Handwritten Notes
On
Nuclear Physics

'NUCLAR PHYSICS'

|1| → Nucleus

[A] → Nucleon = Proton + Neutron

* Mass $= M_p = 1.67 \times 10^{-27}$ kgm $\approx$ [1.007 amu]
* $M_n = 1.68 \times 10^{-27}$ kgm = [1.008 amu]

* $q_p = +e = 1.6 \times 10^{-19}$ cb $= 4.8 \times 10^{-10}$ esu
* [$q_n = 0$]

* Largest unit of charge → Faraday.
* S.I → cb
* c.g.s → s/cb/esu/franklin
* practical unit → abcb, Farady

* Spin quantum no = [$\pm \frac{1}{2}$]*
* Spin angular Momentum = [$\pm \frac{1}{2} \left(\frac{h}{2\pi}\right)$]*
* Formation of proton & neutron in nucleus is explain by quark particle.

* Quark particle →
  - up quark $= q_{up} = \pm \frac{2e}{3}$
  - Down quark $= q_{down} = \pm \frac{e}{3}$

  → * [$m_{down} > m_{up}$]

* Proton = 2 (upquark) + 1 (downquark) $= 2\left(+\frac{2e}{3}\right) + 1\left(\frac{-e}{3}\right) =$ [+e]

* Neuron = 1 (upquark) + 2 (downquark) $= 1\left(+\frac{2e}{3}\right) + 2\left(-e/3\right) =$ [0]

NOTE → Quark partide does not exist in free state that's why quanta of charge still [e].

Imp *** NOTE → * In a stable nucleus ~~protons~~ ~~nucle~~ neutron both are stable particle but in a unstable nucleus both are unstable particle. & outside nucleus proton is stable particle & neutron is unstable.

AIIMS * outside nucleus neutron convert in H^+ & emit β-particle & Antineutrino & behave like radioactive substance of half life 12.5 mint.

$$_0n^1 \longrightarrow {_1H^1} + {_{-1}e^{e}}(\bar{\beta}) + \bar{\nu} \text{ (Antinutrino)}$$

this Rkn. does not obey spin angular Momentum conservation that's why pauli assume another particle is also emmited with β-particle whose spin quantum no. ±1/2, charge & mass no = 0, than particle is called Neutrino & Antineutrino.

* Most unstable particle is → Neutron

|2| → Rebresentation of nucleus →

$_ZX^A$ →
- Z = no. of proton
- A ⇒ No. of nucleon = $N_p + N_n$
- N_n ⇒ A − Z.

|3| → Types of Nucleus →

|a| → Isotropic nucleus → $Z = N_p$ = same, A = different. Ex → $_1H^1, {_1H^2}, {_1H^3}$, No. of proton different

|b| → Isobaric Nucleus → No. of neucleon same = A, Ex → $_6C^{14}, {_7N^{14}}$

|c| → Isotonic/Isoneutronic → same no. of Neutron = same = A − Z
Ex → $_6C^{13}$ $_7N^{14}$ N = 13 − 6 = 7, N = 14 − 7 = 7] same (N)

** |d| → Mirror Nencleus → If no. of proton & neutron is opposite.
$_{Z_1}Y_1^{A_1}$ ⫽ $_{Z_2}Y_2^{A_2}$

NOTE → Those isobar which has opposite & neuron. called mirror nucleus.

|e| → Isodiapher nucleus → difference of Neutron & proton is same.
[Nn − Np = same]

Eg → $_{92}U^{235} \longrightarrow Th^{231}$

p = 92, N = 143, N−P = 51 | p = 90, N = 141, N−P = 51

** |f| → Isoster nucleus → Having same no. of atom & same no. of electron.
Eg → N_2 (14) & CO (14) & CO_2 (22) & N_2O (22)

AIEEE 2005
|g| → Isoelectronic → same no. of e^-.
Eg → O^{-2}, F^-, Ne, Na^+, Mg^{+2}, Al^{+3} ($10e^-$)

|4| → Size of nucleus →

|a| → Radius → Radius of Nucleus ∝ to cube root of Mass no.

