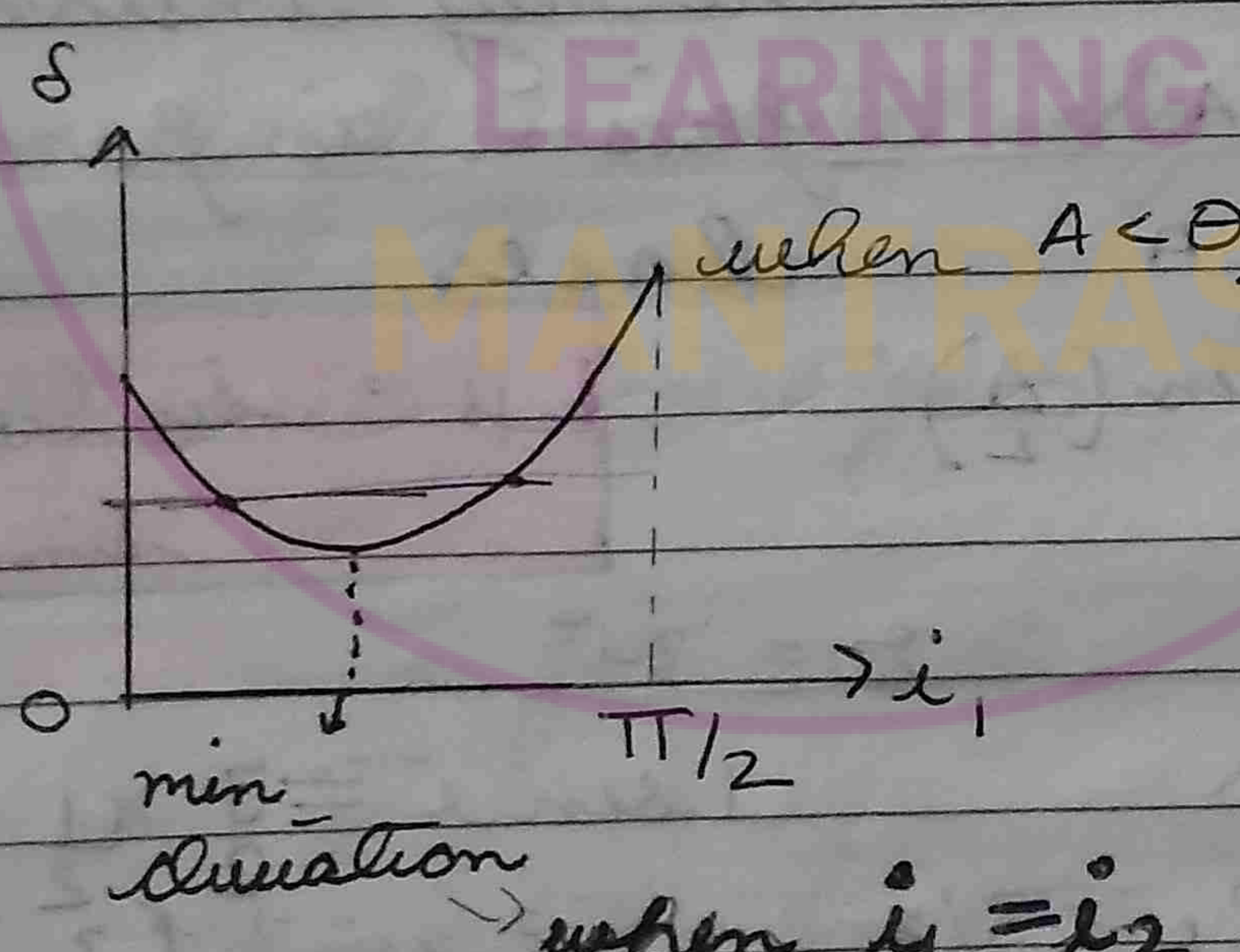
$1 \sin i_1 = \mu \sin r_1 \Rightarrow r_1 = \sin^{-1}\left[\frac{1}{\mu}\sin i_1\right]$

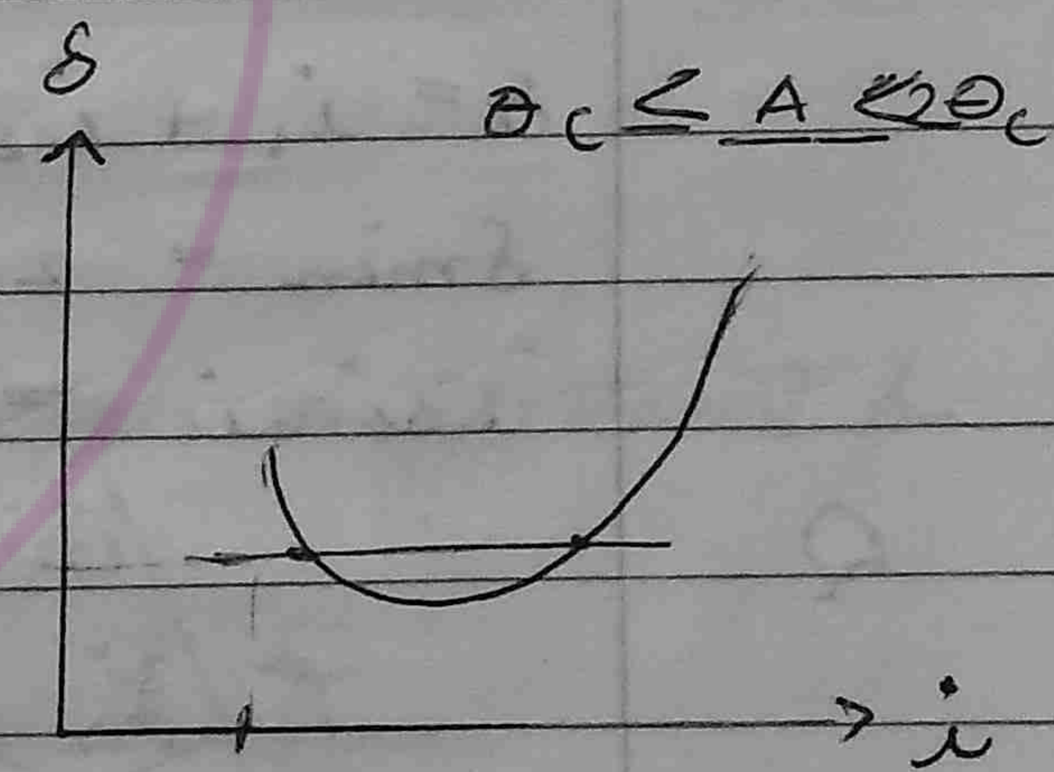
$r_2 = A - r_1$

$\mu \sin r_2 = 1 \sin i_2$

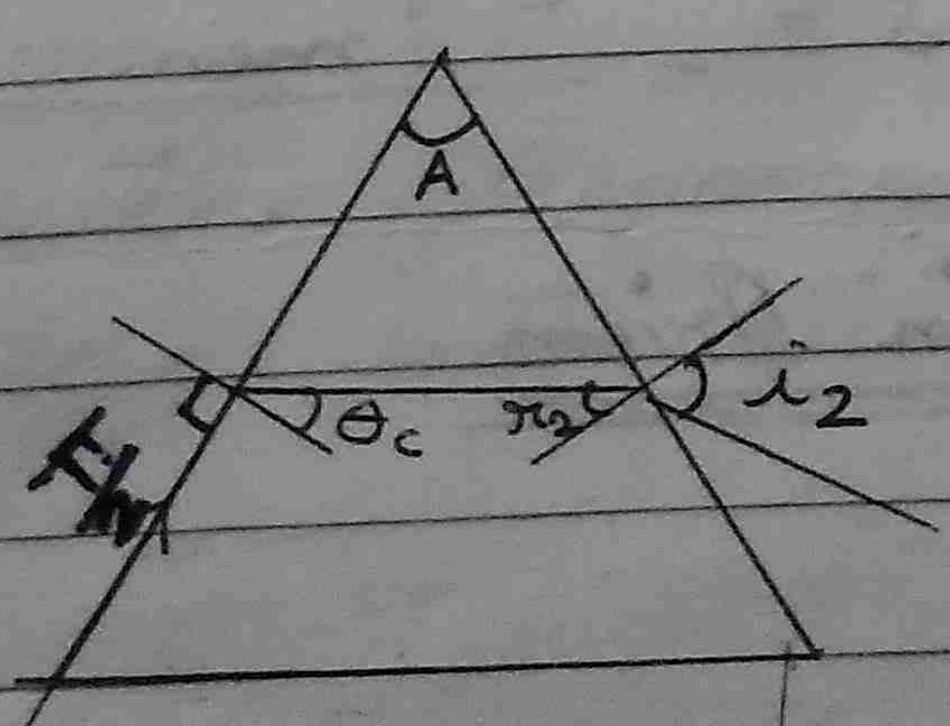
$i_2 = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\sin i_1\right)\right)\right]$

$\Rightarrow \delta = i_1 + i_2 - A = f(i_1)$





- Maximum Deviation



$r_2 = A - \theta_c$

$\mu \sin r_2 = 1 \sin i_2$

$i_2 = \sin^{-1}(\mu \sin(A - \theta_c))$

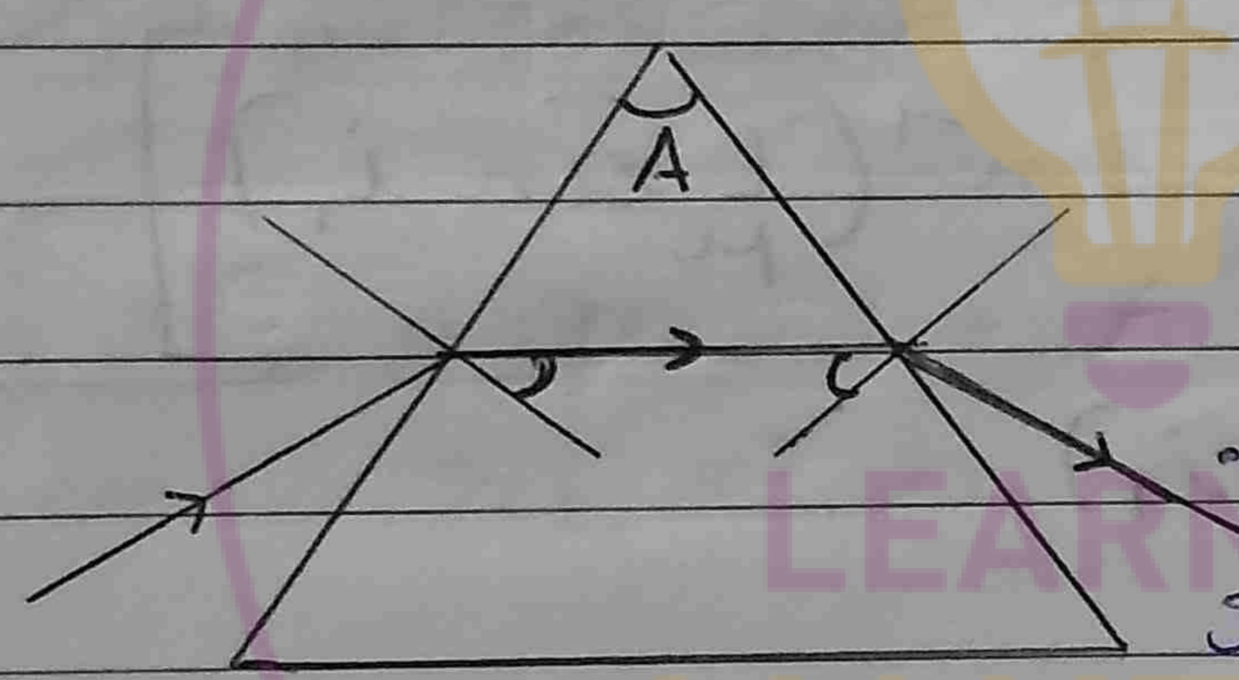
Min deviation ⇒ $\delta_{m} = 2i - A$ ($\because i_1 = i_2$)
Max devn ⇒ $\delta_{m} = \frac{\pi}{2} + i_2 - A$

$$\delta = i_1 + i_2 - A$$

$$\delta_{max} = \frac{\pi}{2} + \sin^{-1}\left(\mu \sin(A - \theta_c)\right) - A$$

- **Minimum Deviation**

condition $i_1 = i_2$



At δ min

$r_1 = r_2 = A/2$

- When deviation is minimum the ray inside the prism is parallel to the base.

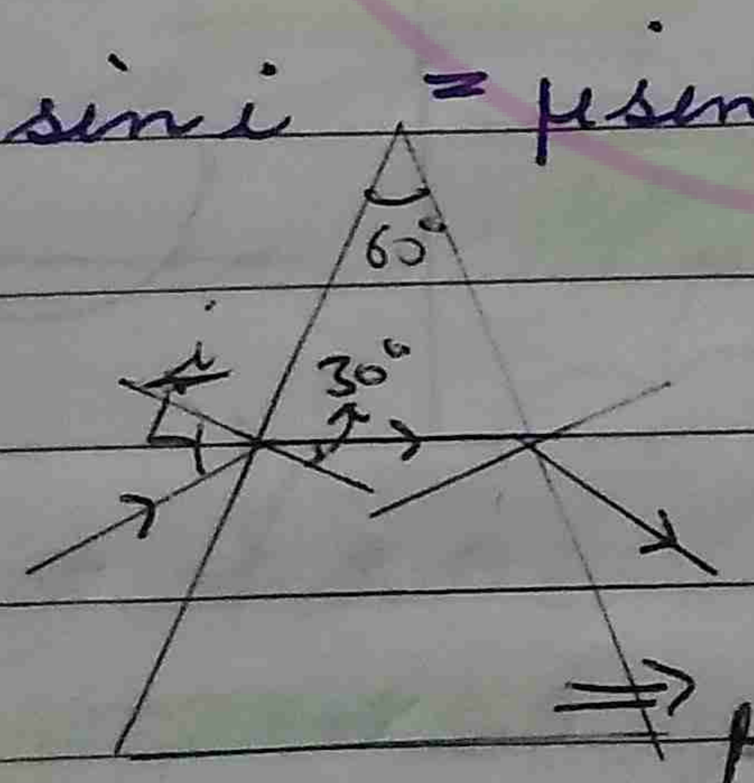
$\delta = i_1 + i_2 - A$

$\delta_{min} = 2i - A$

$1 \sin i = \mu \sin\left(\frac{A}{2}\right)$

$$\mu = \frac{\sin\left(A + \delta_{min}/2\right)}{\sin(A/2)}$$

Q



$r = 30°$

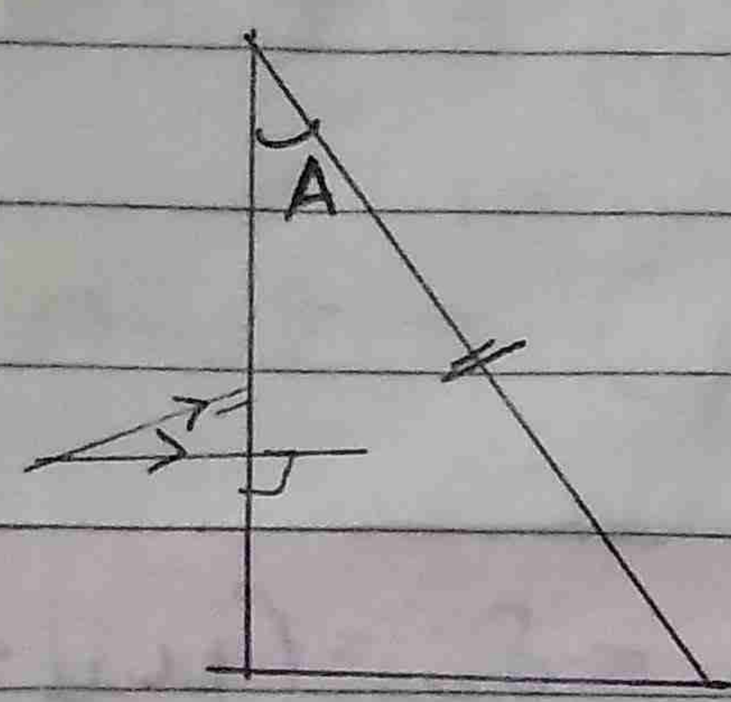
$1 \sin i = \frac{3}{2} \times \frac{1}{2}$

$\Rightarrow \mu = 3/2$ $\quad \Rightarrow i = \sin^{-1}\left(\frac{3}{4}\right)$

$\delta = 2\left(\sin^{-1}\left(\frac{3}{4}\right)\right) - \frac{\pi}{3}$

$\delta = \_\_\_$ min

| A -- small
| i_1 -- small : Thin Prism



$1 \sin i_1 = \mu \sin r_1$

$i_1 = \mu r_1$ --- (i)

$r_2 = A - r_1$ → ⇒ r_2 is small

$\mu \sin r_2 = 1 \sin i_2$

$\Rightarrow i_2 = \mu r_2$ --- (ii)

$\delta = i_1 + i_2 - A \Rightarrow \delta = \mu r_1 + \mu r_2 - A$

$\Rightarrow \delta = \mu A - A \Rightarrow$ $\boxed{A(\mu - 1)}$

$\boxed{\delta = (\mu - 1) A}$ ☆ independent of i

- Prism under white light
- Deviation will be different for different λ
- δ --- min for red
- δ --- max for violet
- Dispersion:
It is the phenomenon of white light splitting into its constituent.

- For dispersion →
→ The two surfaces should not be parallel.
→ The prism should not be hollow

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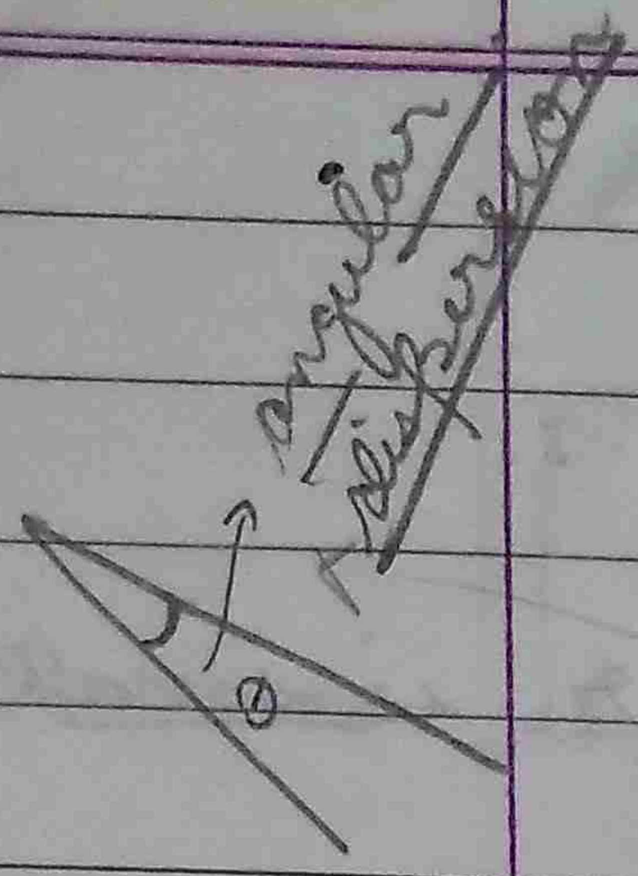
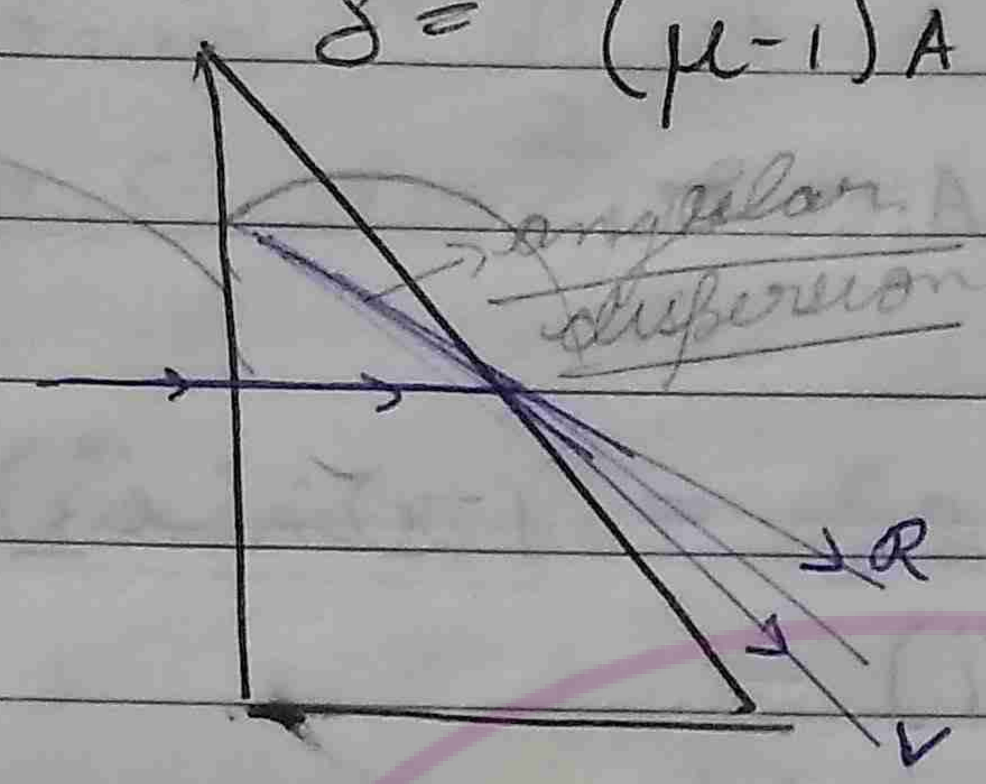
Dispersion



$\delta = (\mu - 1)A$



$$\delta_{mean} = \delta_y = (\mu_y - 1)A$$

(yellow light)

Angular Dispersion – $\theta = \delta_V - \delta_R$ (violet, red)

$\theta = (\mu_V - 1)A - (\mu_R - 1)A$

$$\theta = (\mu_V - \mu_R)A$$

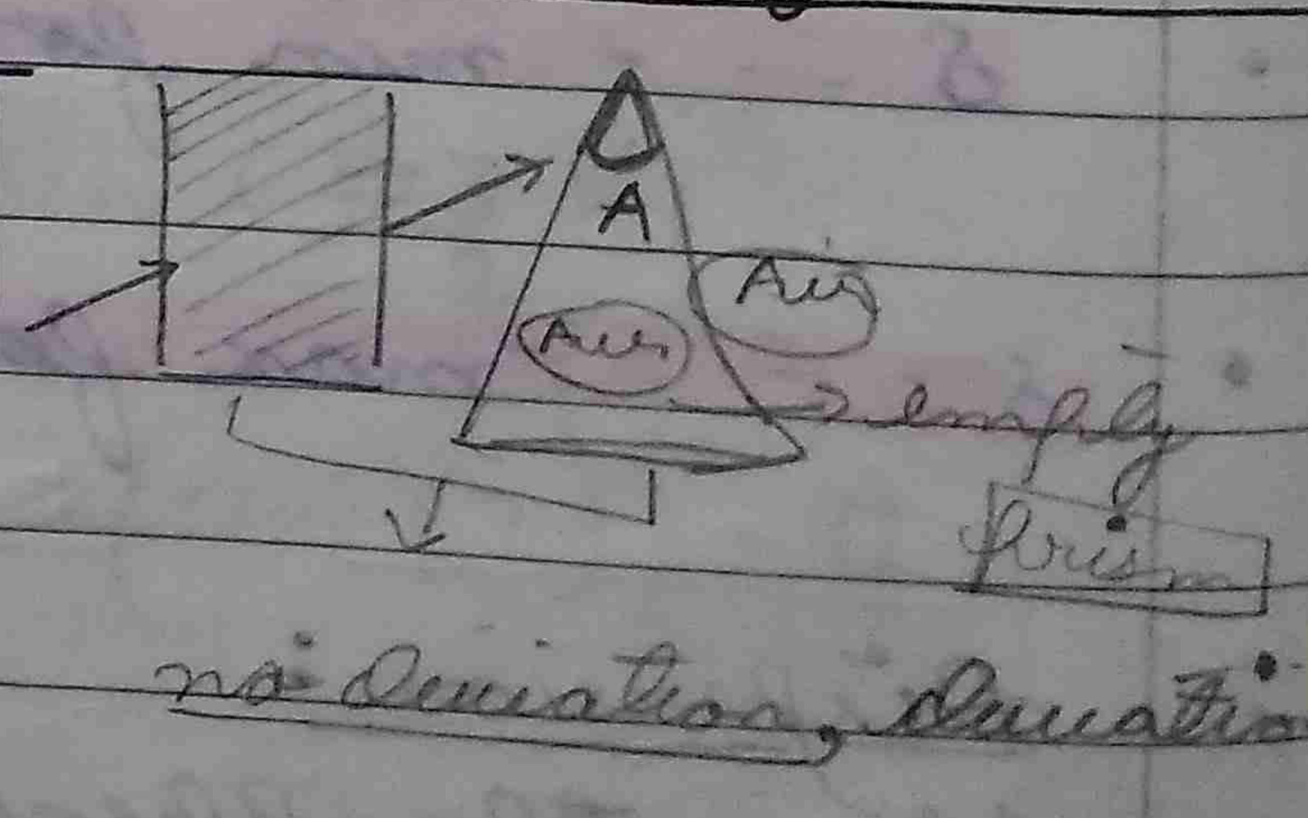
★ dispersive power does not depend on prism angle it depends on material only

- **Dispersive Power**

$$\text{'}\omega\text{'} = \frac{\text{Angular Dispersion}}{\text{Mean deviation}} \Rightarrow \omega = \frac{(\mu_V - \mu_R)A}{(\mu_y - 1)A}$$

$$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1} = \frac{d\mu}{\mu - 1}$$

(dimensionless) (If small)



no deviation, deviation

- A single prism will have both deviation and dispersion.

