Handwritten Notes

on

Moving Charges and Magnetism

- Moving charges and magnetism.

charge → magnetic field

Magnetic field induction ($\vec{B}$)

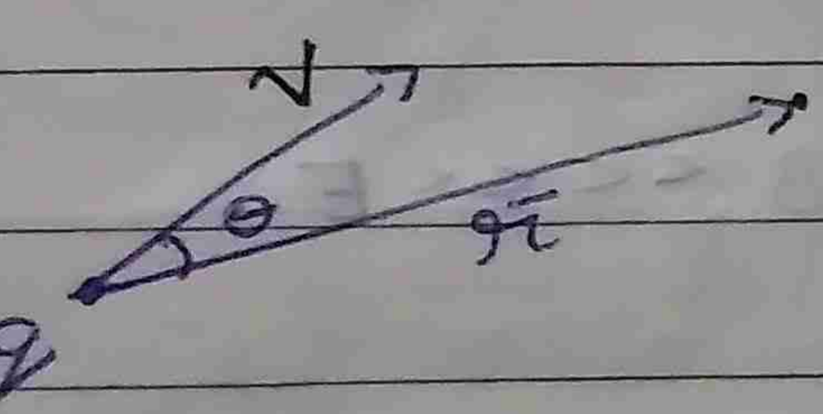
SI unit ---- Tesla

- Charge — Electric field (inherent property)

- charge in uniform motion : Electric field + Magnetic field

- accelerated charge : electric field + magnetic field + radiation

- Magnetic field due to point charge

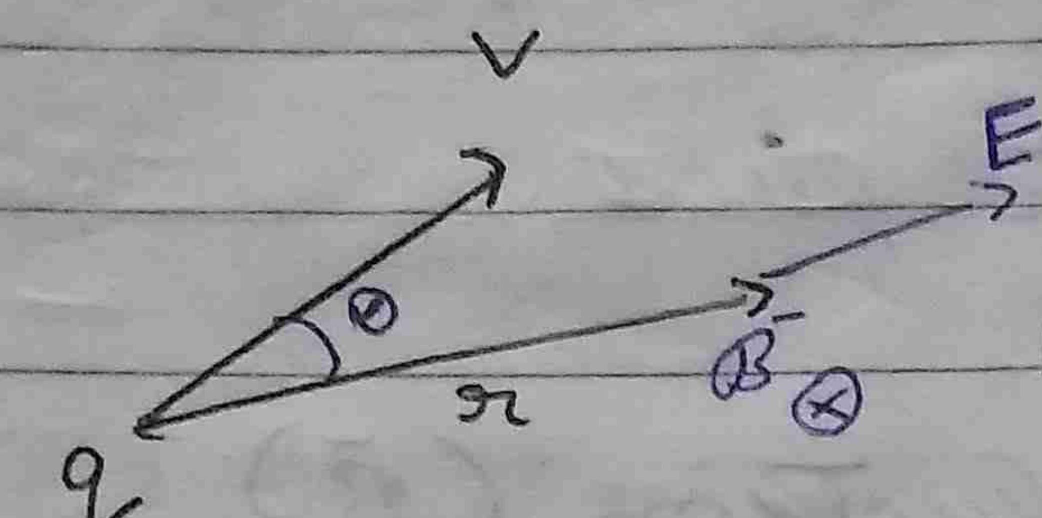


$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q(\vec{v} \times \vec{r})}{r^3}$$

μ_0 = permeability of free space

$\mu_0 = 4\pi \times 10^{-7}$ --- Hm^{-1}

- $\bar{B}$ due to point charge



$$\bar{B} = \frac{\mu_0}{4\pi} \cdot q \frac{(\bar{v} \times \bar{r})}{r^3}$$

- If $v = 0 \Rightarrow B = 0$
- If $\bar{v} \times \bar{r} = 0 \Rightarrow B = 0$
  ($\bar{v} \parallel \bar{r}$, $\theta = 0°$)
- $\bar{B} \perp \bar{v}$; $\bar{B} \perp \bar{r}$

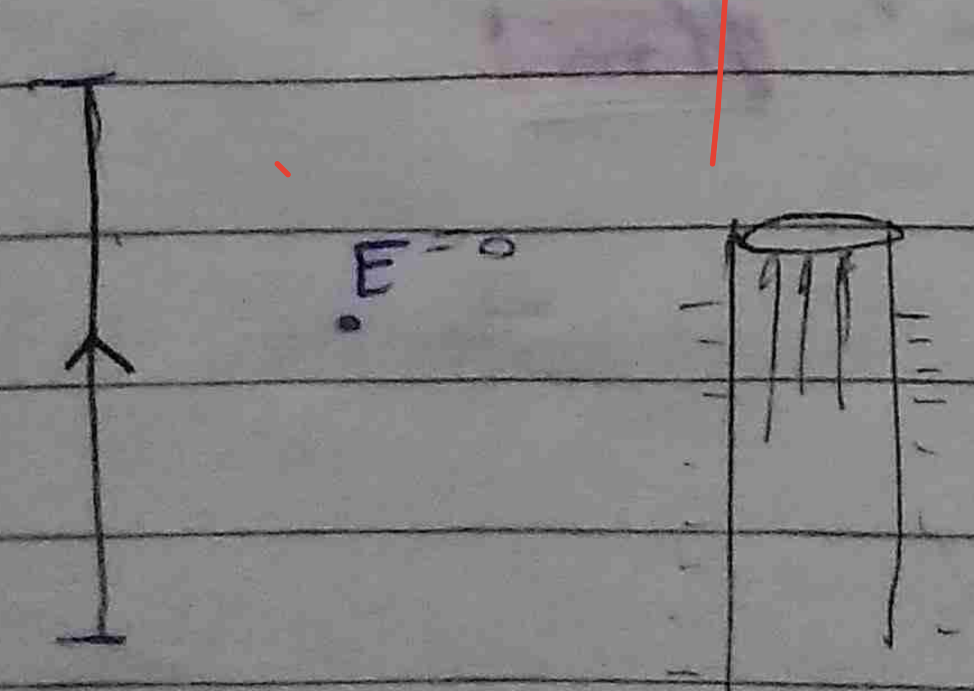
$$\bar{B} = \frac{\mu_0 \varepsilon_0 q}{4\pi\varepsilon_0} \frac{(\bar{v} \times \bar{r})}{r^3} \Rightarrow \bar{B} = \mu_0 \varepsilon_0 \left[\bar{v} \times \frac{q\bar{r}}{4\pi\varepsilon_0 r^3}\right]$$

☆ $\bar{B} = \mu_0 \varepsilon_0 [\bar{v} \times \bar{E}]$ $\quad$ $\bar{B} \perp \bar{E}$

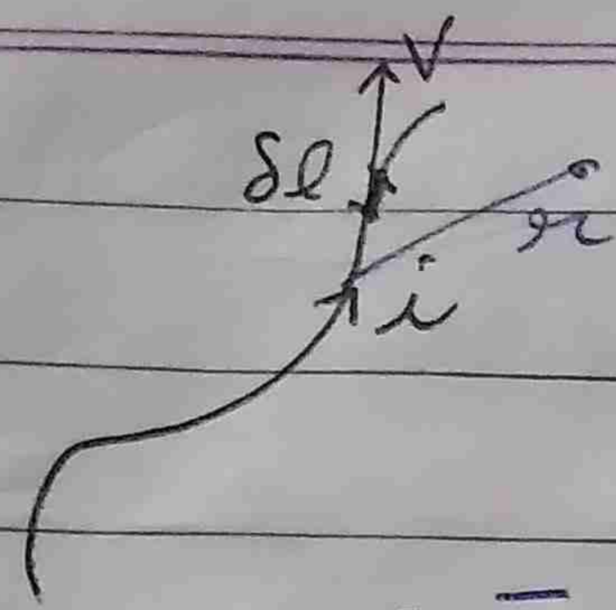
☆ $\mu_0 \varepsilon_0$: $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, m/s$

$\bar{B} = \frac{1}{c^2} [\bar{v} \times \bar{E}] \Rightarrow B <<<< E$

- $\bar{B}$ due to current carrying elements



$\vec{E}$ due to a current carrying wire is zero



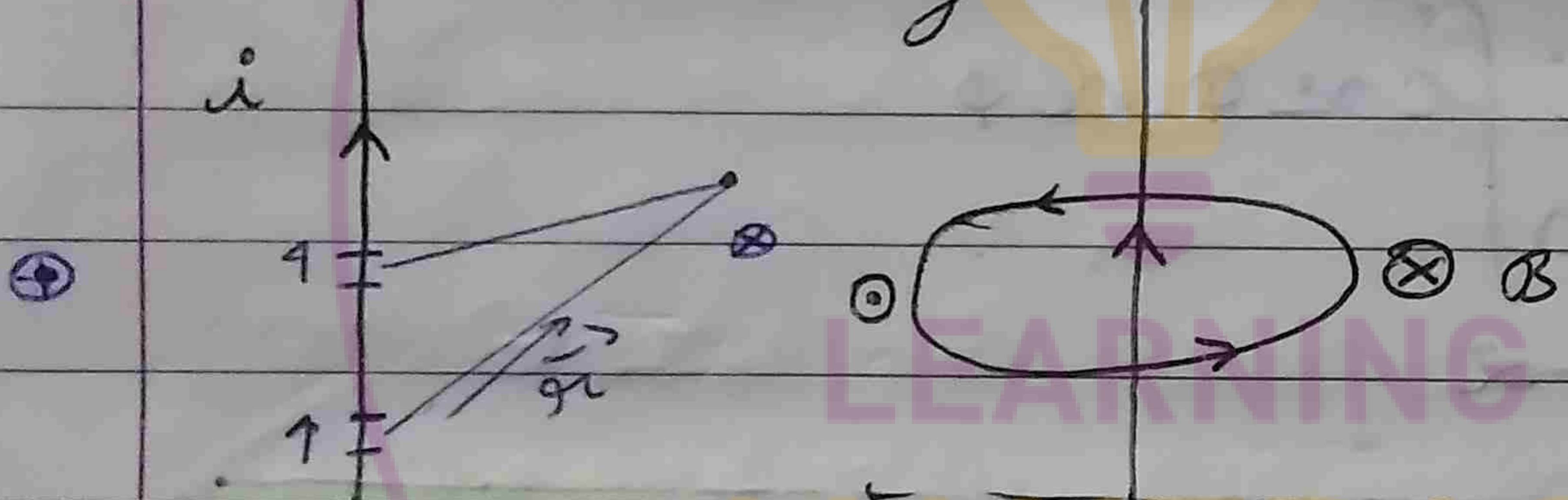
$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\bar{v} \times \bar{r})}{r^3}$$

$$\delta\bar{B} = \frac{\mu_0}{4\pi} \delta q \frac{(\bar{v} \times \bar{r})}{r^3} = \frac{\mu_0}{4\pi} \frac{i\,dt\left(\frac{d\bar{l}}{dt} \times \bar{r}\right)}{r^3}$$

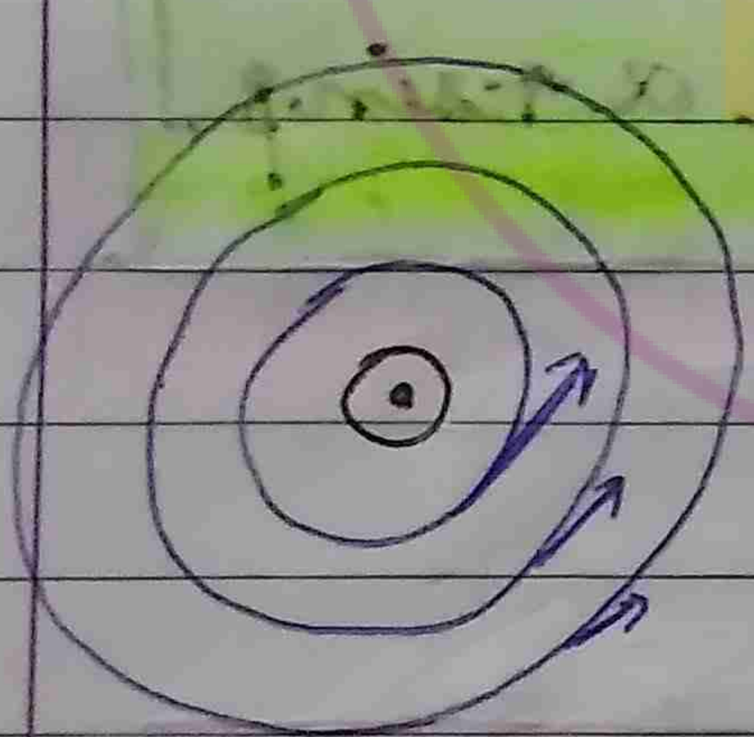
$$\Rightarrow \boxed{\delta\bar{B} = \frac{\mu_0}{4\pi} \frac{i\,(\delta\bar{l} \times \bar{r})}{r^3}} \qquad \boxed{\delta B \propto \frac{1}{r^2}}$$

--- Biot-Savarts law

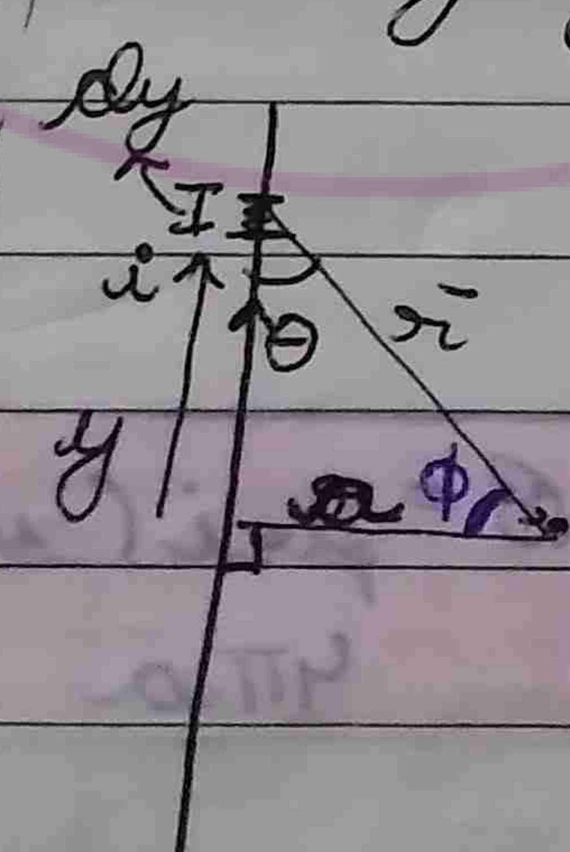
right hand thumb rule -



Thumb -- current
curl --- magnetic field.



1. $\bar{B}$ due to straight current carrying wire



$$\delta\bar{B} = \frac{\mu_0}{4\pi} \frac{i\,(d\bar{l} \times \bar{r})}{r^3}$$

$$\theta + \phi = \frac{\pi}{2}$$

$$\delta\bar{B} = \frac{\mu_0}{4\pi} \frac{i\,dy\; r\sin(\pi - \theta)}{r^3}$$

$$\Rightarrow \delta B = \frac{\mu_0}{4\pi} \frac{i\,dy \sin\theta}{r^2} \quad \text{(iv)}$$

$\sin\theta = \cos\phi$ --- (i)

$y = a\tan\phi$

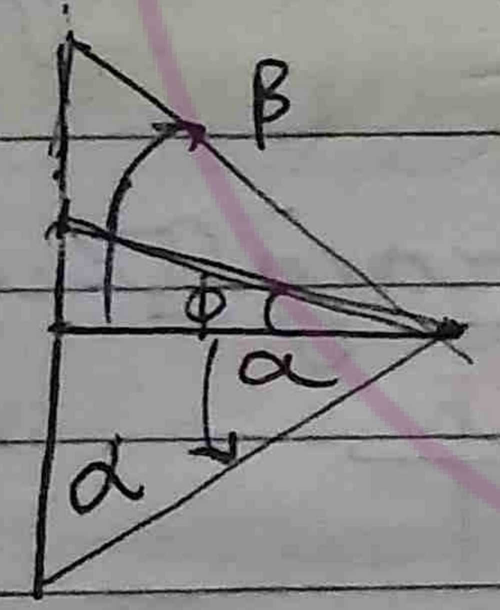
$dy = a\sec^2\phi\, d\phi$ --- (ii)

$r\cos\phi = a$ (iii)

from equation (i), (ii), (iii) and (iv)

$$\delta B = \frac{\mu_0}{4\pi}\frac{i(a\sec^2\phi\, d\phi)\cos\phi}{a^2/\cos^2\phi}$$

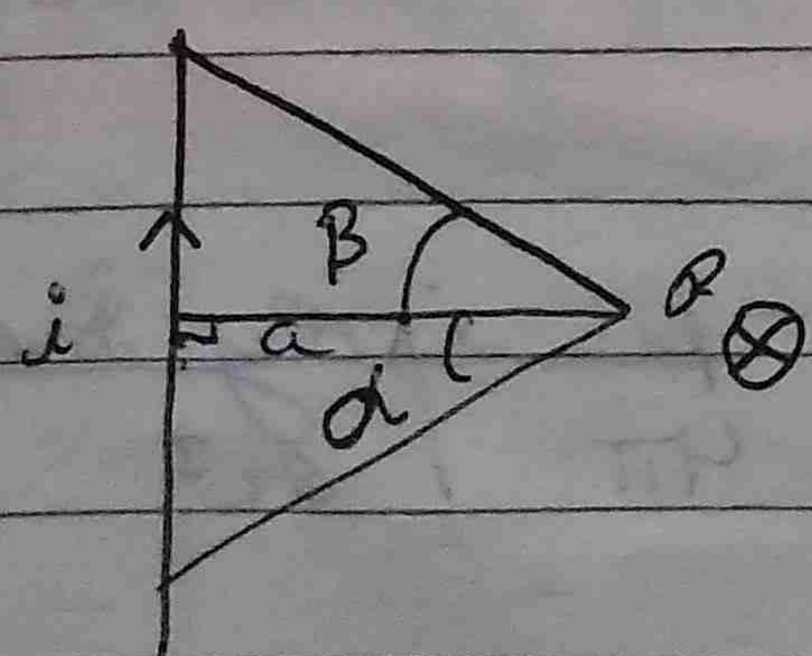
$$\int \delta B = \frac{\mu_0 i}{4\pi a}\int_{-\alpha}^{\beta}\cos\phi\, d\phi$$



$$B = \frac{\mu_0 i}{4\pi a}[\sin\alpha + \sin\beta]$$

Special cases

e.g

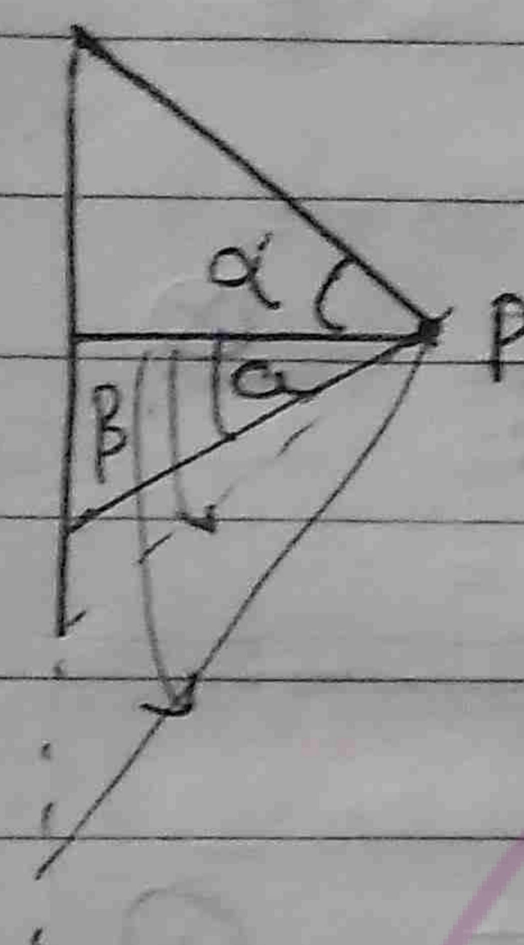


$$B = \frac{\mu_0 i(\sin\alpha + \sin\beta)}{4\pi a}$$

1 If P is at mid point $B = \frac{\mu_0 i \sin\alpha}{2\pi a}$

2 If P is at mid point for infinitely long conductor.

$$\alpha \rightarrow \frac{\pi}{2} \; ; \; \beta \Rightarrow \frac{\pi}{2}$$



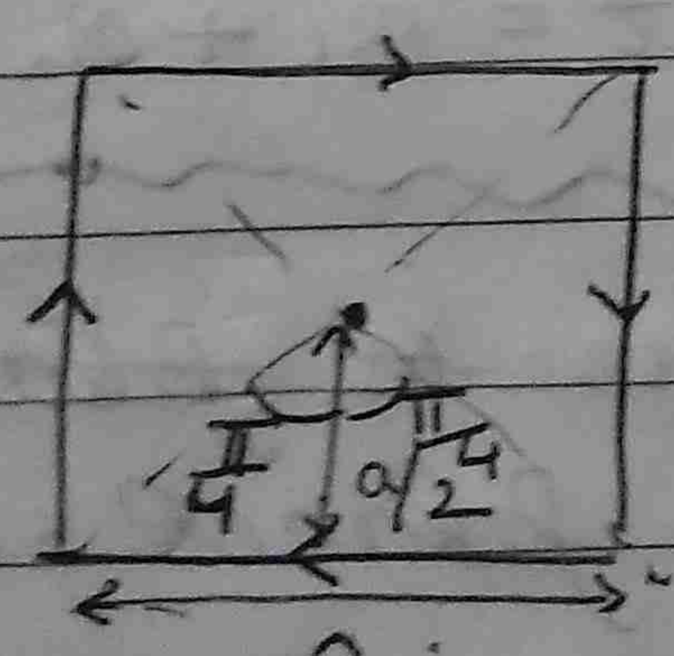
$$B = \frac{\mu_0 i}{2\pi a}$$

3 If P is at one end $\beta = 0$; $B = \frac{\mu_0 i \sin\alpha}{4\pi a}$

4 At one end of an infinitely long conductor

$$B = \frac{\mu_0 i}{4\pi a}$$

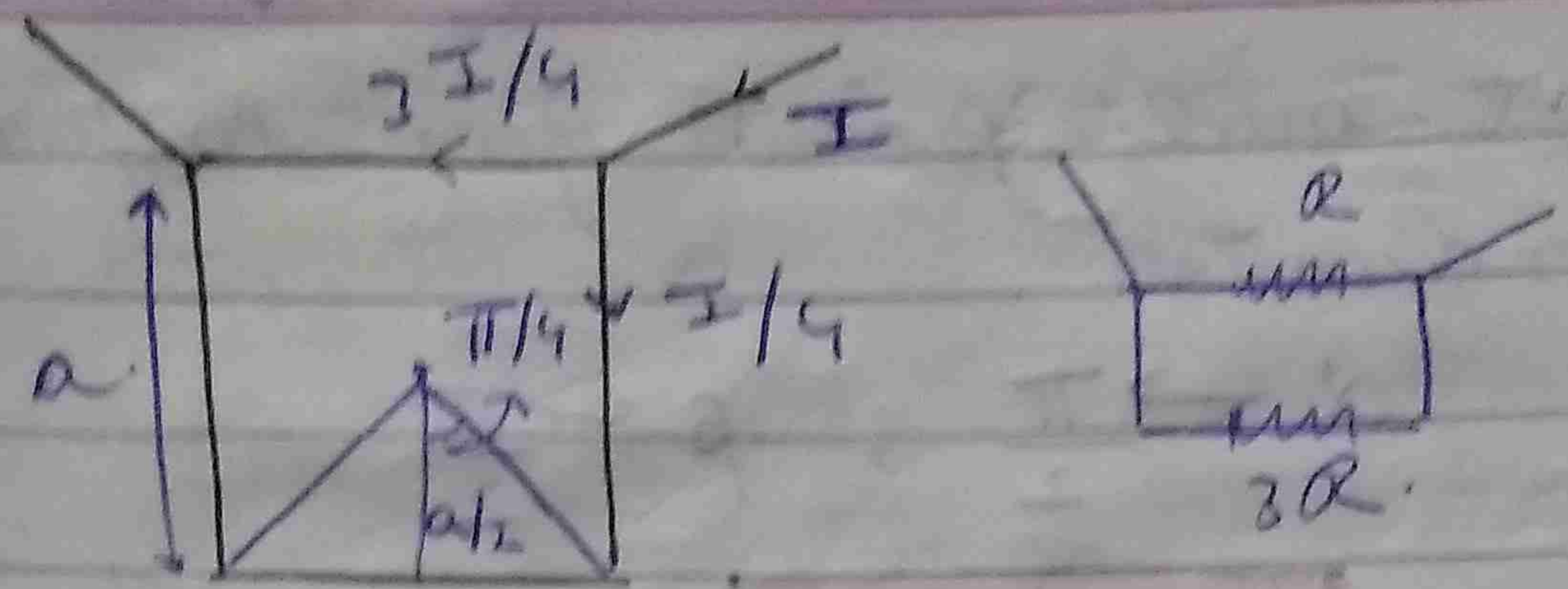
e.g



$$\Rightarrow \frac{4 \times \mu_0 i}{4\pi \frac{a}{2}} \left[\sin\frac{\pi}{4} + \sin\frac{\pi}{4}\right]$$

$$\Rightarrow B = \frac{2\mu_0 i}{\pi a} \frac{2}{\sqrt{2}} \otimes$$

eg

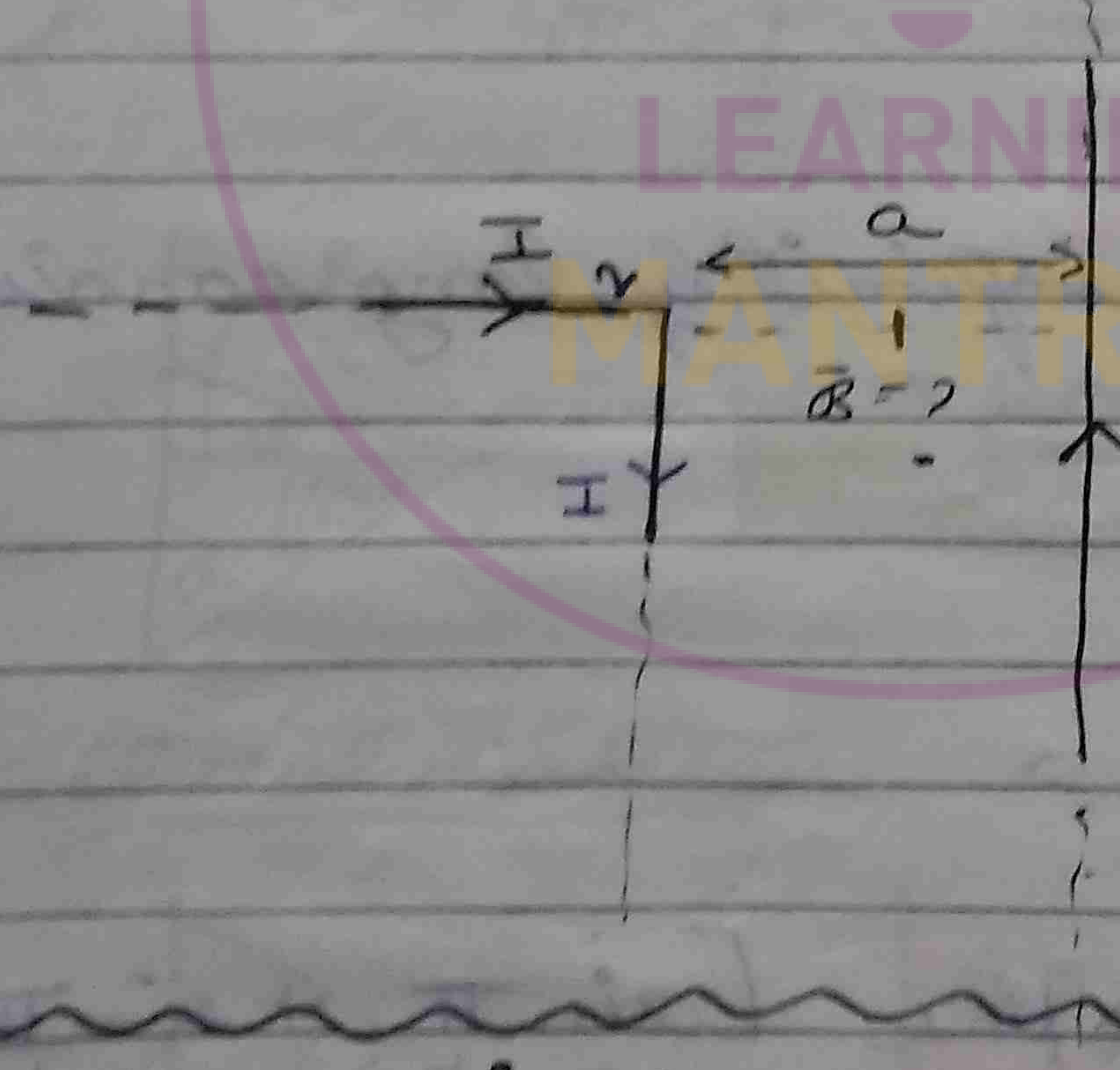


$\frac{I}{4}$ } $B_1 = 3 \times \frac{\mu_0 I/4}{4 \times \frac{a}{2} \pi} 2 \sin \frac{\pi}{4}$ ⊗

$3I/4$ } $B_2 = \frac{\mu_0 \left(\frac{3I}{4}\right)}{4 \times a/2 \pi} 2 \sin \frac{\pi}{4}$ ⊙

$\Rightarrow B_{net} = 0$

eg

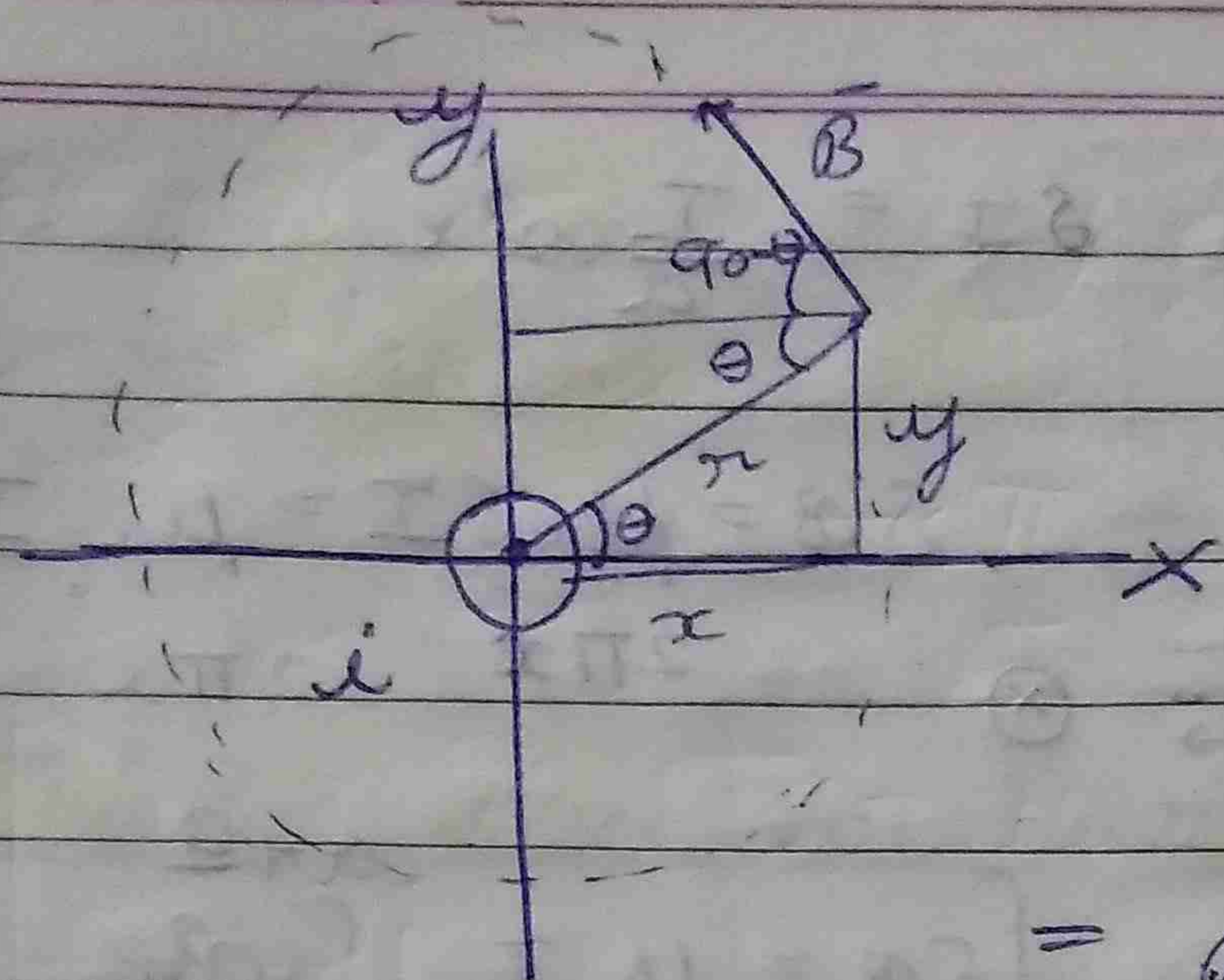


$B_1 = \frac{\mu_0 I}{2\pi a/2}$ ⊙

$B_2 = \frac{\mu_0 I}{4\pi (a/2)}$ ⊙

$B_{net} = \bar{B}_1 + \bar{B}_2$

eg An infinitely long wire ~~is placed~~ carrying current i is placed along z-axis. find $\bar{B}$.



$\vec{B}$ $B = \frac{\mu_0 i}{2\pi r}$

$\vec{B} = [-B\sin\theta\hat{i} + B\cos\theta\hat{j}]$

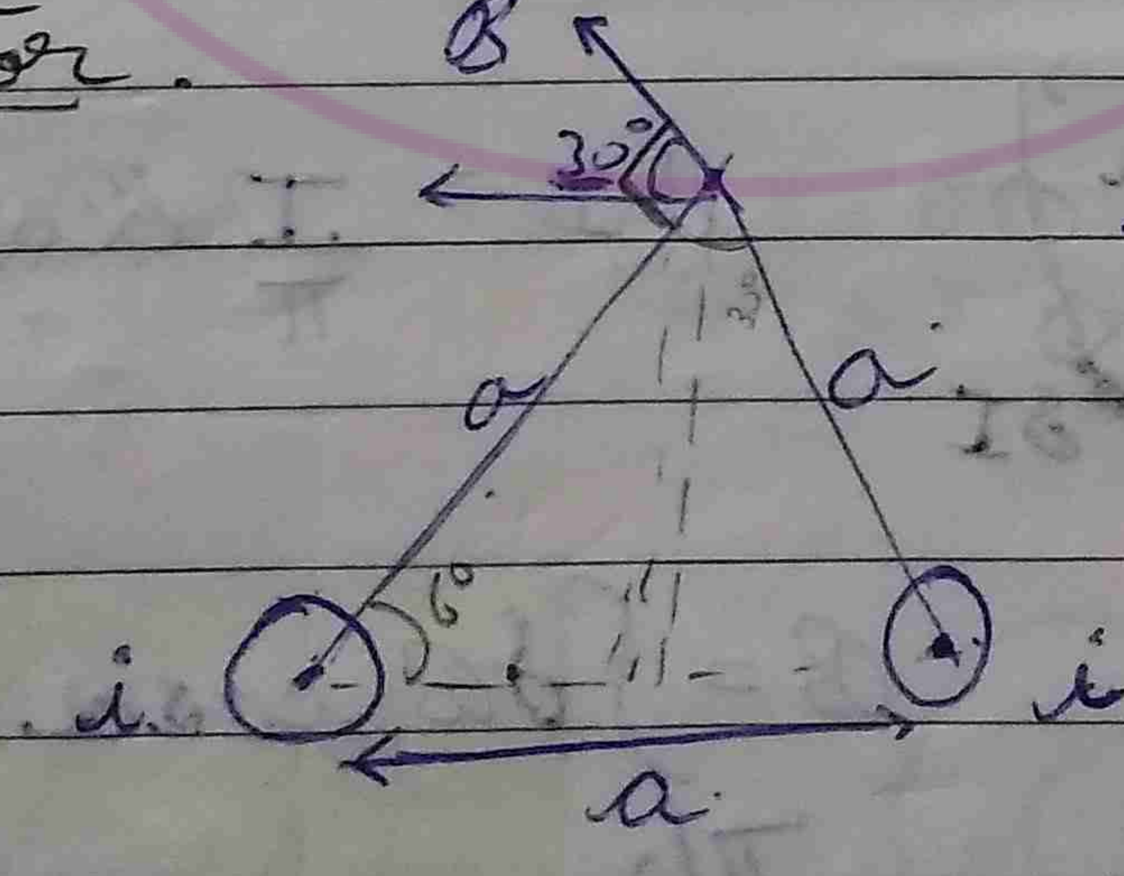
$= B[-\sin\theta\hat{i} + \cos\theta\hat{j}]$

$$= \frac{\mu_0 i}{2\pi\sqrt{x^2+y^2}}\left[\frac{-y}{\sqrt{y^2+x^2}}\hat{i} + \frac{x}{\sqrt{x^2+y^2}}\hat{j}\right]$$

$\Rightarrow \vec{B} = \frac{\mu_0 i}{2\pi(x^2+y^2)}(-y\hat{i} + x\hat{j})$ for first quadrant

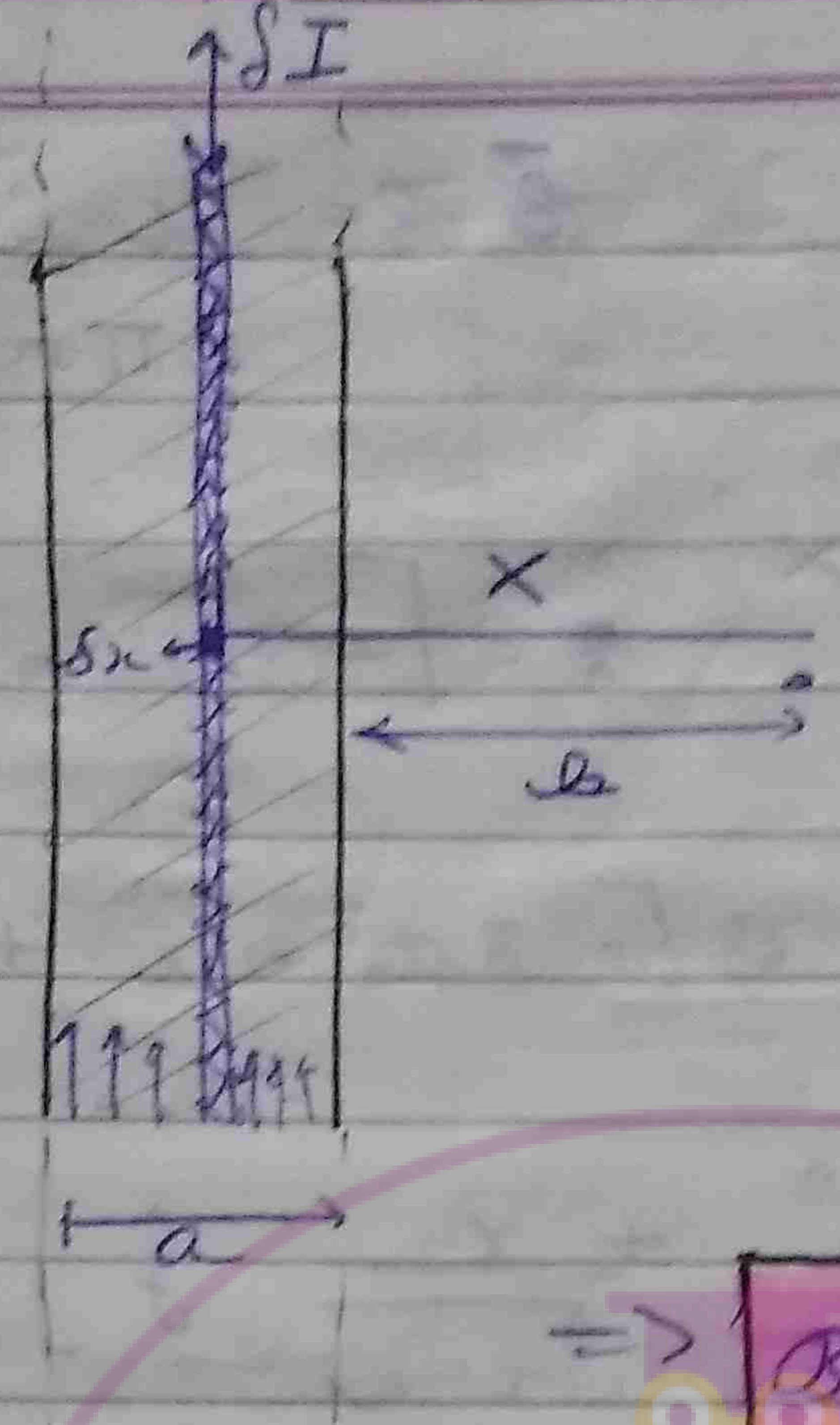
---

Q. Two long wires separated by a, carry current i each in same direction. Find field at a distance 'a' from each conductor.



$2B\cos 30°$

$\sqrt{3}B \Rightarrow \frac{\sqrt{3}\mu_0 I}{2\pi a}$



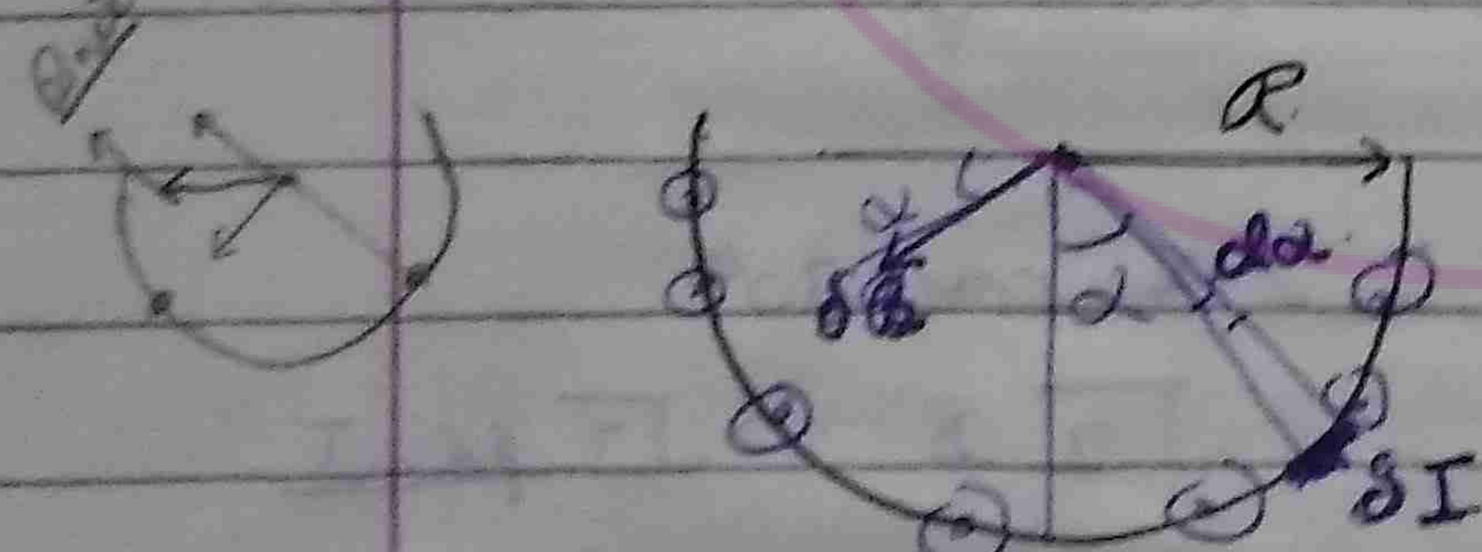
$$\delta I = \frac{I}{a} dx$$

$$\delta B = \frac{\mu_0 \delta I}{2\pi x} = \frac{\mu_0}{2\pi x}\frac{I}{a} dx$$

$$\int \delta B = \frac{\mu_0 I}{2\pi a}\int_b^{a+b} \frac{dx}{x}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi a} \ln\left(\frac{a+b}{b}\right)}$$

Q A long semicylinder of radius R, carries uniform current I. Find the field at the mid of axis



$$\delta I = \frac{I}{\pi} d\alpha$$

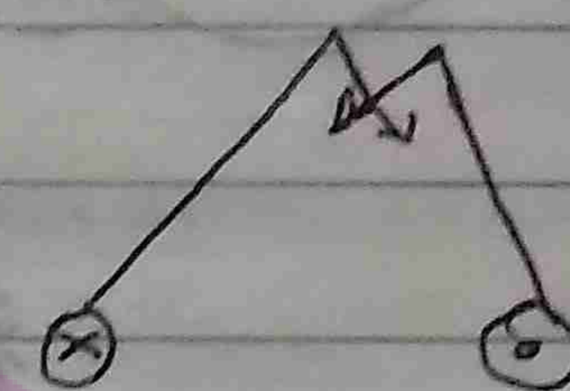
$$\delta B = \frac{\mu_0 \delta I}{2\pi R}$$

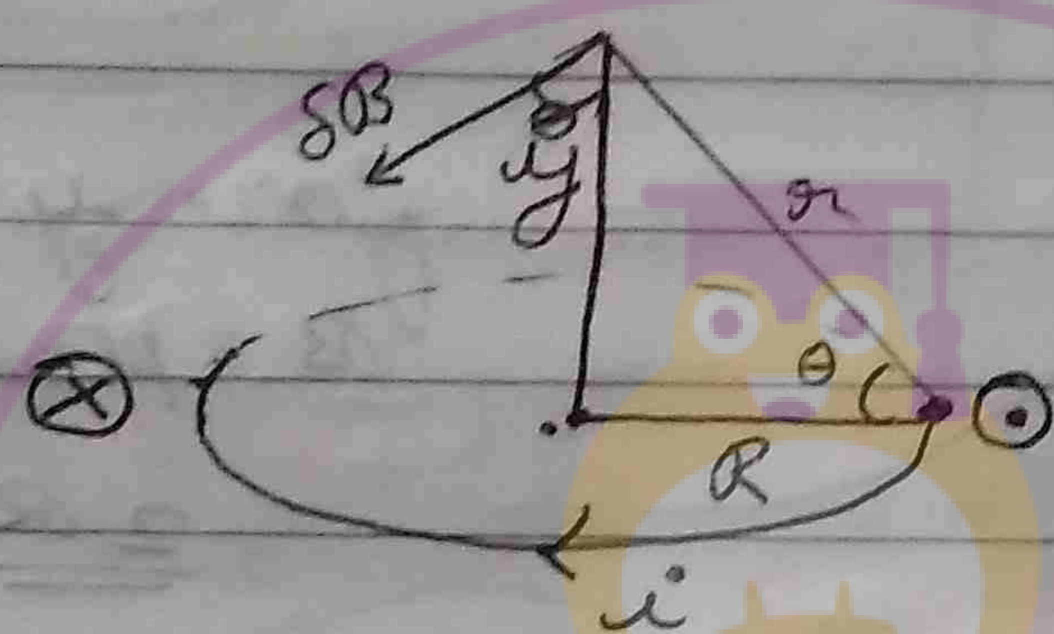
$$B = \int \delta B \cos\alpha \, (-\hat{i})$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\mu_0}{2\pi R}\frac{I}{\pi} d\alpha \cos\alpha$$

$$\frac{\mu_0 I}{2\pi^2 R}\int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha \Rightarrow \frac{\mu_0 I}{\pi^2 R}$$

- Field due to current carrying loop





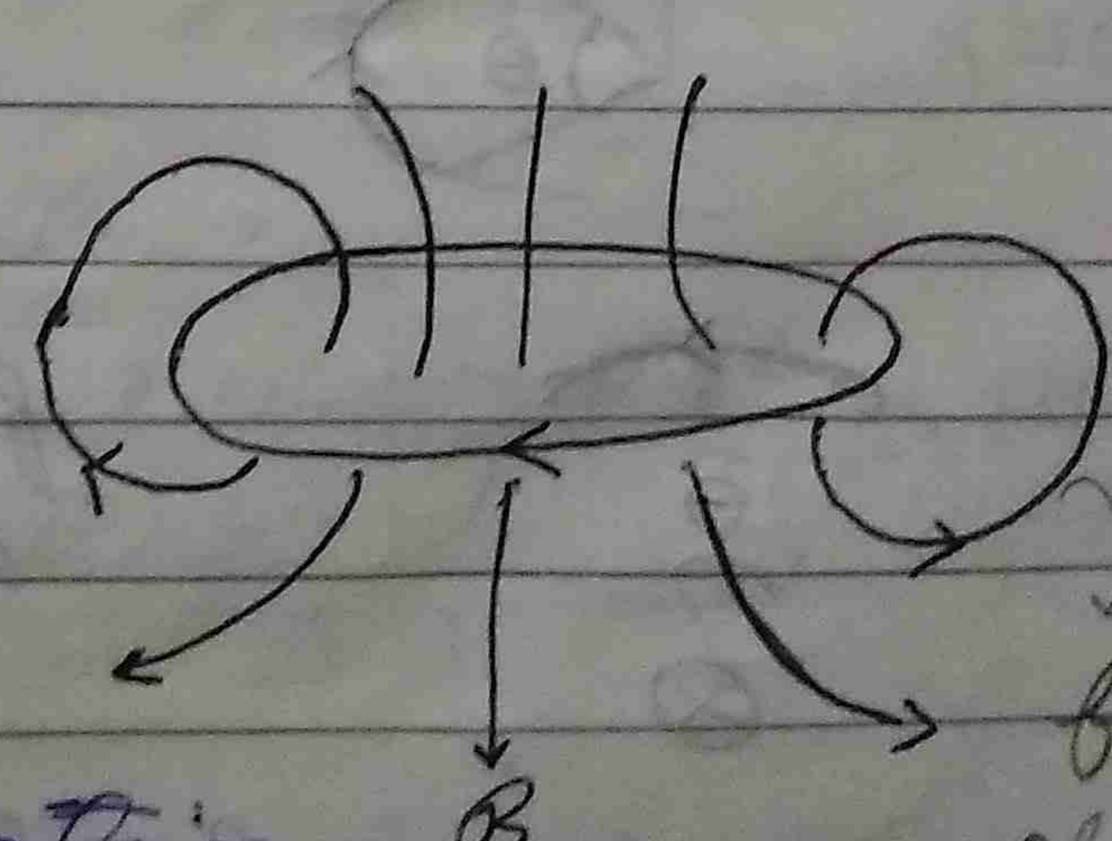
$$\delta \bar{B} = \frac{\mu_0 i}{4\pi} \frac{\delta \bar{l} \times \bar{r}}{r^3}$$

$$\delta B = \frac{\mu_0 i}{4\pi} \frac{\delta l \, r \sin \pi/2}{r^3} = \frac{\mu_0 i \, \delta l}{4\pi r^2}$$

$$\underline{B_{net}} = \int B \cos\theta = \int \frac{\mu_0 i \, \delta l}{4\pi r^2} \frac{R}{r}$$

$$B = \frac{\mu_0 i R}{4\pi r^3}\int \delta l = \frac{\mu_0 i R}{4\pi r^3} 2\pi R = B = \frac{\mu_0 i R^2}{2 r^3}$$

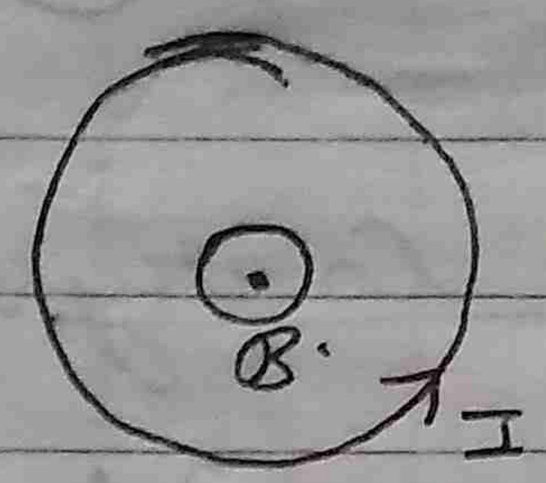
$$B = \frac{\mu_0 i R^2}{2(R^2+y^2)^{3/2}}$$



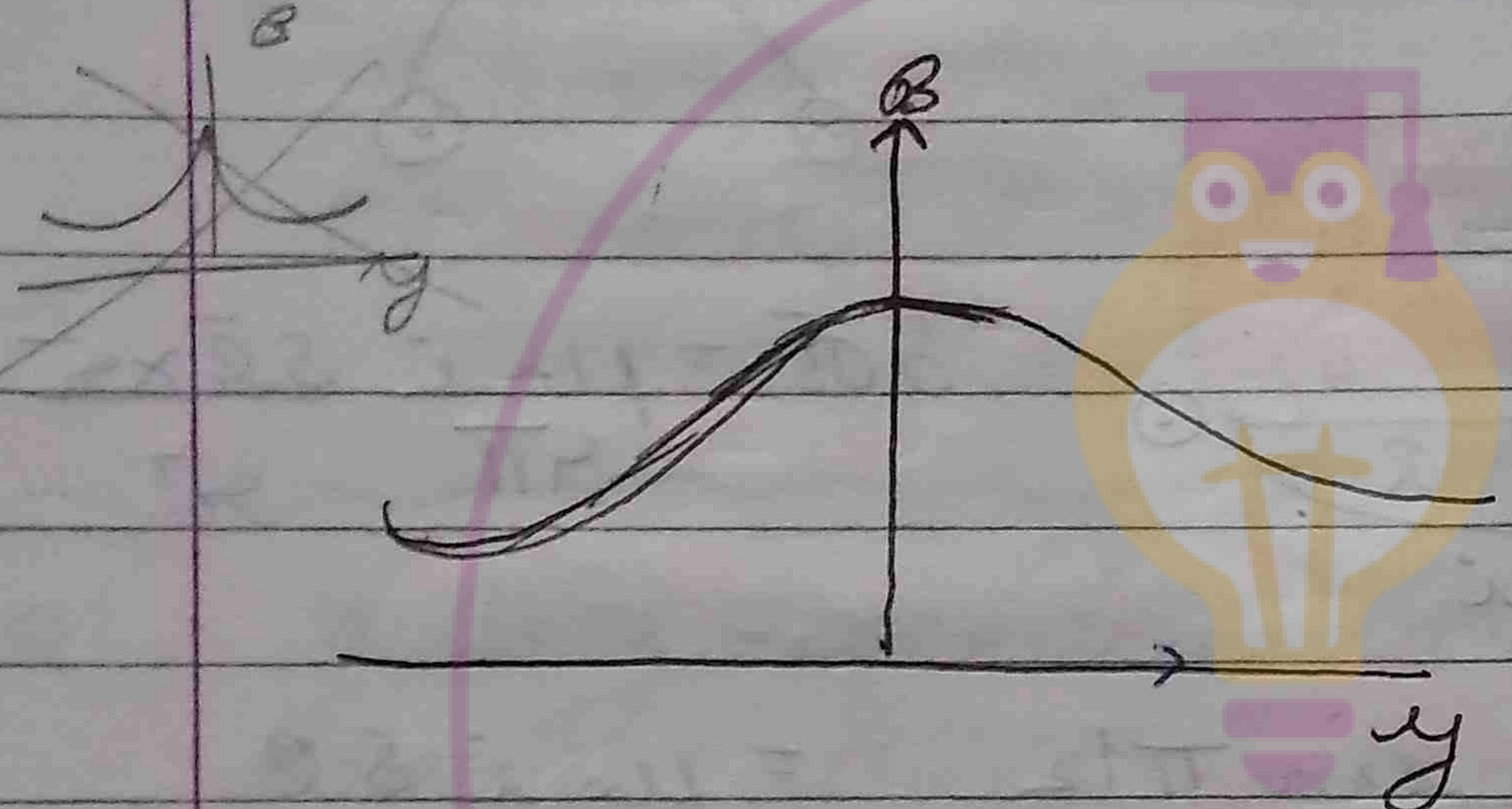
- $\bar{B} \rightarrow$ non-conservative

$$\bar{B}_{centre} = \frac{\mu_0 i}{2R} \times n$$

for n turns



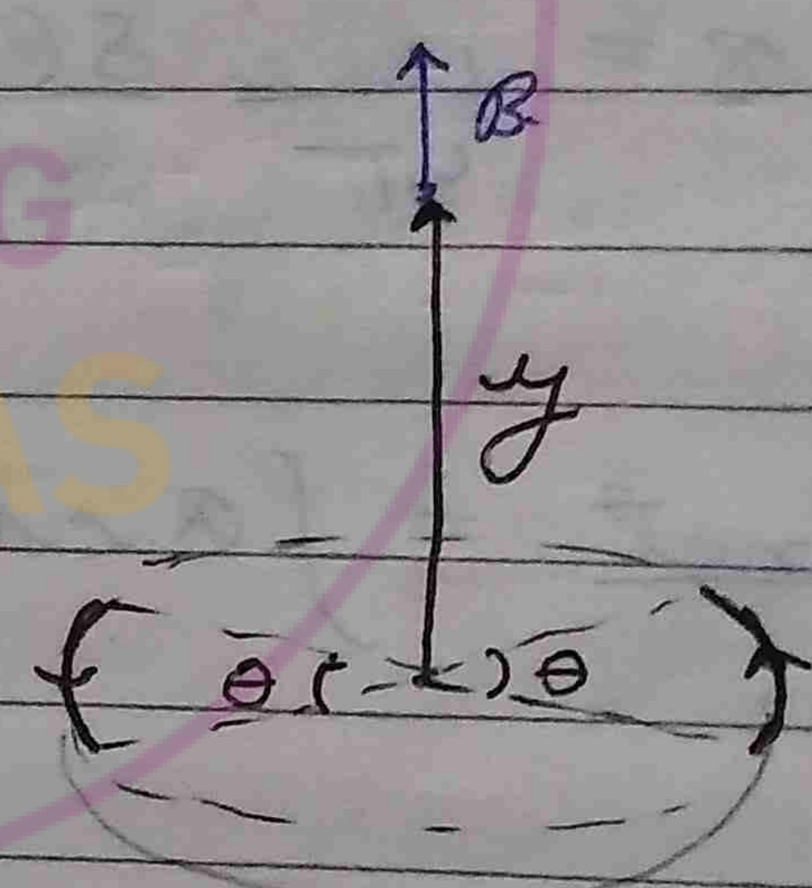
If there are n turns of the wire the field gets added up n times.



for $y << R$

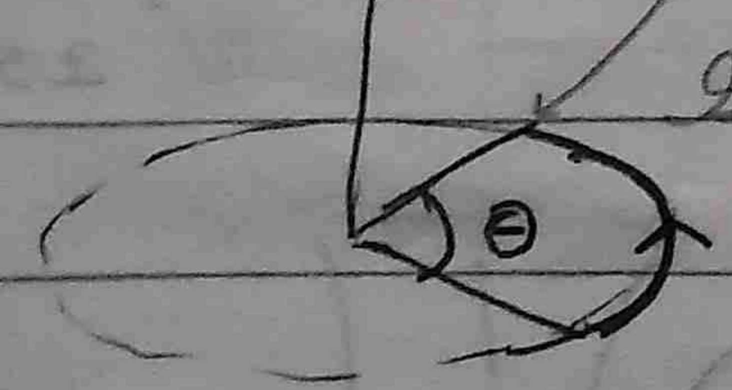
$$B = \frac{\mu_0 i}{2R}$$

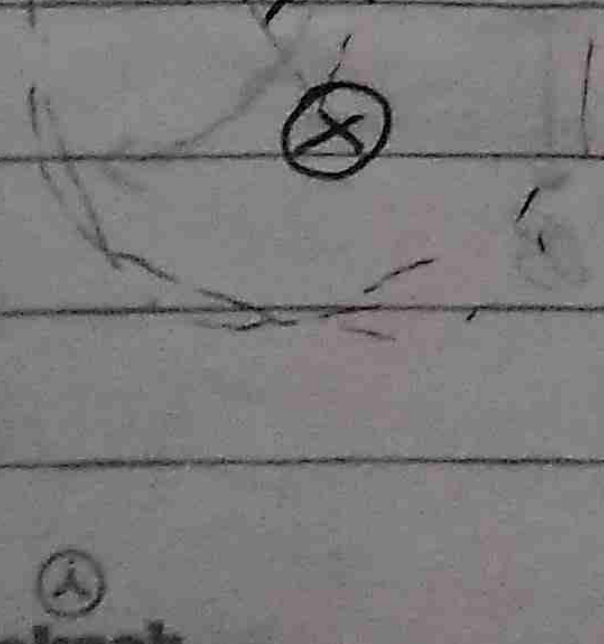
$$B = \frac{\mu_0 i R^2}{2(y^2+R^2)^{3/2}} \times \frac{2\theta}{2\pi}$$



* This result cannot be applied

$\bar{B}$ balance each other horizontal component





for complete loop

$$\bar{B}_{centre} = \frac{\mu_0 i}{2R}$$

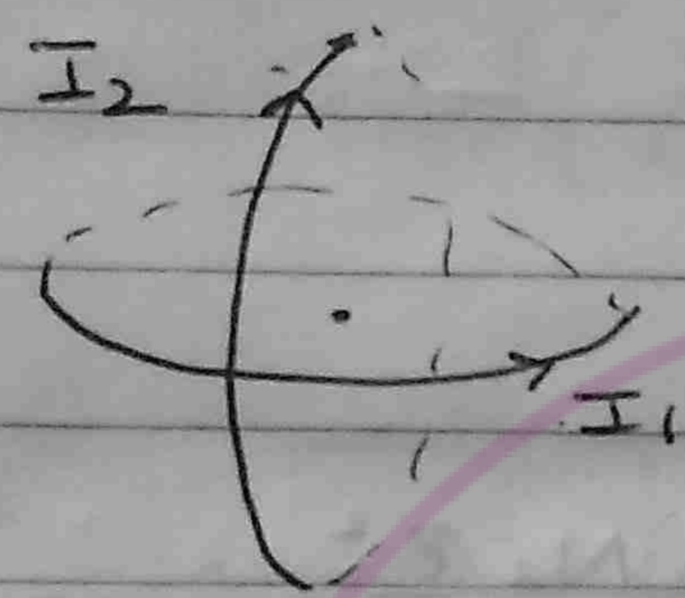
for partial loop

$$\bar{B}_{centre} = \frac{\theta}{2\pi} \frac{\mu_0 i}{2R}$$

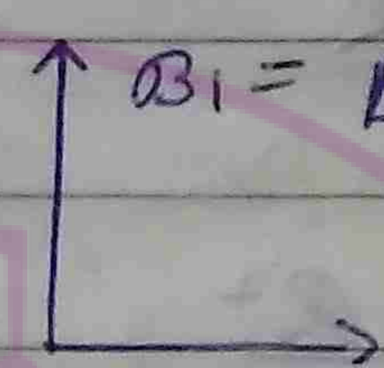
B centre for circle $\frac{\mu_0 i}{2R}$.

$$\boxed{B \text{ centre for semicircle} = \frac{\mu_0 i}{4R}}$$

e.g



field at centre.