* $r_{Nu} \propto (A)^{1/3}$ * $r_{Nu} = r_0 (A)^{1/3}$

* r_0 = Rutherford Nuclear const
$= 1.2 \times 10^{-15}$ m
= 1.2 fermi

|b| → Volume (or Size) → Nucleus behave as non-conducting solid sphere.

$V_{Nu} = \frac{4}{3}\pi r_{Nu}^3$
$= \frac{4}{3}\pi (r_0 A^{1/3})^3$
$V_{Nu} = \frac{4}{3}\pi r_0^3 A \propto A$

* A ↑, r_{Nu} ↑, V_{Nu} ↑ (size ↑)
* Exp. it has been observed that volume of nucleus ∝ mass no.
$\frac{4}{3}\pi R^3 \propto A$

|c| → Mass →

$M_{Nucleus} = M_{Neucleon}$
$= \Sigma M_p + \Sigma M_n$

$M_{Nu} = Zm_p + (A-Z)m_n$
* $m_p = m_n = 1$ amu
$M_{Nu} = Am_p$ = 'A' amu

* $_Z X^A$ — mass of 1 atom or, mass of Nucleus → 'A' amu
$_Z X^A$ — At weight or, mass of 1 mole → 'A' gm

* |d| → Density →

density (d) = $\frac{\text{mass of Nucleus}}{\text{volume of Nucleus}} = \frac{(1.67 \times 10^{-27}) A}{\frac{4}{3}\pi R_0^3 A}$

* $d = 2.3 \times 10^{17}$ kg/m³

*** NOTE → * Density of nucleus is same for all nuclei. (Independent from mass no.)
* As nuclei do not have sharp boundry, the volume of R_0 or r_0 may also have slight deviation from the given volume.
* Density of nucleus is very high while that of atom is very small, as most of the part of atom is empty.

AIR
Nuclear spin → similar to e⁻s nucleon also, rotate about their own axis & they posses a permanent angular momentum & hence the comple Nucleus has some Angular momentum.

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Mass Energy Relation

$$* \boxed{E = mc^2}$$

- E: Energy corresponding to Mass
- m: part of Mass which is convert into Energy.
- c: velo. of light

NOTE → A/c to Einstein system release energy its mass decrese & system absorb Energy its mass increase.

Energy corresponding to Mass –

$$E = mc^2 = m_{amu}(931)MeV = m_{kgm}(9\times10^{16})J \;\; **$$

* 1 kg → $E = 9\times10^{16}J$
* → 1 a.m.u → E = 931 MeV

(a) → Rest Mass Energy [R.M.E] (E_0) →

R.M.E →
- proton, $m_{op} = 1$ a.m.u ⇒ E_{op} ⇒ $\boxed{931 MeV}$ *
- Neutron, $M_{on} = M_{op} = 1$ amu ⇒ E_{on} ⇒ $\boxed{931 MeV}$ *
- Electron, $M_{oe} = 9.1\times10^{-31} = \frac{m_p}{1836} = \frac{1}{1836}$ amu ⇒ E_{oe} ⇒ $\boxed{0.51 MeV}$ *
- positron, $q_{po} = e$, $m_{po} = m_e$ = R.M.E_{po} = R.M.E ⇒ $\boxed{0.51 MeV}$ *
- photon, m_{oph} ⇒ R.M.E_{ph} = $\boxed{0}$ *

(b) → Relative Energy (ER) → Total energy. *

- $\boxed{E_R = m_0c^2\left(1-\frac{v^2}{c^2}\right)^{1/2}}$ *
- $\boxed{E_R = m_0c^2 + \frac{1}{2}m_0v^2}$
- $\boxed{E_R = R.M.E + K.E}$
- $\boxed{E_R > R.M.E}$

$\boxed{E_R = m_Rc^2 \quad m_R > m_0 \quad E_R > E_0}$

K.E के Formula में हमेशा Rest Mass लेता है Aur KVi नहीं लगाते A Relativity theory

Basic process/Defination

(i) → Pair production → High energy photon is absorb by nucleus & configuration* of nucleus remain unchange & nucleus emit pair of electron & positron.

