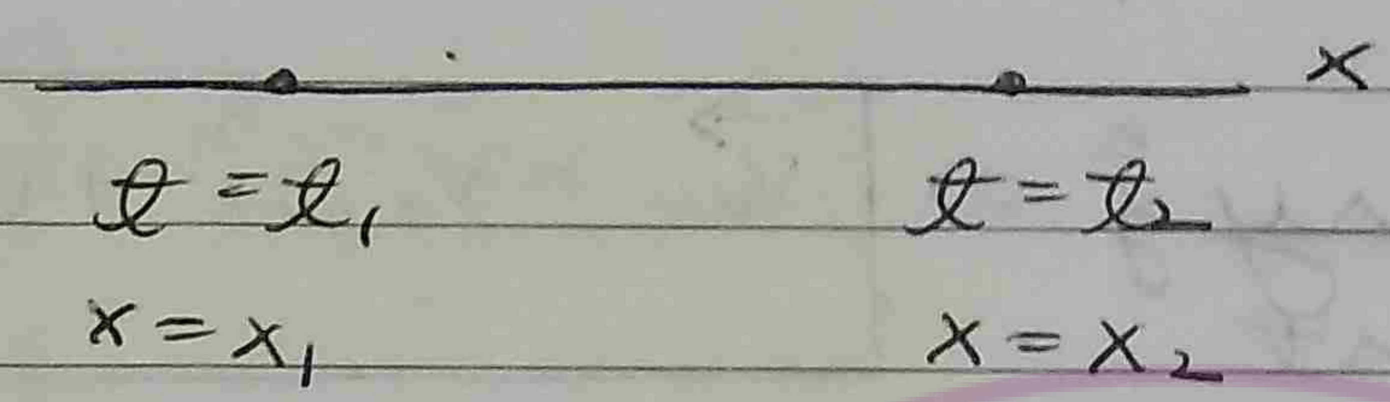
Handwritten Notes

on

Motion in Straight Line

Motion in a straight line

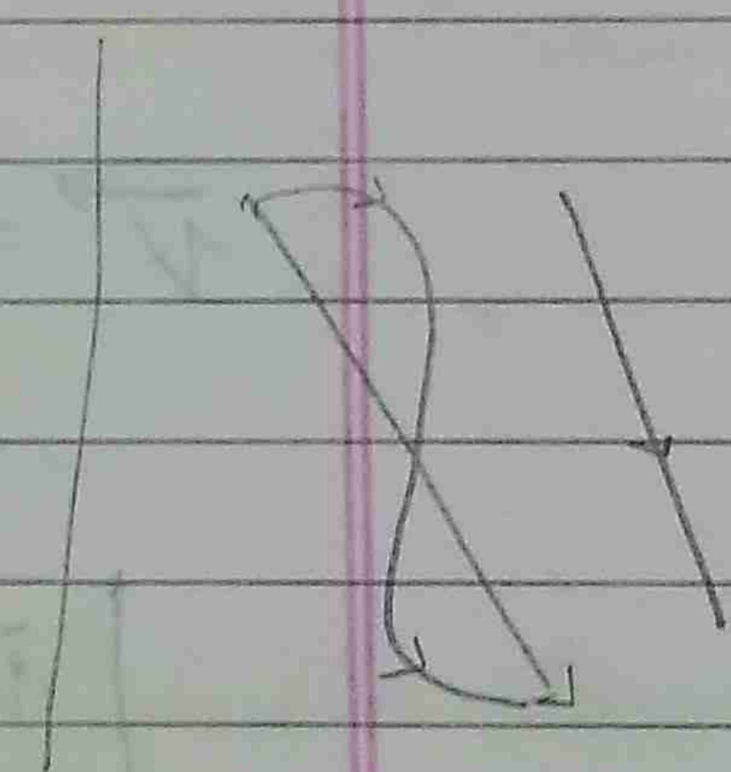
One Dimension



- Average Velocity = $\left| \frac{\text{Displacement}}{\text{time}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \right|$

- Instantaneous velocity = $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

- Average speed = $\frac{\text{distance}}{\text{time}} = \frac{\Delta s}{\Delta t}$

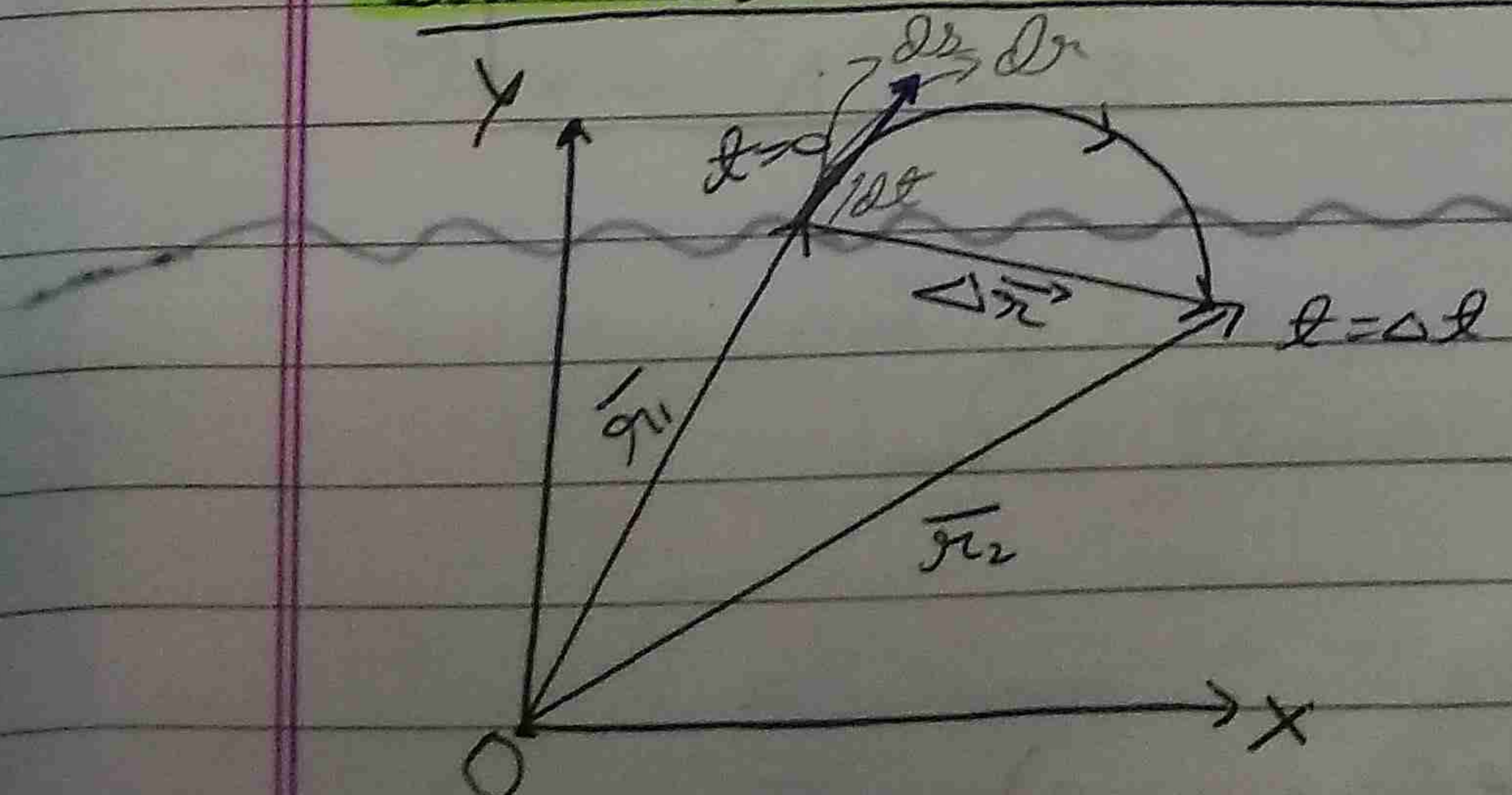


- Instantaneous speed = $u = \frac{ds}{dt}$

Average speed ≥ av. velocity

$\because$ distance ≥ displacement

- Two dimensional motion



$\Delta\vec{r}$ = displacement

$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2$

$\Rightarrow \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$.

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

$$= \Delta x\hat{i} + \Delta y\hat{j}$$

- Average velocity : $\vec{V}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

$$\vec{V}_{av} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} \Rightarrow \boxed{\vec{V}_{av} = V_{x,av}\hat{i} + V_{y,av}\hat{j}}$$

- Instantaneous Velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} \qquad \boxed{v = \frac{d\vec{r}}{dt}}$$

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} \Rightarrow \boxed{\vec{V} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}}$$

$$\boxed{\vec{V} = V_x\hat{i} + V_y\hat{j}} \qquad \boxed{|\vec{V}| = \sqrt{V_x^2 + V_y^2}}$$

|instantaneous velocity| = inst. speed

Ex

$x = 0$ $\quad$ $t = 3\,s$

$t = 0$ $\quad$ $x = 3\,m$.

$x = 4t - t^2$ (m) (t in seconds)

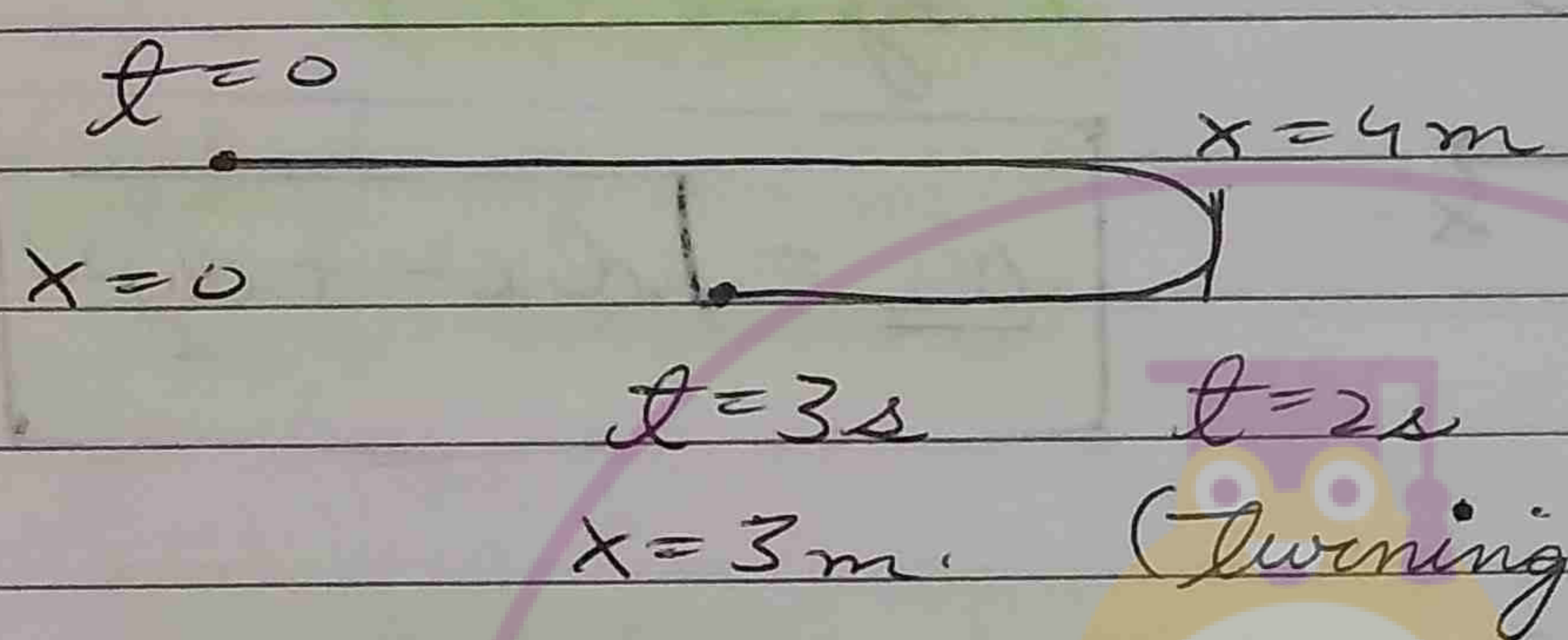
Displacement / Distance in $t = 0$ $t = 3$ seconds.

At $t=0$ $x_1=0$ At $t=3s$ $x_2 = 4\times3-3^2 = 3m$.

$\Rightarrow$ displacement $= x_2 - x_1 = 3m$

$v = \frac{dx}{dt} \Rightarrow \underline{v = 4-2t}$

$v=0$ at $4-2t=0 \Rightarrow t = 2$ seconds.

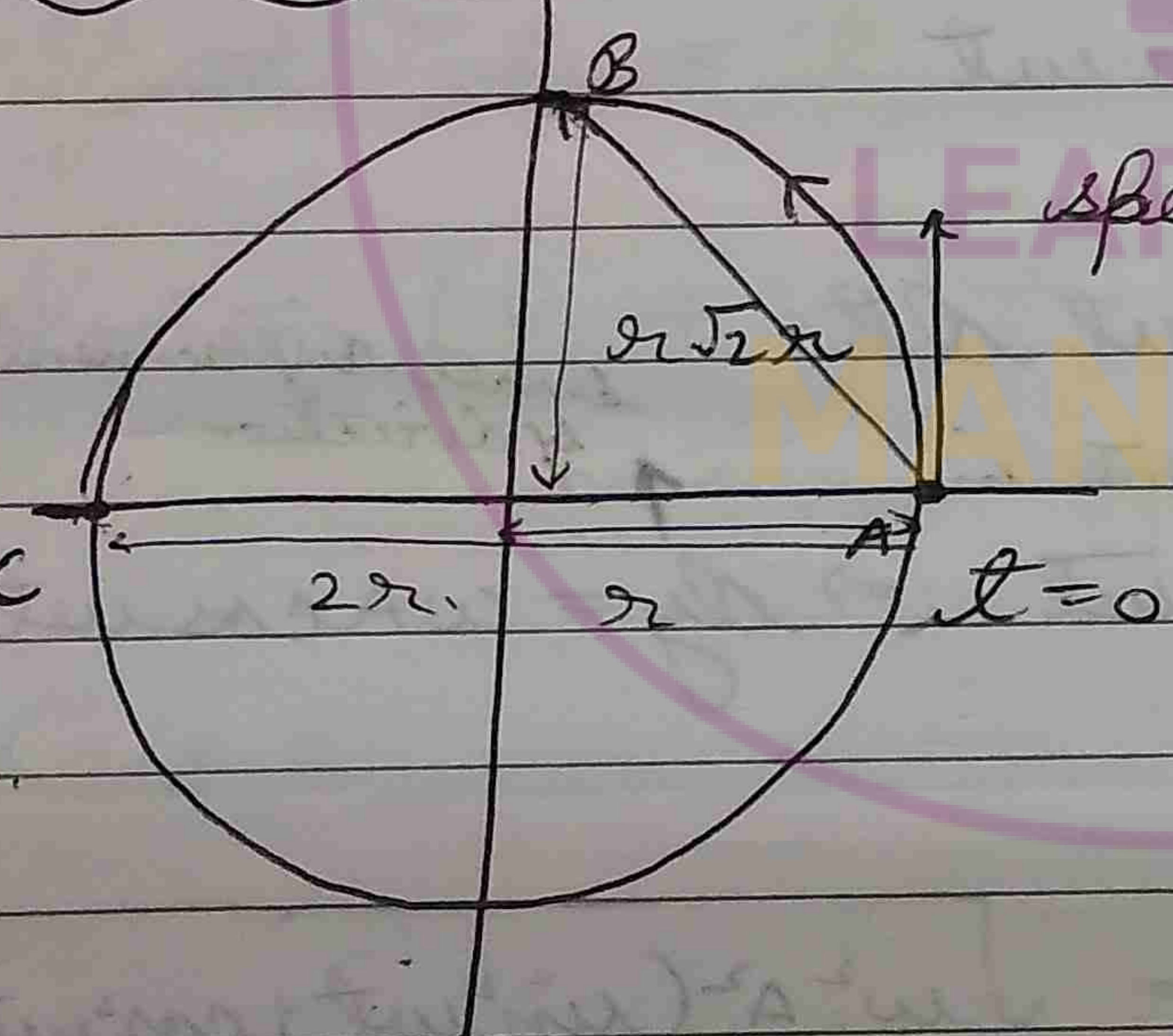


At $t=2s$.
$x = 4\times2 - 2^2 = 4m$

distance $= 4m + 1m$
$= 5m$

- Between $t=0$ and $t=2s$ average velocity and speed are equal. ☆

EX



speed $= v =$ constant.

☆ In this question instantaneous speed and velocity are equal i.e $= v$ but avg speed and velocity will be different.

- From A to B

Average velocity $= \frac{\text{disp.}}{\text{time}} = \frac{\sqrt{2}r}{\pi r/2v} \Rightarrow \boxed{V_{av} = \frac{2\sqrt{2}v}{\pi}}$

time $= \frac{\text{distance}}{\text{speed}} = \frac{\pi r/2}{v} \Rightarrow \boxed{\Delta t = \frac{\pi r}{2v}}$

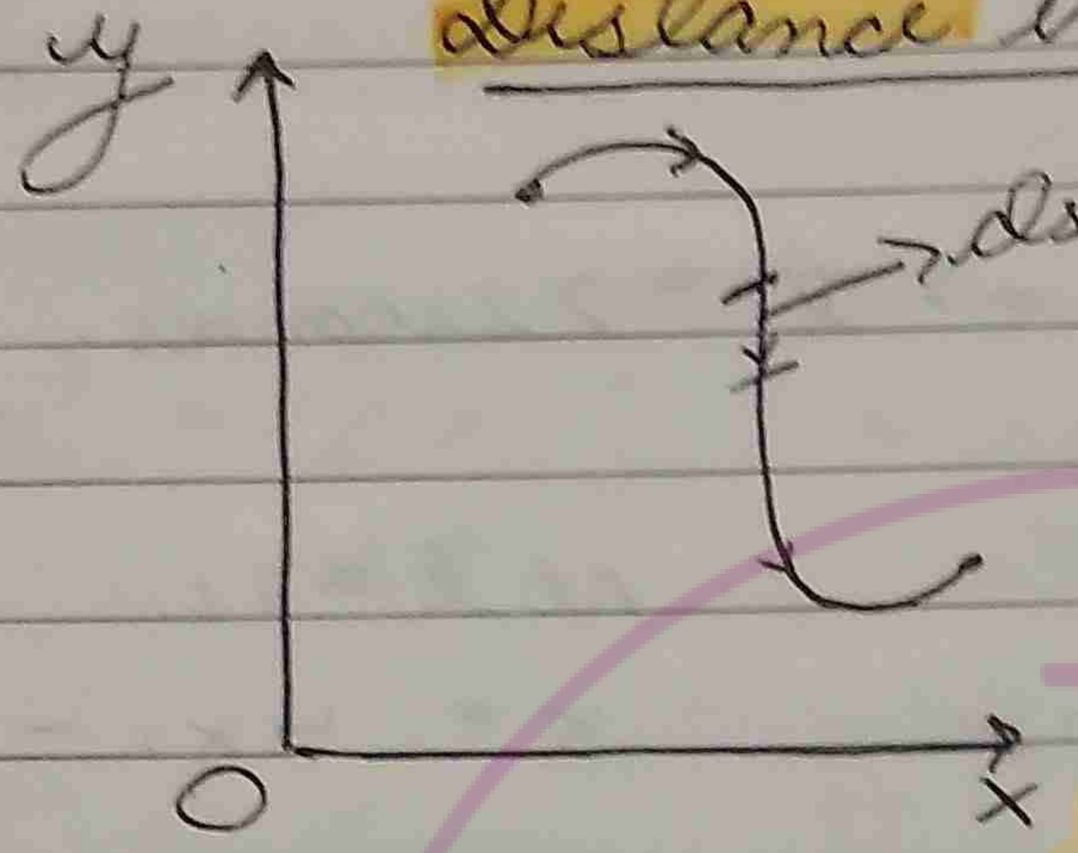
In 2-D cases first change distance in terms of velocity then integrate it.