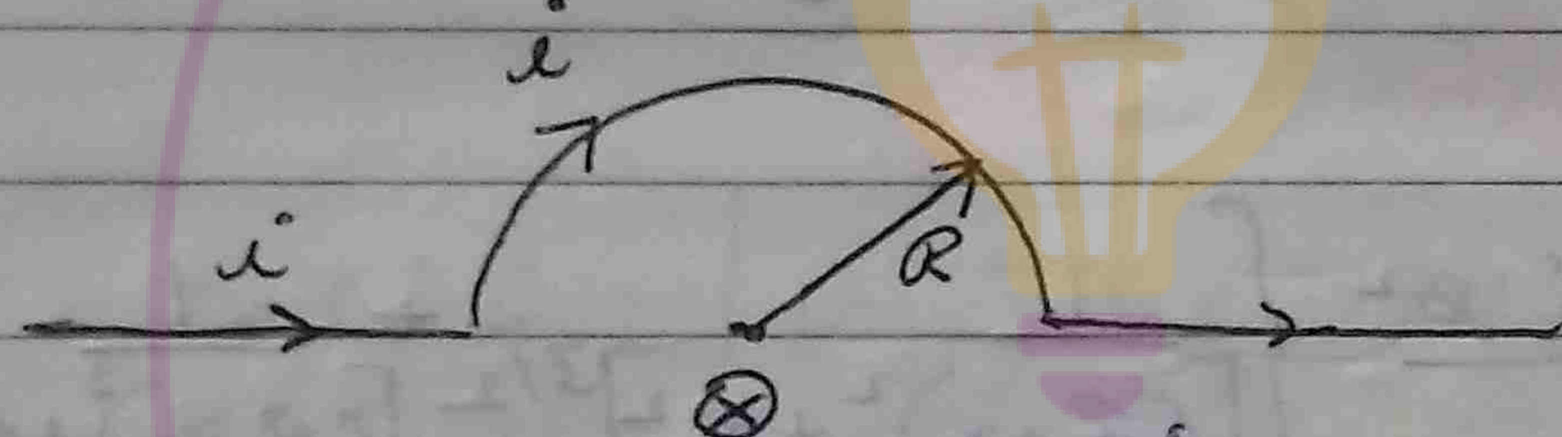


$B_1 = \frac{\mu_0 I_1}{2R}$

$B_{net} = \sqrt{B_1^2 + B_2^2}$

$B_2 = \frac{\mu_0 i_2}{2R}$

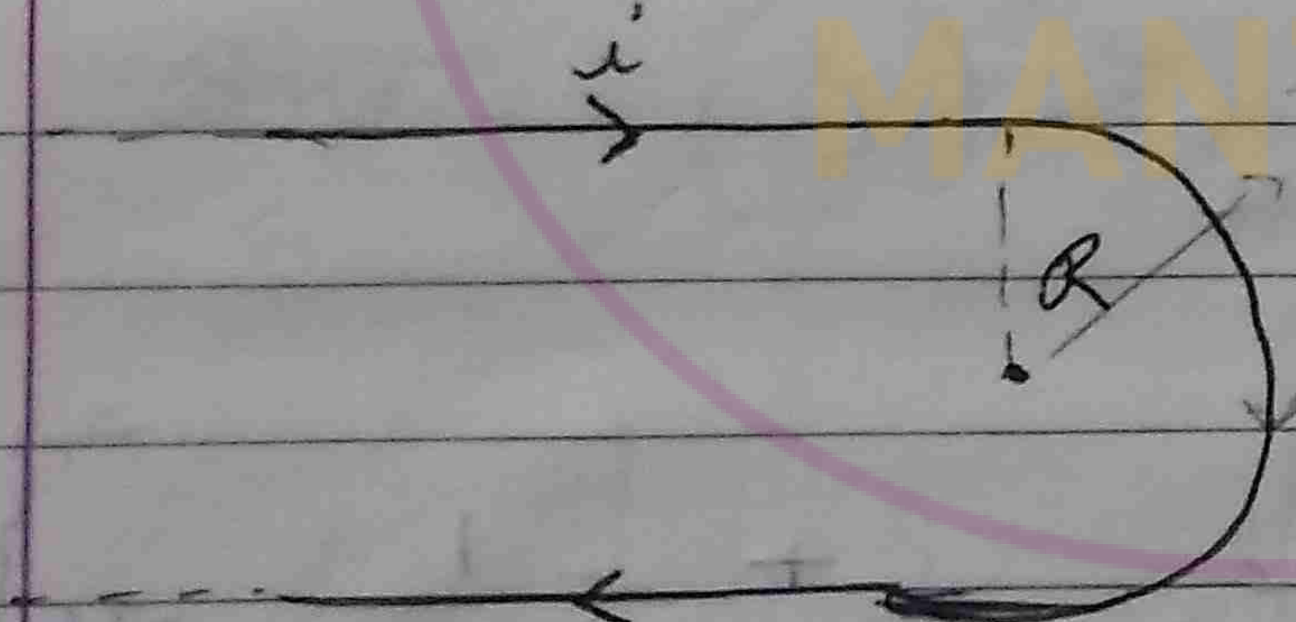
e.g



$\otimes$

$\frac{\mu_0 i}{4R}$

e.g

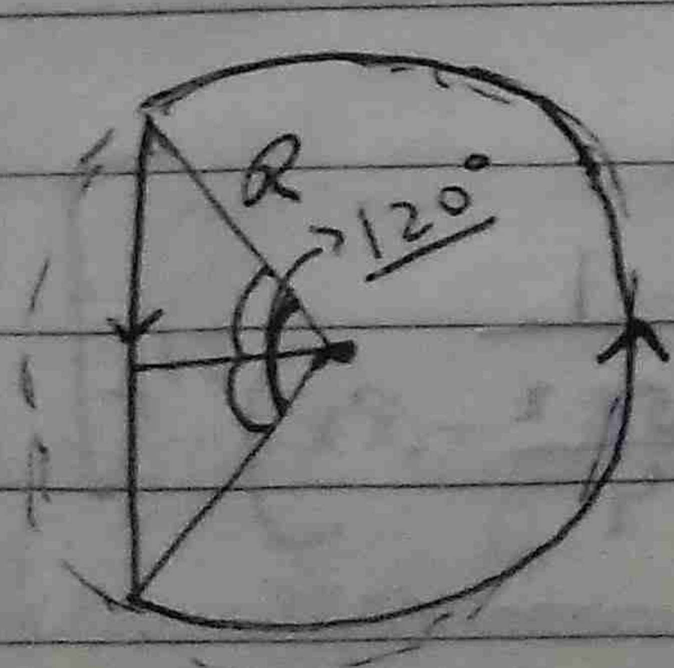


magnetic field at the point.

$\frac{2\mu_0 i}{4\pi r} \otimes + \frac{\mu_0 i}{4R} \otimes$

net field $= \left(\frac{\mu_0 i}{2\pi R} + \frac{\mu_0 i}{4R}\right) \otimes$

e.g ☆

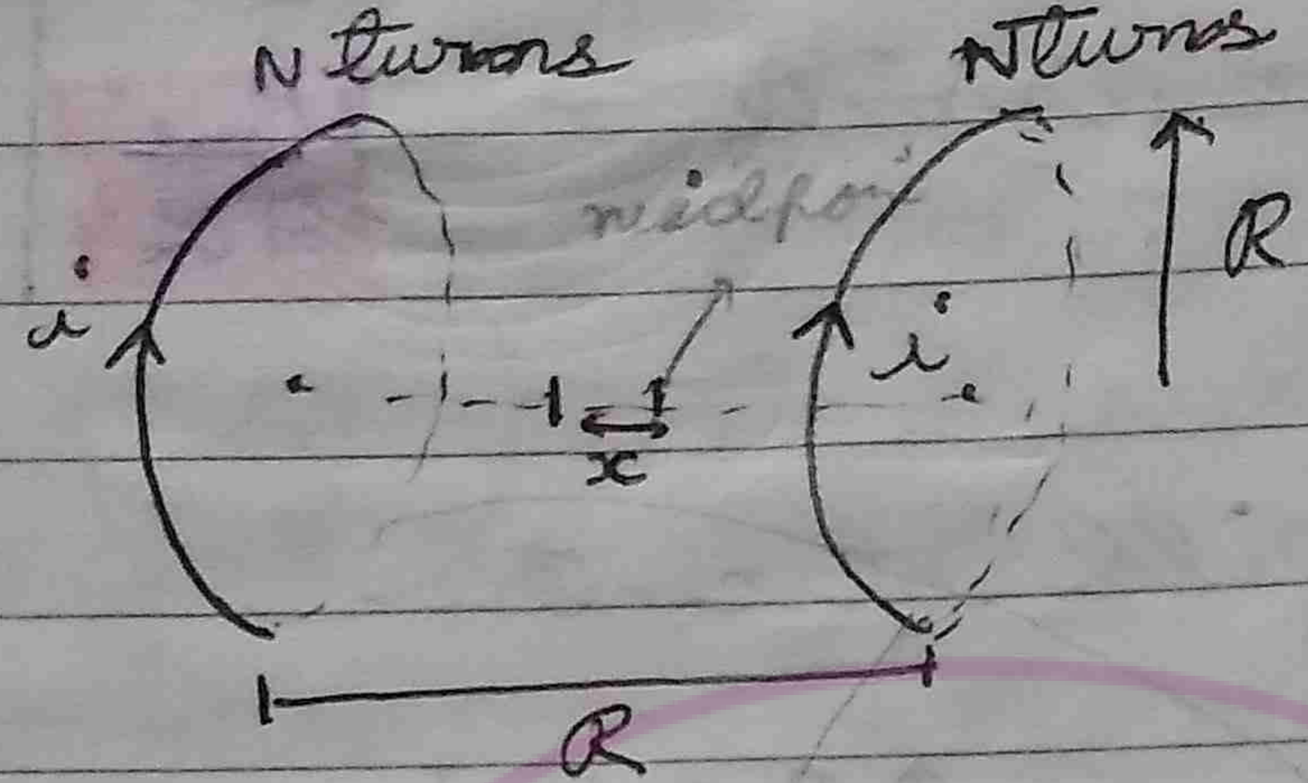


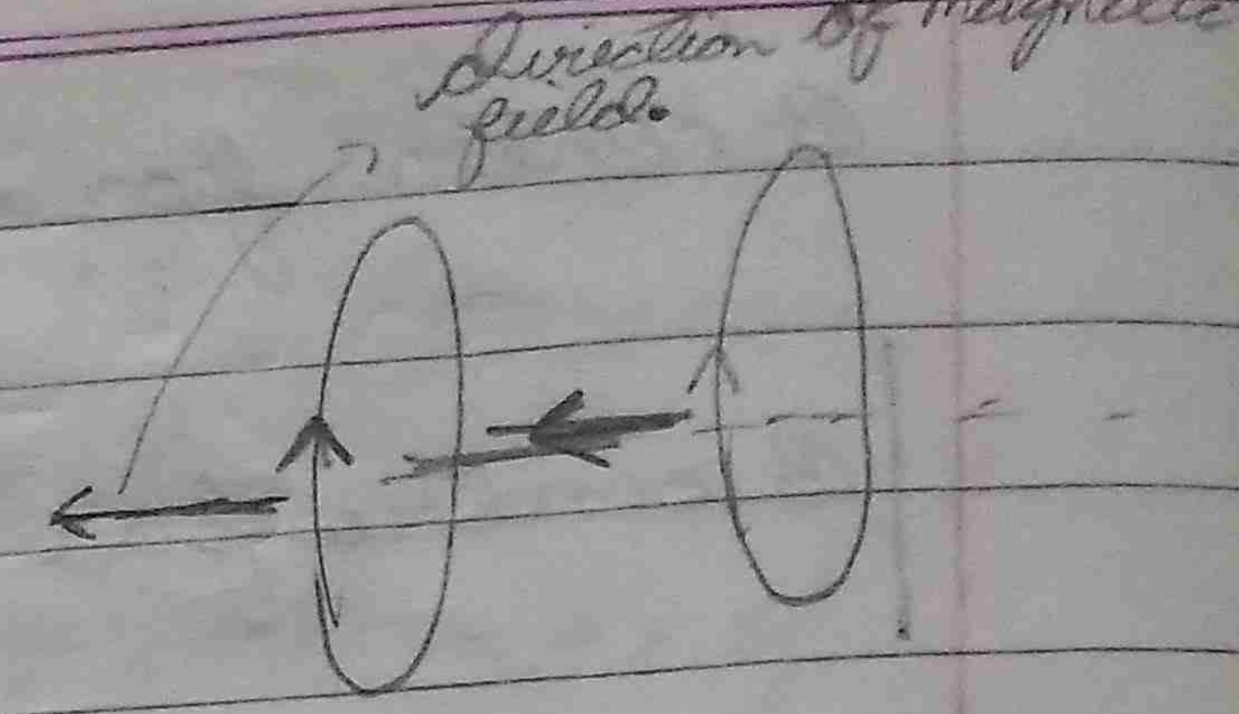
$\bar{B}$ at centre?

$\left(\frac{\mu_0 i}{4\pi R \cos 60°} 2 \sin 60° + \frac{240°}{360°} \frac{\mu_0 i}{2R}\right) \odot$

Name of the Chapter ______ Date ______ Page No.: ______

Helmholtz Coil





$$B = \frac{\mu_0 N i R^2}{2\left\{\left(\frac{R}{2}+x\right)^2+R^2\right\}^{3/2}} + \frac{\mu_0 N i R^2}{2\left\{\left(\frac{R}{2}-x\right)^2+R^2\right\}^{3/2}}$$

$$B_{net} = \frac{\mu_0 N i R^2}{2}\left\{\frac{1}{\left[\left(\frac{R}{2}+x\right)^2+R^2\right]^{3/2}} + \frac{1}{\left[\left(\frac{R}{2}-x\right)^2+R^2\right]^{3/2}}\right\}$$

if $x <<< R$

Bnet

$$\frac{\mu_0 N i R^2}{2}\left\{\frac{1}{\left[\frac{R^2}{4}+x^2+Rx+R^2\right]^{3/2}} + \frac{1}{\left(\frac{R^2}{4}+x^2-Rx+R^2\right)^{3/2}}\right\}$$

(x^2 terms struck out)

$$= \frac{\mu_0 N i R^2}{2}\left[\frac{1}{\left(\frac{5R^2}{4}+Rx\right)^{3/2}} + \frac{1}{\left(\frac{5R^2}{4}-Rx\right)^{3/2}}\right]$$

Signature ______

$$= \frac{\mu_0 N i R^2}{2\left(\frac{5R^2}{4}\right)^{3/2}}\left[\frac{1}{\left(1+\frac{Rx}{\frac{5R^2}{4}}\right)^{3/2}} + \frac{1}{\left(1-\frac{Rx}{5R^2/4}\right)^{3/2}}\right]$$

$$B_{net} = \frac{\mu_0 N i R^2}{2R^3\left(\frac{5}{4}\right)^{3/2}}\left[\frac{1}{\left(1+\frac{4x}{5R}\right)^{3/2}} + \frac{1}{\left(1-\frac{4x}{5R}\right)^{3/2}}\right]$$

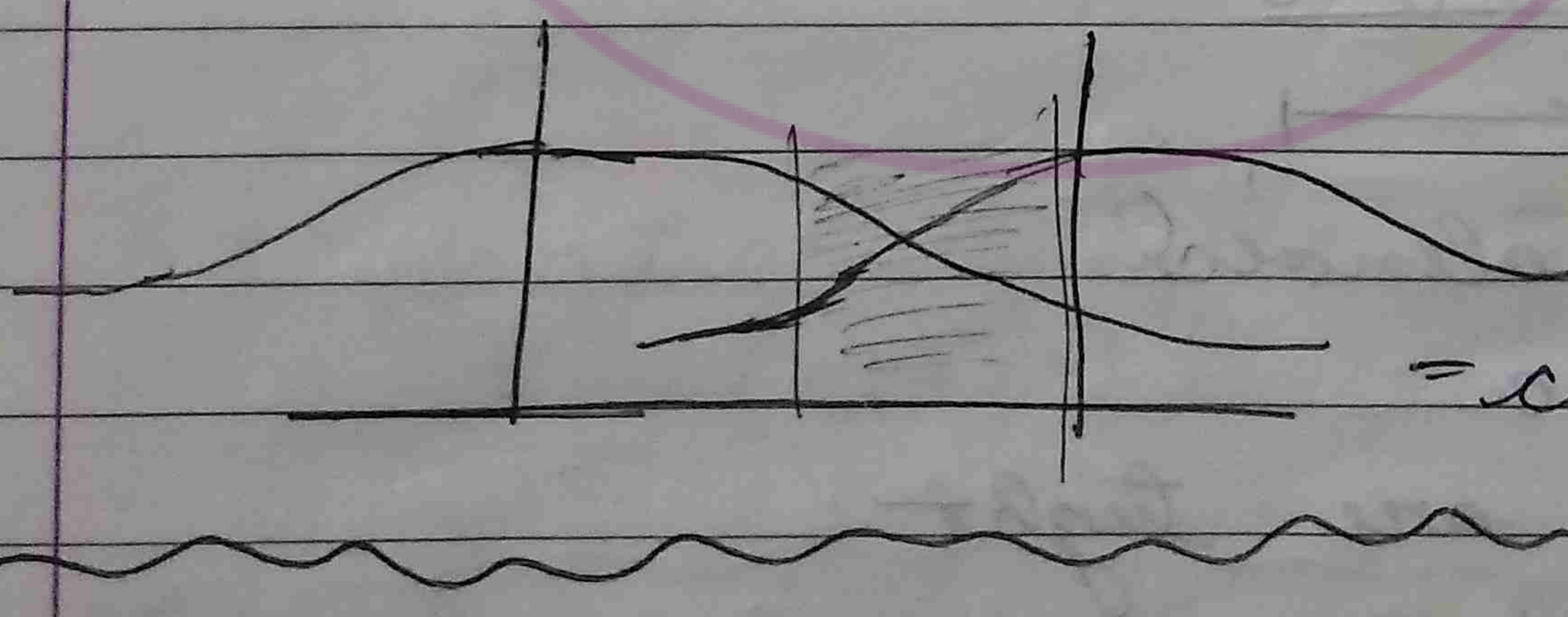
$$= \frac{\mu_0 N i}{2R\left(\frac{5}{4}\right)^{3/2}}\left[\left(1+\frac{4x}{5R}\right)^{-3/2} + \left(1-\frac{4x}{5R}\right)^{-3/2}\right]$$

using Binomial approximation

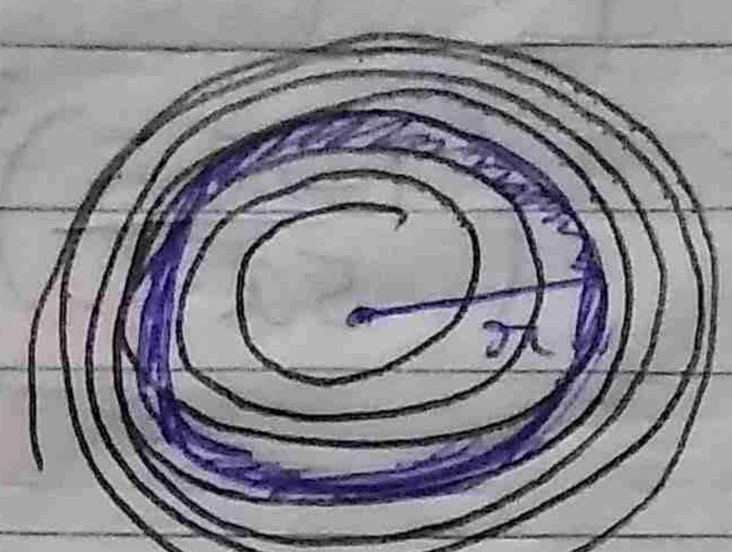
$$= \frac{\mu_0 N i}{2R\left(\frac{5}{4}\right)^{3/2}}\left[1 - \frac{3}{2}\frac{4x}{5R} + 1 + \frac{3}{2}\frac{4x}{5R}\right]$$

$$B_{net} = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 N i}{R} = 0.72\,\frac{\mu_0 N i}{R} = \text{constant}$$

$x << R$



Q A spiral of N turns has inner and outer radii as a and b. Find magnetic field at centre.



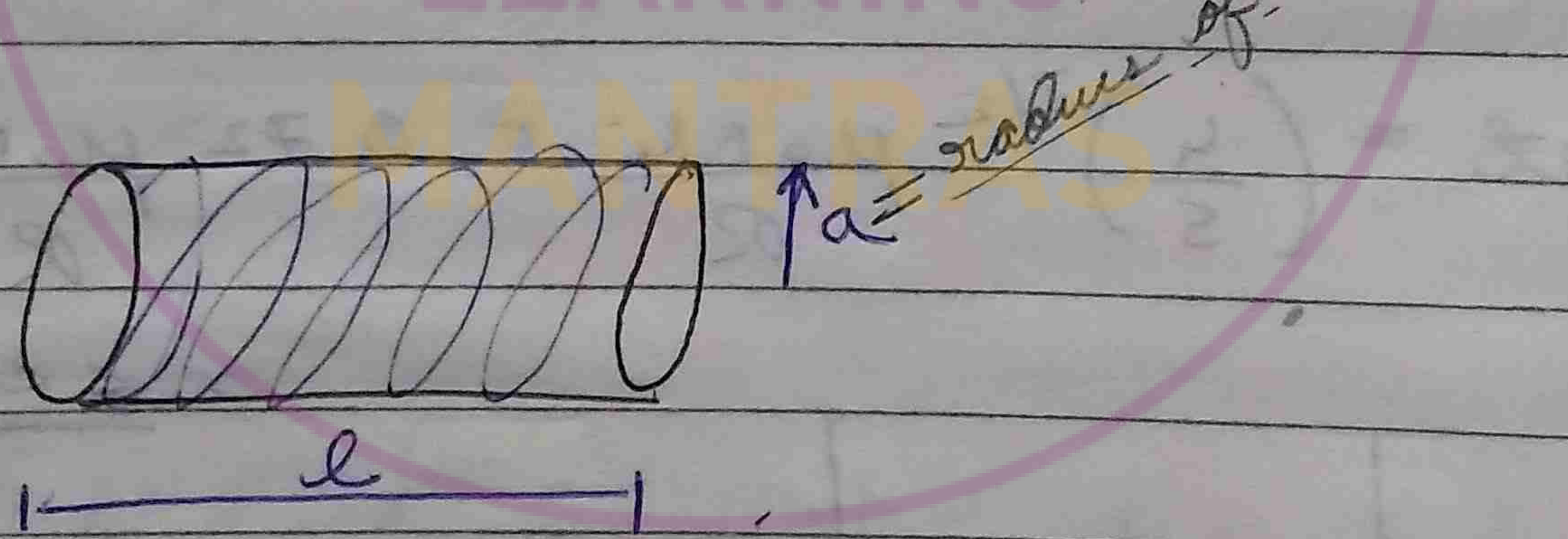
$$\delta N = \frac{N}{(b-a)}\, dr$$

<u>N turns</u> $\delta B = \frac{\mu_0 \delta N i}{2r}$.

$$\delta B = \frac{\mu_0 i}{2r} \frac{N}{(b-a)} dr \Rightarrow B = \frac{\mu_0 N i}{2(b-a)} \int_a^b \frac{dr}{r}$$

$$= \boxed{B = \frac{\mu_0 N i}{2(b-a)} \ln\left(\frac{b}{a}\right)}$$

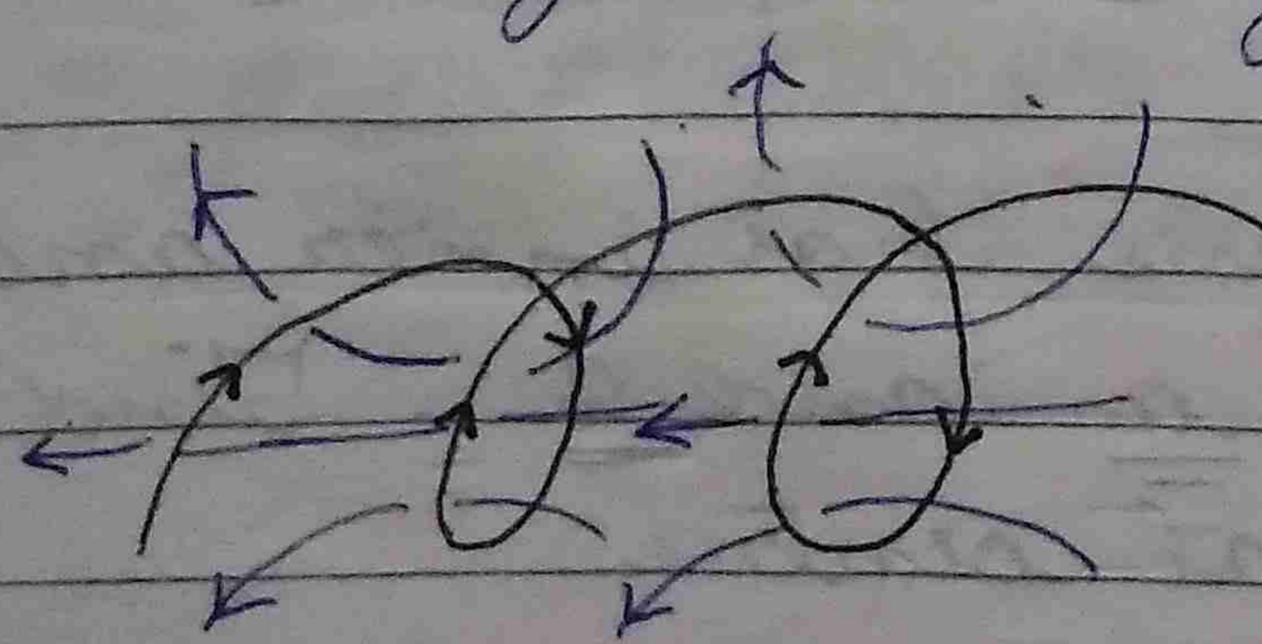
- <u>Solenoid</u>



- Ideal solenoid

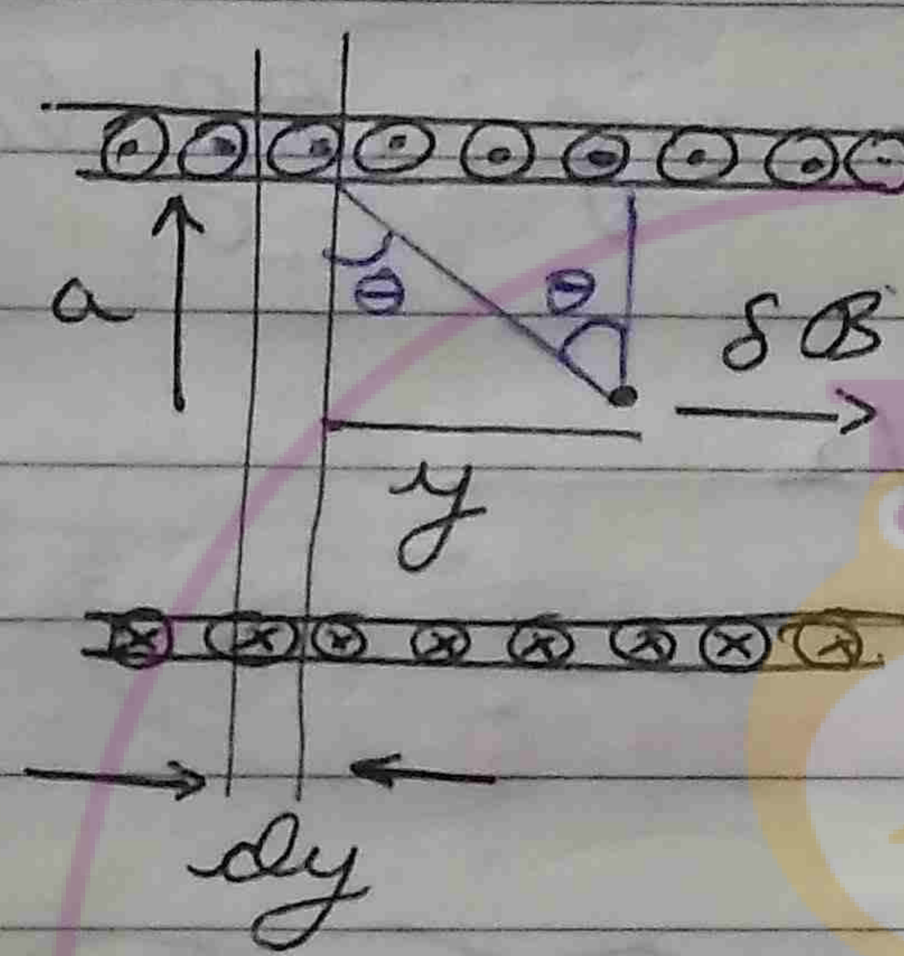
(i) $l >>>> a$

(ii) windings are tight.



- If the solenoid is ideal.

(i) field inside is uniform.
(ii) field outside is zero.



- Let n be no. of turns per unit length.

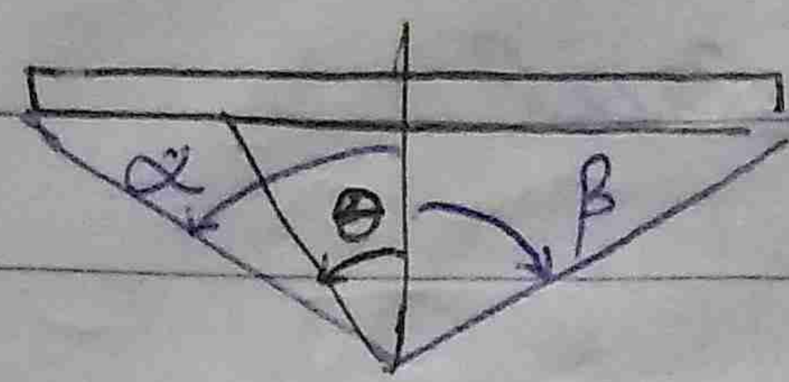
$\delta N = n\,dy$

$$\delta B = \frac{\mu_0 (\delta N) I a^2}{2(a^2+y^2)^{3/2}} = \frac{\mu_0 n I a^2}{2} \frac{dy}{(a^2+y^2)^{3/2}}$$

$\tan\theta = \frac{y}{a}$ $\quad y = a\tan\theta$ $\quad dy = a\sec^2\theta\, d\theta$

$$\delta B = \frac{\mu_0 n I a^2}{2} \frac{dy}{(a^2+y^2)^{3/2}} = \frac{\mu_0 n I a^2}{2} \frac{a\sec^2\theta\, d\theta}{(a^2+a^2\tan^2\theta)^{3/2}}$$

$$\delta B = \frac{\mu_0 n I a^3}{2} \frac{\sec^2\theta\, d\theta}{a^3\sec^3\theta} = \delta B = \frac{\mu_0 n I}{2} \cos\theta\, d\theta$$



$$B = \frac{\mu_0 n I}{2} \int_{-\beta}^{\alpha} \cos\theta \, d\theta$$

$$\Rightarrow \quad B = \frac{\mu_0 n I}{2}\left[\sin\alpha + \sin\beta\right]$$

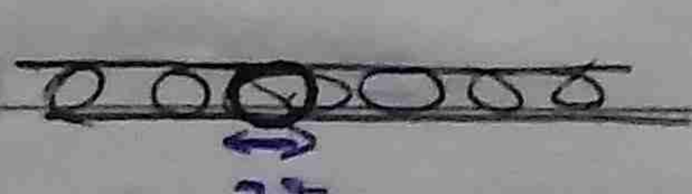
- At mid-point of an infinitely long <u>solenoid</u>:

$$\alpha = \beta \longrightarrow \pi/2$$

$$\Rightarrow \quad B = \mu_0 n I$$

☆ e.g. A <u>wire</u> of radius 'r', is wound over a solenoid (ideal). Find the magnetic field inside, if current is i

$$B = \mu_0 n i = \mu_0 \frac{1}{2r} i$$



<u>Amperes circuital law</u>

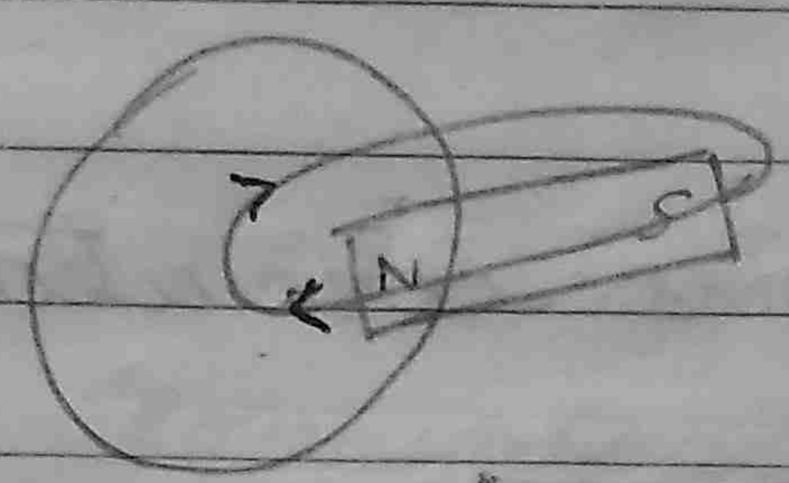
$$\oint \bar{B} \cdot \bar{dl} = \mu_0 I_{\text{inclosed}}$$

: only applicable for steady <u>current</u>.

★ Ampere's circuit law is valid only for steady current.

• Gauss's law in magnetism

$$\boxed{\oint \bar{B}\cdot \delta s = 0}$$ → dipolar magnet



$$\oint \bar{E}\cdot \delta \bar{s} = \frac{q}{\varepsilon_0}$$

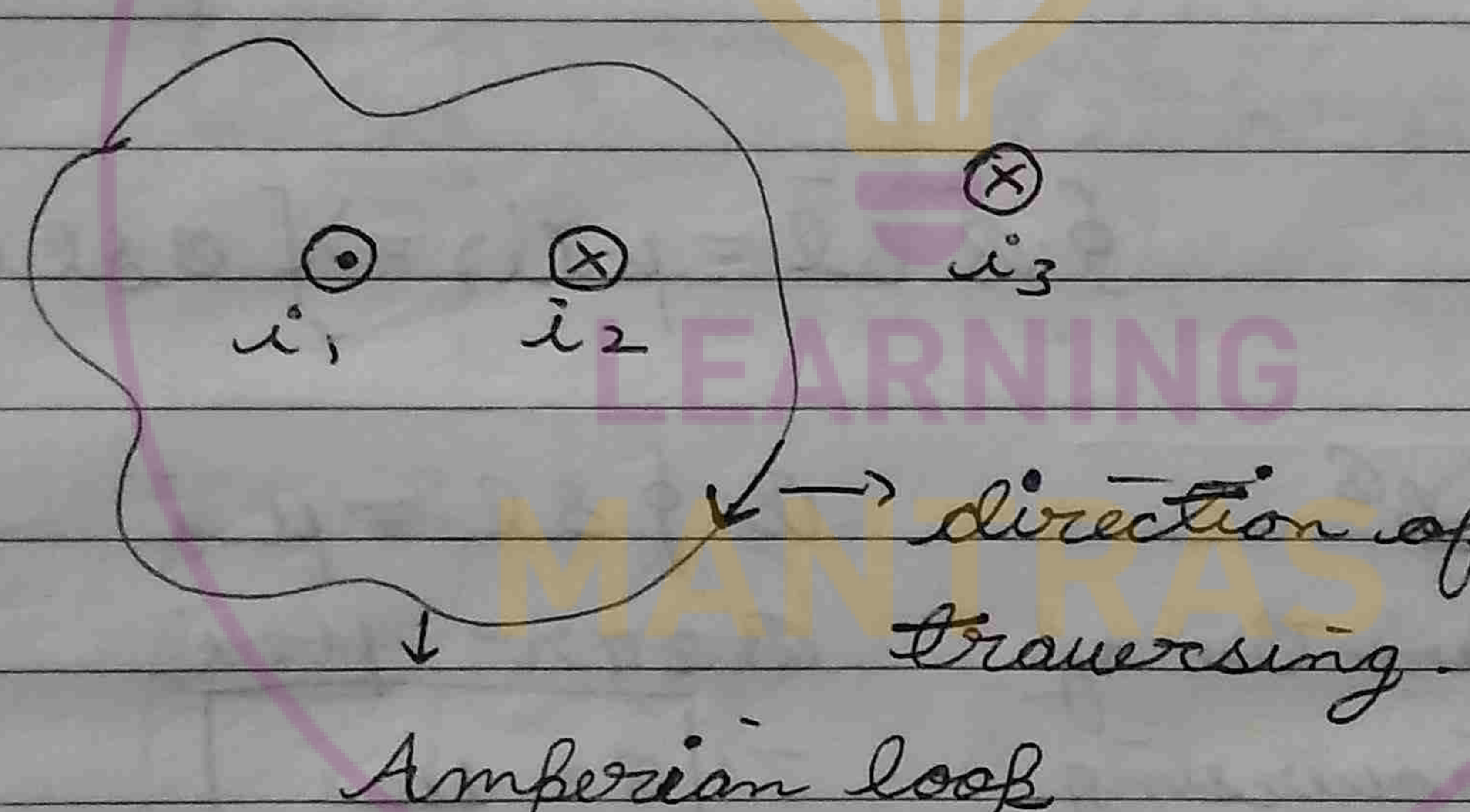
$$\oint \bar{E}\cdot \delta \bar{l} = 0$$

$$\oint \bar{B}\cdot \delta \bar{s} = 0$$

$$\oint \bar{B}\cdot \bar{dl} = \mu_0 I_{enclosed}$$

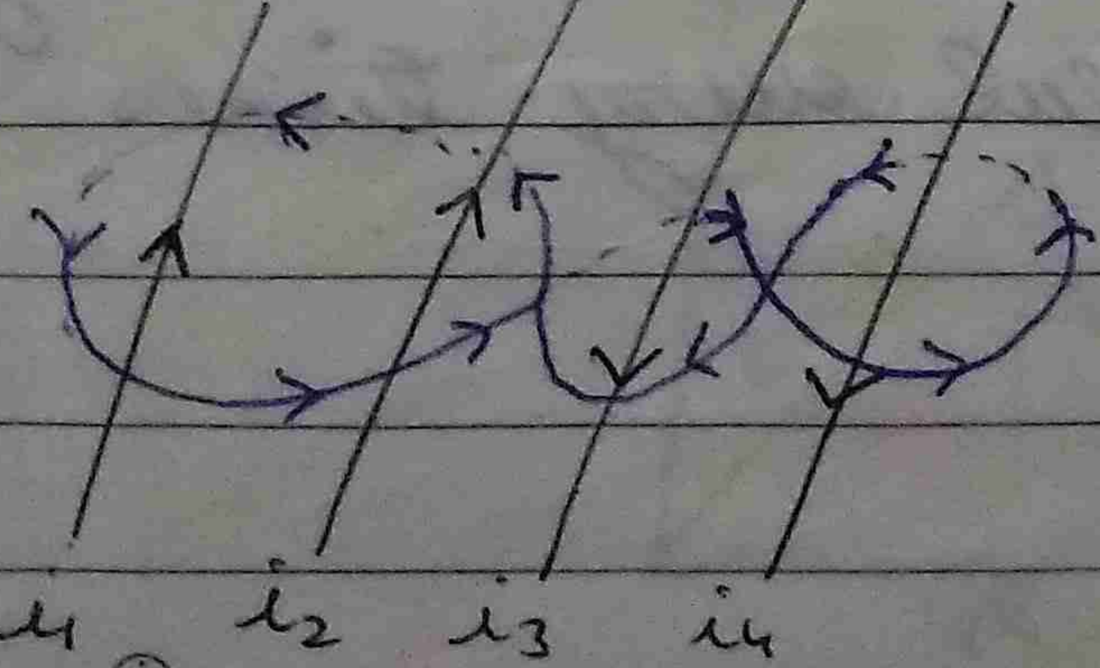
$$\oint \bar{B}\cdot \delta l = \mu_0 I_{enclosed}$$

$\bar{B}$ → field due to all

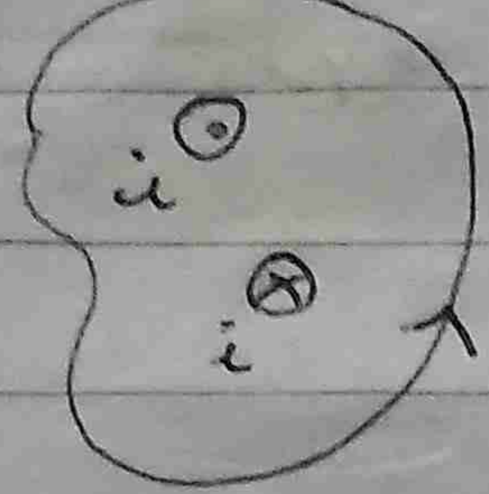


$$\oint \bar{B}\cdot \bar{\delta l} = \mu_0[-i_1 + i_2]$$

• If the field due to current is in the same direction of traversing the current is taken as +ve



$$\oint \bar{B}\cdot \bar{\delta l} = \mu_0[i_1 + i_2 + i_3 - i_4]$$



- ★ $\oint \bar{B}.\bar{\delta l} = 0$

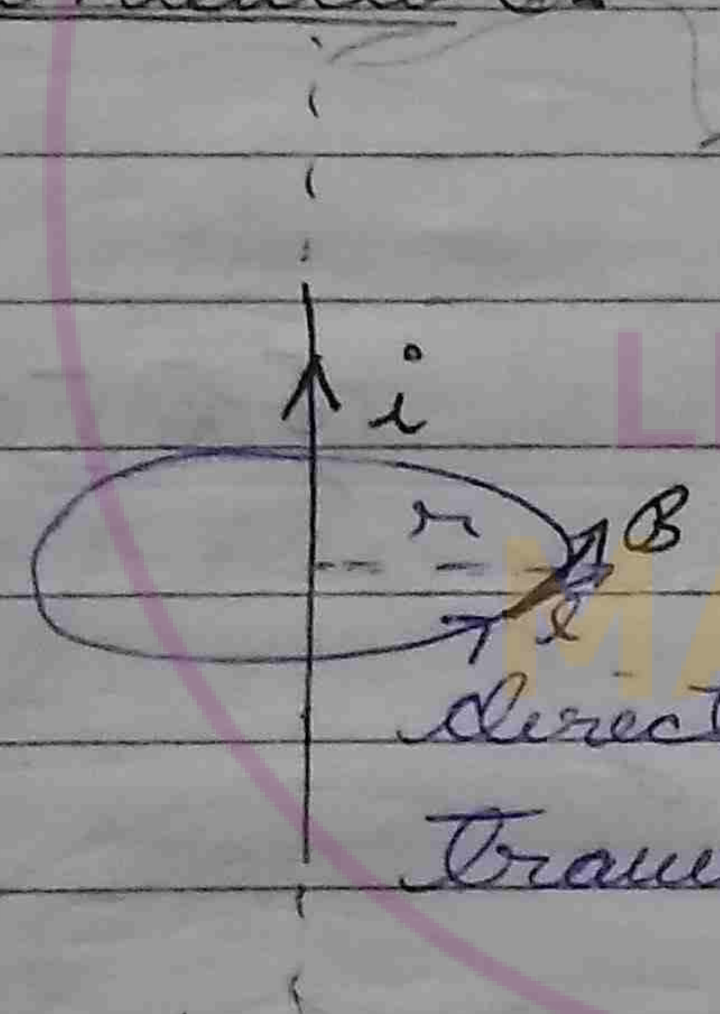
this does not mean that $\underline{B}$ is zero.

- Application of Amperes circuital law.

$$\boxed{\oint \bar{B}.\bar{dl} = \mu_0 I_{enclosed}}$$ I -- steady

- Due to a thin straight infinitely long conductor.

→ short wire very far away

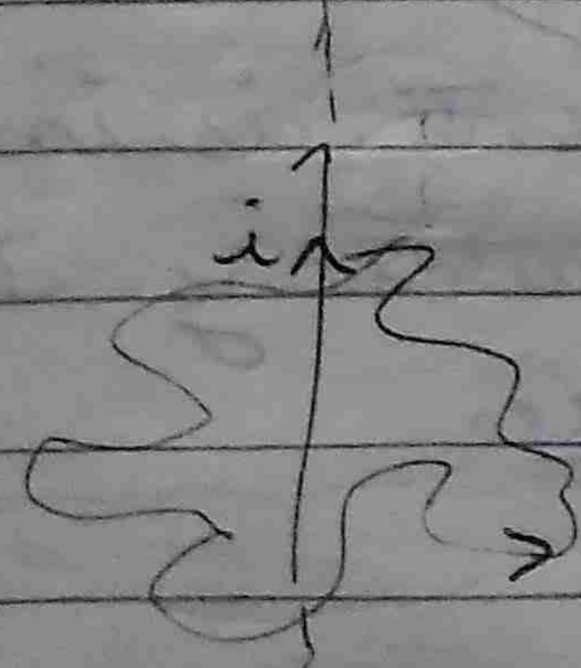


$\oint \bar{B}.\bar{\delta l} = \mu_0(i) \Rightarrow \oint B \delta l \cos 0 = \mu_0 i$

$B \oint \delta l = \mu_0 i$

$B\, 2\pi r = \mu_0 i$

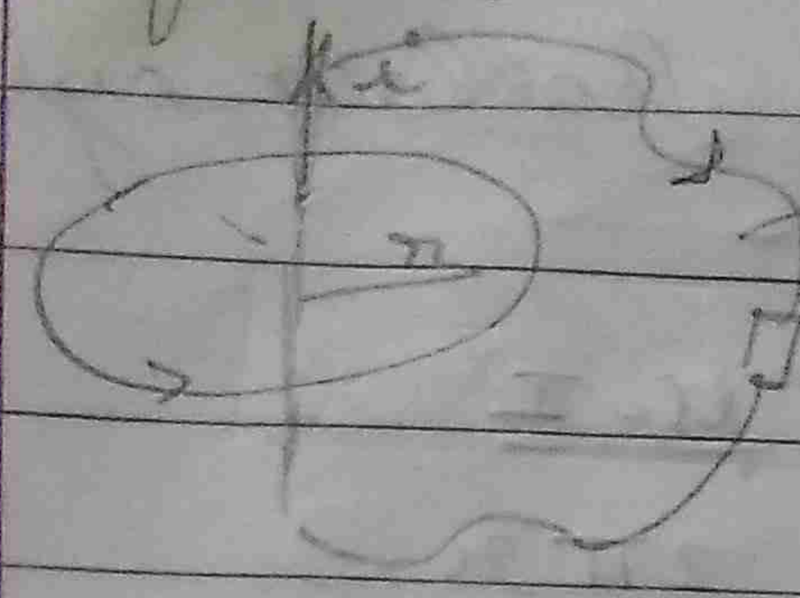
$\Rightarrow \boxed{B = \frac{\mu_0 i}{2\pi r}}$



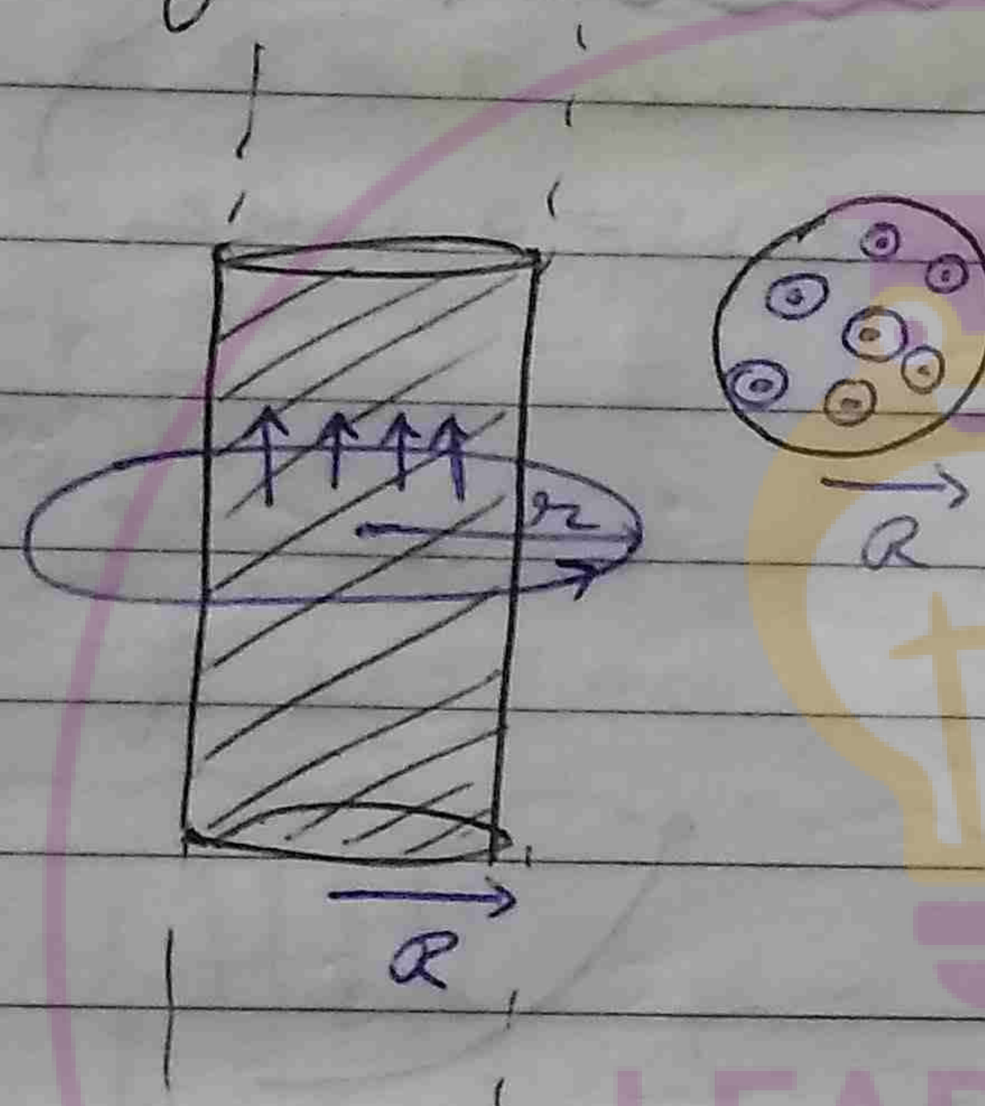
$\oint \bar{B}.\bar{dl} = \mu_0 i$ --- valid

- Amperes law is valid everywhere but not useful every time.

find ...



- $\bar{B}$ due to infinite **solid cylinder** carrying uniform current.