* only done by photon not by neutron, proton etc.

$$* \boxed{E_{ph} = E_{e^-} + E_{po}} = R.ME_{\bar{e}} + KE_E + RME_{po} + KE_{po}$$

$$* \boxed{E_{ph} = 1.02\,MeV + K.E_{e^-} + K.E_{po}}$$

NOTE →
* pair production energy convert in mass but mass energy equivalent is conserved.
* Minimum required energy photon for pair production is 1.02 MeV
* In a nuclear Rxn or, nuclear event charge, mass no, linear momentum, angular momentum, spin quantum no, spin angular momentum & Mass energy Remain conserved means, phy all conservation law is followed.
* $K.E_e = K.E_{po} = 0$ ⇒ $(E_{ph})_{min} = 1.02$ MeV (γ or photon)
* $E_{ph} < 1.02$ MeV ⇒ * pair production not posible.
  * P.E.E, * Crompton Effect] → possible
* $E_{ph} > 1.02$ MeV ⇒ * P.P, * P.E.E, * C.E] → possible.
* P.E.E & C.E atomic event but pair production is nuclear event.

|ii| → **Pair Anhalation** (~~production~~) → Electron & positron come come closer & anhaliate each other release energy in form of Radiation.

AIIMS * In pair anhaliation two γ-photon are emitted & move in a opposite direction (fallow linear momentum conservation).

* In pair anhaliation mass convert into Energy.
* It is Reverce process of pair production.

$E_{e^-} + E_{po} = 2E_{ph} \Rightarrow 0.51 + KE_{e^-} + 0.51 + KE_{po} = 2E_{ph}$

* $$E_{ph} = 0.51\,\text{MeV} + \left(\frac{K.E_{e^-} + K.E_{po}}{2}\right)$$

$K.E_{e^-} = KE_{ph} \Rightarrow$ * $(E_{ph})_{min} = 0.51\,\text{MeV}$

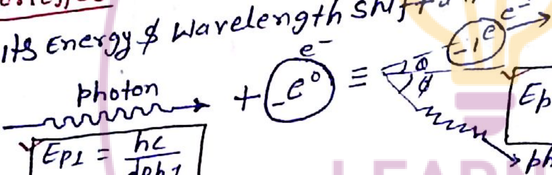
↑ For single For two ⇒ 1.02 MeV

* $$(\lambda_{ph})_{max} = \frac{12400}{0.51 \times 10^6} \text{Å}$$

* Min release of single photon in pair anhaliation is 0.51 MeV & Min release energy 1.02 MeV.

|iii| → **P.E.E** → Photon transfer its 100% energy to single e^- (perfectally Inelastic collision) If it is sufficient to remove the electron than e^- come from metal surface effect is called PEE.

|iv| → **Compton Effect** : → Photon callide elastically with e^- & transfered sum part of its Energy & wavelength shift takes place.



$E_{p1} = \frac{hc}{\lambda_{ph1}}$ $E_{ph} = \frac{hc}{\lambda_{ph2}}$

$E_{ph2} < E_{ph1}$
$\lambda_{ph2} > \lambda_{ph1}$

* compton shift or, λ-shift.

* $$\Delta\lambda = \lambda_{ph2} - \lambda_{ph1} = \frac{h}{m_0 c}(1 - \cos\phi)$$

|v| → **Mass defect (Δm)** → Mass of nucleus is slightly less than from mass of nucleon.

$\Delta m = M_{nucleon} - M_{nucleus}$

* $$\Delta m = Zm_p + (A-Z)m_n - M_{Nu}$$

|vi| → **Binding Energy (B.E)** → Required energy to bound the nucleon in nucleus or, Remove the nucleon from nucleus. (Energy crosponding to mass defect.)