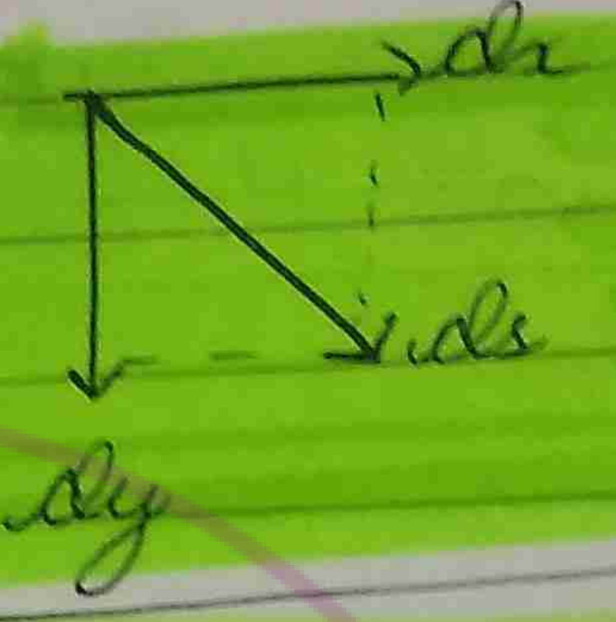
From A to C

$$V_{av} = \frac{2r}{(\pi r/v)} \Rightarrow V_{av} = \frac{2v}{\pi}$$

☆ **Distance in 2-D cases**

Ex





$$ds = \sqrt{dx^2 + dy^2}$$

$x = A\cos\omega t$

$y = A\sin\omega t$

Find distance in interval 0 to t.

$$V_x = \frac{dx}{dt} = -\omega A \sin\omega t$$

$\Rightarrow dx = -\omega A \sin\omega t \, dt$

(dx → small displacement in x direction.)

$$V_y = \frac{dy}{dt} = \omega A \cos\omega t \Rightarrow dy = \omega A \cos\omega t \, dt$$

(dy → small displacement in y direction)

$$\Rightarrow ds = \sqrt{dx^2 + dy^2} = \sqrt{\omega^2 A^2(\sin^2\omega t + \cos^2\omega t)} \cdot dt$$

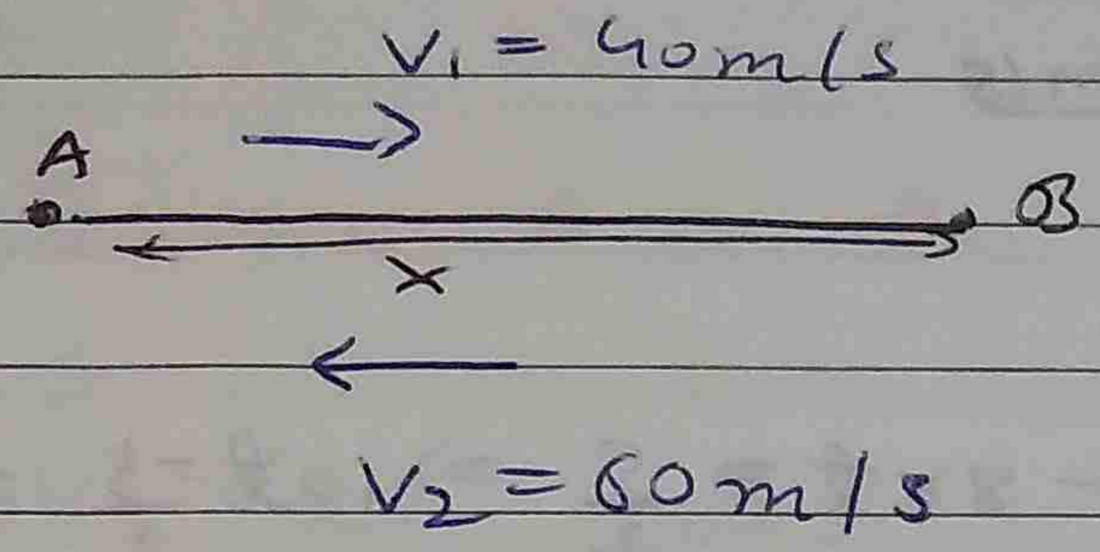
$$ds = \omega A \, dt$$

$$\Rightarrow s = \int ds = \int_0^t \omega A \, dt = \underline{\omega A t}$$

$$ds = \sqrt{(V_x^2 + V_y^2)} \cdot dt$$

- **Average speed** = $\dfrac{\text{total distance}}{\text{total time}}$

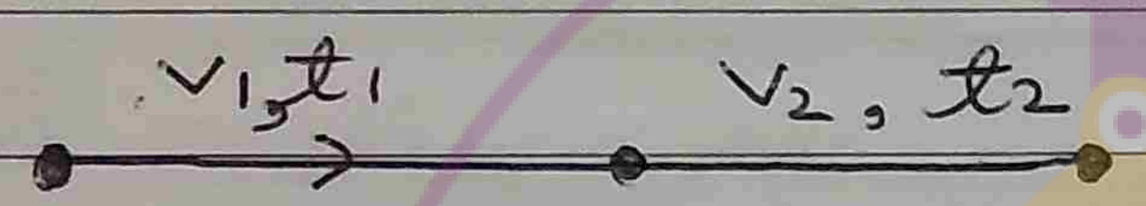
Ex



~~(50 m/s)~~

$$\text{avg. speed} = \frac{2x}{\frac{x}{40} + \frac{x}{60}} = \frac{2 \times 40 \times 60}{40 + 60} = 48 \text{ m/s}$$

- **Case 1**

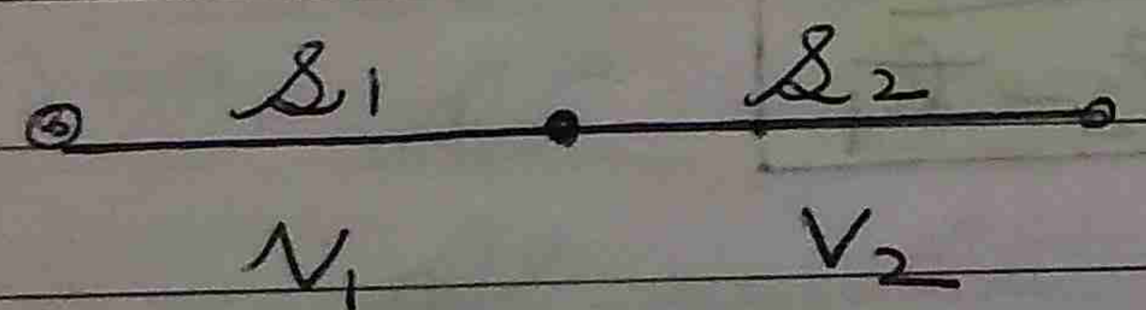


$$V_{av} = \frac{V_1 t_1 + V_2 t_2}{t_1 + t_2}$$

If $t_1 = t_2 = t$

- $V_{av} = \dfrac{(V_1 + V_2)t}{2t} \Rightarrow V_{av} = \dfrac{V_1 + V_2}{2}$

- **Case 2**

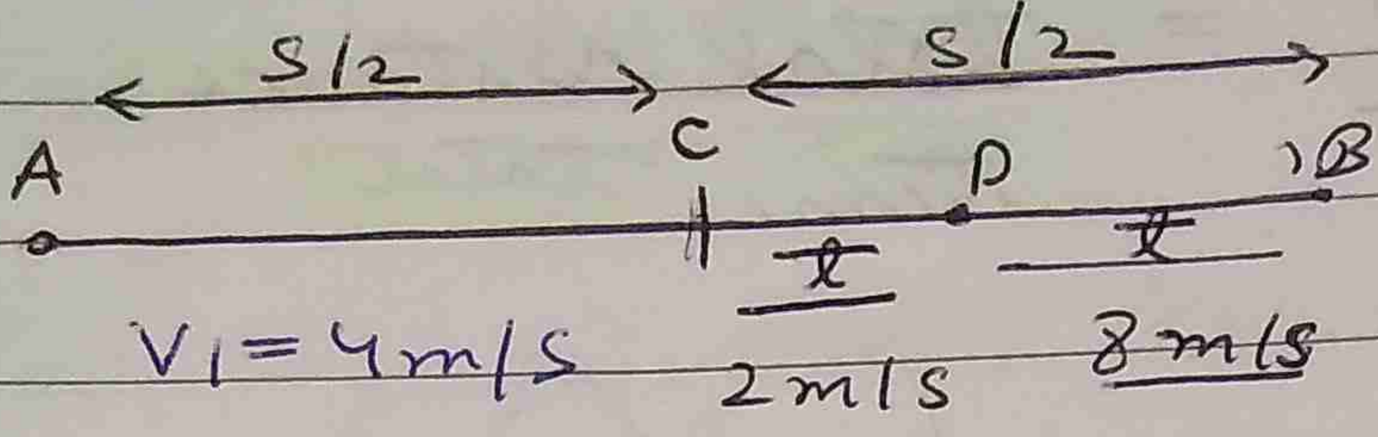


$$V_{av} = \frac{S_1 + S_2}{\frac{S_1}{V_1} + \frac{S_2}{V_2}}$$

- If $S_1 = S_2 = S$

$$V_{av} = \frac{2S}{\frac{S}{V_1} + \frac{S}{V_2}} = \frac{2 V_1 V_2}{V_1 + V_2}$$

Ex



$$t = \frac{s/2}{4} = \frac{s}{8}$$

$$2 \times t + 8 \times t = \frac{s}{2} \Rightarrow 10t = \frac{s}{2} \Rightarrow t = \frac{s}{20}$$

$$V_{av} = \frac{s}{\frac{s}{8} + \frac{s}{20} + \frac{s}{20}} = \frac{1}{\frac{5+2+2}{40}} = \frac{40}{9} \text{ m/s}$$

Method 2

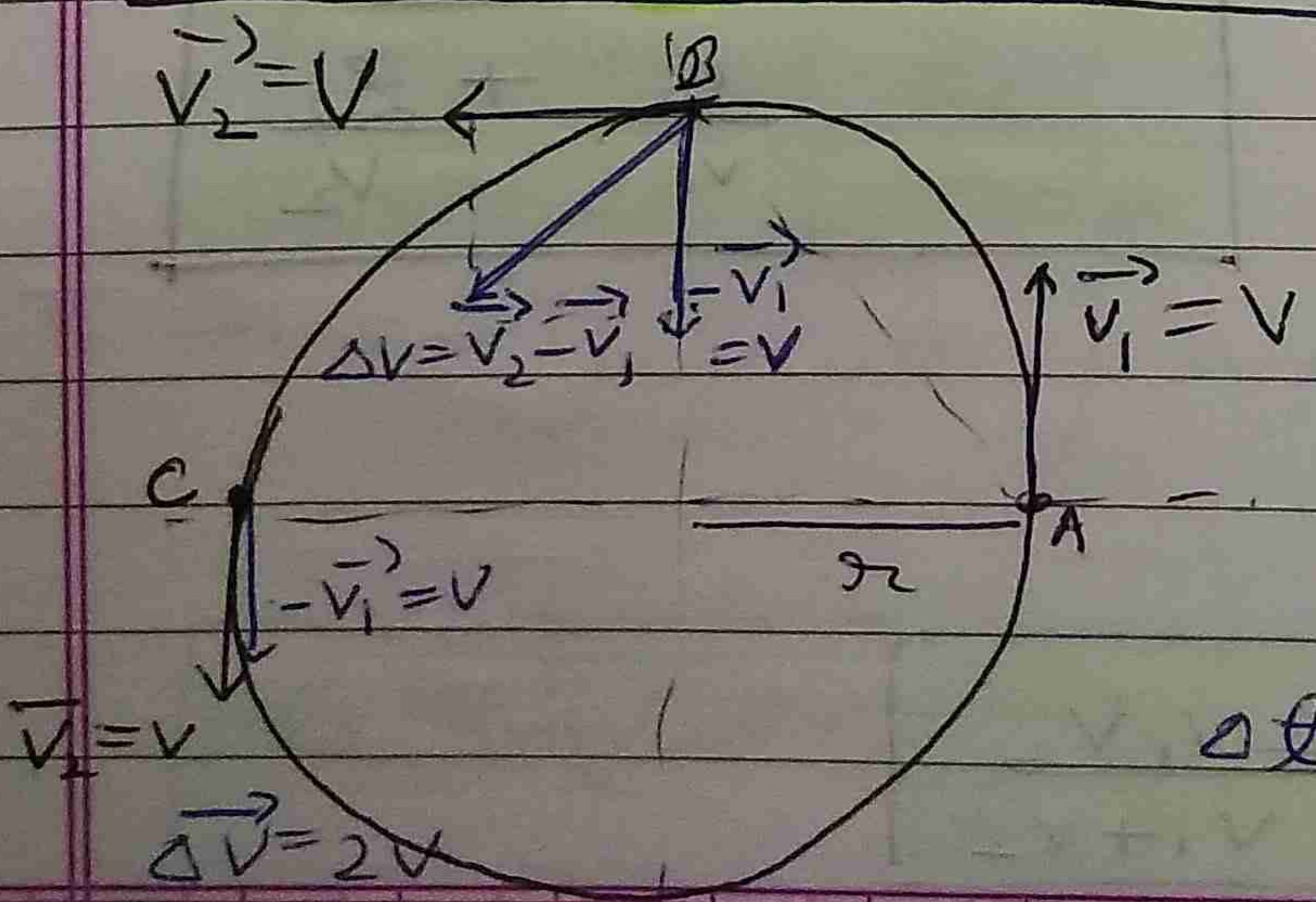
$$\text{av speed } CD = \frac{2+8}{2} = 5$$

$$\text{av speed } AB = \frac{2 \times 5 \times 4}{5+4} = \frac{40}{9} \text{ m/s}$$

- **Acceleration** → rate of change of velocity (vector) w.r.t time.

Vector

$$\vec{a}_{av} = \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V_2} - \vec{V_1}}{t_2 - t_1}$$



Between A, B

$$|\Delta \vec{V}| = \sqrt{2} V$$

$$\Delta t = \frac{\pi r/2}{V} = \frac{\pi r}{2V}$$

$$= |a_{av}| = \frac{\sqrt{2}v}{\pi r/2v} = \frac{2\sqrt{2}v^2}{\pi r}$$

- Between A and C

$$|\Delta v| = 2v \qquad \Delta t = \frac{\pi r}{v}$$

$$|a_{av}| = \frac{2v}{\frac{\pi r}{v}} = \frac{2v^2}{\pi r}$$

- **Instantaneous acceleration**

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad \boxed{\vec{a} = \frac{d\vec{v}}{dt}} \qquad \because \vec{v} = \frac{dr}{dt} \Rightarrow \boxed{\vec{a} = \frac{d^2r}{dt}}$$

- **For 1D case**

$$v = \frac{dx}{dt} \qquad \Rightarrow a = \frac{d^2x}{dt^2}$$

- **For 2D cases**

$$\vec{r} = x\hat{i} + y\hat{j} \Rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\Rightarrow \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} \quad : \quad v = \sqrt{v_x^2 + v_y^2}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} \quad : \quad a = \sqrt{a_x^2 + a_y^2}$$

Q $x = t^3 - 6t^2 + 9t$

Find →

a velocity and acceleration at time $t = 1$

b 't' at which $v = 0$

c 't' at which $a = 0$

☆d time interval in which $v = +ve$.

☆e " " $v = -ve$.

a(i) $v = \frac{dx}{dt} = 3t^2 - 12t + 9$ (ii) $a = \frac{dv}{dt} = 6t - 12$

b $v = 0$ if $3t^2 - 12t + 9 = 0$

$3(t-1)(t-3) = 0 \Rightarrow t = 1s$ and $t = 3s$.

c $a = 0$ if $6t - 12 = 0 \Rightarrow t = 2s$.

★d $v = +ve$ if (a) $t < 1$ $t < 3$ $= t < 1$

(b) $t > 1$ $t > 3$ $= t > 3$

$\boxed{t < 1s \text{ and } t > 3s}$ velocity is +ve

~~1s <~~

e. $v = -ve$ if

$\boxed{1s < t < 3s}$

- General equations of motion in 1 D

Displacement (Position) $\xrightarrow{\text{Dif}^n}$ velocity $\xrightarrow{\text{Dif}^n}$ acceleration; acceleration $\xrightarrow{\text{Integ}^n}$ velocity $\xrightarrow{\text{Integ}^n}$ Displacement

(i) $V \longrightarrow x$

$$\frac{dx}{dt} = v \Rightarrow \int_{x_0}^{x} dx = \int_0^t v\,dt \Rightarrow x - x_0 = \int_0^t v\,dt$$

Ex $v = t^2 + t$ given at $t = 1$ s $x = 2$ m

$$\frac{dx}{dt} = t^2 + t \Rightarrow \int_{x=2}^{x} dx = \int_{t=1}^{t} (t^2+t)\,dt$$

$$\Rightarrow [x]_2^x = \left[\frac{t^3}{3} + \frac{t^2}{2}\right]_1^t \Rightarrow x - 2 = \left(\frac{t^3}{3} + \frac{t^2}{2}\right) - \left(\frac{1}{3} + \dots\right.$$

$$\Rightarrow x = \frac{t^3}{3} + \frac{t^2}{2} - \frac{5}{6} + 2 \Rightarrow x = \frac{t^3}{3} + \frac{t^2}{2} + \frac{7}{6}$$

(ii) a as f(t)

$$a = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt \Rightarrow v - v_0 = \int_0^t a\,dt$$

Ex $a = 2t$

given at $t = 0$, $v = 3$ m/s

$$\Rightarrow \frac{dv}{dt} = 2t \Rightarrow \int_3^v dv = \int_0^t 2t\,dt = v - 3 = [t^2]$$

$$\Rightarrow v = t^2 + 3$$

$a = f(x)$

$\Rightarrow a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx} \Rightarrow$ $\boxed{a = v\frac{dv}{dx}}$

$\Rightarrow \int_{x_i}^{x} a\,dx = \int_{v_i}^{v} v\,dv$

$\Rightarrow \frac{v^2 - v_1^2}{2} = \int_{x_1}^{x} a\,dx$

Ex $a = 2x$ Find velocity as a function of x. given that at $x = 2m$ $v = 4m/s$

$\Rightarrow v\frac{dv}{dx} = 2x \Rightarrow \int_{4}^{v} v\,dv = \int_{2}^{x} 2x\,dx$

$\Rightarrow \frac{v^2}{2} - \frac{4^2}{2} = x^2 - 2^2 \;\Big|\; \Rightarrow \frac{v^2}{2} = x^2 - 4 + 8$

$\Rightarrow v = \sqrt{2x^2 + 8}$

- $v = \frac{dx}{dt} \Rightarrow x = \int_{x_1}^{x} dx = \int_{t_1}^{t} v\,dt$

- If $a = f(v)$

$\Rightarrow \frac{dv}{dt} = f(v) \Rightarrow \int_{v_1}^{v} \frac{dv}{f(v)} = \int_{t_1}^{t} dt$

Ex $a = 4 - 2v$ (given $t=0$, $v=0$)

Find $v = f(t)$?

$\frac{dv}{dt} = 4 - 2v \Rightarrow \int_{v=0}^{v} \frac{dv}{4-2v} = \int_{t=0}^{t} dt$

$\int \frac{dx}{a-bx} = \frac{\ln(a-bx)}{-b}$

$\Rightarrow \left[\frac{\ln(4-2v)}{-2}\right]_0^v = [t]_0^t$

$\ln a - \ln b \downarrow \ln \frac{a}{b}$

$\Rightarrow \ln\left[\frac{4-2v}{4}\right] = -2t \Rightarrow \frac{4-2v}{4} = e^{-2t}$

$\Rightarrow 4 - 2v = 4e^{-2t} \Rightarrow 2v = 4 - 4e^{-2t} \Rightarrow \boxed{v = 2(1 - e^{-2t})}$

- Uniformly accelerated motion (in 1 dimension)

$\boxed{a = \text{constant}}$ $\quad \frac{dv}{dt} = a = \text{constant}$

$\int_u^v dv = \int_0^t a\,dt$ At $t=0$ $v=u$ (initial velocity)

$\Rightarrow [v]_u^v = a[t]_0^t \Rightarrow v - u = at \Rightarrow \boxed{v = u + at}$

- displacement / Position

$\frac{dx}{dt} = v = u + at \Rightarrow \int_{x_0}^{x} dx = \int_0^t (u+at)\,dt$

• $(x - x_0) = s$

$$\Rightarrow [x]_{x_0}^{x} = \left[ut + \frac{at^2}{2}\right]_0^t$$

final coordinate → x, initial coordinate → x_0

$$\Rightarrow x - x_0 = ut + \frac{1}{2}at^2$$

$x - x_0$ → s (displacement)

$$\Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$$

for position $\boxed{x = x_0 + ut + \frac{1}{2}at^2}$

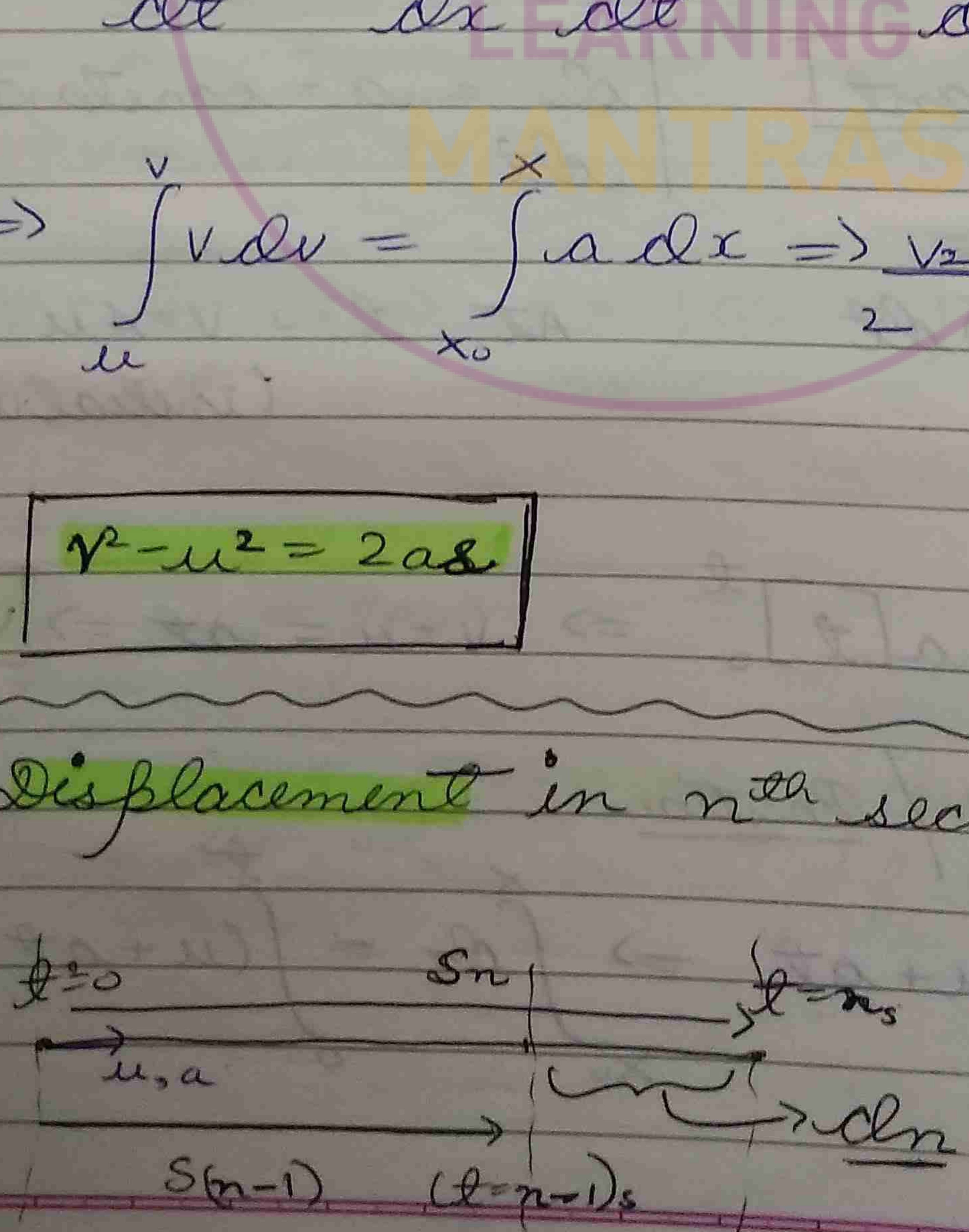
• **Velocity in terms of displacement**

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow \int_u^v v\,dv = \int_{x_0}^{x} a\,dx \Rightarrow \frac{v^2}{2} - \frac{u^2}{2} = a\underbrace{(x - x_0)}_{s}$$

$$\boxed{v^2 - u^2 = 2as}$$

• **Displacement in n^{th} second.**



$$d_n = S_n - S_{n-1}$$

$$\Rightarrow d_n = \underbrace{\left\{un + \frac{1}{2}an^2\right\}}_{S_n} - \underbrace{\left\{u(n-1) + \frac{1}{2}a(n-1)^2\right\}}_{S_{n-1}}$$

$$\Rightarrow d_n = u(n-(n-1)) + \frac{1}{2}a(n^2-(n-1)^2)$$

$$\boxed{d_n = u + \frac{a}{2}(2n-1)}$$

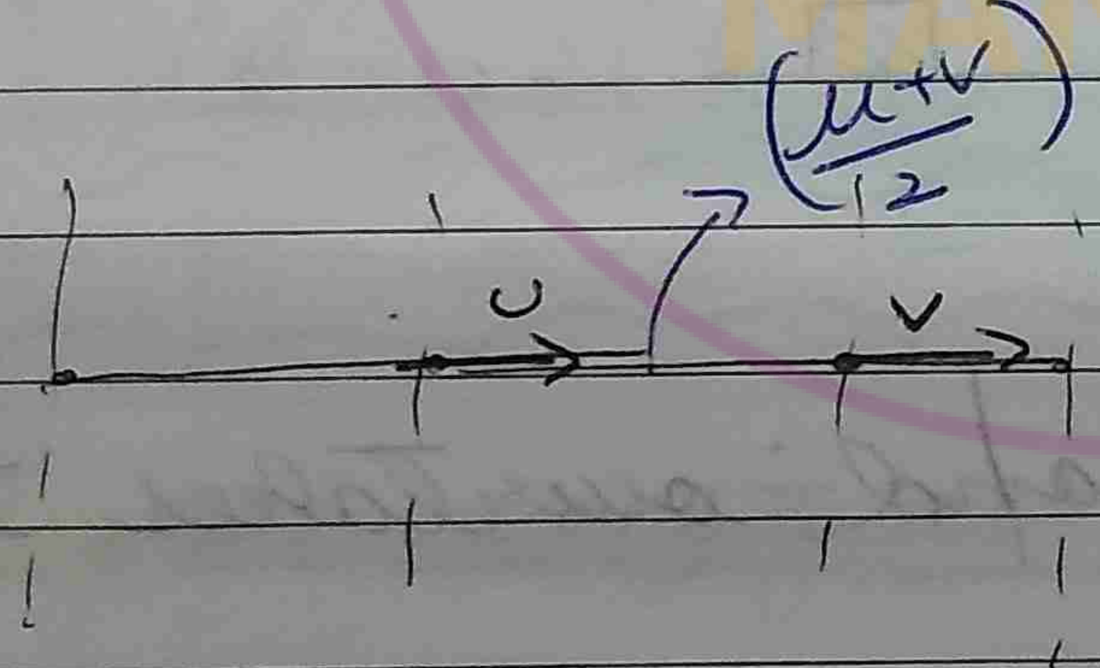
This formula gives **displacement** in nth second and not distance.