(i) $r > R$

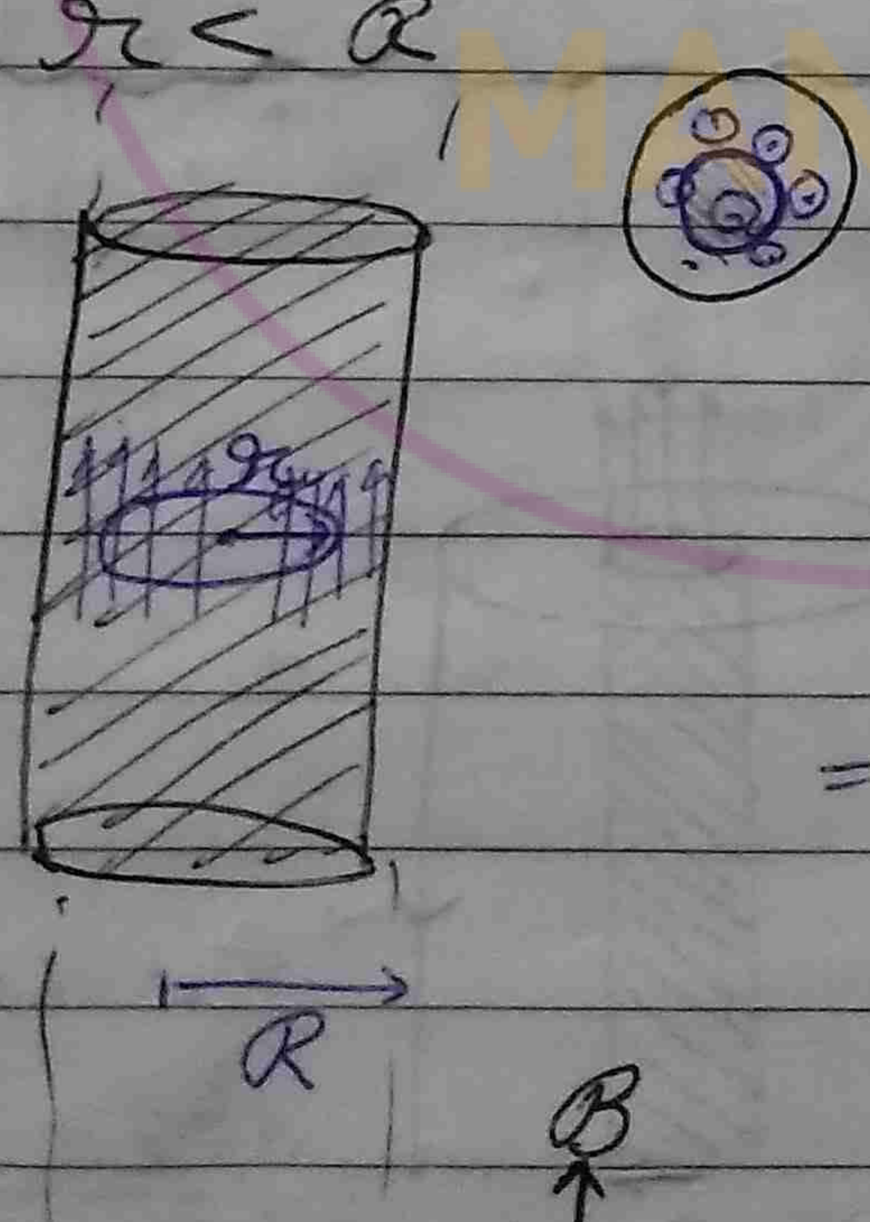
$$\oint \bar{B}\cdot\bar{\delta l} = \mu_0 I$$

$$\oint B\,\delta l \cos 0^\circ = \mu_0 I$$

$$B \int \delta l = \mu_0 I$$

$$B\,2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

(ii) $r < R$

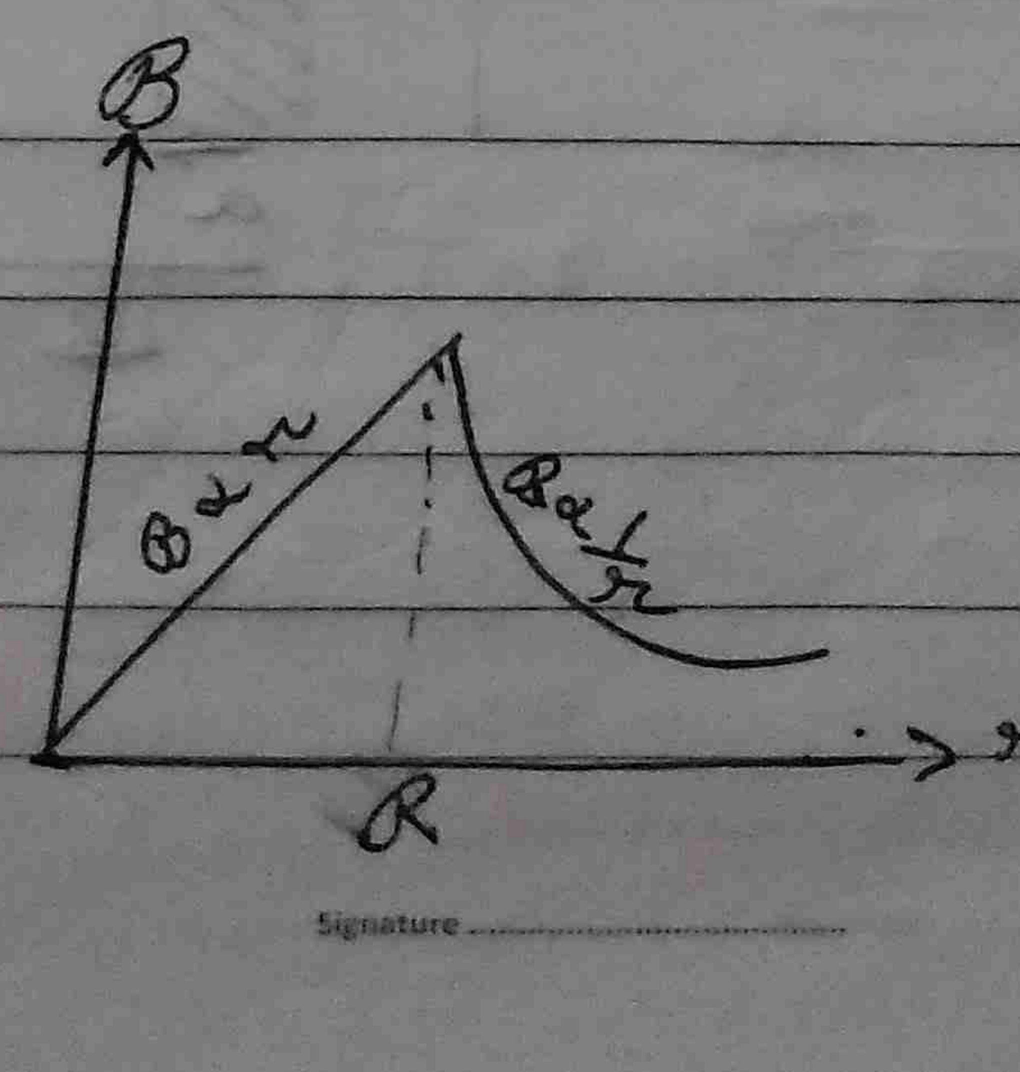


$$\oint \bar{B}\cdot\bar{\delta l} = \mu_0 I_{enclosed}$$

$$B\cdot 2\pi r = \mu_0 \frac{I}{\pi R^2} \times \pi r^2$$

$$\Rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

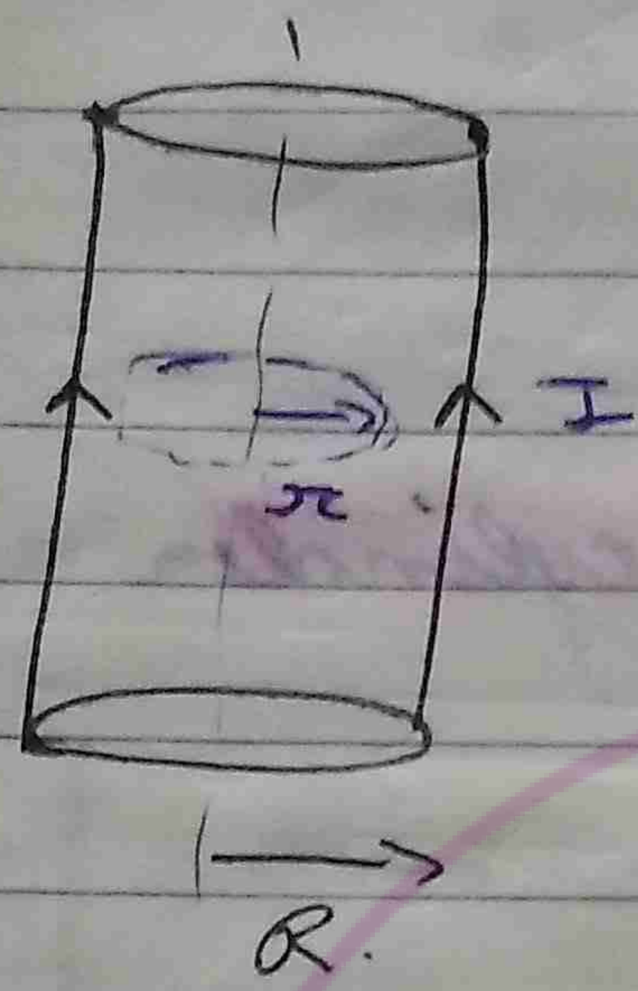
$r < R$; $B = \frac{\mu_0 I r}{2\pi R^2}$
$r > R$; $B = \frac{\mu_0 I}{2\pi r}$



★ Not discontinous

★ B = max at surface

- $\bar{B}$ due to infinitely long hollow cylinder

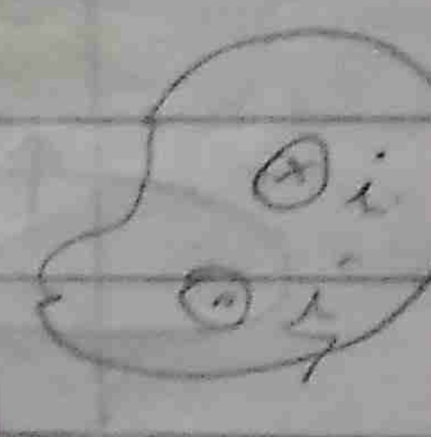


$r > R \quad B = \frac{\mu_0 I}{2\pi r}$

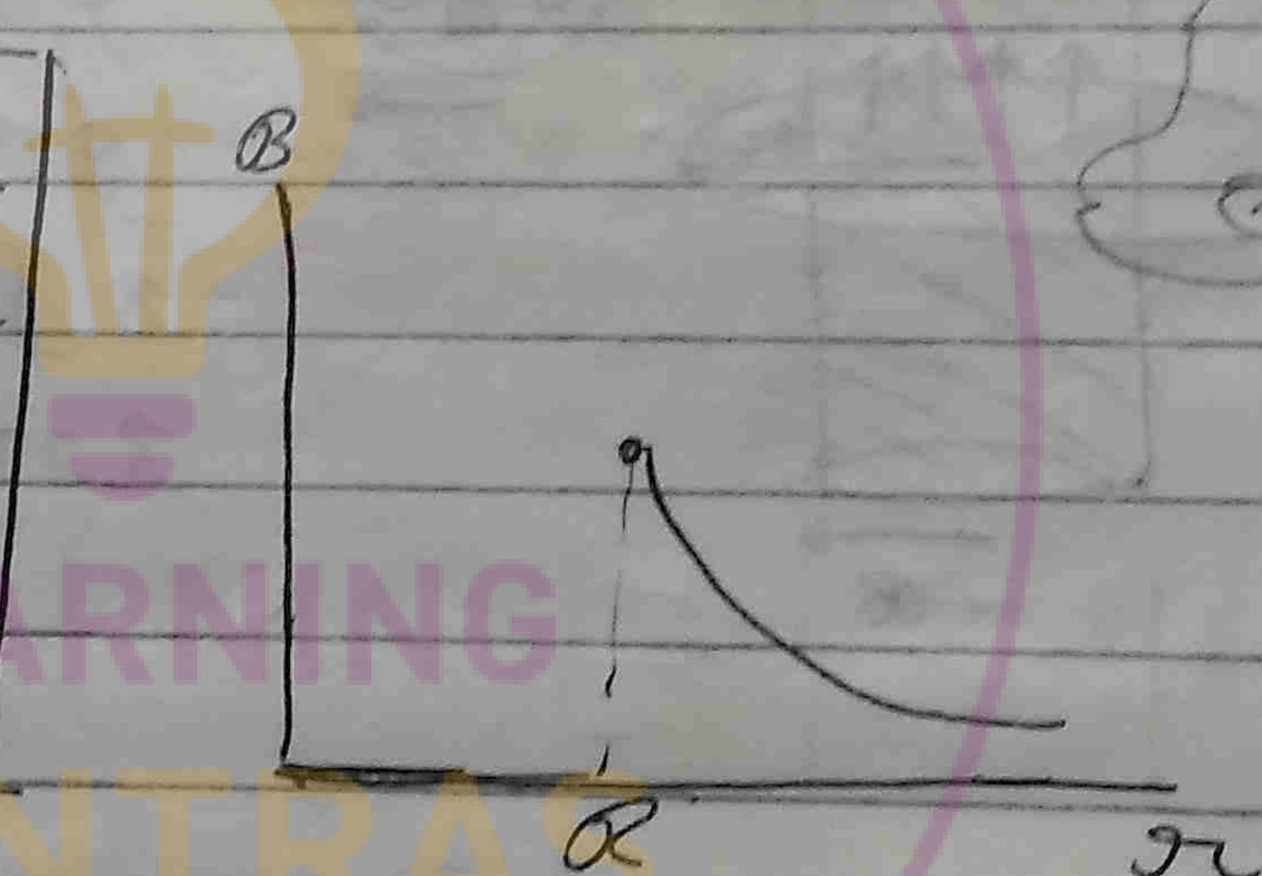
$r < R \quad \oint \bar{B}.\delta\bar{l} = 0$

*here $B = 0$

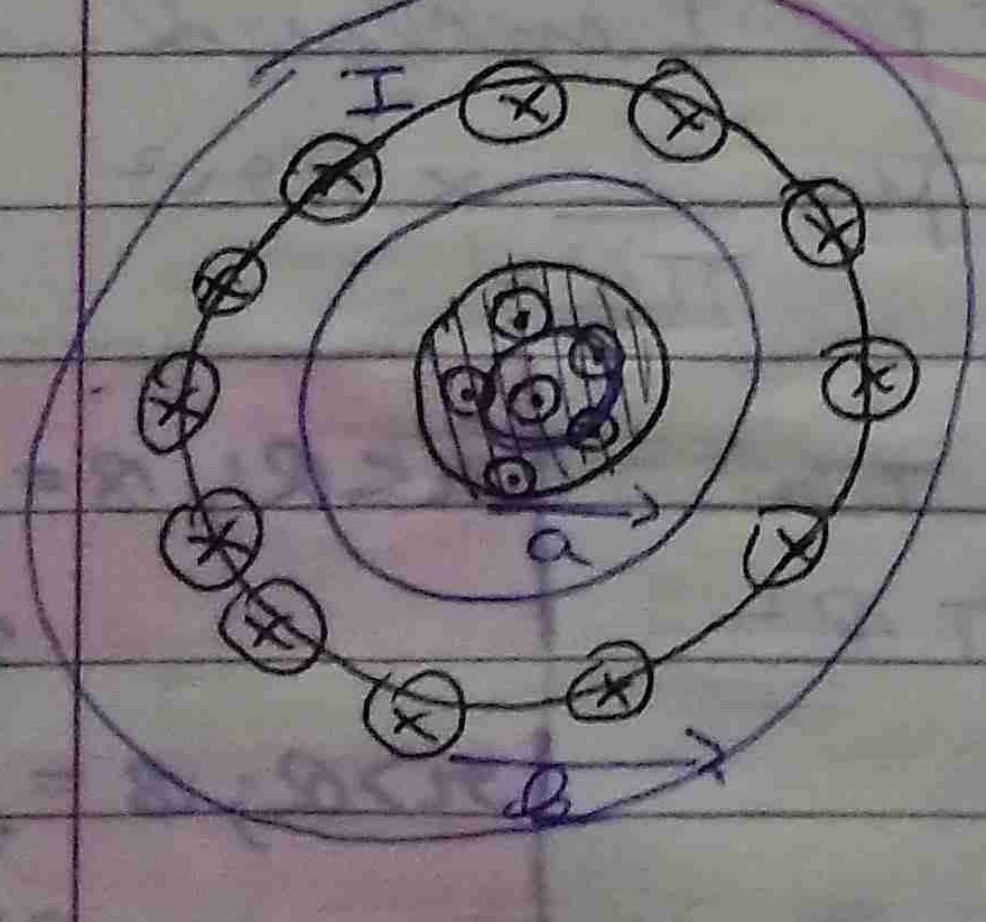
$\oint B\delta l \cos 0 = 0$ when

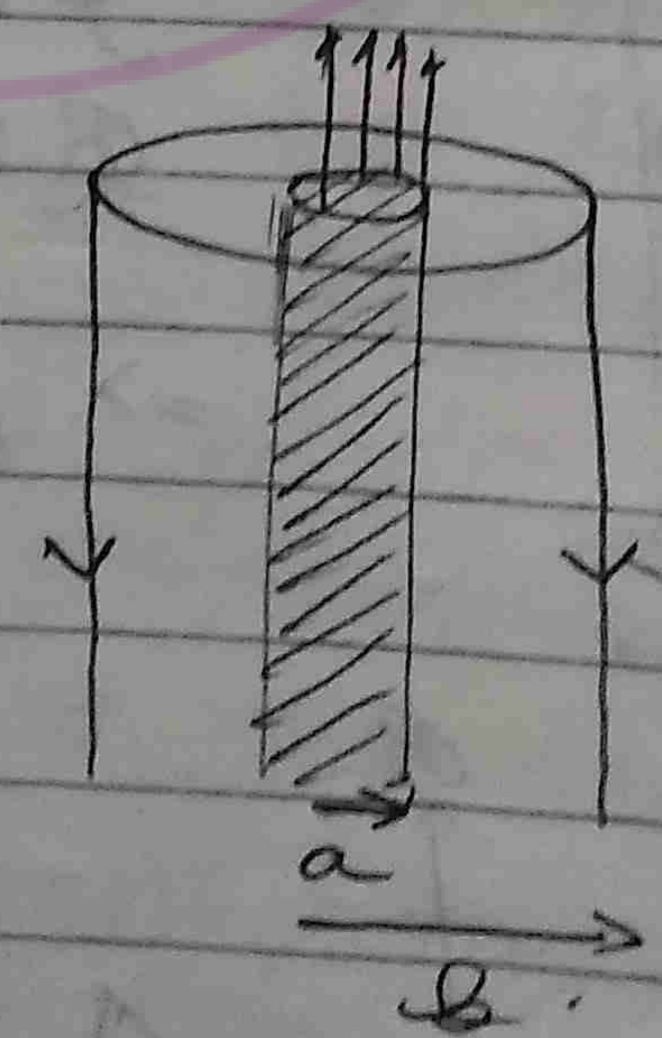


$r > R;\ B = \frac{\mu_0 I}{2\pi r}$
$r < R;\ B = 0$



- Co-Axial Cabale





I is same but opposite in direction.

(i) $r < a$

$$\oint \bar{B}\cdot\delta\bar{l} = \mu_0 \frac{I}{\pi a^2} \times \pi r^2 \Rightarrow B \times 2\pi r = \frac{\mu_0 I r^2}{a^2}$$

$$= \frac{\mu_0 I r}{2\pi a^2}$$

(ii) $a < r < b$.

$$\oint B\cdot\delta l = \mu_0 I \Rightarrow B 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

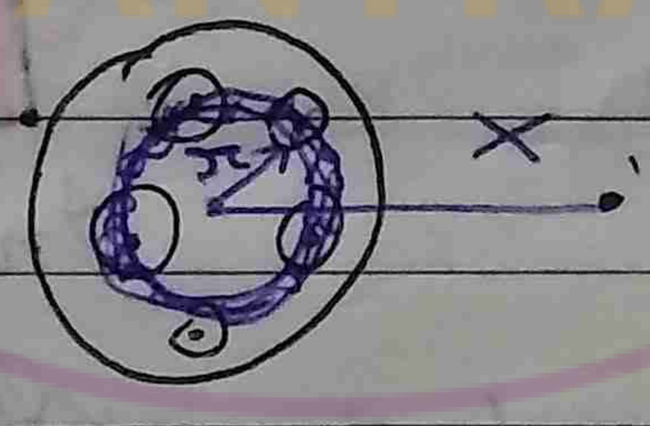
(iii) $r > b$

$$\oint B\cdot\delta l = 0 \quad \Rightarrow B = 0$$

---

e.g

$\bar{J} = \bar{J_0} r \; Am^{-2} \; [0 \le r \le a]$



(i) $x > A$

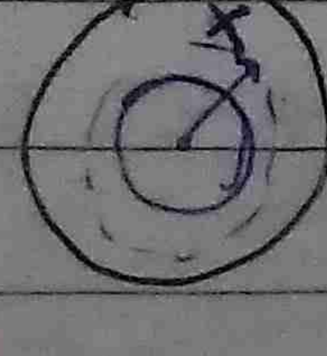
$$\oint \bar{B}\cdot\bar{\delta L} = \mu_0 I_{enclosed}$$

$$B 2\pi x = \mu_0 \int J \delta s$$

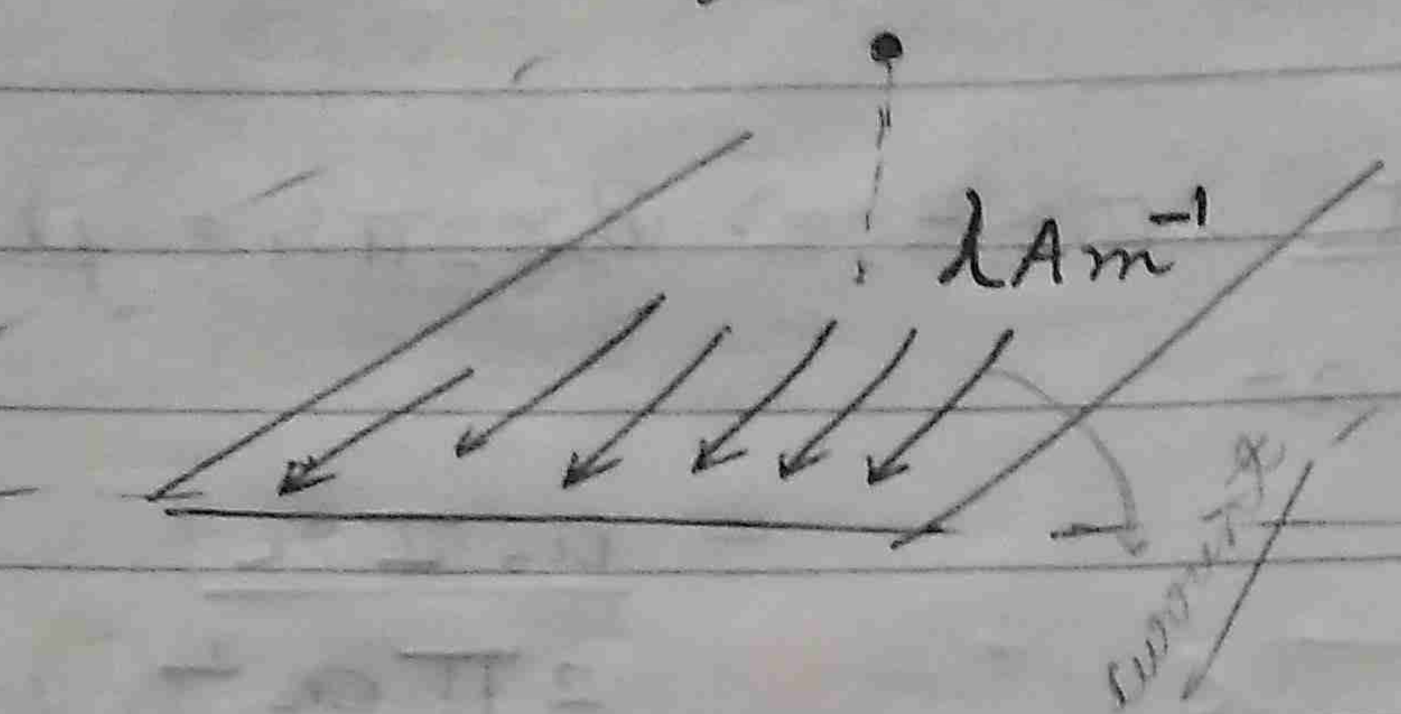
$$= \mu_0 \int_0^a J_0 r \, 2\pi r \, dr$$

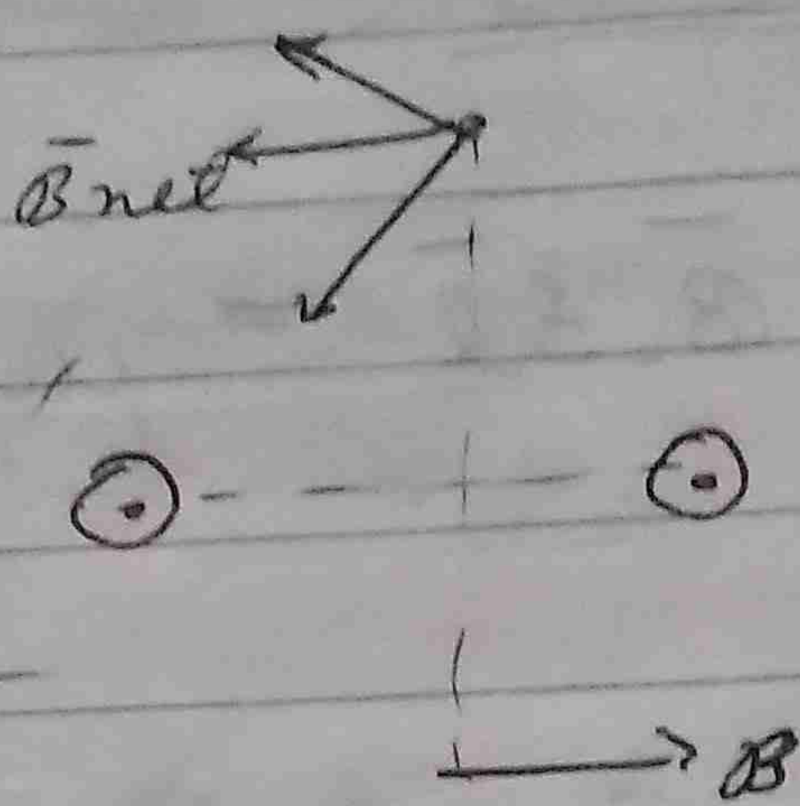
(ii) $x < A$

$$\oint \bar{B}\cdot\delta\bar{l} = \mu_0 \int J \delta s \Rightarrow B 2\pi x = \mu_0 \int_0^x J_0 r \, 2\pi r \, dr$$

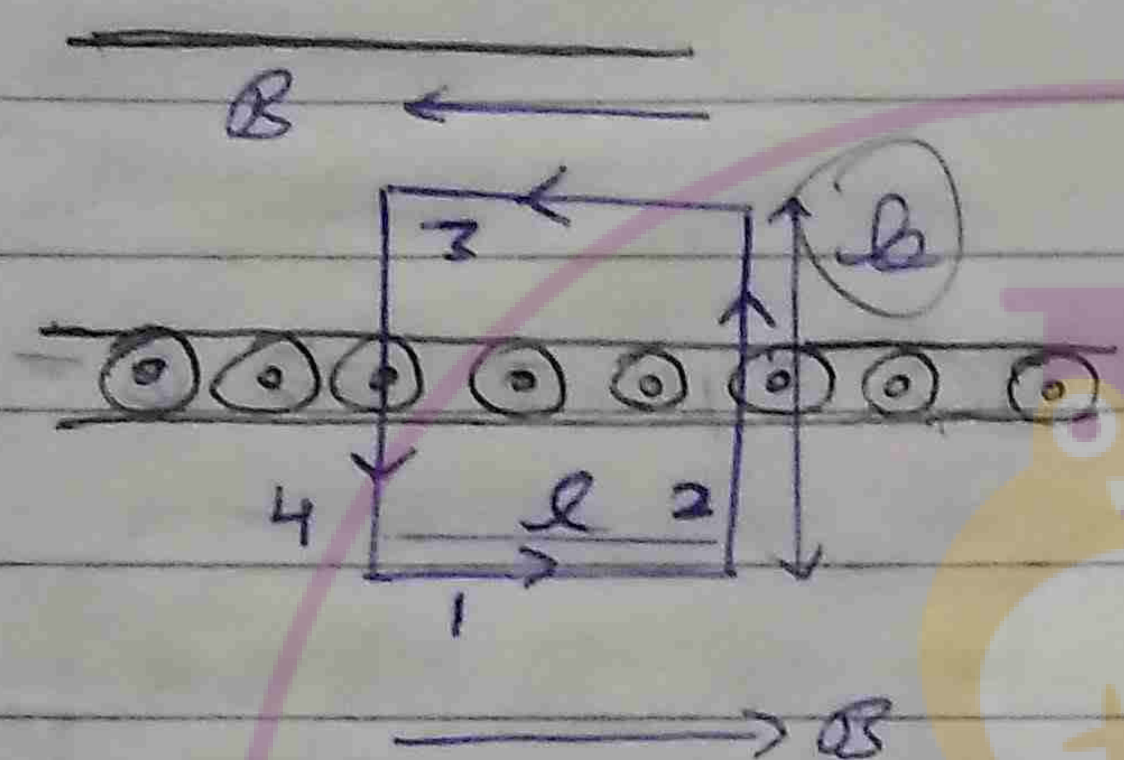


Due to infinite sheet





λ = surface current density



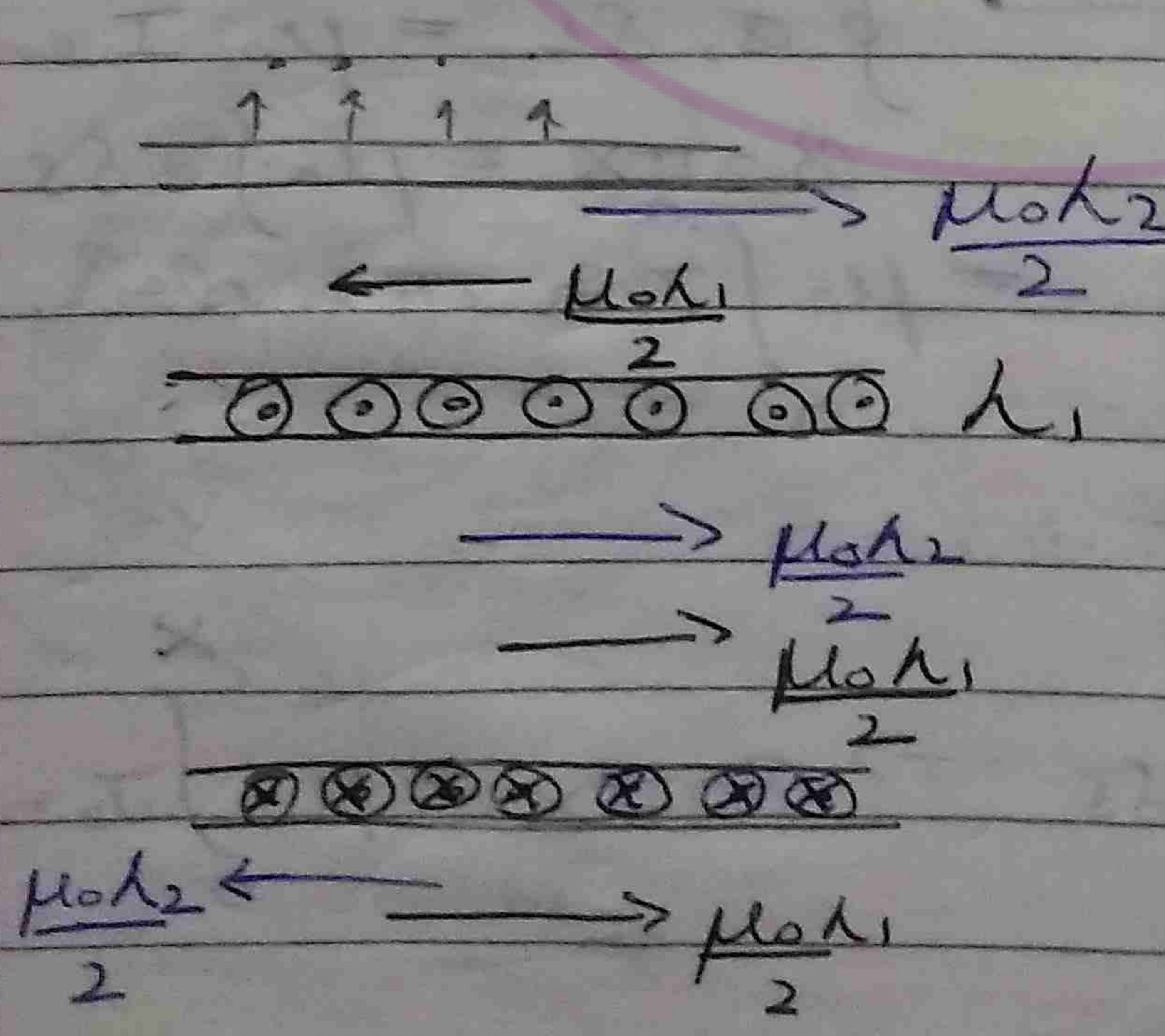
$$\oint \bar{B}.\delta\bar{l} = \mu_0 \lambda l$$

$$\int_1 \bar{B}.\delta\bar{l} + \int_2 \bar{B}.\delta\bar{l} + \int_3 \bar{B}.\delta\bar{l} + \int_4 \bar{B}.\delta\bar{l} = \mu_0 \lambda l$$

$$\bar{B}_1.\bar{l}_1 + \bar{B}_3.\bar{l} = \mu_0 \lambda l$$

$$\Rightarrow Bl + Bl = \lambda \mu_0 l \Rightarrow \boxed{B = \frac{\mu_0 \lambda}{2}}$$ independent of B

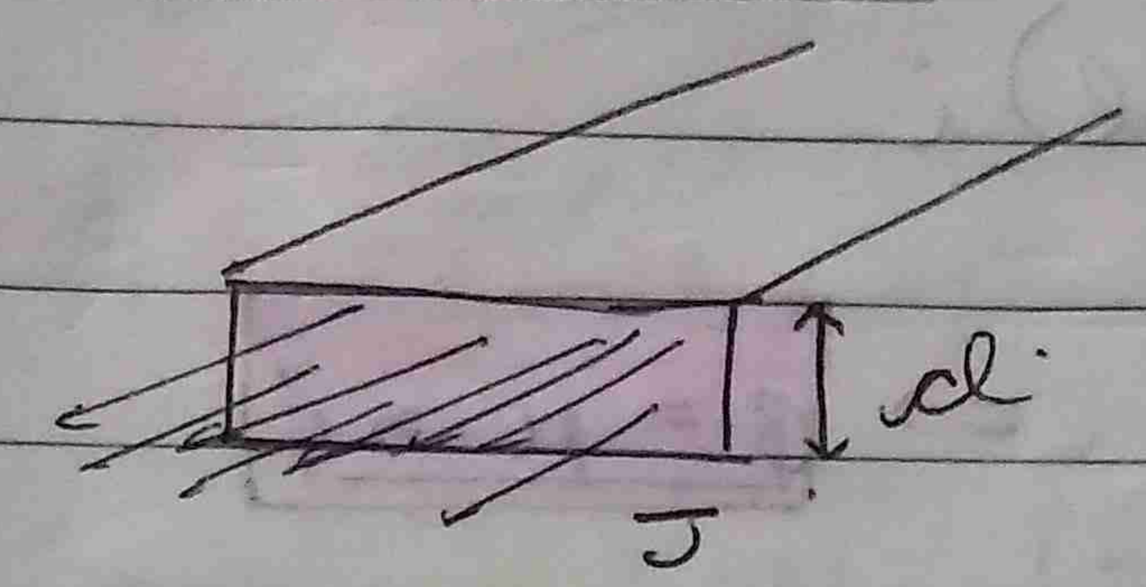
Independent of distance from the sheet.



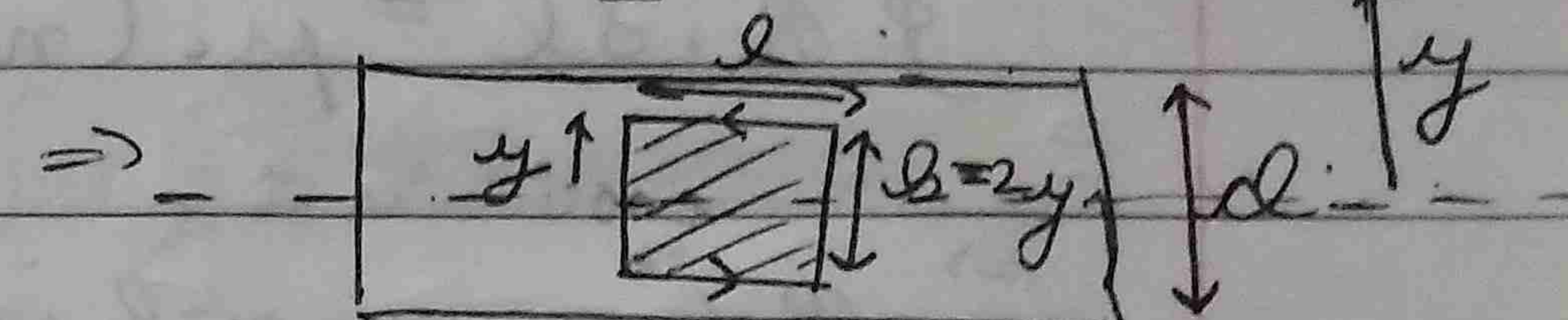
J = Volume current density.

current = J × area

- **Due to slab**



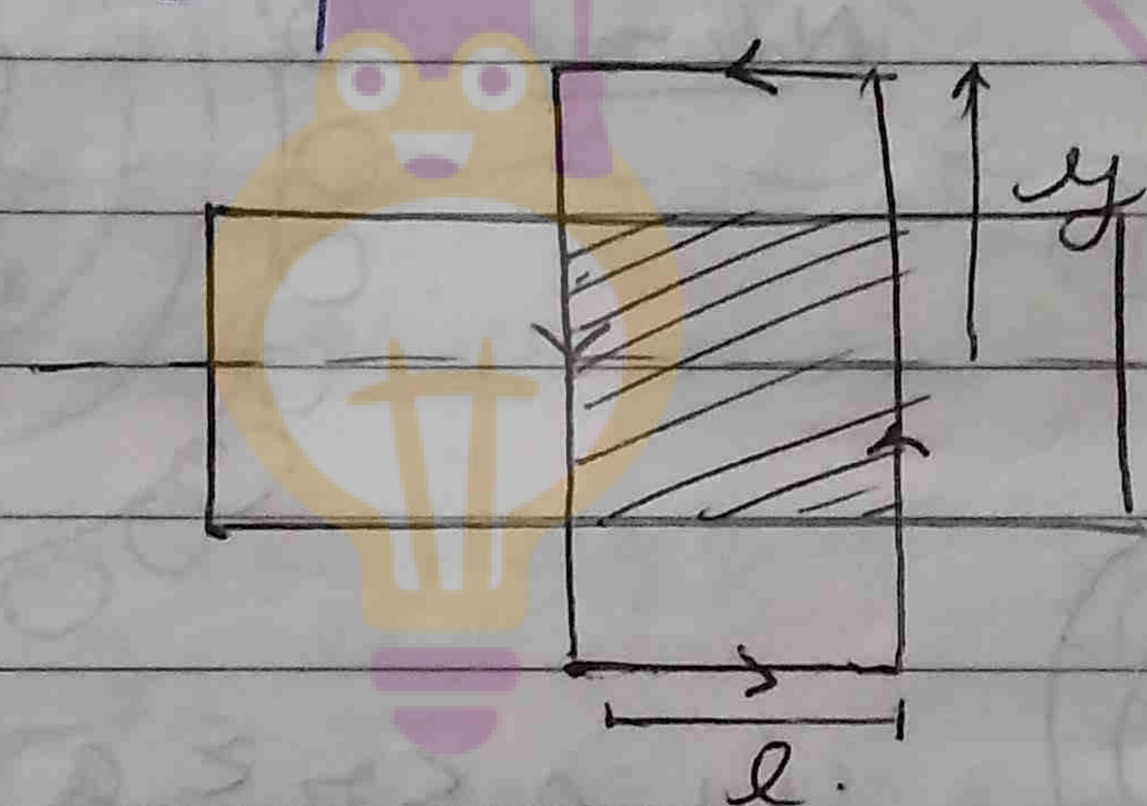
(i) $y < \frac{d}{2}$



$$\oint \bar{B}\cdot d\bar{l} = \mu_0 J l 2y \Rightarrow 2Bl = \mu_0 J 2ly$$

$$\Rightarrow \boxed{B = \mu_0 J y}$$

(ii) $y > \frac{d}{2}$



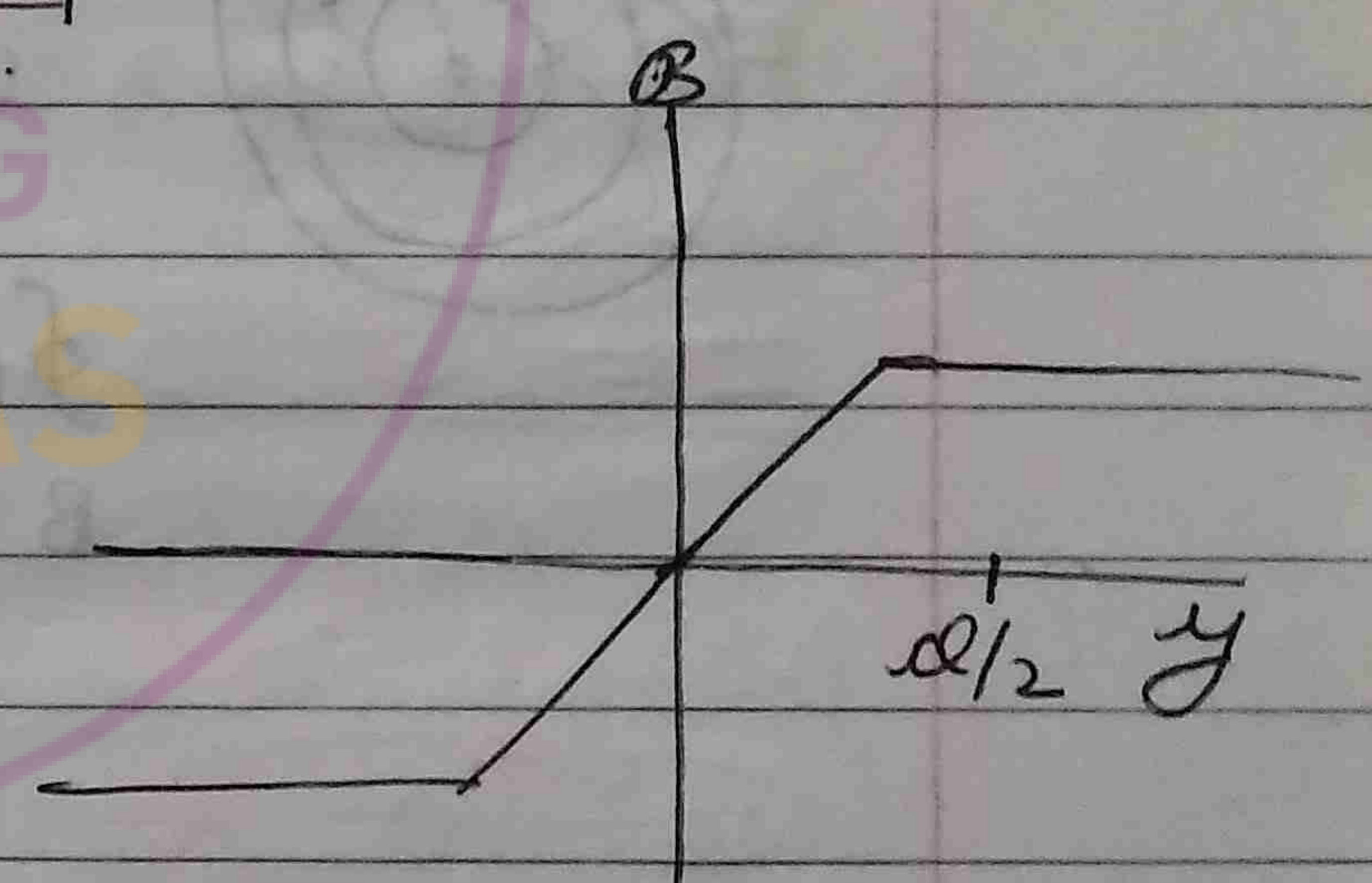
$$\oint \bar{B}\cdot d\bar{l} = \mu_0 J d l$$

$$2Bl = \mu_0 J d l$$

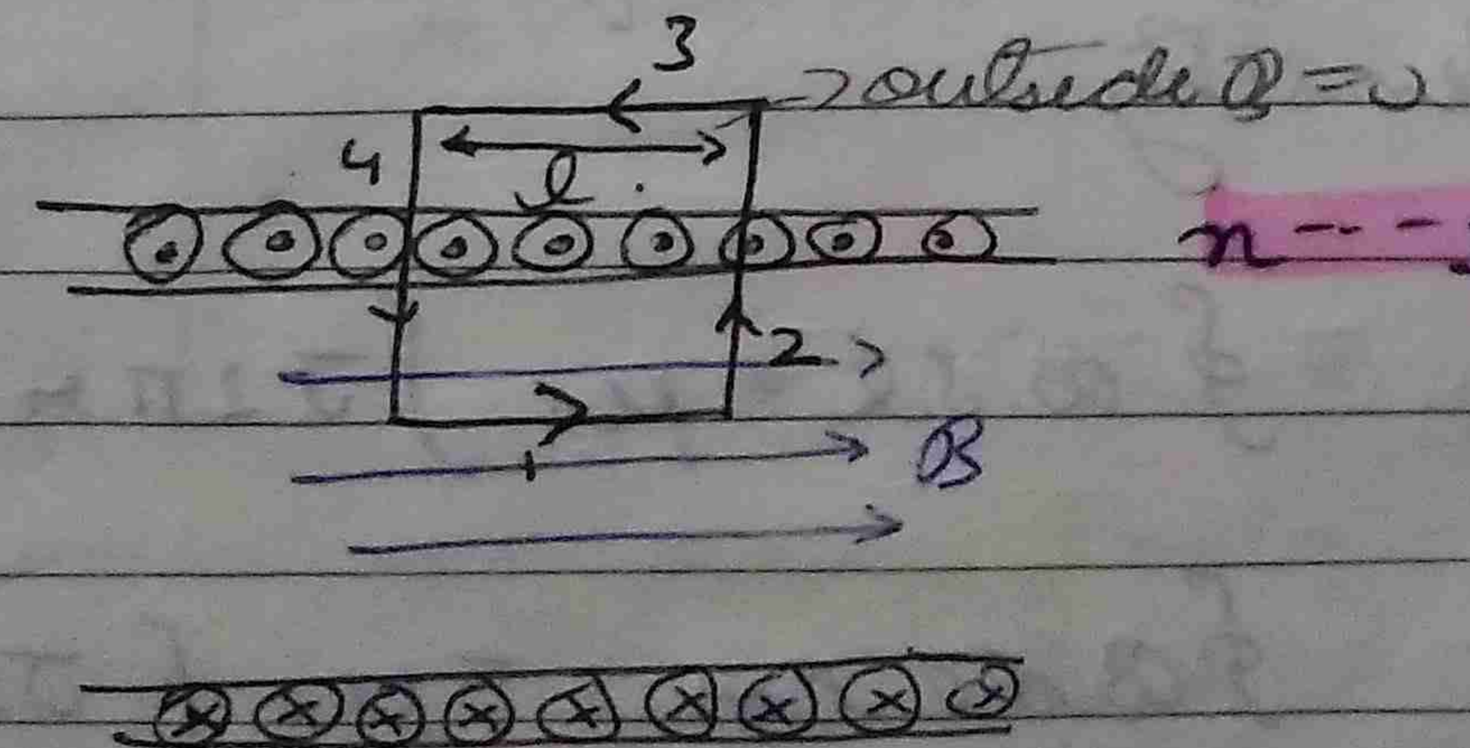
$$\Rightarrow \boxed{B = \frac{\mu_0 J d}{2}}$$

$y < \frac{d}{2}$; $B = \mu_0 J y$

$y \geq \frac{d}{2}$; $B = \frac{\mu_0 J d}{2}$



- **Due to solenoid (ideal)**

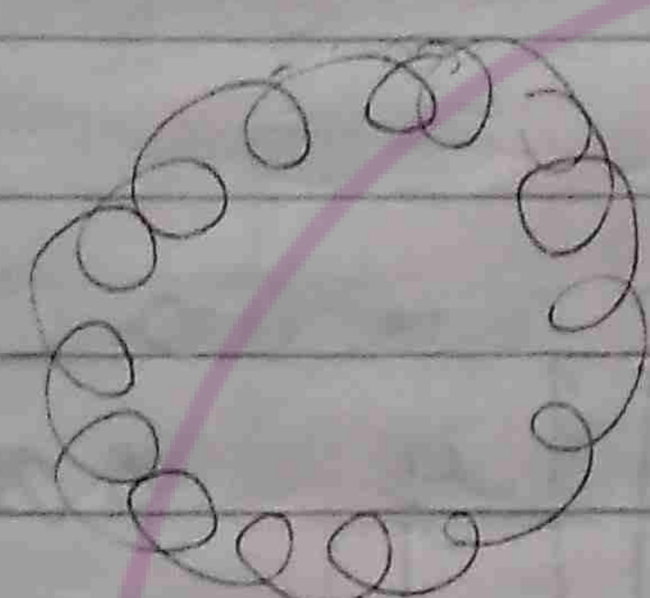


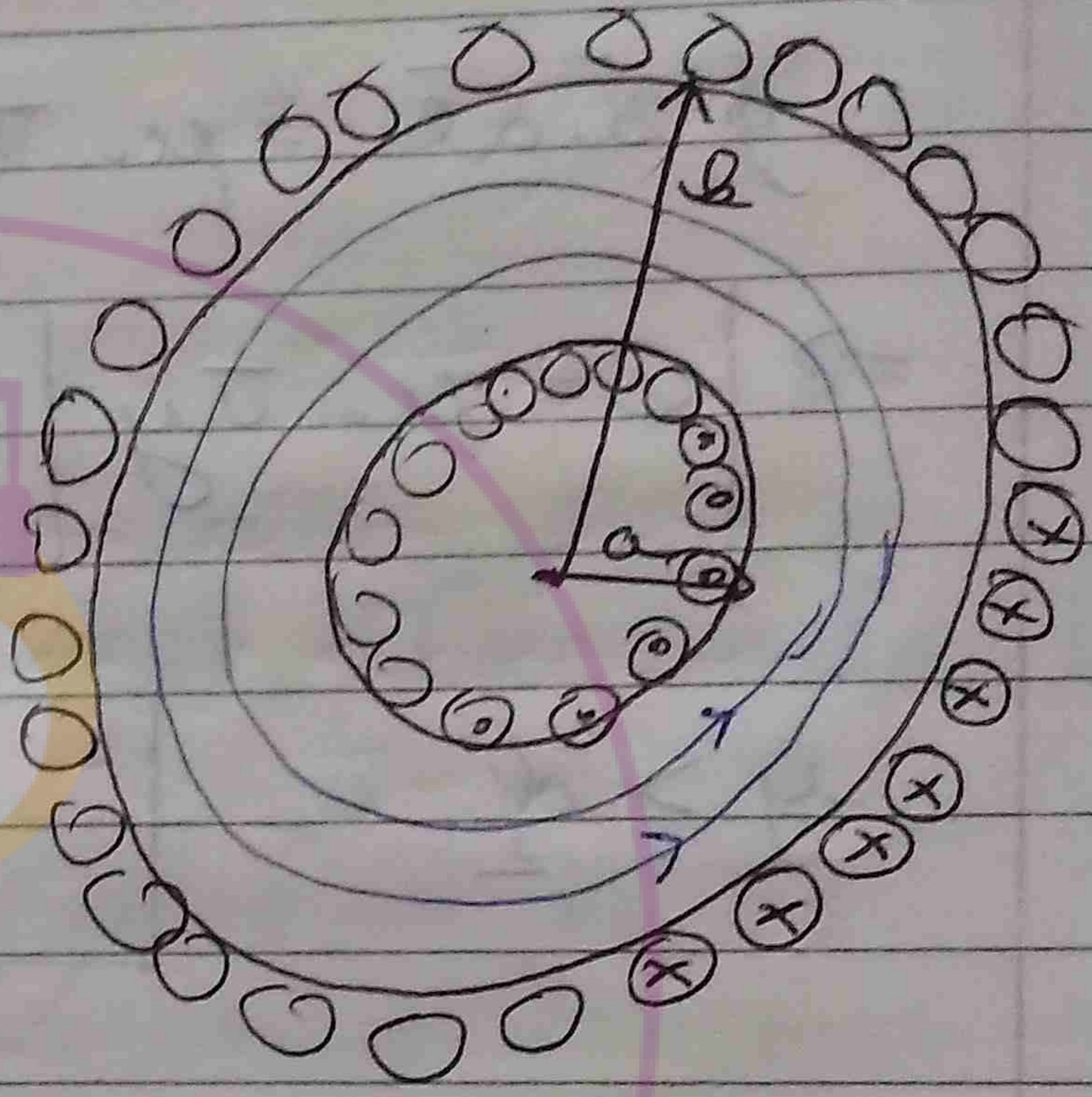
n --- no. of turns / length

$$\oint \bar{B} \cdot \delta l = \mu_0 (nl) i$$

$$\Rightarrow B l \cos 0^\circ = \mu_0 n l i \Rightarrow \boxed{B = \mu_0 n i}$$

- **Toroid**



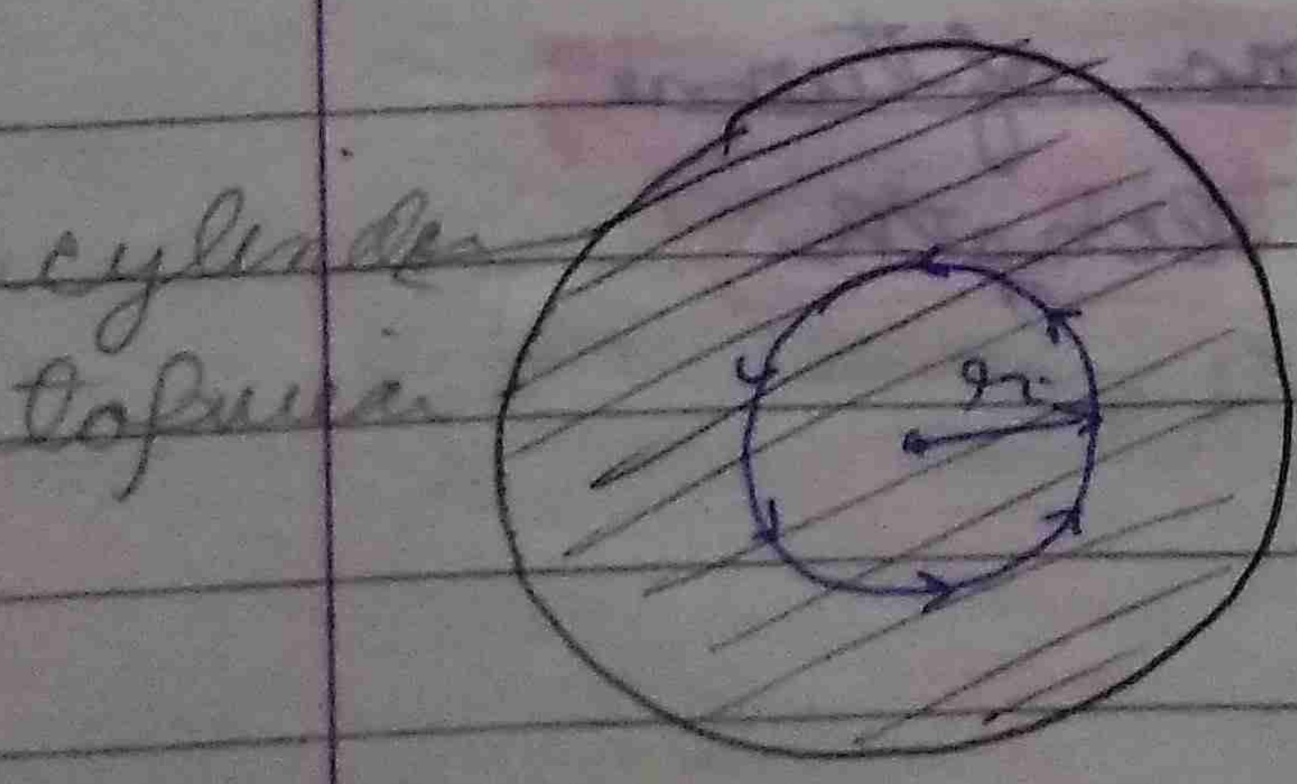


(i) $a < r < b$

$$\oint B \cdot \delta l = \mu_0 N i$$

$$B\, 2\pi r \cos 0^\circ = \mu_0 N i \Rightarrow \boxed{B = \frac{\mu_0 N i}{2\pi r}}$$