* $$B.E = \Delta mc^2 \simeq \Delta m(931)\,\text{MeV}$$ (Δm in a.m.u)

* $$B.E = [Zm_p + (A-Z)m_n - M_{Nu}]c^2$$

NOTE → B.E of free nucleon & Hydron nucleus is zero.

|vii| → **Binding Energy per nucleon (B.E/A)** → Averge energy required to remove the single nucleon from nucleus.

** $$\frac{B.E}{A} = \frac{\Delta mc^2}{A} = \frac{[Zm_p + (A-Z)m_n - M_{nu}]c^2}{A}$$

NOTE → * Stability of Nucleus $\propto \left(\frac{B.E}{A}\right)$

* $\left(\frac{B.E}{A}\right)_{Max} = Fe^{56} = 8.8 \frac{MeV}{Nucleus}$
* Most stable nucleus in P.T ⇒ Fe^{56}
* " " Element " " " ⇒ He
* " " " Radioactive series
* " " " "] ⇒ Pb/Bi

or, Last e^- of Radioactive series

* Stability of Nucleus → $Fe^{56} > O^{16} > U^{235} > He^4 > {}_1H^1$

**** Binding Energy curve

curve b/w B.E per nucleon & corresponding no. of nucleon.

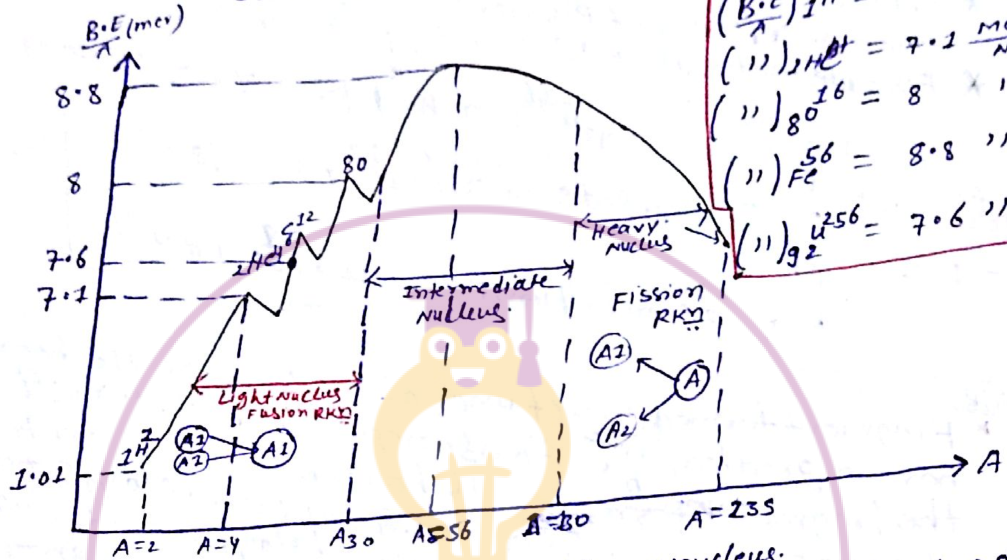
$\left(\frac{B.E}{A}\right)_{1}H^2 = 0.01 \frac{MeV}{Nucleon}$

$(\,'' \,)_2He^4 = 7.1 \frac{MeV}{Nucleon}$

$(\,'' \,)_8O^{16} = 8$ "

$(\,'' \,)_{Fe}^{56} = 8.8$ "

$(\,'' \,)_{92}U^{256} = 7.6$ "



* $\left(\frac{B.E}{A}\right)$ of Intermediate nucleus is more than & heavy nucleus.

Imp * Heavy nucleus split in two or, more than two comparable mass nucleus & release Energy reaction is called fission RKn.

* In presence of High temp (10^7K) & High pressure (more than 10^6 atm) light nucleus come closer & form a comparable mass nucleus & Release energy rkn is fusion RKn.

Imp * In both rkn $\left(\frac{B.E}{A}\right)$ of product nucleus is more than Reactant & energy Relase in both RKn.
product of fission RKn is intermediate & of fusion...