- **Average velocity**

$$V_{av} = \frac{s}{t} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{1}{2}at$$

$$= \frac{2u + at}{2} = \frac{u + (u + at)}{2} \Rightarrow \boxed{V_{av} = \frac{u+v}{2}}$$ ★

applicable for any time interval



Results

$$\vec{v} = \vec{u} + \vec{a}t$$
$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$
$$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$$
$$d_n = u + \frac{a}{2}(2n-1)$$
$$V_{av} = \frac{\vec{u}+\vec{v}}{2}$$

so have to be written with proper sign (v, s, a, u are all vectors)

$$x = x_0 + ut + \frac{1}{2}at^2$$

· At the point where overtaking is taking ... coordinate of bus and car would be same.

Ex A body moves 5m in 3rd second and 9 m in 7th second. find distance moved between $t = 5s$ and $t = 8s$.

$5 = u + \frac{a}{2}(5)$

$9 = u + \frac{a}{2}(13)$

$-4 = -4a \Rightarrow a = 1 m/s^2$

$\Rightarrow 5 = u + \frac{5}{2}$

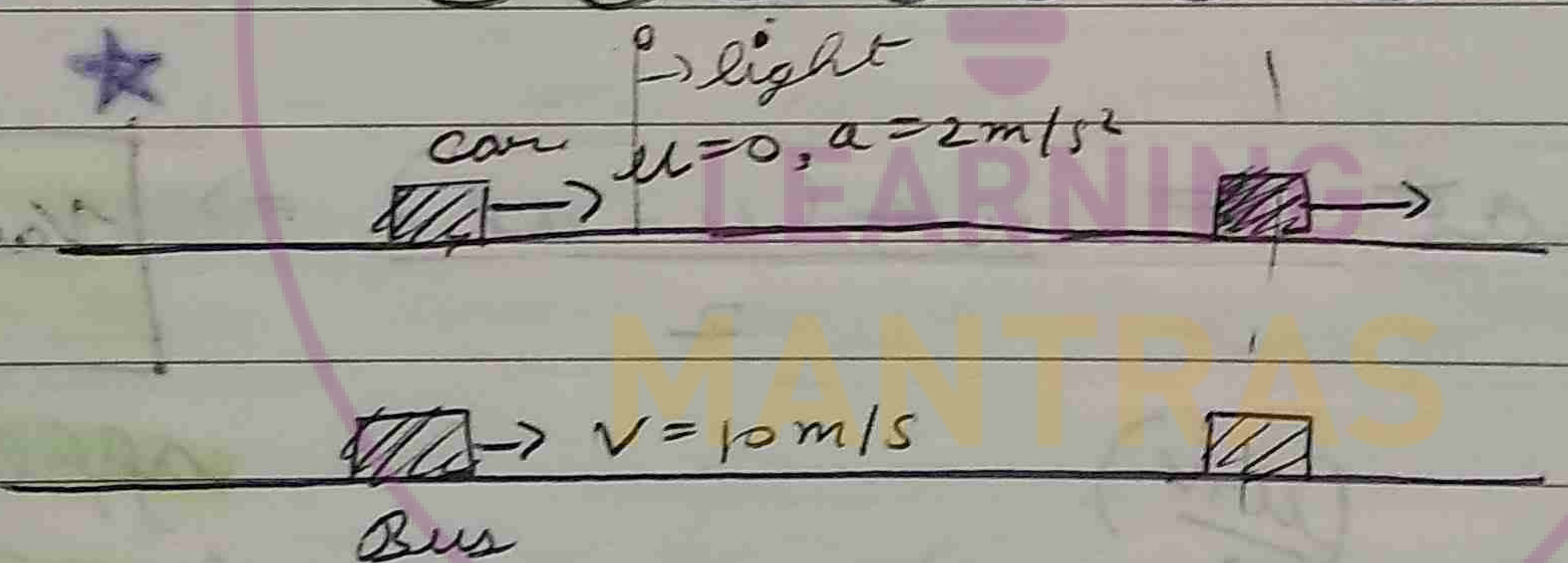
$\Rightarrow u = 5/2$

required disp $= S_8 - S_5$

$= \left\{\frac{5}{2} \times 8 + \frac{1}{2} \times 1 \times 8^2\right\} - \left\{\frac{5}{2} \times 5 + \frac{1}{2} \times 1 \times 5^2\right\}$

$= (20 + 32) - \left(\frac{25}{2} + \frac{25}{2}\right) \Rightarrow 27m$

Ex ★



a. Time at which car overtakes the bus?

b. Velocity of car at that moment?

c. Distance from the light?

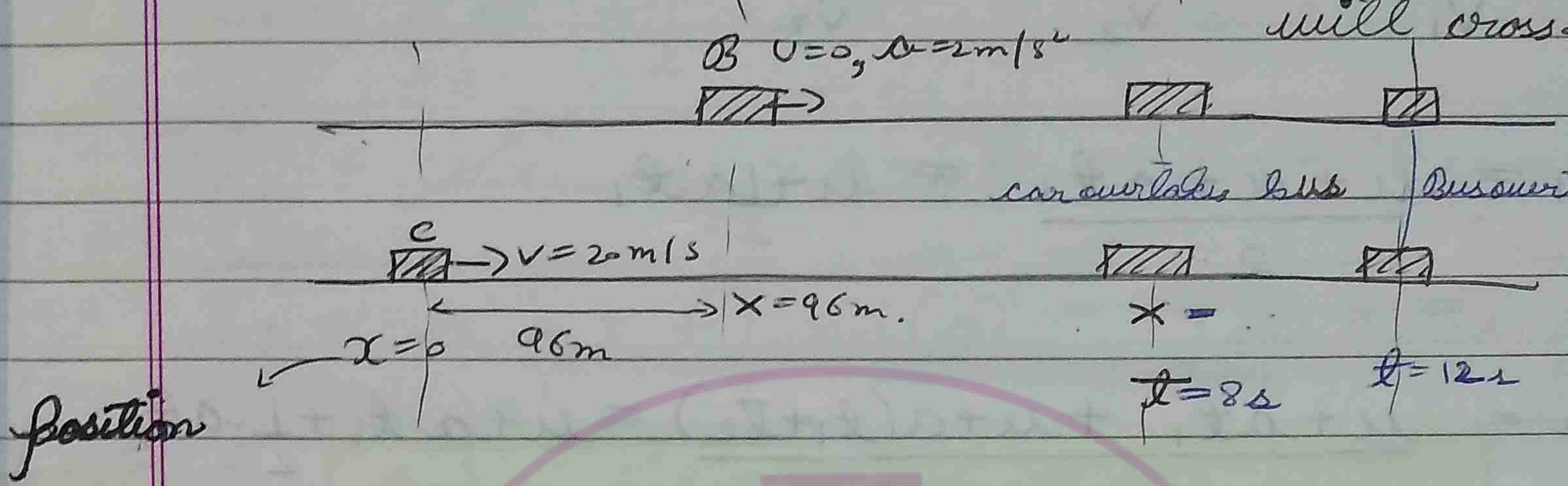
a. $S_{car} = S_{bus}$

$= 0.t + \frac{1}{2}.2.t^2 = 10t \Rightarrow t = 10$ seconds.

b $v = u + at = 0 + 2 \times 10 = 20\,m/s$

c $s_{bus} = 10 \times 10 = 100\,m$ $\quad S_{car} = 0 \times 10 + \frac{1}{2} \times 2 (10)^2 = 100$

- Coordinate method
- At what time they will cross each other



$X_{car} = X_{bus} \Rightarrow X_{0car} + S_{car} = X_{0bus} + S_{bus}$

$0 + 20t = 96 + 0.t + \frac{1}{2} . 2t^2$

$\Rightarrow t^2 - 20t + 96 = 0$

$\Rightarrow (t-12)(t-8) = 0 \Rightarrow t = 8s, 12s$

Ex initial velocity u

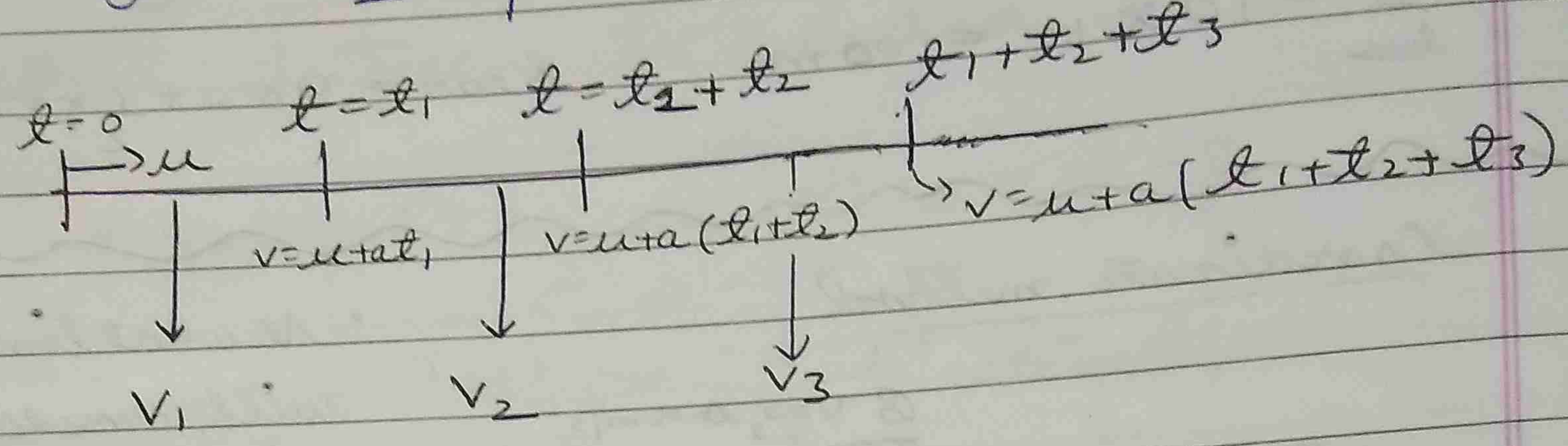
- First $t_1 \rightarrow$ av. velocity $= v_1$
- next $t_2 \rightarrow$ av. velocity $= v_2$

next $t_3 \rightarrow$ av. velocity $= v_3$

$V_{av} = \frac{V_1 + U_1}{2}$

Find $\frac{V_1 - V_2}{V_2 - V_3} = ?$

$$V_{avg} = \frac{u+v}{2}$$



$$V_1 = \frac{u+u+at_1}{2} = u+\frac{1}{2}at_1$$

$$V_2 = \frac{u+at_1+u+a(t_1+t_2)}{2} = u+at_1+\frac{1}{2}at_2$$

$$V_3 = u+at_1+at_2+\frac{1}{2}at_3$$

$$\frac{V_1-V_2}{V_2-V_3} = \frac{-\frac{1}{2}at_1-\frac{1}{2}at_2}{-\frac{1}{2}at_2-\frac{1}{2}at_3} = \boxed{\frac{t_1+t_2}{t_2+t_3}}$$

- **Important results:**

$\boxed{u=0}$

$\frac{u+v}{2}$

$u=0, a$

displacement in first second $d_1 = u^{\,0} + \frac{a}{2}(2n-1) = d_1 = \frac{a}{2}$

" in second second $d_2 = u^{\,0} + \frac{a}{2}(2\times2-1) \Rightarrow d_2 = \frac{3a}{2}$

$d_3 = u^{\,0} + \frac{a}{2}(5) \Rightarrow d_3 = \frac{5a}{2}$

displacement in n^{th} second $\quad |d_1 : d_2 : d_3 \ldots = 1 : 1 : 3 : 5 \ldots|$

(The second term of the ratio is written as 2 and crossed out, with 1 written over it.)

- $s = ut + \frac{1}{2}at^2$

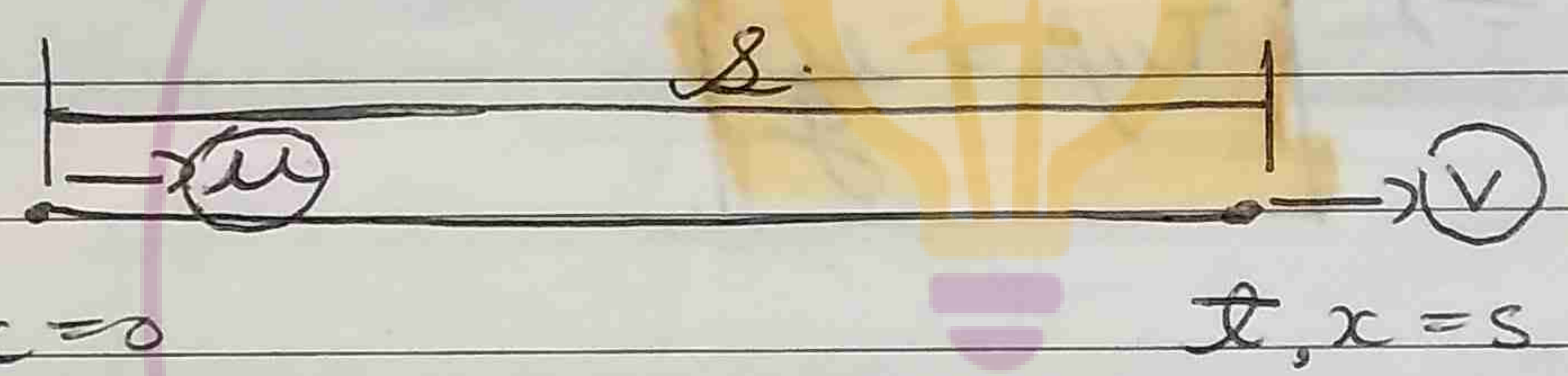
$s_1 = u^{\nearrow 0} \times 1 + \frac{1}{2} \times a \times 1^2 = \frac{1}{2}a$

$s_2 = u^{\nearrow 0} \times 2 + \frac{1}{2} \times a \times (2)^2 = 2a = 4s_1 = 2^2 s_1$

$s_3 = u^{\nearrow 0} \times 3 + \frac{1}{2} \times a(3)^2 = \frac{9}{2}a = 9s_1 = 3^2 s_1$

Total displacement $\Rightarrow \quad |s_1 : s_2 : s_3 \ldots = 1^2 : 2^2 : 3^2 \ldots|$

★ **Ex**



: Find velocity (a) at $x = \frac{s}{2}$ (b) at $t = \frac{t}{2}$

a $= \quad v^2 = u^2 + 2as \quad$ (i) $\Rightarrow as = \frac{v^2 - u^2}{2}$

let velocity at $\frac{s}{2} = v_1$

$v_1^2 = u^2 + 2a \times \frac{s}{2} \quad$ (ii) $\Rightarrow v_1^2 = u^2 + \frac{v^2 - u^2}{2}$

$\Rightarrow \frac{v^2 + u^2}{2} \quad \Rightarrow \quad \boxed{v_1 = \sqrt{\frac{v^2 + u^2}{2}}}$

b $\quad a = \frac{v - u}{t}$

let v_2 be velocity at $\frac{t}{2}$

$v_2 = u + a\frac{t}{2} = u + \frac{v - u}{t} \times \frac{t}{2} \Rightarrow \boxed{v_2 = \frac{v + u}{2}}$ ← valid for all time interval

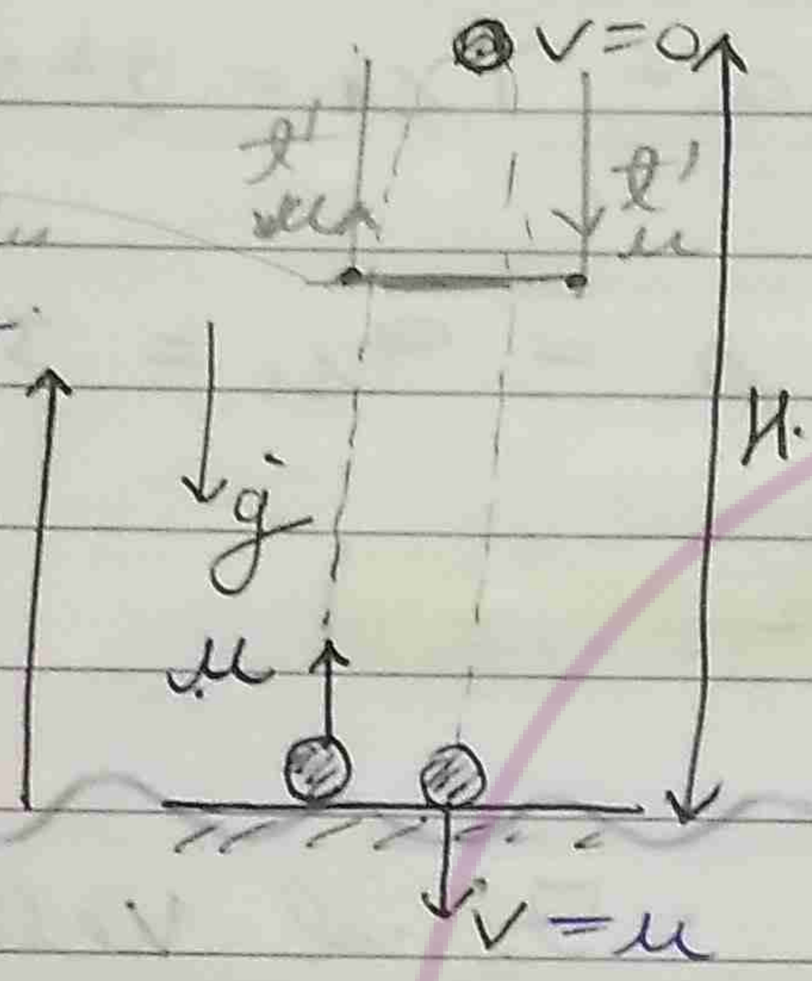
Freely falling Body

$W = mg$

$\Rightarrow a = \frac{F}{m} = g \approx 9.8/10\ m/s^2$

$v = u + at$
$s = ut + \frac{1}{2}gt^2$
$v^2 = u^2 + 2as$

• Case i :



even from the point our concept of Tup = Tdown and v=u can be applied (ii)

Tup = time to reach maximum height

$v = u + at = 0 = u + (-g).T_{up}$

$\Rightarrow \boxed{T_{up} = \frac{u}{g}}$

$v^2 = u^2 + 2as \Rightarrow 0^2 = u^2 + 2(-g)H$

$\Rightarrow \boxed{H = \frac{u^2}{2g}}$

• Downward motion

T down

$s = ut + \frac{1}{2}at^2 = -H = \cancel{0.T_{down}} + \frac{1}{2}(-g)T_{down}^2$

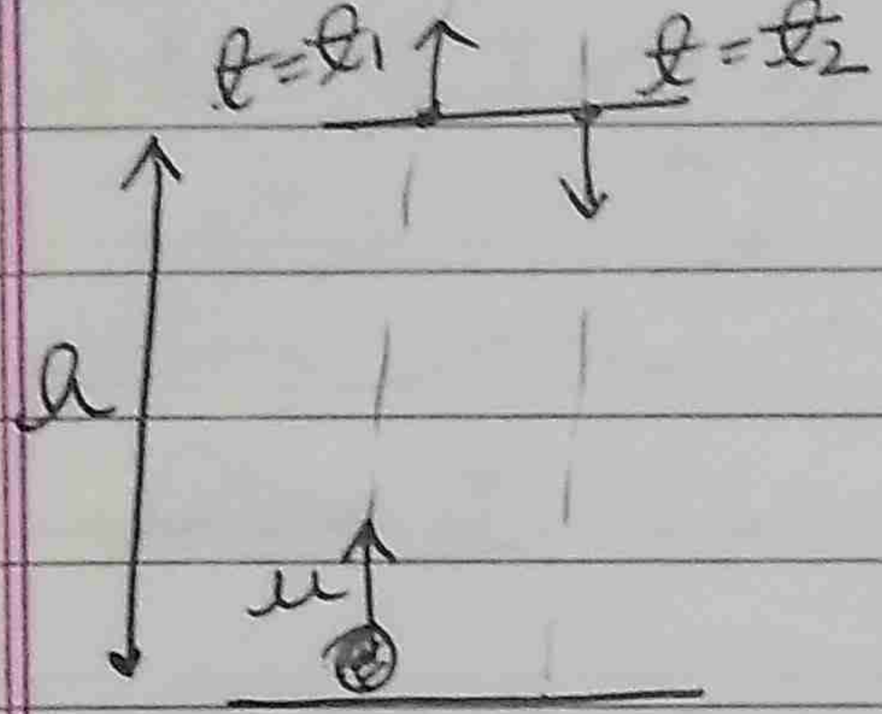
$\Rightarrow T_{down} = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2.u^2/2g}{g}} = \sqrt{\frac{u^2}{g^2}} = \frac{u}{g}$

$\Rightarrow \boxed{T_{up} = T_{down}}$

- $v = u + at = 0 + (-g) \cdot \frac{u}{g} = -u$

$\Rightarrow$ $v = u$

$t_{up} = t_{down}$



Height at instant t.