A radially symmetric magnetic field, $B = B_0 r^{\alpha}$ is present find the current density



$$\oint B \cdot \delta l = \mu_0 \int J\, 2\pi r\, dr$$

$$\oint B\, dl \cos 0 = \mu_0 \int J\, 2\pi r\, dr$$

$$B \oint dl = \mu_0 \int J 2\pi r \, dr$$

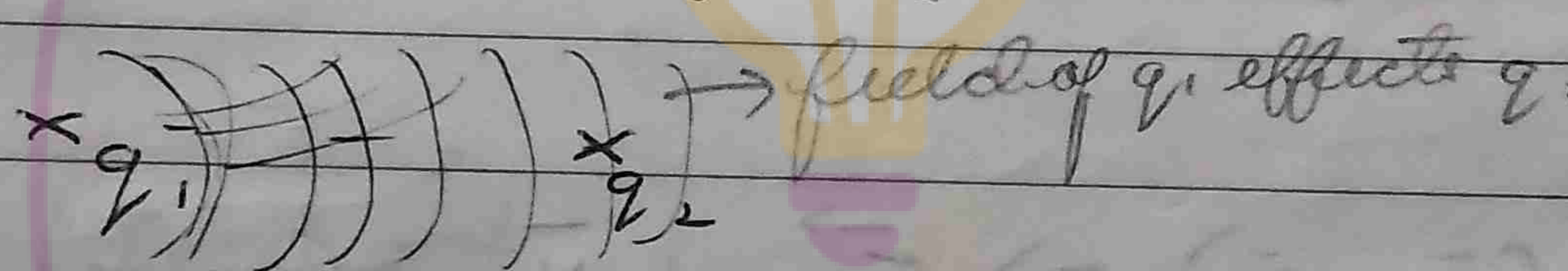
$$B_0 r^{\alpha} 2\pi r = \mu_0 \int J 2\pi r \, dr$$

$$B_0 r^{\alpha+1} = \mu_0 \int J r \, dr$$

differentiating w.r.t to r.

$$\Rightarrow B_0 (\alpha+1) r^{\alpha} = \mu_0 J r$$

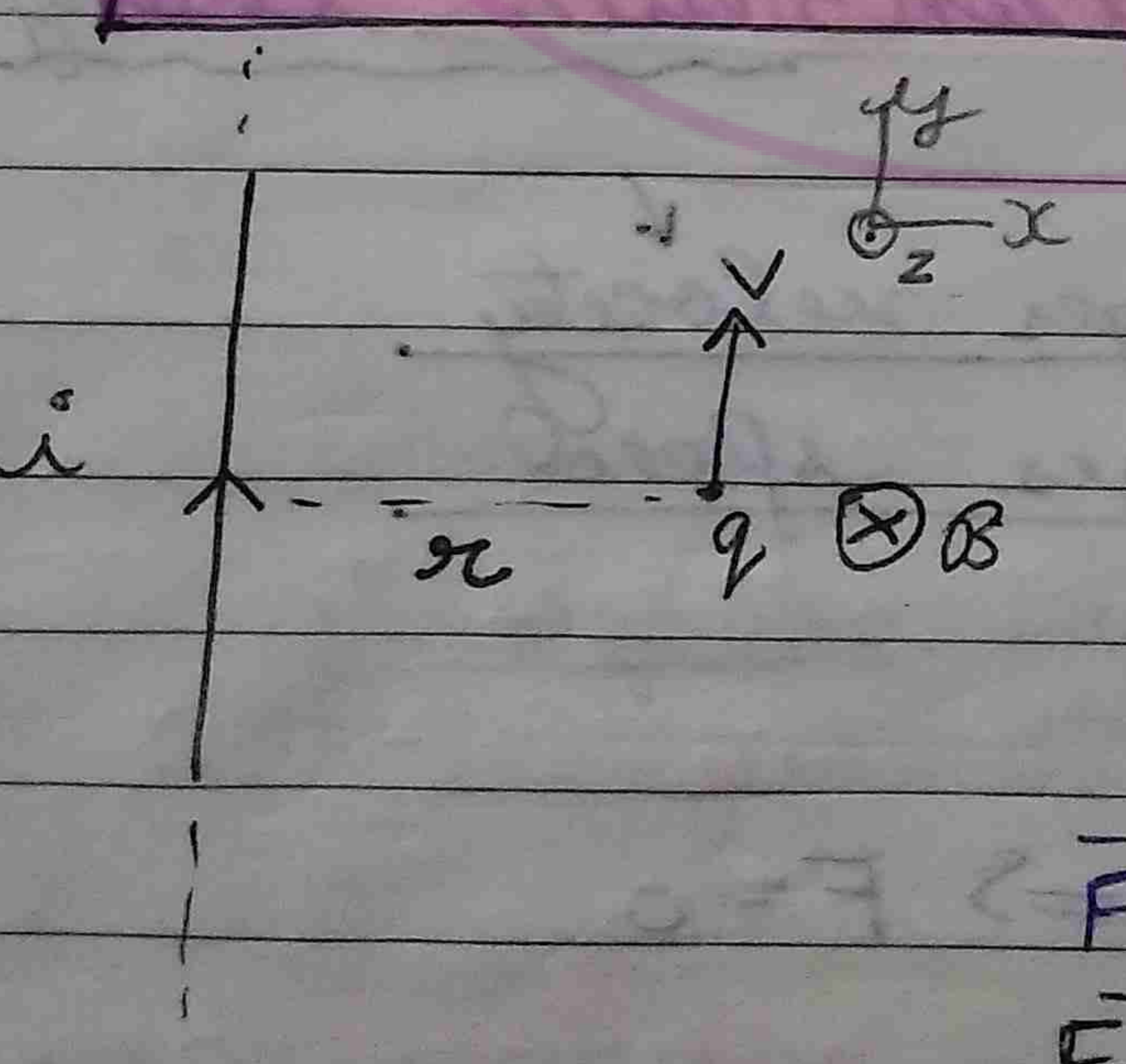
- **Magnetic force**



- Point charge

$$\boxed{\bar{F}_m = q(\bar{v} \times \bar{B})}$$

e.g



- Find the force on point charge.

$$\bar{F}_m = q(\bar{v} \times \bar{B})$$ (magnetic field due to wire)

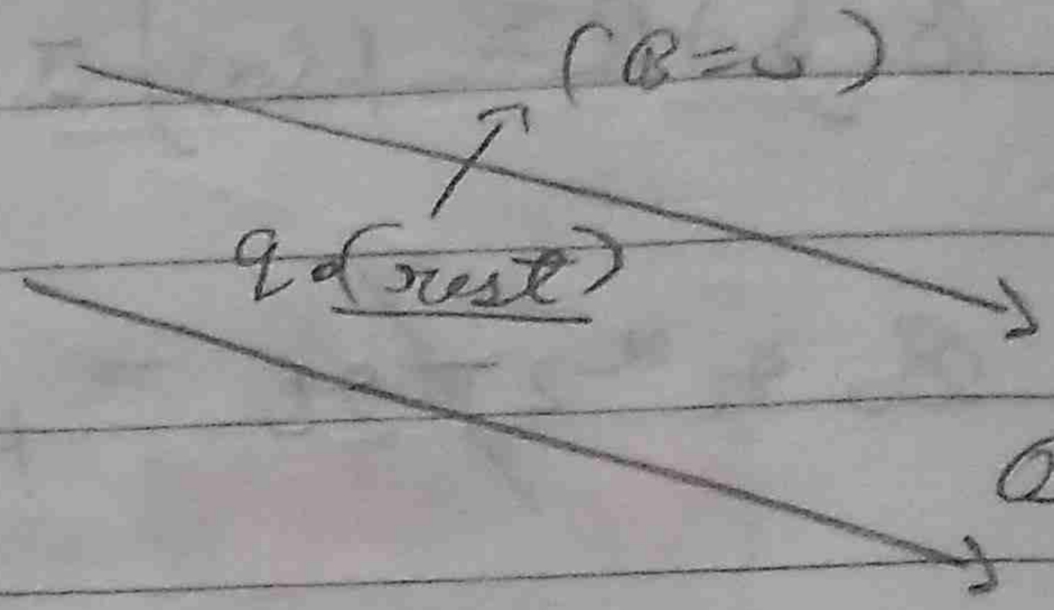
$$= q(v\hat{j} \times B(-\hat{k}))$$

$$\bar{F}_m = -qvB\,\hat{i}$$

$$\bar{F}_m = -\frac{qv\mu_0 i}{2\pi r}(\hat{i})$$

for force field of other is required

- Point charge



$\bar{F}_m = q(\bar{v} \times \bar{B})$

If $v = 0 \Rightarrow F_m = 0$

- A static charge will not experience force in magnetic field.

☆ A static charge neither produces magnetic field, nor experiences magnetic force in external field.

- $\bar{F}_m = q(\bar{v} \times \bar{B}) \Rightarrow \vec{F}_m \perp \bar{v}$ always.

Work done by magnetic force is always **ZERO**

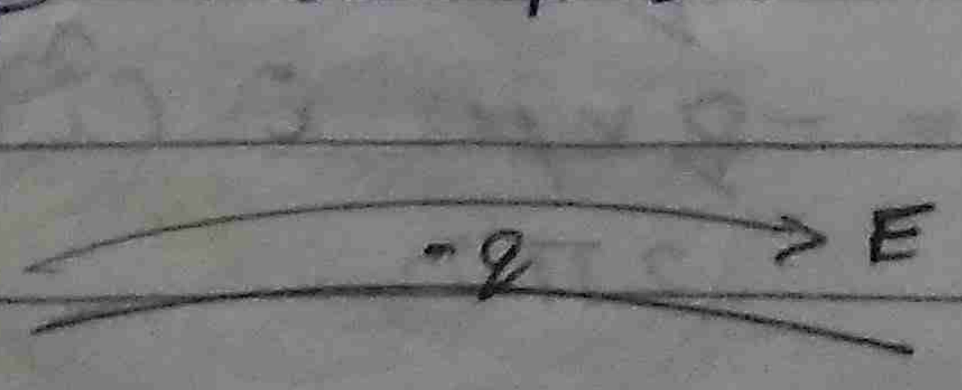
- Magnetic force can never change speed.

Force $\Rightarrow$ changes velocity

work $\Rightarrow$ changes speed.

$\Rightarrow$ only direction changes

- If $\bar{v}$ is $\parallel$ to $\bar{B}$ $\Rightarrow F = 0$



$F = qE$

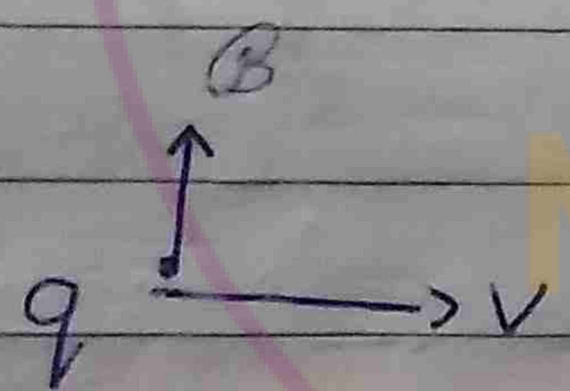
- In a given field (B)

☆ F = 0 → v = 0
→ $\theta = 0°, \underline{180°}$

- <u>Direction</u> → cross product
  - fleming left hand rule.

Thumb → magnetic force
index → field
middle → velocity.

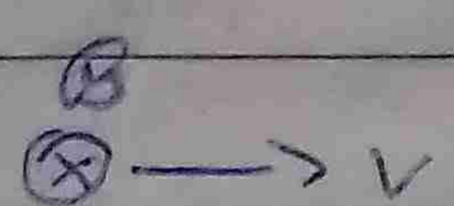
- <u>Magnetic force</u>



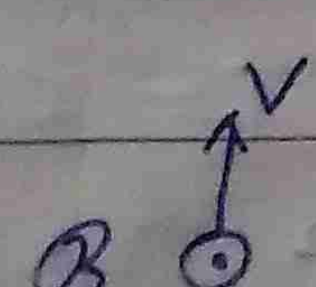
$\bar{F}, \bar{v}, \bar{B}$

$\bar{F} = q(\bar{v} \times \bar{B})$

⊙

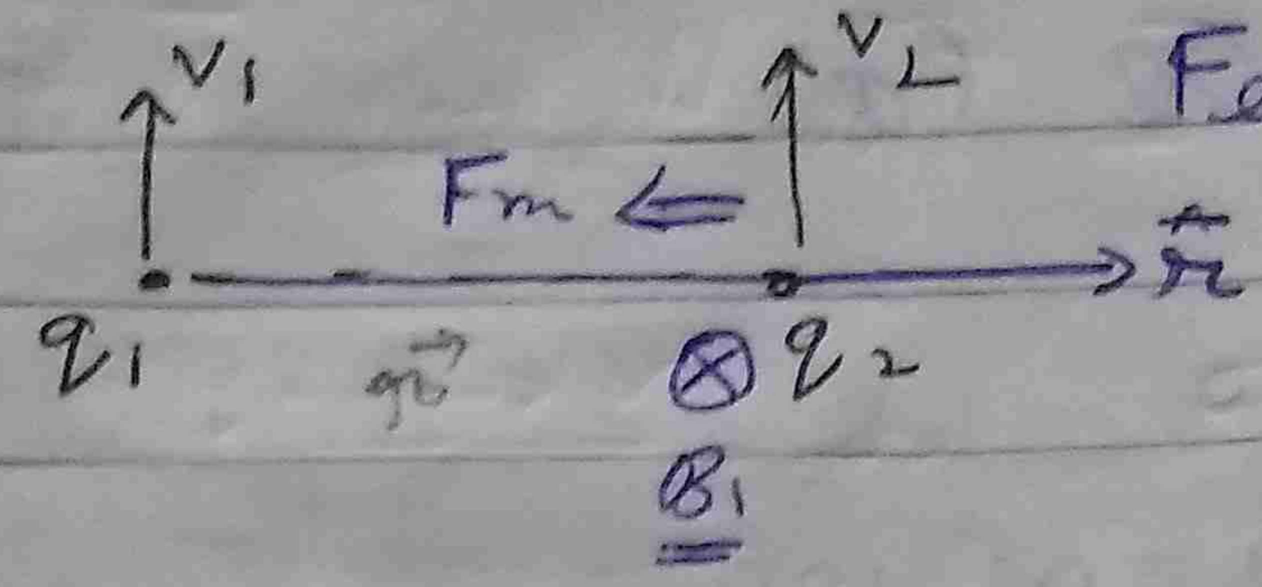


$\uparrow \; \bar{v} \times \bar{B}$



$\Rightarrow \bar{v} \times \bar{B}$

☆ Two parallel beams of protons moving in same direction will magnetically attract but will overall repel each other because electrostatic force is much more than magnetic force.



$$F_{electric} = \frac{k q_1 q_2}{r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$\bar{B}_1 = \frac{\mu_0}{4\pi} q_1 \frac{(\bar{v}_1 \times \bar{r})}{r^3} \Rightarrow B_1 = \mu_0 \frac{q_1 v}{r^2}$$

$$\bar{F}_2 = q_2(\bar{v}_2 \times \bar{B}_1) \text{ — Attractive!}$$

$$= q_2 v_2 B_1 = \frac{\mu_0 q_1 q_2 v_1 v_2}{4\pi r^2}$$

$$\frac{F_e}{F_m} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \times \frac{4\pi r^2}{\mu_0 q_1 q_2 v_1 v_2} \Rightarrow \frac{F_e}{F_m} = \frac{1}{\mu_0 \varepsilon_0 v_1 v_2} = \frac{c^2}{v_1 v_2}$$

$\Rightarrow F_e >>> F_m$

$\Rightarrow$ **Net force is repulsive**

☆ $\underset{q_1}{\times}$ — r — $\underset{q_2}{\times}$ when charge is at rest

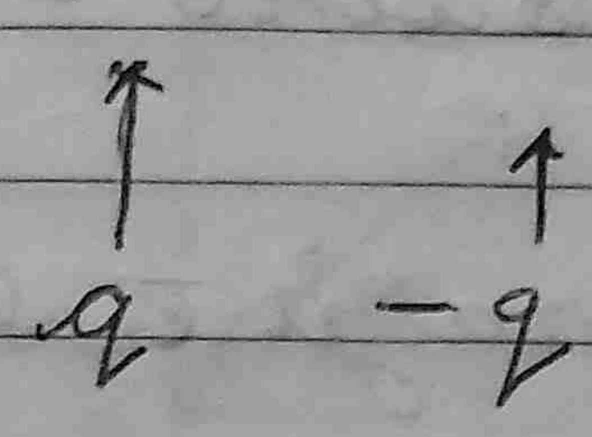
$$F_{net} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

☆ $\underset{q_1}{\times}$ (moving with v) — r — $\underset{q_2}{\times}$ (moving with v) when charges are moving

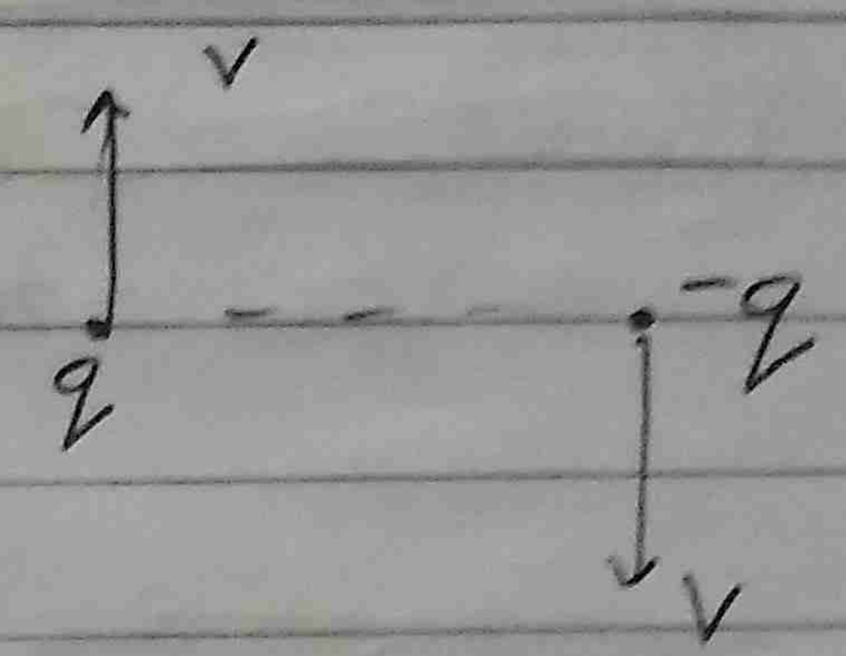
$$F_{net} < \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

- Even the momentum carried by magnetic field has to considered while studying newtons IIIrd law.

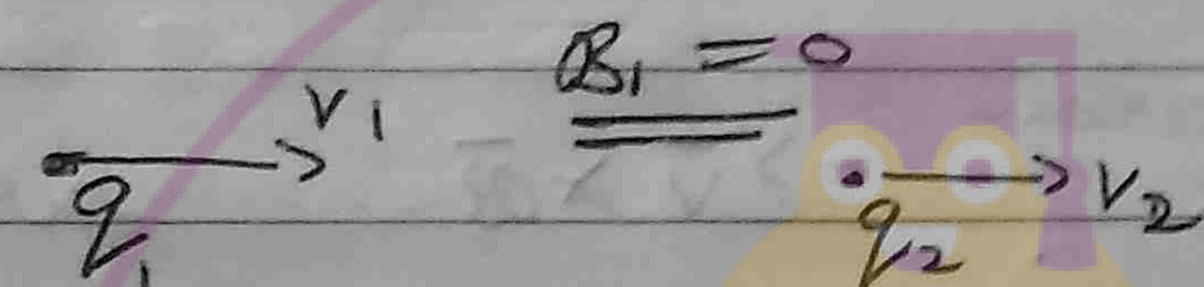
- **Two like charges, moving parallel will magnetically attract.**



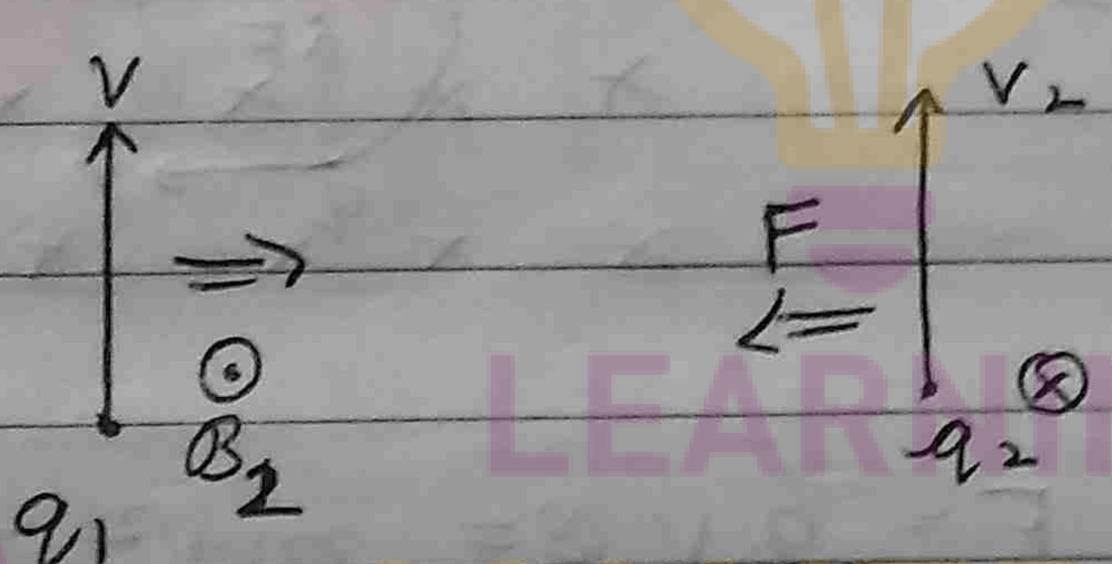
magnetically repell

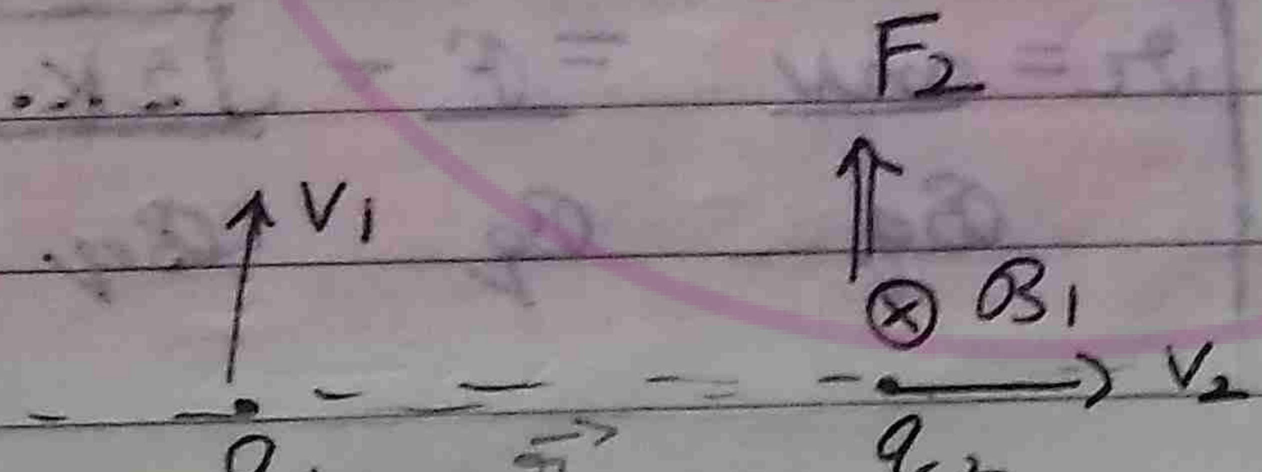


magnetically attract



neither attract nor repell.

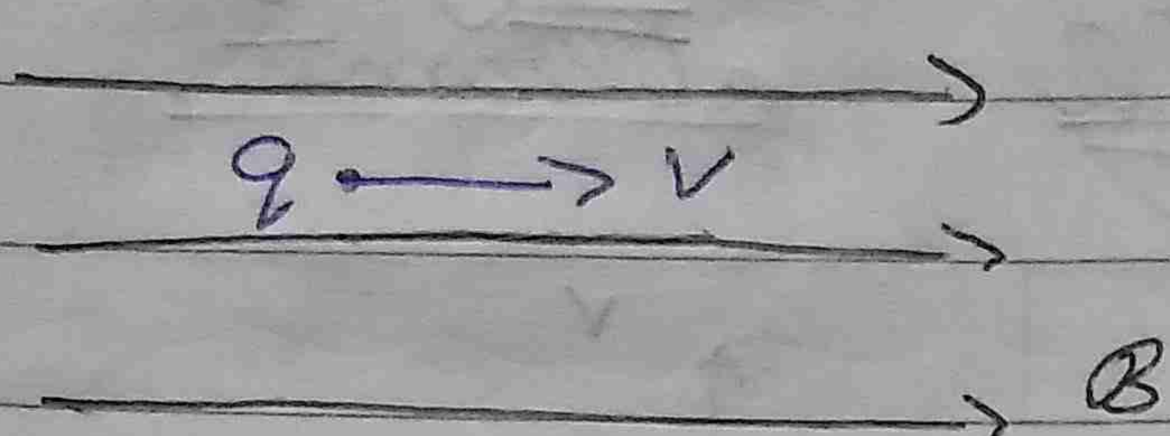
- 

- 

Here the newtons third law is not obeyed

- **Motion of point charge in uniform magnetic field**

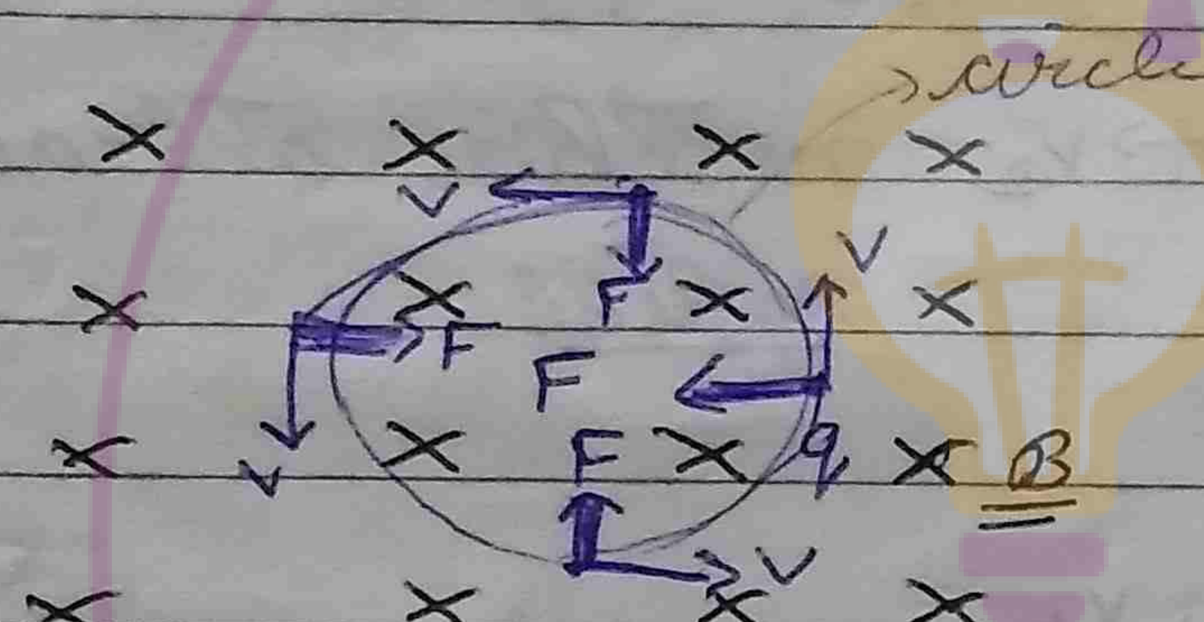
1. If $\bar{V}$ is $\parallel$ to $\bar{B}$.



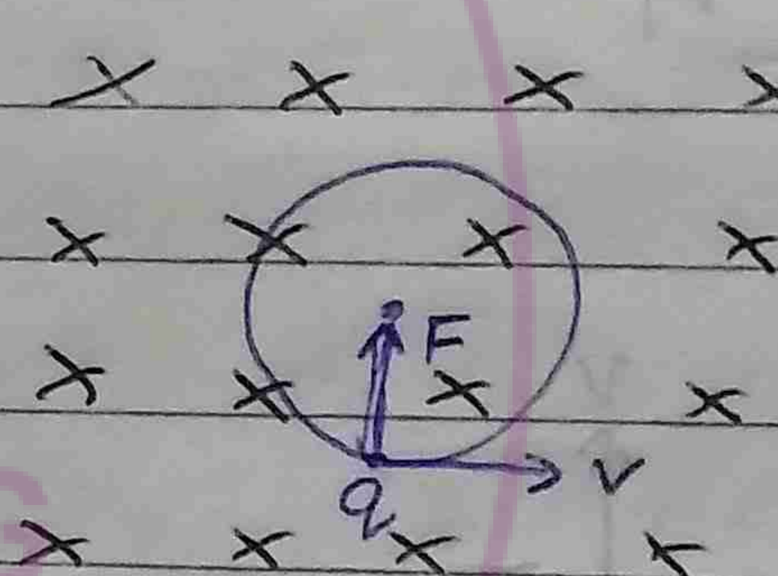
$F = qVB\sin\theta = 0$

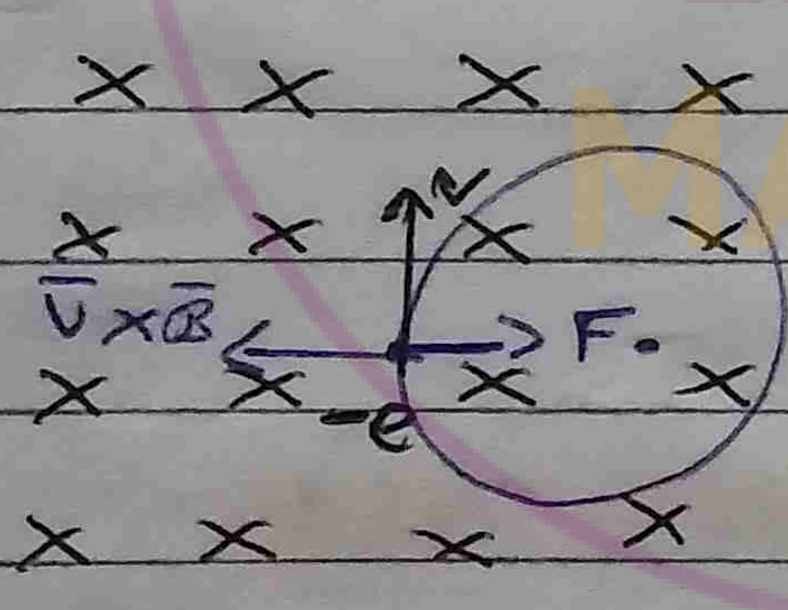
--- motion is straight line.

2. If $\bar{V}$ is $\perp$ B



$\Rightarrow \bar{V} \times \bar{B}$ --- circle ($\theta = 90^\circ$)

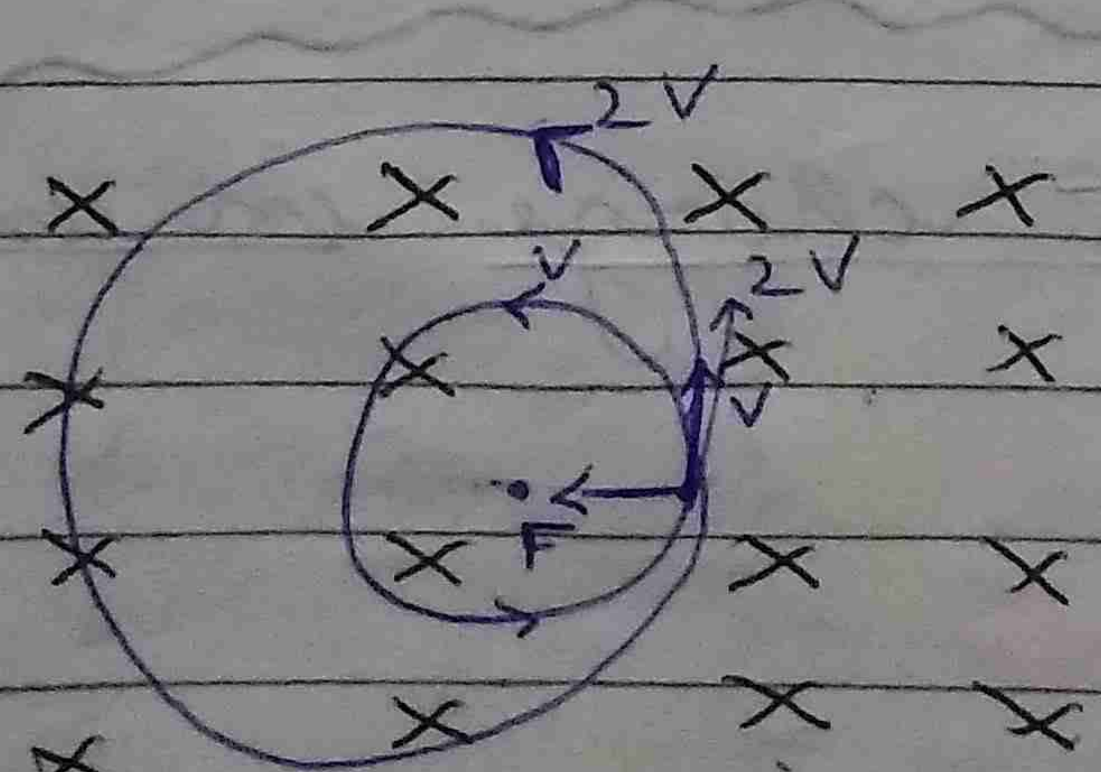




$F = qVB = \frac{mv^2}{r} = r = \frac{mv}{Bq}$ ($\theta = 90^\circ$)

$$r = \frac{mv}{Bq} = \frac{P}{Bq} = \frac{\sqrt{2K.E\,m}}{Bq}$$

• for given particle $r \propto v$

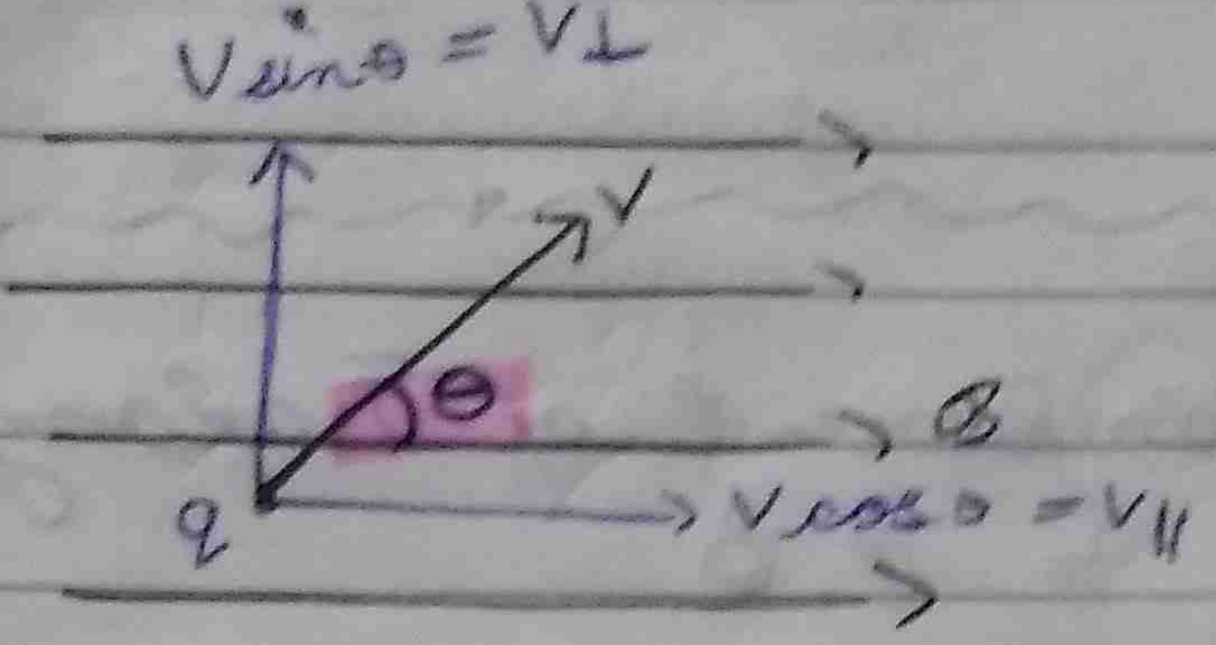


$T = \frac{2\pi r}{v}$; $T = \frac{2\pi m}{Bq}$

- independent of v.
- independent of r.

uniform circular motion

- If v is inclined with $\bar{B}$



$V_{\parallel}$ - - - st line

$V_{\perp}$ - - - circular

$$r = \frac{mV_{\perp}}{Bq}$$

$$\text{pitch} = V_{\parallel} T = V_{\parallel} \frac{2\pi m}{Bq}$$

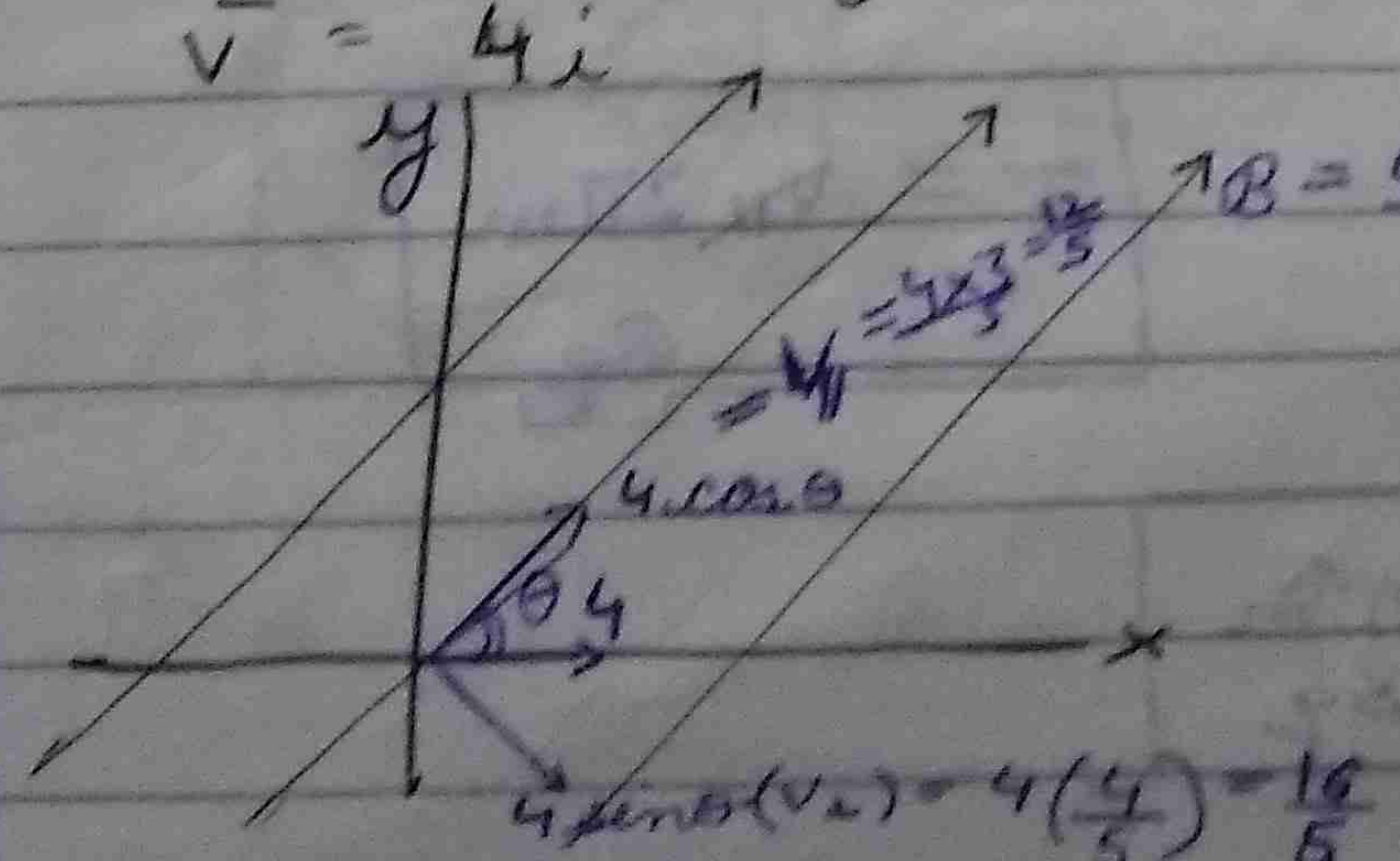
- If $\bar{v}$ is ∥ to $\bar{B}$ - path is straight line
- $\bar{v}$ is ⊥ to B - - circle
- $\bar{v}$ is ∠(inclined) to B - - - helix

---

* e.g. $\bar{B} = 3\hat{i} + 4\hat{j}$

$\bar{v} = 4\hat{i}$

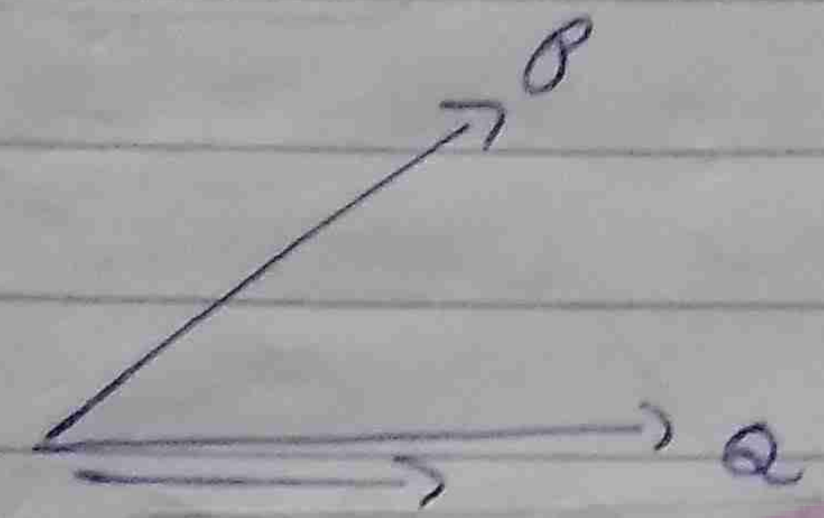
$\tan\theta = \frac{4}{3}$



$$r = \frac{mV_{\perp}}{Bq} = \frac{m(16/5)}{(5)q}$$

$T = \frac{2\pi m}{Bq}$; pitch $= \left(\frac{12}{5}\right) T$

(T → time period)



Component of $\bar{P}$ along $\bar{Q}$
$= \bar{P} \cdot \hat{Q}$

e.g. $\bar{v} = 4\hat{i}$; $\bar{B} = 7\hat{i}$ -- straight line

$\bar{v} = 2\hat{i} - 3\hat{j}$; $\bar{B} = 4\hat{i} - 6\hat{j}$ -- straight line

$\bar{v} = \hat{i} - 2\hat{j}$; $\bar{B} = 8\hat{i} + 4\hat{j}$ -- circle (dot product is zero $\theta = 90°$)

$\bar{v} = (\hat{i} + 3\hat{j})$; $\bar{B} = 4\hat{i} + 3\hat{j}$ -- helix

$V_{\parallel} = \bar{v} \cdot \hat{B} = (\hat{i} + 3\hat{j}) \cdot \frac{(4\hat{i} + 3\hat{j})}{5} = \frac{13}{5}$

$V_{\parallel}^2 + V_{\perp}^2 = v^2$ $\quad \left(\frac{13}{5}\right)^2 + V_{\perp}^2 = 10$

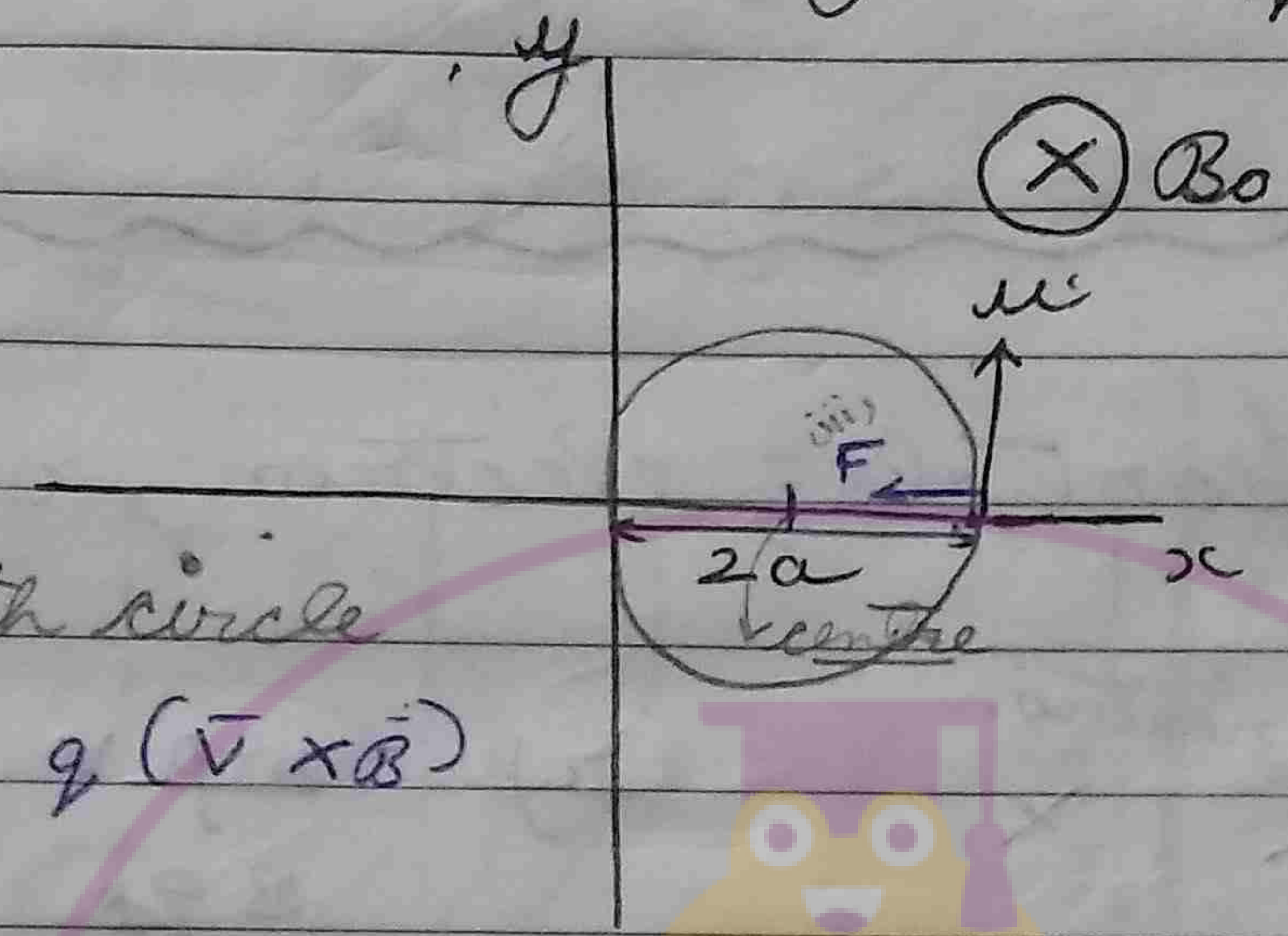
$B = 5$

$r = \frac{m V_{\perp}}{Bq}$

$T = \frac{2\pi m}{Bq}$ (~~$V_{\parallel}$~~ not req.)

$\text{Pitch} = V_{\parallel} \frac{2\pi m}{Bq}$

Q. A uniform magnetic field $-B_0\hat{k}$ exists in space. A particle (q, m) at $(2a, 0, 0)$ is given velocity $\frac{Bqa}{m}\hat{j}$.

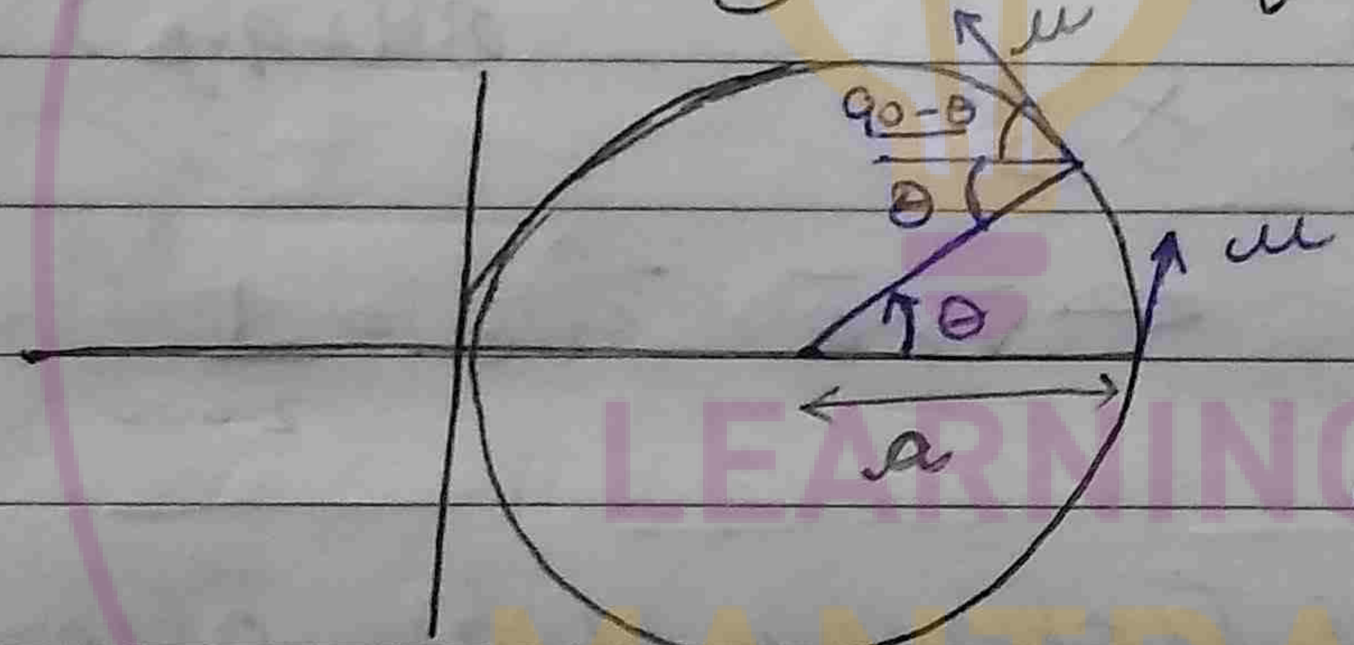


$r = \frac{mv}{Bq} = \frac{m}{Bq}\left(\frac{Bqa}{m}\right) = a$

path circle

$F = q(\bar{v} \times \bar{B})$

(i) Find velocity as f(t)



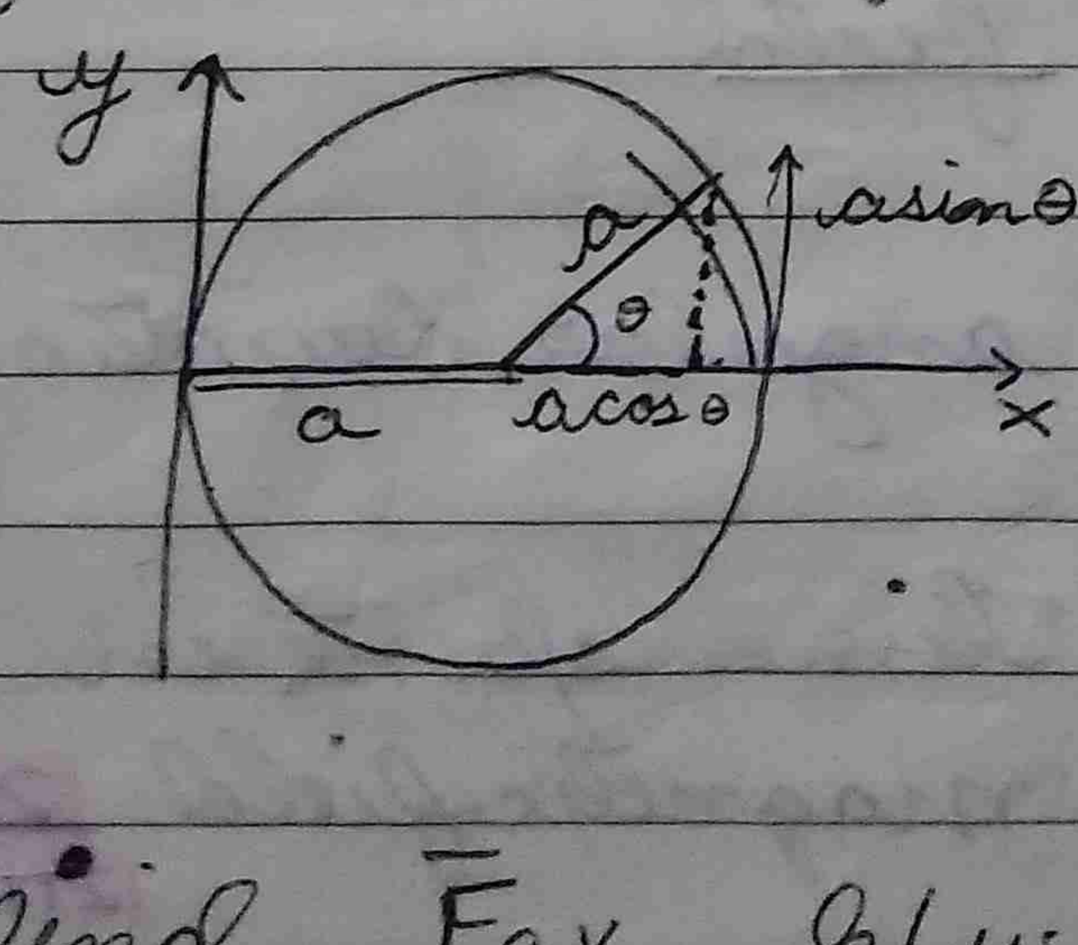
$\bar{V} = -u\sin\theta\hat{i} + u\cos\theta\hat{j}$

$= u(-\sin\theta\hat{i} + \cos\theta\hat{j})$

where $u = \frac{Bqa}{m}$

$\because \omega = \frac{2\pi}{T}$ (time period of charge) $\quad \theta = \omega t = \left(\frac{Bq\,t}{m}\right) \quad \left(\because T = \frac{2\pi m}{Bq}\right)$

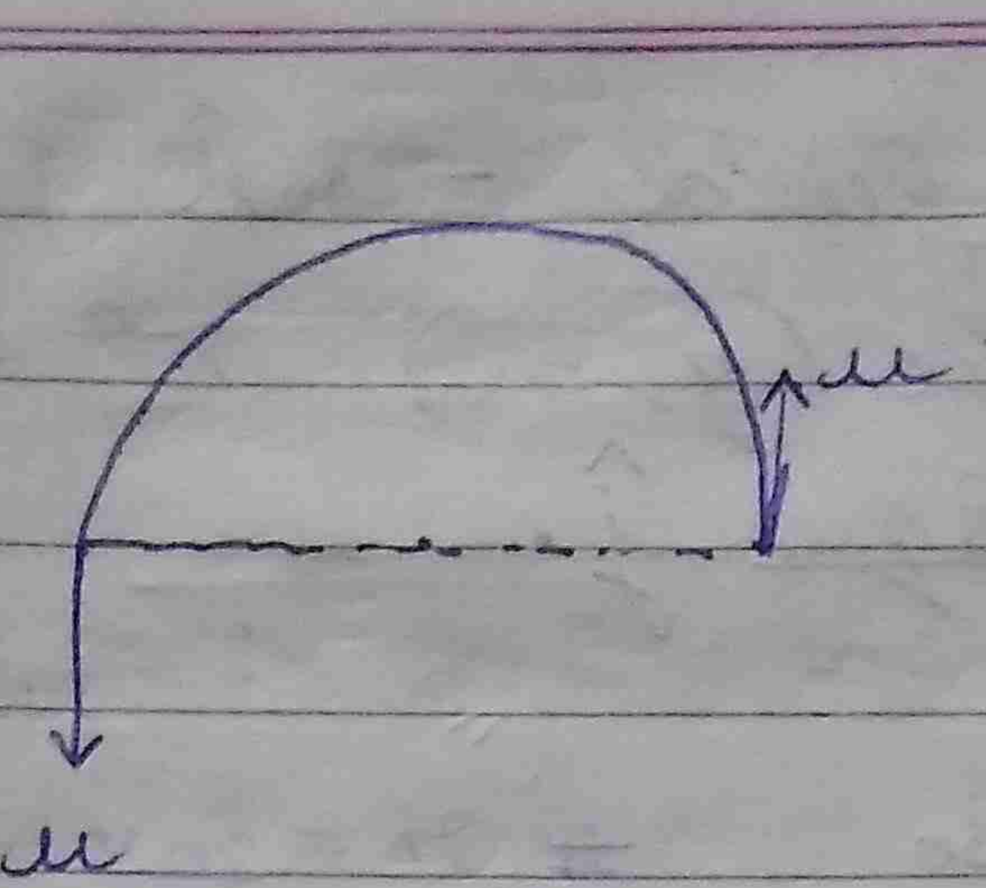
(ii) find position vector as f(t)