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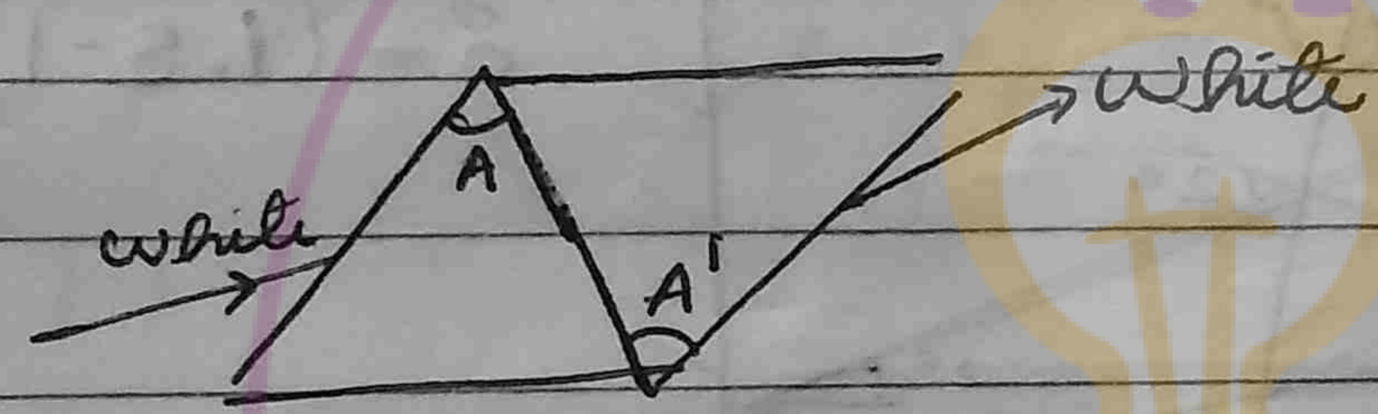
$$\delta_{net} = (\mu_y - 1)A + (\mu_y' - 1)A'$$

$$\theta_{net} = (\mu_v - \mu_R)A + (\mu_v' - \mu_R')A'$$

- **Deviation without Dispersion**

$$A' = -\frac{(\mu_v - \mu_R)A}{(\mu_v' - \mu_R')} \quad \text{condition}$$



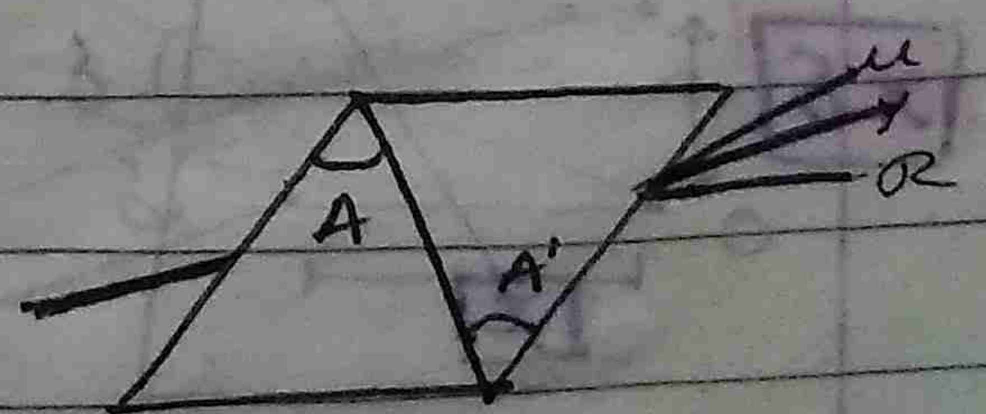
$$\delta_{net} = (\mu_y - 1)A - (\mu_y' - 1)\frac{(\mu_v - \mu_R)A}{(\mu_v' - \mu_R')}$$

$$= (\mu_y - 1)A\left[1 - \frac{\mu_v - \mu_R}{\mu_y - 1}\,\frac{\mu_y' - 1}{\mu_v' - \mu_R'}\right]$$

$$\delta_{net} = \delta\left[1 - \omega/\omega'\right] \text{ or } (\mu_y - 1)A\left[1 - \frac{\omega}{\omega'}\right]$$

- **Dispersion without deviation**

condition $$A' = -\frac{(\mu_y - 1)A}{(\mu_y' - 1)}$$



Signature ....

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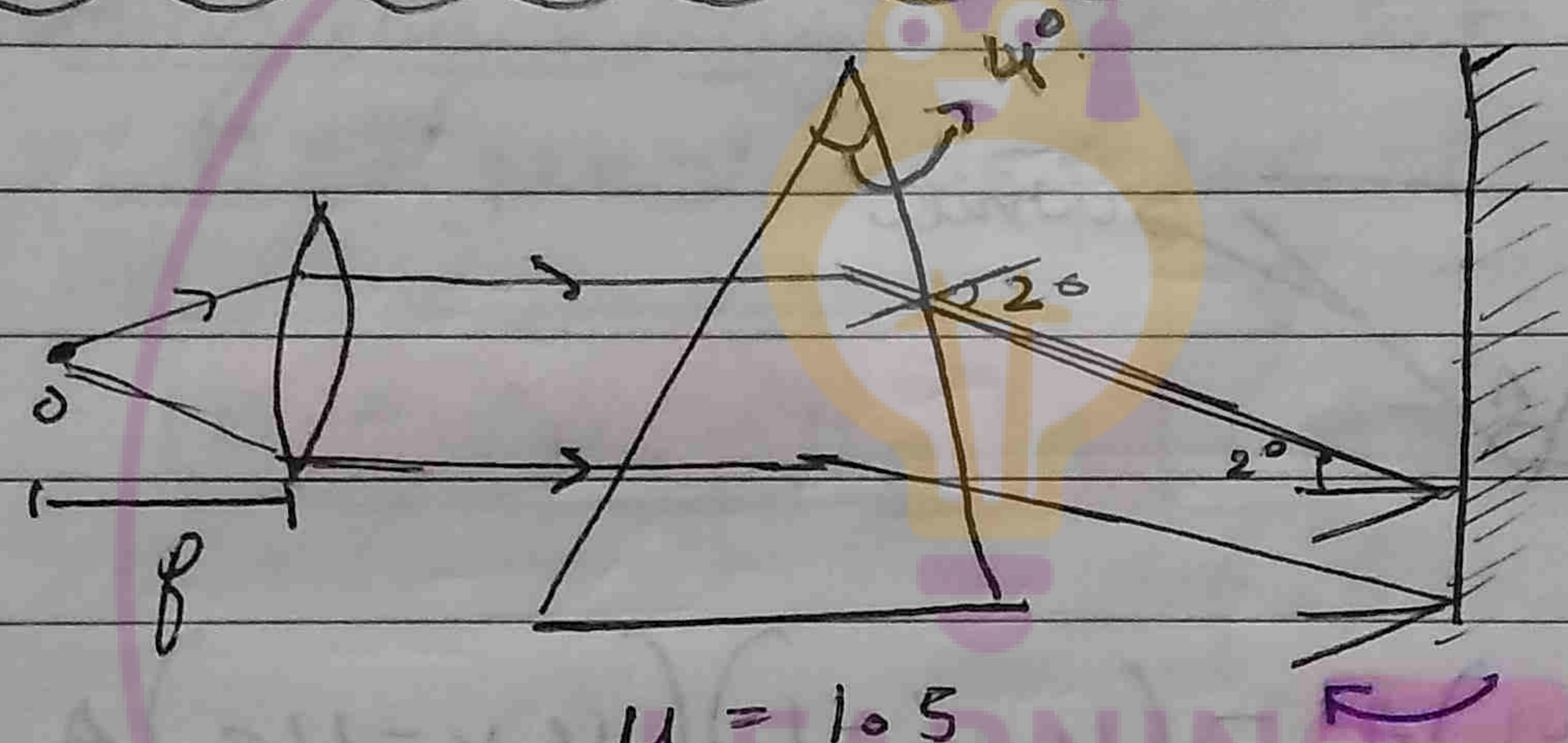
$$\theta_{net} = (\mu_v - \mu_R)A - \frac{(\mu_v' - \mu_R')(\mu_y - 1)A}{(\mu_y' - 1)}$$

$$\theta_{net} = (\mu_v - \mu_R)A\left[1 - \frac{\mu_v' - \mu_R'}{\mu_v - \mu_R}\,\frac{\mu_y - 1}{\mu_y' - 1}\right]$$

$$\boxed{\theta_{net} = \theta\left[1 - \frac{\omega'}{\omega}\right]} \quad \text{or} \quad (\mu_y - 1)A\left[\omega - \omega'\right]$$

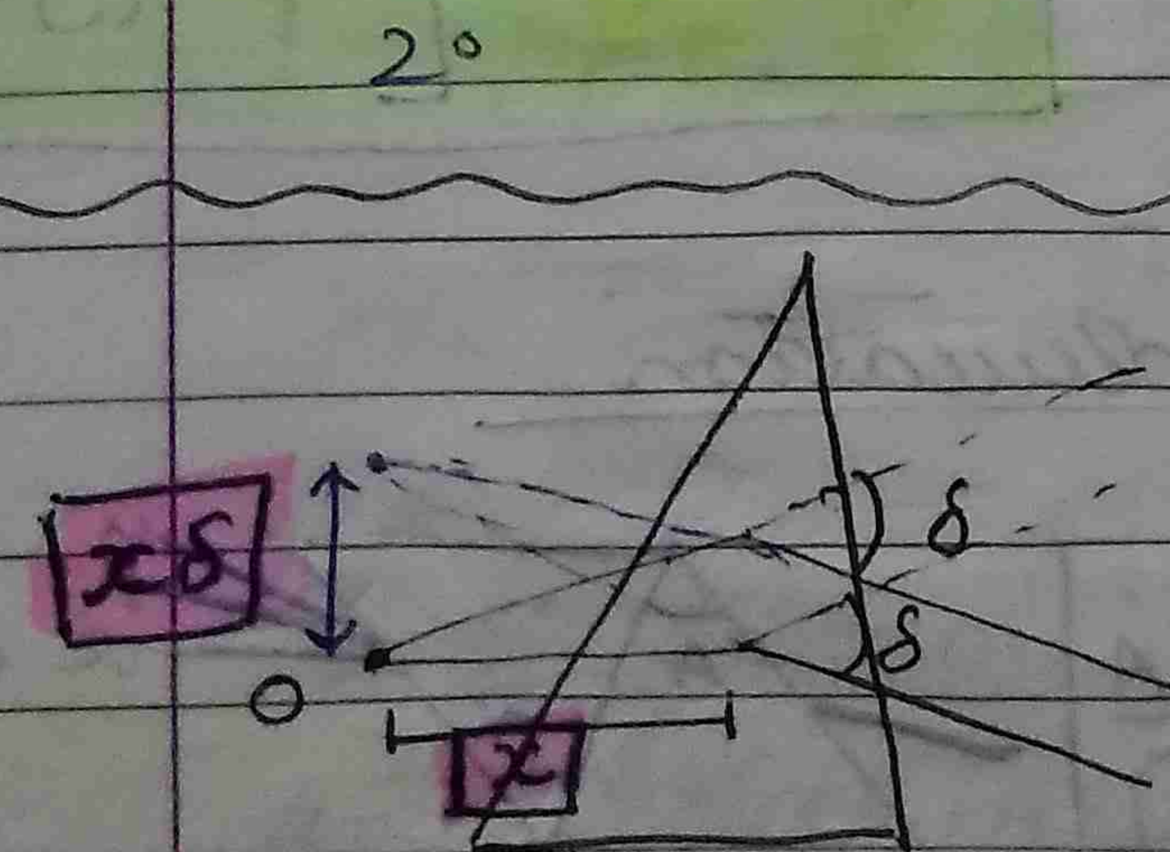
e.g



$\delta = (1.5 - 1)\,4° = 2°$

By what degree should the mirror be rotated so that final image coincides with object

If angle of incidence is zero the final image will coincide with object.
so the mirror should be rotated by 2°



here $\delta = (\mu - 1)A$

The prisms shifts the object upwards by δx.
i.e object is shifted by $x(\mu - 1)A$

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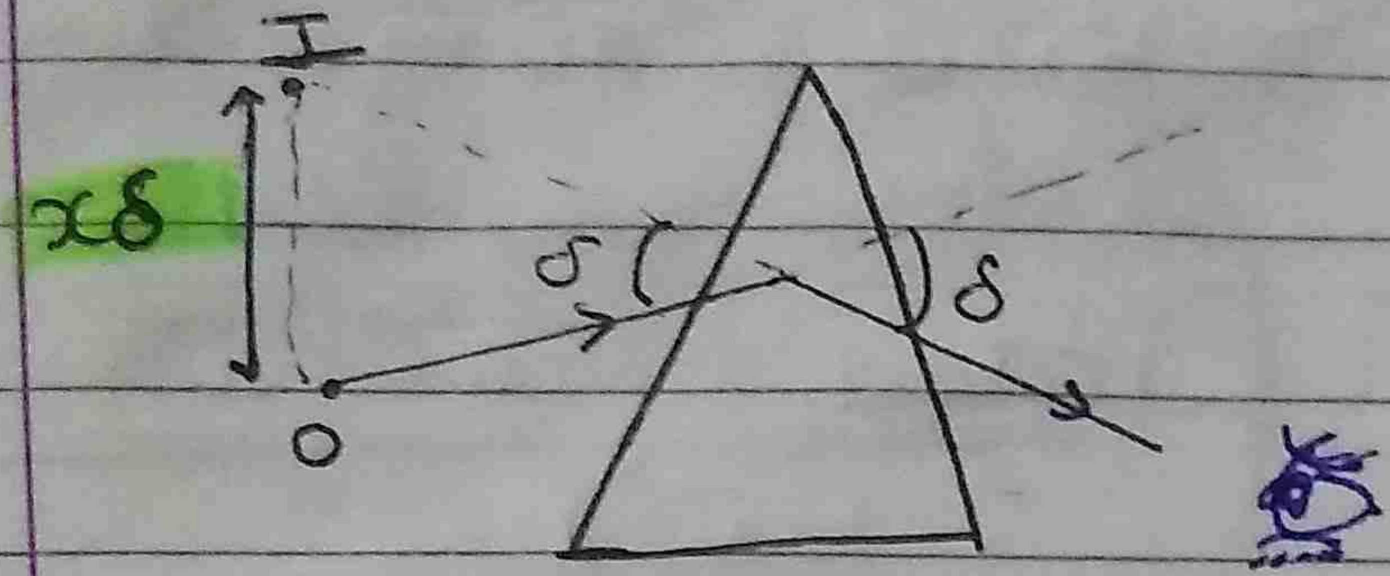
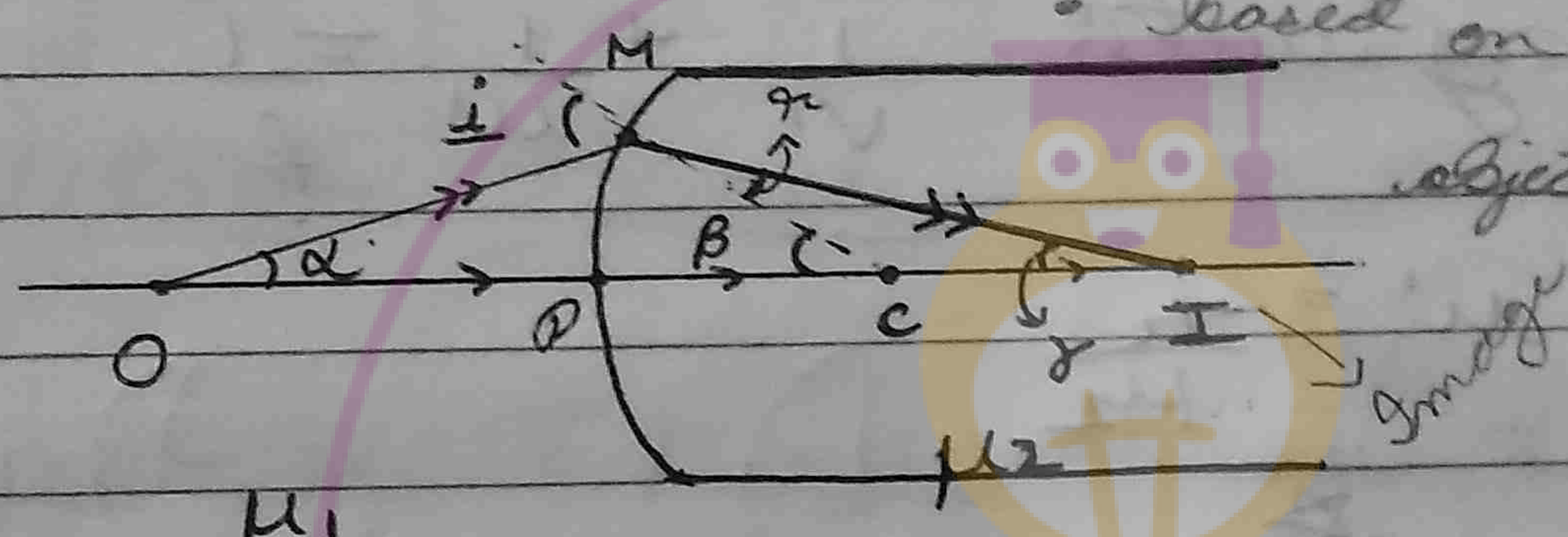


- Refraction Through Curved Surface



- based on assumption that object are point object
- Angles are very small.

$$\mu_1 \sin i = \mu_2 \sin r \Rightarrow \mu_1 i = \mu_2 r \quad \text{--- (i)}$$

$$i = \alpha + \beta \quad \text{--- (ii)} \qquad \beta = r + \gamma \quad \text{--- (iii)}$$

$$r = \beta - \gamma \quad \text{(iv)}$$

from (i), (ii) and (iv)

$$\mu_1(\alpha + \beta) = \mu_2(\beta - \gamma)$$

$\tan\alpha \quad \tan\beta \qquad \tan\beta - \tan\gamma$

$$\mu_1\left(\frac{MP}{PO} + \frac{MP}{PC}\right) = \mu_2\left(\frac{MP}{PC} - \frac{MP}{PI}\right)$$

$$\Rightarrow \mu_1\left(\frac{1}{PO} + \frac{1}{PC}\right) = \mu_2\left(\frac{1}{PC} - \frac{1}{PI}\right)$$

$$= \mu_1\left(\frac{1}{-u} + \frac{1}{R}\right) = \mu_2\left(\frac{1}{R} - \frac{1}{+v}\right)$$

$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}} \quad (a)$$

$$\frac{1}{f} = \frac{\mu_2 - \mu_1}{R} = \text{Power}$$

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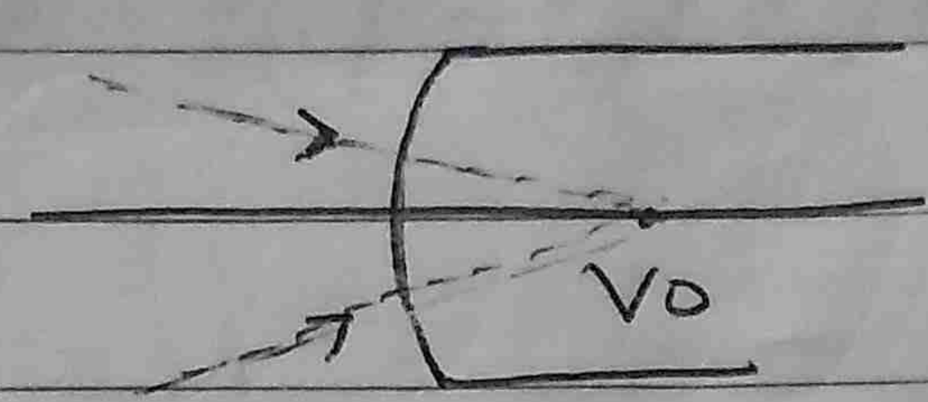


μ_1 is that medium where incident rays first strike.

- If $R \longrightarrow \infty$

$$\frac{\mu_2}{v} = \frac{\mu_1}{u}$$

comparing with (a)

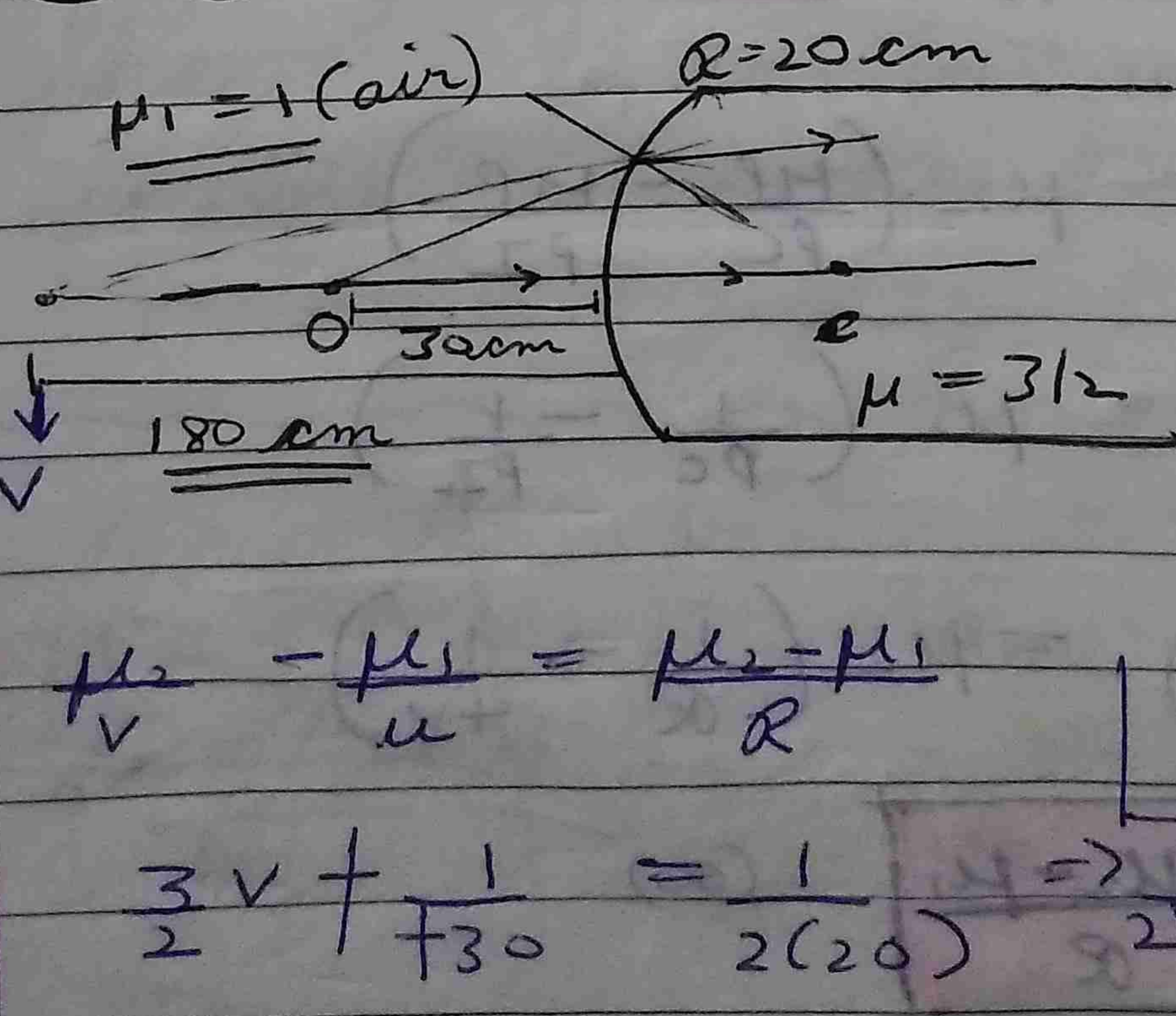
$$\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f}$$

$$v' = \frac{v}{\mu_2} \qquad u' = \frac{-u}{\mu_1}$$

$$m = -\frac{v'}{u'} = \boxed{\frac{v}{u}\frac{\mu_1}{\mu_2}}$$

- Here If $v = +ve \Rightarrow$ image is real.
  If $v = -ve \Rightarrow$ image is virtual

e.g.