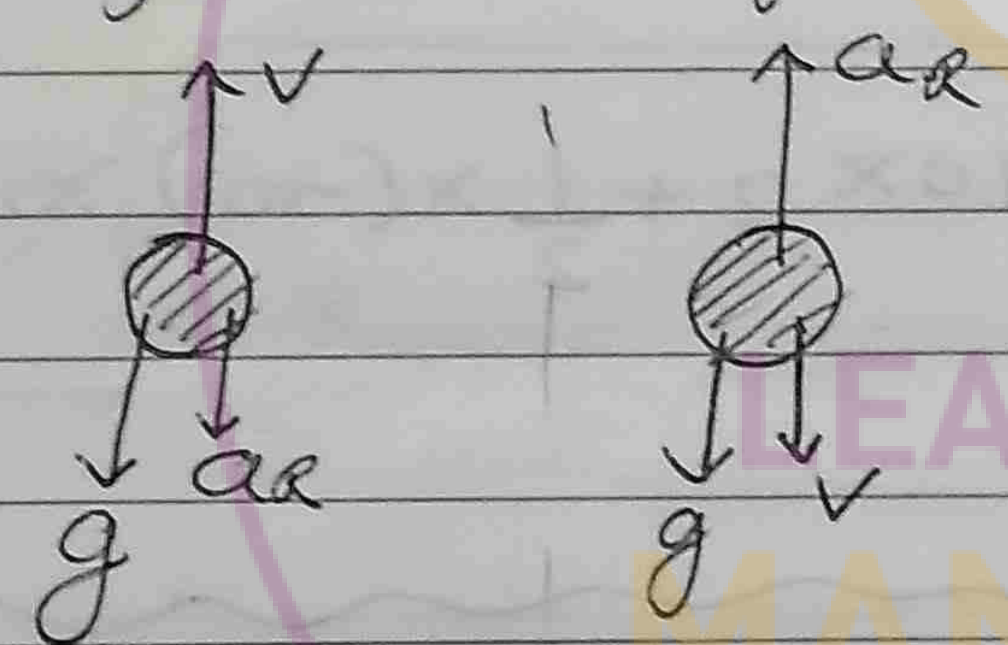
$s = ut + \frac{1}{2} at^2 \Rightarrow +h = ut + \frac{1}{2}(-g)t^2$

$\Rightarrow$ $h = ut + \frac{1}{2}(-g)t^2$

$\Rightarrow$ $h = ut - \frac{1}{2} gt^2$ if t is given

$\Rightarrow$ $t = t_1$ and t_2 if h is given

- <u>In presence of air resistance</u>



$\Rightarrow$ $a_{net} > g$ $a_{net} < g$

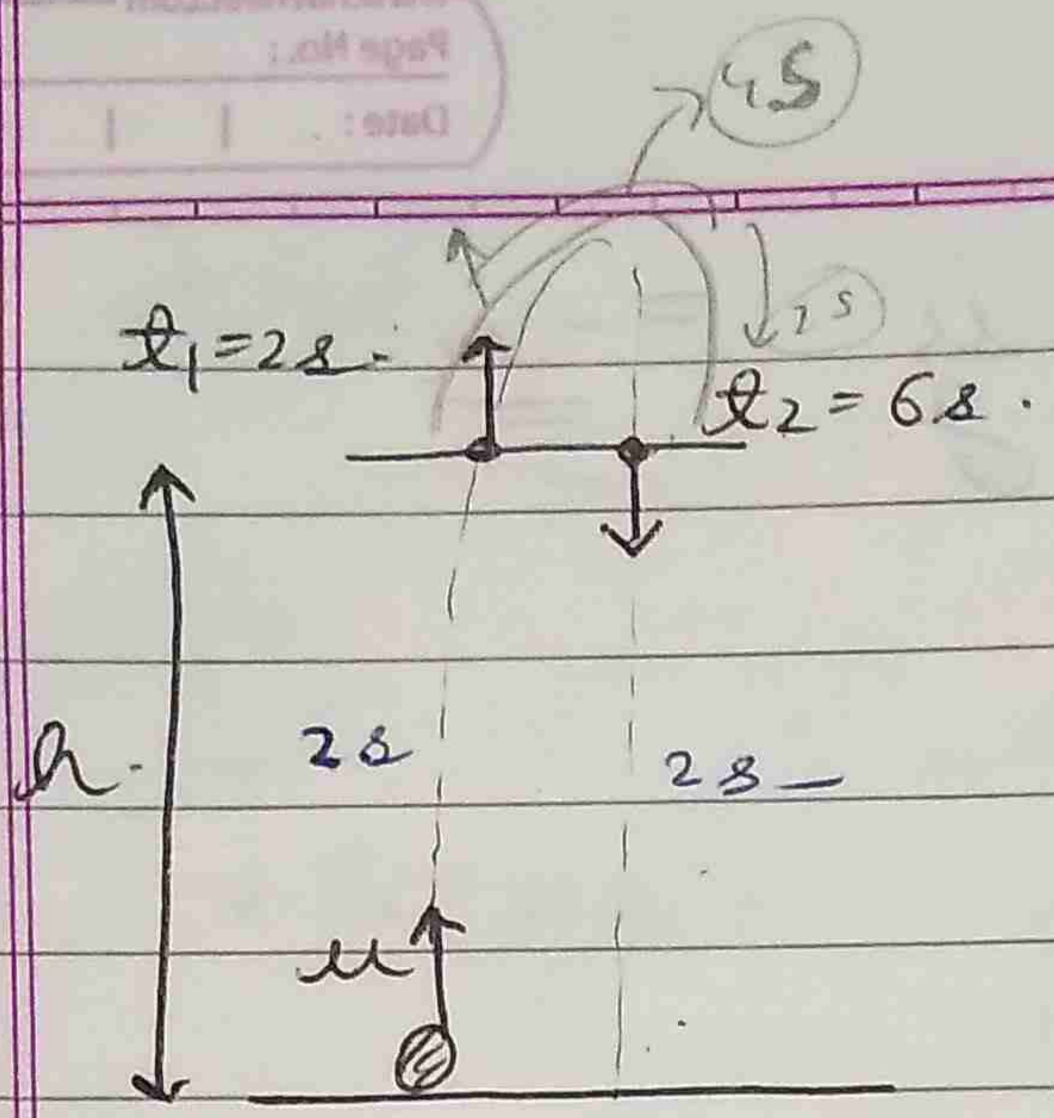
$t_{up} < \frac{u}{g}$ $t_{down} > \frac{u}{g}$ $\Rightarrow$ $t_{up} < t_{down}$

velocity when object reaches surface. $v < u$ velocity with which the object was thrown.

$\because$ downward acceleration is less, downward velocity is less.

- There is continuous loss of energy due to air drag.

Q



Find

a| Total time T

b| u

c| H (Max height)

d| h

a) $T = t_1 + t_2 = 8s$

b) $t_{up} = t_{down} = \frac{T}{2} = 4s$

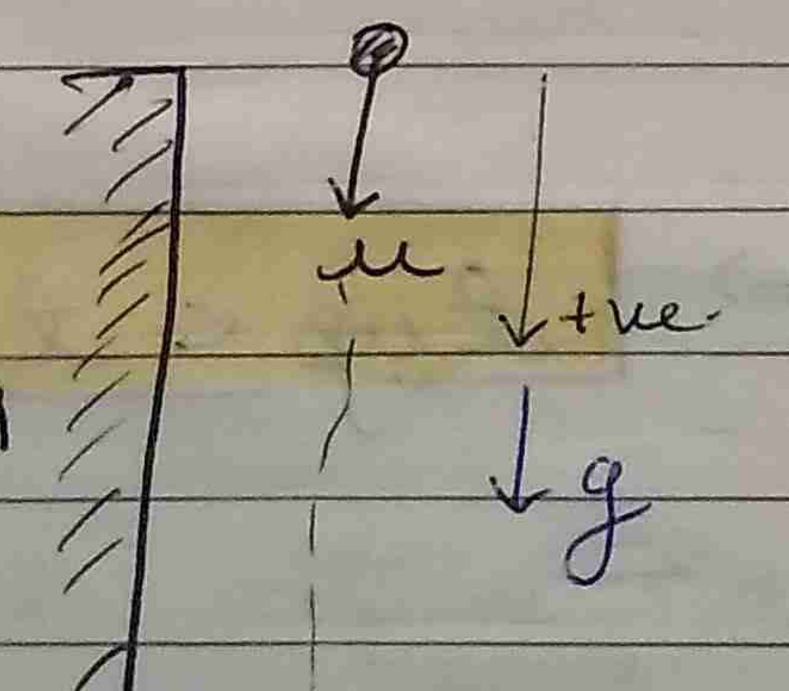
$\Rightarrow \frac{u}{g} = 4s \Rightarrow u = 40 m/s$

c) $H = \frac{u^2}{2g} = \frac{(40)^2}{2 \times 10} = 80m$

d) $h = ut + \frac{1}{2}at^2 = 40 \times 2 + \frac{1}{2} \times (-10) \times 2^2$

$= 80 - 20 = 60\ m$

- **Case 2**



- Time of flight

$s = ut + \frac{1}{2}at^2$

$|+H = u.T + \frac{1}{2}gT^2|$

$\Rightarrow T = \text{---}$

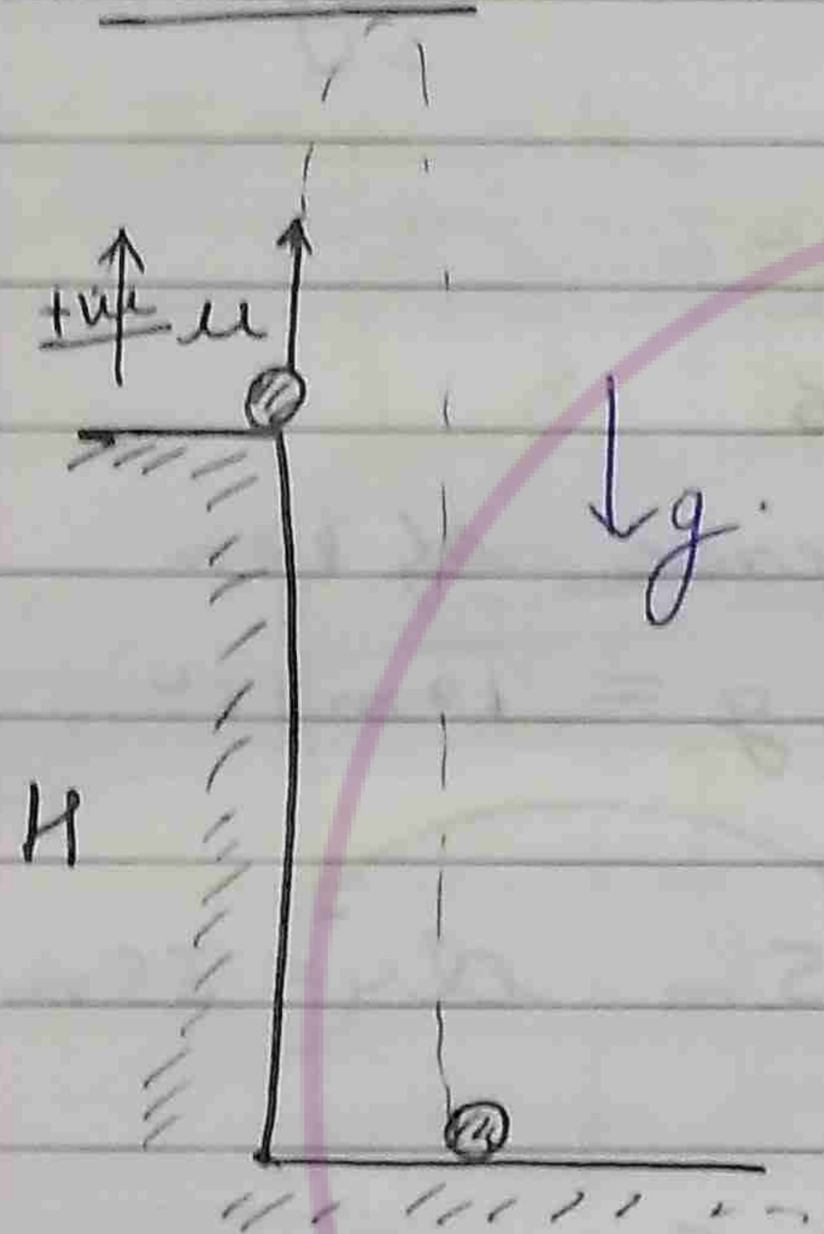
- Final v

$$v^2 = u^2 + 2as = u^2 + 2g \cdot H \Rightarrow \boxed{v = \sqrt{u^2 + 2gh}}$$

→ Special case : $u = 0$ (dropping / releasing)

$$H = 0 \cdot T + \frac{1}{2} g T^2 \Rightarrow \boxed{T = \sqrt{\frac{2H}{g}}} \qquad \boxed{v = \sqrt{2gh}}$$

- Case 3



- Time of fall ?
- final velocity ?

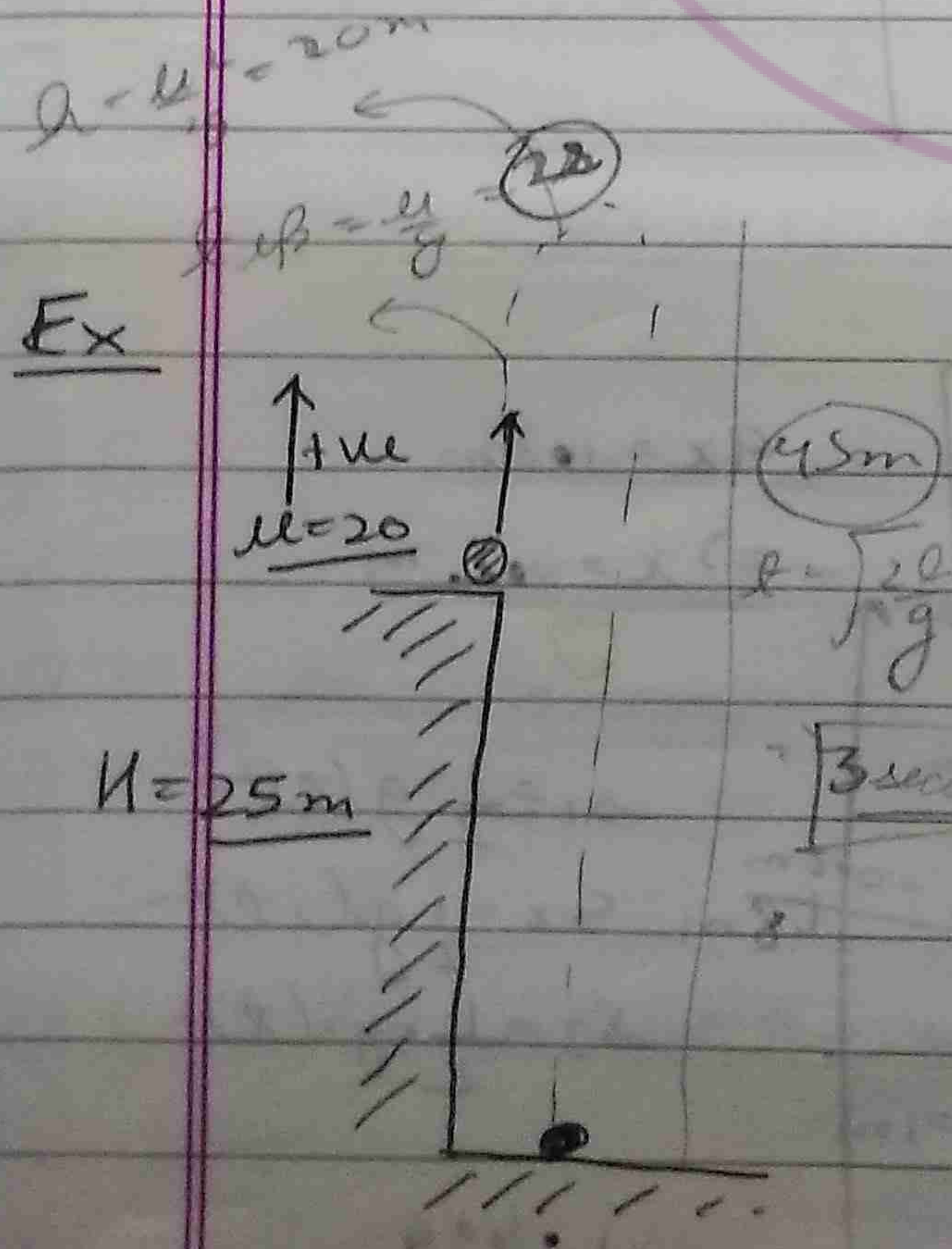
$u = +u$
$s = -H$
$a = -g$

$$s = ut + \frac{1}{2} at^2 = -H = u \cdot T - \frac{1}{2} g T^2$$

$$\Rightarrow \frac{1}{2} g T^2 - ut - H = 0$$

$$\Rightarrow T = \frac{u \pm \sqrt{u^2 + 2gH}}{g}$$

Ex



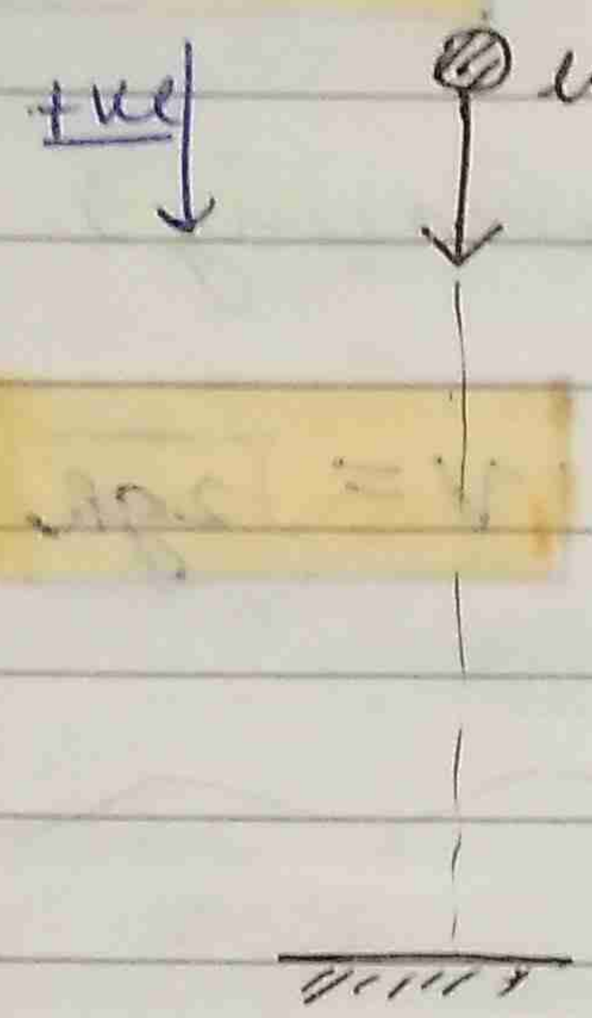
$$s = ut + \frac{1}{2} at^2 \Rightarrow -25 = 20t + \frac{1}{2}(-10)$$

$$\Rightarrow 5t^2 - 20t - 25 = 0$$

$$\Rightarrow t = \frac{20 + \sqrt{400 + 500}}{10} = 5 \text{ seconds}$$

- distance travelled in last second of upward journey = Distance travelled in first second of Downward journey

- Displacement of a freely falling body from rest :-



- In $t = 1s$

$s_1 = ut + \frac{1}{2}at^2$

$= \frac{1}{2} \times g \times (1)^2 = 5m$

$t = 2s$ $\quad s_2 = \frac{1}{2} \times g \times 2^2 = 2g = 20m$

$s_3 = \frac{1}{2} \times g \times 3^2 = \frac{9}{2}g = 45m$

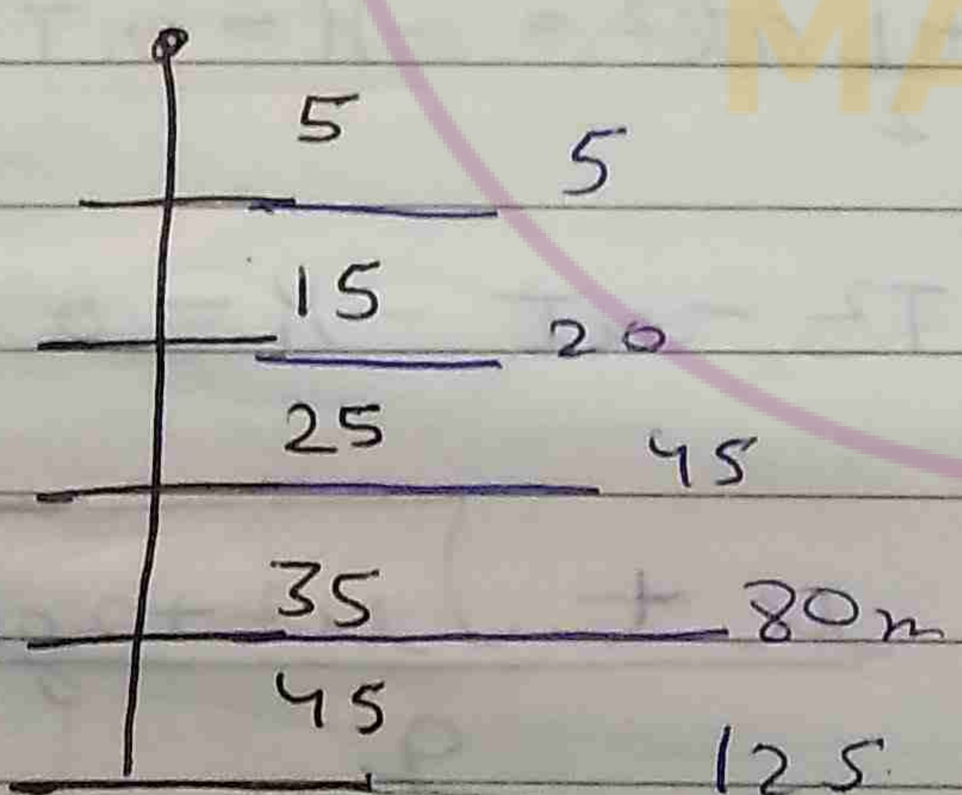
$s_1 : s_2 : s_3 = 1^2 : 2^2 : 3^2 : 4^2$

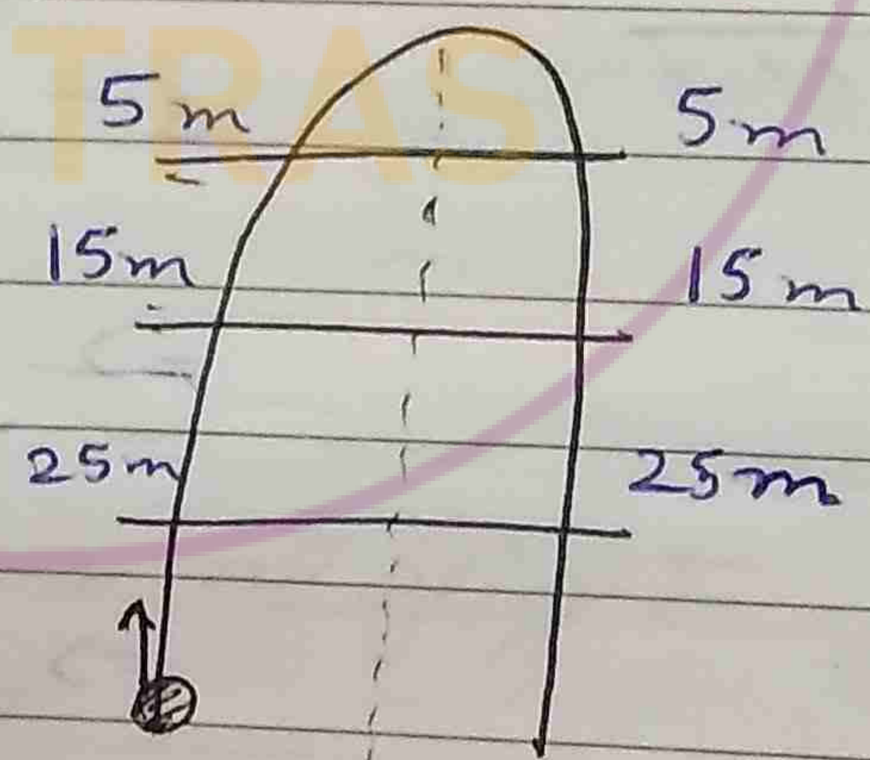
$= 1 : 4 : 9 : 16$

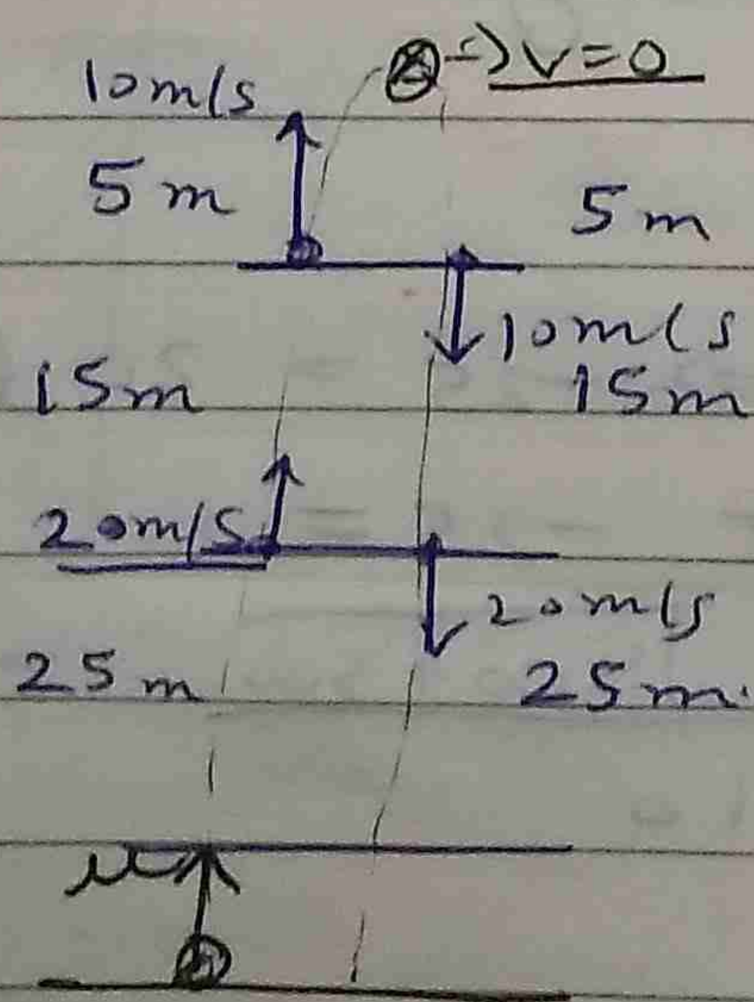
$s_4 = \underline{80m}$ $\qquad$ Increase of 10 m because $g = 10 m/s^2$