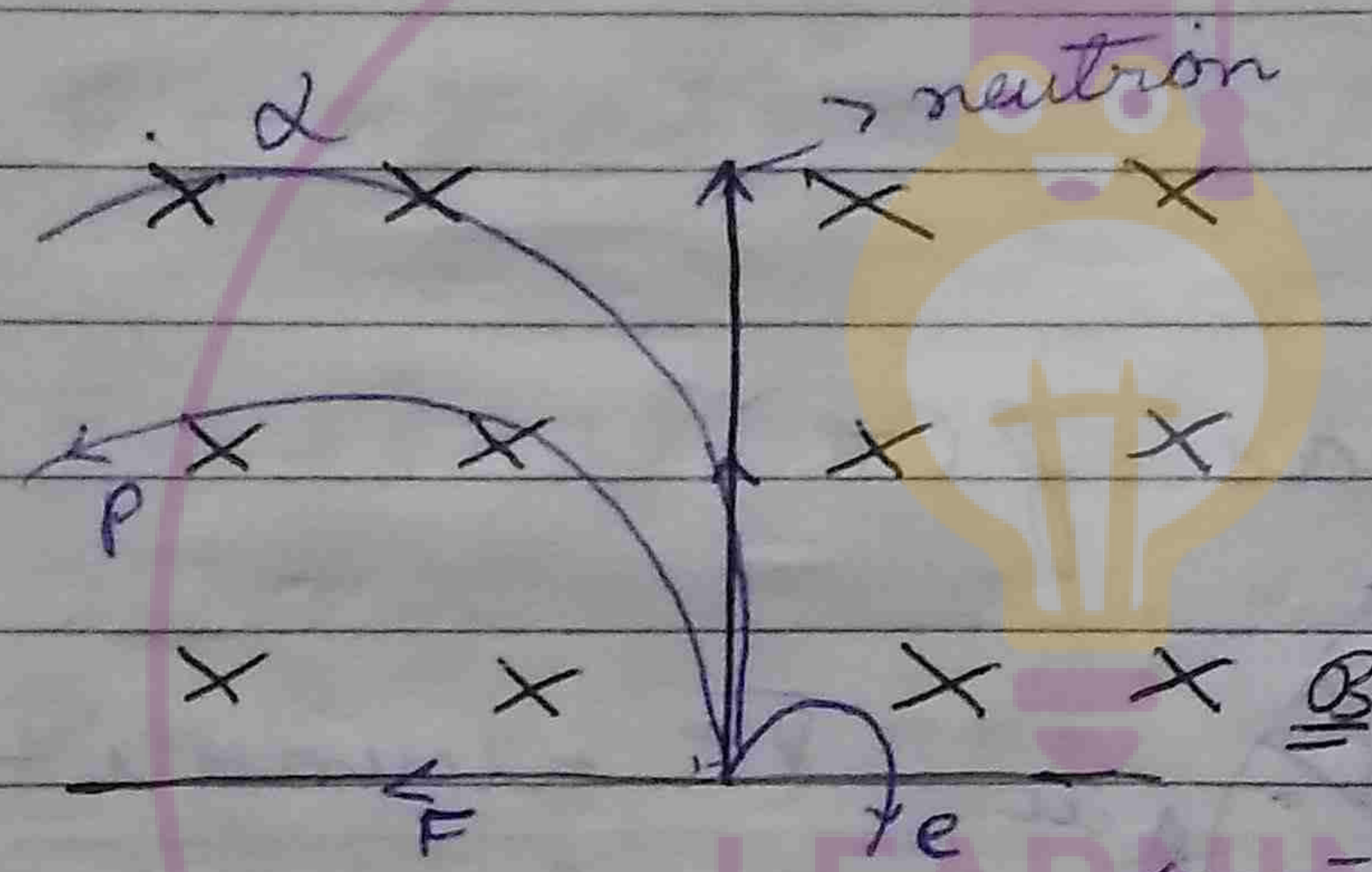
$\bar{r} = (a + a\cos\theta)\hat{i} + a\sin\theta\hat{j}$

(iii) find $\bar{F}_{av}$ b/w $t = 0$ to $t = \frac{\pi m}{Bq}$

$$\bar{F}_{av} = \frac{\Delta \bar{p}}{\Delta t} = \frac{-2mu}{\frac{\pi m}{Bq}}\hat{j}$$



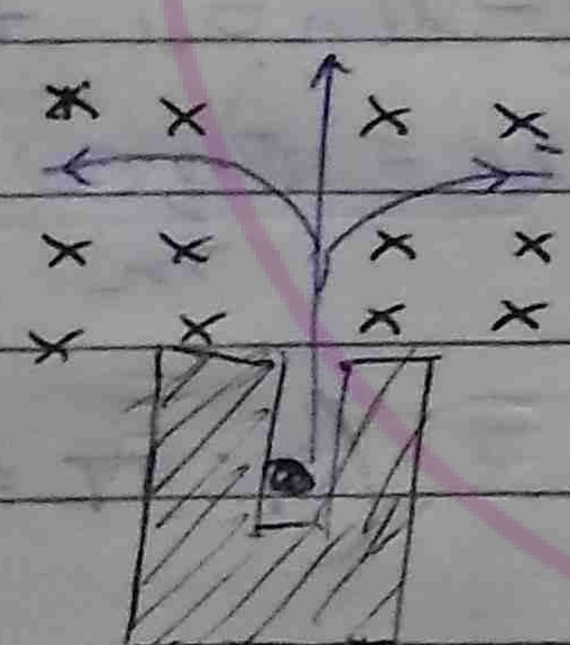
- proton, α particle, electron, neutron



$$r_p = \frac{m_p v}{B q_p}$$

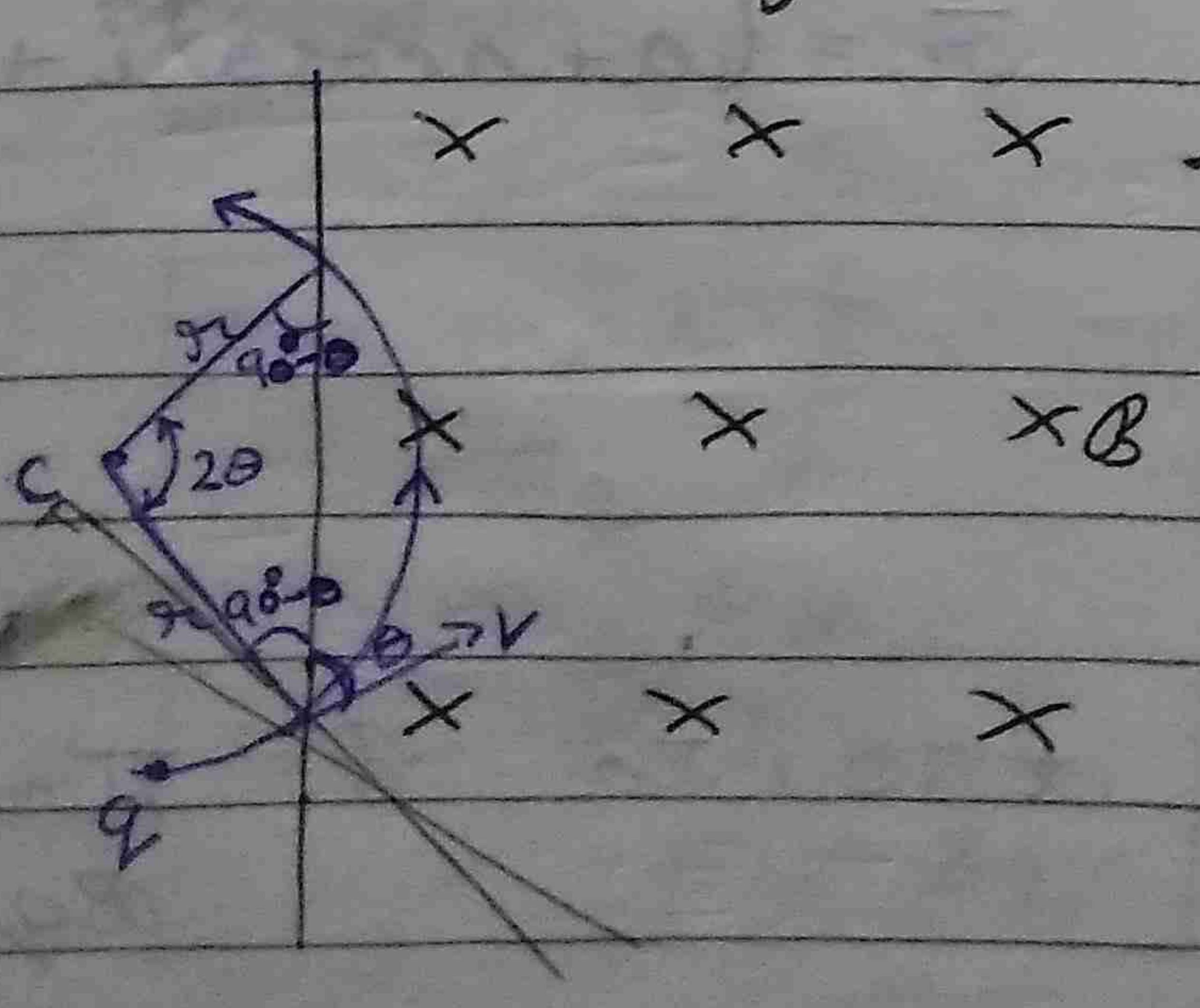
$$r_\alpha = \frac{(4m_p)v}{B(2q_p)} = 2r_p$$

$$r_e = \frac{1}{2000} r_p \text{ or } \frac{1}{1840} r_p$$

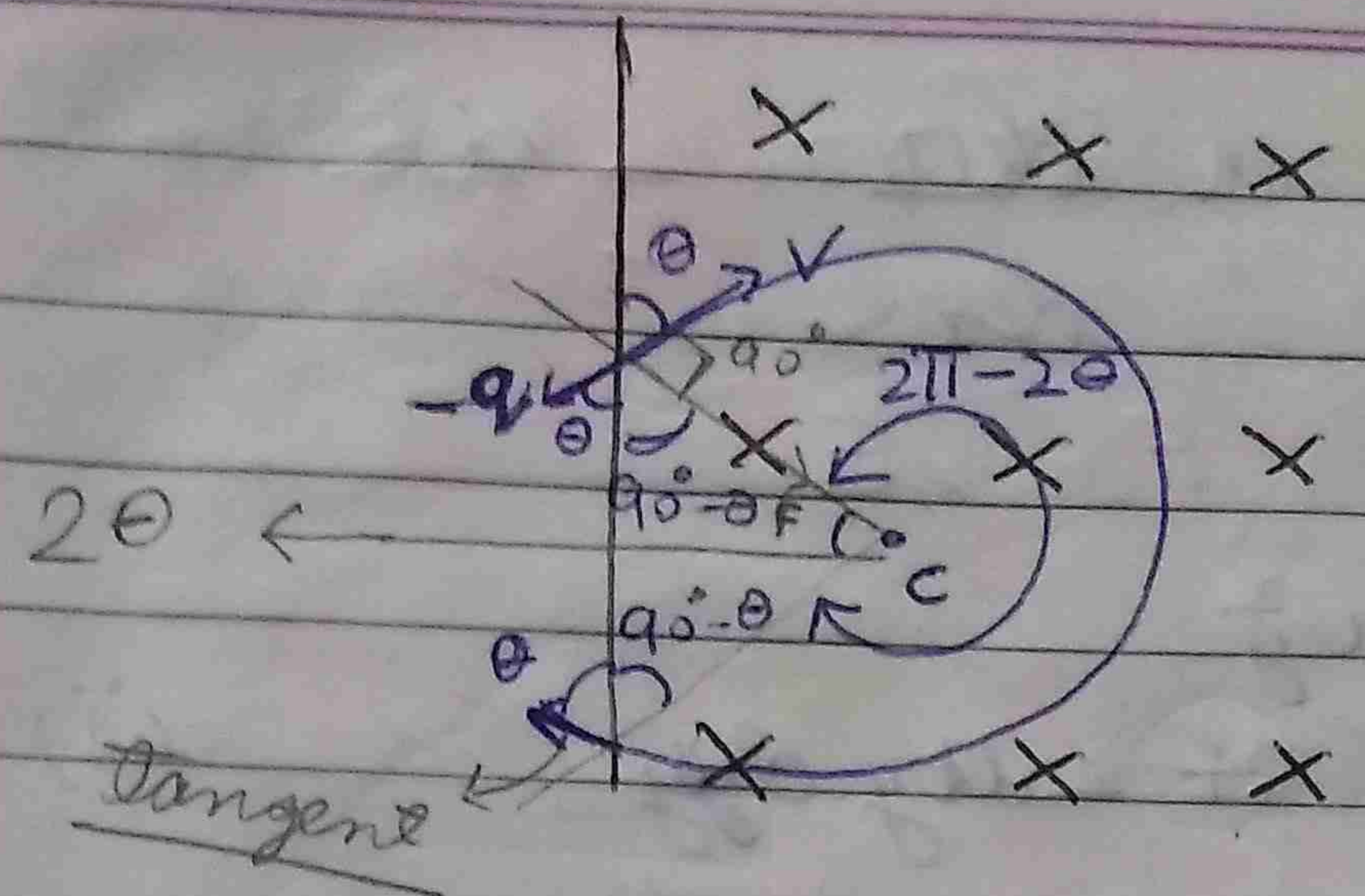


- This technique was also used to find isotopes.

- Finite magnetic field

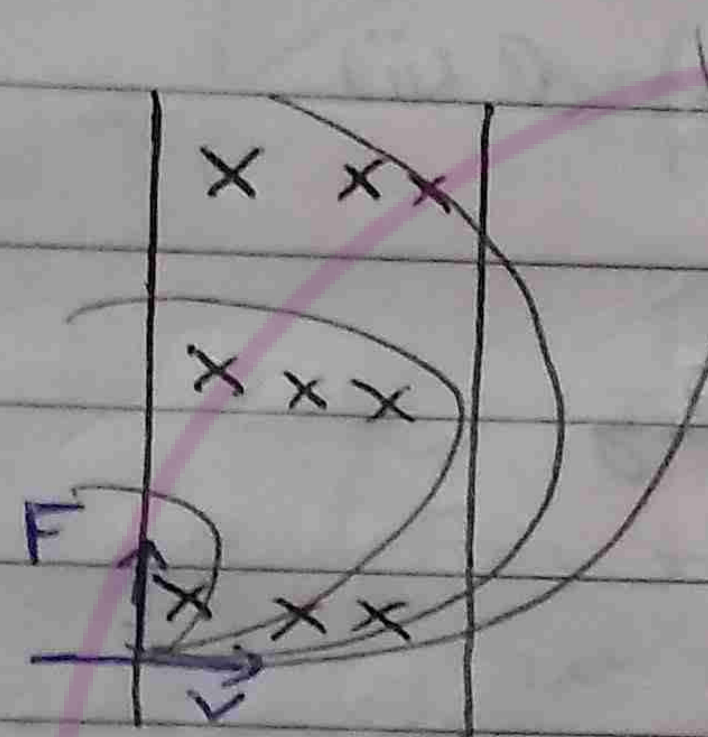


- angular deviation = 2θ
- time spent in magnetic field $= \frac{2\theta}{2\pi}\,\frac{2\pi m}{Bq}$

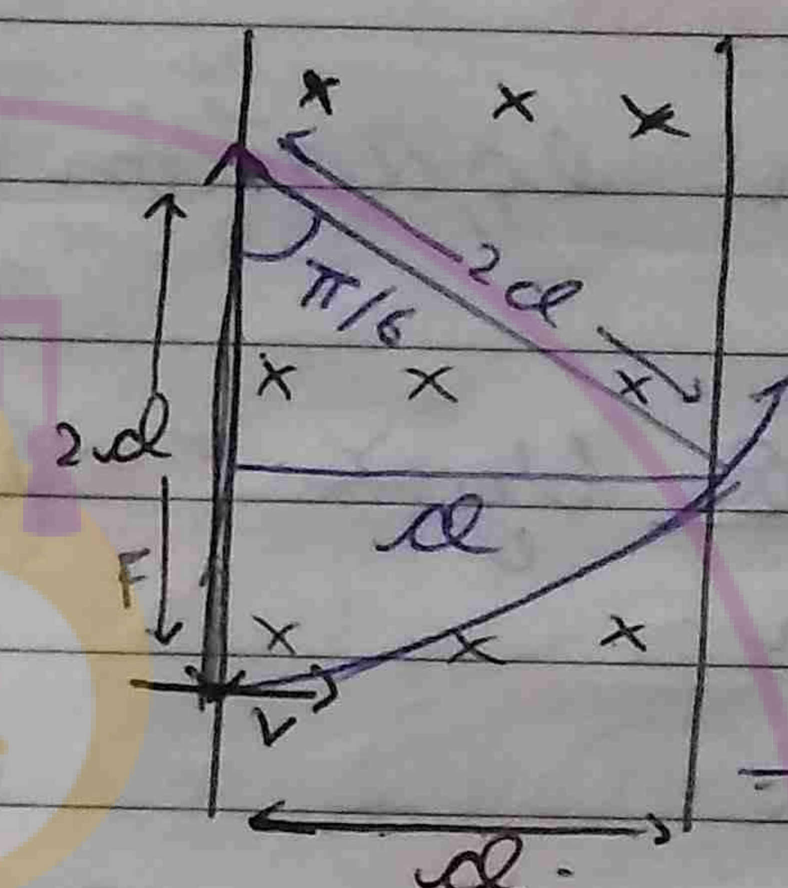


Time spent = $\frac{2\pi - 2\theta}{2\pi} \times \frac{2\pi m}{Bq}$

$\frac{2\pi - 2\theta}{2\pi} \times \frac{2\pi m}{Bq}$



#



$V = \frac{2Bqd}{m}$

$r = \frac{mv}{Bq} = 2d$

⇒ Time spent =

$\frac{\pi/6}{2\pi} \times \frac{2\pi m}{Bq}$

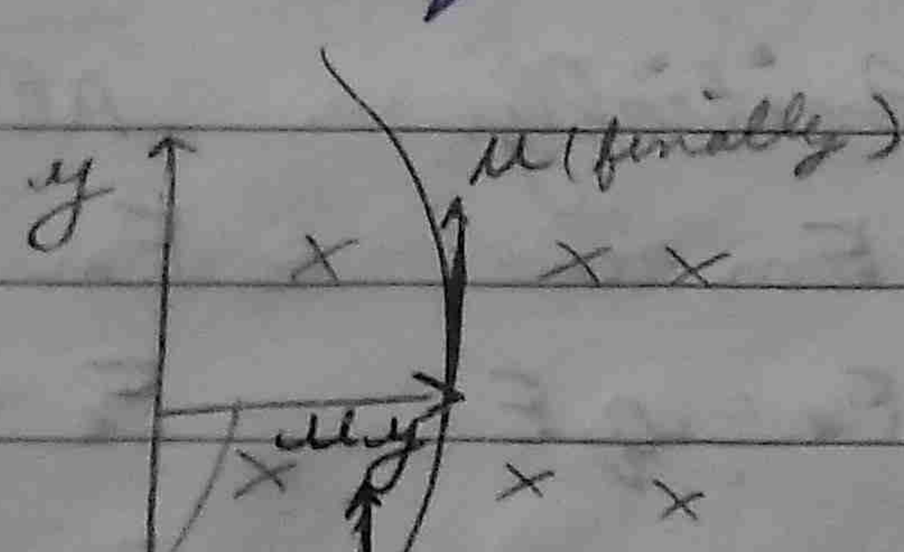
Non-uniform magnetic field

$\bar{B} = -B_0 x \hat{k}$

- A particle (q, m) at orgin has velocity $u\hat{i}$. Find the max distance from y axis.

⇒ $\bar{F} = q(\bar{V} \times \bar{B})$

This formula is valid for uniform / non-uniform magnetic field.



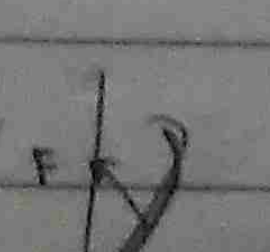
$\bar{F} = q(u_x\hat{i} + u_y\hat{j}) \times (-B_0 x \hat{k})$

$\bar{F} = -qB_0(-u_x x\hat{j} + u_y x\hat{i})$

$\bar{F} = -qB_0 u_y x(\hat{i}) + qB_0 u_x x(\hat{j})$

$a_x = \frac{-qB_0}{m} u_y x = u_x \frac{du_x}{dx}$

$a_y = \frac{qB_0}{m} u_x x = u_y \frac{du_y}{dy}$



max distance from y axis

$a = v\frac{dv}{dx}$

Name of the Chapter ______ Date ______ Page No.: ______

$$-\frac{qB_0}{m}u_y x = u_x \frac{du_x}{dx} \quad \text{(i)}$$

$$u^2 = u_x^2 + u_y^2$$

$$0 = 2u_x du_x + 2u_y du_y \quad \text{--- (ii)}$$

from equation (i) and (ii)

$$-\frac{qB_0}{m}u_y x = -u_y \frac{du_y}{dx}$$

$$= \int_0^{x_{max}} dx\, x = \frac{m}{B_0 q}\int_0^{u} du_y = \frac{x_{max}^2}{2} = \frac{mu}{B_0 q}$$

- Combined electromagnetic field

$\bar{F} = q\bar{E} + q\bar{v}\times\bar{B}$ --- Lorentz force equation.

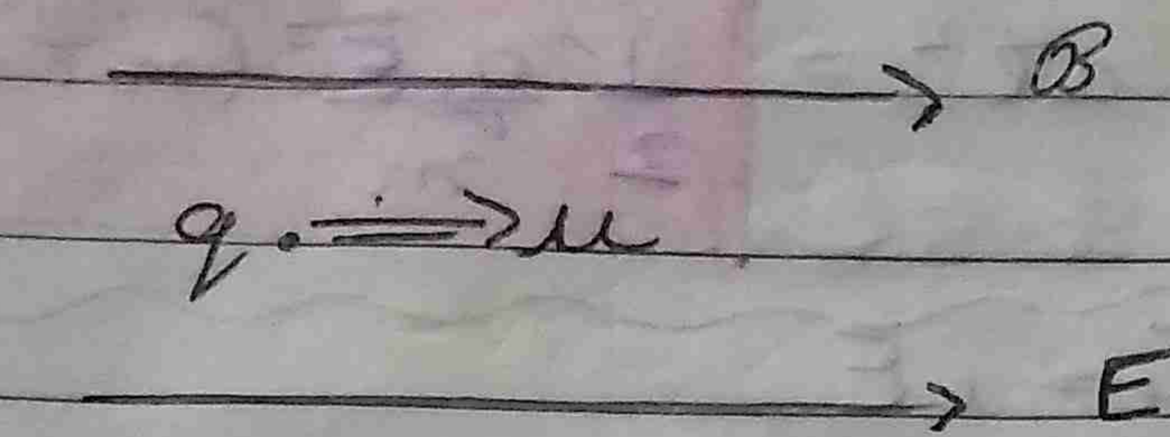
q.(rest) → B, → v, $F_e = qE$ → E

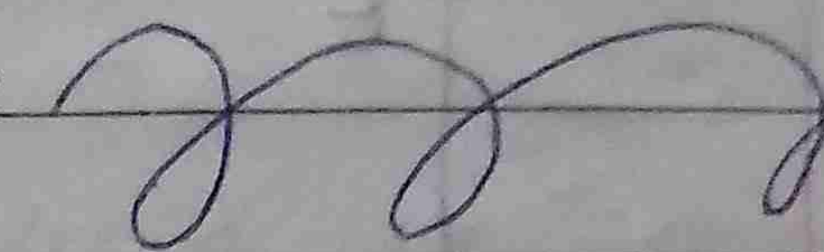
	Initially	At 't'
	$F_m = 0$	$F_m = 0$
	$F_e = qE$	$F_e = qE$

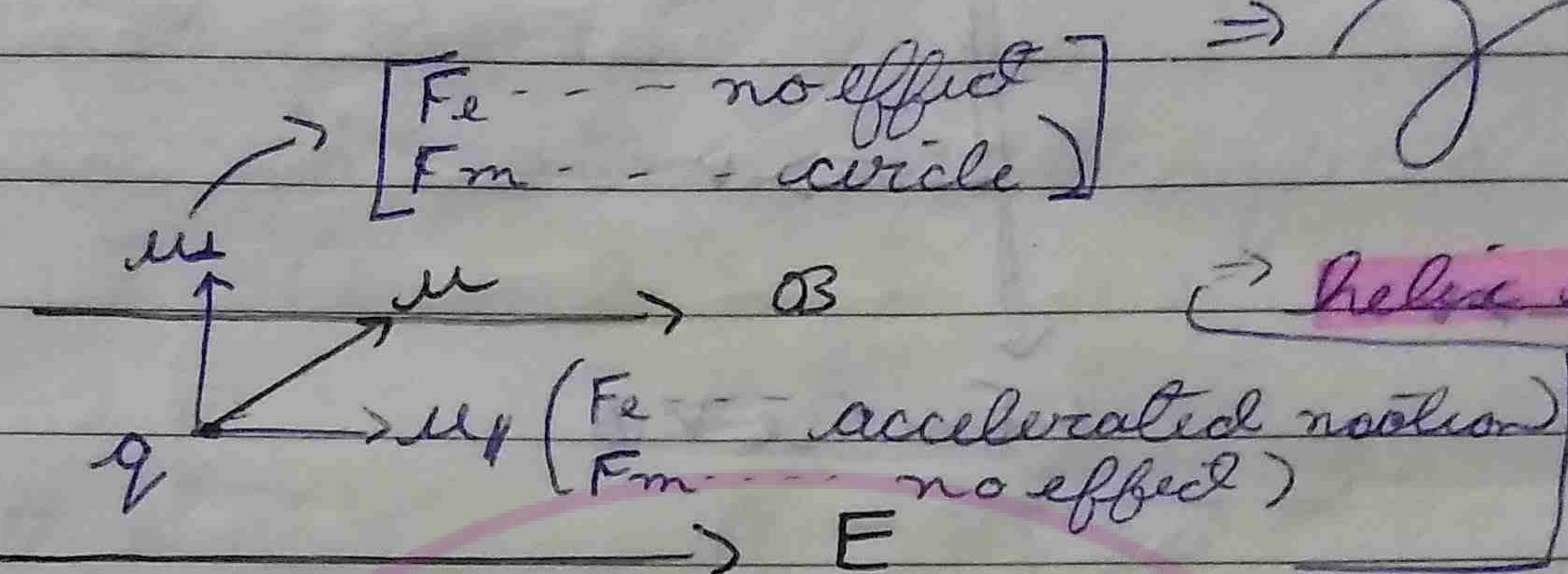
⇒ st. line accelerated motion.

$v = u + at$

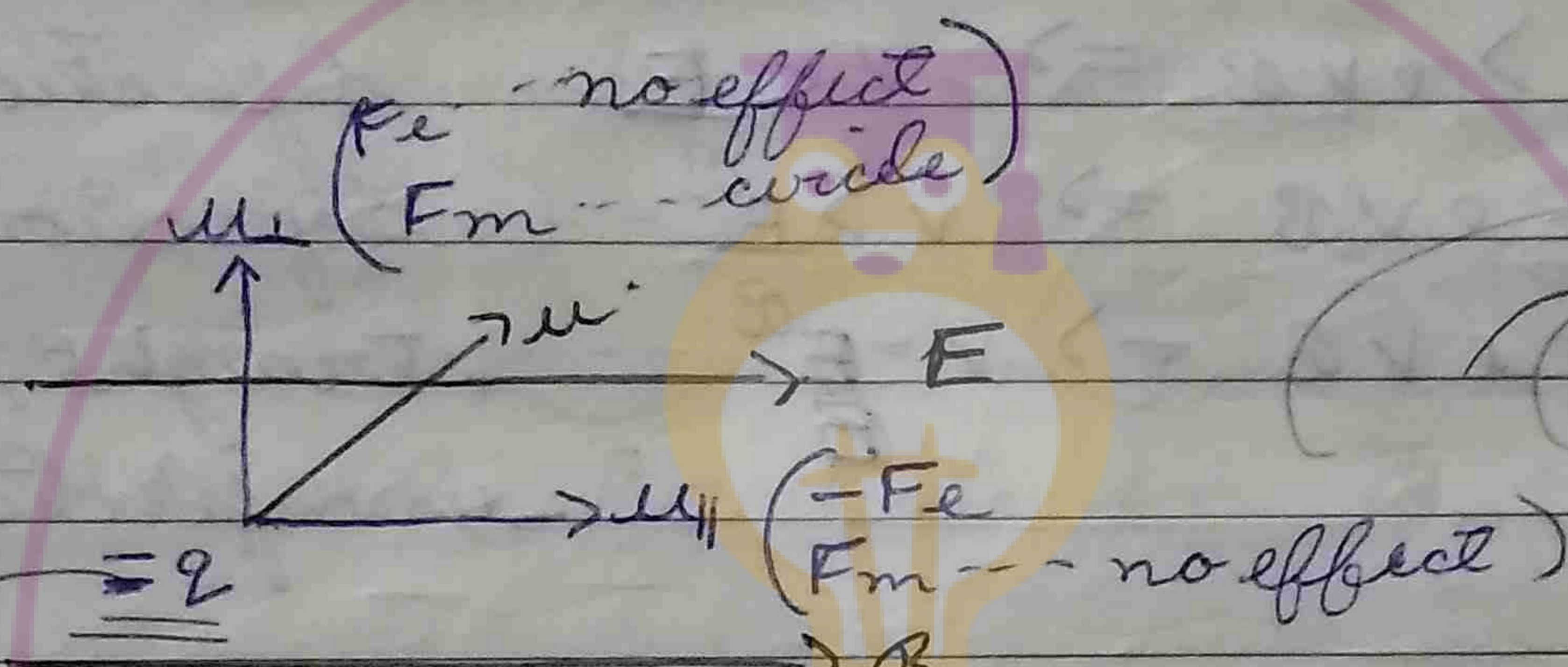
$v = 0 + \frac{qE}{m}t$

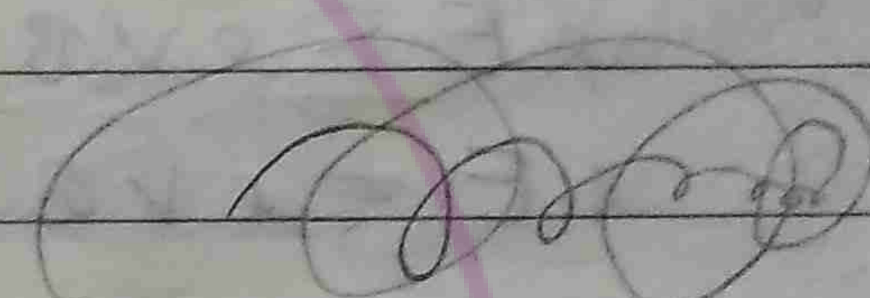






⇒ helix of increasing pitch but radius remains same.

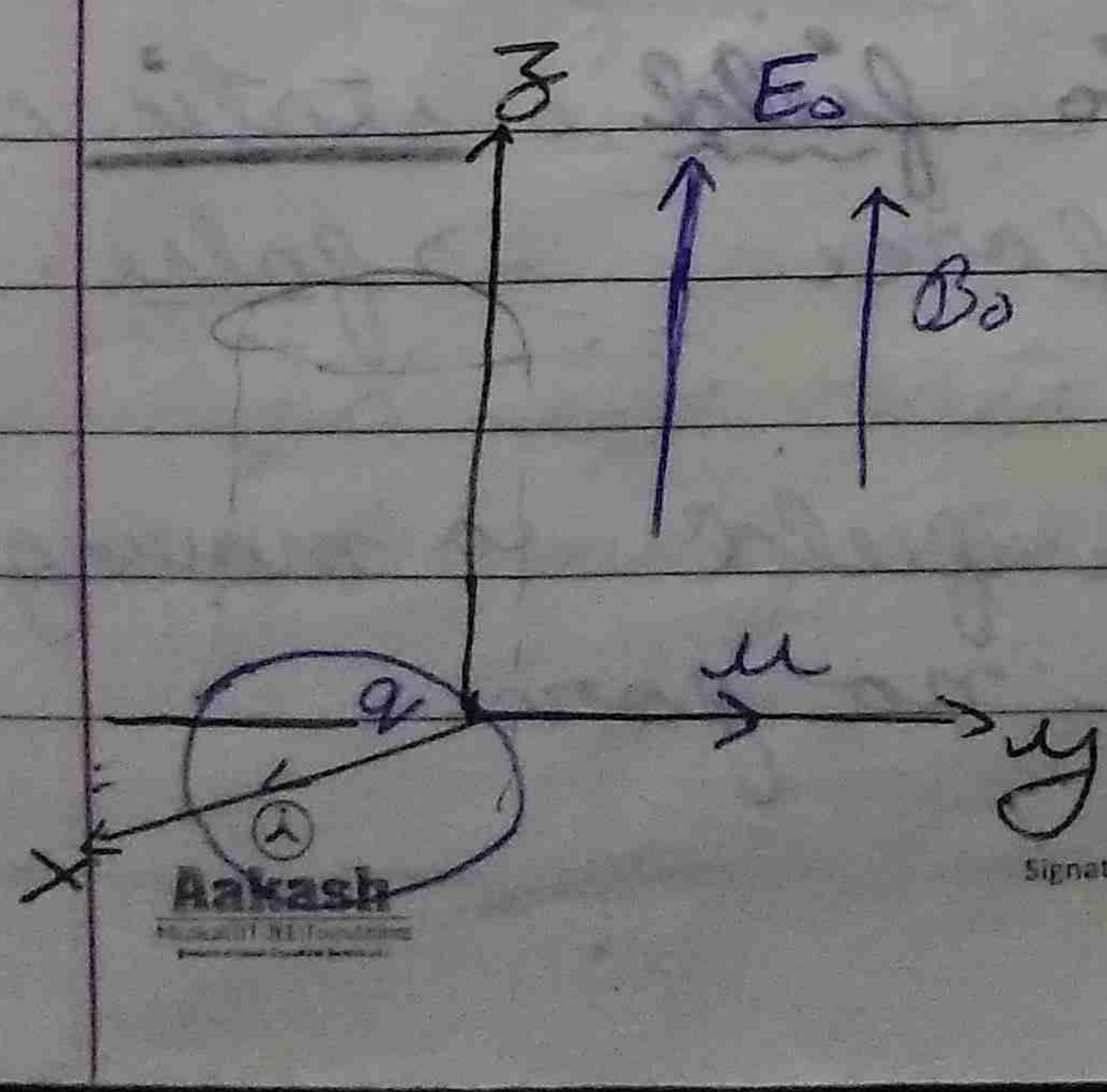


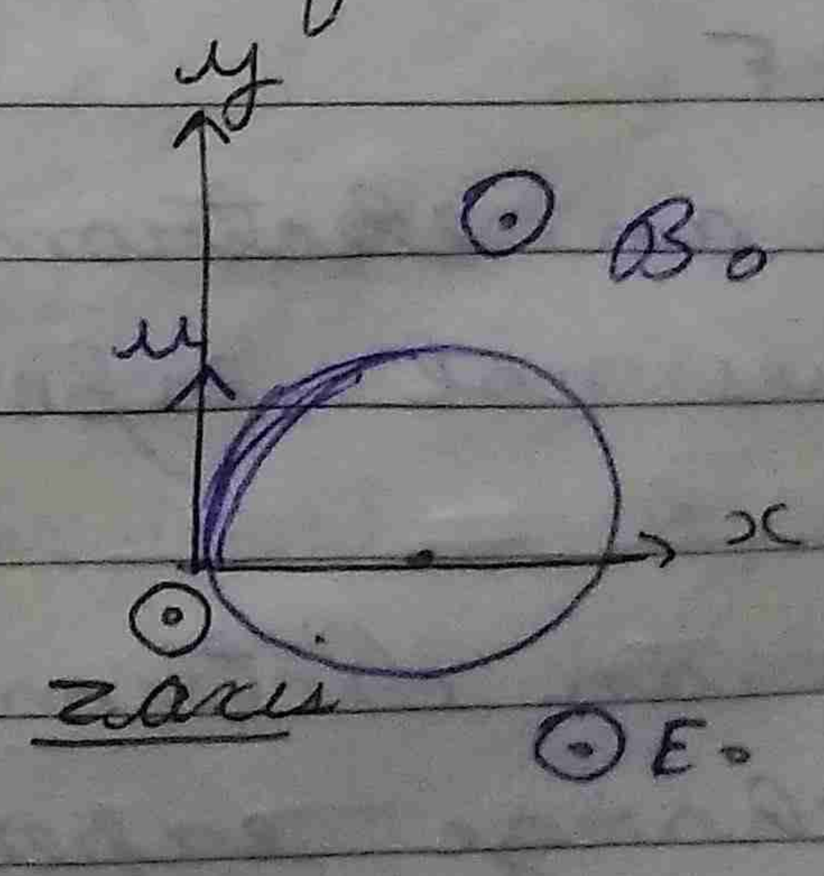


helix of first decreasing pitch then increasing in −ve direction.

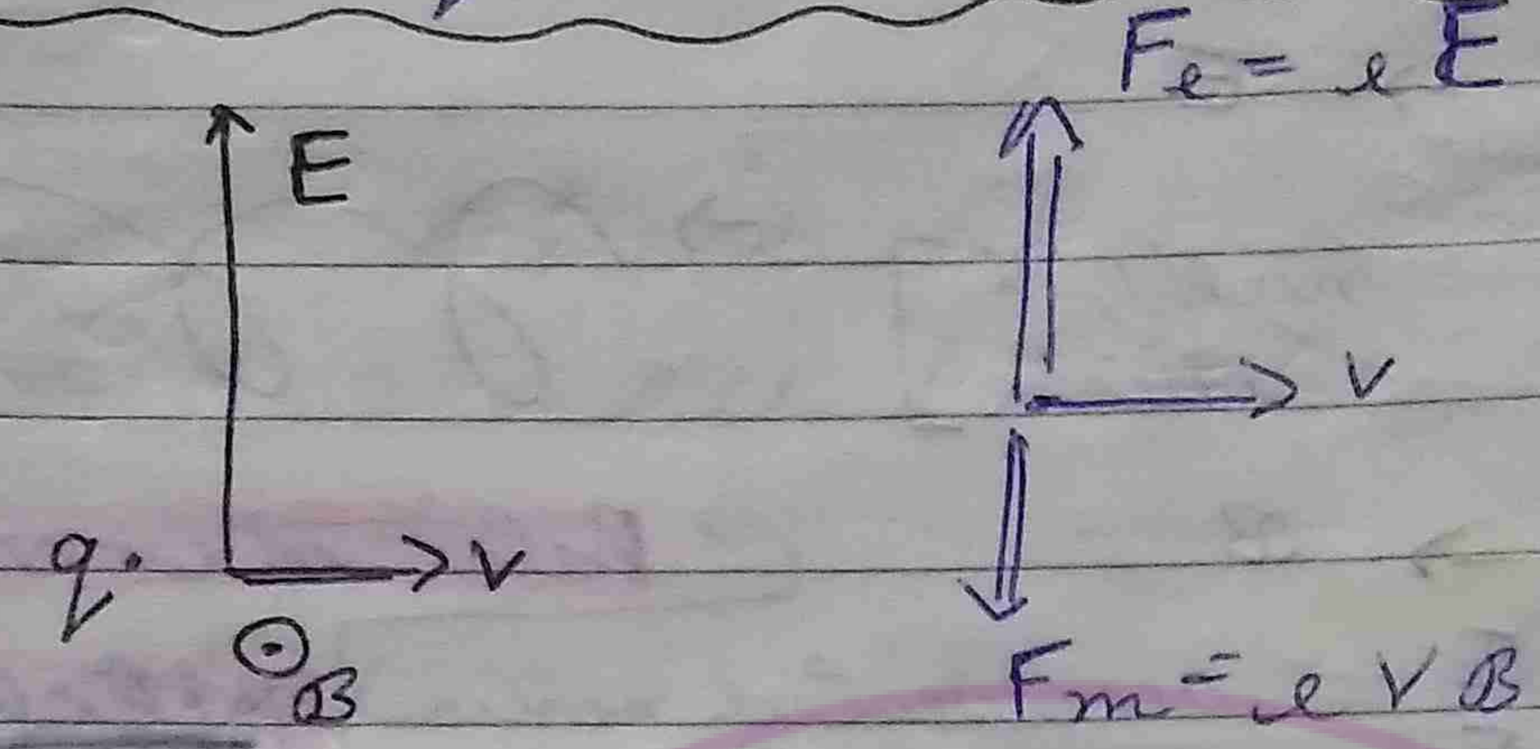
$E = E_0\hat{k}$; $\bar{B} = B_0\hat{k}$

$\bar{u} = u\hat{j}$. Find the z-coordinate when it hits z axis for n^{th} time.





$T = \frac{2\pi m}{B q}$ $\quad z = \frac{1}{2} a t^2 = \boxed{\frac{1}{2} \frac{q E_0}{m} (nT)^2}$



- If $eE > evB \Rightarrow v < \frac{E}{B}$ --- deflection up.
  $eE < evB \Rightarrow v > \frac{E}{B}$ --- deflection down.
  $eE = evB \Rightarrow v = \frac{E}{B}$ --- straight, undeflected

$\bar{F} = q\bar{E}$; $\bar{F}_m = q(\bar{v} \times \bar{B})$

- If charge particle is present in electric field
  -- Must experience force.

- If a charge particle is in magnetic field --- may / may not experience force.

T or F

- In an electromagnetic field a static charge does not experience force --> false.

- In an electromagnetic field a moving charge experiences no force.

☆ Only electric force is responsible for ΔK.E

$\bar{V} \parallel \bar{B} \rightarrow F \neq 0$

$\bar{V}$ (inclined) $\angle \bar{B} \Rightarrow$ → $F = 0$ / → $F \neq$

F_{net}

• Field in electromagnetic field is zero:

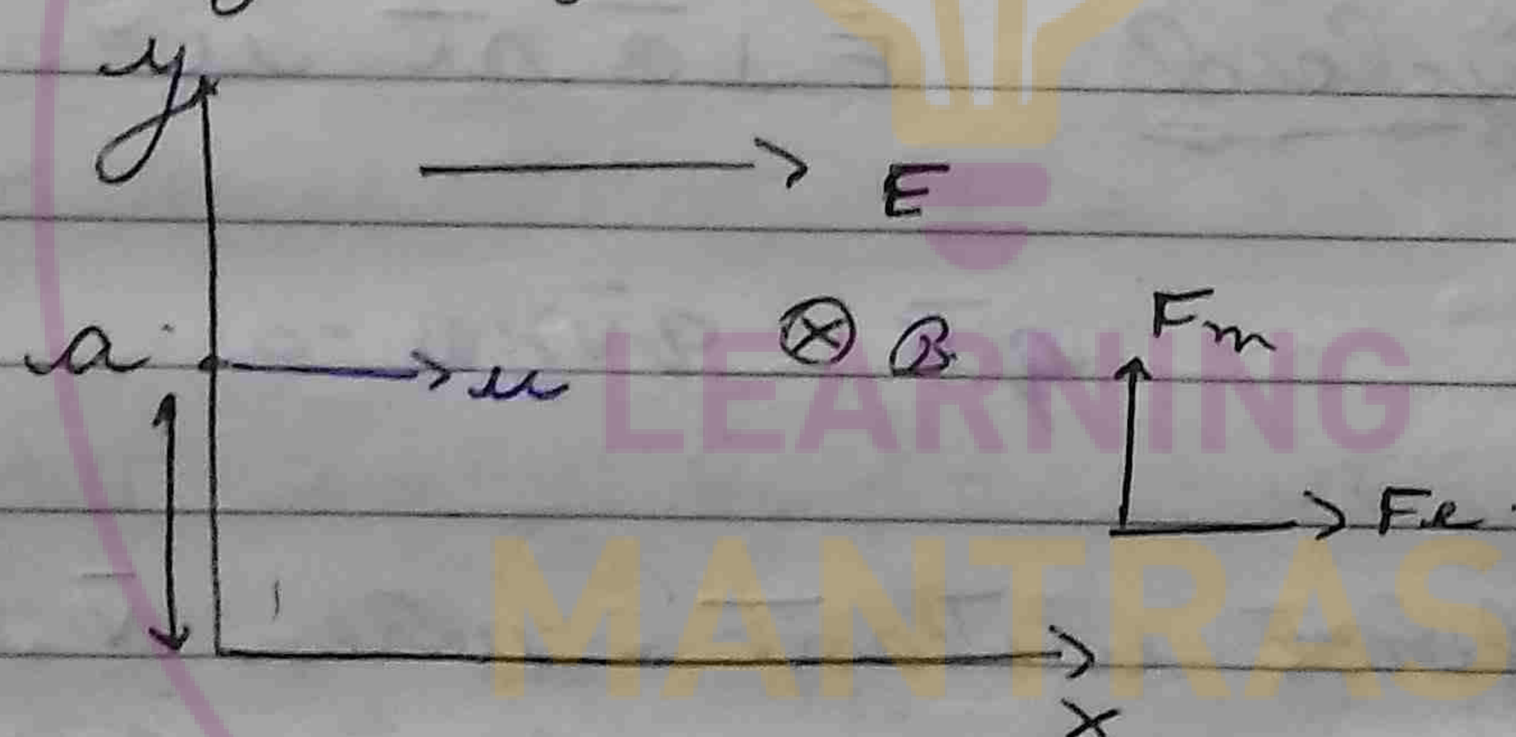
a. $\bar{V} \perp \bar{B}$ (could be true or not)

b. $\bar{V} \perp \bar{E}$ ✓ $\quad q\bar{E} + q\bar{V}\times\bar{B} = 0$

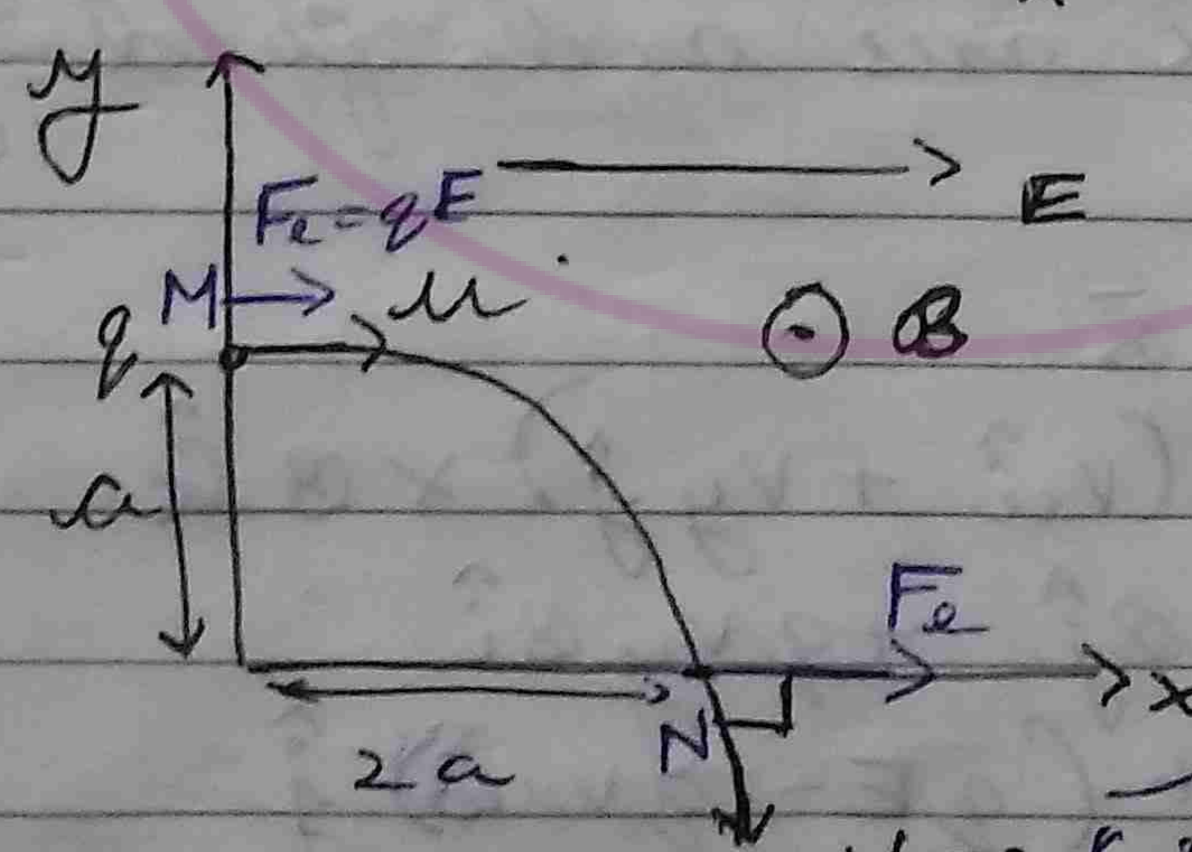
c. $\bar{E} \perp \bar{B}$ ✓ $\quad \bar{E} = -\bar{V}\times\bar{B}$

d. $V = \frac{E}{B}$ (could be true). $\quad E = VB\sin\theta$ → because it may have different angle

• Charged particle in $\bar{B} + \bar{E}$.



☆



W all forces = ΔK

☆ W electric force = ΔK.

$qE \times 2a = \frac{1}{2}(m(V^2 - u^2))$

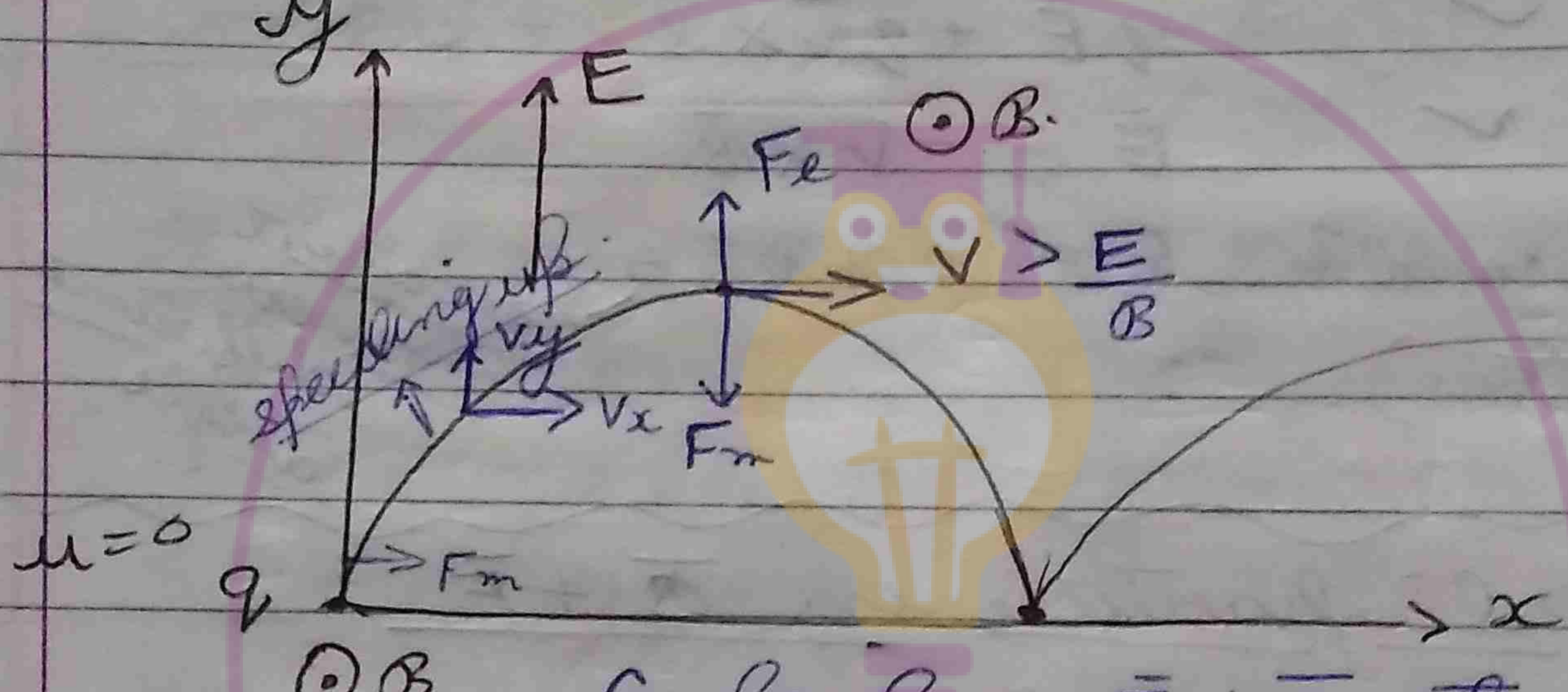
Find the rate of work done by electric and magnetic force at M and N.

$P = \bar{F}.\bar{V}$

P by magnetic force = 0

P by electric force at M = $F_e u \cos 0°$

at N = $F_e u \cos 90° = 0$



Cycloid $\bar{E} \perp \bar{B}$ at $u = 0$

$q\bar{E} + q\bar{v} \times B = 0$

- Find the first instant when $\bar{v}$ is parallel to x axis and find y max.