$\mu_1 = 1$; $\mu_2 = 3/2$

$u = -30$ $\quad R = +2$

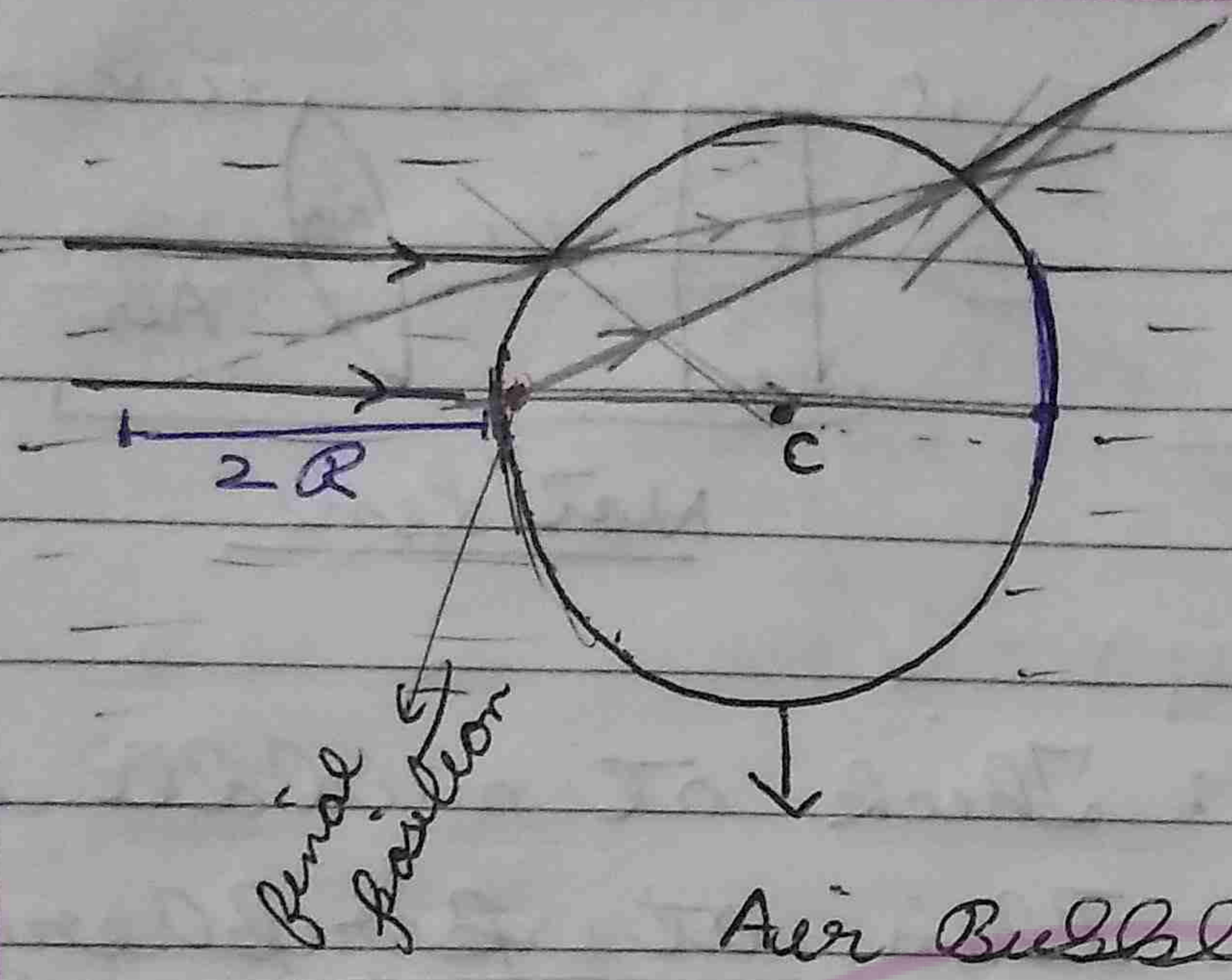
find v ?

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{3}{2v} + \frac{1}{+30} = \frac{1}{2(20)} \Rightarrow \frac{3}{2v} = \frac{1}{40} - \frac{1}{30} = \frac{30-40}{1200}$$

$$\Rightarrow v = -180$$

$\mu_{water} \Rightarrow \frac{4}{3}$ $\frac{3}{2}$



$\rightarrow \mu = \frac{4}{3}$ $\frac{3}{2}$

where will the set of parallel rays converge.

- <u>Water</u> → <u>air</u>

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} - \frac{3/2}{\infty} = \left(\frac{-1}{2}\right) \times \frac{1}{(+R)}$

$\Rightarrow \frac{1}{v} = \frac{-1}{2R} \Rightarrow \underline{v = -2R}$

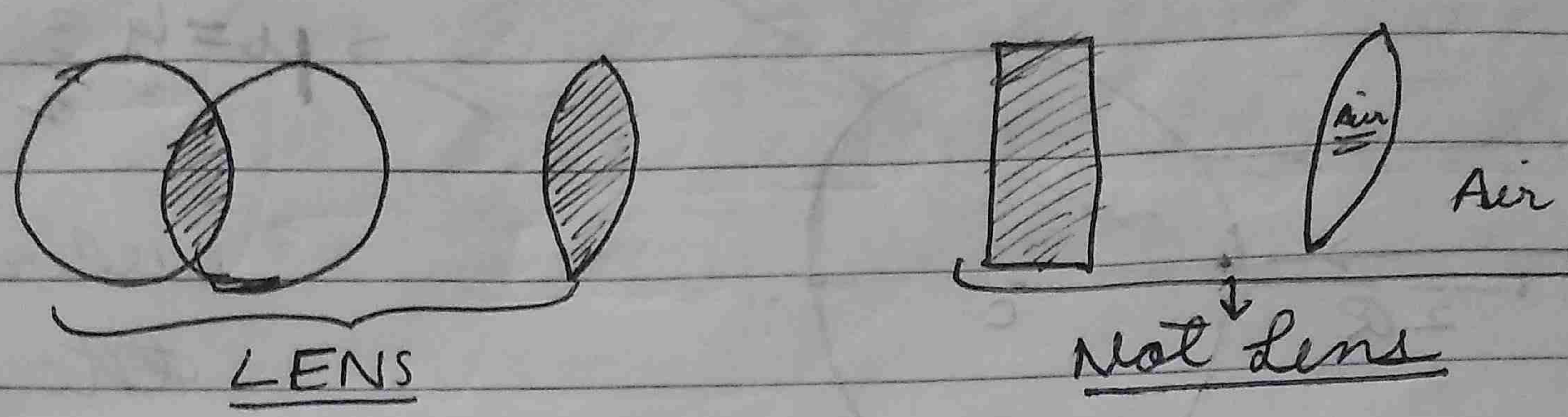
- <u>Air</u> → <u>Water</u>

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{3}{2v} + \frac{1}{+4R} = \frac{1}{2} \times \left(\frac{1}{-R}\right)$

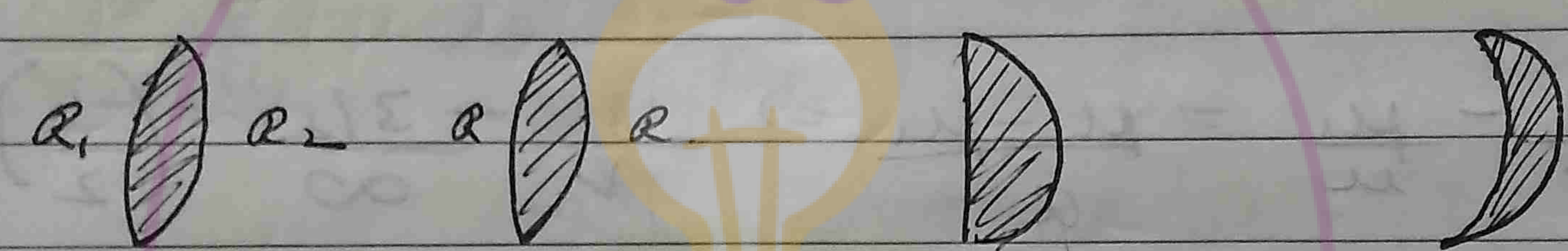
$\frac{3}{2v} = \frac{-1}{2R} - \frac{1}{4R} \Rightarrow \frac{3}{2v} = \frac{-3}{4R} \Rightarrow \underline{v = -2R}$

<u>LENS</u>

- An arrangement where the two refracting surfaces are not parallel and its μ is <u>different</u> from the surrounding.

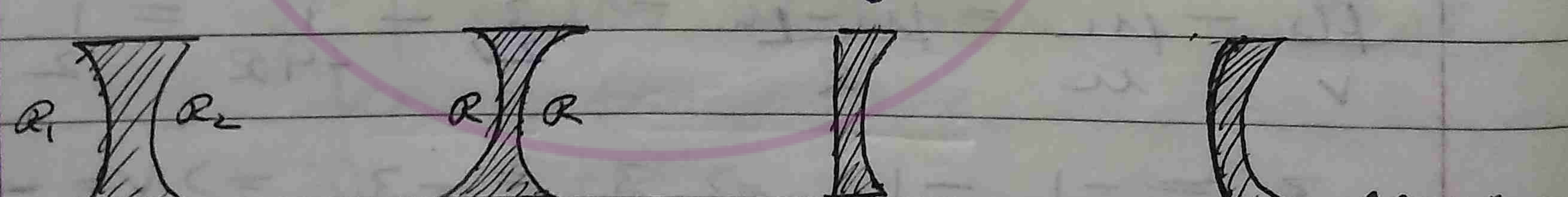


- Types : (a) Convex : Thick at middle and (converging) Thin at periphery.
  most of the time (not always)



Biconvex Equiconvex Plano-convex Concavo-convex

(b) Concave lens : Thin at middle thick, at periphery.

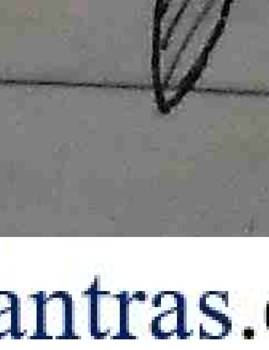


Biconcave Equiconcave Planoconcave convexo concave

- Lens Maker's Formula

Applicable when

(i) lens is thin

(ii) both media are identical (i.e. media surrounding both sides  of lens are identical).

$$\frac{\mu_{medium}}{f} = \text{power}$$

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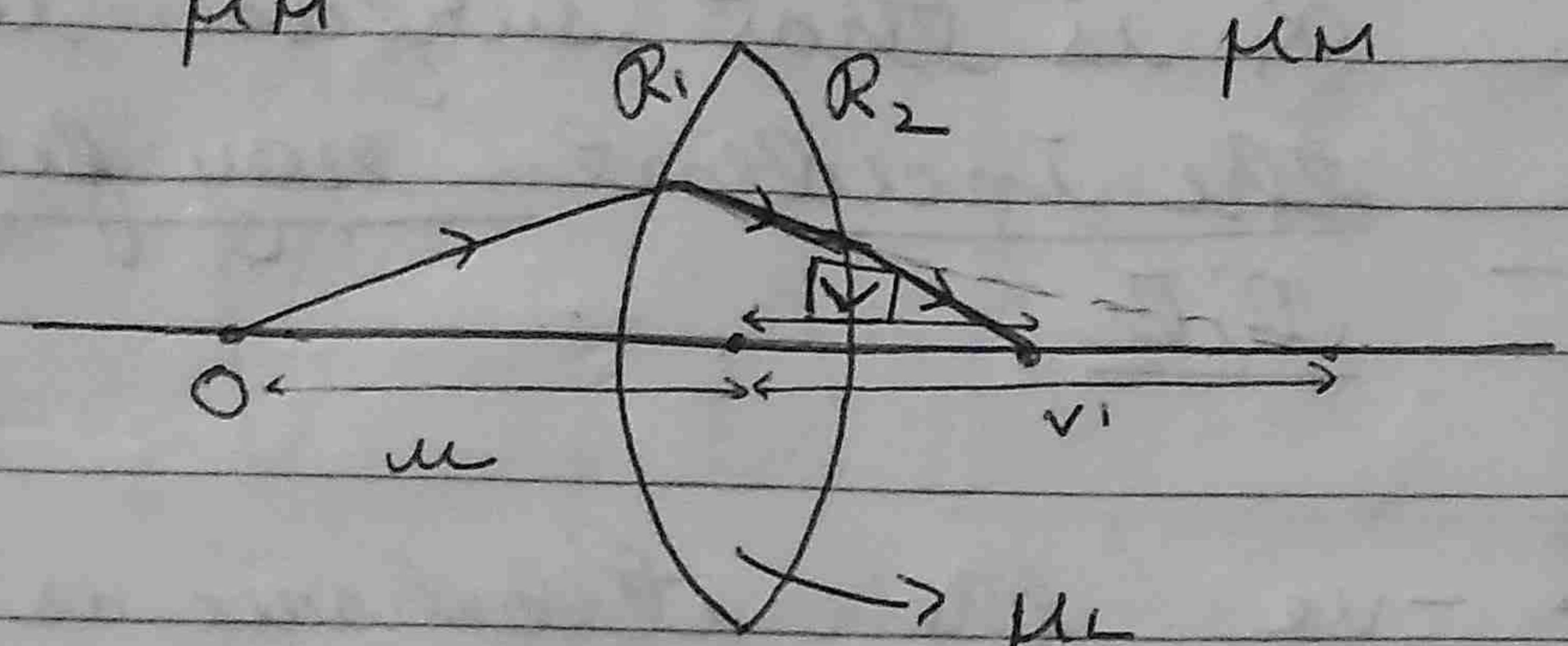


Here $\frac{\mu_L}{\mu_M} = \mu$ (let)

M ⟶ L
Medium → Lens

$$\frac{\mu_L}{V_1} - \frac{\mu_M}{u} = \frac{\mu_L - \mu_M}{R_1} \quad \text{--- (i)}$$

L ⟶ M

$$\frac{\mu_M}{V} - \frac{\mu_L}{V_1} = \frac{\mu_M - \mu_L}{R_2} \quad \text{--- (ii)}$$

adding equation (ii) and equation (i)

$$\frac{\mu_M}{V} - \frac{\mu_L}{V_1} + \frac{\mu_L}{V_1} - \frac{\mu_M}{u} = \frac{\mu_M - \mu_L}{R_2} + \frac{\mu_L - \mu_M}{R_1}$$

$$\frac{1}{V} - \frac{1}{u} = \frac{1-\mu}{R_2} + \frac{\mu - 1}{R_1}$$

- $$\frac{1}{V} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

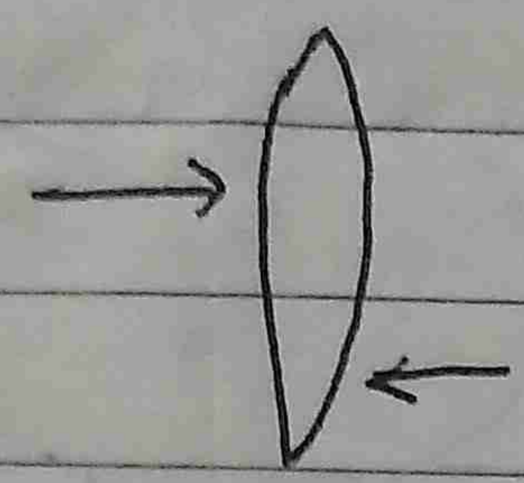
→ If $u \longrightarrow \infty$ then $V \longrightarrow f$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ -- lens makers formula.

$\mu = \frac{\mu_L}{\mu_M}$

↳ can be write as

$\frac{1}{} - \frac{1}{}$

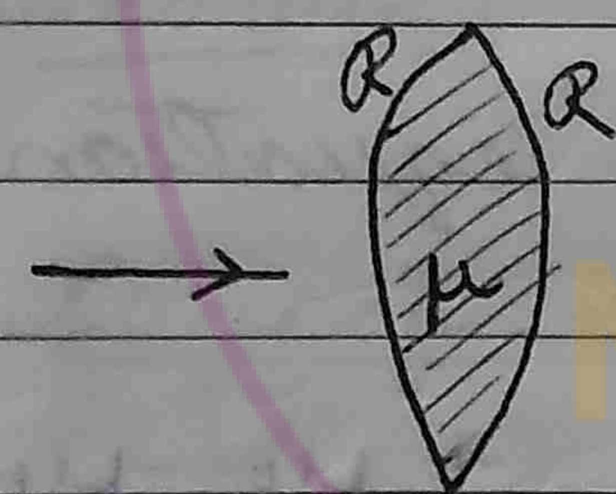


R_1 is that surface where the incident ray first hits

If f (focal length) = $-ve$ then it behaves as diverging lens.

If f = $+ve$ then it behaves as converging lens.

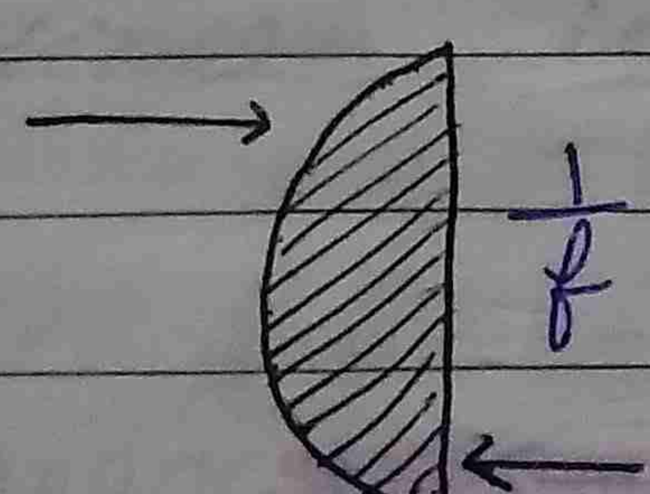
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- 

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{+R} - \frac{1}{-R}\right)$$

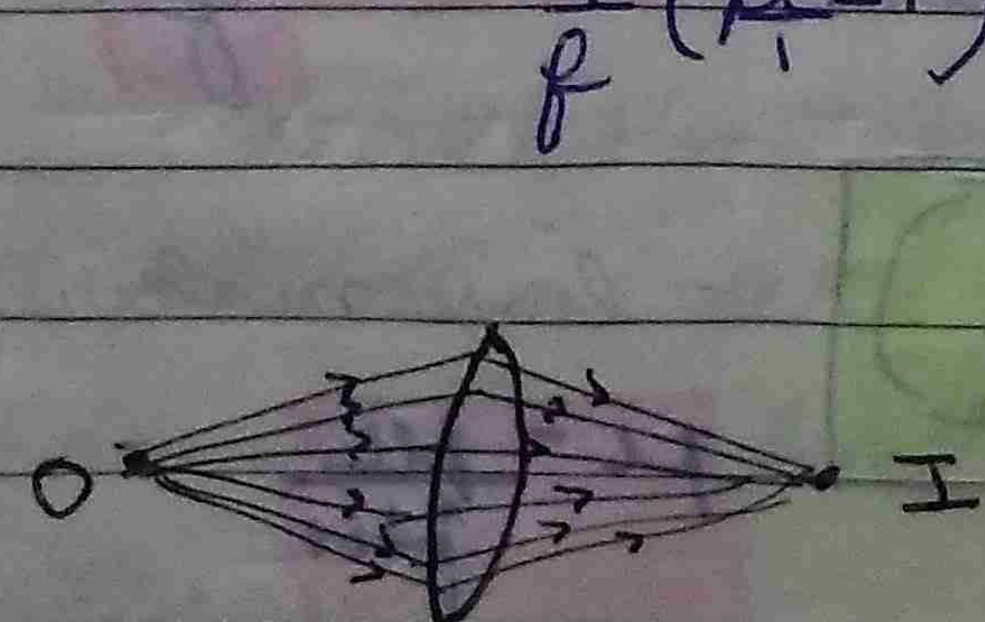
$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$

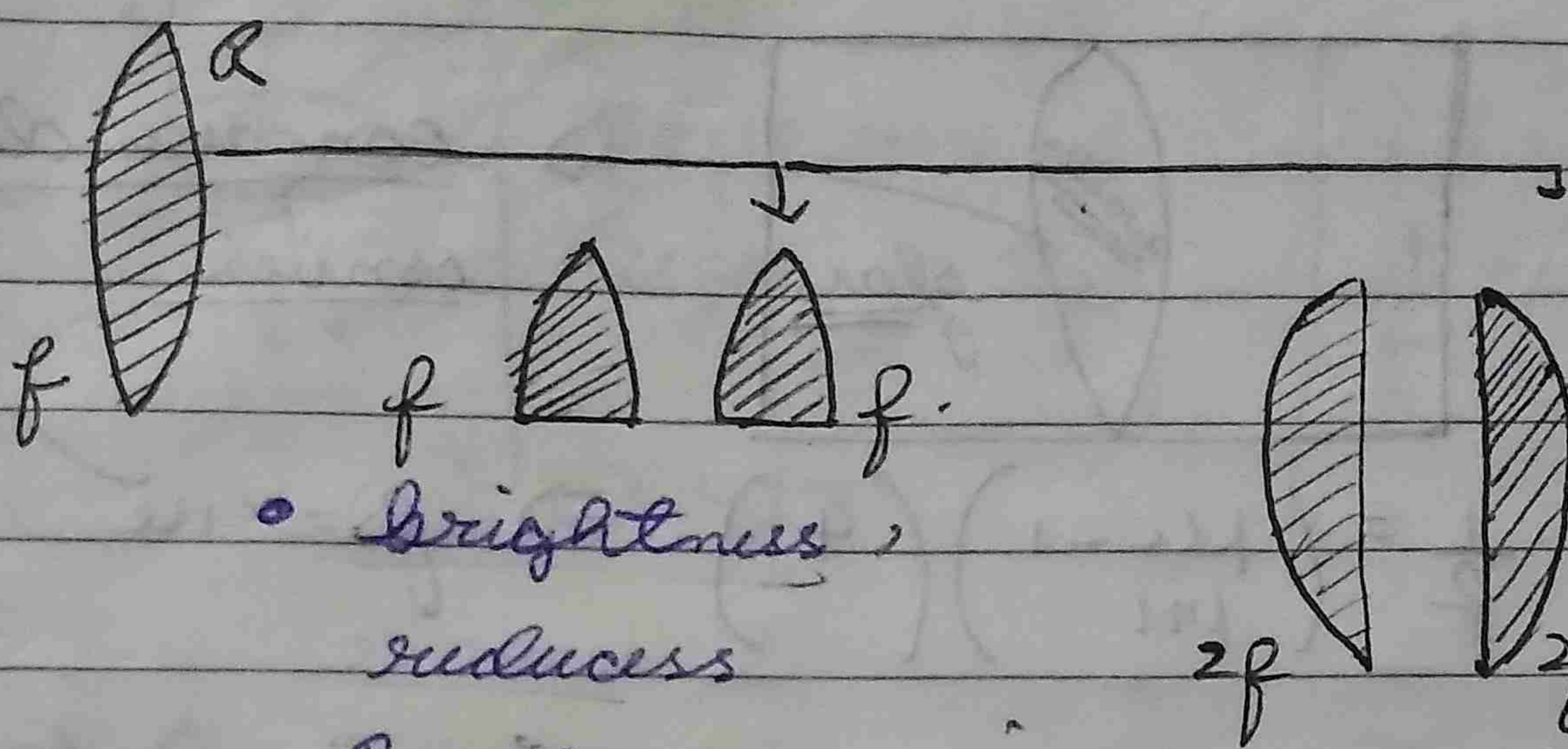
∴ convex converging

- 

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{+R} - \frac{1}{\infty}\right) = (\mu - 1)\left(\frac{1}{R}\right)$$

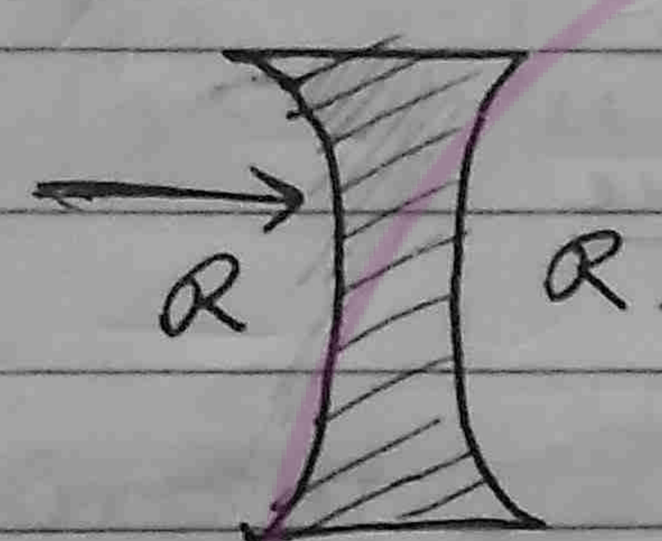
$$\frac{1}{f} (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = (\mu - 1)\left(\frac{1}{R}\right)$$





- Brightness reduces
- Position remain same

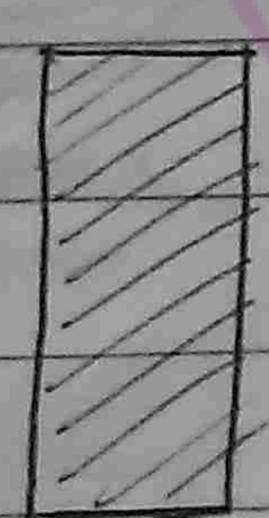
- Brightness remains same
- position different