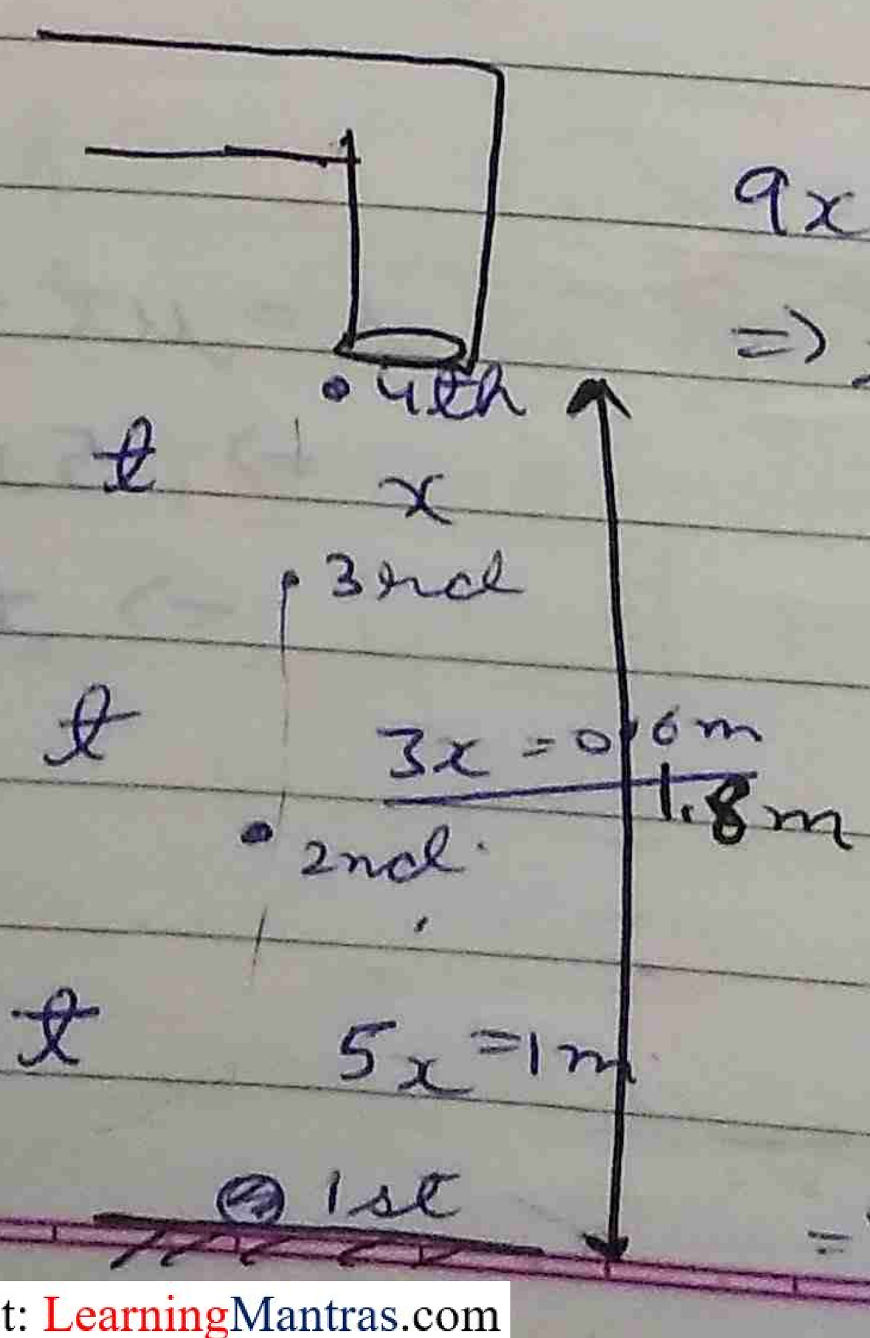
displacement in nth second

$d_1 = 5 \quad d_2 = 15 \quad d_3 = 25m \quad d_4 = 35m$









$9x = 1.8m$

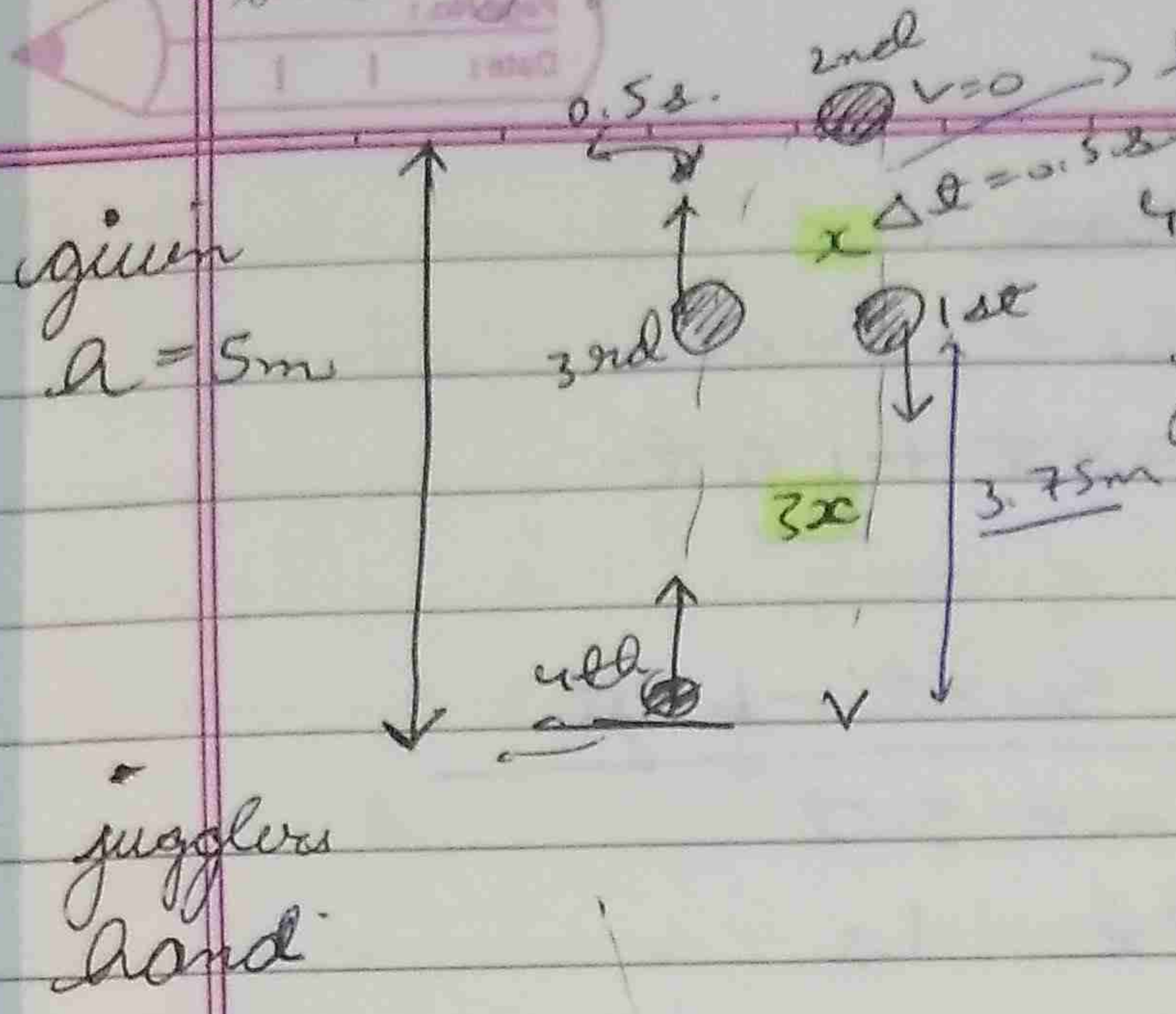
$\Rightarrow x = 0.2m$

$s_1 = \frac{1}{2}g(3t)^2$

$s_2 = \frac{1}{2}g \times (2t)^2$

$s_3 = \frac{1}{2} \times g \times (t)^2$

$\Rightarrow 1 : 4 : 9$



4 balls maintained by the juggler in air thrown at equal intervals.

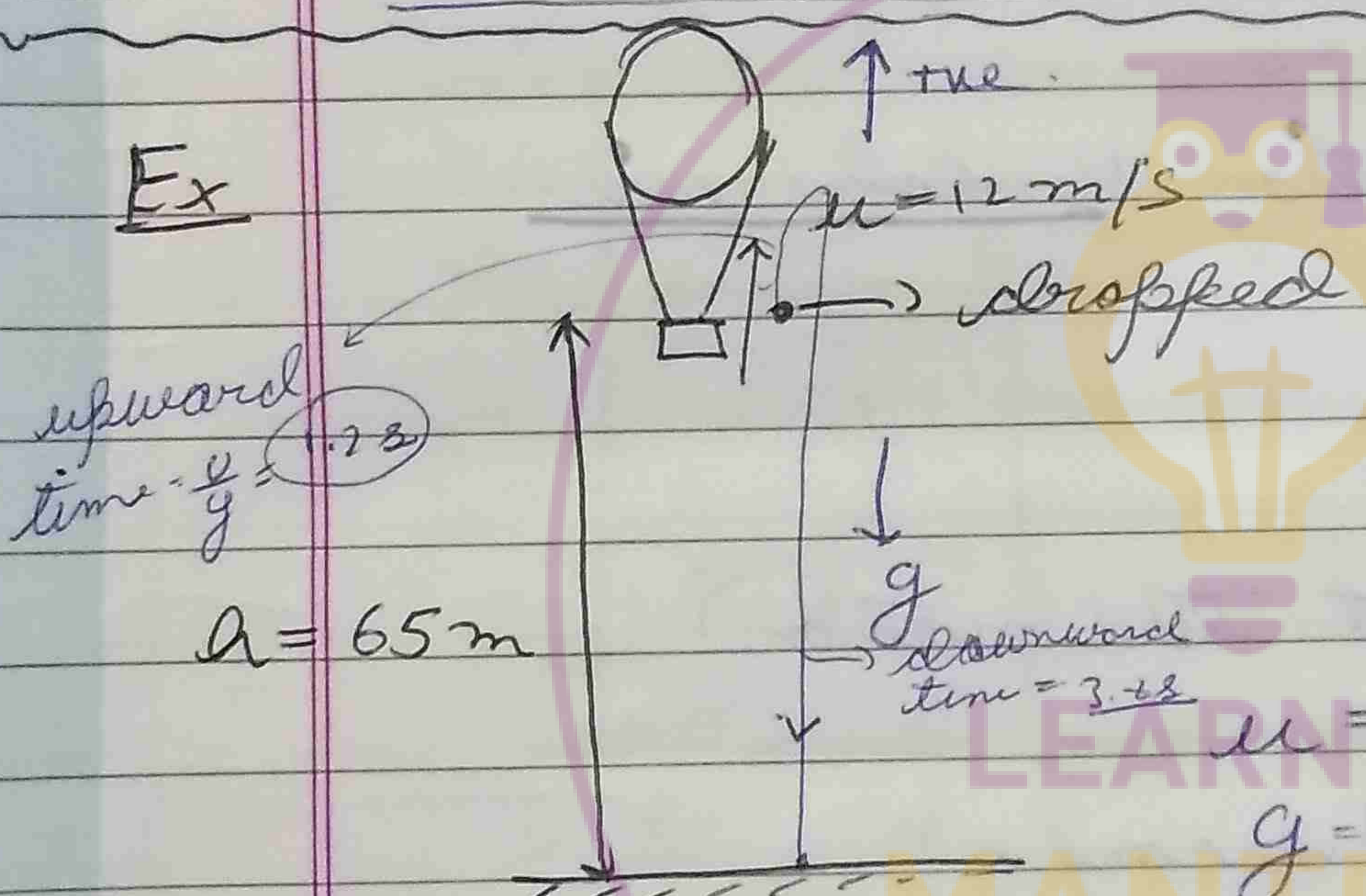
Total time of flight $= \frac{2u}{g} = \frac{2 \times 10}{10}$

$h = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gh} = \sqrt{100} = 10 m/s$

time interval between two consecutive balls $= \frac{2}{4} = 0.5$ ($\frac{\text{Total time}}{\text{no. of balls}}$)

$x + 3x = 4x = 5m \Rightarrow x = 1.25m$

Ex



Find
a) Time of fall
b) displacement and distance
c) velocity (final)

$u = +12 m/s$

$g = a = -10 m/s^2$

$s = -65 m$

(a) $s = ut + \frac{1}{2}at^2 \Rightarrow -65 = 12t - 5t^2$

$\Rightarrow 5t^2 - 12t - 65 = 0 \Rightarrow t = \frac{12 + \sqrt{144 + 1300}}{10}$

= 5 seconds

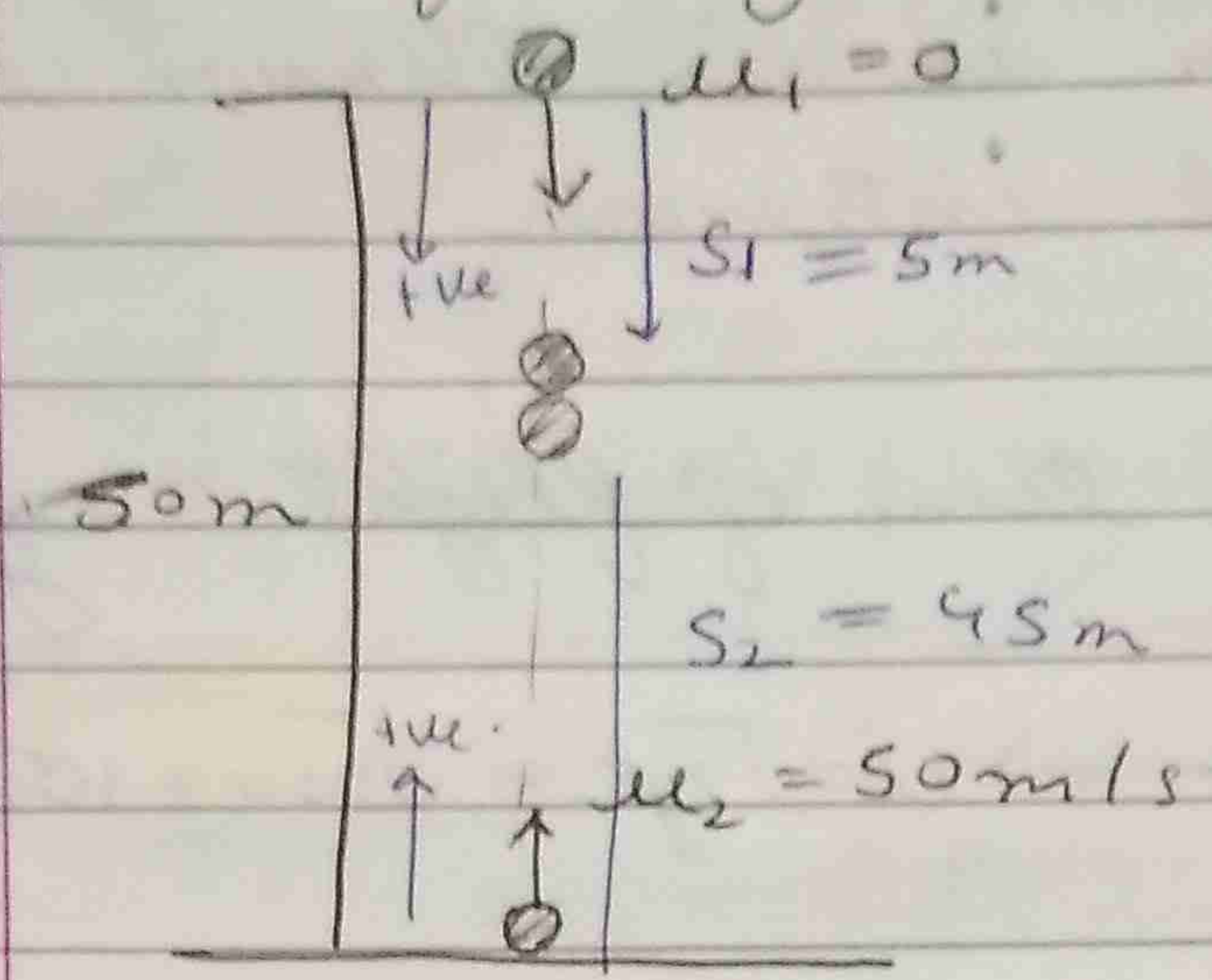
(b) Displacement = 65m (downward)

distance $= 65 + 2 \times \frac{u^2}{2g} = 79.4 m$

(c) $v = u + at = 12 - 10 \times 5 = -38 m/s$

☆ The thrown body acquires the velocity of parent body not its acceleration.

Ex (a) time of meeting ? (b) where ? Method 1



$S_1 + S_2 = 50m$

$S_1 = 0 \times t + \frac{1}{2} g t^2$

$S_2 = 50 \times t - \frac{1}{2} g t^2$

$50m = 50t$

$\Rightarrow t = 1s$

Method 2

for both bodies they have same acceleration

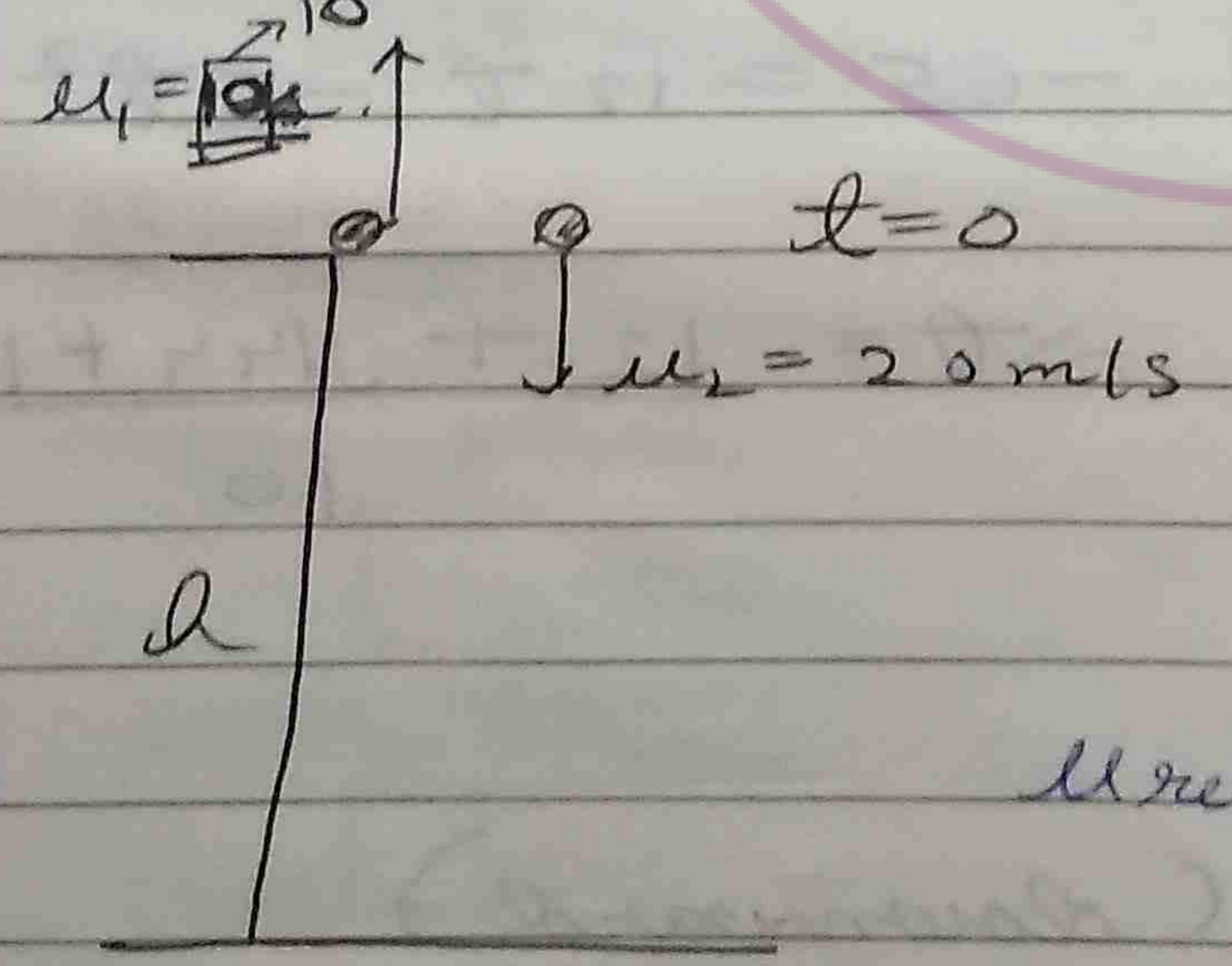
$\Rightarrow$ relative acceleration = 0

$\rightarrow$ throughough the motion Vrelative will be same

$V_{rel} = 50 m/s$
(throughout the motion)

$t = \frac{S_{rel}}{V_{rel}} = \frac{50}{50} = 1$ second.