* Fusion RKn is possible in light nucleus & product of nucleus is Intermediate & Heavy.
light nucleus.

* Fission RKn is possible in light nucleus & product of nucleus is Intermediate & Heavy.

Imp * For any stable Nucleus → Rest Mass of all its constituents > Rest mass of nucleus.

***** Release Energy in a Nuclear Reaction

Case-I → If Mass of product & Reactant are given. →

* Reactant ⟶ Product + Energy.
* change in mass $\Delta m = M_{Rea} - M_{prod}$.
* $E = \Delta mc^2 = \Delta m (931)$ MeV (Δm in a.m.u)

Case-II → If Rest mass Energy is given →

Reactant ⟶ Product + Energy

$R.M.E_R = R.M.E + E$

$R.M.E_R - R.M.E_P$

$E = RME_R - R.M.E_P$

* $R.M.E_R > R.M.E_P \Rightarrow E = \oplus ve$ (Release)
* $R.M.E_R < R.M.E_P \Rightarrow E = \ominus ve$ (absorb)

Case-III → If B·E is given → $B\cdot E = -R\cdot M\cdot E$

$E = B\cdot E_p - B\cdot E_R$

* $B\cdot E_p > B\cdot E_R \Rightarrow E = \oplus ve$ (Release)
* $B\cdot E_p < B\cdot E_R \Rightarrow E = \ominus ve$ (absorb)
* $B\cdot E_p = B\cdot E_R \Rightarrow E = 0$

1 day = 86400 sec
1 yr = $\pi \times 10^7$ sec

Nuclear Reaction

|1| → Fission Reaction → * Release energy per fission 200 MeV.
* Release Energy per nucleon per fission $= \frac{200}{235} = 0.8 \frac{MeV}{Neucleon}$
* Approx 1% of mass convert in energy remaning 99.9% part convert in mass of Ba, Kr & nucleon & its K·E.
* Fisiable Material → $U^{235}, U^{238}, Pu^{239}, Th^{222}$

$_0n^1 + {}_{92}U^{235} \longrightarrow {}_{92}U^{236*} \longrightarrow Ba + Kr + 3({}_0n^1) + \text{Energy}.$

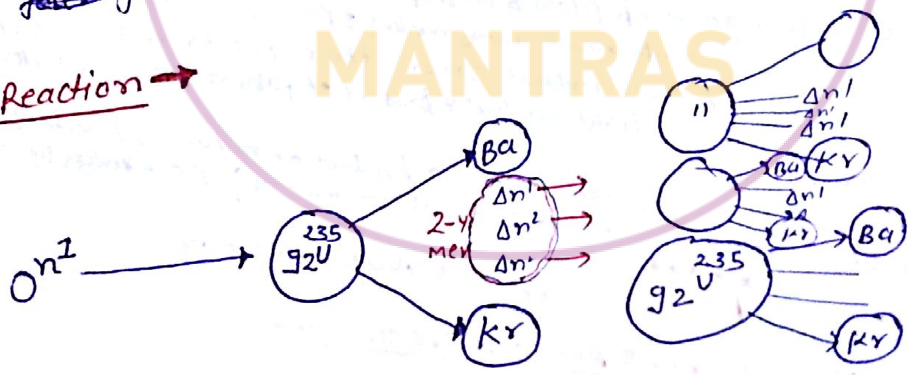
** # $P = 1\,KW$ — Per fission → 200 MeV (Release energy) → Fission Rate ⇒ 3.1×10^{13} fission/sec
→ Mass consumption Rate $= 12 \times 10^{-12}$ kgm/sec

** 2016 AIIMS ** A/R

NOTE → Fragment formed are of unequal masses bcoz the heavy nuclei have a greater $\frac{n}{p}$** ratio as compaired to light nucli., thus the fragment formed will have more neutron to maintain this ratio. genrally fragment form one belongs to family with heigher $\frac{n}{p}$ value & another belong to moderate $\frac{n}{p}$ value, few extra neutron are emitted as soon as fragment are formed.