$\bar{F} = q\bar{E} + q\bar{v} \times \bar{B}$

$= qE\hat{j} + q(v_x\hat{i} + v_y\hat{j}) \times B\hat{k}$

$\bar{F} = qE\hat{j} - qv_xB\hat{j} + qv_yB\hat{i}$

$\bar{F} = (qv_yB)\hat{i} + (qE - qv_xB)\hat{j}$

$\frac{dv_x}{dt} = \frac{qB}{m} v_y$ (i)

$$\frac{dV_y}{dt} = \frac{q}{m}(E - V_x B) \quad \text{---(ii)}$$

Differentiating equation (ii)

$$\frac{d^2V_y}{dt^2} = -\frac{qB}{m}\frac{dV_x}{dt} \quad \text{---(iii)}$$

from equation (i) and equation (iii)

$$\frac{d^2V_y}{dt^2} = -\frac{qB}{m}\left(\frac{qB}{m}\right)V_y \Rightarrow \frac{d^2V_y}{dt^2} + \frac{q^2B^2}{m^2}V_y = 0 \quad \text{(iv)}$$

We know that $\frac{d^2x}{dt^2} + \omega^2 x = 0 \Rightarrow x = A\sin(\omega t + \delta)$

$$\frac{d^2V_y}{dt^2} + \frac{q^2B^2}{m^2}V_y = 0 \Rightarrow \frac{d^2V_y}{dt^2} + \omega^2 V_y = 0$$

where $\boxed{\omega = \frac{qB}{m}}$

$V_y = A\sin(\omega t + \delta)$

At $t = 0 \Rightarrow V_y = 0 \Rightarrow 0 = A\sin(0 + \delta)$

$\Rightarrow \delta = 0 \Rightarrow V_y = A\sin\omega t$

$\frac{dV_y}{dt} = A\omega\cos\omega t$

At $t = 0$ $\quad \frac{qE}{m} = A\frac{qB}{m}\cos(0) \Rightarrow A = \frac{E}{B}$

$$\boxed{V_y = \frac{E}{B}\sin\omega t}$$

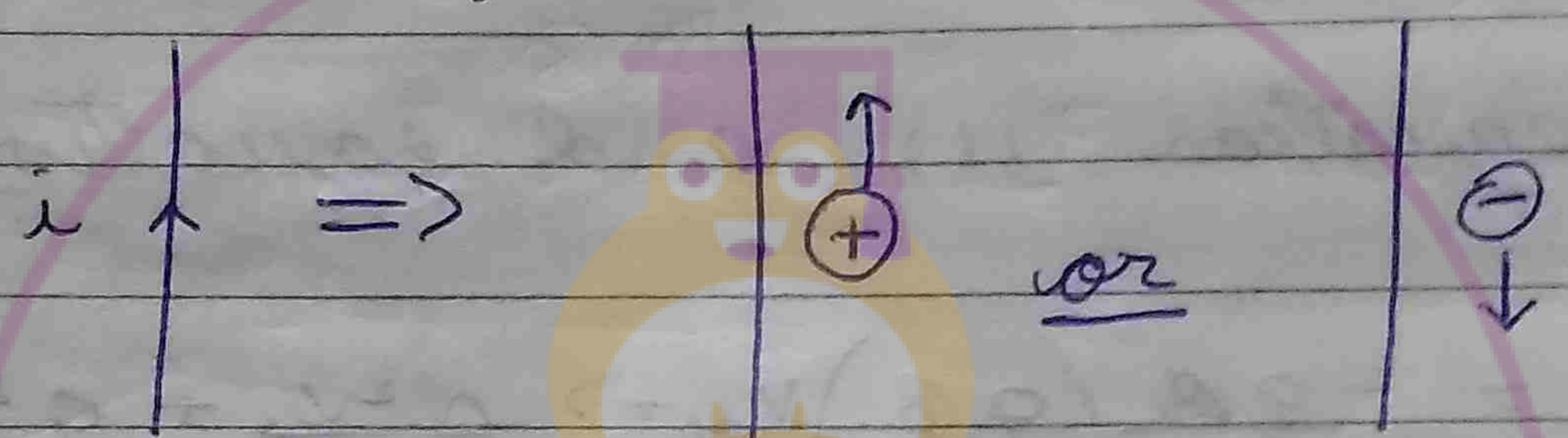
• Hall effect is used to determine nature of charge carri

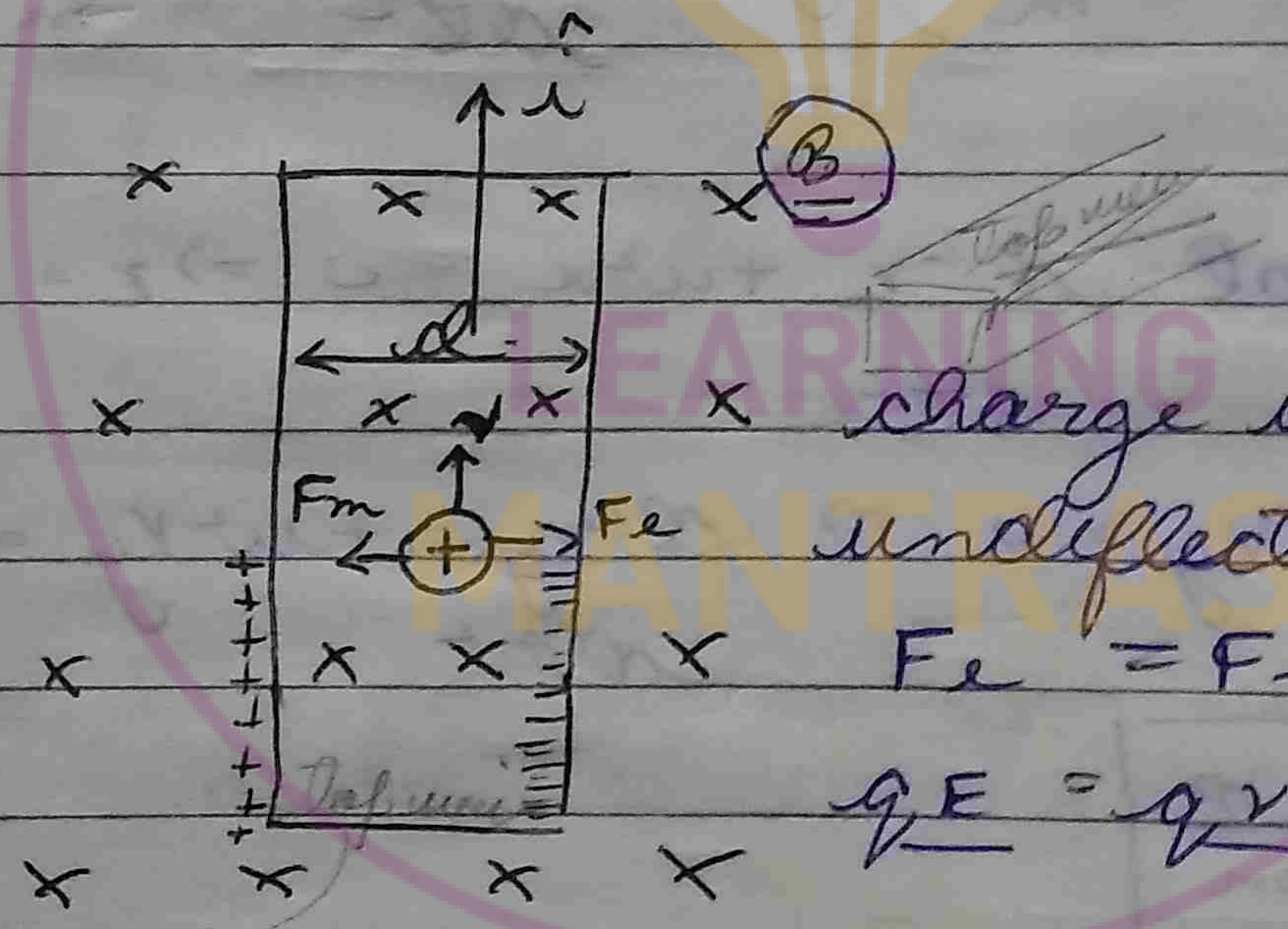
when $v_y = 0$

$\omega t = n\pi \Rightarrow t = \frac{\pi}{\omega}$

$\frac{dy}{dt} = \frac{E}{B}\sin\omega t \Rightarrow \int^{y_{max}} dy = \frac{E}{B}\int_0^{\pi/\omega} \sin\omega t \, dt$

• **Hall Effect**



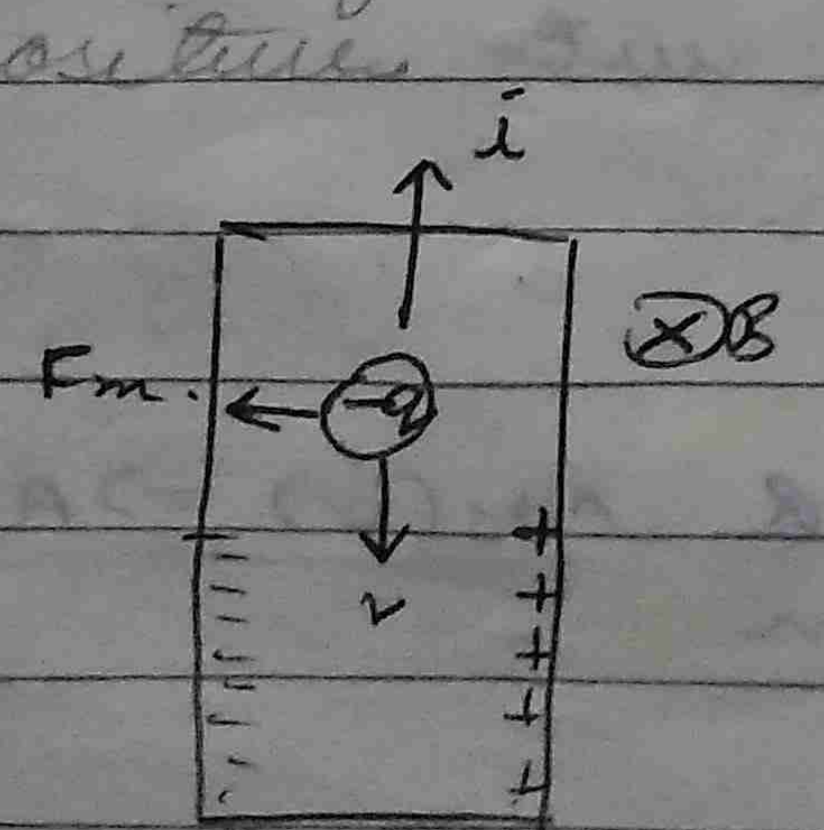


charge will move undeflected when

$F_e = F_m$

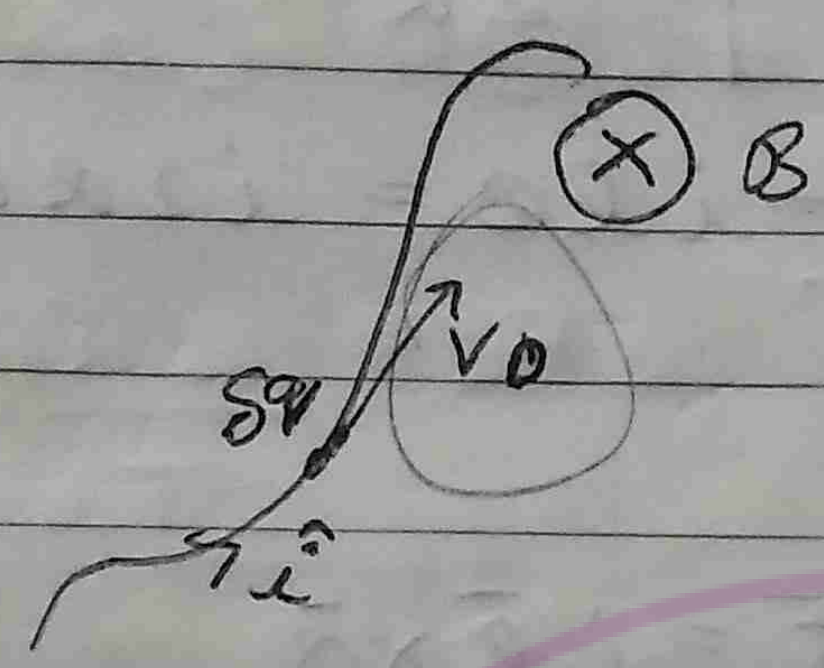
$qE = qvB$

when potential of left is higher than right charge carrier will be positive.



when the potential of right is higher than left then charge carrier is negative.

- Current carrying conductor in magnetic field.



$$\bar{F} = q\,\bar{v} \times \bar{B}$$

$$\delta \bar{F} = \delta q\, \bar{v}_0 \times \bar{B}$$

$$= \delta q \frac{\delta \bar{l}}{dt} \times \bar{B}$$

$$\Rightarrow \boxed{\delta \bar{F} = i\, \delta \bar{l} \times \bar{B}}$$

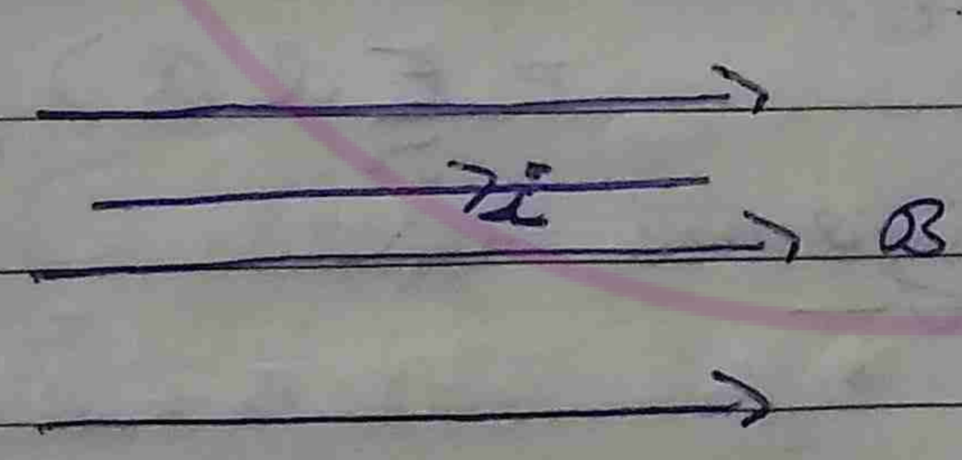
$$\bar{F} = \left(\int i\, \delta \bar{l} \times \bar{B}\right)$$

- If $\bar{B}$ is uniform $\bar{F} = \int (i\, \delta \bar{l} \times \bar{B})$

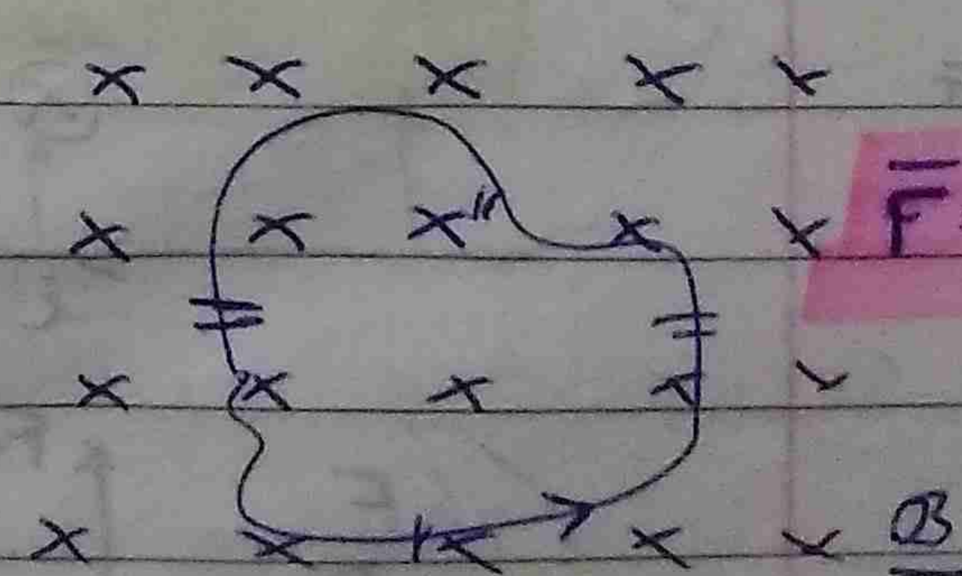
$$\bar{F} = \left(\int i\, \delta \bar{l}\right) \times \bar{B} \Rightarrow \boxed{\bar{F} = i\, \bar{l} \times \bar{B}}$$

- If $\bar{B}$ is uniform : $\bar{F} = i\, \bar{l} \times \bar{B}$

- If $\bar{l} \parallel \bar{B} \Rightarrow F_m = 0$

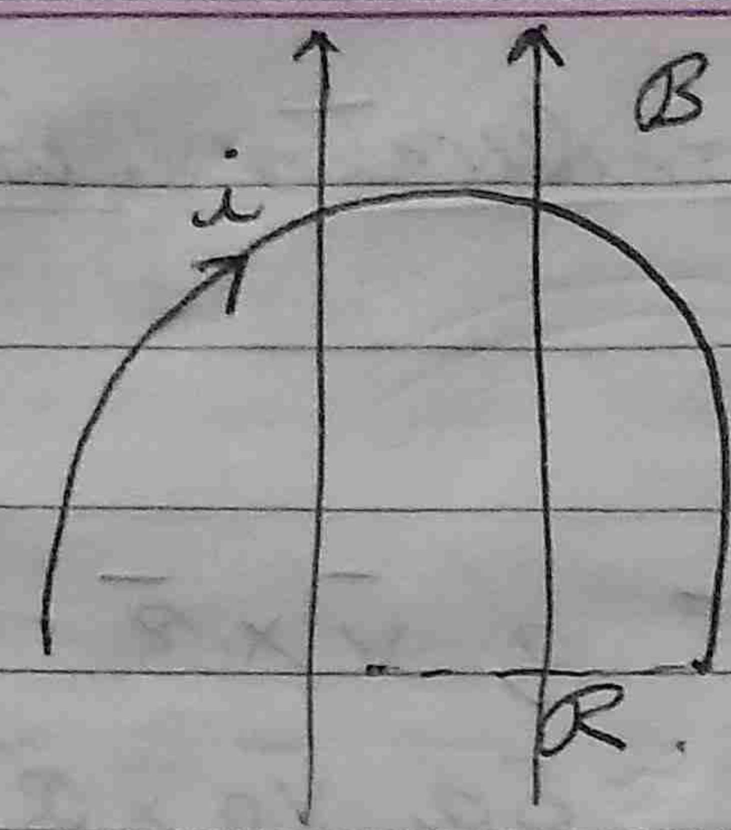


\# for loop of any shape : $\bar{F} = 0$

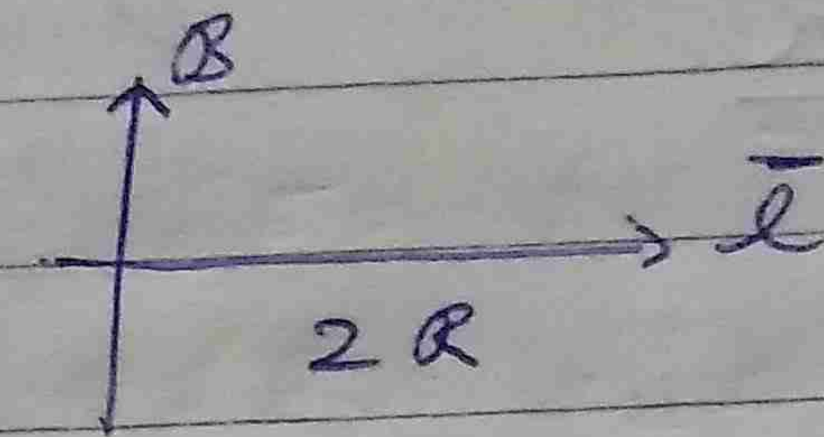


- net force is zero but individual parts experience force.

e.g.

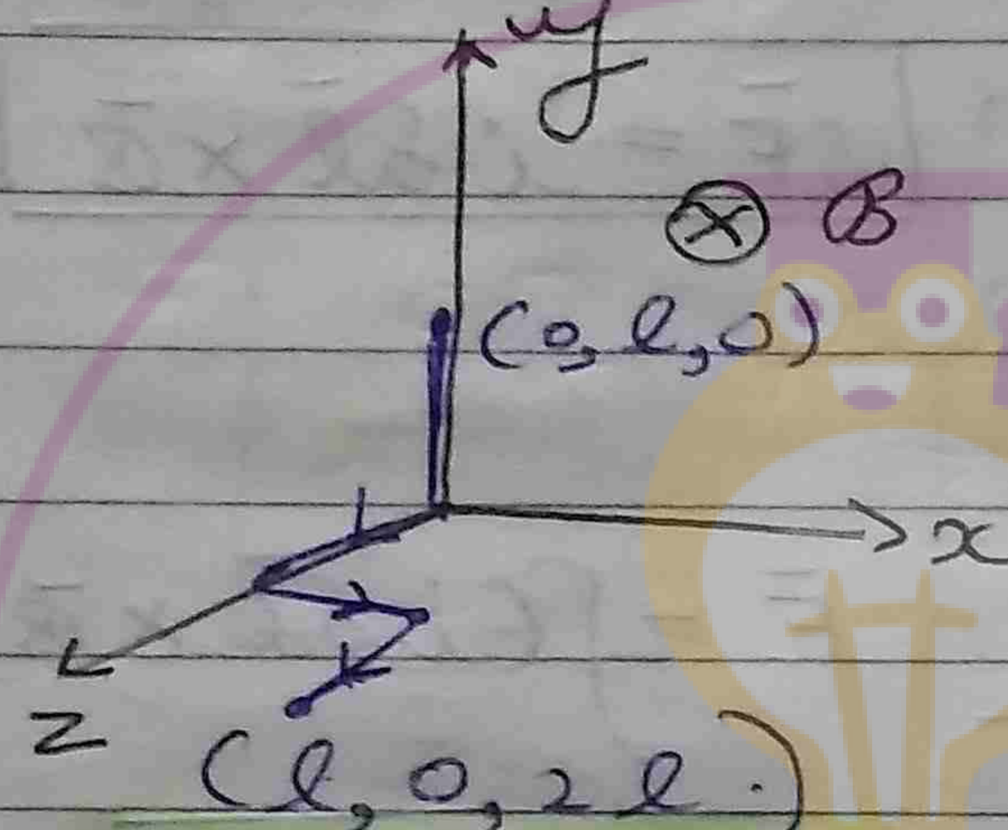


$$\bar{F} = i\bar{l} \times \bar{B}$$



$$F = ilB = i2RB \odot$$

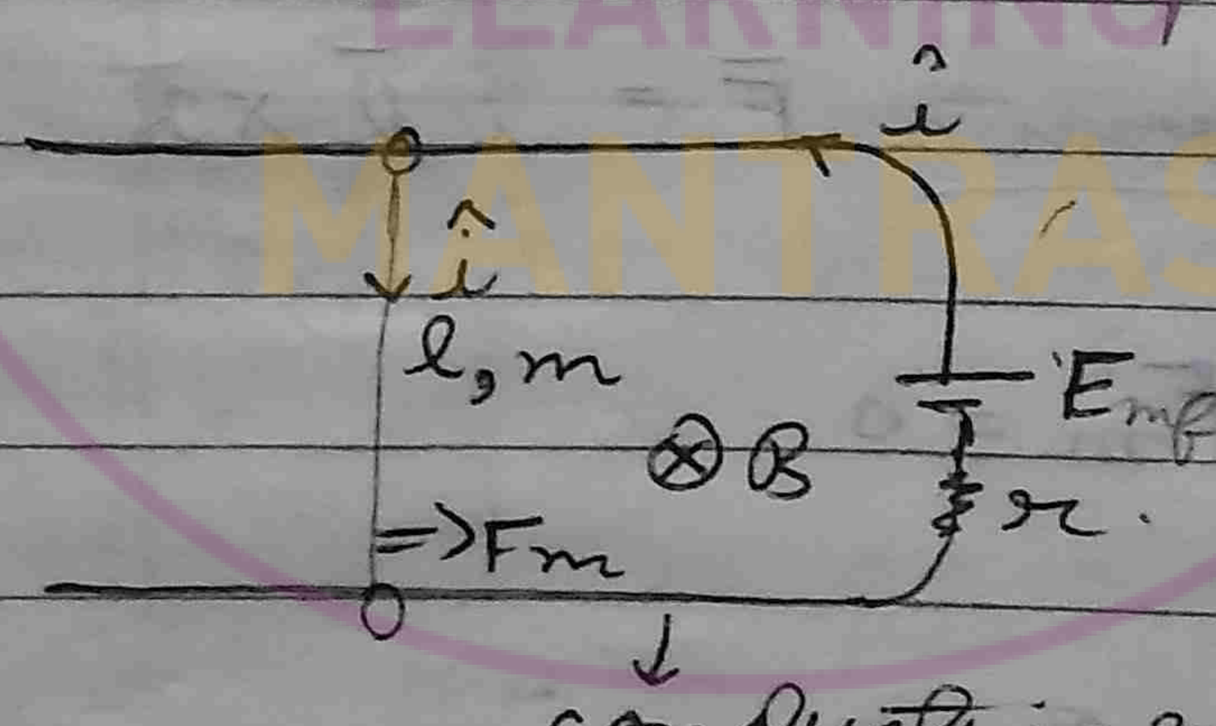
★



$$\bar{F} = i\bar{l} \times \bar{B}$$

$$F = i(l\hat{i} - l\hat{j} + 2l\hat{k}) \times (-B$$

#



μ = coefficient of friction

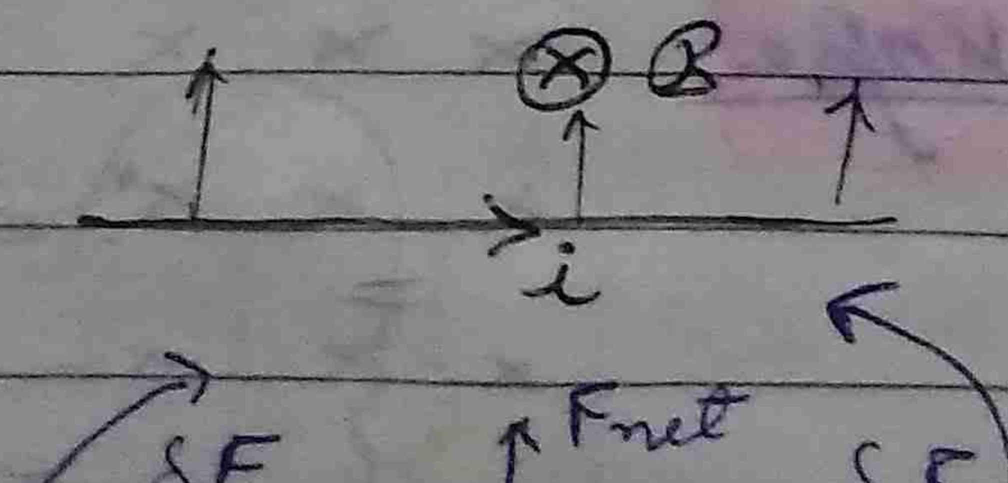
$$F_m = ilB$$

$$= \frac{E}{r} lB$$

$$= \frac{E}{r} lB > \mu mg$$

if the conductor has to move

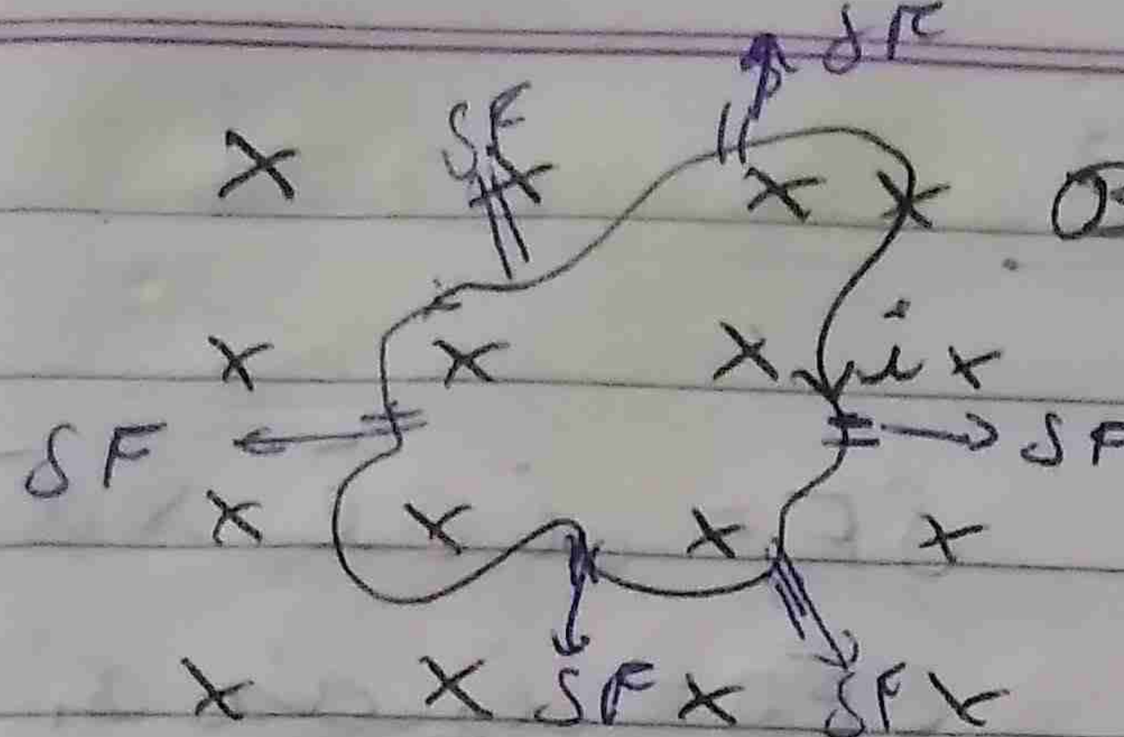
#



$F = ilB$ - - - μf

net torque is zero

#



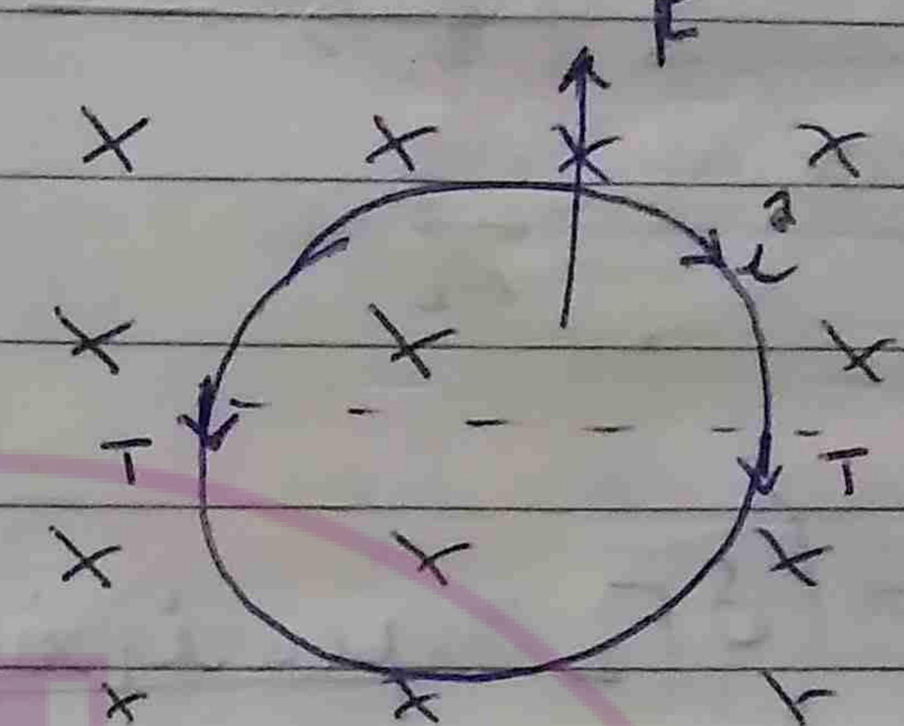
Total length of conductor is l

F_{net} on conductor is 0

Find the tension

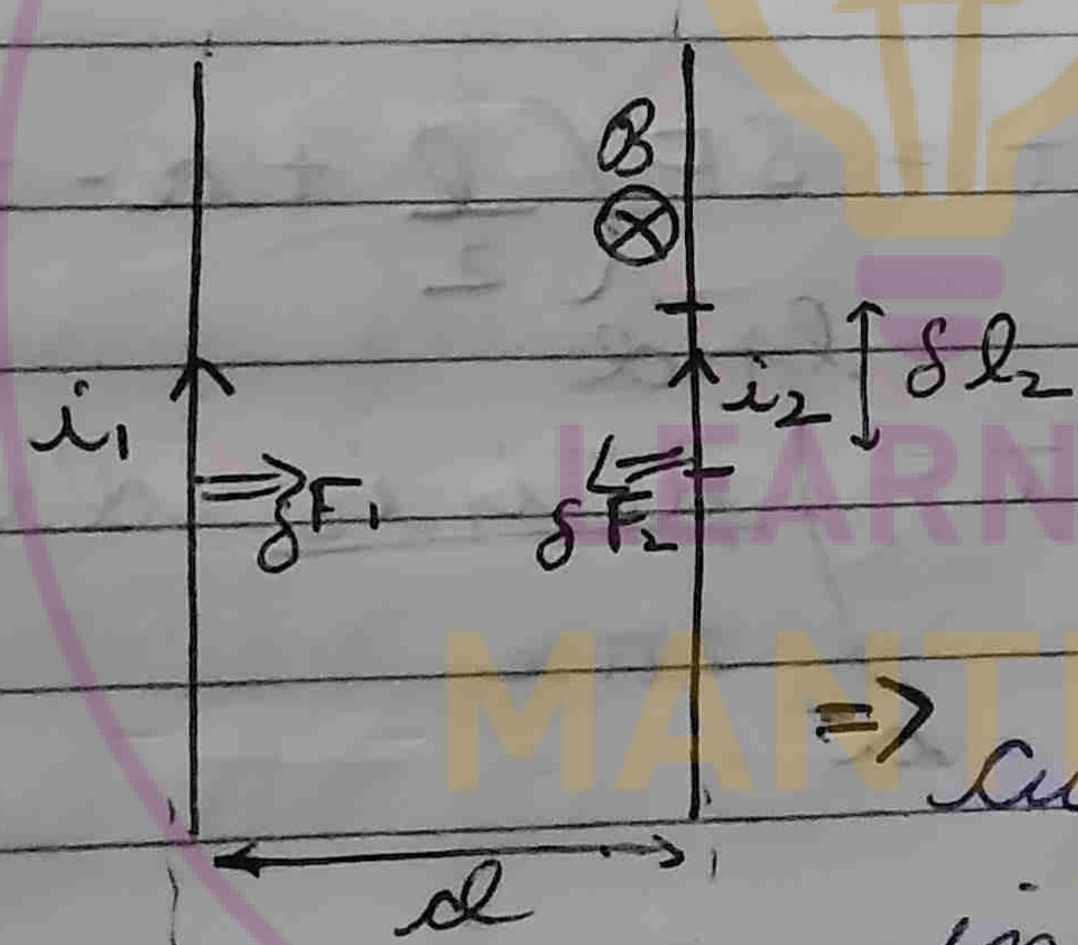
$F_{s.c} = i\,2R B = 2T$

$\Rightarrow i \frac{2L}{2\pi} B = 2T$



$$T = \frac{BiL}{2\pi}$$

Force between parallel current wire



$B_1 = \frac{\mu_0 i_1}{2\pi d}$; $\delta F_2 = i_2 \delta \bar{l}_2 \times \bar{B}_1$

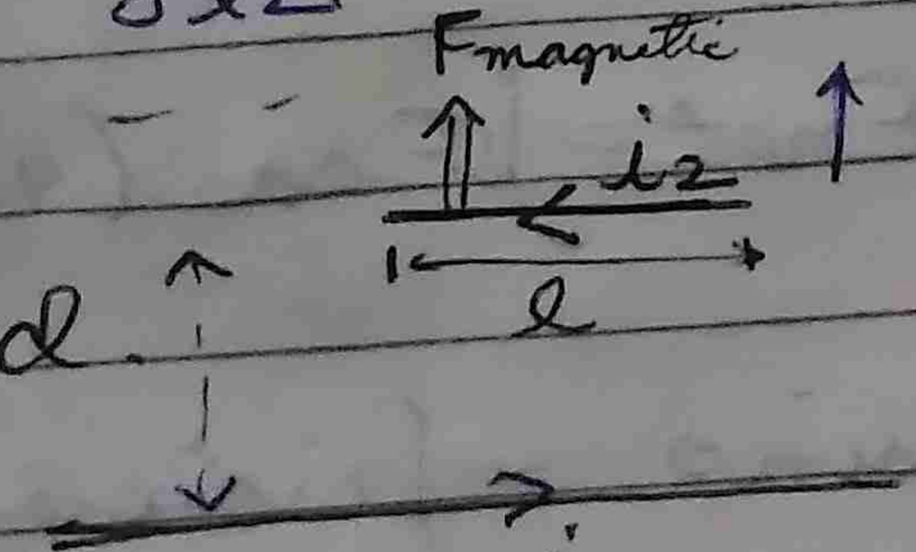
$= i_2 \delta l_2 B_1$

$\Rightarrow$ current carrying wire in same direction would attract.

$\delta F_2 = i_2 \delta l_2 B_1$

$\frac{\delta F_2}{\delta l_2} = i_2 B_1$ $\Rightarrow$ $\frac{\delta F}{\delta l} = \frac{\mu_0 i_1 i_2}{2\pi d}$

Ex-

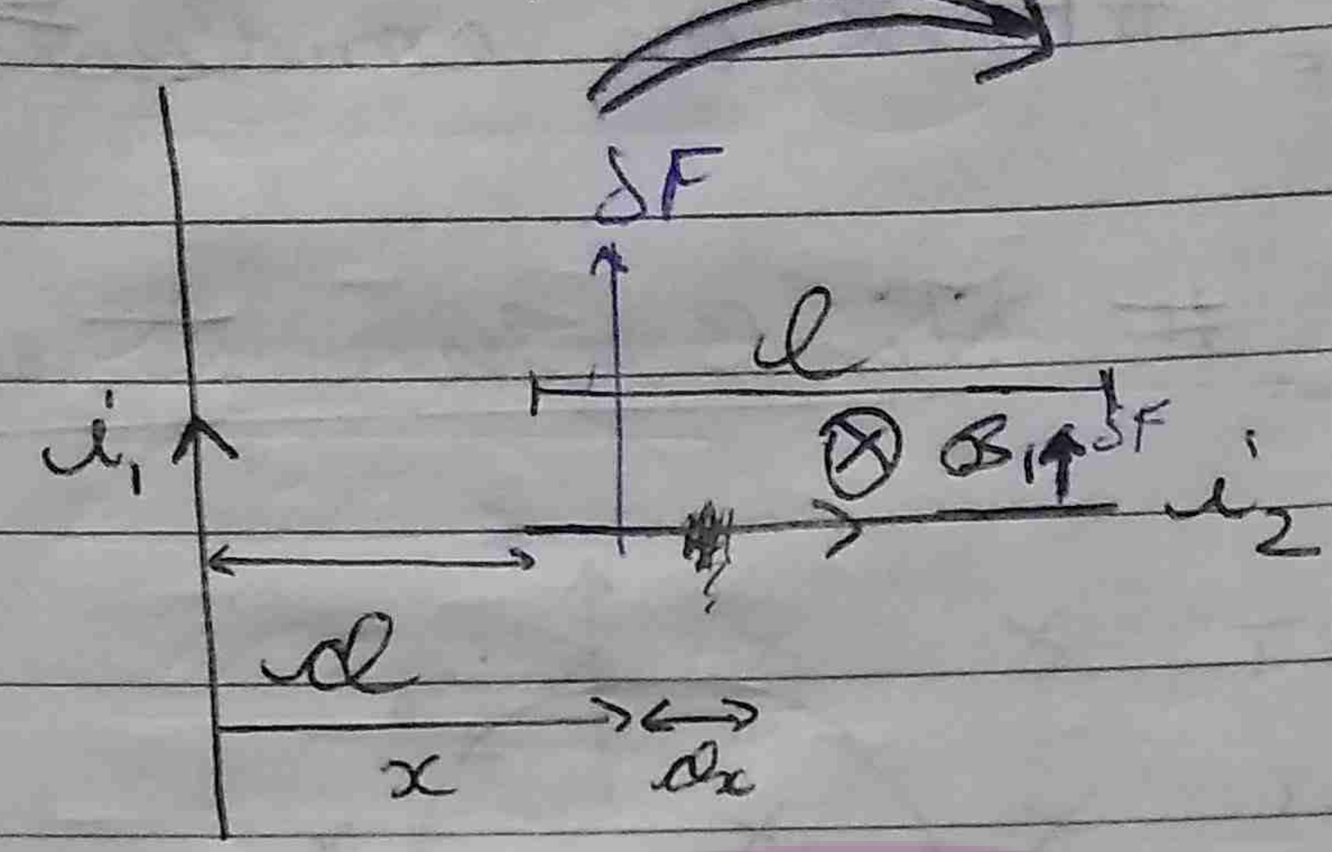


If the rod is at rest

$F_{magnetic} = mg$

$\frac{\mu_0 i_1 i_2}{2\pi d} l = mg$

Non uniform field

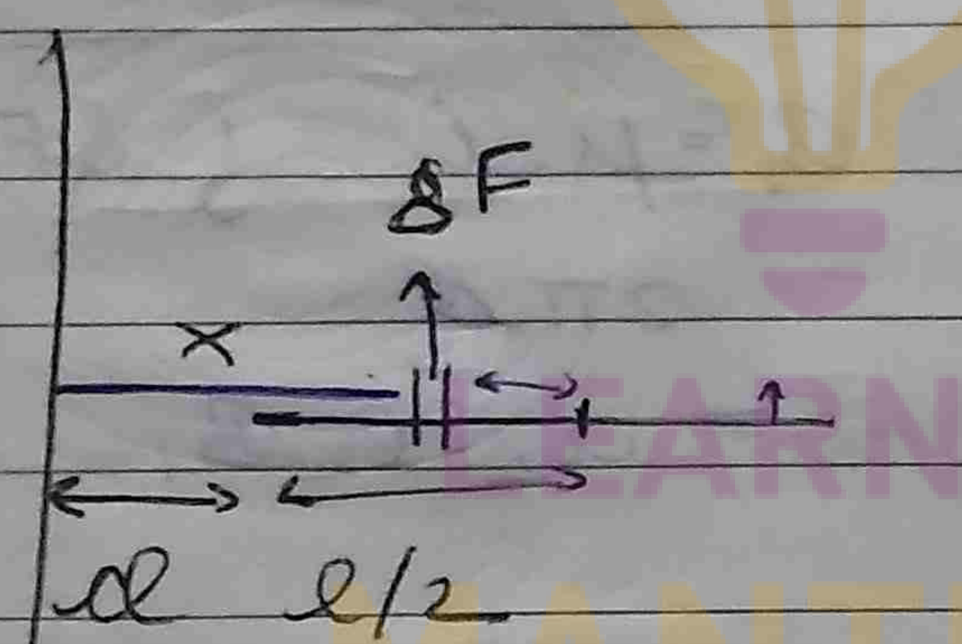


$\delta F = i\,\delta \bar{l} \times \bar{B}$

$\delta F_2 = i_2\,dx\,B_1$

$\delta F_2 = \frac{\mu_0 i_1 i_2\,dx}{2\pi x}$

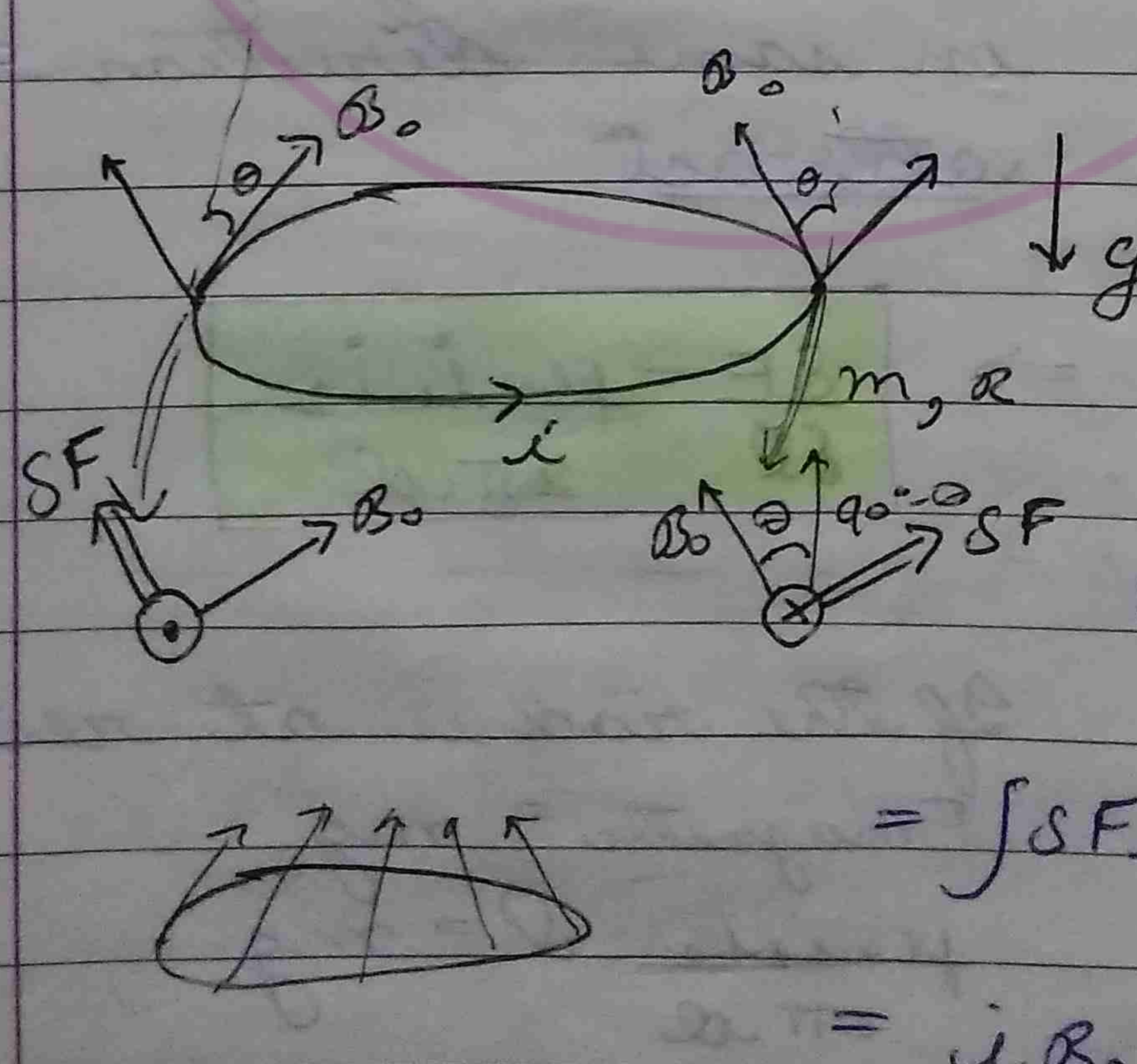
$$F = \int \delta F_2 = \frac{\mu_0 i_1 i_2}{2\pi}\int_{d}^{d+l}\frac{dx}{x} = \boxed{\frac{\mu_0 i_1 i_2}{2\pi}\ln\left(\frac{d+l}{d}\right)}$$



$\tau = \delta F\left(\frac{l}{2} + d - x\right)$

$$\tau = \int_{d}^{l+d}\frac{\mu_0 i_1 i_2}{2\pi x}\,dx\left(\frac{l}{2} + d - x\right)$$

e.g.



$\delta \bar{F} = i\,\delta\bar{l} \times \bar{B}$

$\delta F = i\,dx\,B_0$

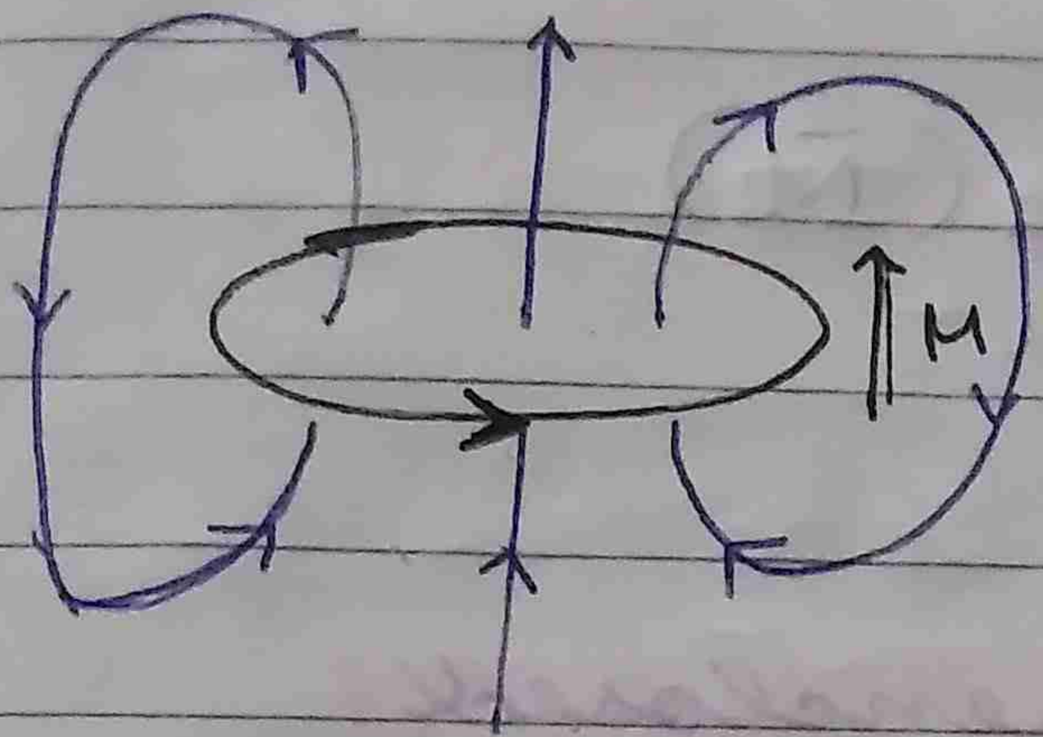
$F_{net} = \int \delta F \cos(90-\theta)$

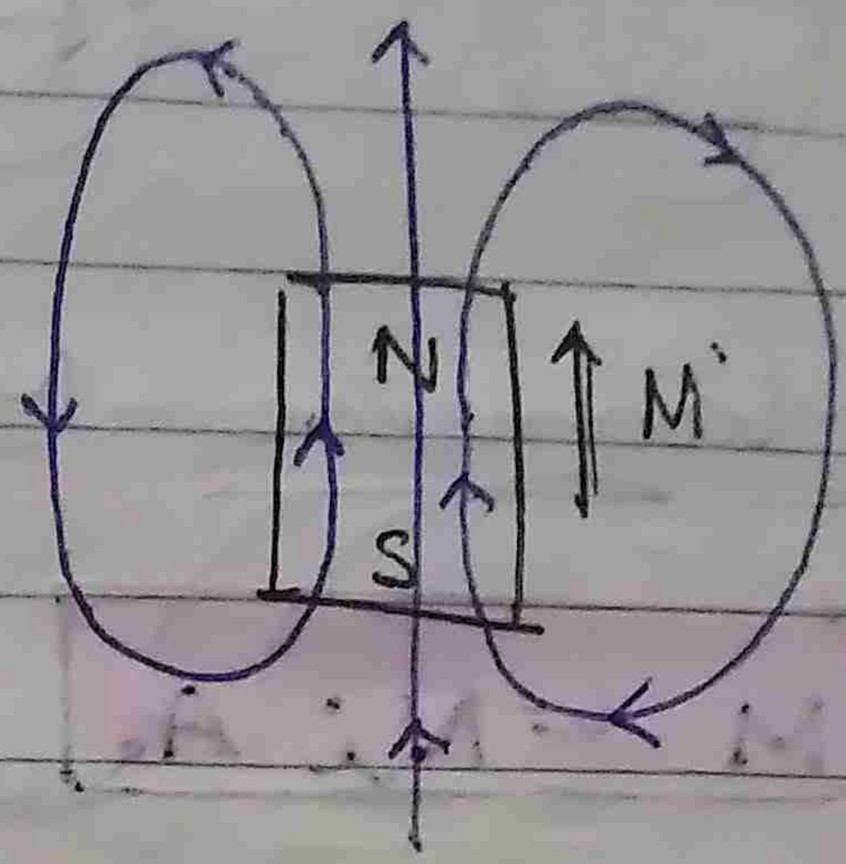
$= \int \delta F \sin\theta = \int i\,dx\,B_0 \sin\theta$

$= i B_0 \sin\theta\, 2\pi R$

If at rest → $i B_0 \sin\theta\, 2\pi R = mg$

Magnetic Dipole





- A current carrying loop of any shape behaves as a magnetic dipole.

- difference between field, force, dipole

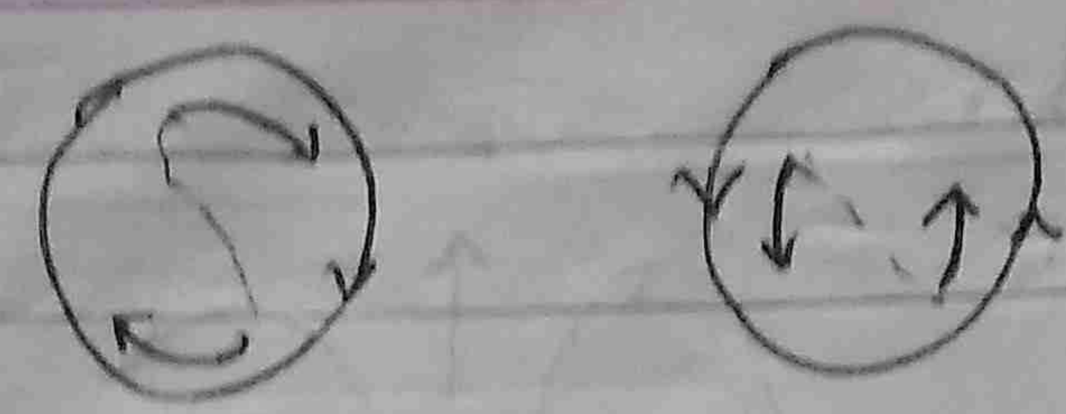
- A current carrying conductor produces magnetic field.