Ex

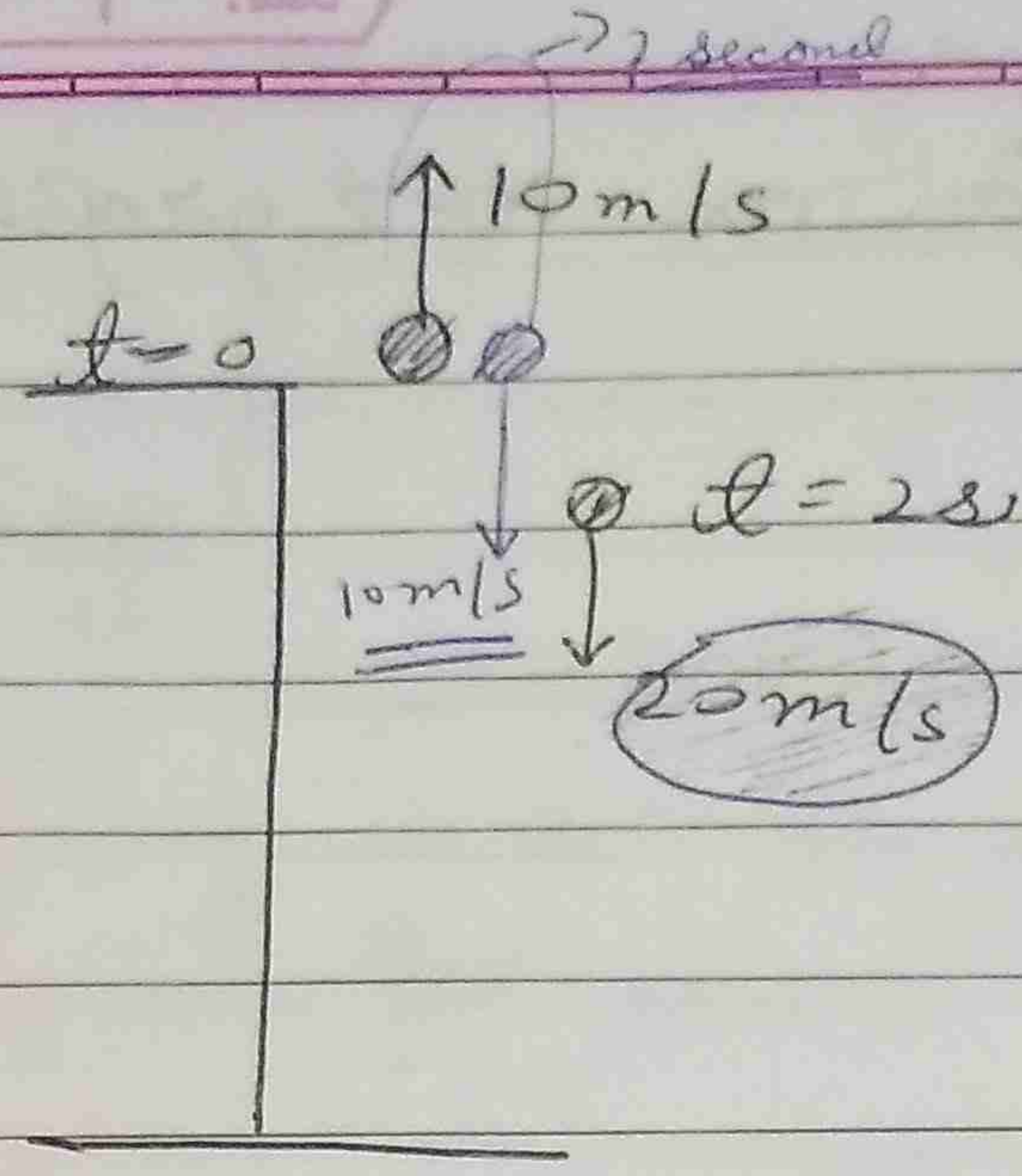


find distance b/w balls at $t = 2$ seconds

$u_{rel} = 20 + 10 = 30 m/s$
= constant

distance between two balls after 2 seconds = $30 \times 2 = 60m$

Ex →



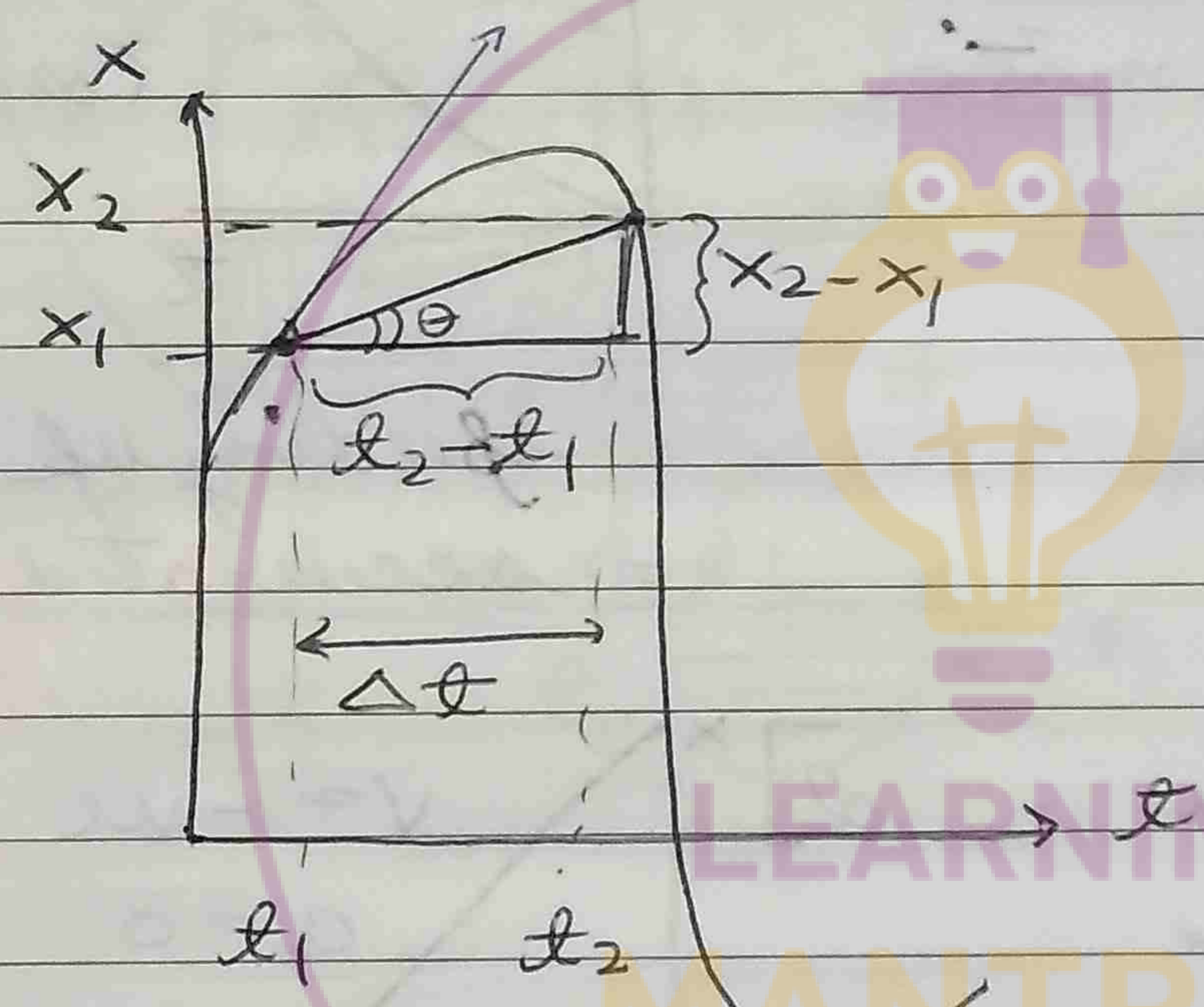
Find distance at $t = 5$ seconds

$v_{rel} = 20 - 10 = 10$ m/s

$\Delta t = 3$ seconds

$\Rightarrow$ distance $= 10 \times 3 = 30$ m

- **Position-time graph:** (x-t graph)



Average velocity

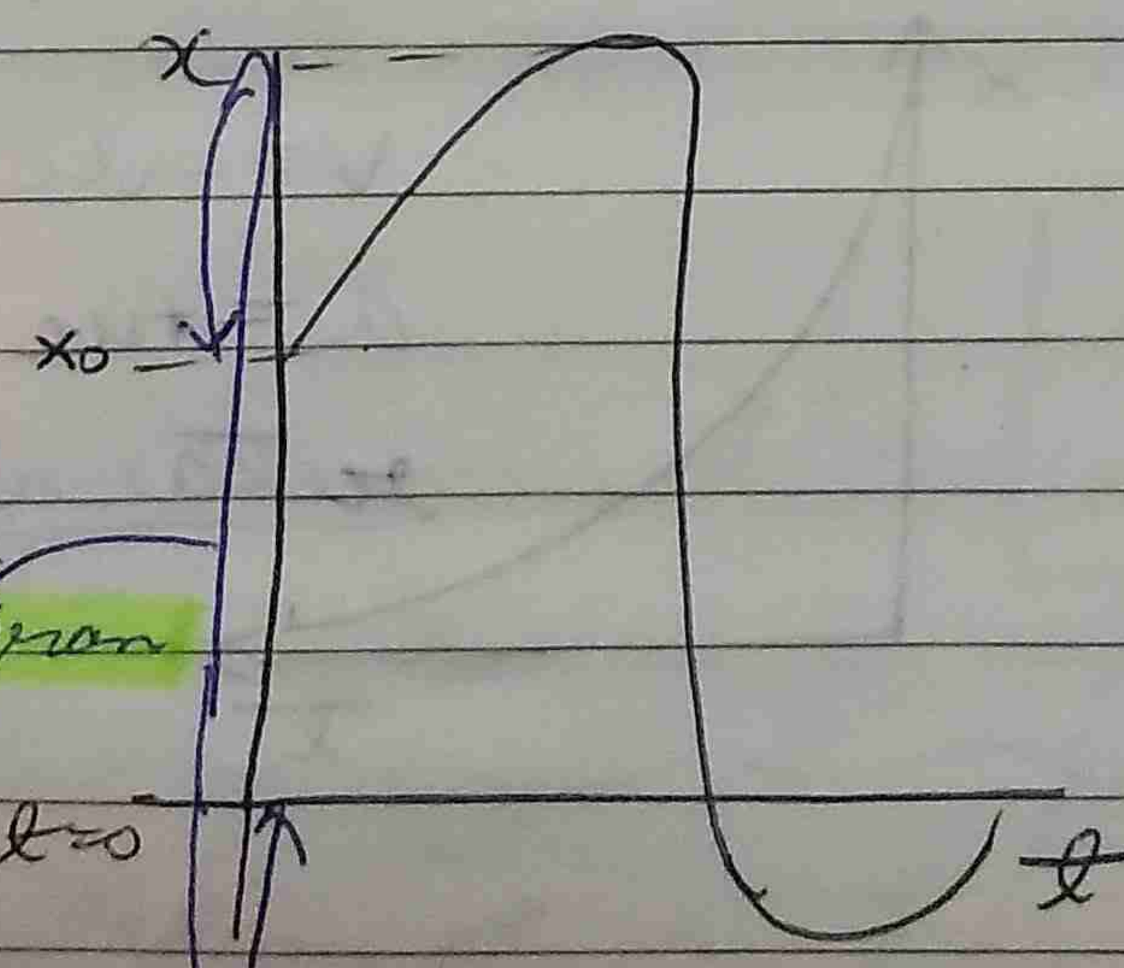
$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

$\Rightarrow v_{av} = \tan\theta$ = slope of chord

- Instantaneous velocity

$v = \frac{dx}{dt}$ = slope of tangent

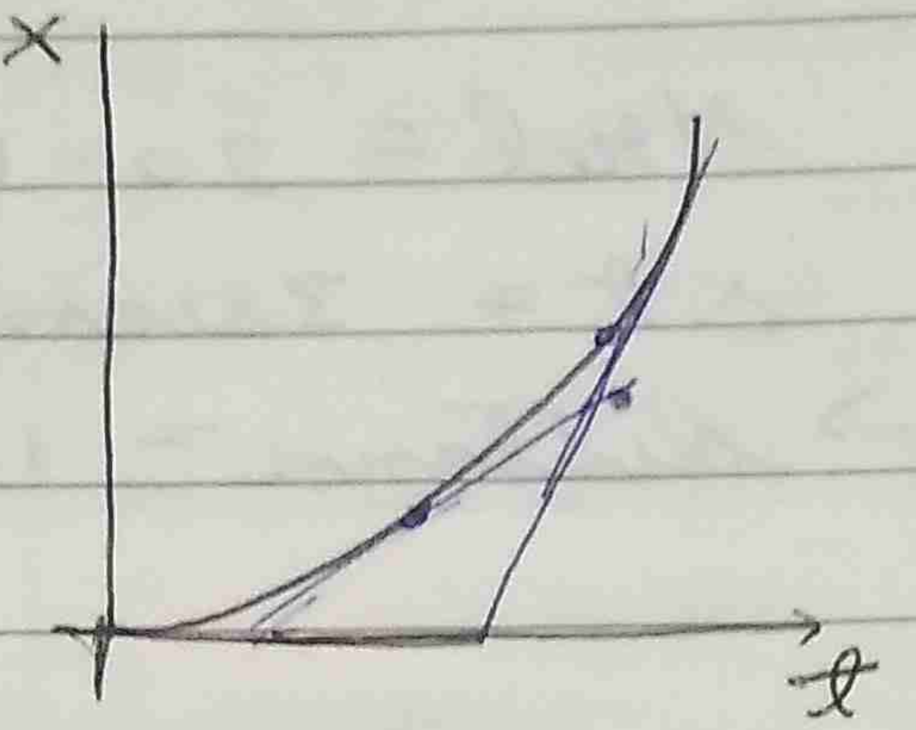
- **distance**



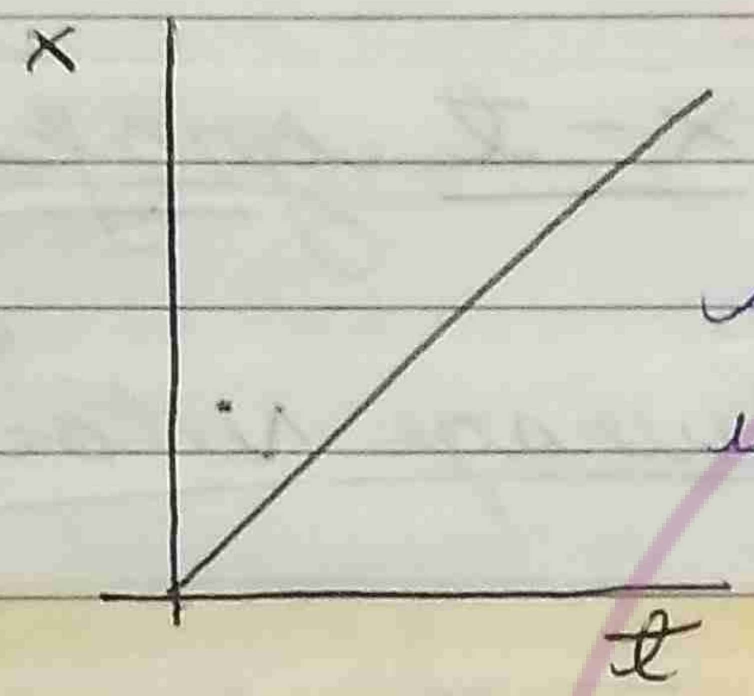
Motion diagram → calculate its length = Distance

☆ Position time graph can be projected on x-axis to find distance

☆ Nature of acceleration on x-t graph

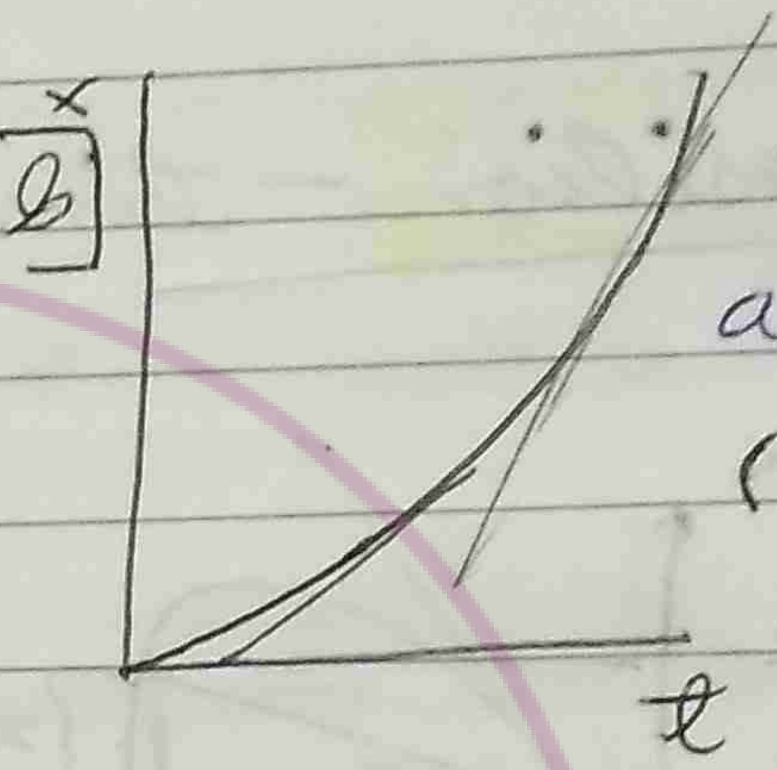


a



v = +ve
a = 0
uniform motion

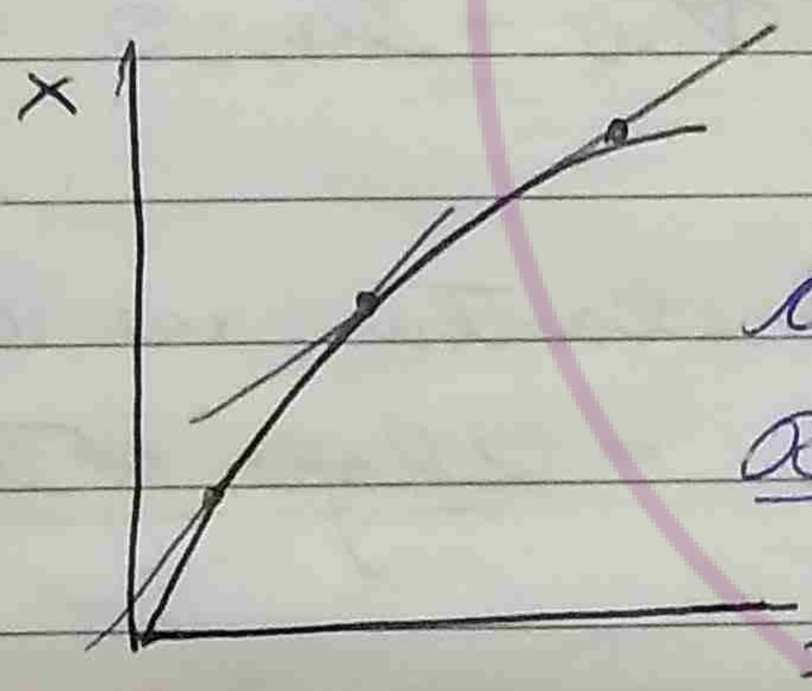
b



v = +ve
a = +ve
(same sign)

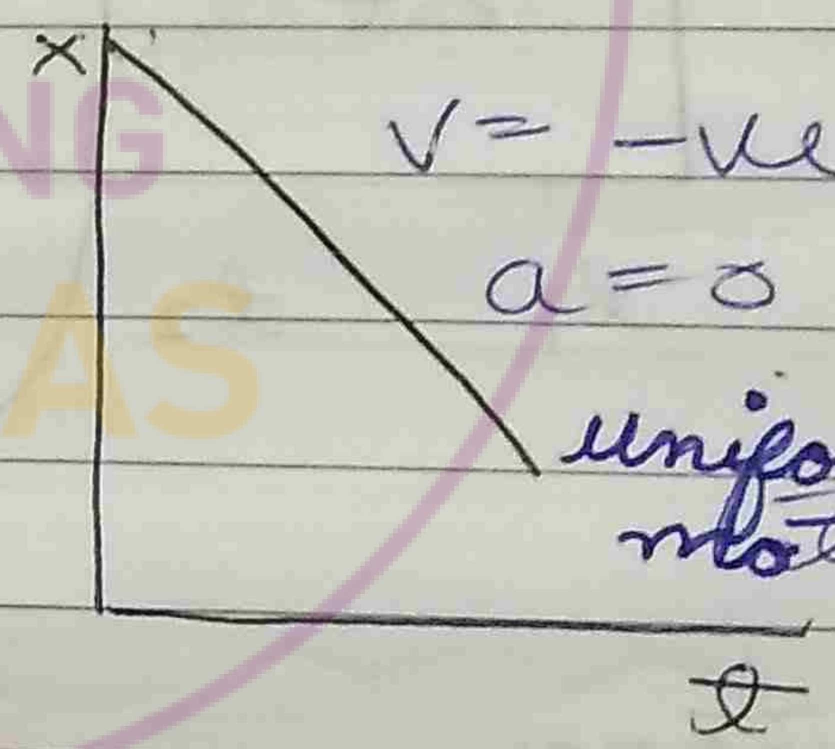
speeding up
⇒ accelerated

c



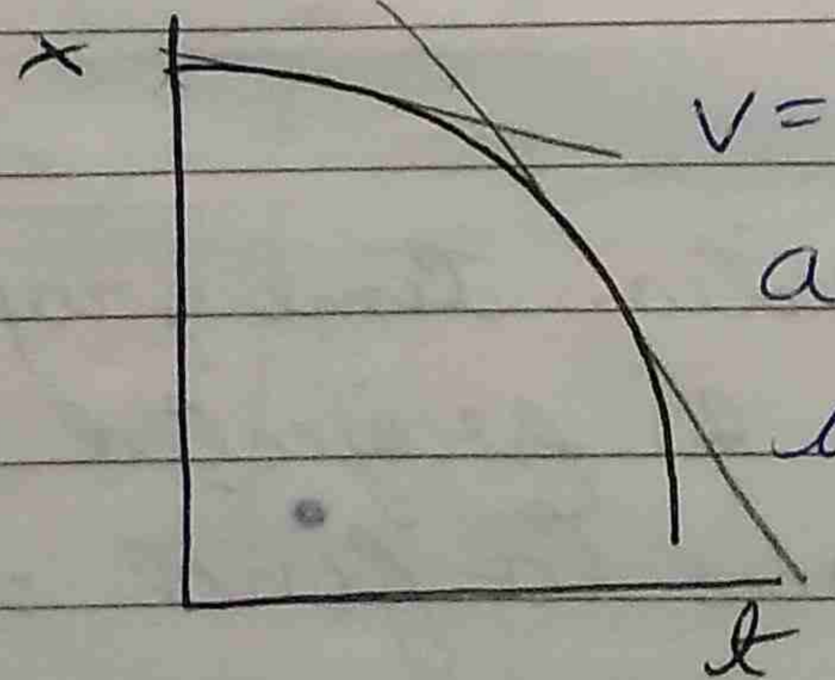
v = +ve
a = -ve
Retarded
(opposite sign)