** *** AIIMS

Chain Reaction →



Difficulty in chain Reaction

** |1| → In a natural urenium ratio of urenium isotopes $U^{233} : U^{235} : U^{238}$ is $0.3 : 0.7 : 99$ & Required energy for fission of U^{238} & U^{235} Resp. 7 - 8 MeV & less than 1eV but energy of secondry neutron is 2 - 4 MeV.**

Probability of neutron absorbtion is 99% & due to hight K·E of neutron move with very high velocity (10^6 m/sec)* & leakout from fissable material.

Removal Action →

|a| → To increase probability of collision with U^{235} concentration ~~condition~~ of U^{235} nucleus is increrease (Inrch process). (concentration of U^{235} in inrich uranium is 3% (Max)).

** AIIMS 2015

|b| → To decrease energy of neutron, substance moderator are used.
* D_2O (Best Moderator), H_2O, BaO, parafin wax (Hydrocarbon).
  - Why? → When comparable masses body collide elastically max energy transfer takes place. Mass of deutron is in order of mass of neutron thats why it absorb max energy of neutron.

!! *** But, H_2O is not best Moderator?? !!

Neutron (m)	Moderator (m_2)	$\frac{\Delta K.E_1}{K.E_1}$	Remain Energy
1 a.m.u	$_1H^1$ = 1 amu	→ 100%	0 ✓
1 a.m.u	$_1H^2$ = 2 a.m.u	→ $\frac{4(1)(2)}{(1+2)^2}$ = 8/9 * (2003 AIIMS)	$\frac{1}{9}$ ✓

that's why H_2O is not a best Moderator!!

|c| → To maintain minimum 1 neutron in a fissable material size of fissable fuel is degien as a critical size & critical mass (10 kg).

AIR * → U^{238} does not participate in nuclear chain reaction bcoz compound nucleus forms convert it self in plutonium.

$$ {}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{92}U^{236} \longrightarrow {}_{56}Ba^{246} + {}_{36}Kr^{92} + 3\,{}_0n^1 $$

or,

$$ \longrightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 $$

- ${}_0n^1$: Slow moving neutron, K.E = 0.04 eV {also called thermal neutron}
- ${}_{92}U^{236}$: compound nucleus in higher quantum state.

Neutron production factor (K) / Reproduction factor (r)

Ratio of secondry active neutron to pri. active neutron.

- Neuclear bom or, atom bomb ← Eg ← * If K > 1 ⇒ uncondroled chain Reaction ⇒ m > critical mass.
- Nuclear Reactor ← Eg ← * If K = 1 ⇒ controled chain Rxn ⇒ m = critical mass.
- * If K < 1 ⇒ Rate of chain Reaction decrease ⇒ m < critical mass.

Imp NOTE →
* If all neutron are active than chain Reaction is G.P.
* Number of nutron produce after 'n' heat = $N^{th} = 3^N$
* Number of nutron produce Rate & absorbtion Rate ∝ volume of fissable material.
* Nutron production Rate ∝ surface.
* Nuctron leakout Rate ∝ surface.

Controled chain Reaction

Eg → Nuclear Reactor.

* control Rod → cadmium, graphite.
* Moderators → D_2O, H_2O, Ba, Parafin.
* Fusable Material → U^{235}, U^{238}, Pu^{239}, Th^{222}
* Pb → Best Radiation absorber.

Uncondrolled chain Reaction

Eg → Nuclear bomb

* Fusable material → H, He, Be, B, C, N.

Breader Reactor → Fast nutrongenerate fuel aswell as energy.

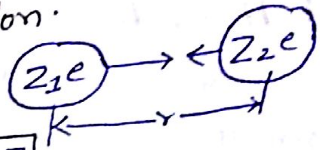
Fusion Reaction → Presence of High temp. & High pressure light nucleus come closer & form comparable mass nucleus.