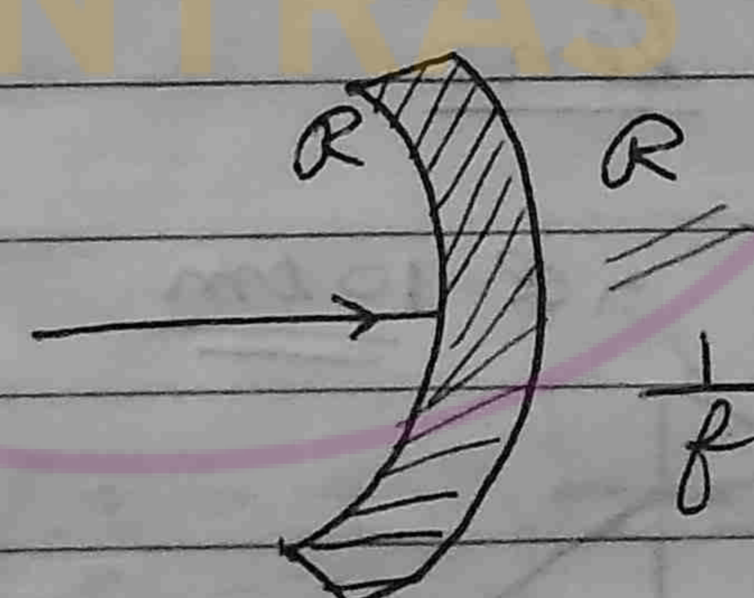
$$\frac{1}{f} = (\mu_1 - 1)\left(\frac{1}{-R} - \frac{1}{+R}\right)$$

$$\Rightarrow \frac{1}{f} = -(\mu - 1)\left(\frac{2}{R}\right)$$

∴ concave diverging



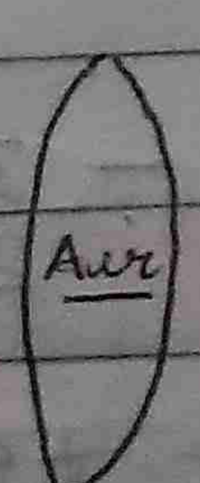
$\frac{1}{f} = 0$



$\frac{1}{f} = 0$

$R_1 = -R$; $R_2 = -R$

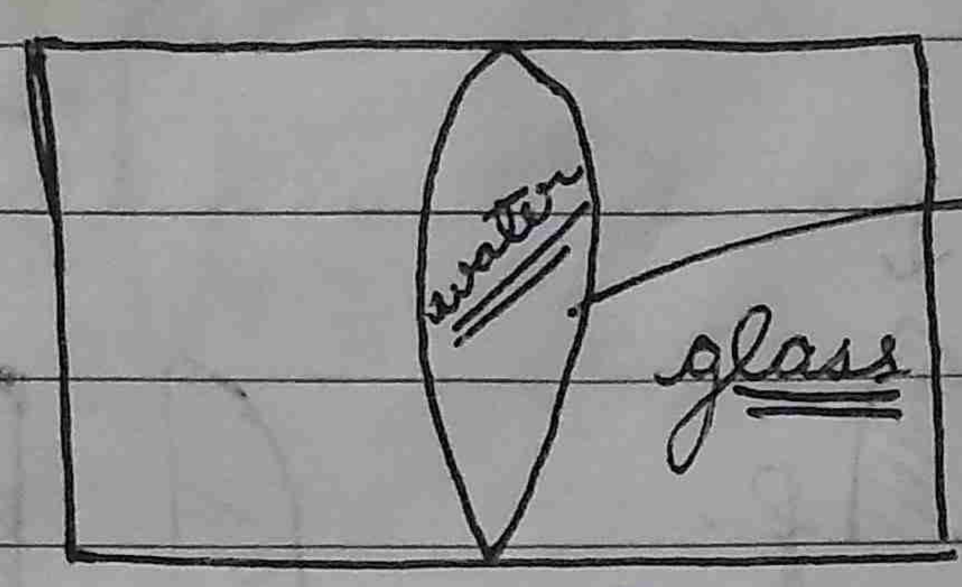
☆ e.g. goggles have both surface of same radii, hence Power = 0



Air

$\mu = \frac{\mu_L}{\mu_M} = 1 \Rightarrow \frac{1}{f} = 0$

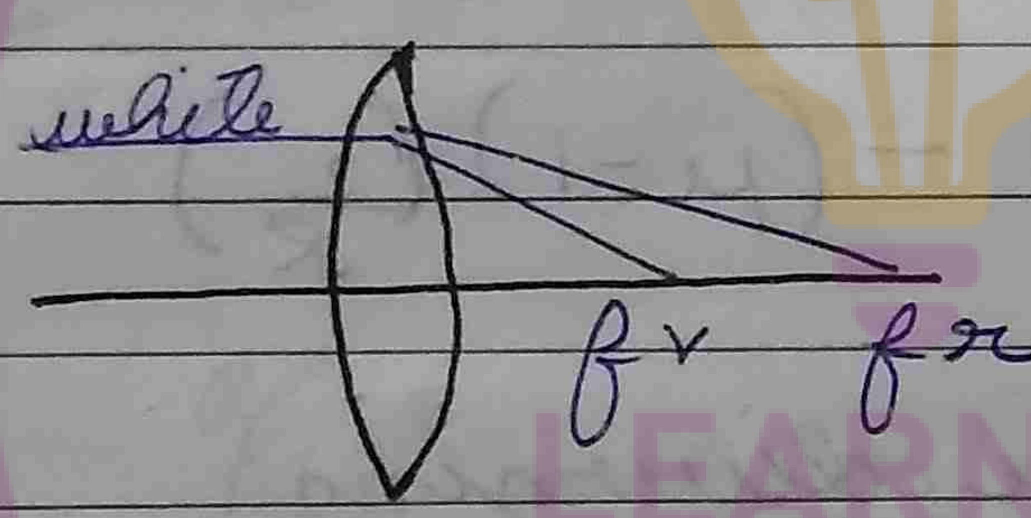
- focal length : a point where parallel light rays converge.



→ ~~concave~~ convex diverging

$$\frac{1}{f} = \left(\frac{\mu_L}{\mu_M} - 1\right)(+) \Rightarrow f = -ve$$

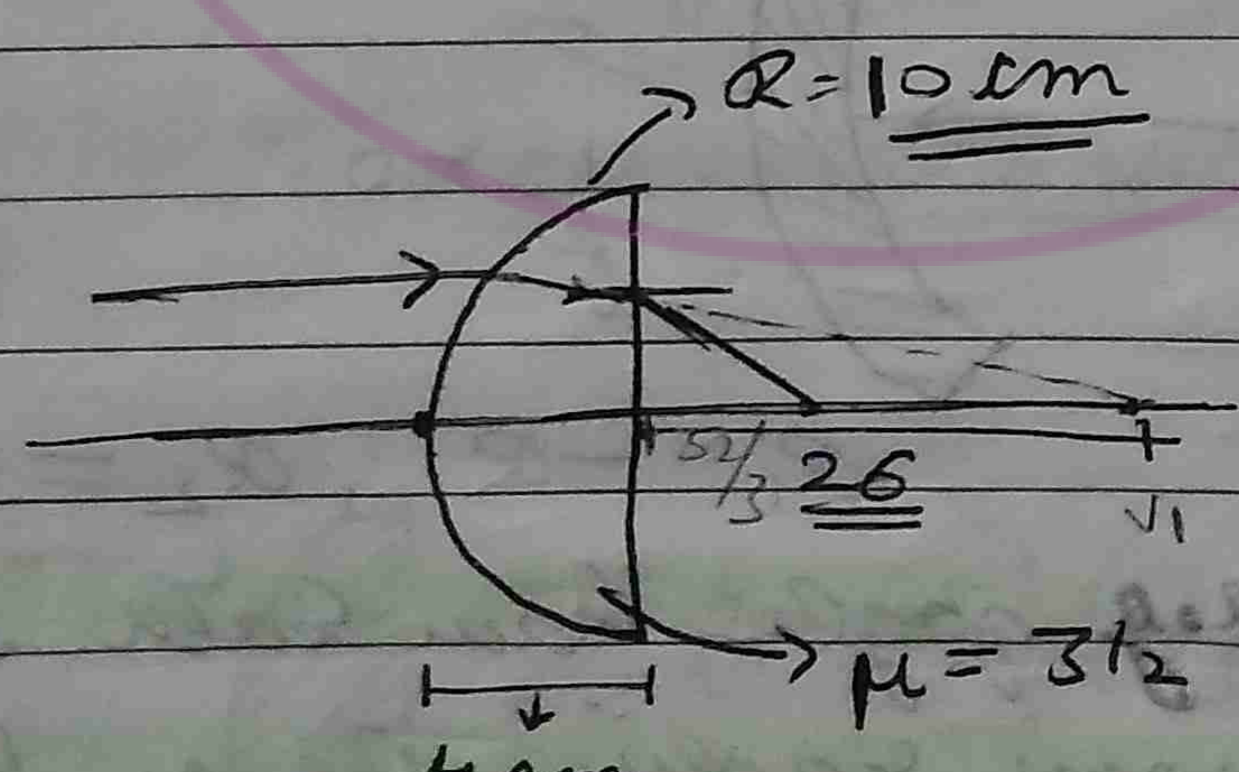
☆ $$\frac{1}{f} = \left(\frac{\mu_L}{\mu_M} - 1\right)\left(\begin{matrix} + \\ - \end{matrix}\right)$$ + → convex, − → concave

- 

$\mu_r < \mu_v$

$\Rightarrow f_r > f_v$

- Thick lens



- A → G : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{3}{2v_1} - 0 = \frac{1}{2(+10)}$$

$$v_1 = +30$$

G → A

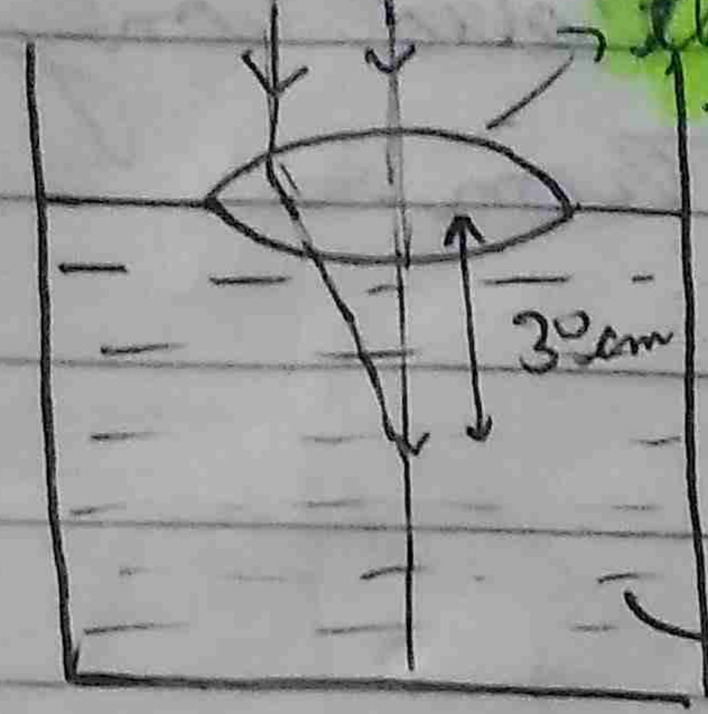
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = 0$$

$$\frac{1}{v} - \frac{3}{2(+26)} = 0$$

$$v = \frac{+52}{3}$$

↓ focal length

Medium surrounding the lens are different.



$R = 10\,cm$

$\mu_{glass} = 3/2$

$$\frac{1}{f} = \left(\frac{\mu_L}{\mu_M} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- A → G

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{3}{2v} - 0 = \frac{1}{2(+10)} \Rightarrow v = 30$$

- G → W

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{4}{3v} - \frac{3}{2(+30)} = \frac{\frac{4}{3} - \frac{3}{2}}{-10}$$

$\Rightarrow v =$

focal length where parallel rays are converging.

- Lens Formula

$$m = \frac{f - v}{f} = \frac{f}{f + u} = \frac{v}{u}$$

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$ lens formula

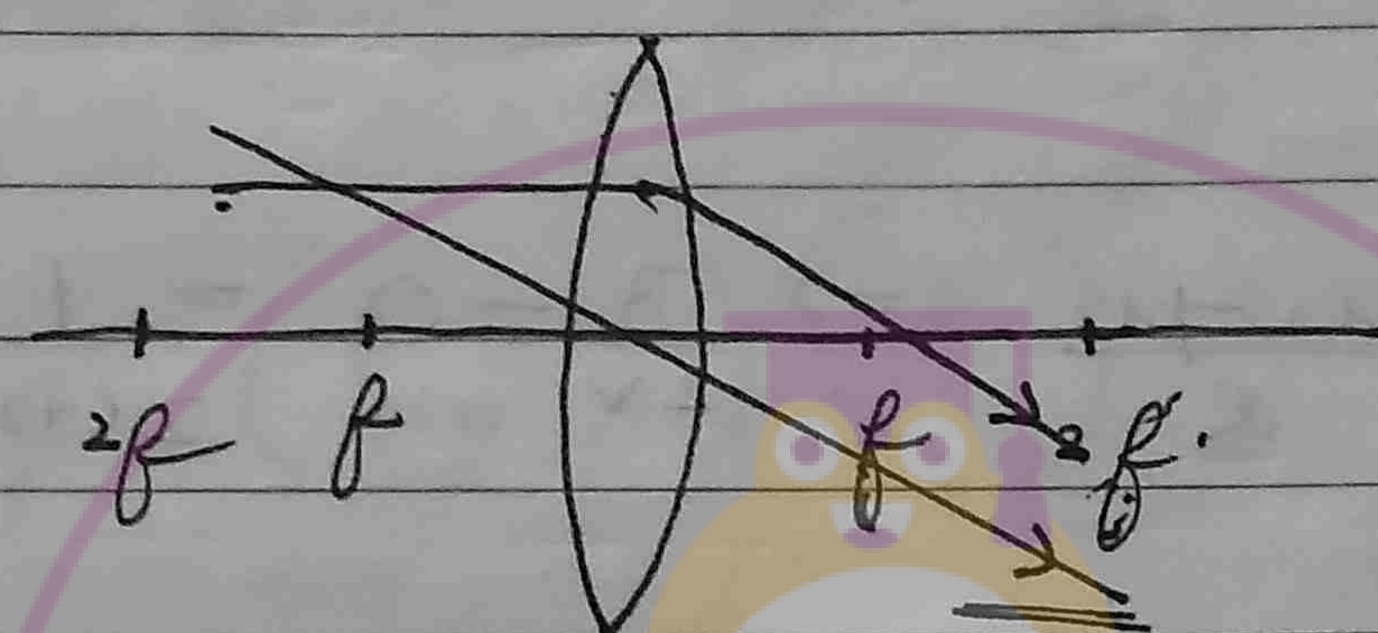
$$P = +\frac{1}{f}$$

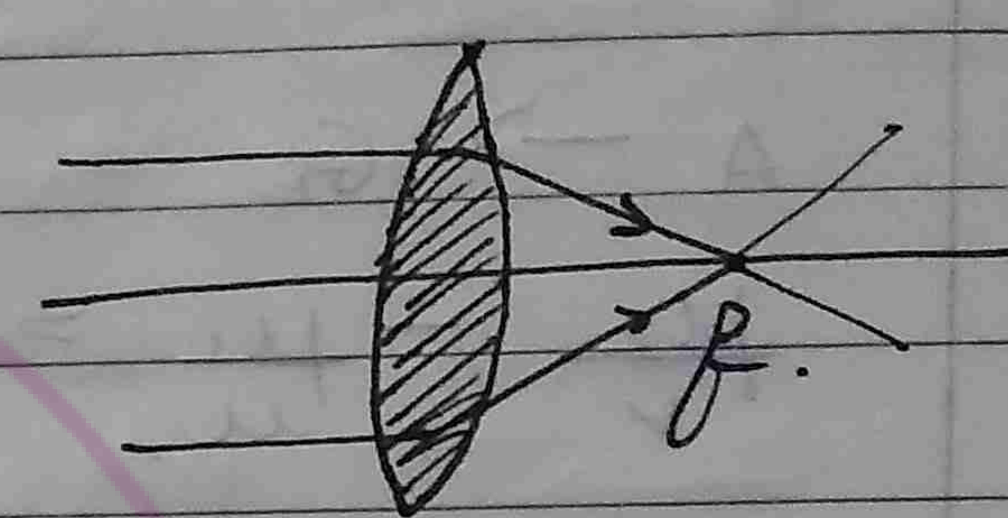
$$m = \frac{I}{O} = +\frac{v}{u}$$ → use only when image is ⊥ to axis

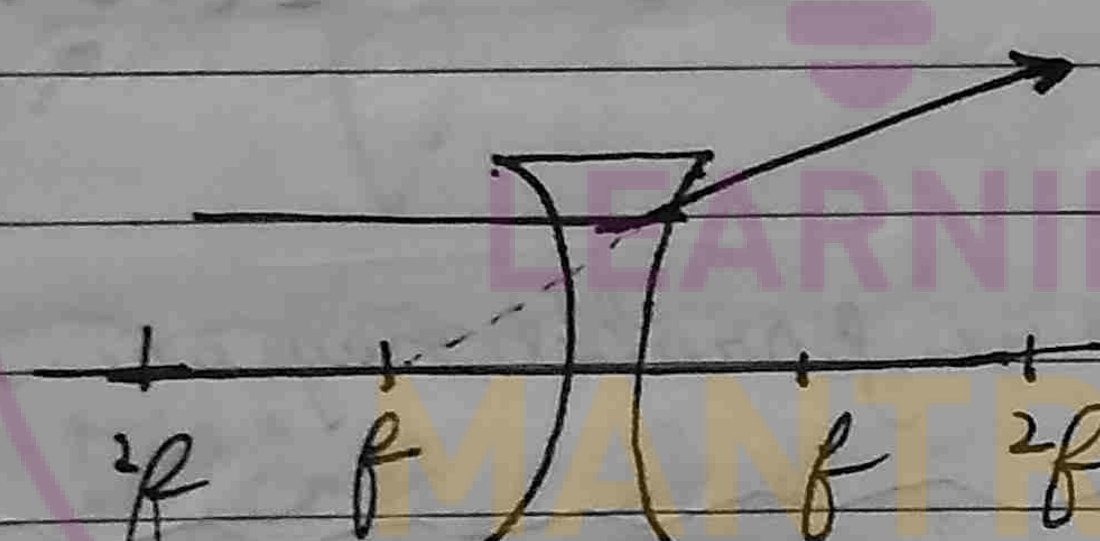
If $v = +ve$ — — — Real. $v = -ve$ virtual

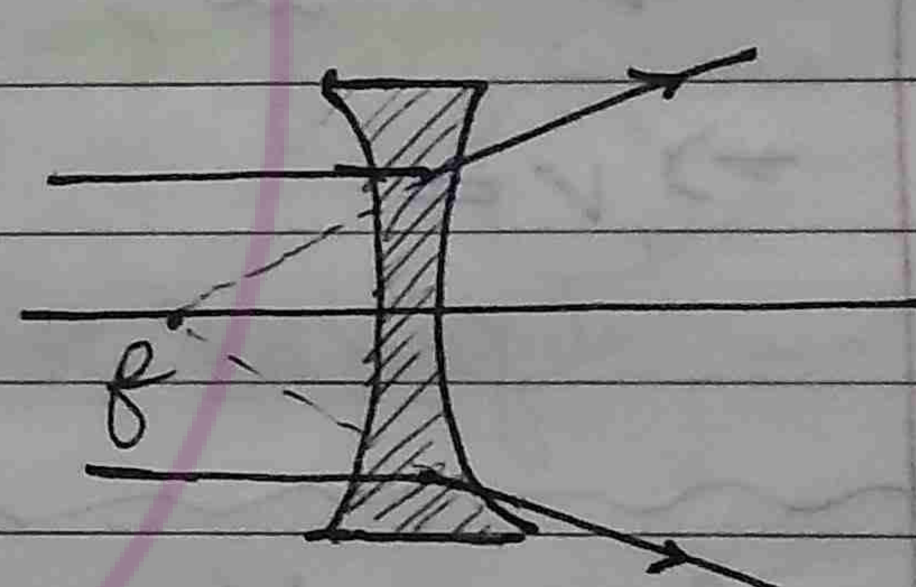
- These formulas are valid only for paraxial approximation.