d

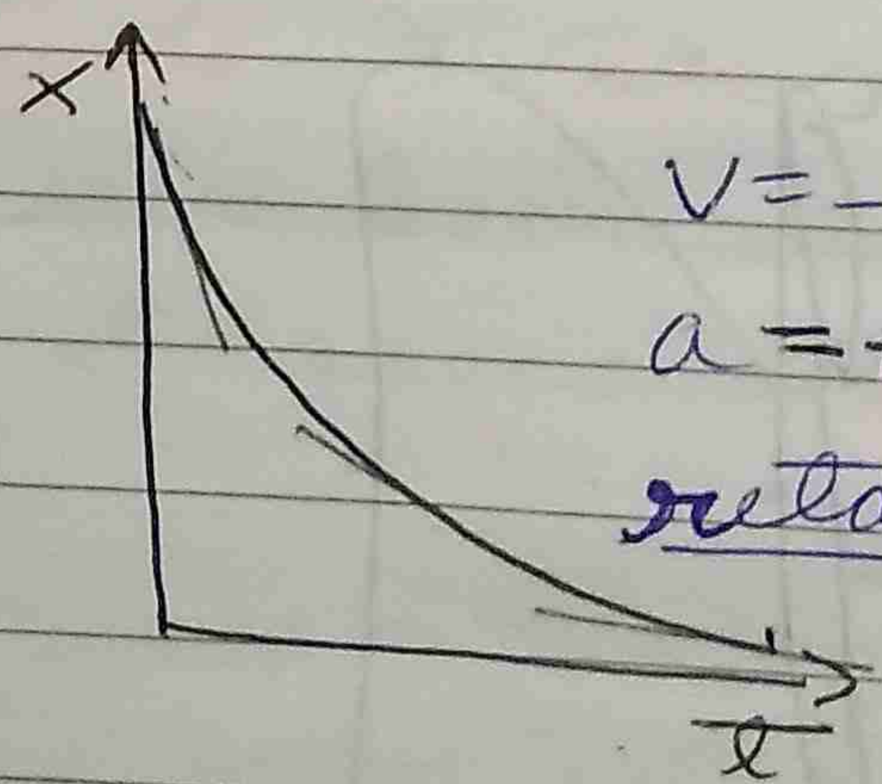


v = -ve
a = 0
uniform motion

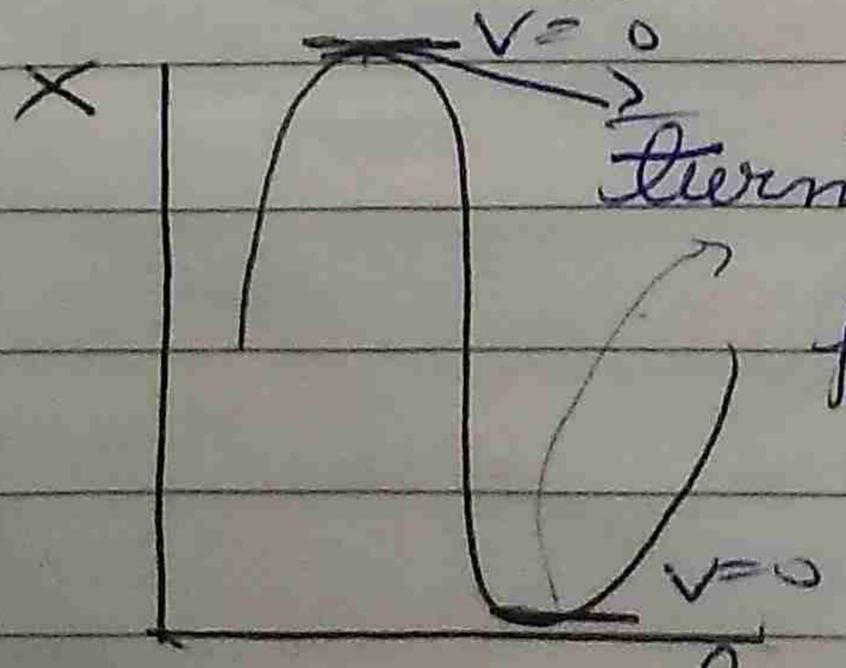
e



v = -ve
a = -ve
acceleration

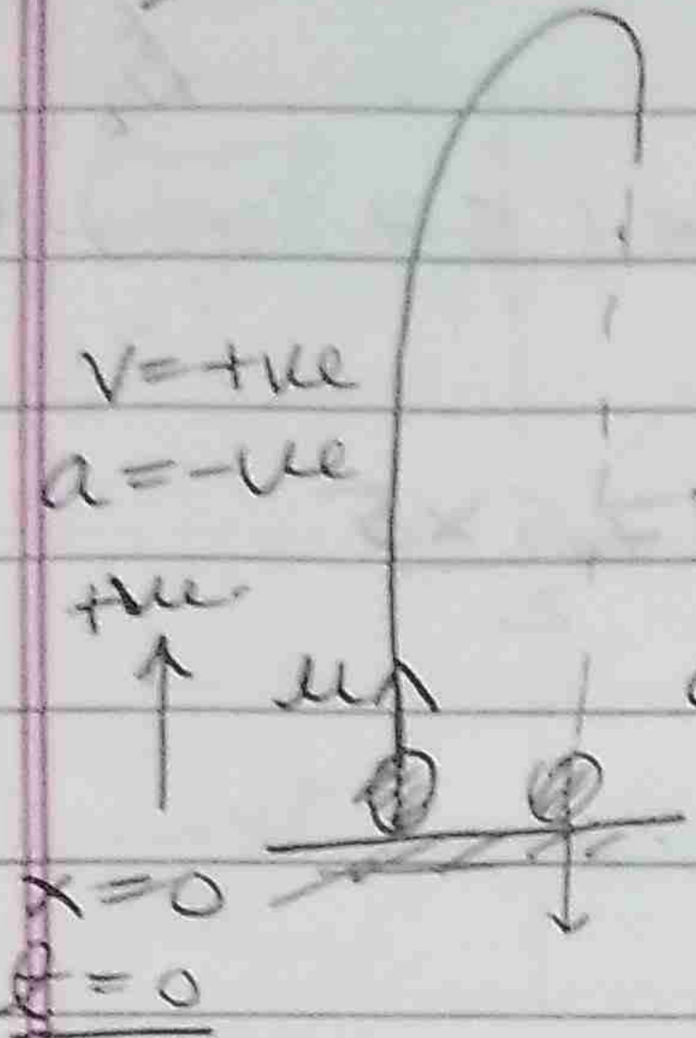


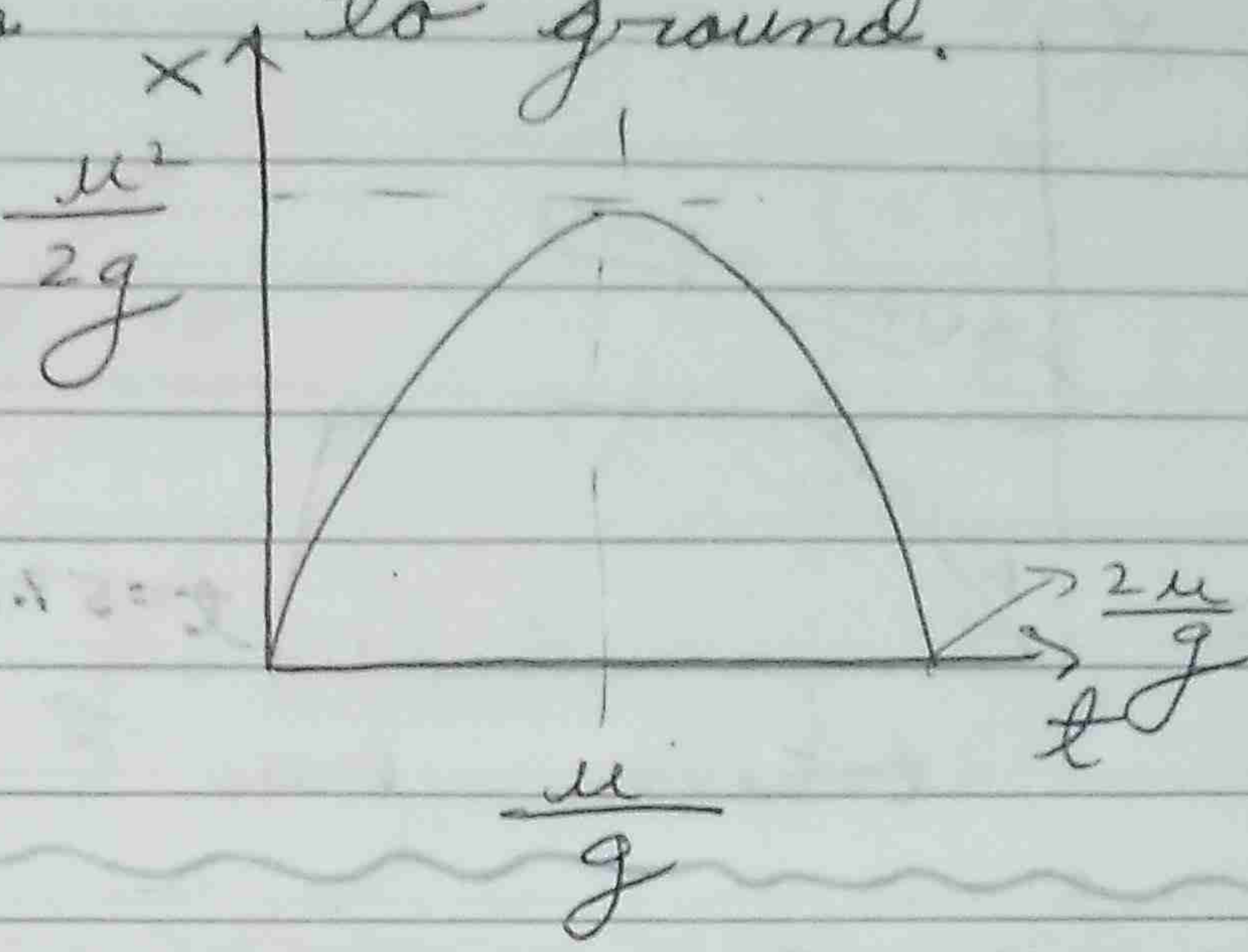
v = -ve
a = +ve
retarded



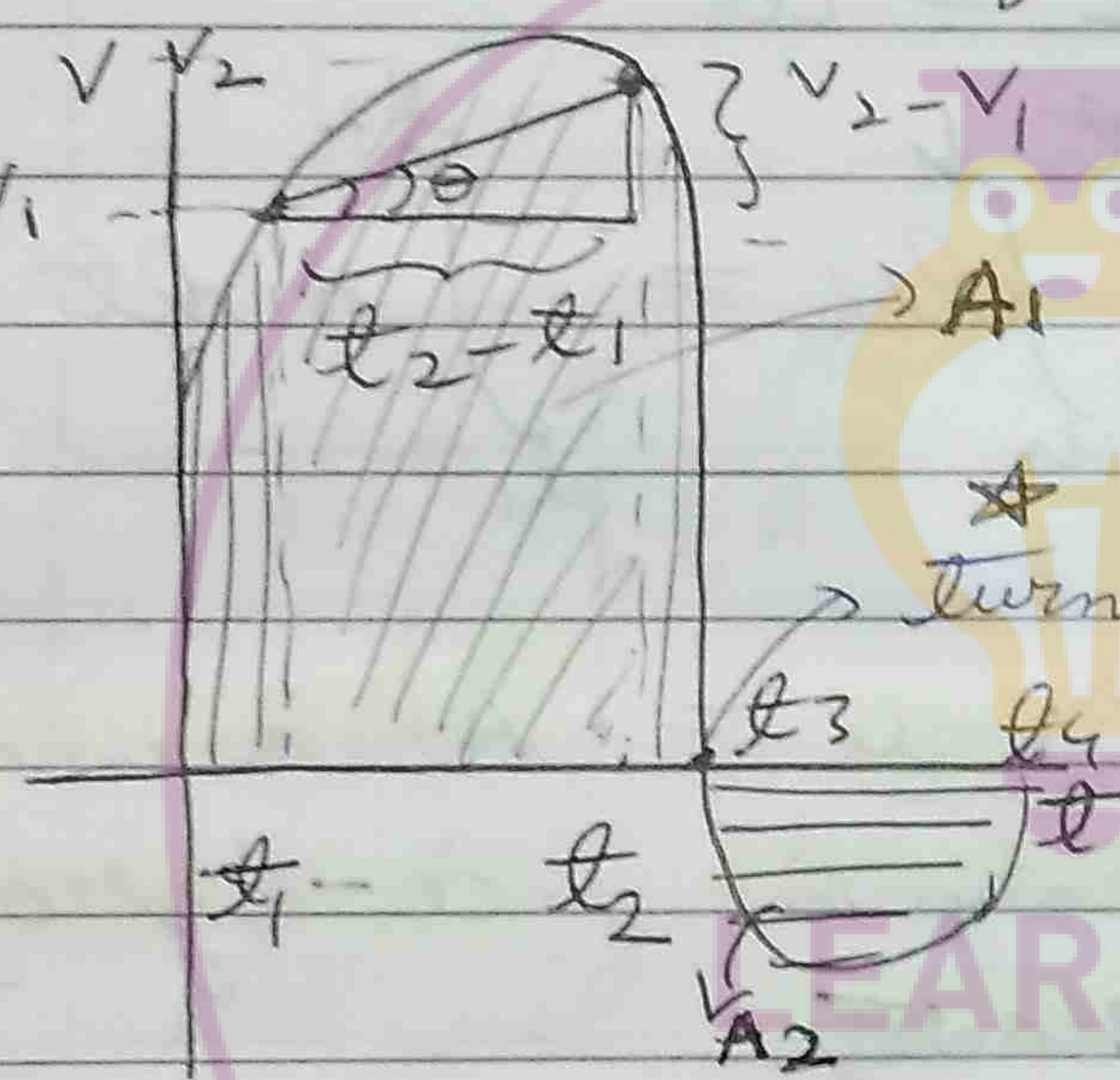
turning point

Ex → Plot x-t graph till the body returns to ground.





Velocity time graph (V−t)



· Average acceleration

$$a_{av} = \frac{V_2 - V_1}{t_2 - t_1} = \tan\theta$$

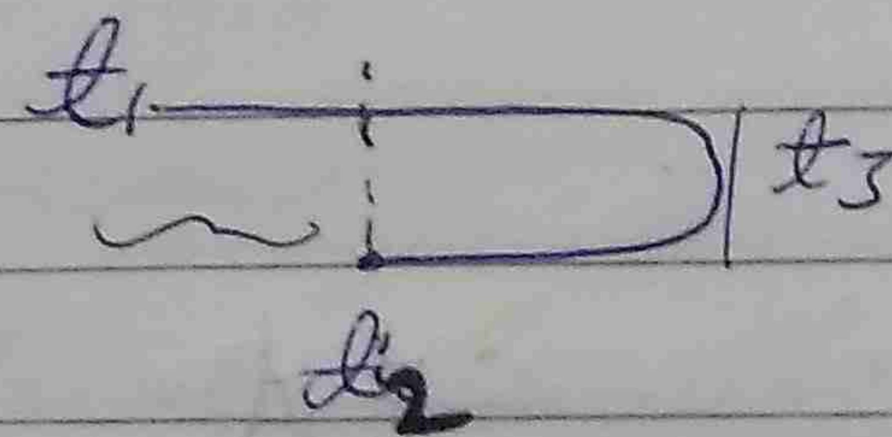
☆ turning point = slope of chord.

· Instantaneous acceleration

a = slope of tangent

· **Displacement** ⇒ $x_2 - x_1$

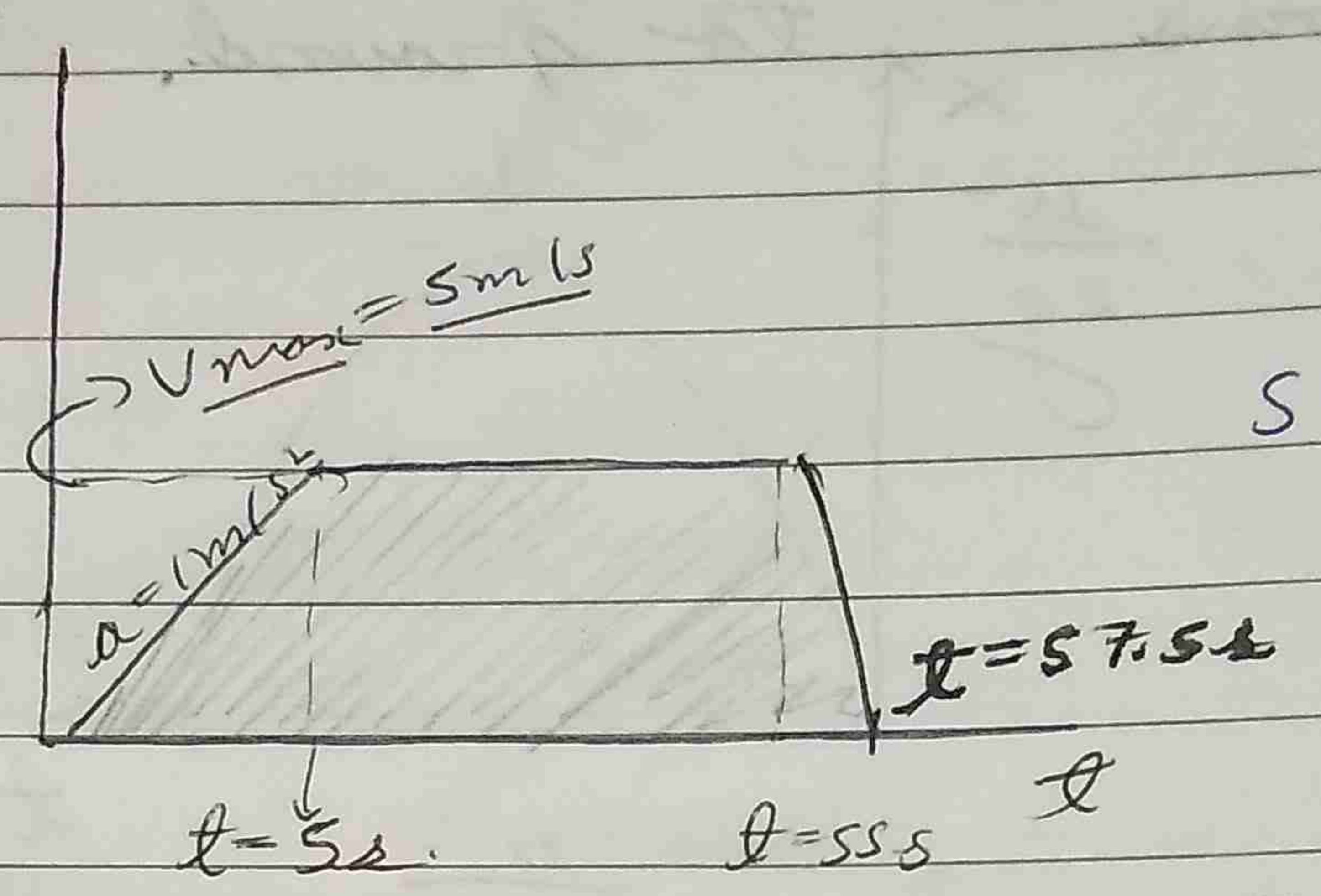
$$V = \frac{dx}{dt} \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt \Rightarrow \underbrace{x_2 - x_1}_{\text{disp.}} = \text{Area under V-t graph}$$



displacement = $A_1 - A_2$
distance = $A_1 + A_2$

Q Motion of a train

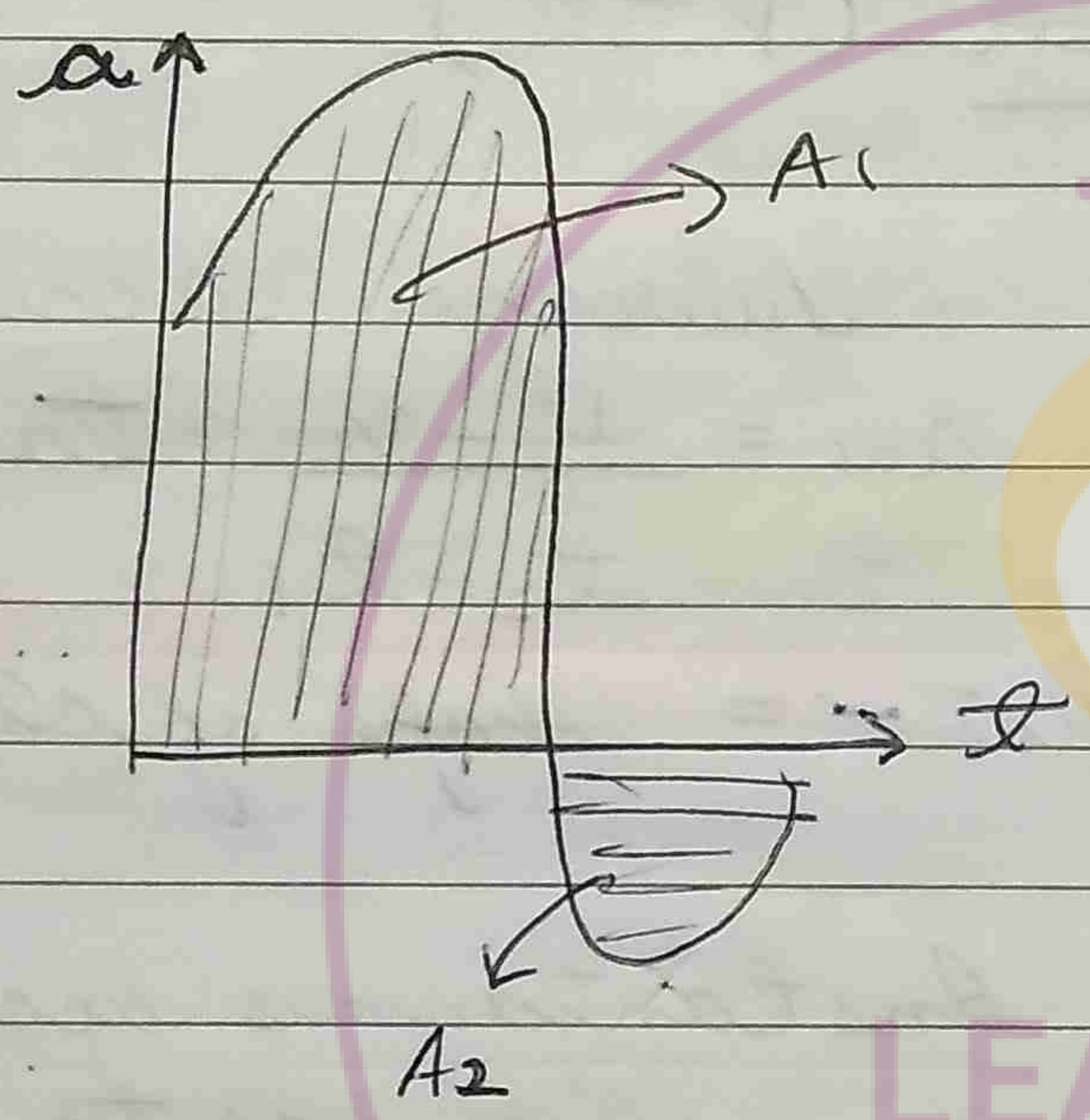
acceleration ±1 m/s² till t_1 = 5 s; Then moves under constant velocity for 5 s then retards at a rate 2 m/s to come to rest. Distance travel