(Fusion Reaction → constructive use → sun & star; Distructive use → Nuclear bomb)

|i| → Release Energy per fusion ⇒ 28 MeV

*|ii| → Per Nucleon → Fission ⇒ 0·8 MeV/Nucleon ; Fusion ⇒ 7 MeV/Nucleon.

|iii| → condition of fusion Reaction.



* $T \cdot E \geq E \cdot P \cdot E$

** $T \geq \frac{1}{3K}\left[\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}\right]$

or, stability of nucleus. Unstability of nucleus :→ A/c to nuclear force In a nucleus electrostatic Repulsion force b/w proton & proton is 10^{34} times more than gravitational attractive force but all nucleus are not unstable it means another attractive force is also present b/w nucleon called nuclear force. (which are 50-60 times stronger than electrostatic forces.

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Properties of Nuclear force →

|i| → Generally it is attractive nature force & act b/w nucleon inside a nucleus.

A/R |ii| → They *donot depend on charge, hence nuclear force b/w any pair of neutron & proton is same i.e n-n or, n-p or, p-p but net force b/w them $F_{n-n} = F_{n-p} > F_{p-p}$ A/R → due to Electrostatic Repulsion.

* F_{p-p} → F_C → ⊕ve ; F_g, F_N → ⊖ve

$F_{p-n} \simeq F_{n-n}$ → F_g, F_n → ⊖ve.

* donot obey law of superposition

** AIIMS 2016 |iii| → It is non central force.

A → |iv| → It is non-conservative force.

R → |v| → Nuclear force depend on distance b/w neucleon but it does not obey Invese square law. or, Exponential law.

2016 JIPMER |vi| → It is strongest force in a natural forces but its Range is very short. (Range = 10^{-14} m)*

**|vii| → Unlike gravitational or, electrostatic force force they donot have any formula for calculation.

**|viii| → When the distance b/w nucleons is less than 0·5 fermi, they become strongally Repulsive.

*|ix| → Net force b/w proton-proton in stable nucleus (Inside) is attractive & outside nucleus is Repulsive.

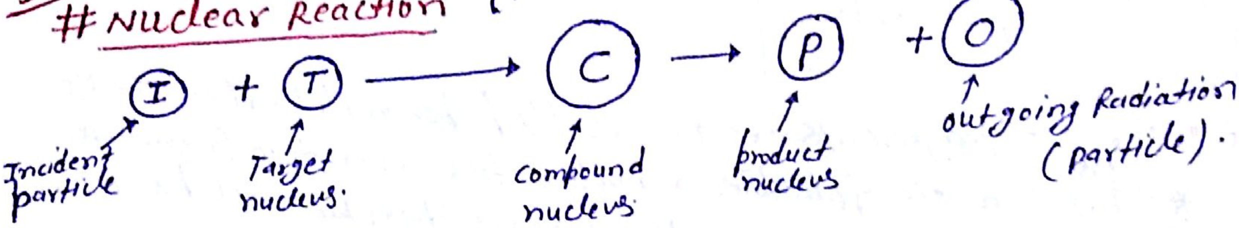
A/C to Neutron & proton Ratio

$N \geq Z \Rightarrow$ Nucleus stable
$N < Z \Rightarrow$ Nucleus unstable.

A/C to ...

Even-Even > even-odd > odd-odd

IIT
Nuclear Reaction (No role of e^- in this Reaction)



Q - value of a Nuclear Reaction -

$$\underset{K \cdot E_i}{I} + \underset{(Rest)}{T} \longrightarrow \underset{(K \cdot E_p)}{P} + \underset{(KE_o)}{O}$$