- Special Rays



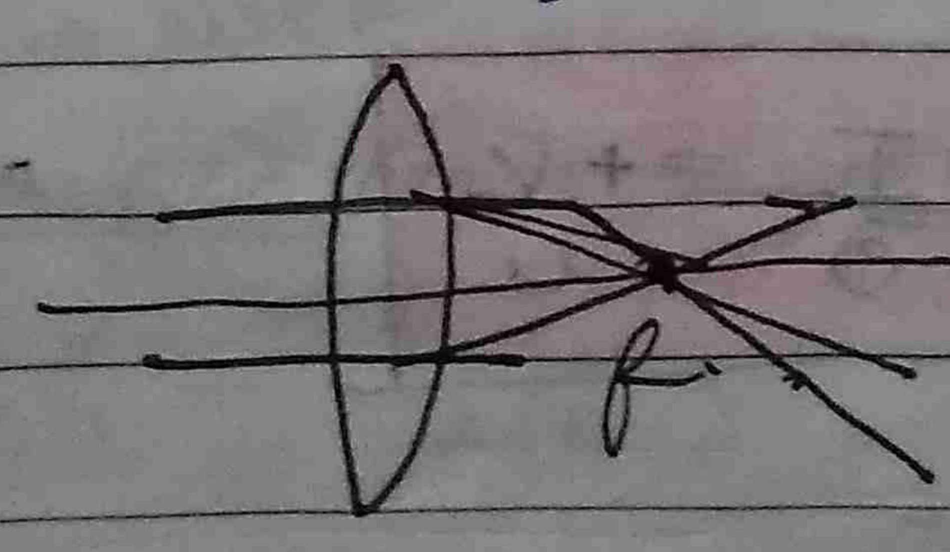


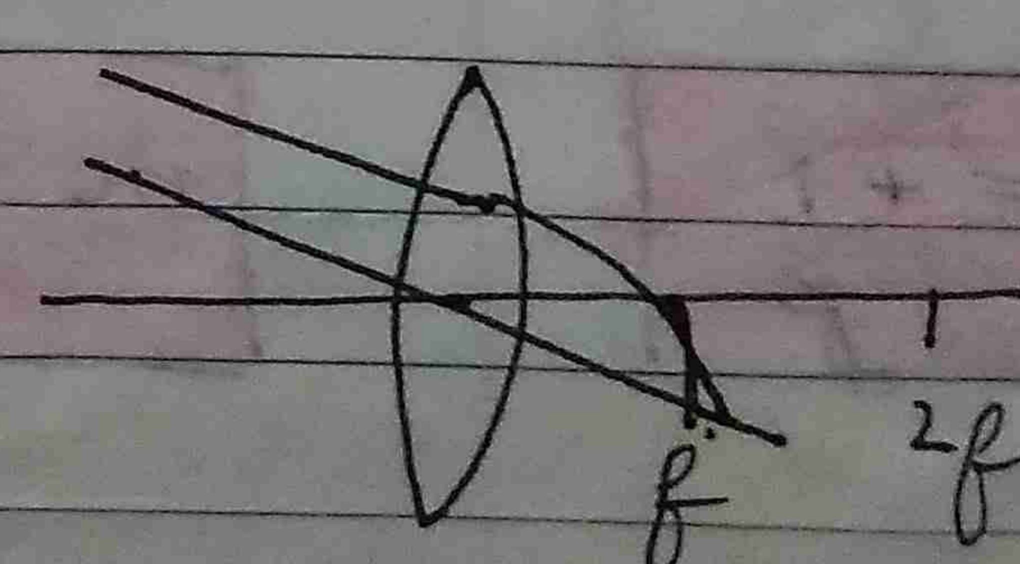




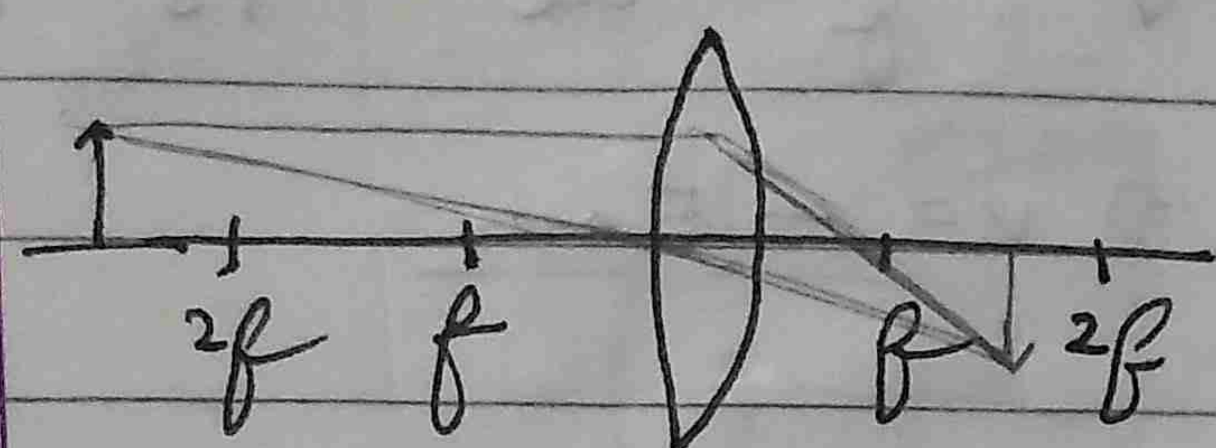
- Image tracing

- when object is at infinity : Image will form at f



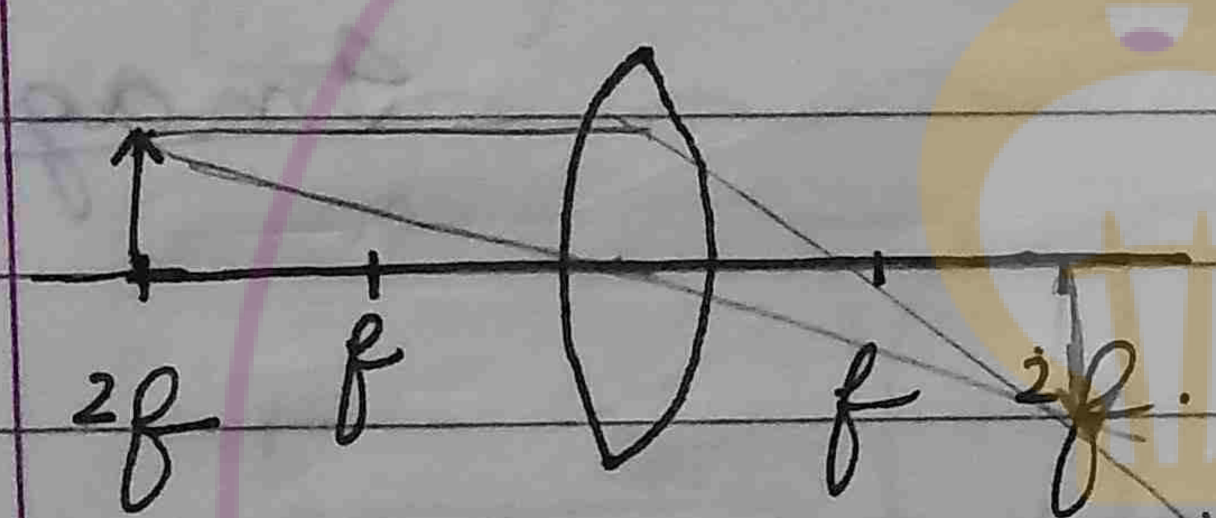


- Object between ∞ and 2f

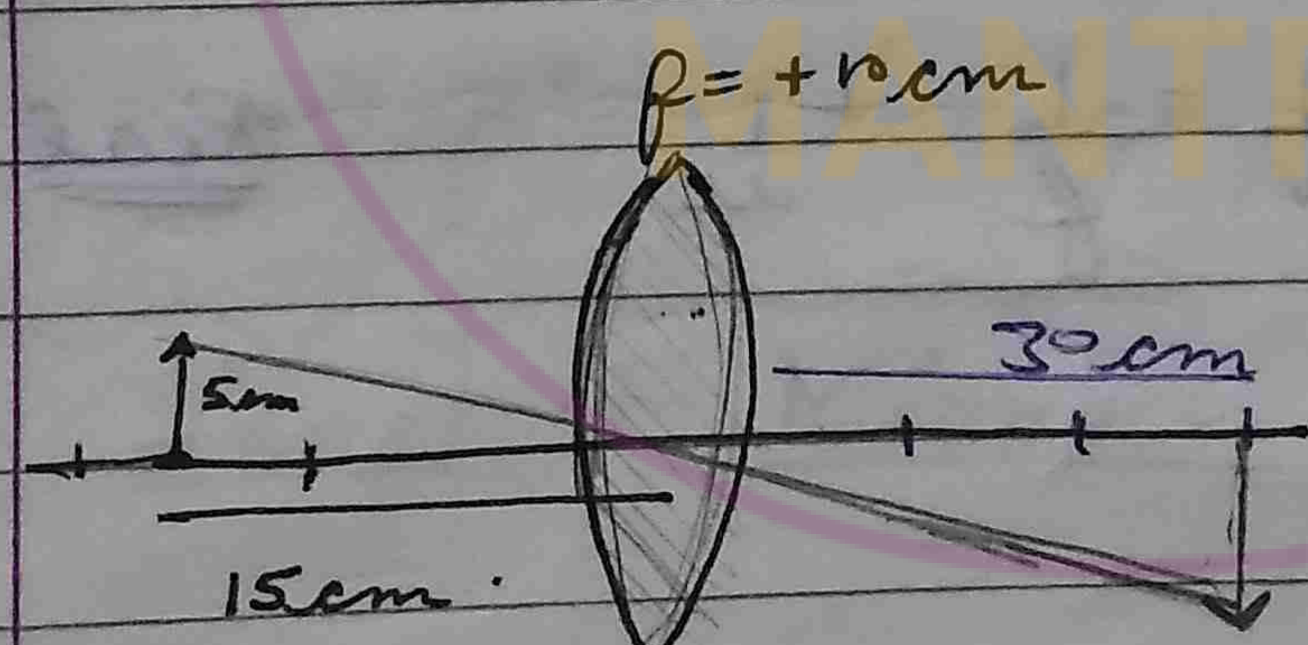


: Image formed b/w F and 2F
Real, inverted and diminished

- Object at 2f.

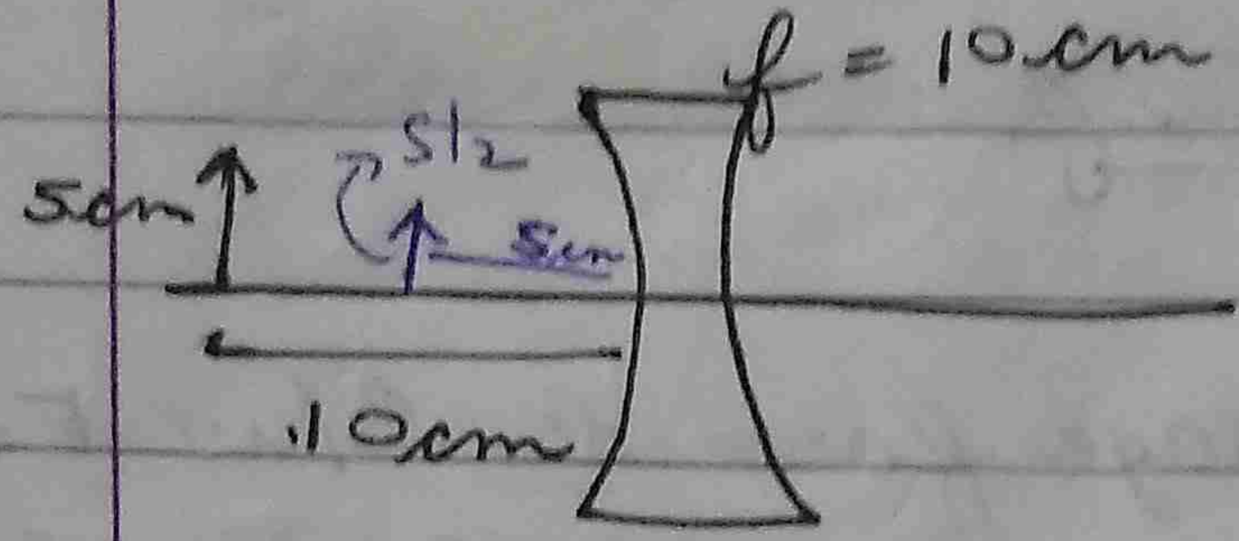


- Numericals



$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{+10} + \frac{1}{-15} \Rightarrow v = +30$

$m = \frac{v}{u} = \frac{I}{O} \Rightarrow \frac{+30}{-15} = \frac{I}{+5} \Rightarrow I = -10\text{ cm}$



$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = -\frac{1}{10} - \frac{1}{10}$$

$\Rightarrow v = -5cm$

$m = \frac{I}{O} = \frac{v}{u} \Rightarrow \frac{I}{5} = -\frac{5}{-10} \Rightarrow I = 5/2$

Convex lens $f = +$

— Image

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

☆ Real object $\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ → Real / → virtual

☆ Virtual object $\Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$ - - - Real

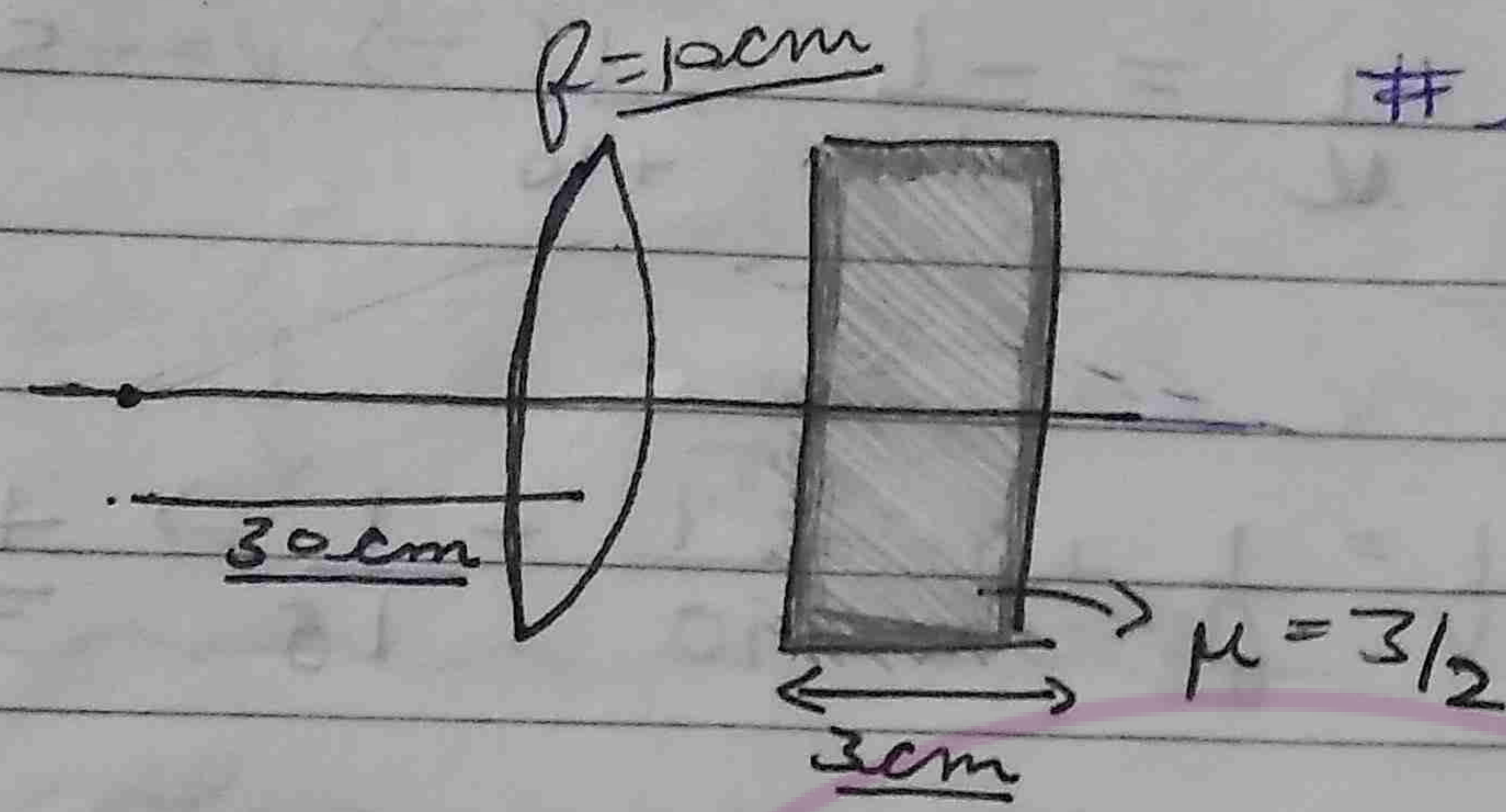
Concave lens

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{-1}{f} + \frac{1}{u}$$

☆ Real object $\frac{1}{v} = \frac{-1}{f} - \frac{1}{u} \Rightarrow$ virtual

☆ Virtual object $\frac{1}{v} = \frac{-1}{f} + \frac{1}{u}$ → Real / → virtual

lens + slab

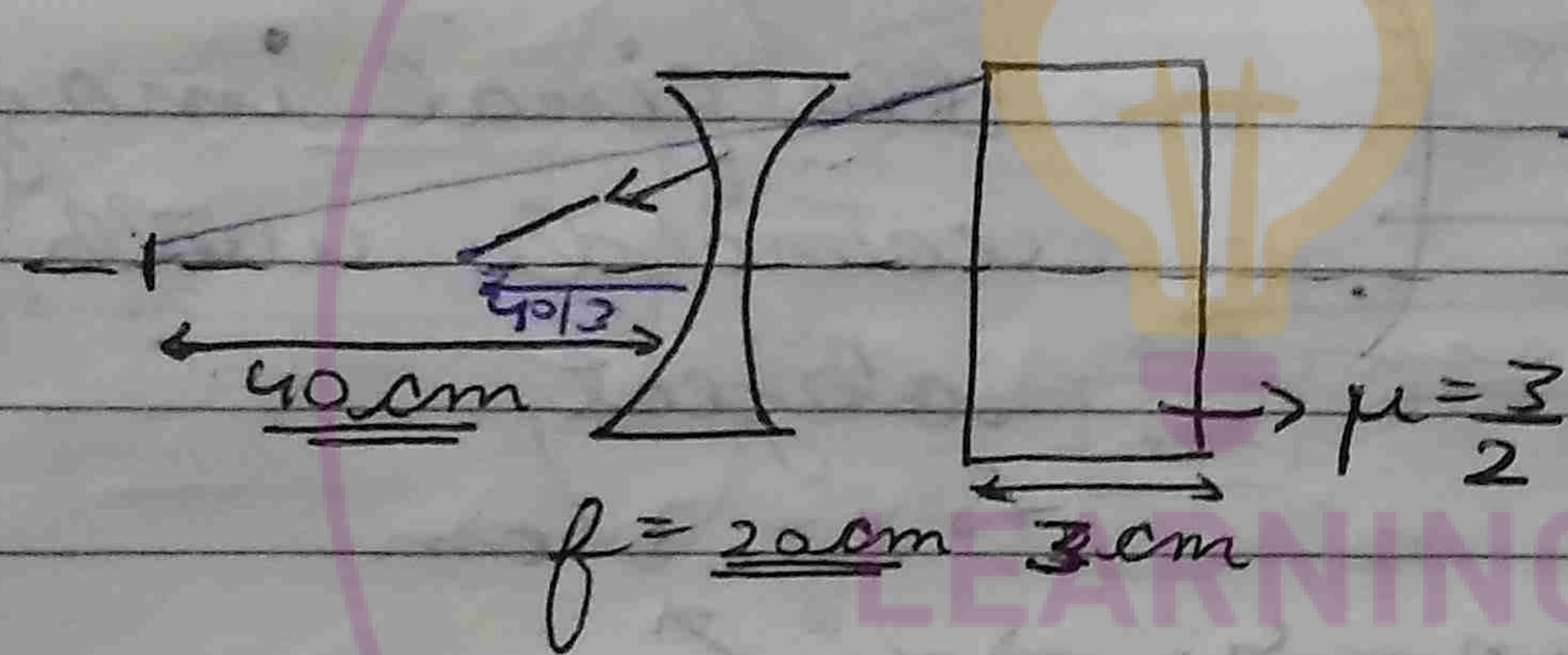


slab; shift $= t\left(1-\frac{1}{\mu}\right) = 3\left(1-\frac{1}{3/2}\right)$
$= 1cm$

lens $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} \Rightarrow v = +15$

$\Rightarrow$ final position = 16cm



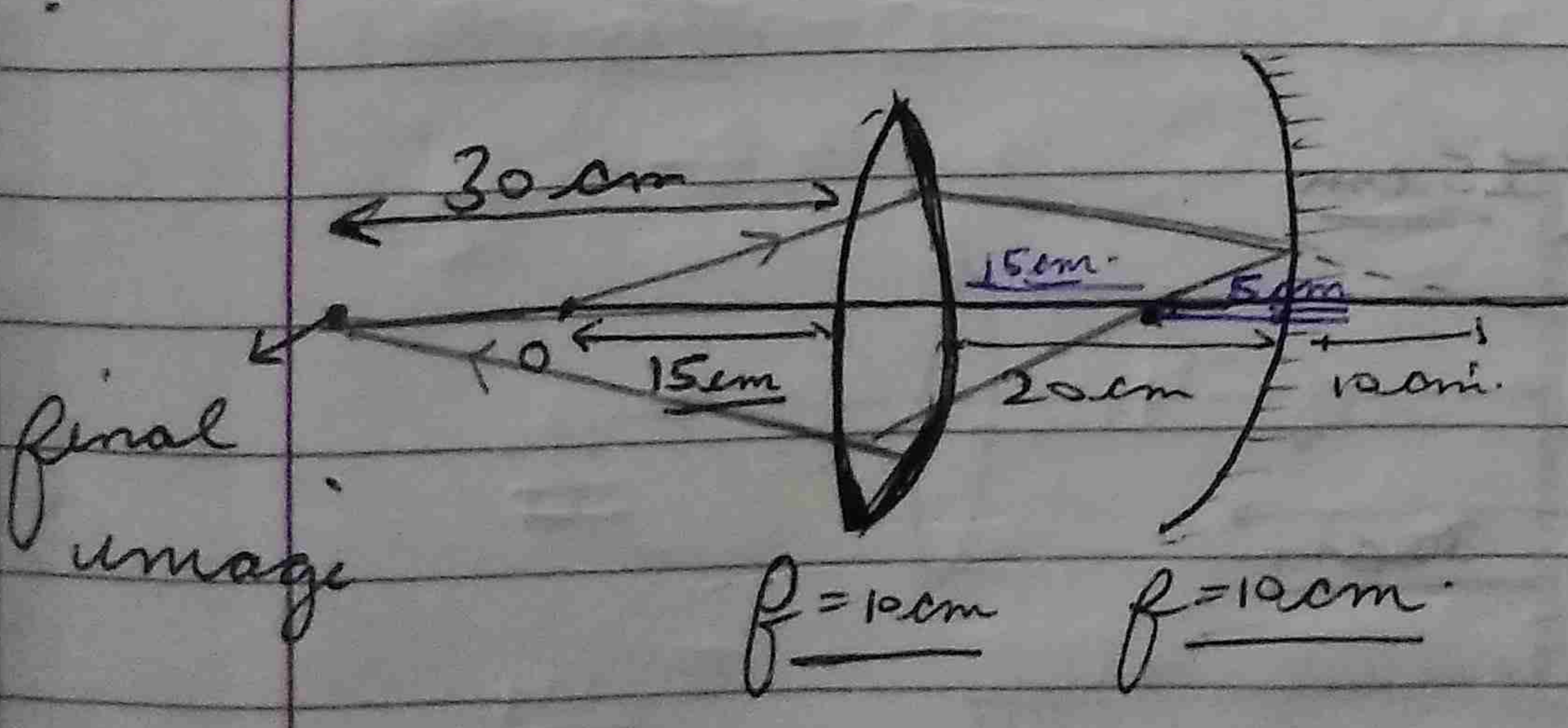
$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-20} + \frac{1}{-40}$

$\Rightarrow \frac{1}{v} = \frac{-3}{40} \Rightarrow v = \frac{-40}{3}$

$\Rightarrow$ final image position $= \left(\frac{40}{3} - 1\right)$ from lens

Lens + Mirror

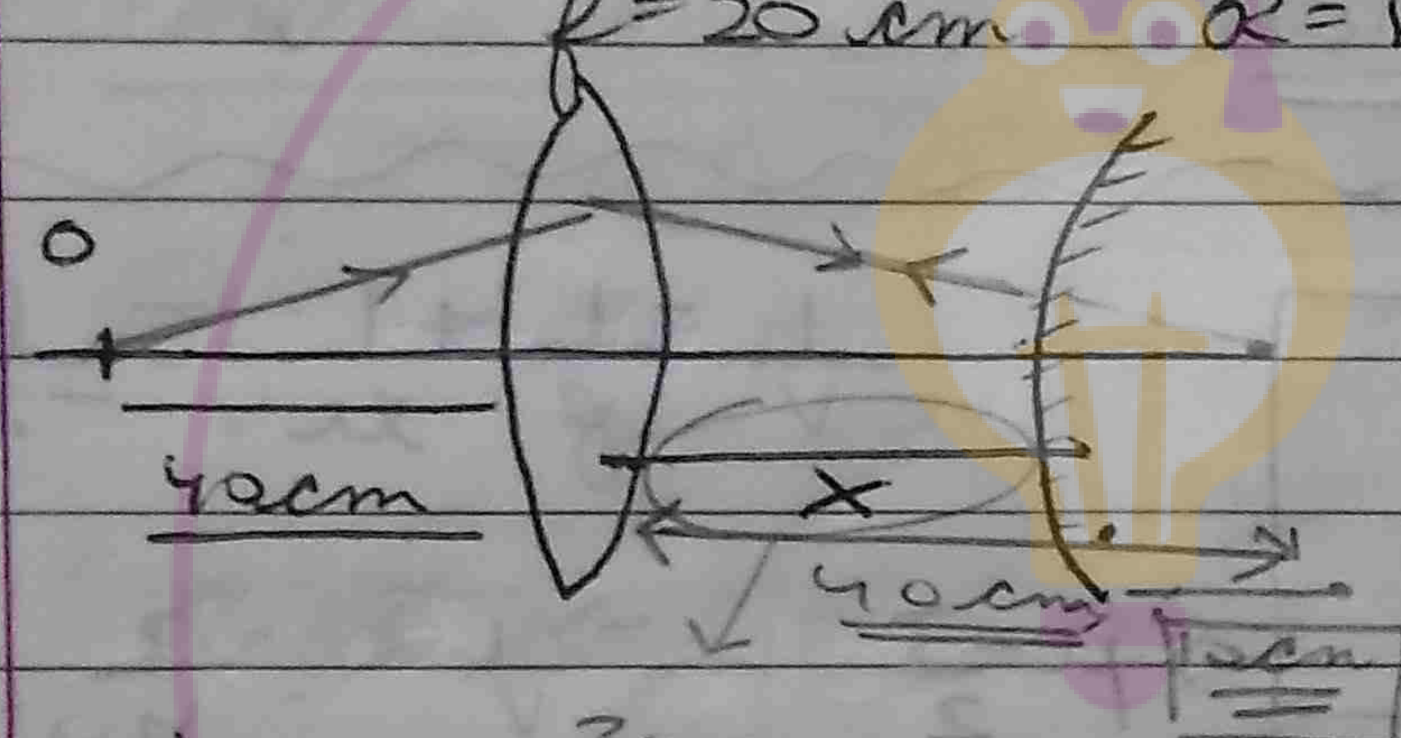
☆



Lene $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{+10} - \frac{1}{15} \Rightarrow v = +30$

mirror $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{-1}{10} - \frac{1}{+10} \Rightarrow v = -5.$

lens (2nd time) $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{+10} - \frac{1}{15} \Rightarrow +30$



Find x so that the final image coincides with object

30cm

$\Rightarrow x = 30$ cm

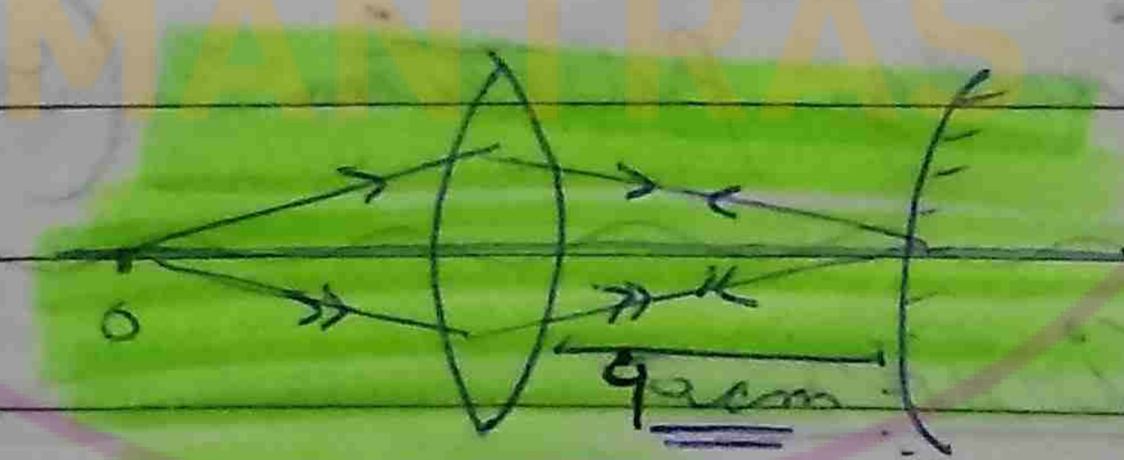
$\frac{1}{v} \Rightarrow \frac{1}{20} - \frac{1}{40}$