- A current carrying conductor experiences force in Bext

- A current carrying loop behaves as a magnetic dipole

C.W ⇒ acts as S
a.c.w ⇒ acts as N

✡ The magnetic moment is directed from C.W(S) to a.c.w(N)

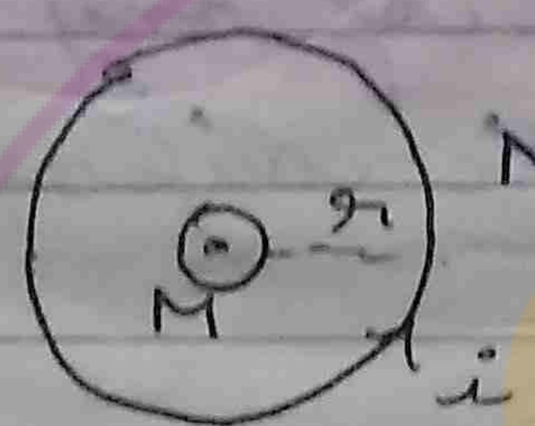


• **Magnetic Moment ($\bar{M}$)**

$$M = NiA$$

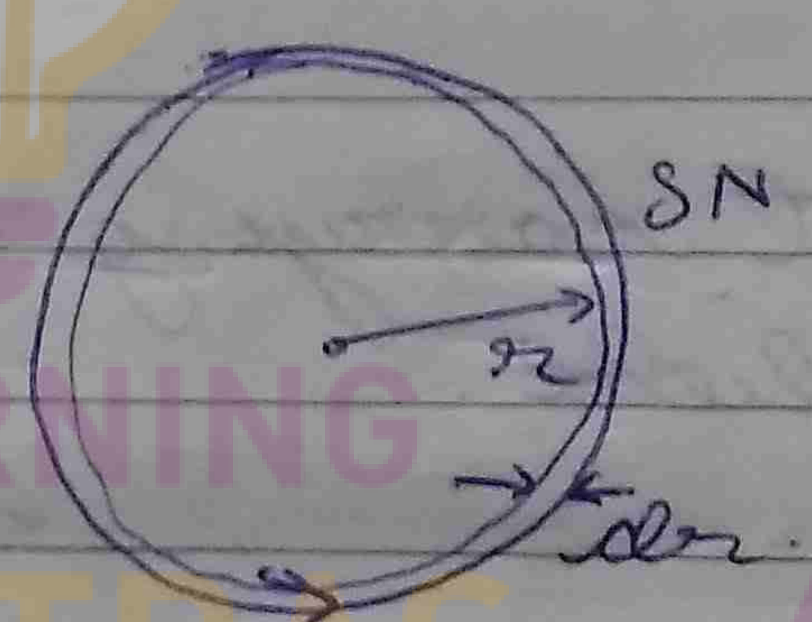
A → Area enclosed

•



$N = 2$ $\quad M = NiA = (2)(i)(\pi r^2)$ ... Amp

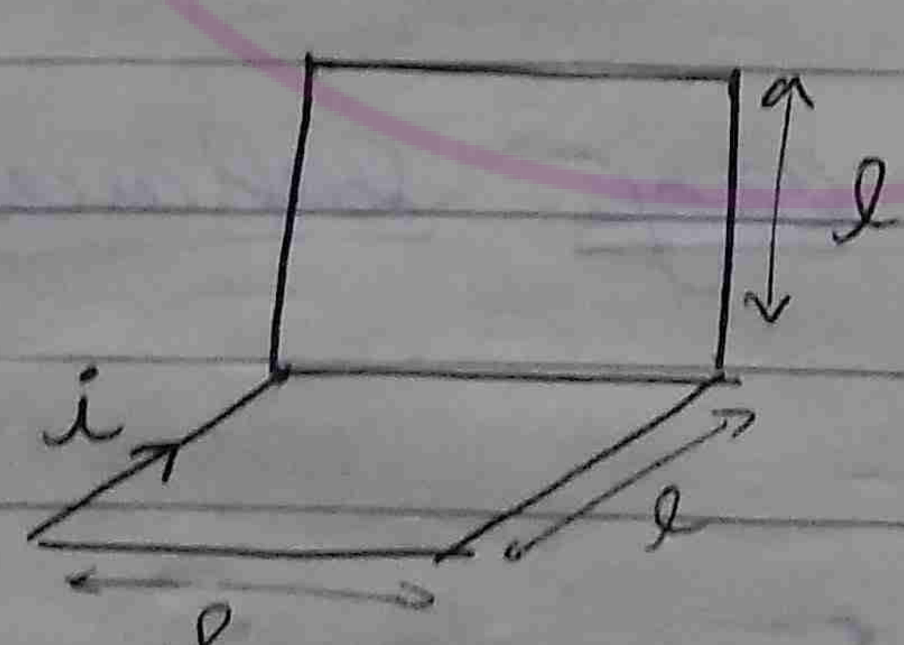
✡

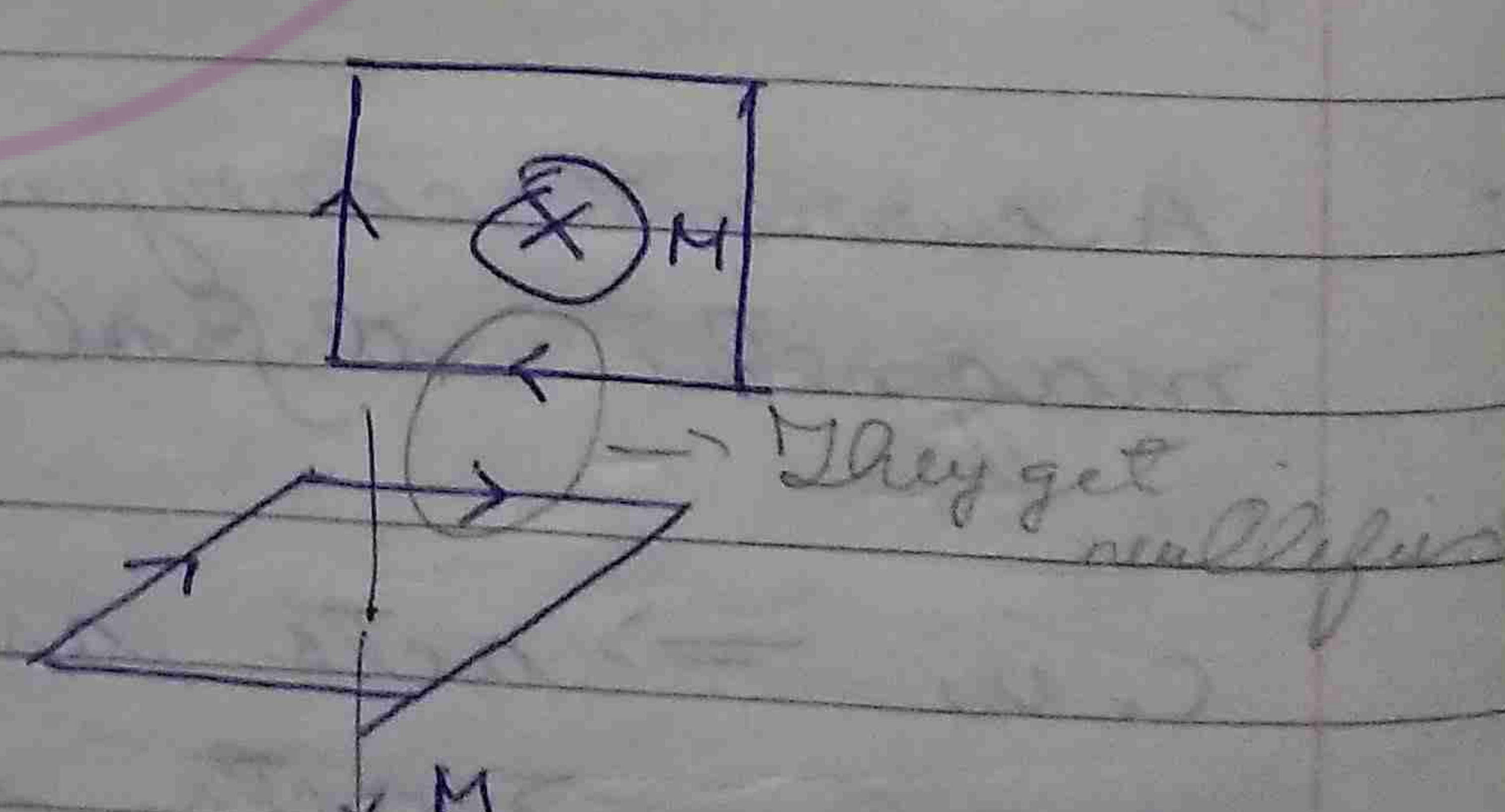


$$\delta M = \delta N I \pi r^2$$

$$\int \delta M = \int_0^R \frac{N}{R} dr \, \pi r^2$$

#

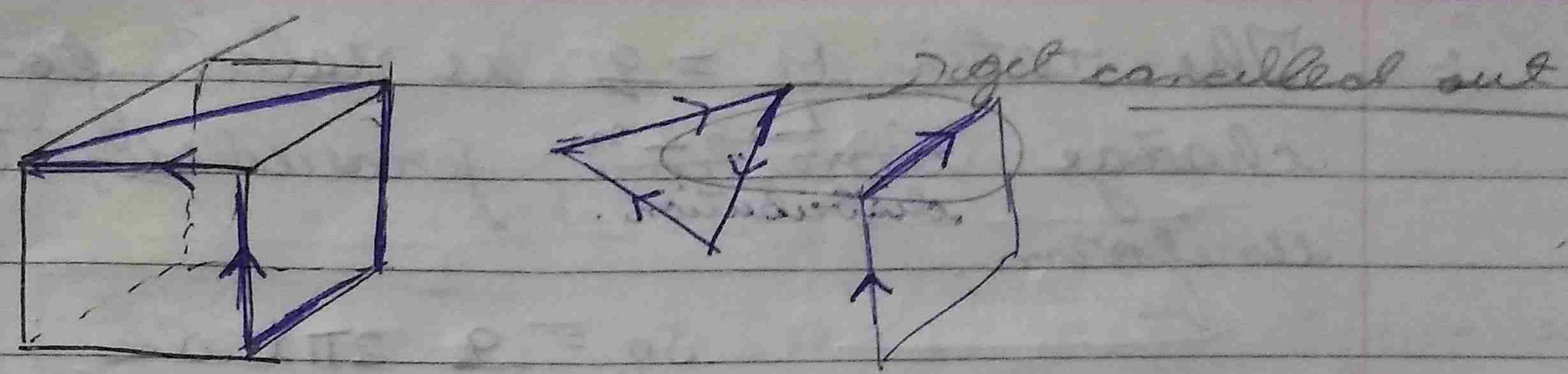


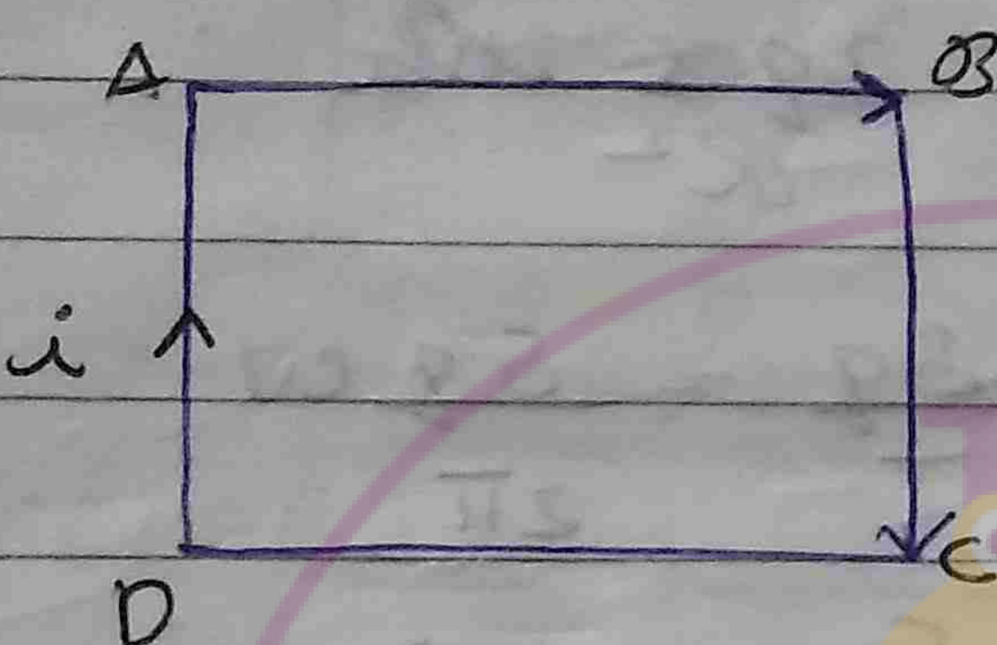


$$M = NiA = (1)(i)(l^2)$$

$$M_{net} = \sqrt{2} M$$

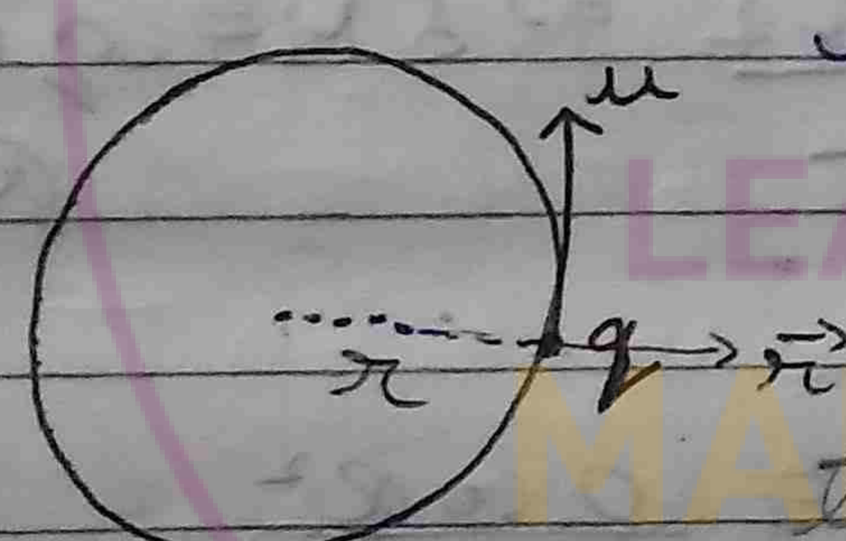
eg





$\bar{M} = i(\bar{AB} \times \bar{BC}) = i(\bar{BC} \times \bar{CD})$

- **Circulating charges**



$i = \dfrac{q}{\frac{2\pi r}{u}} = \dfrac{qu}{2\pi r}$ (time period)

$M = NiA = \dfrac{qu\pi r^2}{2\pi r} \Rightarrow \bar{M} = \dfrac{qur}{2} \odot$

$\bar{L} = \bar{r} \times \bar{p} \Rightarrow \bar{L} = rmu \odot$

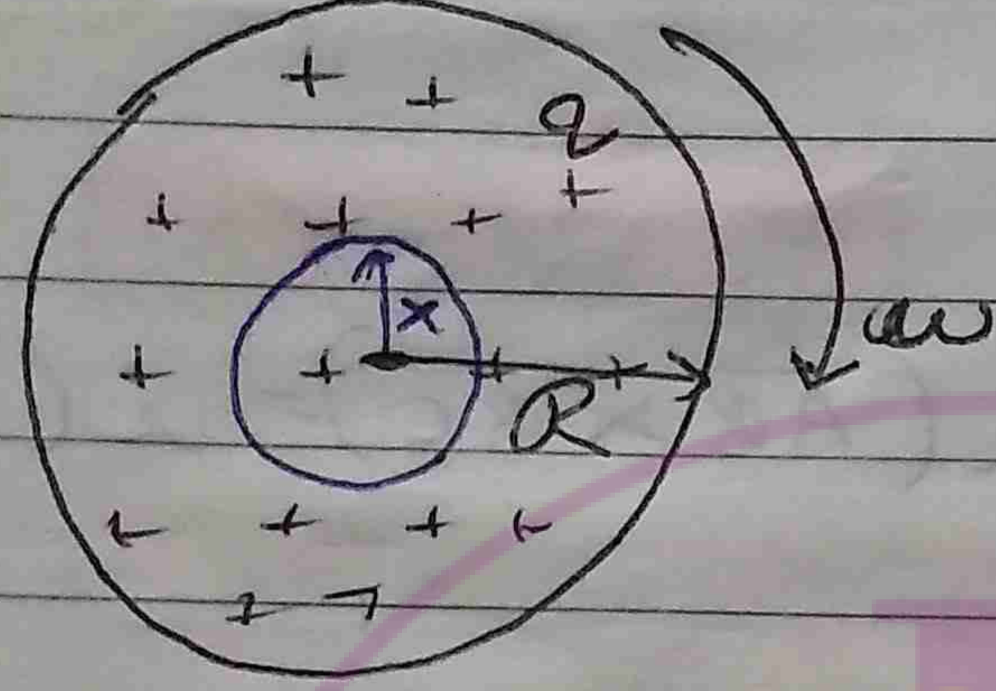
- If q is +ve $\Rightarrow \bar{M} \uparrow\uparrow \bar{L}$ (parallel)
- If q is −ve $\Rightarrow \bar{M} \uparrow\downarrow \bar{L}$ (antiparallel)

$\dfrac{M}{L} = \dfrac{qur/2}{rmu} \Rightarrow \dfrac{M}{L} = \dfrac{q}{2m} \Rightarrow \boxed{\bar{M} = \dfrac{q}{2M}\bar{L}}$

(specific charge; M → mass)

gyromagnetic ratio

- The ratio $\frac{M}{L} = \frac{q}{2m}$ is valid for any charge ~~direction~~ distribution provided it is uniform.



$$\delta q = \frac{q}{\pi R^2} 2\pi x \, dx$$

$$\delta q = \frac{2qx}{R^2} dx$$

$$\delta I = \frac{\delta q}{T} = \frac{\delta q \, \omega}{2\pi}$$

$$\delta M = \delta I \pi x^2 \Rightarrow \frac{\delta q \, \omega}{2\pi} \pi x^2$$

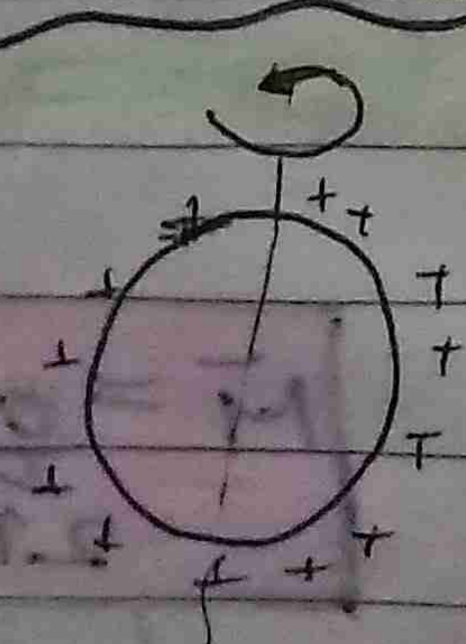
$$\delta M = \frac{(2qx\,dx)}{R^2} \frac{\omega x^2}{2} \Rightarrow \delta M = \frac{q\omega}{R^2} x^3 dx$$

$$\int \delta M = \frac{q\omega}{R^2} \int_0^R x^3 dx = \frac{q\omega R^2}{2}$$

$$\Rightarrow M = \frac{q\omega R^2}{(2)(2)}$$

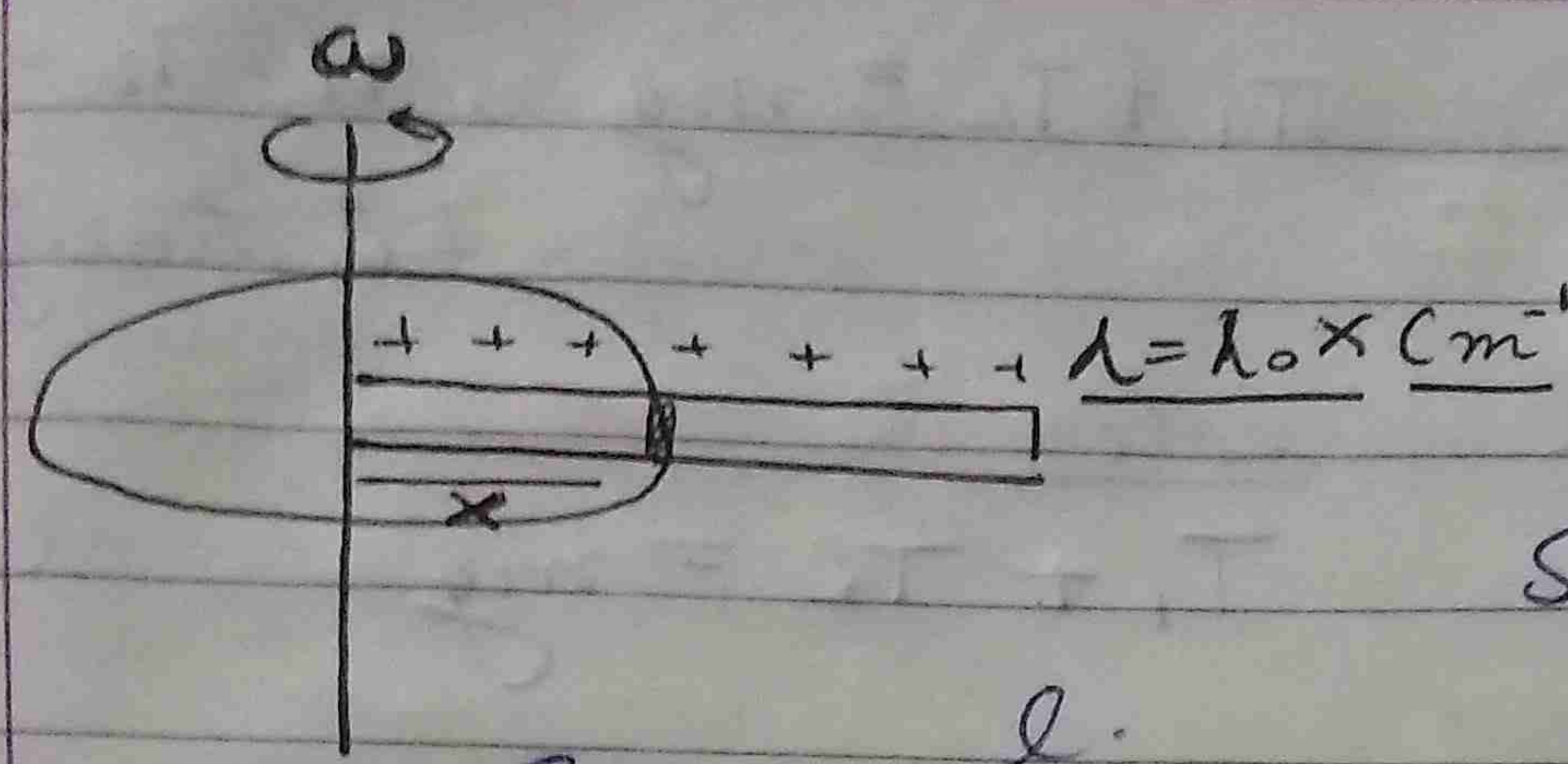
$$L = I\omega = \frac{mR^2}{2}\omega \Rightarrow \boxed{\frac{M}{L} = \frac{q}{2m}}$$

☆ #



$$M = \frac{Lq}{2m} = \frac{I\omega q}{2m} = \boxed{\frac{2}{3} mr^2 \frac{\omega q}{2m}}$$

#



$\delta I = \frac{\delta q}{2\pi}\omega$

$\delta M = \delta I \pi x^2 = \frac{\delta q \omega \pi x^2}{2\pi}$

$\int \delta M = \int_0^l \frac{(\lambda_0 x)dx \, \omega x^2}{2}$

- Loop in magnetic field (uniform)

- A dipole in uniform E

(i) $F_{net} = 0$

(ii) $\bar{\tau}_{net} = \bar{p} \times \bar{E}$

(iii) $U = -\bar{p} \cdot \bar{E}$

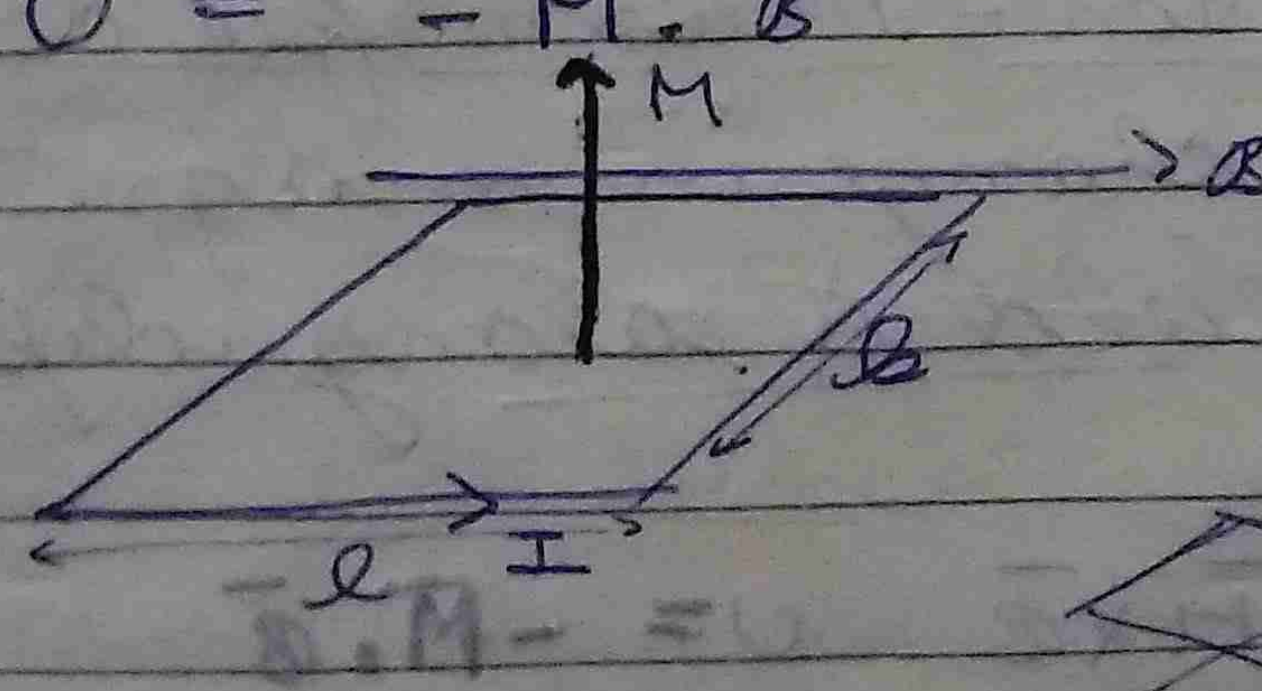
- If a current carrying loop is in uniform magnetic field

(i) $F_{net} = 0$ $(F = (i\,\bar{l} \times B)$ where $\bar{l} = 0)$

2 $\bar{\tau} = \bar{M} \times \bar{B}$

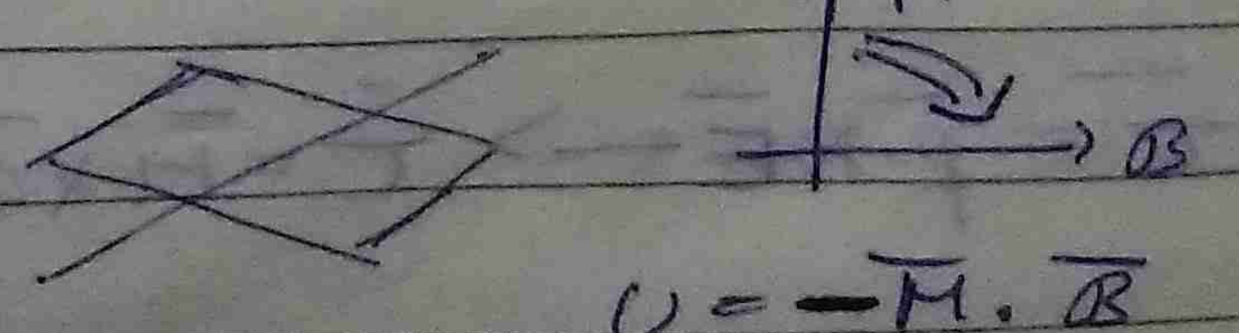
3 $U = -\bar{M} \cdot \bar{B}$

☆



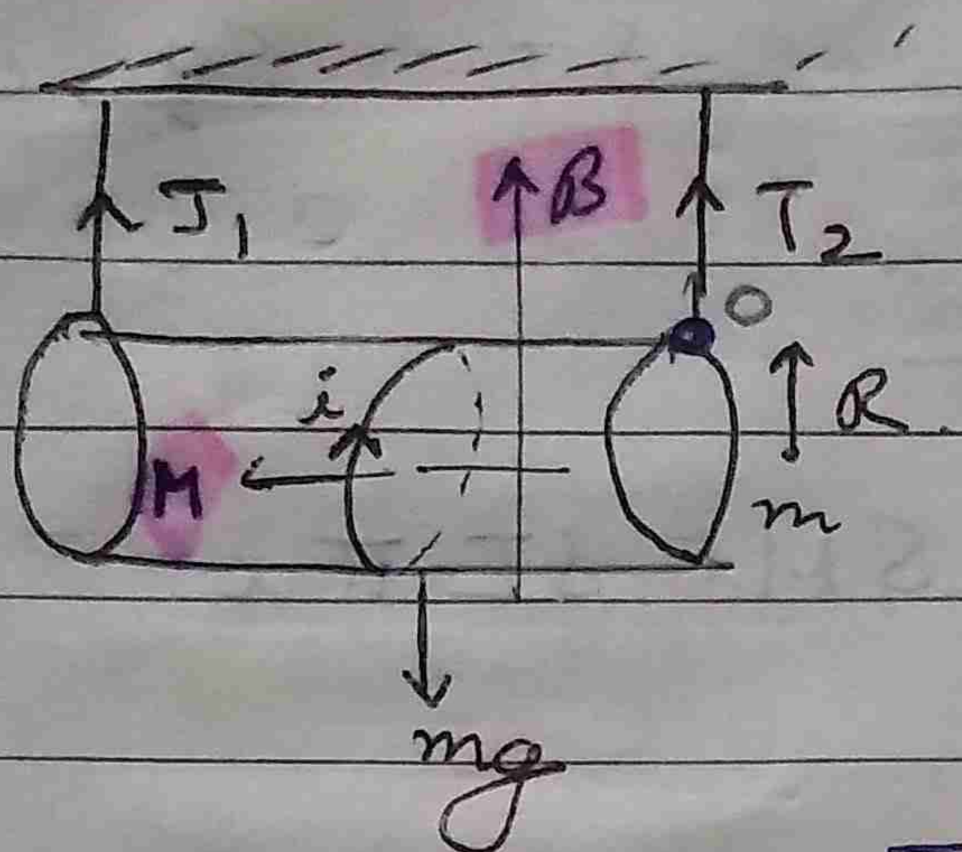
$F = 0$

$\tau = MB \sin \pi/2$



$U = -\bar{M} \cdot \bar{B} = 0$

#



$T_1 + T_2 = mg$; $T_1 = T_2 = \frac{mg}{2}$ (initially)

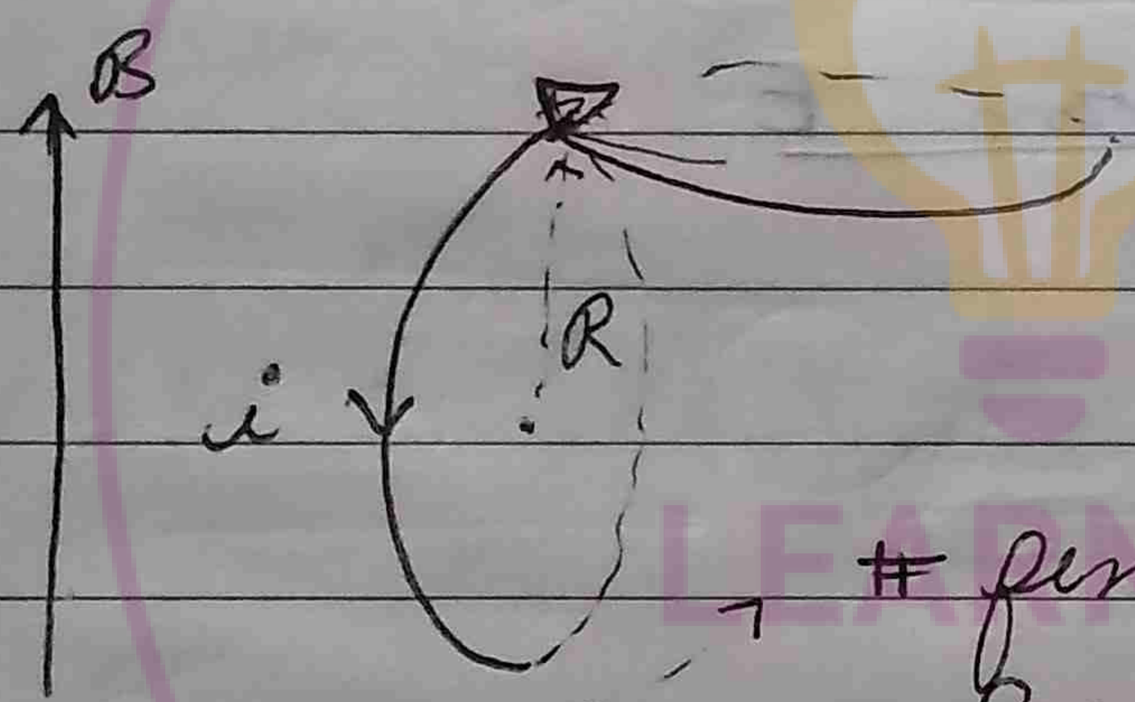
<u>later</u>

$T_1 + T_2 = mg$ --- (i)

$T_1 l - mg\frac{l}{2} + MB = 0$ (ii)

$\Rightarrow T_1 = \frac{mg}{2} - \frac{MB}{l} < \frac{mg}{2}$

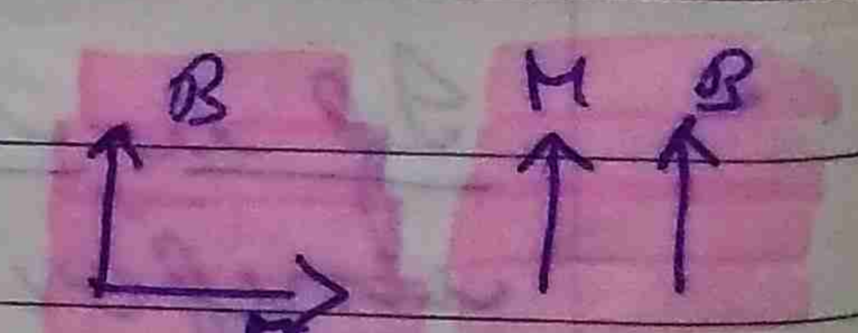
☆



• can move in vertical <u>plane</u>

find ω of ring when it becomes horizontal.

$I = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}MR^2$



$W_{ext} + W_{nc} = \Delta K + \Delta U$

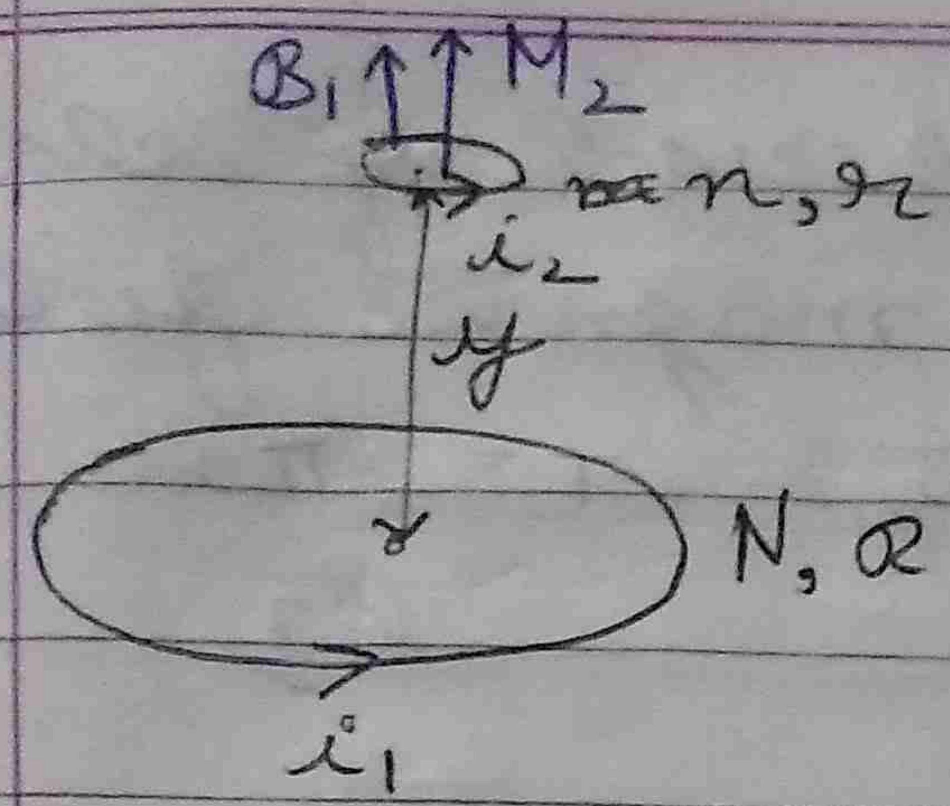
$0 = \left(\frac{1}{2}I\omega^2 - 0\right) + (mg\,R) + (-MB)$

• <u>Magnetic dipole in non-uniform field</u>

$F = M\frac{dB}{dl}$ → change in magnetic field along dipole.

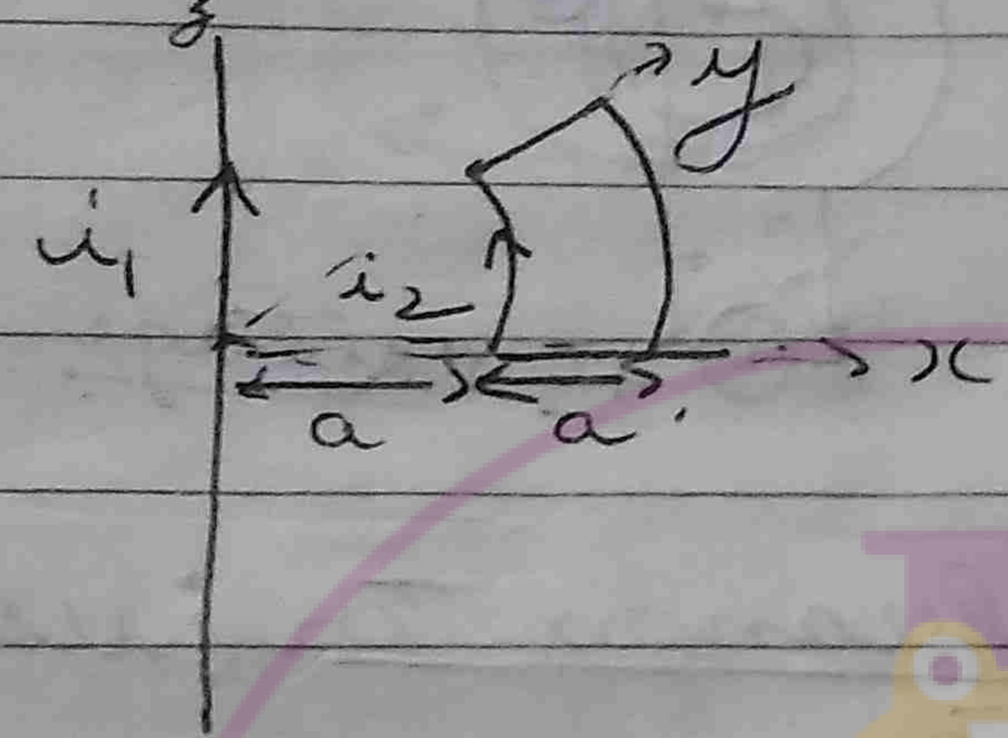
$\bar{\tau} = \bar{p} \times \bar{E} \longrightarrow \bar{\tau} = \bar{M} \times \bar{B} \quad U = -\bar{M}\cdot\bar{B}$

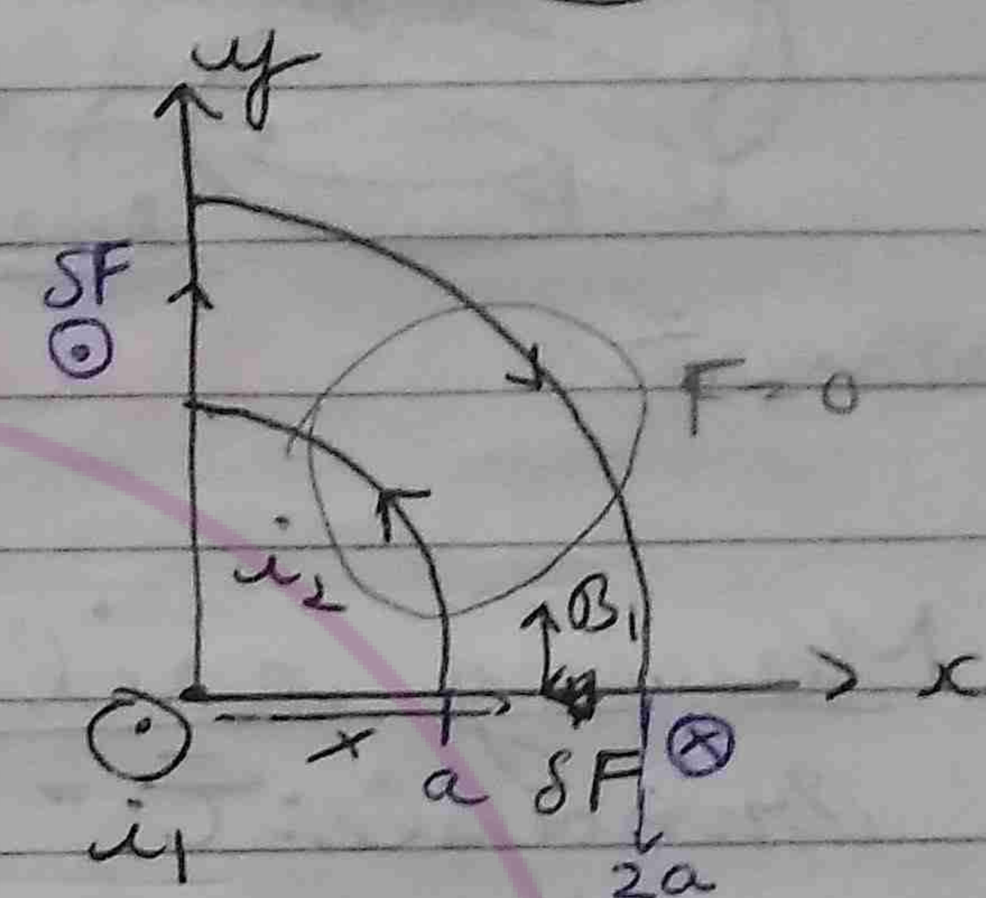
e.g. find force on smaller loop



$$F = M_2 \frac{dB}{dy} = M_2 \frac{d}{dy}\left(\frac{\mu_0 N i_1 R^2}{2(R^2+y^2)^{3/2}}\right)$$

#





$$\delta F = i_2\, \delta \bar{l} \times \bar{B} \Rightarrow \delta F = i_2\, dx\, B_1 \otimes$$

$$\delta F = i_2\, dx \frac{\mu_0 i_1}{2\pi x}$$

$$\delta \tau = (\delta F)(\sqrt{2}x) \Rightarrow \delta \tau = \frac{\mu_0 i_1 i_2 \sqrt{2}\, x\, dx}{2\pi x}$$

$$= \frac{\mu_0 i_1 i_2}{\sqrt{2}\pi} \int_a^{2a} dx$$

- **Cyclotron**

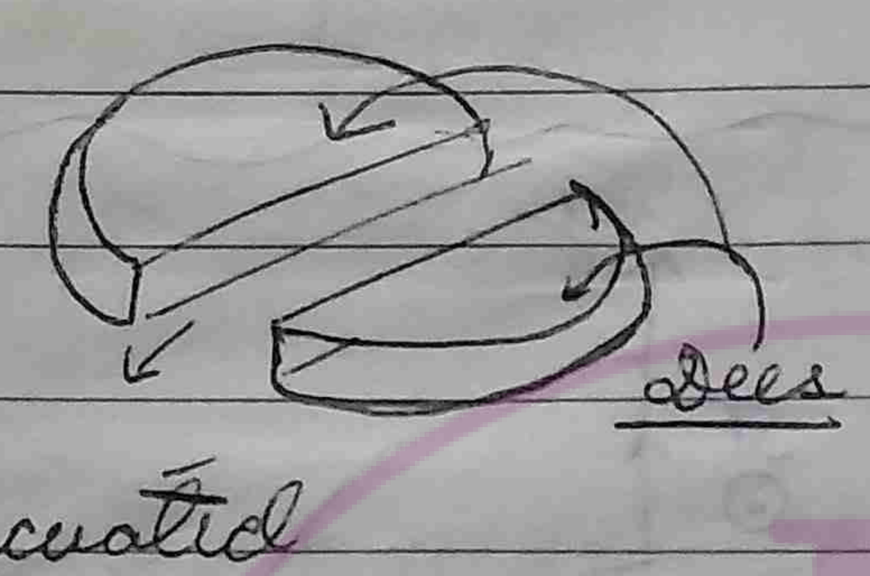
  Lawrence and Living stone 1934

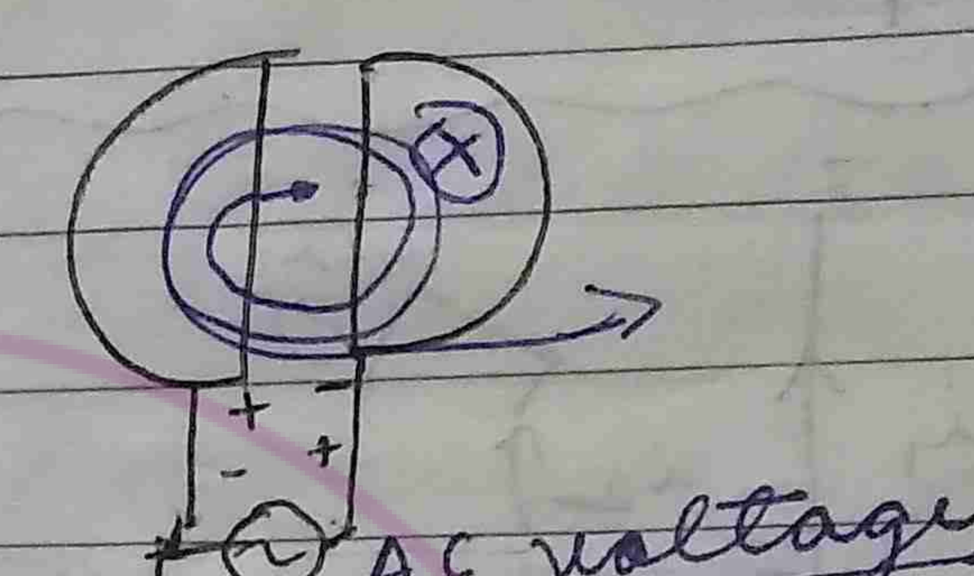
→ Used to accelerate charged particles to a very high speed.

- Cannot be used for light particles because their speed becomes comparable to light and mass increases.

Name of the Chapter ______ Date ______ Page No.: ______

Principle: The time period of charged particle in uniform magnetic field is independent of velocity. $T = \frac{2\pi m}{Bq}$

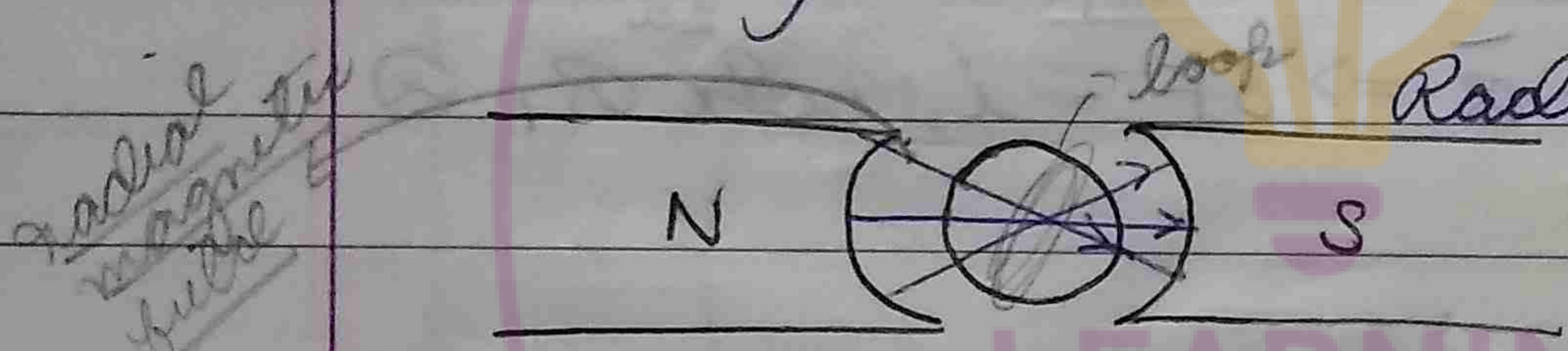




- Moving coil Galvanometer: used to measure current

Principle: $\bar{\tau} = \bar{M} \times \bar{B}$

Radial magnetic field $\Rightarrow \vec{M} \perp B$



At equilibrium $\tau = k\theta$ (k → spring constant)

$MB = k\theta$ $NiAB = k\theta \Rightarrow$ $i = \frac{k}{NBA}\theta$

current sensitivity $= \frac{\theta}{i}$

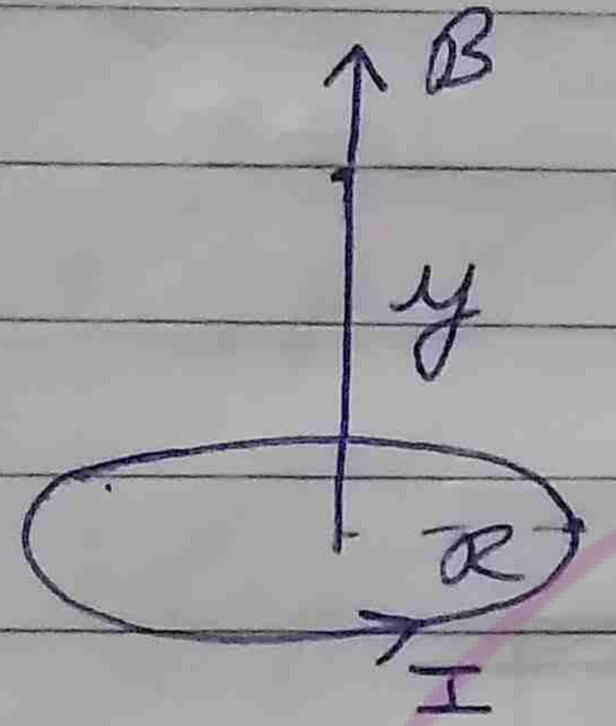
Voltage sensitivity $= \frac{\theta}{iR}$ $V.S = \frac{C.S}{R}$

• Magnetic pole strength (μ) ≈ charge in electrostatics

Magnetism of Materials

10^4 G (gauss) = 1 T

• Permanent magnet



$$B = \frac{\mu_0 I r^2}{2(y^2 + r^2)^{3/2}}$$

If $y >>> r$, let $M = \pi r^2 I$

(i) $B = \frac{\mu_0}{4\pi} \frac{2M}{y^3}$

(ii) If this loop is placed in uniform magnetic field ; $\bar{\tau} = \bar{M} \times \bar{B}$

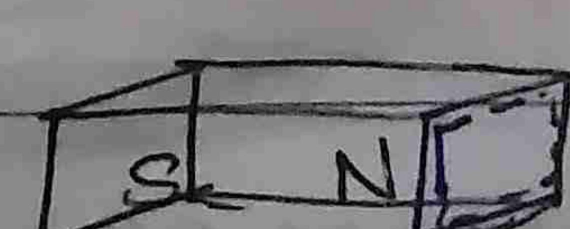
Electrostatics : $\bar{p}$ (dipole) ; $\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\bar{p}}{y^3}$ $\bar{\tau} = \bar{p} \times \bar{E}$

+ve charge ≡ N pole

−ve charge ≡ S pole

• Magnetic pole strength (μ) (A-m)

• μ depends on



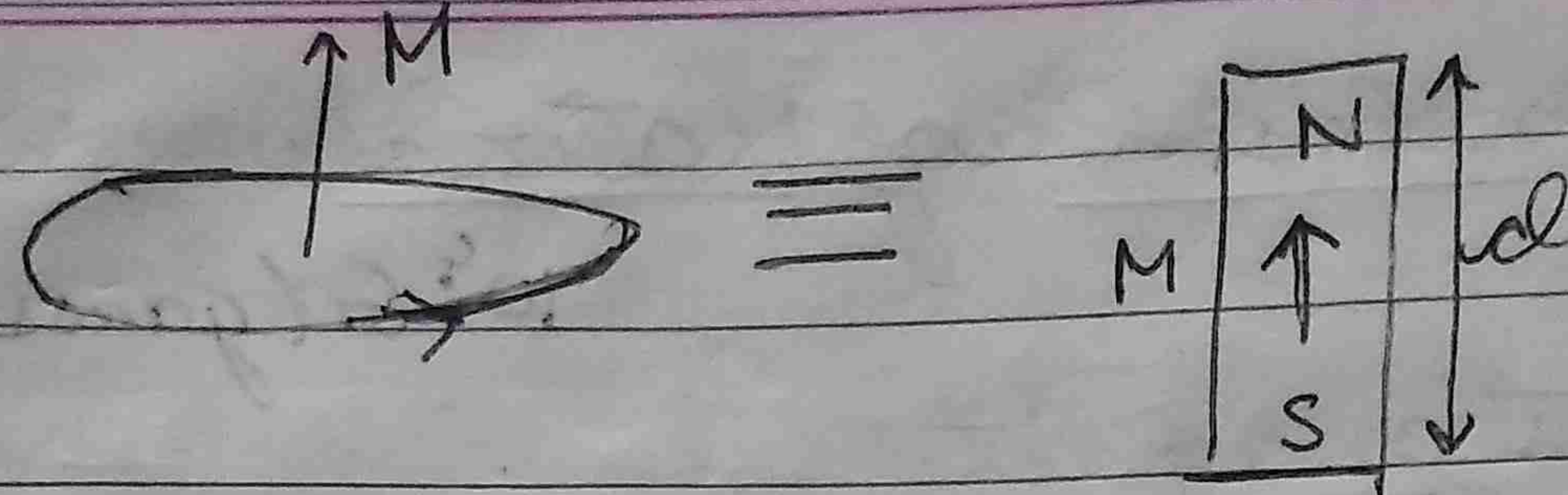
(i) Intensity of magnetisation

(ii) Area of poles

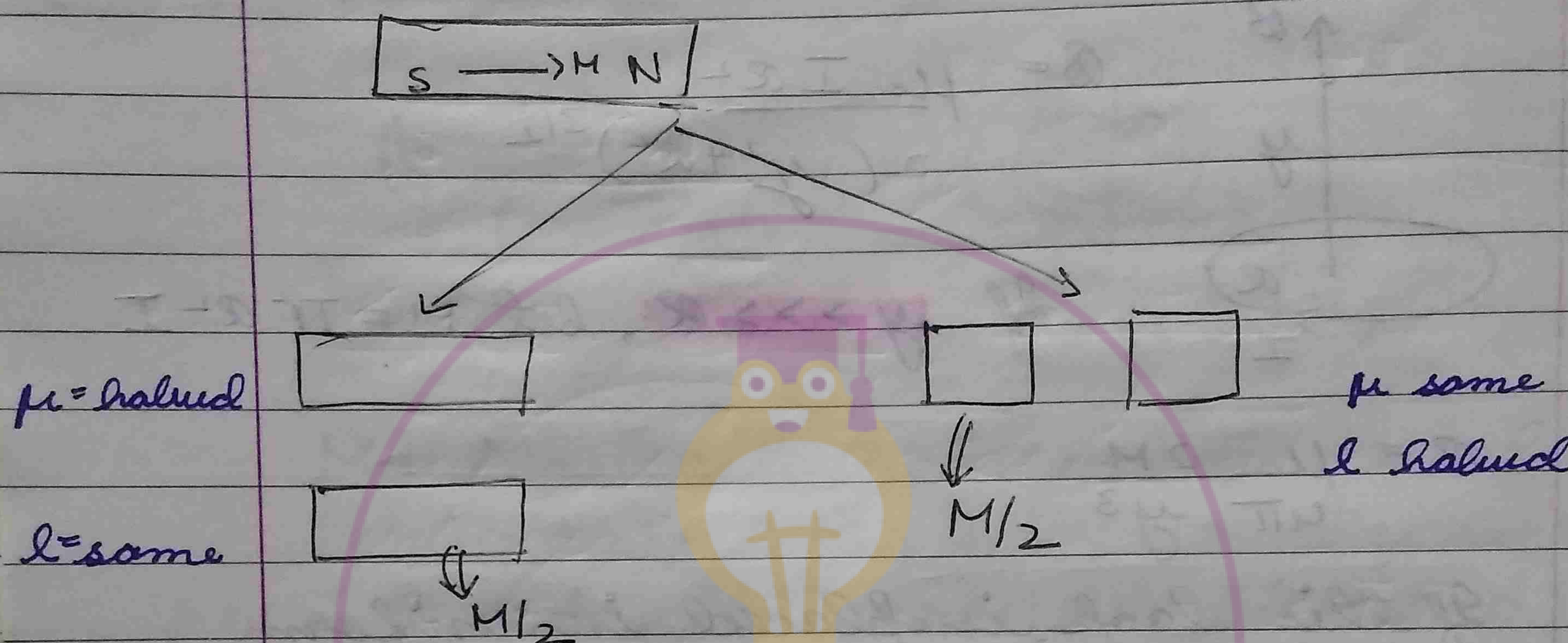
(iii) μ $\bar{F} = \mu \bar{B}$ (similar to $\bar{F} = q\bar{E}$)

(iv) $B = \frac{k\mu}{r^2}$ where $k = \frac{\mu_0}{4\pi}$

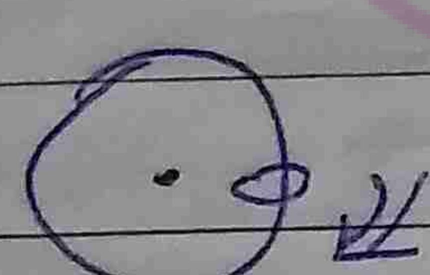
#

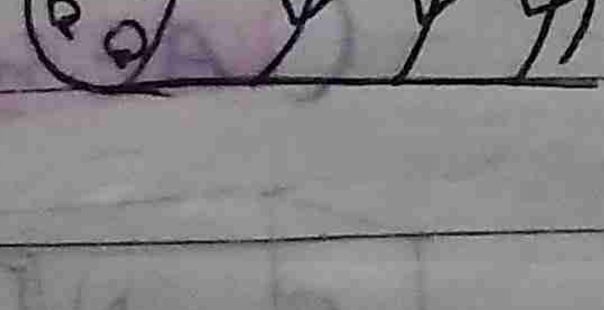


; $M = \mu . d$ ---- Am^2



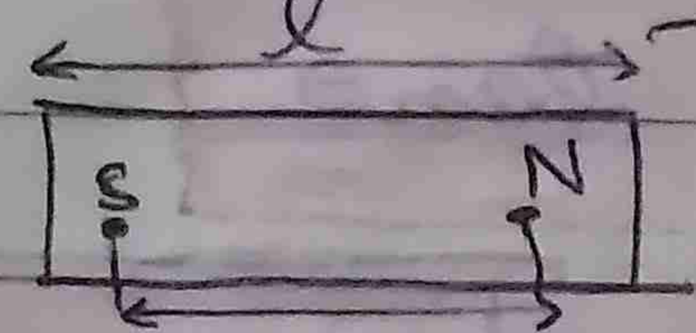
- <u>Cause of magnetism in permanent magnet</u>
- Every atom has revolving electron which constitutes current

 || permanent magnetic moment.

 (i) If revolving electrons are in same plane.

(ii) sense of revolution is identical.