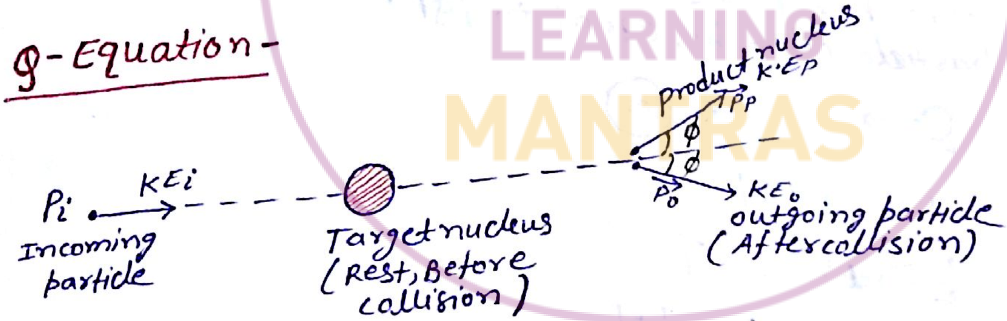
$$Q = K \cdot E_p + KE_o - KE_i$$

If mass of $I, T, P, \& O$ are $m_i, m_t, m_p \& m_o$, then from conservation of Energy.

$$Q = (\Delta m)c^2$$

Imp
* If $m_i + m_t > m_p + m_o \Rightarrow Q > 0 \Rightarrow$ Exergonic Reaction
* If $m_i + m_t < m_p + m_o \Rightarrow Q < 0 \Rightarrow$ Endergonic Reaction.

Q-Equation-



From Momentum conservation

$$\vec{P_i} = \vec{P_p} + \vec{P_o}$$
$$\vec{P_p} = \vec{P_i} - \vec{P_o}$$
$$P_p^2 = P_i^2 + P_o^2 - 2\vec{P_i} \cdot \vec{P_o}$$
$$P_p^2 = P_i^2 + P_o^2 - 2P_i P_o \cos\theta$$
$$\because K \cdot E = P^2/2m$$

$$P_i^2 = 2m_i (K \cdot E_i)$$
$$P_p^2 = 2m_p (K \cdot E_p)$$
$$P_o^2 = 2m_o (KE_o)$$

$$Q = K \cdot E_p + K \cdot E_o - KE_i$$

$$Q = \left(1 + \frac{m_o}{m_p}\right) K \cdot E_o - \left(1 - \frac{m_i}{m_p}\right) K \cdot E_i - \frac{2}{m_p}\sqrt{m_i m_o KE_i KE_o} \cos\theta$$

If outgoing particle is scattered at angle $\pi/2$.

$$Q = \left(1 - \frac{m_o}{m_p}\right) KE_o - \left(1 - \frac{m_i}{m_p}\right) K \cdot E_i$$

A body of mass 'M' at Rest, it explodes in two particle m1 & m2, calculate energy of fragments of the body in terms of 'Q'.

$$Q = \frac{P_1^2}{2}\left[\frac{m_1+m_2}{m_1 m_2}\right]$$

$$K.E_1 = \frac{P_1^2}{2m_1} = \frac{Q m_2}{m_1 m_2}$$

$$K.E_2 = \frac{P_2^2}{2m_2} = \frac{Q m_1}{m_1+m_2}$$

NOTE → * K.E of fragments are inversally propessnal to their masses.

* This analysis applicable in nuclear fission in two fragments.

* If nucleus is converted in three parts then we will have 2 equation & three unknowns. This was the reason of birth of nutrino & antinutrino particle during such experiment missing energy & momentum where assigned to particles nutrino & anti-nutrino.

Threshald Energy of an Endoergic Reaction –

To initiate an endoergonic Reaction the K.E of incoming particle must be greater than a threshald value. The K.E should overcome the ⊖ve Q value as as – some part of it also used to provide K.E to the product nuclei & outgoing particle.

In centre of mass from total momentum of particle is zero, hence K.E with respect to centre of mass of incoming particle must be equal to |Q|.

$$K.E' \geq |Q|$$

$$\frac{1}{2} M_{red} v^2 \geq |Q|$$

$$\frac{1}{2}\left(\frac{mM}{m+M}\right) v^2 \geq |Q|$$

$$\frac{1}{2} m v^2 \geq \left(\frac{m+M}{M}\right)|Q|$$

$$K.E_{particle} \geq \left(1+\frac{m}{M}\right)|Q|$$