$\frac{1}{v} = \frac{1}{20} - \frac{1}{40}$

$v \Rightarrow +40$ $\quad x + R = 40 \Rightarrow x =$ ↗

$\frac{1}{v} = \frac{1}{20} + \frac{1}{-40}$

or



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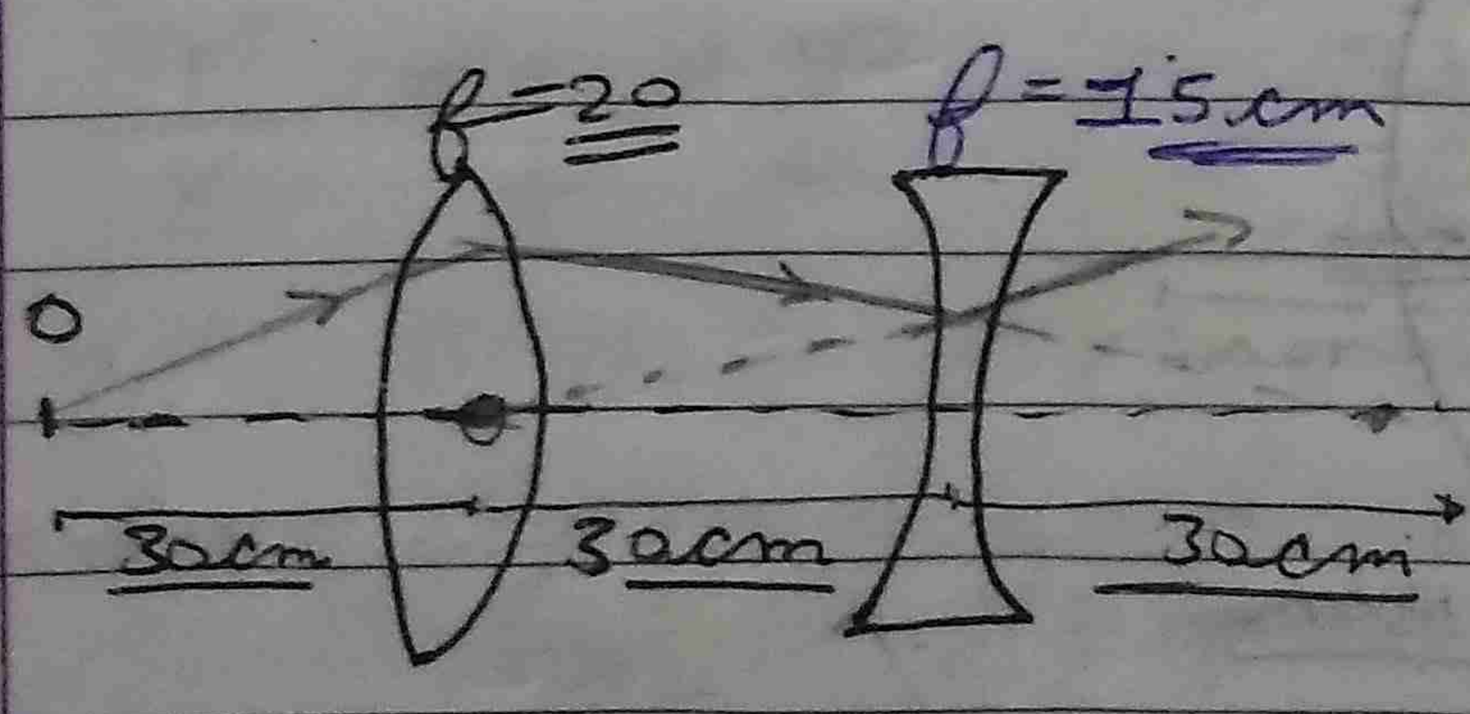
lens + lens



convex

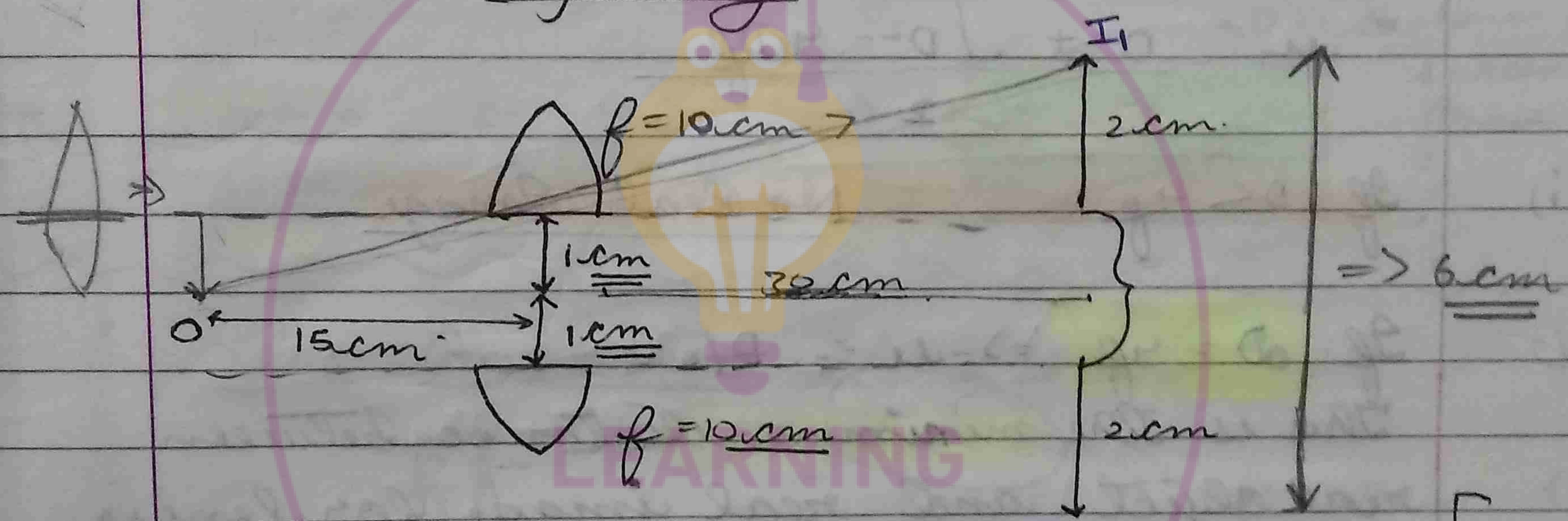
$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{+20} - \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{30-20}{600} \Rightarrow v = 60$

Concave

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{-1}{15} + \frac{1}{30}$$

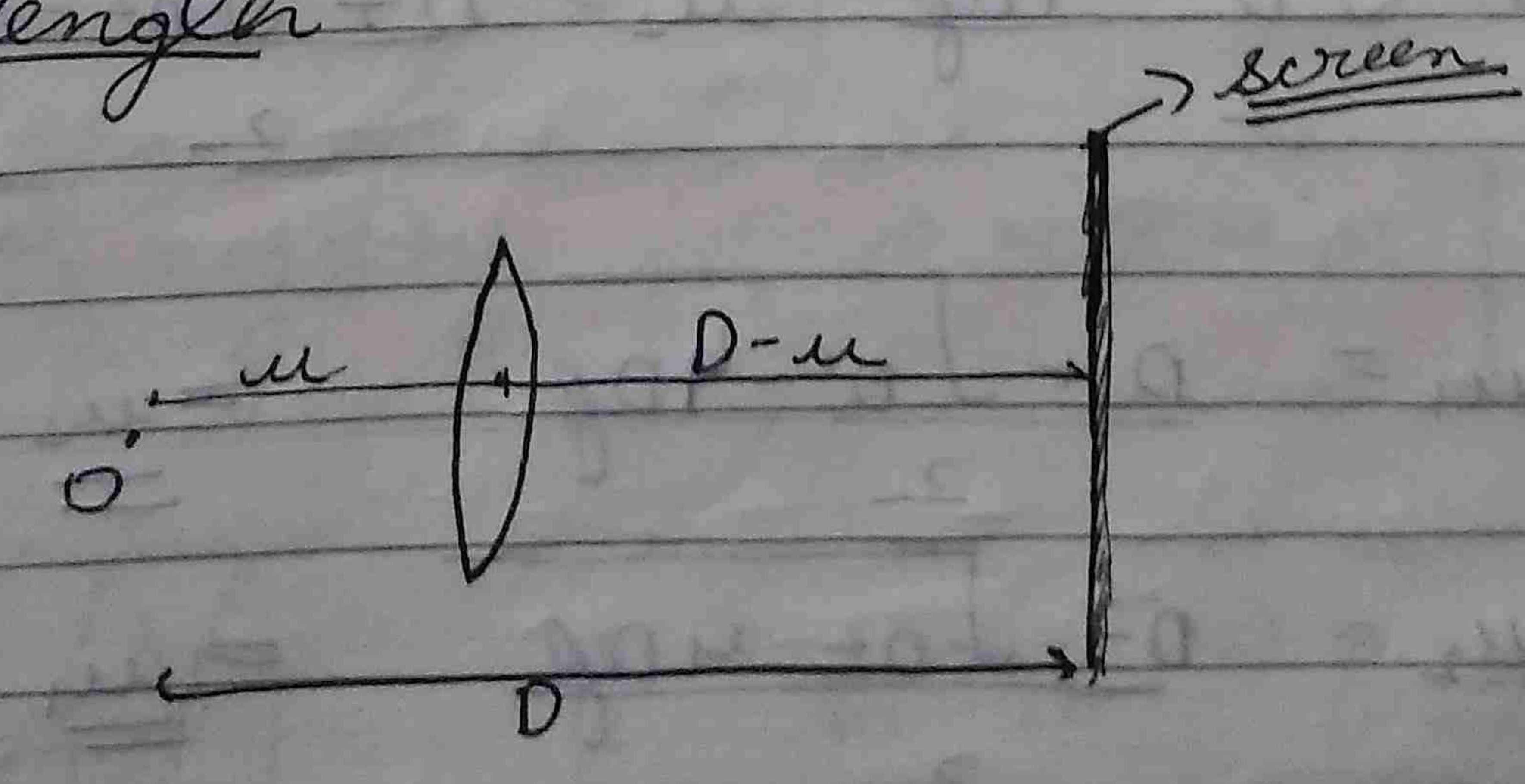
$$\Rightarrow \frac{1}{v} = \frac{-2+1}{30} \Rightarrow v = -30$$

Lens splitting



lens 1: $u = -15$, $O = -1\,cm$ $\Big\} \Rightarrow v = +30$, $I = +2$ $\left[\frac{+30}{-15} = \frac{I}{-1}\right]$

- Displacement method of calculating focal length



- For a concave mirror minimum distance between object and image is 0.

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$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{(D-u)} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{D}{u(D-u)} = \frac{1}{f} \Rightarrow Df = Du - u^2$$

$$u^2 - Du + Df = 0$$

$$\Rightarrow u = \frac{D \pm \sqrt{D^2 - 4Df}}{2}$$

(i) If $D < 4f$ --- No Real Image

(ii) If $D = 4f \Rightarrow u = D/2$

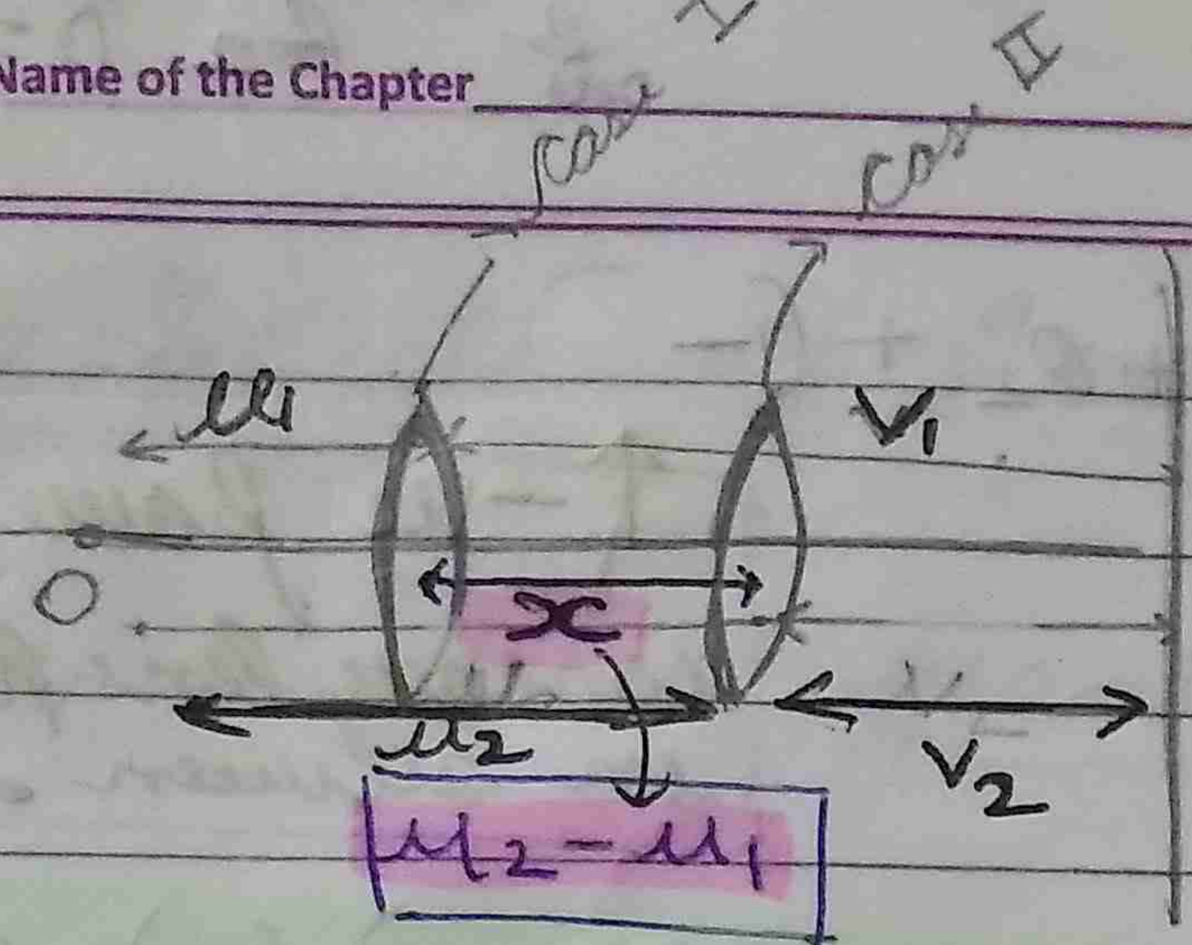
This is the minimum distance between real object and real image for lens is $4f$.

(iii) If $D > 4f$

$$u_1 = \frac{D - \sqrt{D^2 - 4Df}}{2} \quad u_2 = \frac{D + \sqrt{D^2 - 4Df}}{2}$$

$$v_1 = D - u_1 = \frac{D + \sqrt{D^2 - 4Df}}{2} = u_2$$

$$v_2 = D - u_2 = \frac{D - \sqrt{D^2 - 4Df}}{2} = u_1$$



Now

$$m_1 = \frac{v_1}{u_1} \qquad m_2 = \frac{v_2}{u_2}$$

$$\Rightarrow m_1 m_2 = 1$$

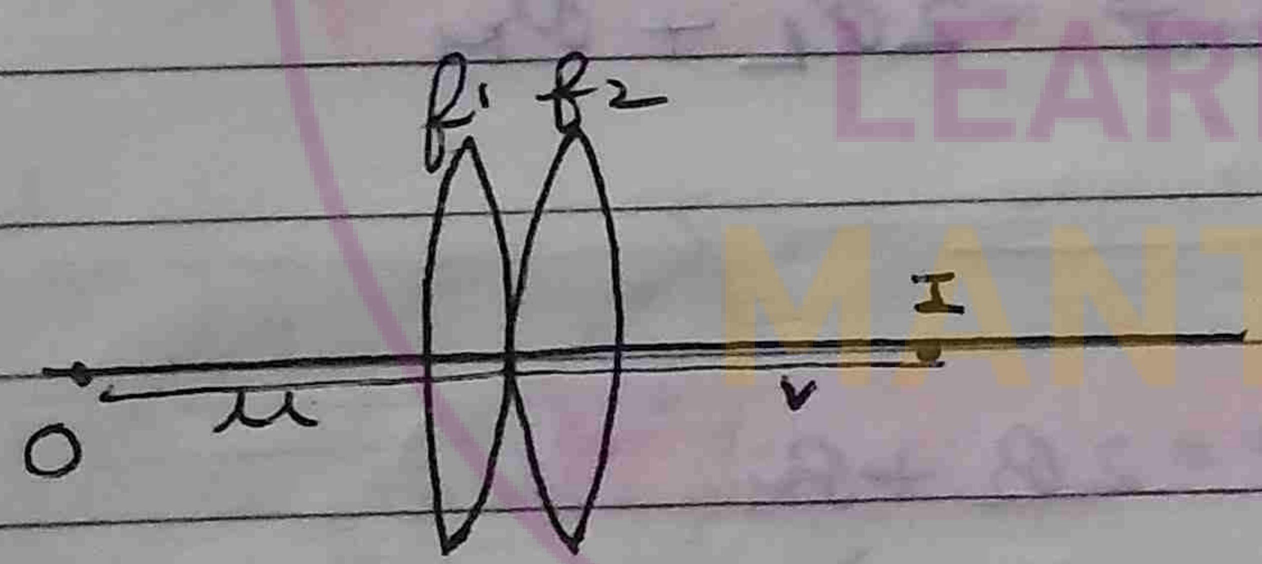
$$\Rightarrow \frac{I_1}{O} \times \frac{I_2}{O} = 1 \Rightarrow O = \sqrt{I_1 I_2}$$

$$x = u_2 - u_1 = \sqrt{D^2 - 4Df}$$

- x is the separat. b/w the lens
- D is distance of screen from objec

$$x^2 = D^2 - 4Df \Rightarrow f = \frac{D^2 - x^2}{4D}$$

- **Lens Combination**



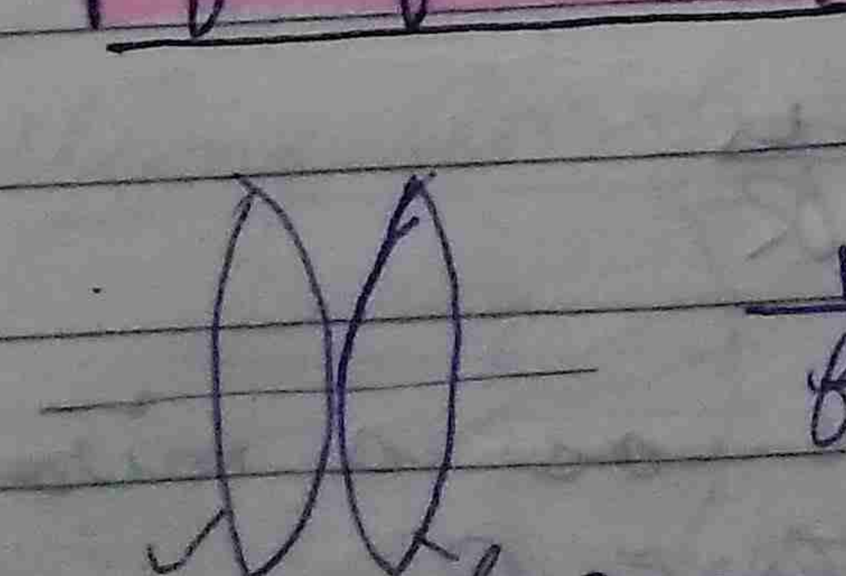
Lens$_1$ $\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$ (i)

Lens$_2$ $\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$ - - (ii)

adding (i) and (ii) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \; ; \; P = P_1 + P_2$$

e.g.



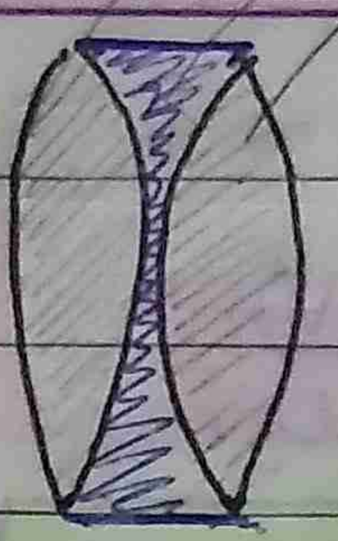
$\frac{1}{f} = \frac{1}{10} + \frac{1}{20} \Rightarrow f = \frac{20}{3}$

$P = P_1 + P_2$

* If $\mu_1 = \mu_2 \Rightarrow \frac{1}{f} = 0$ and it acts as a glass slab.

$P = P_1 + P_2 - dP_1P_2$

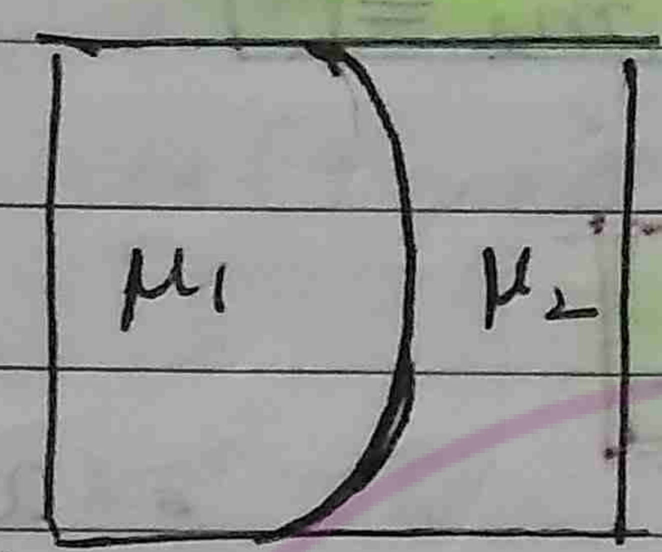
$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$



$P = P_1 + P_2 + (-)$

↑ -ve power of concave lens present in between.

#

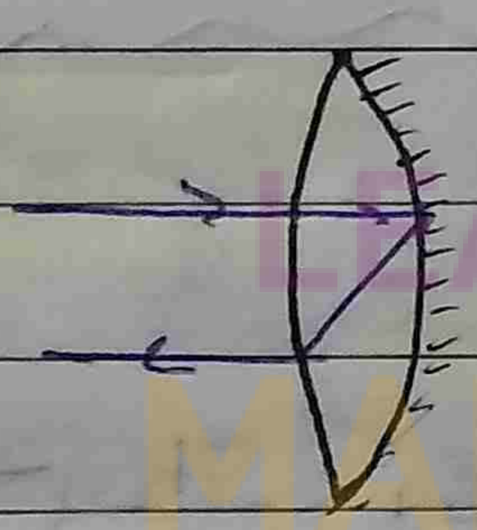


Treat this as case of individual lenses

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = (\mu_1 - 1)\left(\frac{1}{R}\right) - (\mu_2 - 1)\frac{1}{R}$

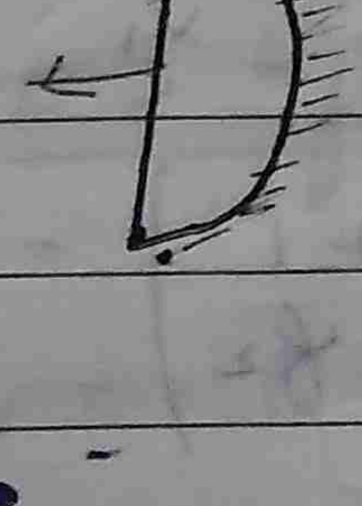
☆ $\frac{1}{f} = (\mu_1 - \mu_2)\frac{1}{R}$

<u>Silvering of lens</u>



$\Rightarrow L + M + L$

$P_{net} = 2P_L + P_M$



$P_{net} = 2P_L + P_M$

<u>Lens</u> $\frac{1}{f(L)} = (\mu - 1)\left(\frac{1}{R}\right)$ $P_L = +\frac{1}{f_L}$

<u>Mirror</u> $f(M) = -R/2$; $P_M = \frac{-1}{f_m} = \frac{2}{R}$

$P_{net} = 2P_L + P_M$

$= 2(\mu - 1)\frac{1}{R} + \frac{2}{R} = \frac{2\mu}{R}$

Overall arrange behaves as a mirror (concave) of focal length $= \frac{R}{2\mu}$

Defects of Images

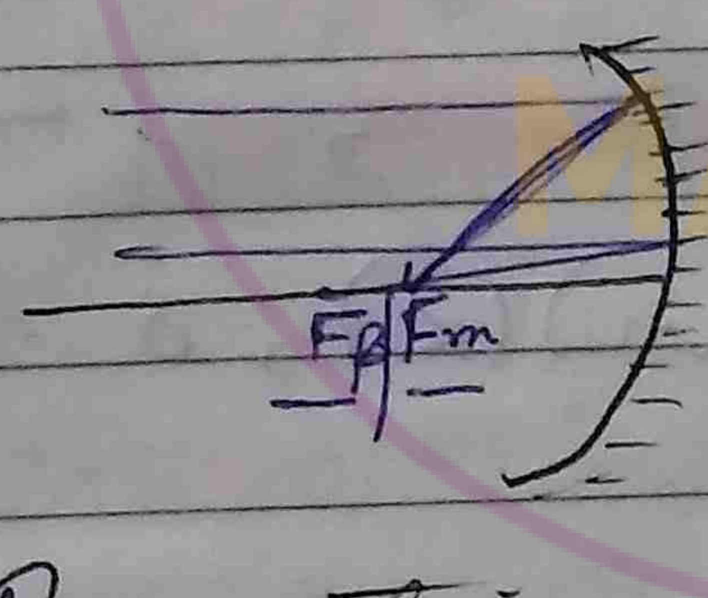
(i) Defect due to geometry : Achromatic Aberration

(ii) Defect due to wavelength : Chromatic aberration

$f_{mirror} = \frac{R}{2}$; $\frac{1}{f_{lens}} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

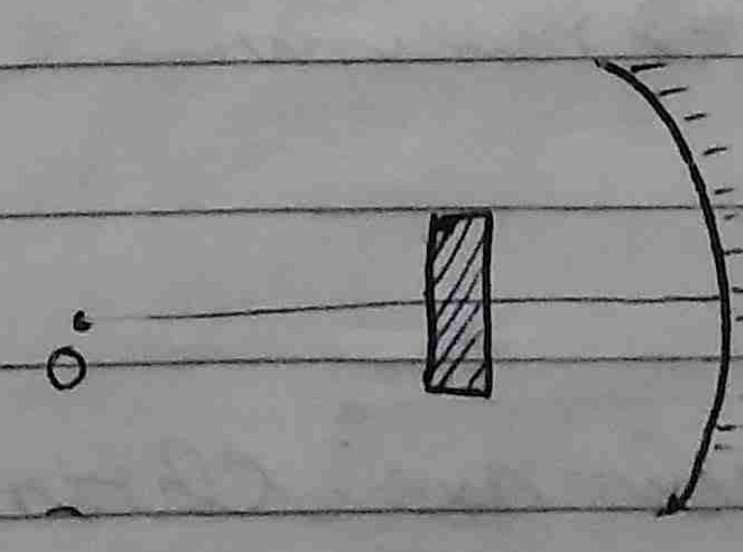
• Images due to mirror are free from chromatic aberration. Because focus does not depend on wavelength of light.