- **Field due to permanent magnet**



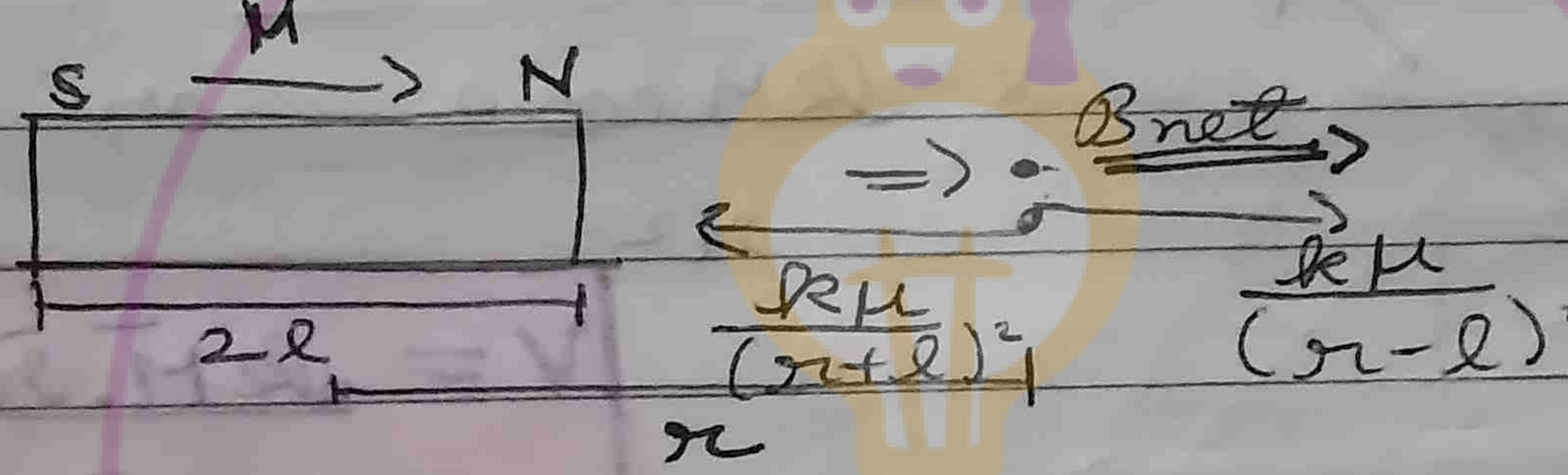
l → geometrical length

l' → effective length

$$\frac{l'}{l} \approx 0.85$$

$M = \mu l'$

- Magnetic field at axial position (End-on)



$$\frac{k\mu}{(r+l)^2} \qquad \frac{k\mu}{(r-l)^2}$$

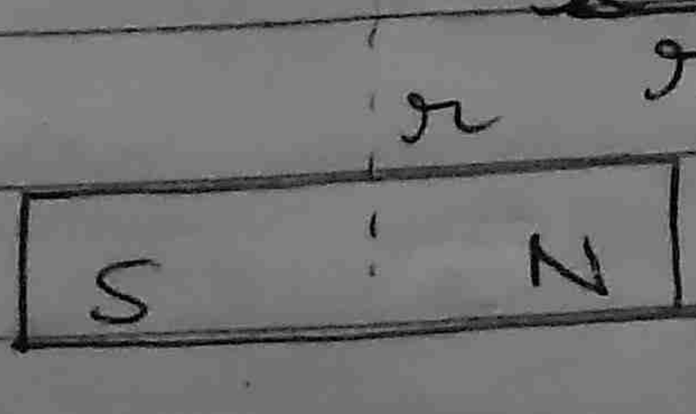
$$B_{net} = \frac{k\mu}{(r-l)^2} - \frac{k\mu}{(r+l)^2} = \boxed{\frac{k\mu\, 4rl}{(r^2-l^2)^2}}$$

- **Ideal dipole** : $r >>> l$.

$$\Rightarrow B_{net} = \frac{k\mu\, 4l}{r^3} = \frac{2k\mu\, 2l}{r^3} = \boxed{\frac{2kM}{r^3}}$$

$$\Rightarrow \vec{B}_{net} = \frac{2k\vec{M}}{r^3} \text{ ; where } k = \frac{\mu_0}{4\pi}$$

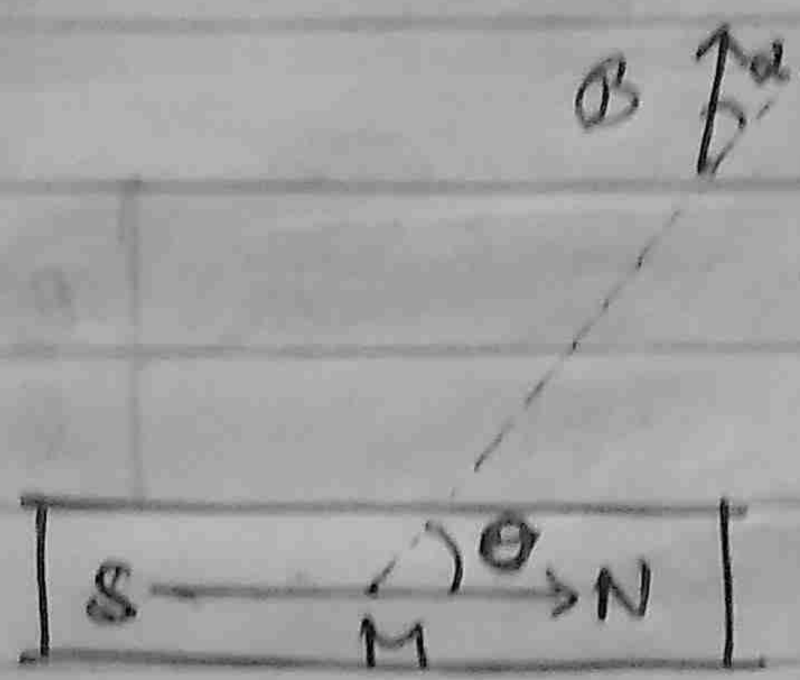
- Equatorial position (Broad side on)



$$B = \frac{kM}{r^3}$$

$$\boxed{B_{net} = \frac{-k\vec{M}}{r^3}}$$

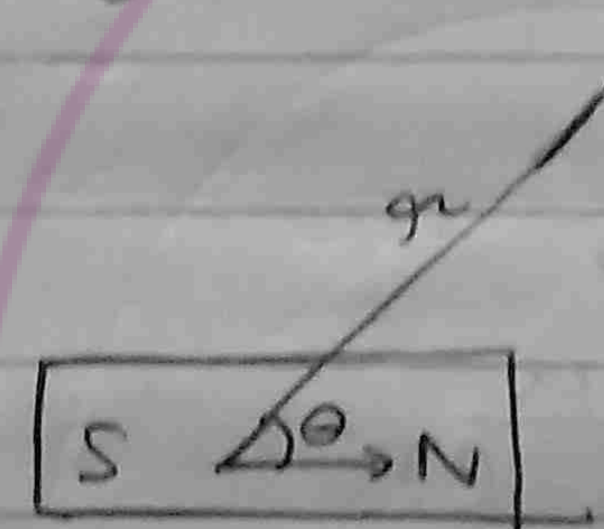
- **Magnetic field at generalised position**



$$\tan\alpha = \frac{1}{2}\tan\theta$$

$$B = \frac{km}{r^3}\sqrt{1+3\cos^2\theta}$$

- **Magnetic Scalar Potential**

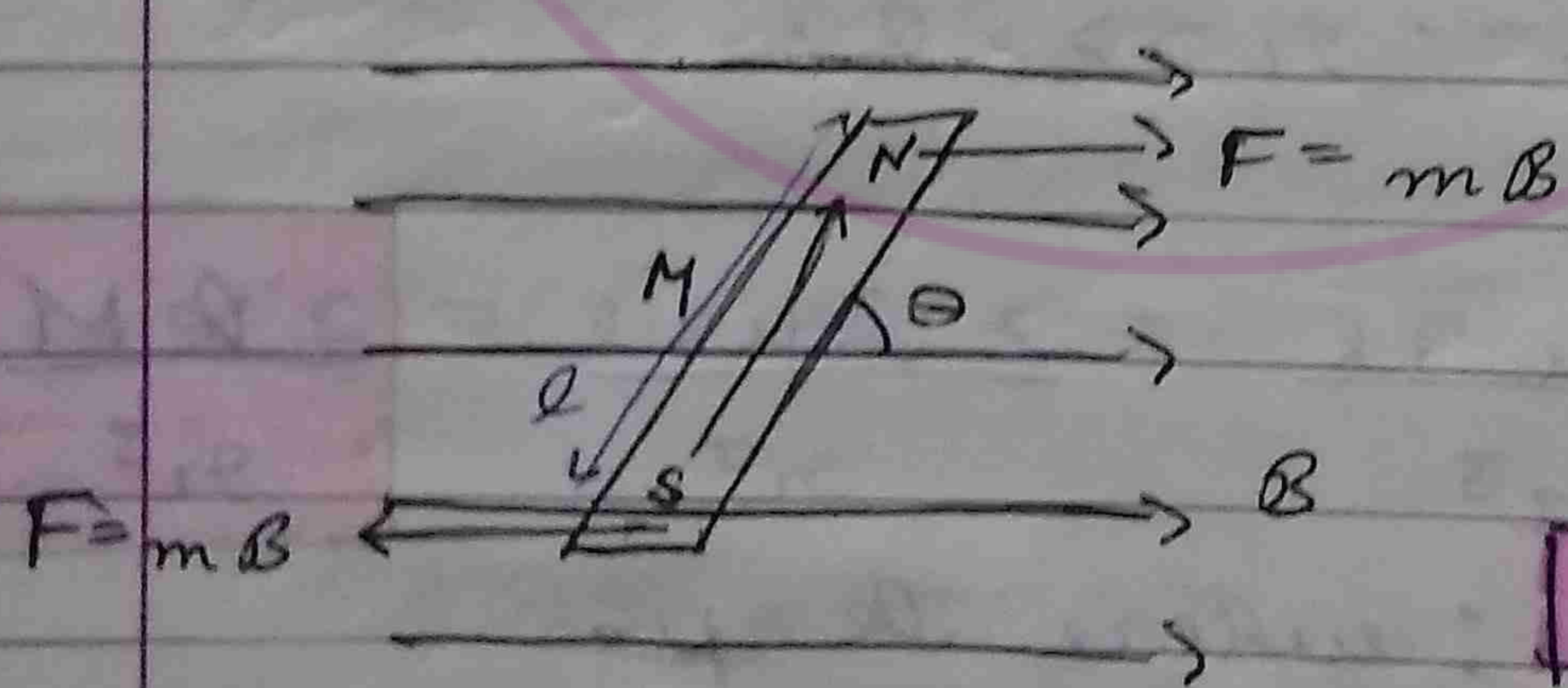


$$V = \frac{kM\cos\theta}{r^2} \quad \text{or}$$

$$V = \frac{k\bar{M}\cdot\bar{r}}{r^3}$$

- **Magnetic in external field**

$\mu \equiv m$



$F_{net} = 0$

$\tau = mB.l\sin\theta$

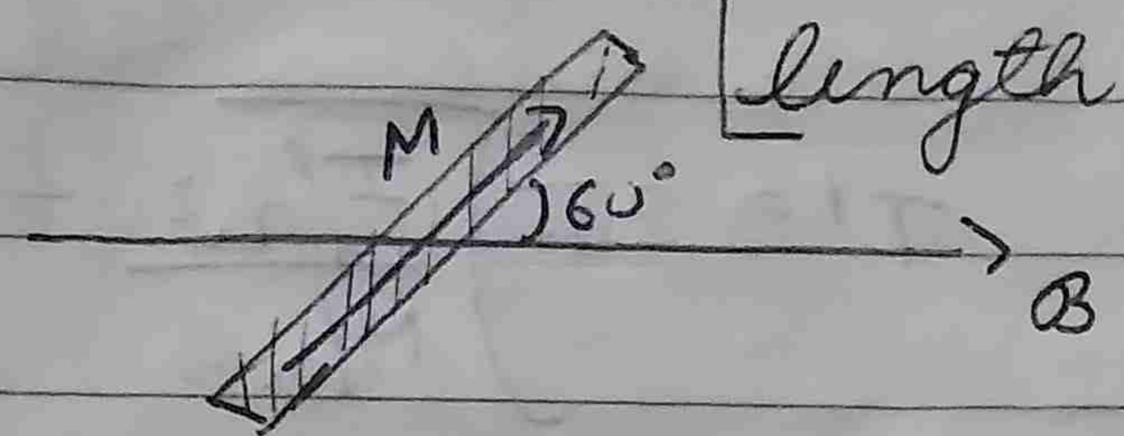
$= MB\sin\theta$

$$\bar{\tau} = \bar{M}\times\bar{B}$$

$$PE = -\bar{M}\cdot\bar{B}$$

Numericals

| Mass : m | length : l |

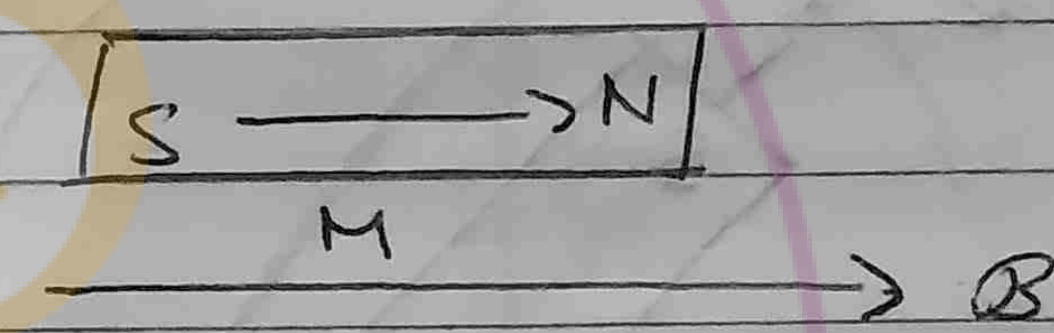


• Find 'ω', when the magnet becomes parallel to magnetic field.

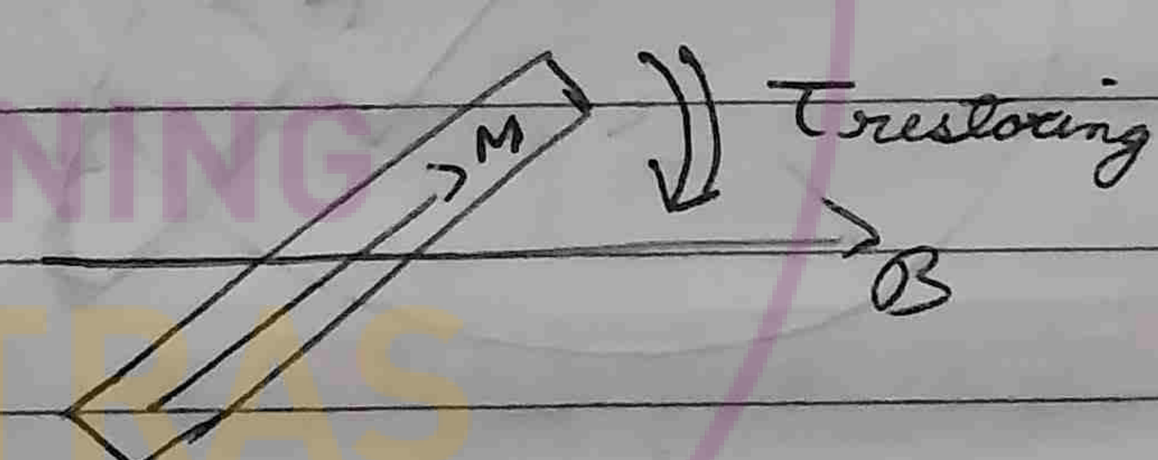
E const ⟹ $K_I + U_I = K_f + U_f$

$$0 - \frac{MB}{2} = \frac{1}{2}\left(\frac{ml^2}{12}\right)\omega^2 - MB\cos 0°$$

• Initially



⟹ If disturbed



$\tau = MB\sin\theta$

∴ only oscillatory not SHM.

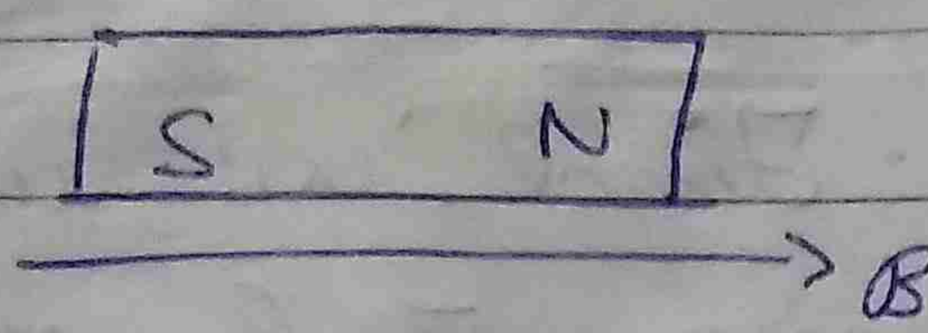
• If θ is small $\tau = MB\theta$ ⟹ $T = 2\pi\sqrt{\frac{I}{MB}}$

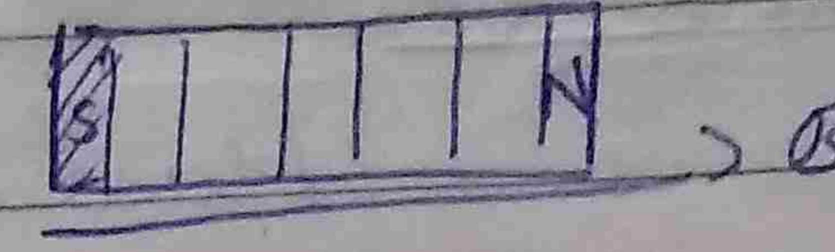
$(\because MB = I\omega^2)$

A magnet has time period T in ext. field. Find the new period if magnet is cut into n equal parts ⊥ to axis.

★ If bar magnet is broken into n equal parts perpendicular to its length then time period of each part becomes $T' = \frac{T}{n}$

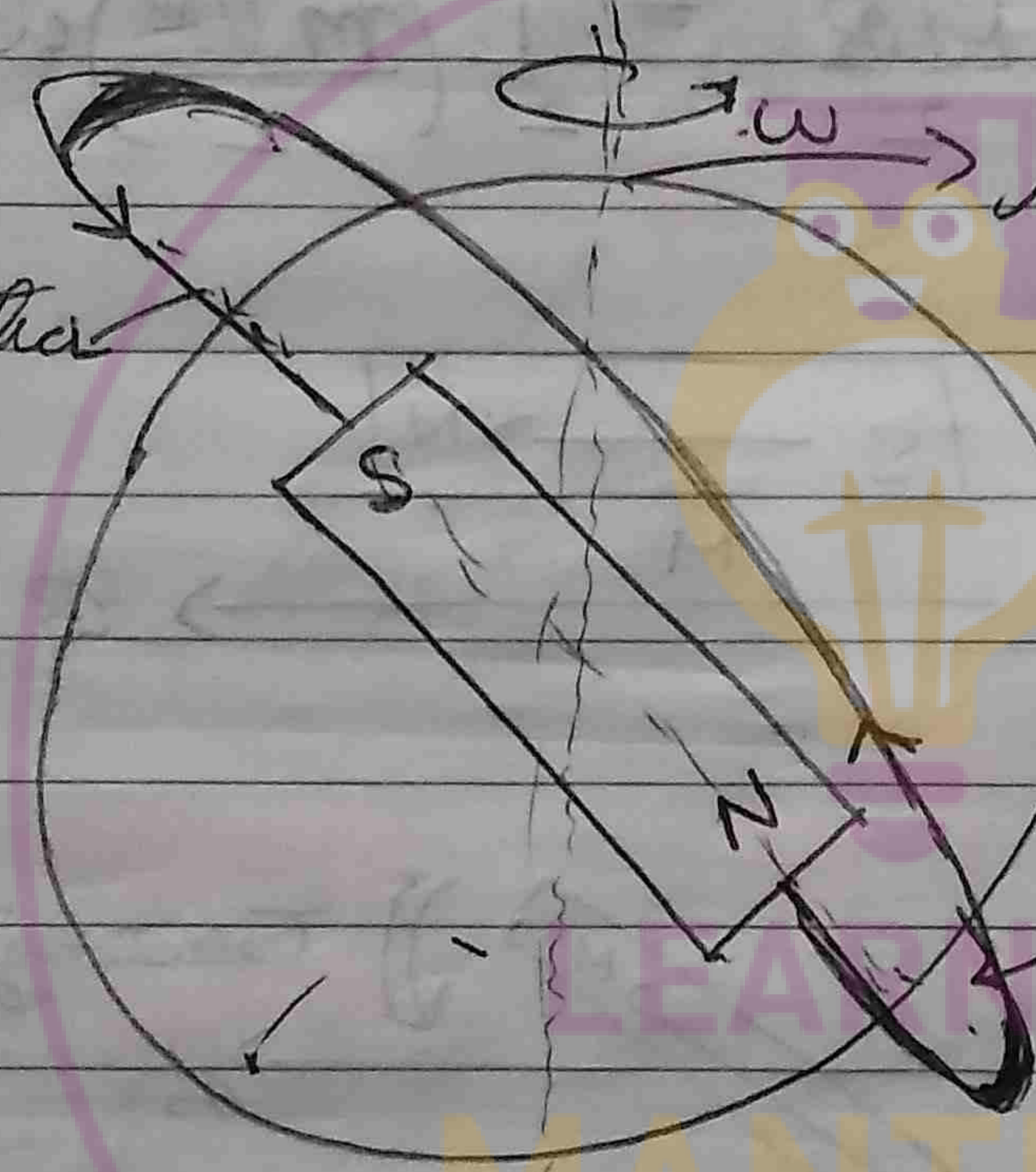
$\because I = ml^2 = \frac{m}{n} \times \left(\frac{l}{n}\right)^2$





$$T = 2\pi\sqrt{\frac{I}{MB}} \qquad T' = 2\pi\sqrt{\frac{I/n^3}{M/n\,B}} = \frac{T}{n}$$

- ## Terrestrial Magnetism



- Geographical meridan at 'P'

→ It is the plane passing through P and Axis of rotation.

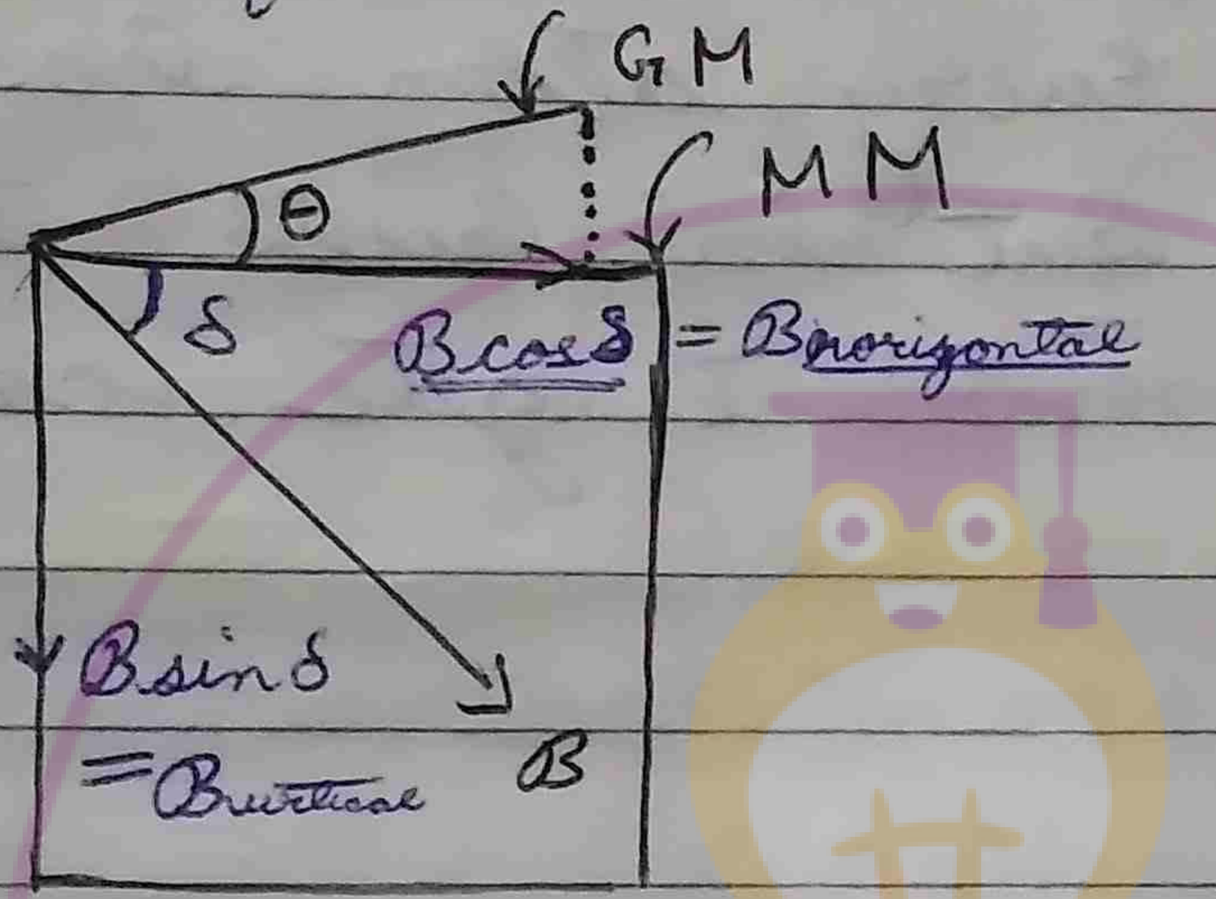
- Magnetic meridan at 'P'

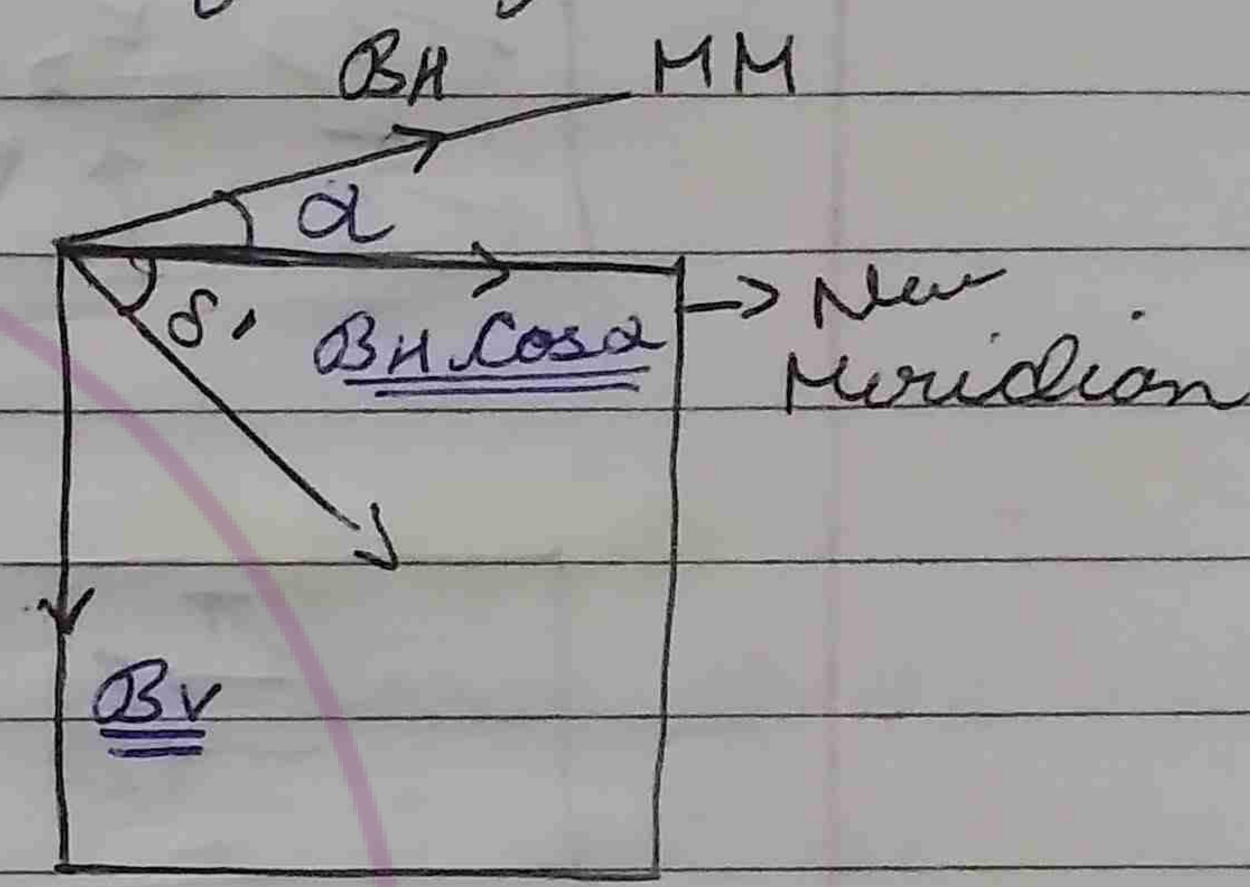
→ It is the plane passing through P and magnetic axis.

- Declination(θ): It is the angle between Geographical meridan (GM) and magnetic meridan (MM) at point P.

- Inclination or dip (δ): It is the angle b/w Earth's field and horizontal.

δ = 0 for equator = π/2 for poles





$B = \sqrt{B_V^2 + B_H^2}$ ← $\tan\delta' = \frac{B_V}{B_H\cos\alpha} = \frac{\tan\delta}{\cos\alpha}$

$\tan\delta = \frac{B_V}{B_H}$

$\delta' > \delta$; If $\alpha = \pi/2$; $\delta' = \pi/2$

↓ because $\cos\theta \le 1$; because $\tan\frac{\pi}{2} = \infty$

- Magnetism in matter

(i) There is magnetic moment due to circulating electron.

(ii) Intrinsic angular momentum of electron (spin).

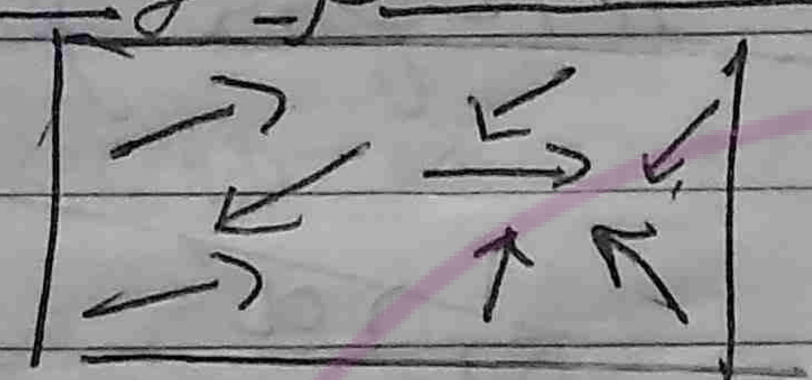
→ there is magnetic moment due to spin as well.

• Favourable condition – low temperature
– high ext. magnetic field

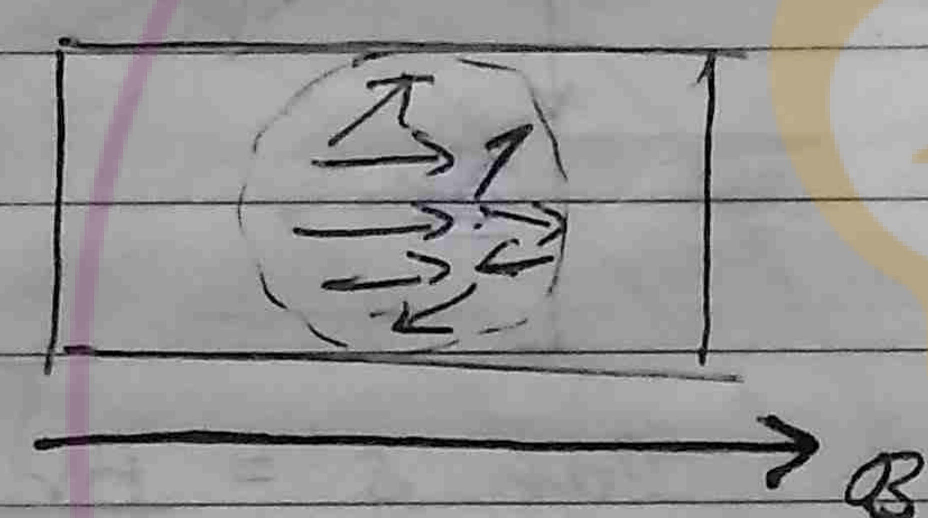
(iii) A nucleus also has spin and hence magnetic moment.

Material
- Having permanent $\bar{M}$
- Not having permanent $\bar{M}$

(i) **Having permanent $\bar{M}$.**



Every atom has moment. But in bulk the net moment gets cancelled.

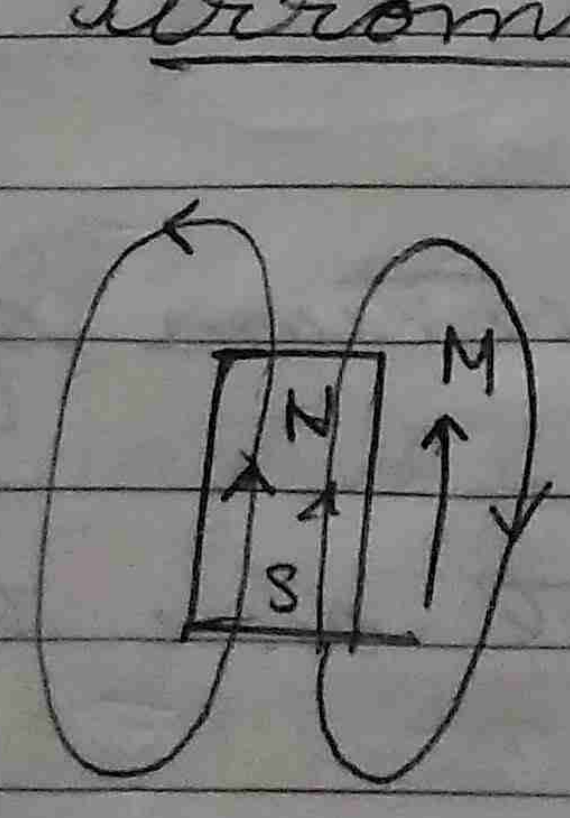


• Intensity of magnetisation depends on:

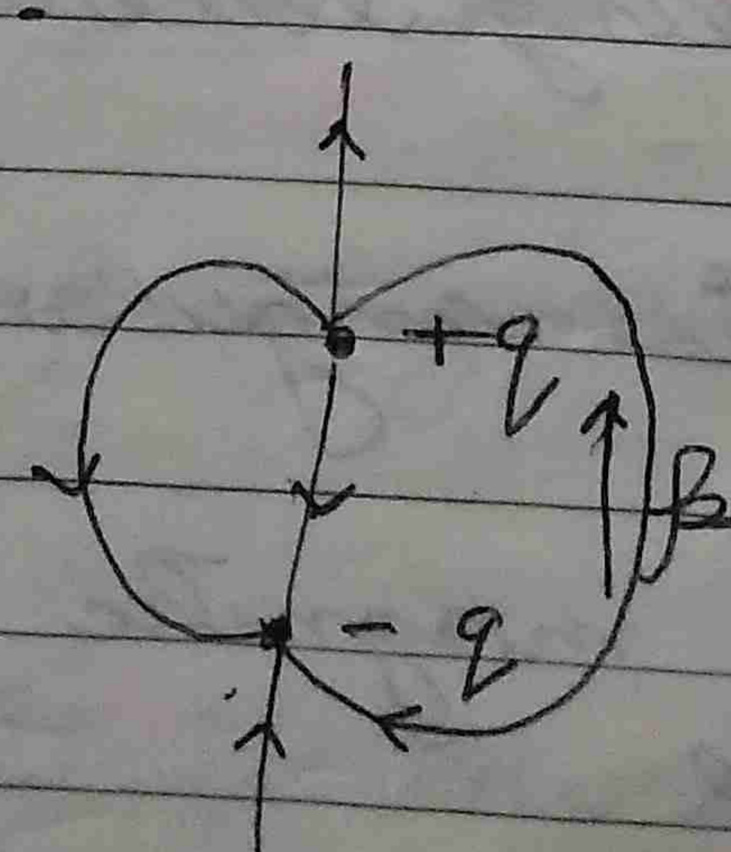
(i) strength of external field

(ii) temperature

• Paramagnetic, Diamagnetic and Ferromagnetic.



Magnetic dipole

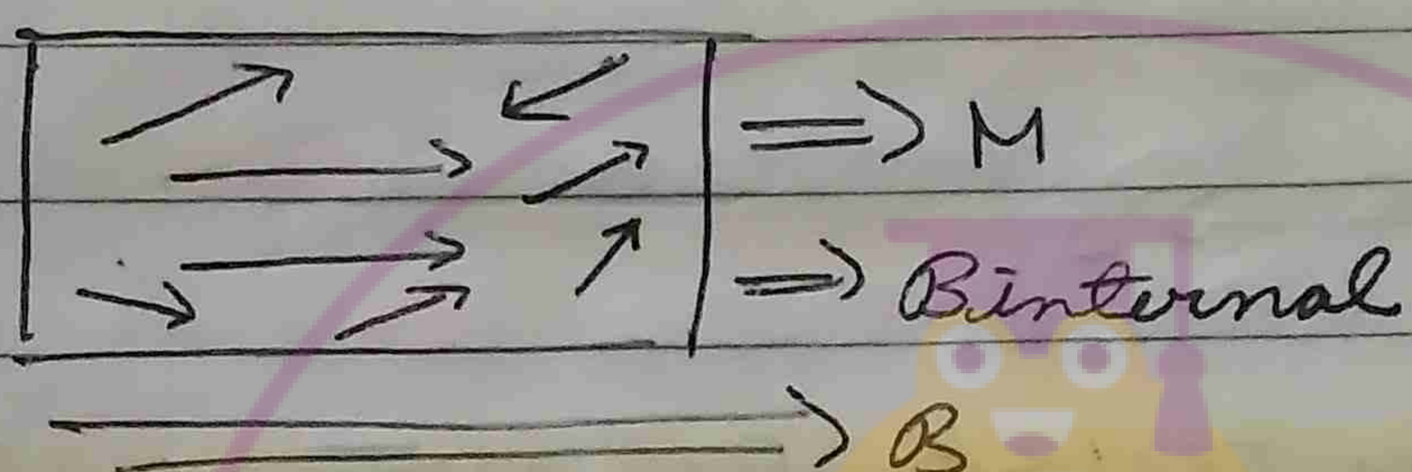


Electric dipole

- **Within the dipole:**

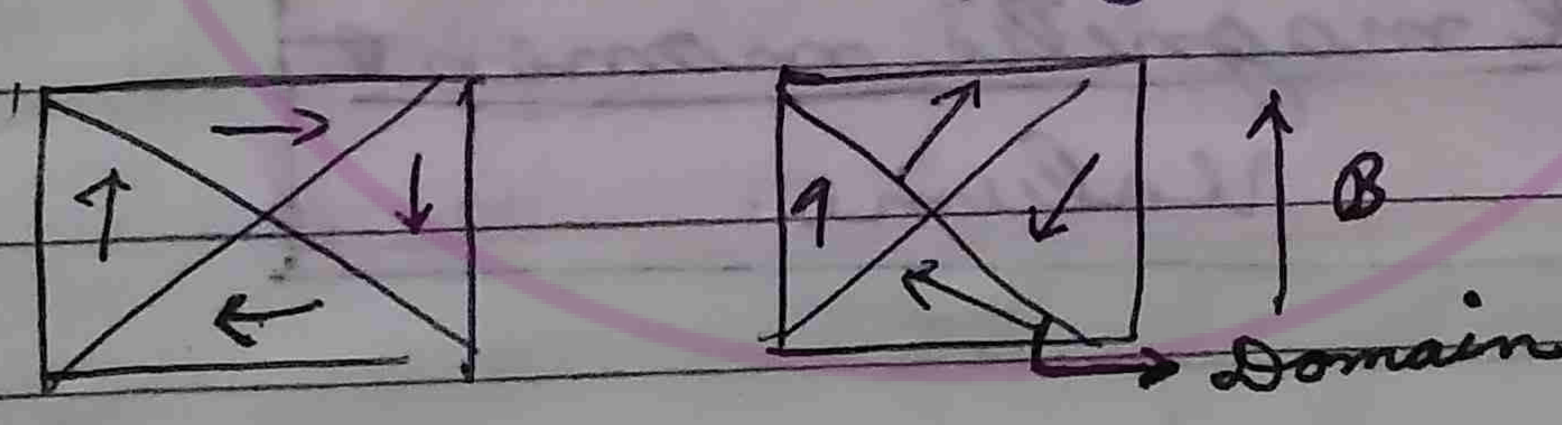
$\bar{M} \uparrow\uparrow \bar{B}$ (parallel) $\bar{p} \uparrow\downarrow \bar{E}$ (antiparallel)

- Atoms having permanent magnetic moment ; paramagnetic



→ If paramagnetic materials are kept in external field, the net field inside the material would increase

- In some materials the net field increases sharply ; ferromagnetic material

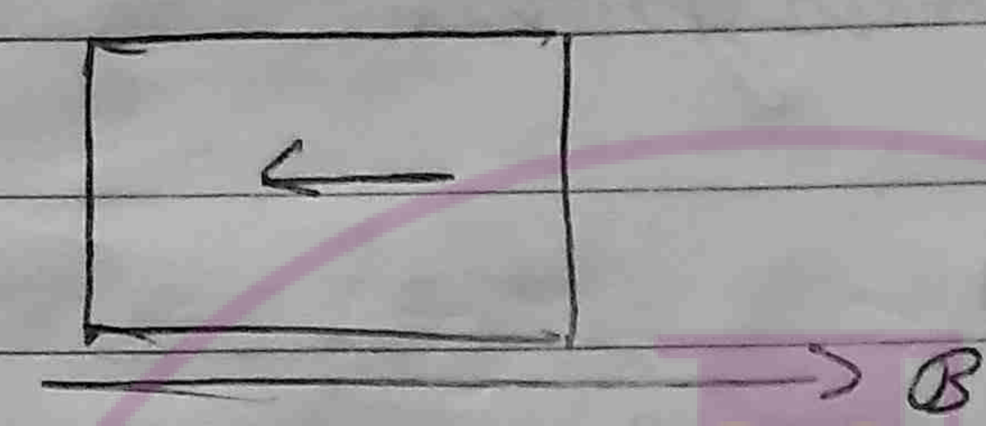


- **Diamagnetism.**

→ These materials do not possess permanent ~~dipole~~ magnetic moment.

- Lenz's law →

→ When external field is applied, due to lenz's law: the net magnetic field is slightly lowered.

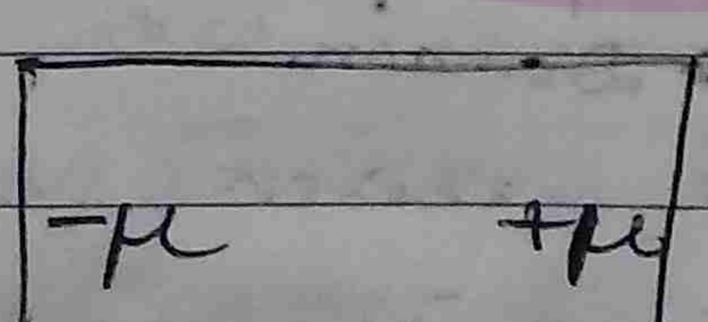


→ Diamagnetism is a property exhibited by all, but this property is suppressed in paramagnetic and ferromagnetic.

- Intensity of magnetisation ($\bar{I}$)

$$\bar{I} = \frac{\text{Net magnetic moment}}{\text{Volume}}$$

e.g



$$I = \frac{\mu l}{A l} = \frac{\mu}{A} \quad \frac{(\text{Amp-metre})}{(\text{metre})^2}$$

- $\bar{H}$ (Magnetic intensity or magnetizing field).

- $\bar{I}$ --- Intensity of magnetisation

- The expression $\bar{I} = \chi \bar{H}$ is not valid for ferromagnetic material.
- χ – depends on nature of material and temperature.

susceptability : ease of being magnetised

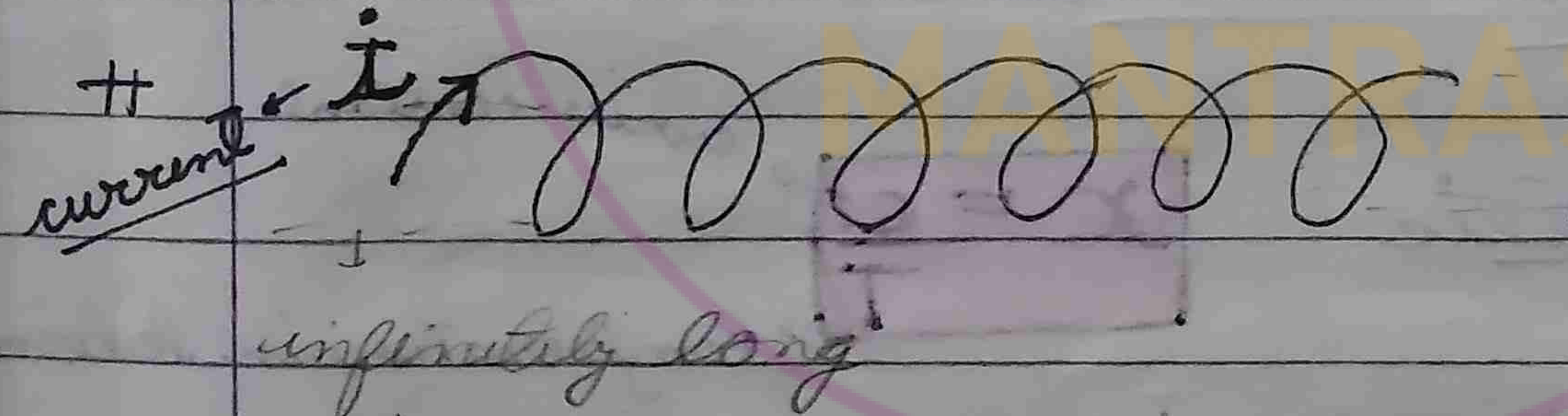
$\bar{B}$ – – net field (magnetic induction vector).

$$\bar{H} = \frac{\bar{B}}{\mu_0} - \bar{I}$$

- If material is very long or toroid then $\bar{H}$ is only due to external agent although the material may be magnetised.

- For vacuum, $\bar{I} = 0 \Rightarrow \boxed{\bar{B} = \mu_0 \bar{H}}$

$$d\bar{H} = \frac{1}{4\pi} \frac{i\, d\bar{l} \times \bar{r}}{r^3} \quad \left(\because \frac{\vec{B}}{\mu_0} = \vec{H} \right)$$



- Find H

$B = \mu_0 n i = \mu_0 H$

$\Rightarrow H = ni$

This value won't change even if we place a long rod inside because the spring and rod are infinite and its ends won't be considered.

- Magnetic Susceptibility (χ) only for pm, dm

$\bar{I} \propto \bar{H}$; $\bar{I} = \chi \bar{H}$

for vacuum ; $\chi = 0$

for diamagnetic $\chi = -ve$ for paramagnetic $\chi = +ve$

μ_R = μ relative
μ_0 = μ of vaccum
μ_m = μ of material

$\Rightarrow \mu_R = \frac{\mu_m}{\mu_0}$

$\chi \propto \frac{1}{T}$

• Magnetic susceptibility decreases with rise in temperature.

$\bar{H} = \frac{\bar{B}}{\mu_0} - \bar{I} \Rightarrow \boxed{\bar{B} = \mu_0(\bar{H} + \bar{I})}$

$= \bar{B} = \mu_0(\bar{H} + \chi\bar{H}) \Rightarrow \bar{B} = \mu_0(1+\chi)\bar{H}$

$= \bar{B} = \mu_0 \mu_R \bar{H} = \boxed{\bar{B} = \mu \bar{H}}$

(μ_R: relative permeability; μ: permeability of material)

$\mu_R = 1 + \chi$ → 1 : vacuum
→ > 1 p.m
→ < 1 d.m

without medium $\mu_0 n i = \bar{B}$
with material $\bar{B} = \mu n i$

• **Curies law**

• paramagnetic : $\boxed{\chi = \frac{C}{T}}$ (C → curies constant; T → temperature in kelvin)

– ferromagnetic : T_c -- curie point
A f.m magnetic substance becomes p.m if heated beyond curie point

$\boxed{\chi = \frac{C'}{T - T_c}}$ (for ferromagnetic substance beyond curie temperature)

χ p.m ∝ 1/T d.m -- no dependence
f.m -- complex behaviour

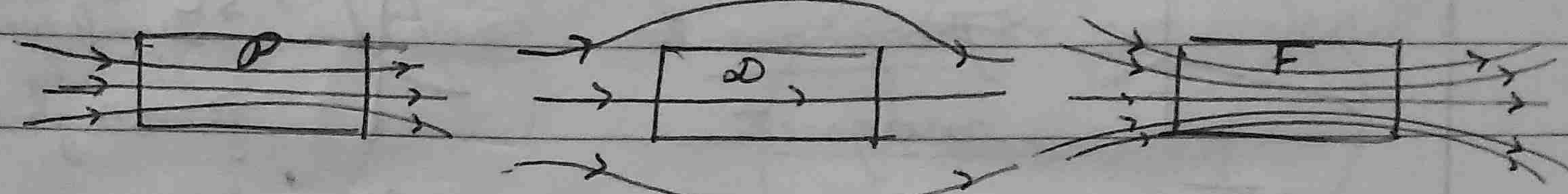
because induction ... always ... whether ...

$\mu = \mu_0(1+\chi_m)$ $\frac{\mu}{\mu_0} = (1+\chi_m)$

$\mu_r = (1+\chi_m)$

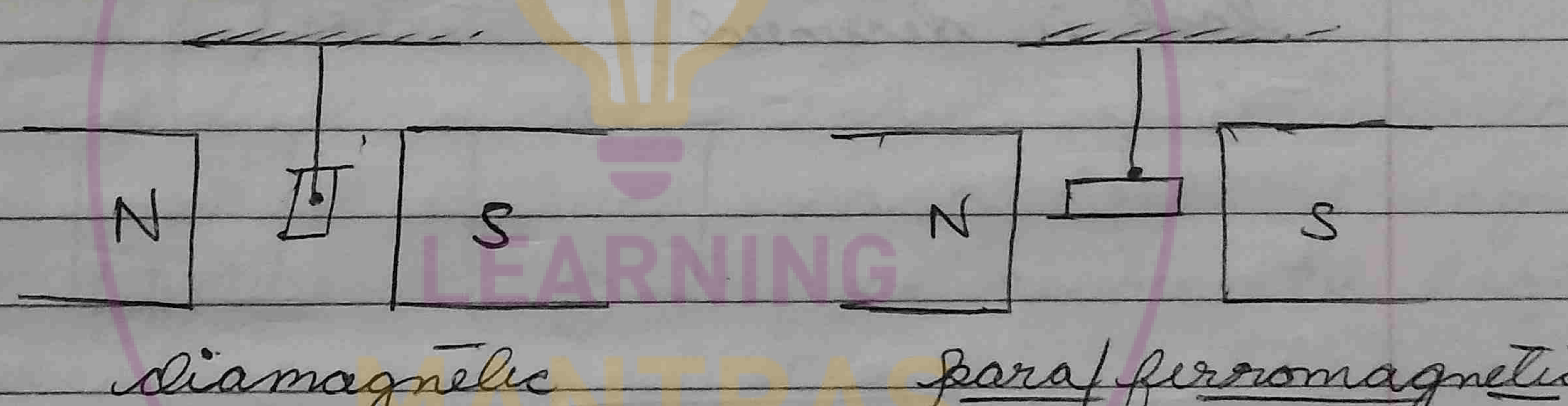
- Properties of para, dia and ferromagnet

1 Bnet

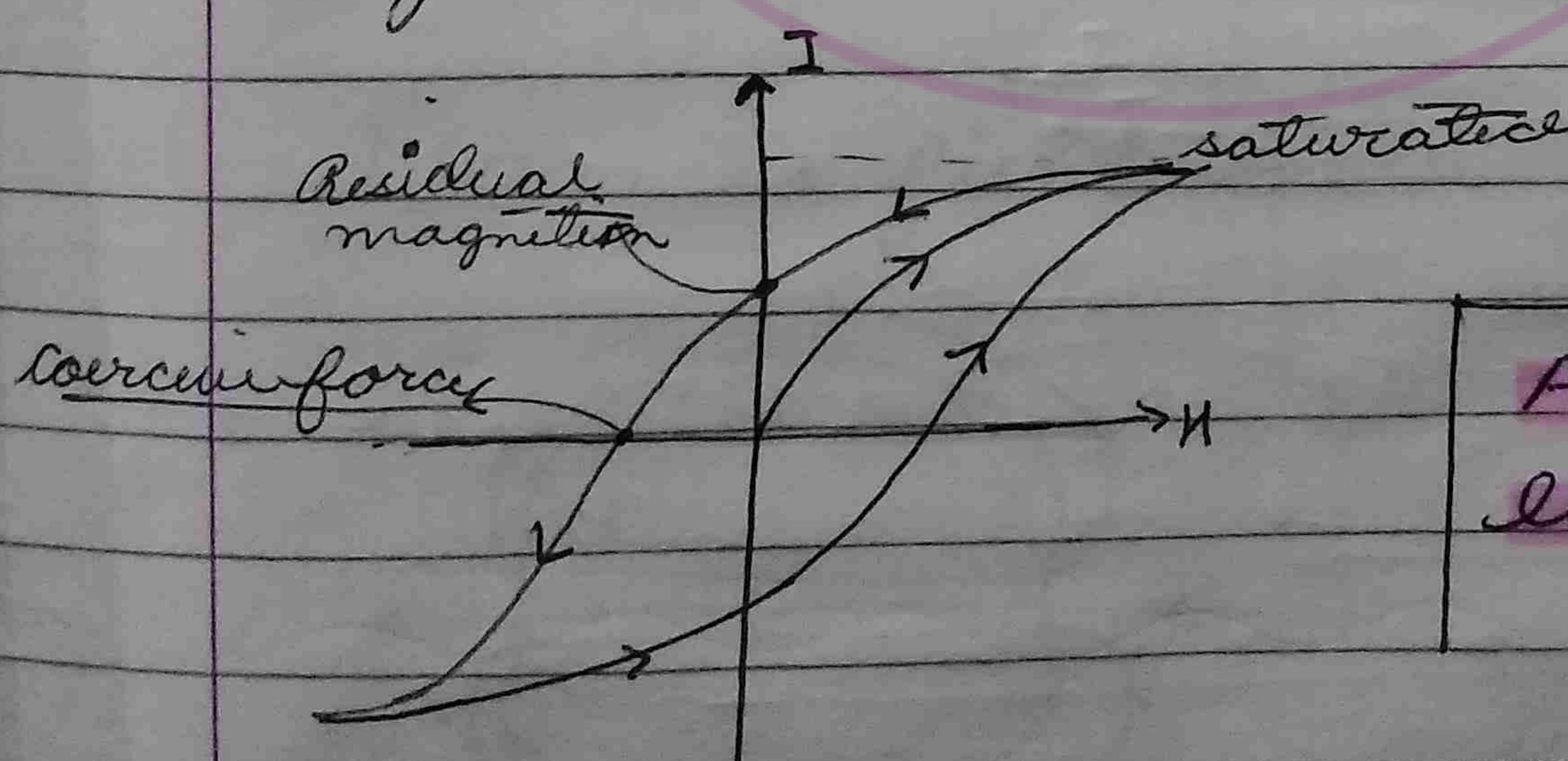


2 Paramagnetism or diamagnetism may be found in solids, liquids or gases. But ferromagnetism is normally found only in solids.

3



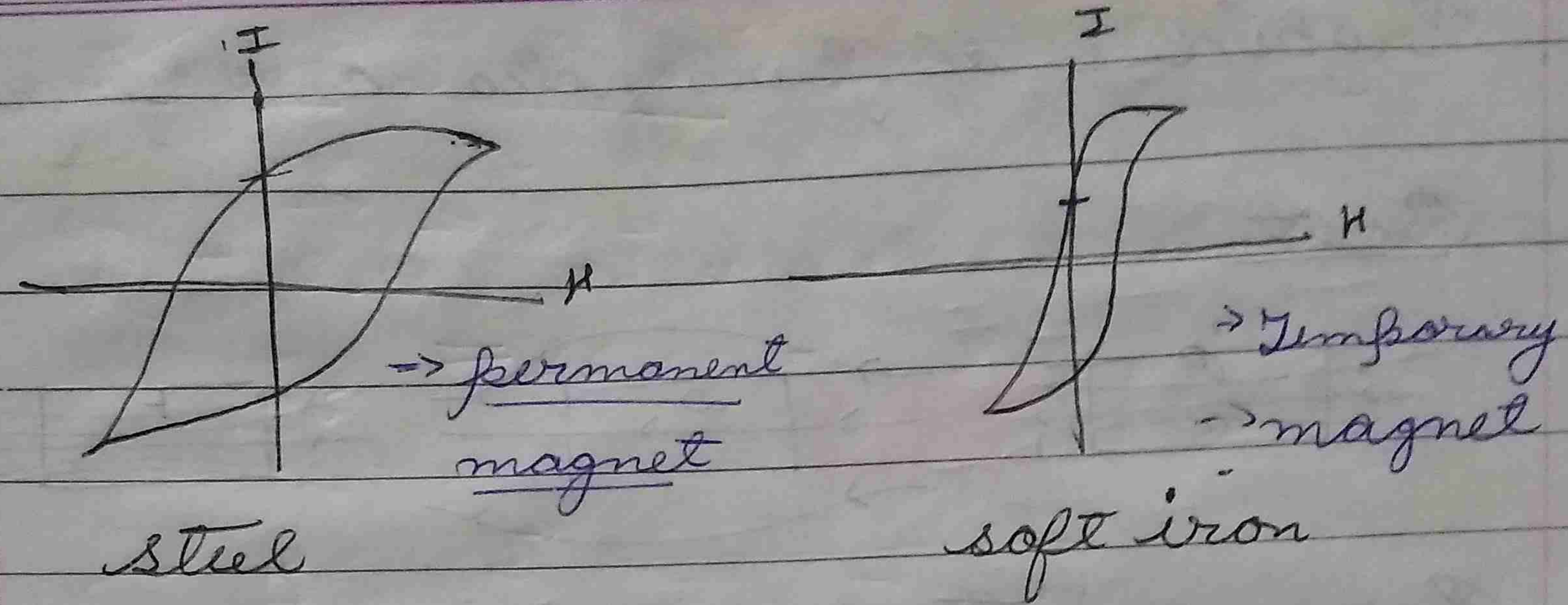
- Hystereses



Area of hystereses loop $\propto$ Thermal Energy / Volume

⇒ Where area of loop is greater, in one cycle energy loss in terms of thermal energy would be greater.

Name of the Chapter ____________ Date ____________ Page No.: ______



- Hysterises loss is reduced by using sof iron as the area of hysterises loop is decreased.