Mirror : Achromatic aberration → spherical aberration.



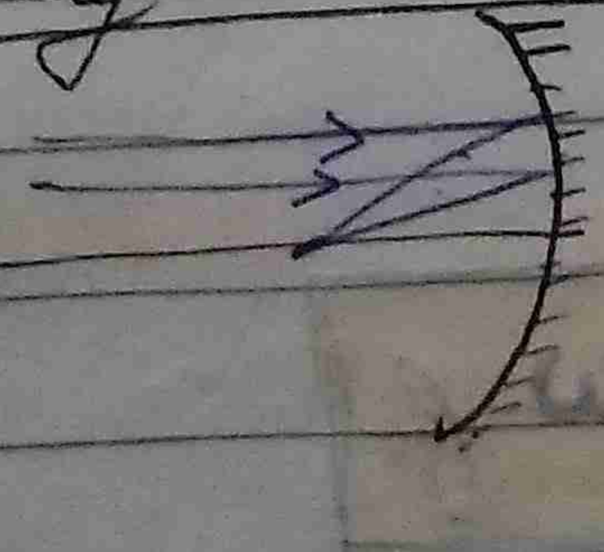
Prevention

1. Using stops



But this lowers intensity

2. Using Parabolic mirror



☆ To remove chromatic aberration
$f_v - f_r = 0$

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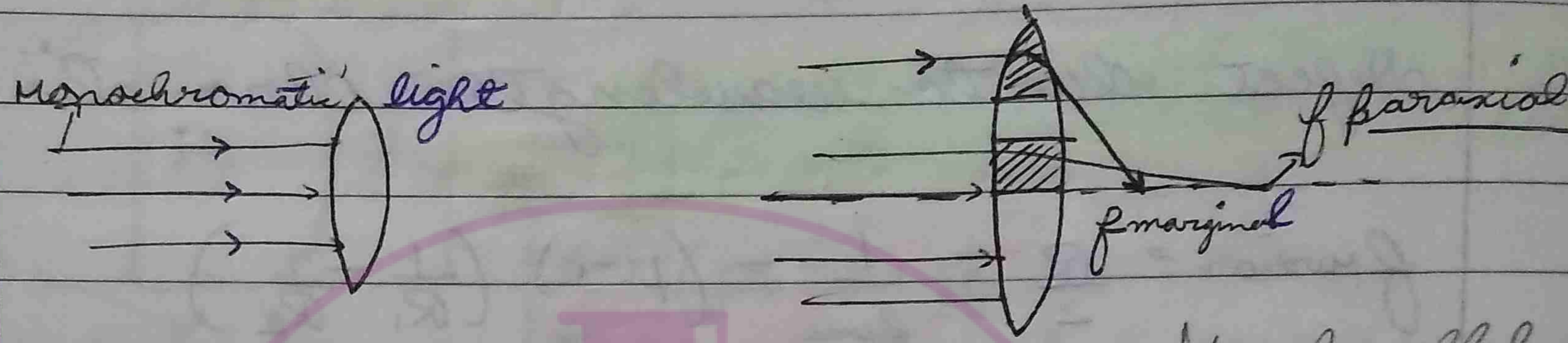
Defect of Images

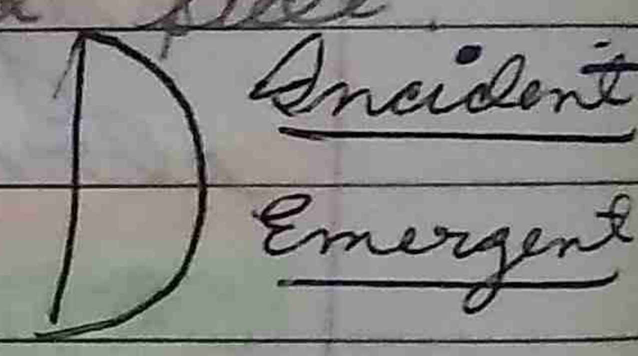
- **Achromatic Aberration**



More parallel rays will face curved side.

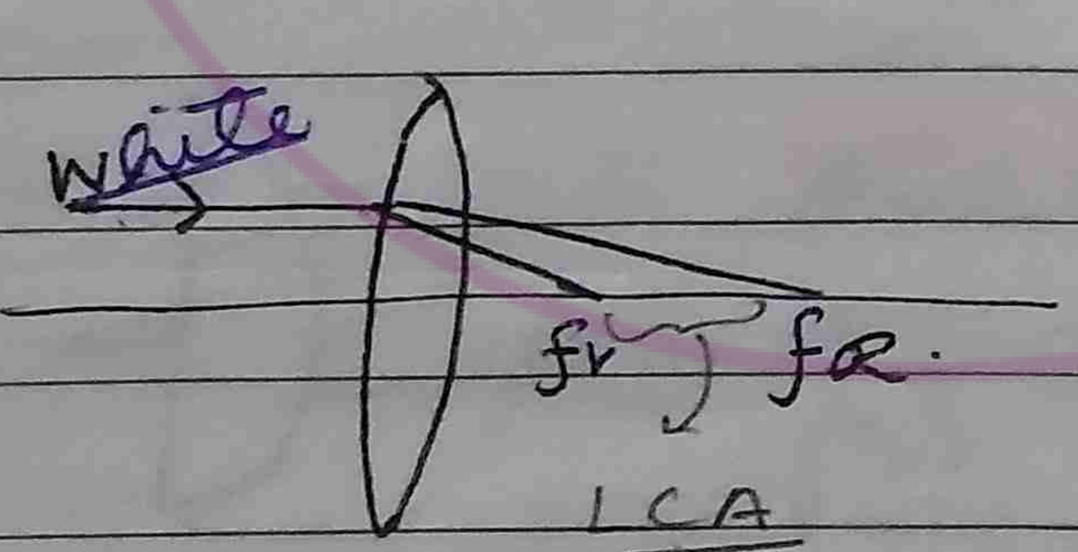
A paraxial < A marginal

$\delta = (\mu - 1)A \Rightarrow \delta_M > \delta_p$



- **Prevention**: (i) Using stops
  (ii) Using plano convex lens

- **Chromatic Aberration**



$\frac{1}{f} = (\mu - 1)(K)$; $\delta = (\mu - 1)A$

$K = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\mu_{red} \Rightarrow$ minimum $\Rightarrow$ $f_{red} \Rightarrow$ maximum.
$\delta_{red} \Rightarrow$ minimum

- **Longitudinal Chromatic aberration (LCA)**

$$|f_v - f_R = \Delta f = df|$$

$$f_v - f_R = \omega f$$

L.C.A = dispersive power × focal length of lens
$\omega \times f$

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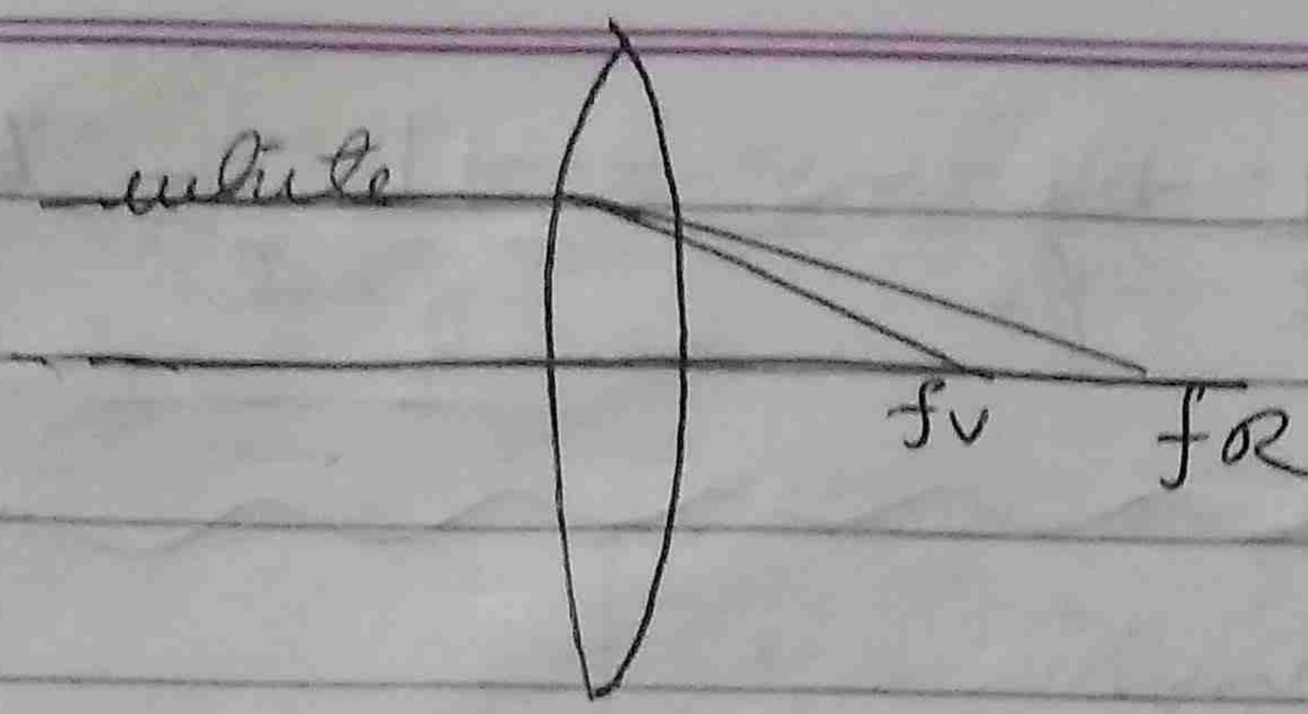


$\frac{1}{f} = (\mu - 1)(K)$ --- (i) differentiating (i)

$\left| -\frac{df}{f^2} \right| = K\, d\mu$ = (ii)

dividing (i) and (ii) ⇒ ω = dispersive power

$\frac{df}{f^2 \cdot \frac{1}{f}} = \frac{K\, d\mu}{(\mu-1)(K)} = \frac{df}{f} = \frac{d\mu}{\mu - 1}$

difference in refractive index of violet and red

mean refractive index

⇒ $LCA = df = \omega f$

here $\omega \Rightarrow \left(\frac{\mu_v - \mu_R}{\mu_y - 1} \right)$

→ for a single lens LCA can not be zero.

So two lens are joined to have zero LCA

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow -\frac{df}{f^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$

$0 = \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} \Rightarrow \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$

= Achromatic doublet

So while making achromatic doublet, two lens of different material should be used

↑ i.e same material

- If $\omega_1 = \omega_2 \Rightarrow \frac{1}{f} + \frac{1}{f_2} = 0 \rightarrow \frac{1}{f_{net}} = 0$

---

e.x Required ; $f = 10\,cm$.

crown glass = $\omega_c = 0.014$

flint glass = $\omega_f = 0.028$

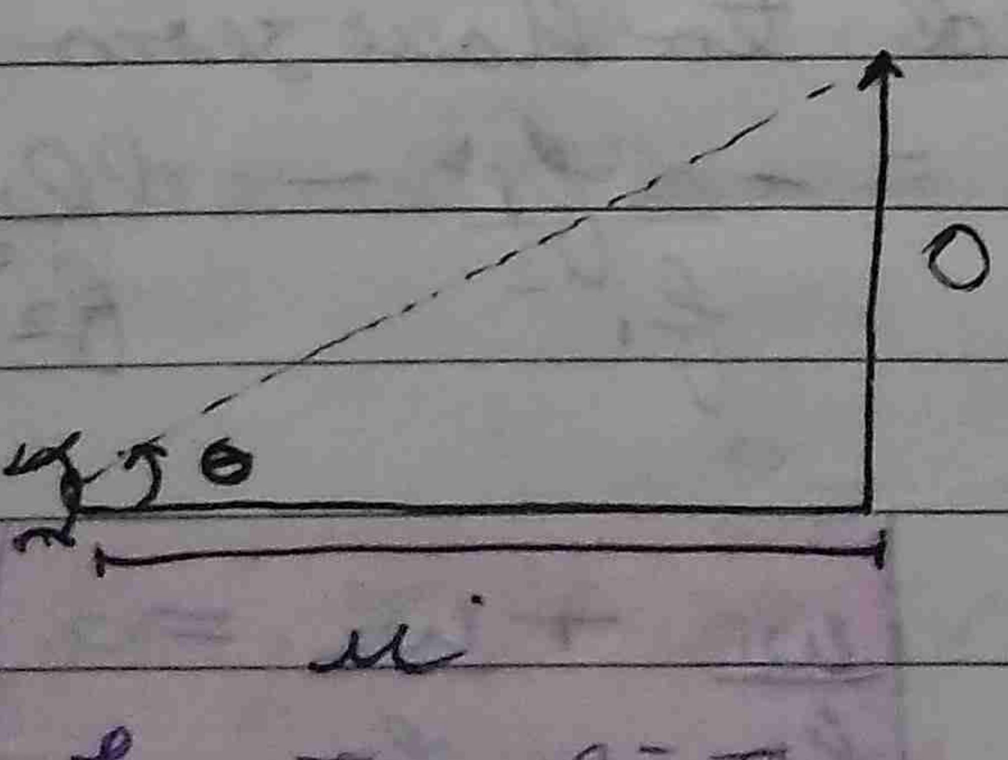
$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10}$ (i) $\quad \frac{0.014}{f_1} + \frac{0.028}{f_2} = 0$ (ii)

$\Rightarrow \frac{1}{f_1} = \frac{-2}{f_2}$ (iii)

solve → we get f_1 and f_2

- Optical Instrument

→ Visual angle



$\theta = \frac{O}{u}$

Least distance of distinct vission (LDDV) or D = (25 cm)

☆ In simple microscope angular magnification depends on u but not f.

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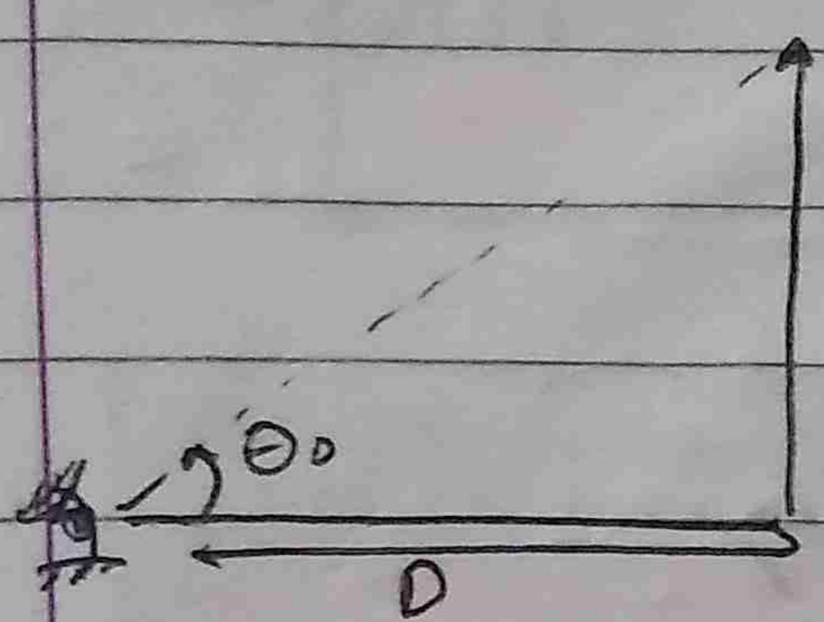
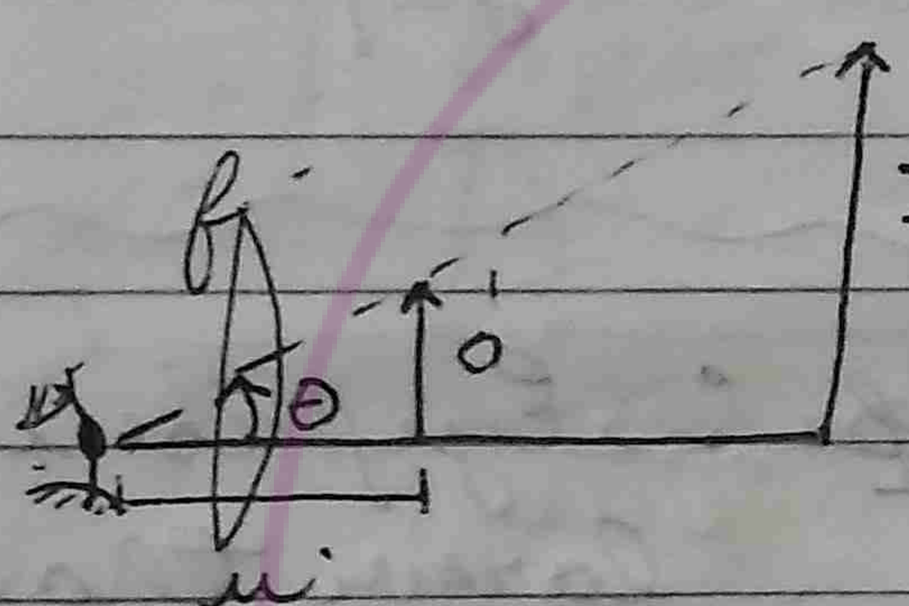
- **Simple microscope or magnifier**



θ_0 = max visual angle of unaided eye

$$\theta_0 = +\frac{O}{D}$$



Let $u < D$

$|f| > |u|$

virtual, erect, magnified

$$\theta = +\frac{O}{u} > \theta_0 = +\frac{O}{D}$$

- **Angular magnification or magnifying power**

$$AM = \frac{\theta}{\theta_0} = \frac{D}{u} > 1$$

e.g. if $u = 10 \Rightarrow A.M = 2.5\times$

- The image should be between LDDV and infinity.

- **Normal adjustment**; $v = \infty$

$u = f$; $AM = \frac{D}{f}$

- When image is at **LDDV**

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-D} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{u} = \frac{1}{f} + \frac{1}{D}$$

$$M_e = \frac{Df_e}{D+f_e}$$

Signature ______

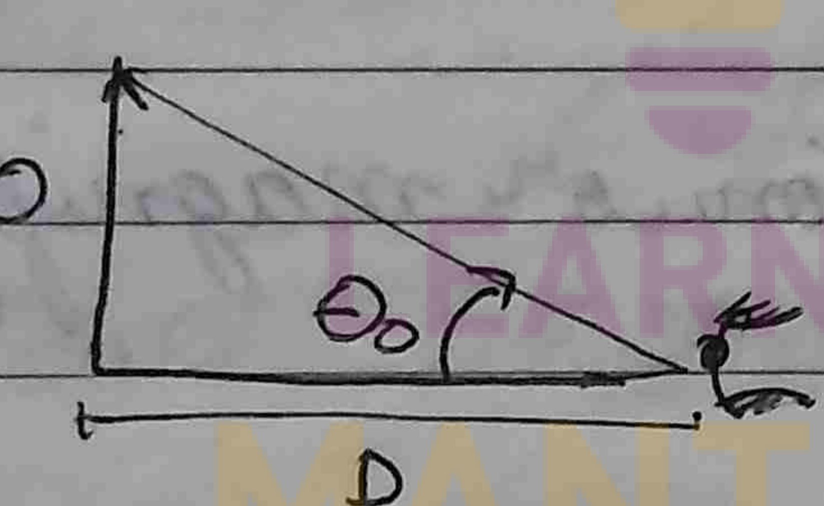
$$AM = \frac{D}{u} = D\left(\frac{1}{D} + \frac{1}{f}\right) = 1 + \frac{D}{f}$$

$$AM = \frac{D}{u}$$

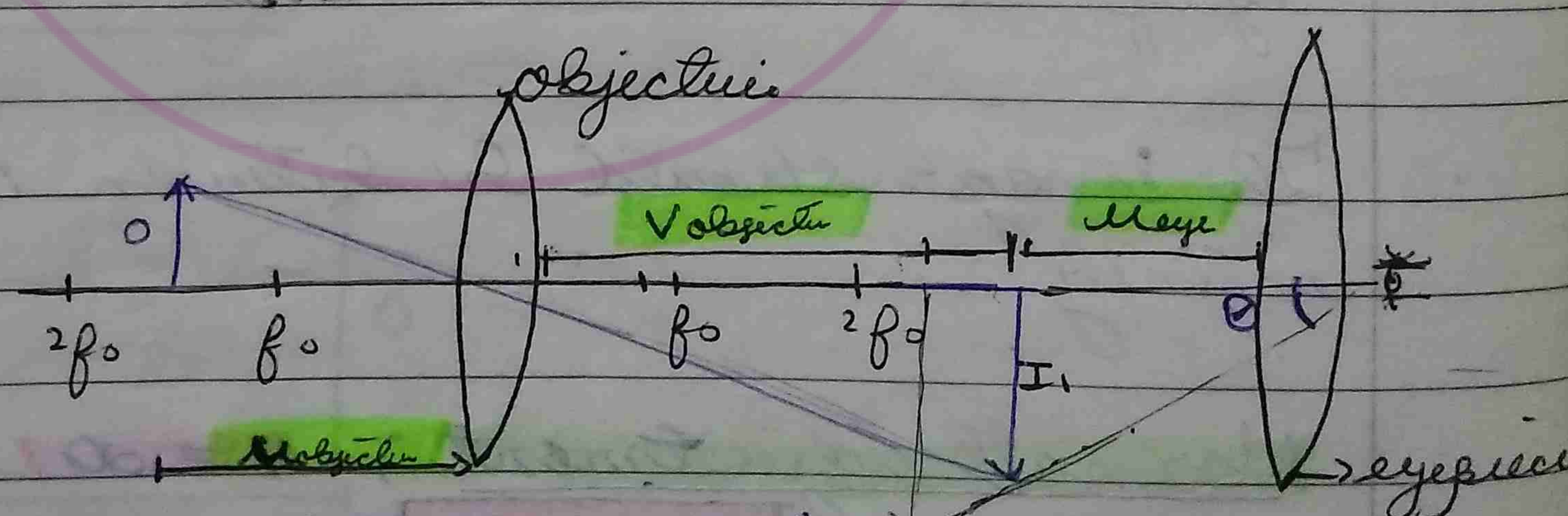
$$(AM)_{min} = \frac{D}{f} \rightarrow \text{Relaxed eye}$$

$$(AM)_{max} = 1 + \frac{D}{f} \rightarrow \text{strained eye / LDDV}$$

Compound Microscope: Eyepiece is larger than objective



$$\theta_0 = \frac{O}{D}$$



• Intermediate image (I_1), $= R$, I, Magn

Observer

$\theta = -\frac{I_1}{M_e}$ LDDV

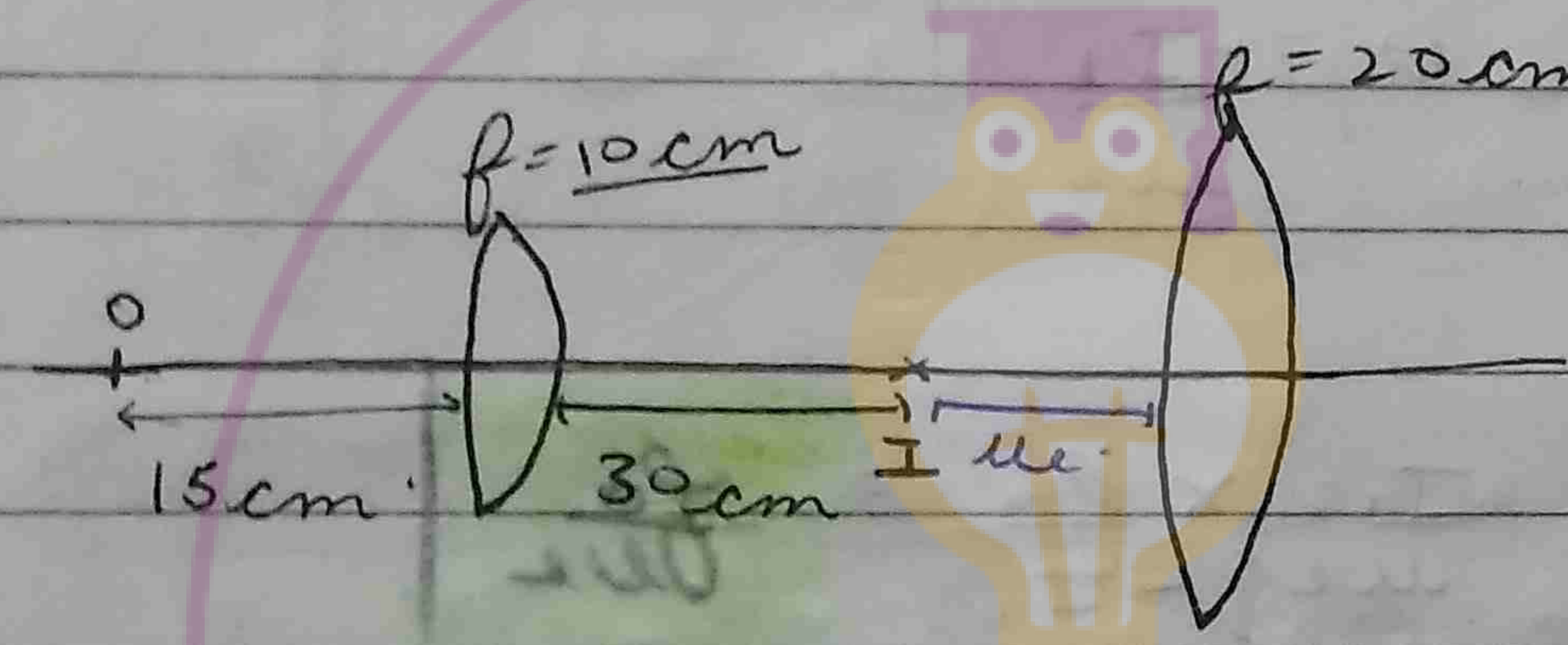
$AM = \frac{\theta}{\theta_0} = \frac{-I_1}{u_e} \frac{D}{O} \Rightarrow AM = \frac{-I_1}{O} \frac{D}{u_e}$

$\Rightarrow$ $AM = -\frac{v_o}{u_o} \frac{D}{u_e}$ (v_o → v objective)

$(AM)_{min} = \frac{-v_o}{u_o} \frac{D}{f_e}$

$AM(max) = \frac{-v_o}{u_o}\left(1 + \frac{D}{f_{e}}\right)$

e.x



$L = v_o + f_e$

- find AM and length of microscope so that final image is at 30 cm

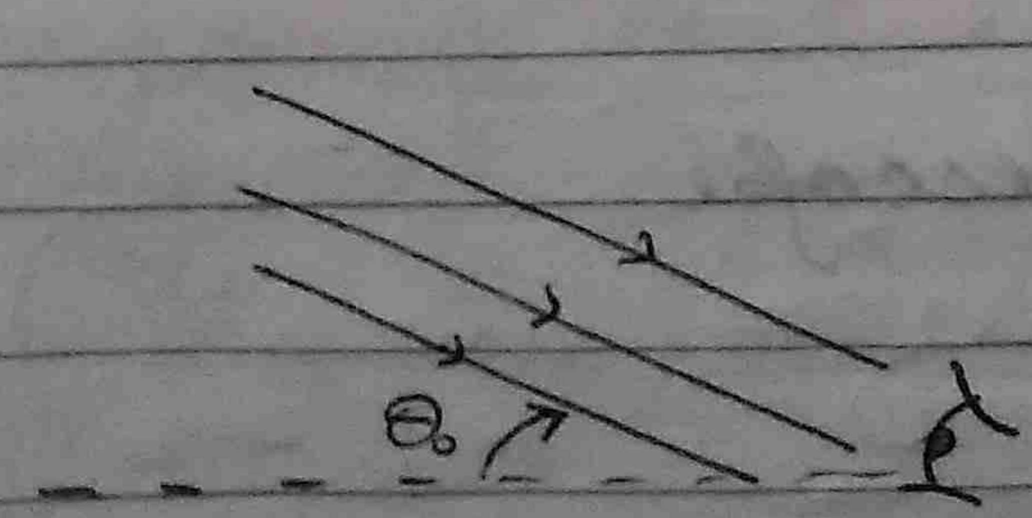
Eye piece $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$\rightarrow \frac{1}{-30} - \frac{1}{-u_e} = \frac{1}{+20} \Rightarrow \frac{1}{u_e} = \frac{1}{20} + \frac{1}{30} = \frac{1}{u_e} = \frac{50}{600} = \frac{1}{12}$

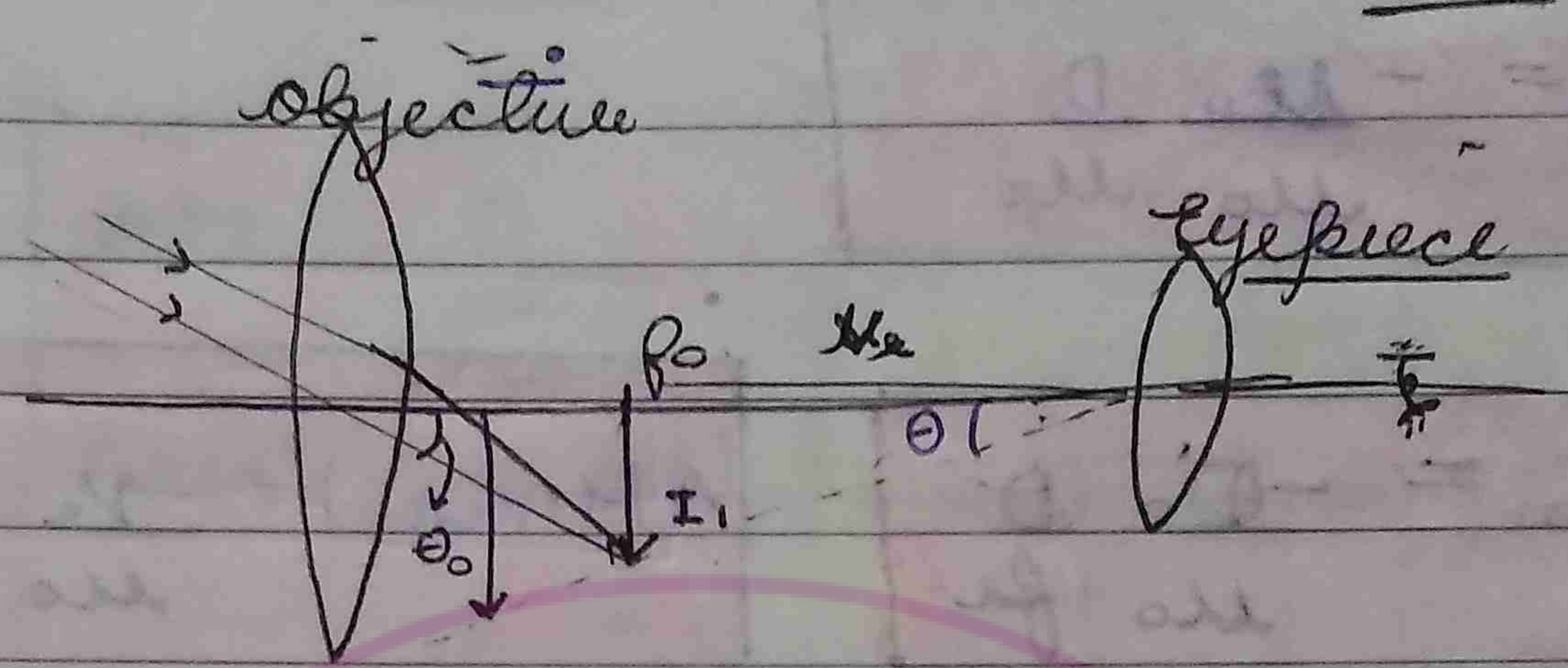
$u_e = 12$

$AM = -\frac{v_o}{u_o} \frac{D}{u_e} = \frac{-30}{15} \frac{25}{12} = -5x$

- Telescope



- **Astronomical Telescope : Objective larger than eyepiece**



$\theta_o = \frac{I_1}{f}$ $\quad \theta = \frac{-I_1}{u_e}$

$$AM = \frac{\theta}{\theta_o} = \frac{I_1}{u_e} \times \frac{f_o}{-I_1} = \frac{-f_o}{u_e}$$

- Normal adjustment

$u_e = f_e$; $AM_{min} = \frac{-f_o}{f_e}$

- when image is at LDDV

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

$$AM_{max} = -\frac{f_o}{u_e} = -f_o\left[\frac{1}{f_e} + \frac{1}{D}\right]$$

- Terrestrial Telescope.

★ 3 lenses used in terrestrial microscope

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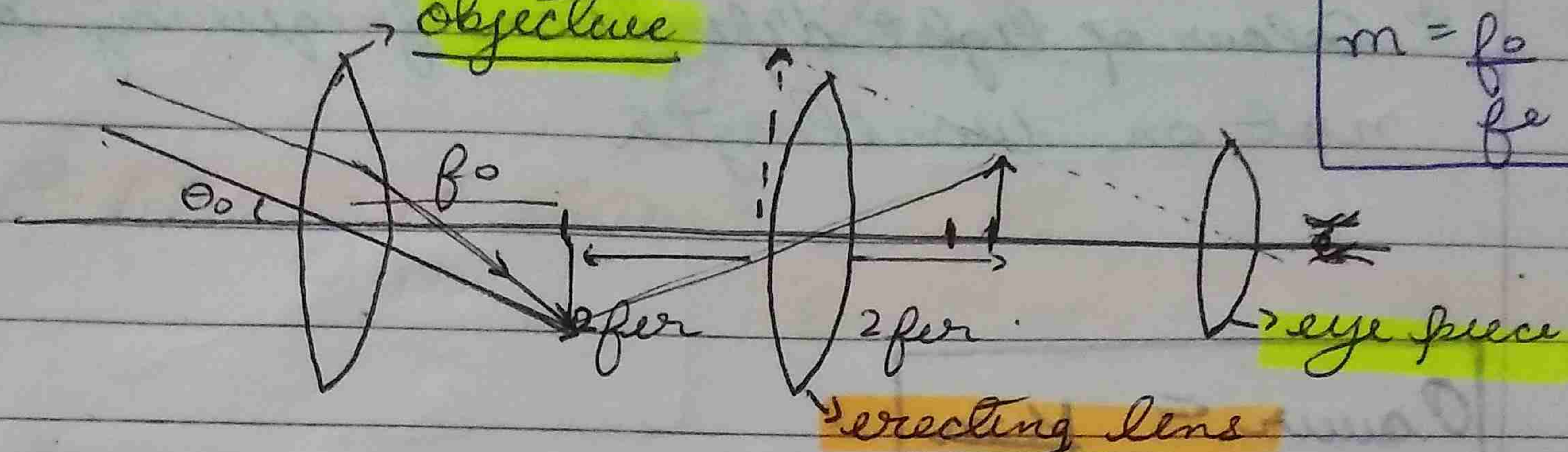

$m = \frac{f_0}{f_e}$

$L_{\text{terrestrial telescope}} = L_{\text{astronomical telescope}} + 4f_{er}$

- Eye
- Defect of Vision

(i) Normal Range 25 to ∞ [for normal person : near point = 25, far point = ∞]

far sightedness → Hypermetropia

convex lens

(ii) short sightedness → Myopia

Concave lens

e.g. far point = 3m

★ Image of lens is object of eye.

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-3} - \frac{1}{-\infty} = \frac{1}{f_l} \Rightarrow f_l = -3m$

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☆ Colour of light depends on frequency and not on wavelength

$$\boxed{\text{Power} = \frac{\mu_m}{f}}$$

$$\text{Power} = (\mu_L - \mu_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{\mu_m}{f} = (\mu_L - \mu_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{\mu_L}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

☆ Power

$$= \frac{P_1}{P_2} = \frac{(\mu_L - \mu_{M_1})}{(\mu_L - \mu_{M_2})}$$

☆ focal length ⇒ $\frac{f_1}{f_2} = \frac{(\mu_{L/2} - 1)}{(\mu_{2/1} - 1)}$

Longitudinal Chromatic aberration $= f_{red} - f_{violet} = \omega f = \left(\frac{\mu_v - \mu_r}{\mu_{ye} - 1}\right)(f_y)$ (f: focal length of lens)

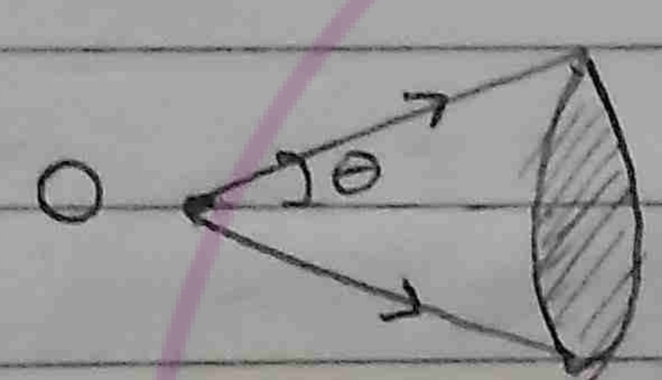
$$\boxed{d = \omega_1 f_1 + \omega_2 f_2}$$ If $\omega_1 = \omega_2$ ⇒ $d = f_1 + f_2$

$R.P \propto \sqrt{V}$

Resolving Power

Resolving Power = $\dfrac{1}{\text{limit of resolution}}$

- Limit of resolution of a normal human eye = 1 minute $= \dfrac{1^\circ}{60} = \dfrac{1}{60} \times \dfrac{\pi}{180}$ (radian)



μ = Refractive index of the medium in which the object has been kept

$\dfrac{2\mu \sin\theta}{1.22\lambda}$

θ = half the angle formed by the object on objective lens.

λ = wavelength of light used.

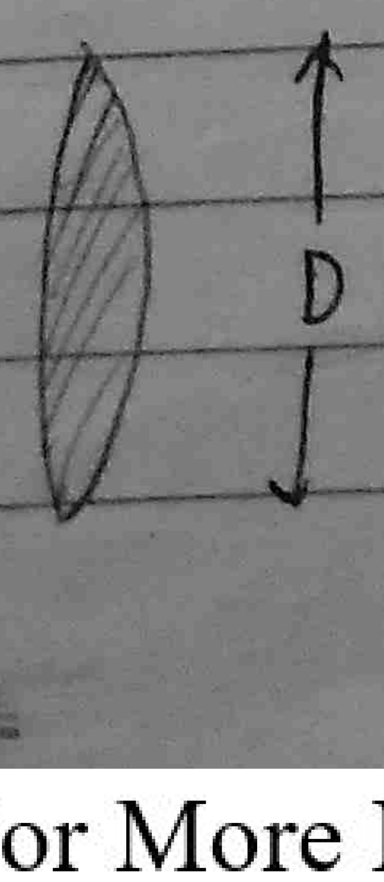
- Electron Microscope

$\lambda = \dfrac{h}{\sqrt{2meV}}$

$R.P = \propto \dfrac{1}{\lambda} \propto \sqrt{\text{voltage}}$

★★ For electron microscope its R.P is directly proportional to $\sqrt{V}$

Resolving Power of a Telescope



$\dfrac{D}{1.22\lambda}$

D = diameter of the objective

λ = wavelength of light used

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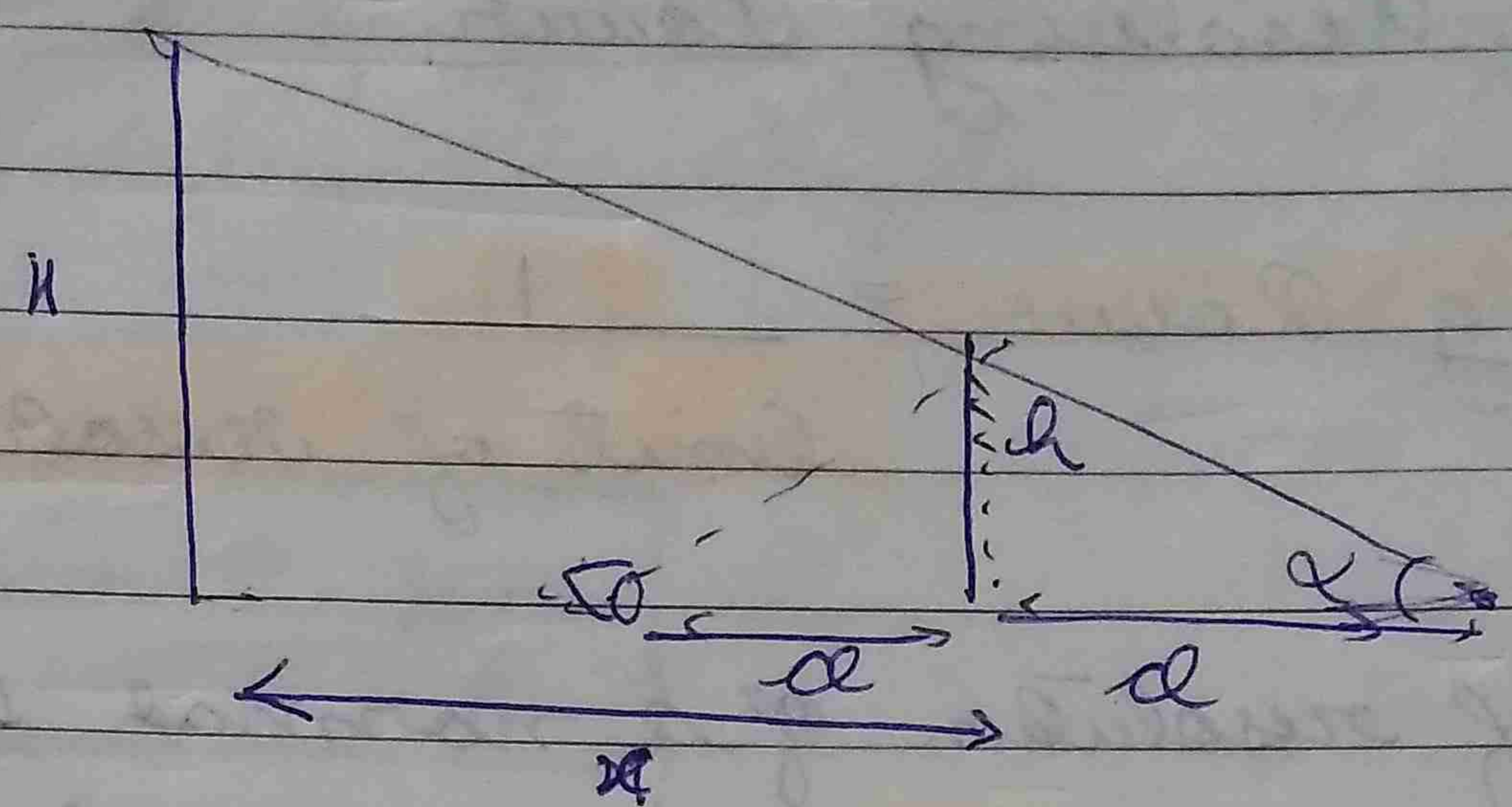


$$\tan \alpha = \frac{H}{x + d} = \frac{h}{d}$$

$$h = \frac{H d}{x + d} \Rightarrow \frac{H}{\frac{x}{d} + 1}$$

h ↓ height of mirror required

- ## Coefficient of coupling

If all the flux due to '2' passes through the coil 1, flux linkage is maximum.

$\phi_{11} = \phi_{12}$ $\phi_{22} = \phi_{21}$

$L_1 I_1 = M I_2$ (i) $L_2 I_2 = M I_1$ (ii)

$\Rightarrow L_1 L_2 I_1 I_2 = M^2 I_1 I_2$

$\Rightarrow$ $\boxed{M_{MAX} = \sqrt{L_1 L_2}}$ Mutual inductance when flux linkage is maximum

- ## Coefficient of coupling

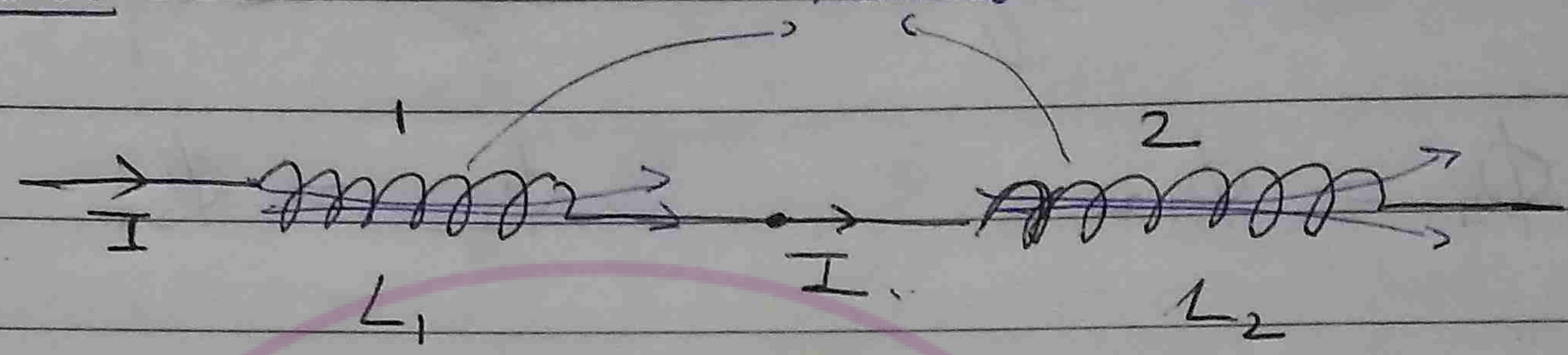
$K = \dfrac{\text{Mutual Inductance b/w two coils}}{\text{Maximum possible mutual inductance.}}$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

•

- **Equivalent inductance through mutually coupled coils**

- **Series** mutual inductance = M.



$$\phi_1 = L_1 I_1 + M I_2$$
$$= L_1 I + M I$$

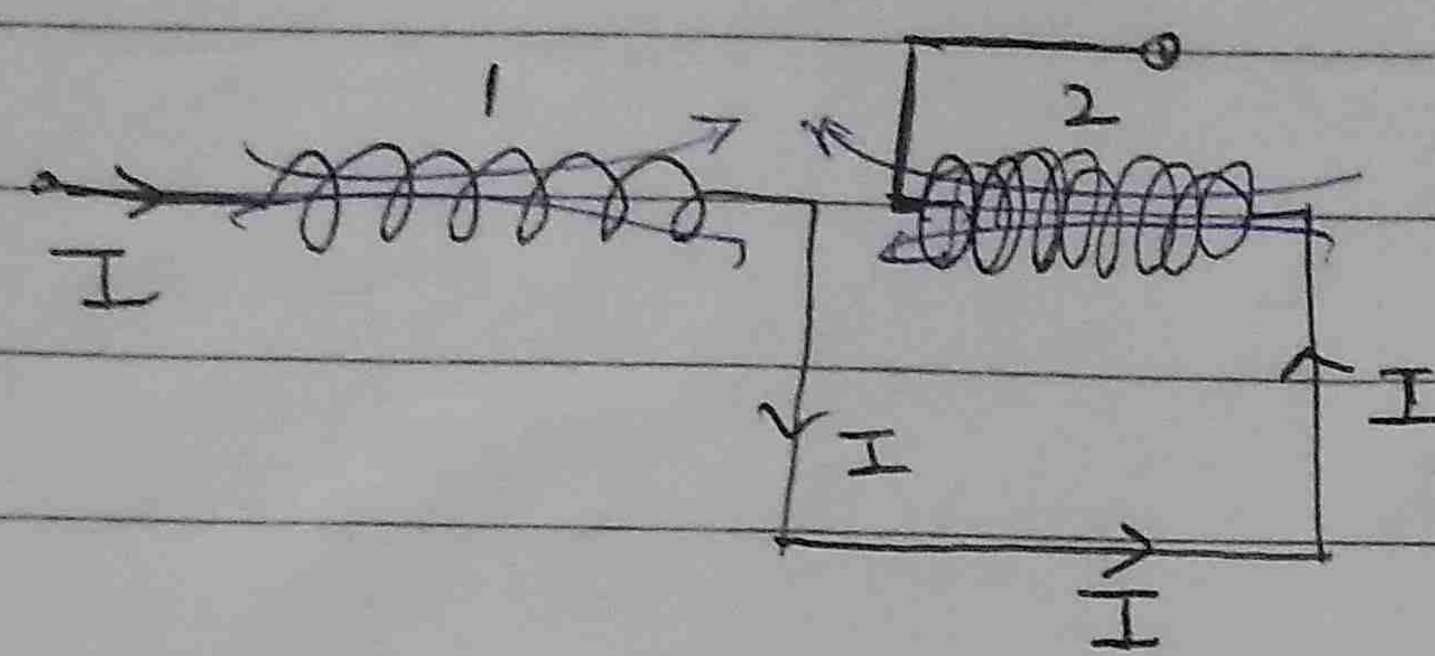
$$V_1 = -\frac{d\phi_1}{dt} = -L_1 \frac{dI}{dt} - M\frac{dI}{dt}$$

$$\phi_2 = L_2 I + M I$$
$$V_2 = -L_2 \frac{dI}{dt} - M\frac{dI}{dt}$$

$$V = V_1 + V_2$$

$$V = -L_{eq}\frac{dI}{dt} = \left(-L_1\frac{dI}{dt} - M\frac{dI}{dt}\right) + \left(-L_2\frac{dI}{dt} - M\frac{dI}{dt}\right)$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2 + 2M}$$



$\phi_1 = L_1 I_1 - M I_2 = L_1 I = M I$

$\Rightarrow V_1 = -\frac{d\phi}{dt} = -L_1 \frac{dI}{dt} + M \frac{dI}{dt}$

$V_2 = -L_2 \frac{dI}{dt} + M \frac{dI}{dt}$

$-L_{eq} \frac{dI}{dt} = \left(-L_1 \frac{dI}{dt} + M \frac{dI}{dt}\right) + \left(-L_2 \frac{dI}{dt} + M \frac{dI}{dt}\right)$

$\Rightarrow -L_{eq} = -(L_1 + L_2) + 2M$

$\boxed{L_{eq} = L_1 + L_2 - 2M}$

General formula $L_{eq} = \boxed{L_1 + L_2 \pm 2M}$