$s = \frac{1}{2} \times (57.5 + 50) \times 5$

$s \Rightarrow \frac{107.5 \times 5}{2}$

- **Accleration - time (a-t)**



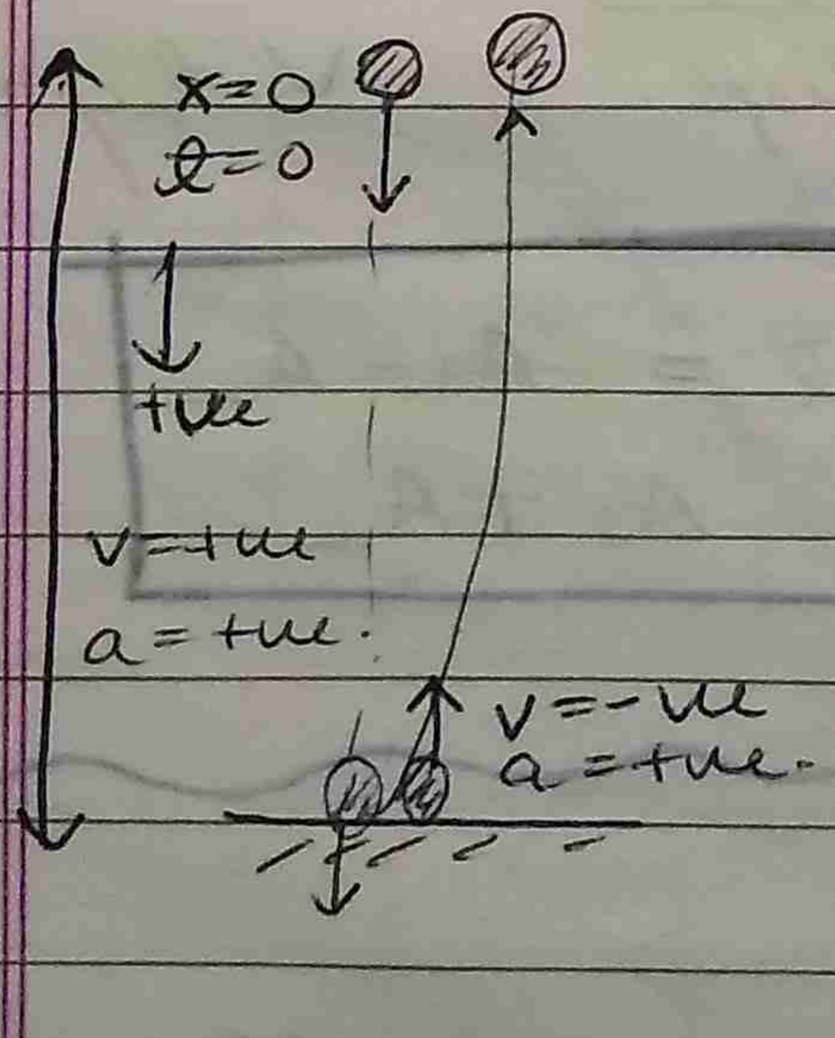
$\frac{dv}{dt} = a$

$\Rightarrow \int_{u}^{v} dv = \int_{t_1}^{t_2} a\,dt$

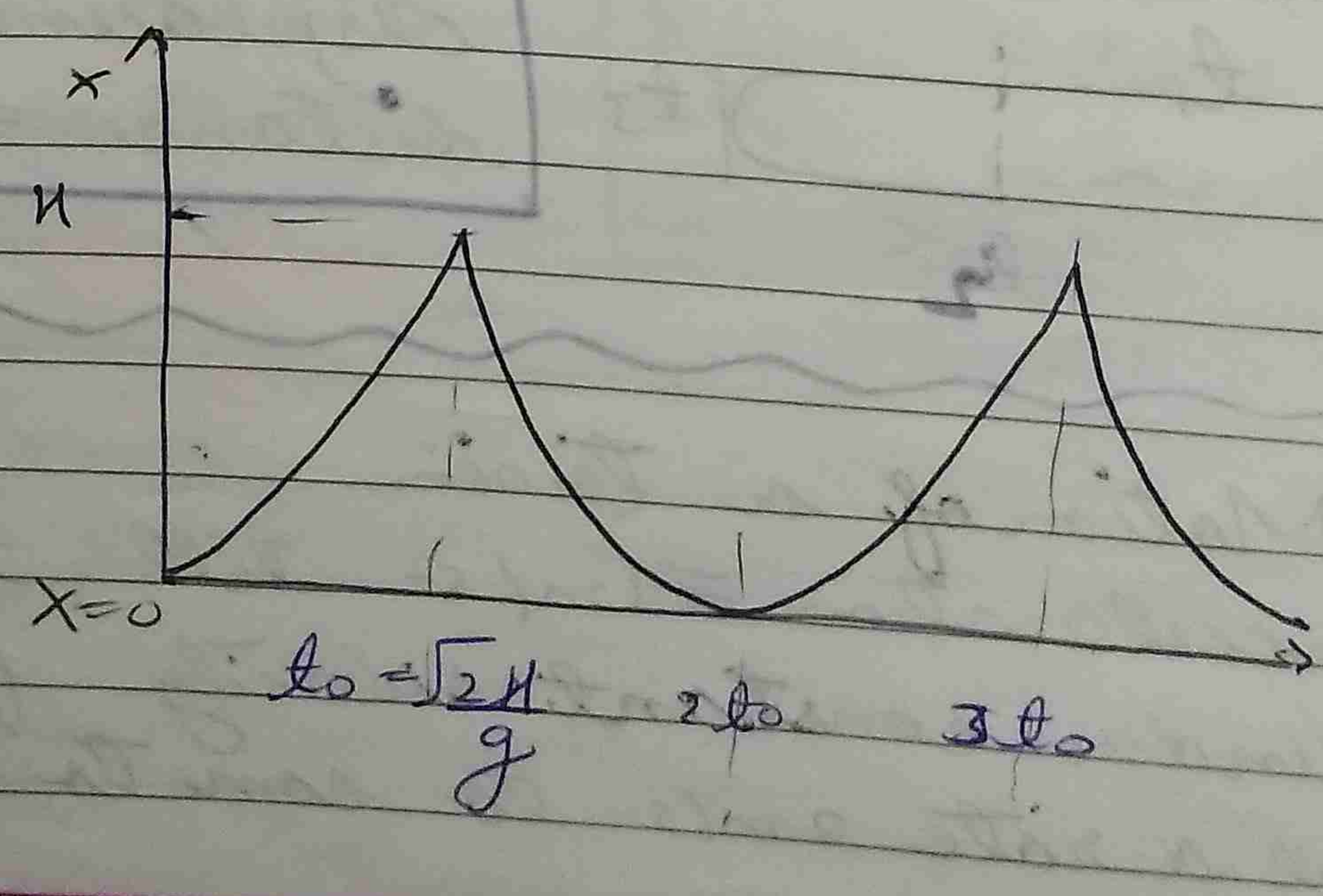
$\Rightarrow v - u$ = Area under a-t graph
★ change in velocity

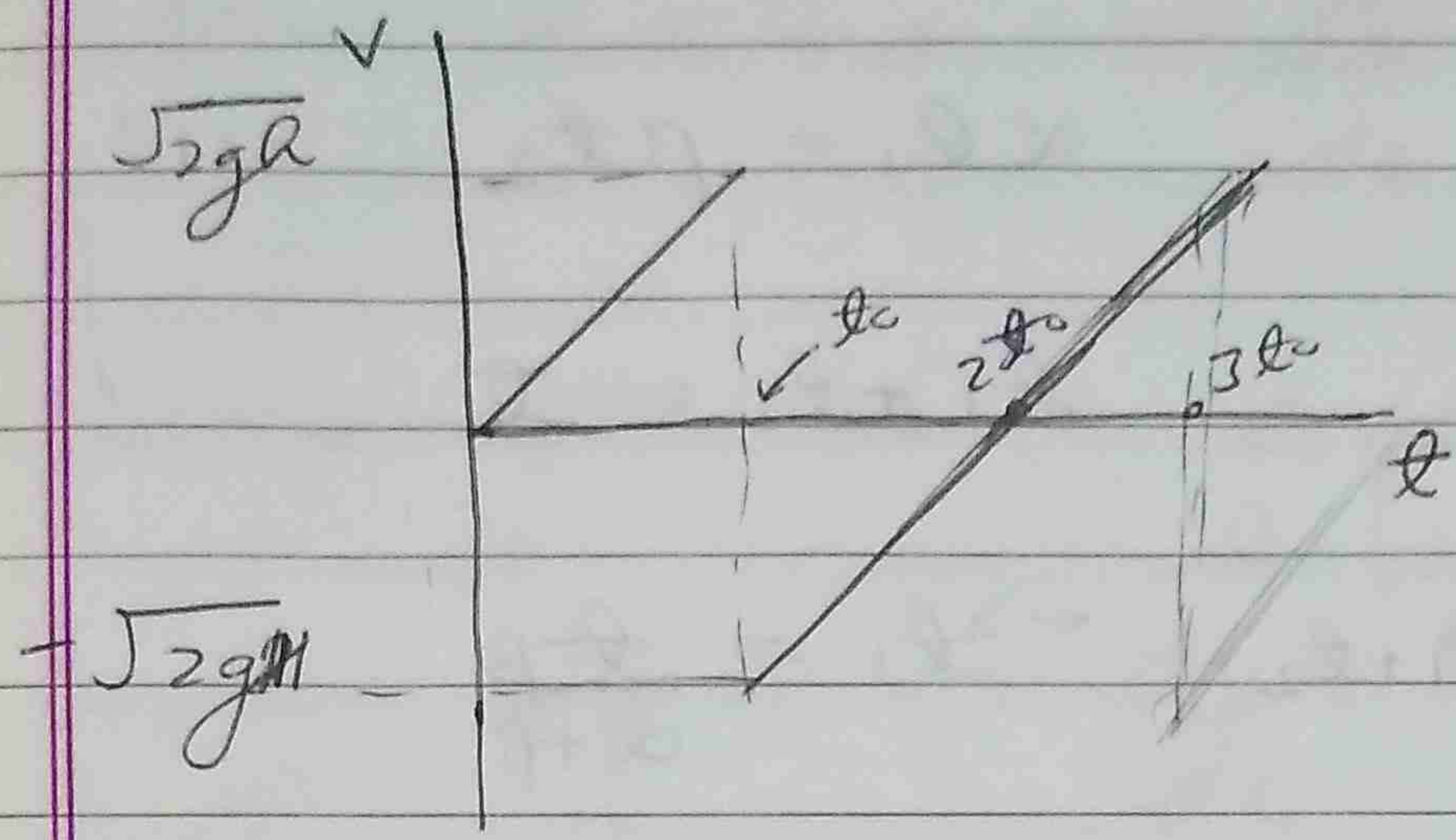
$\Rightarrow$ change in velocity $= A_1 - A_2$

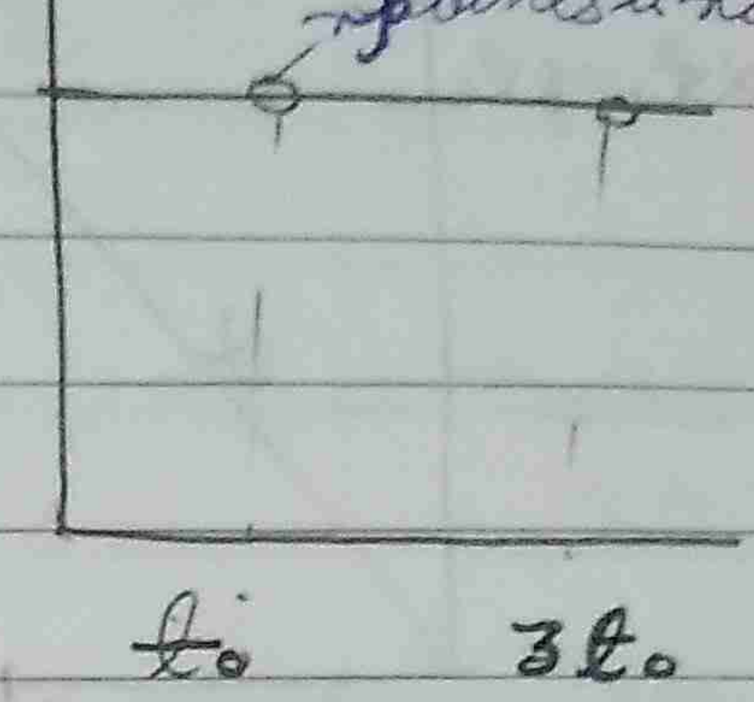
Ex



- Plot x-t, v-t, at graphs

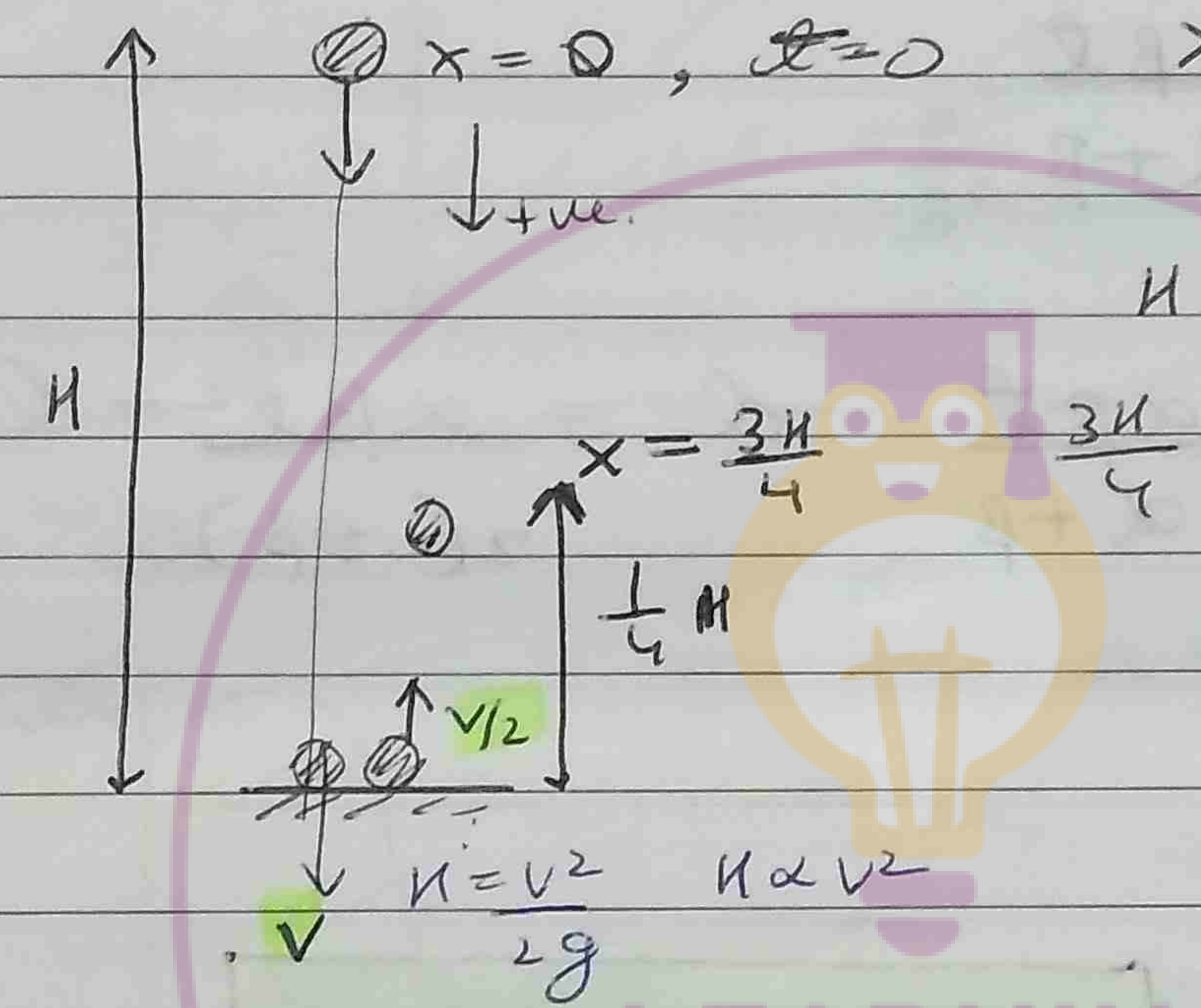


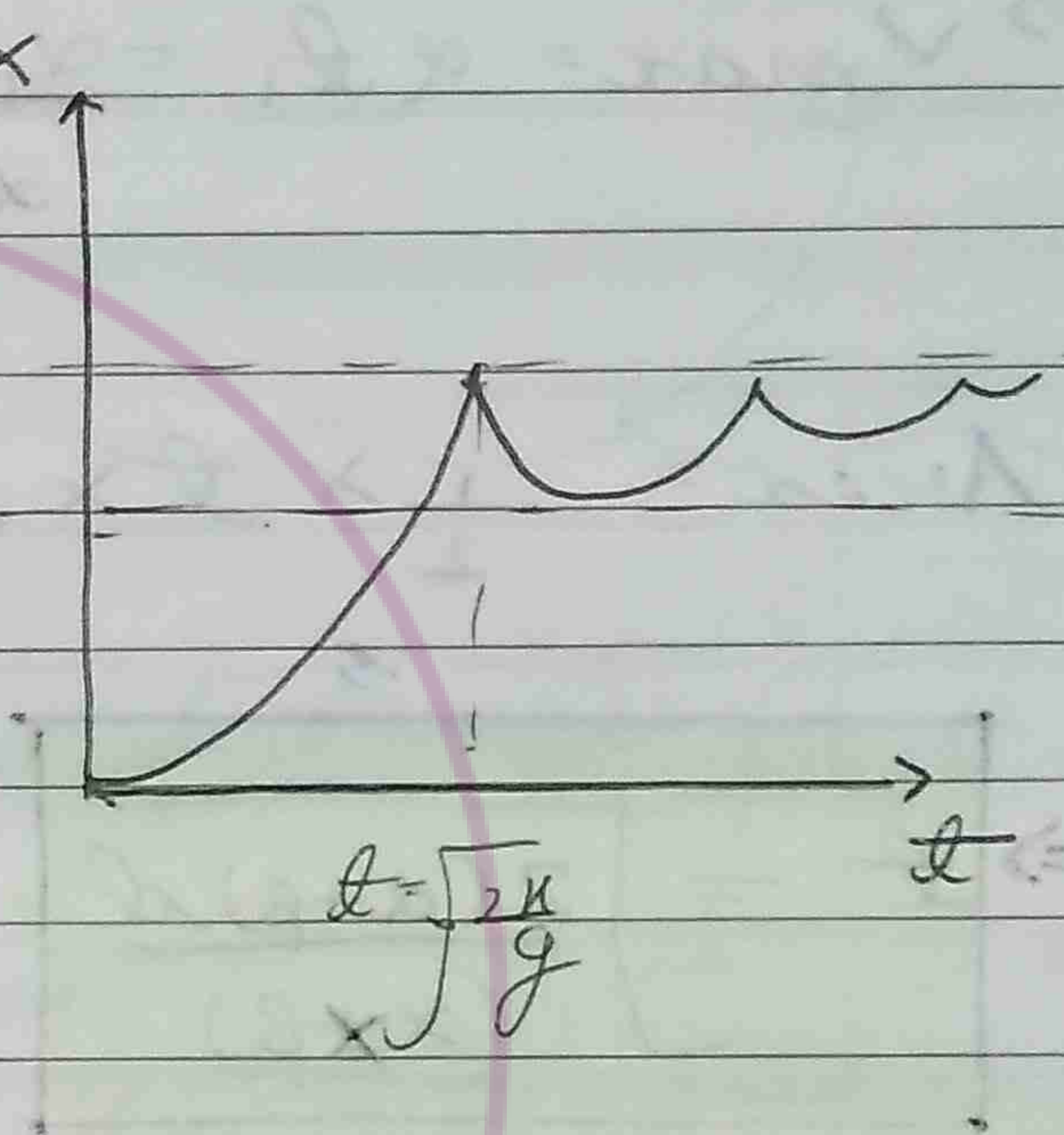


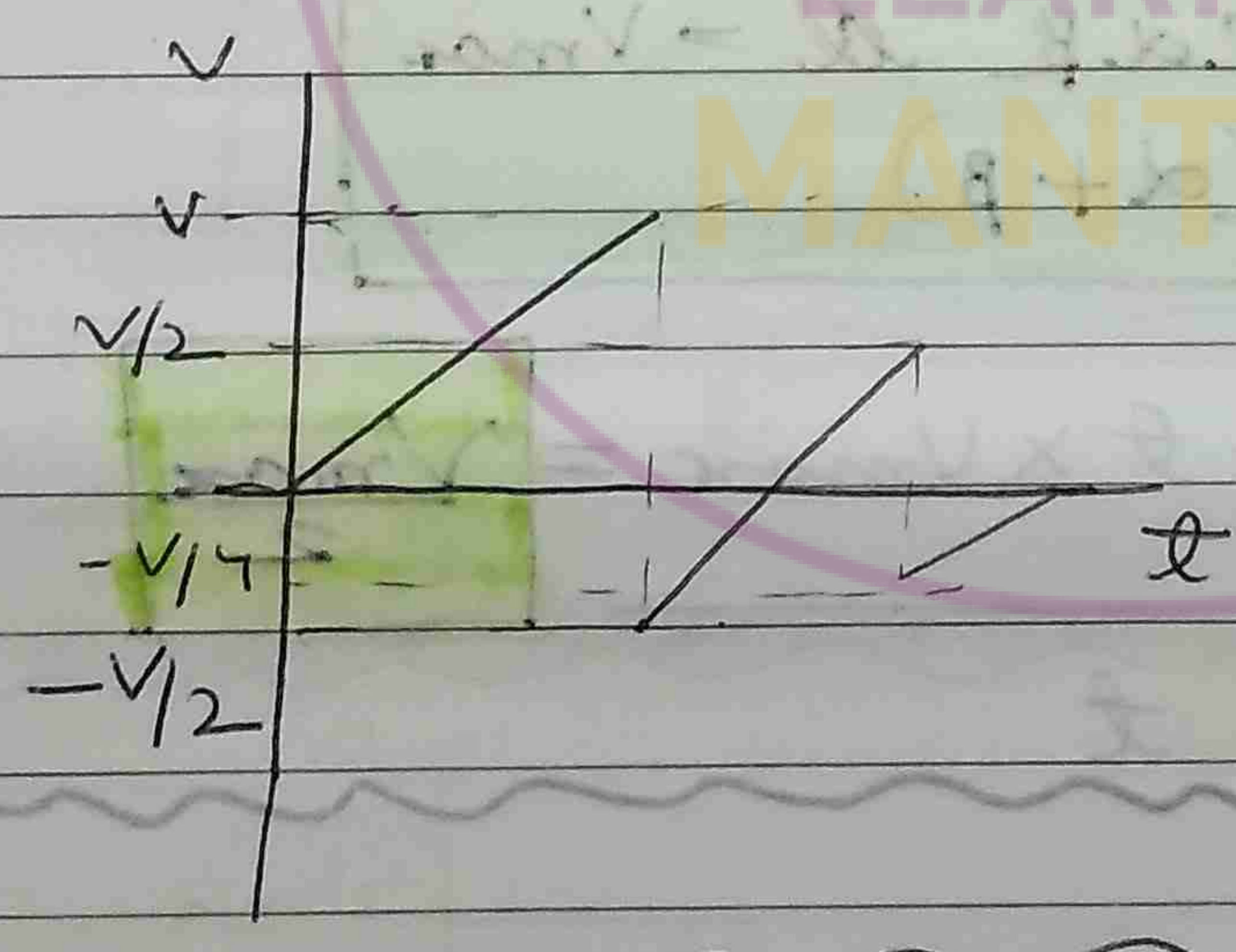


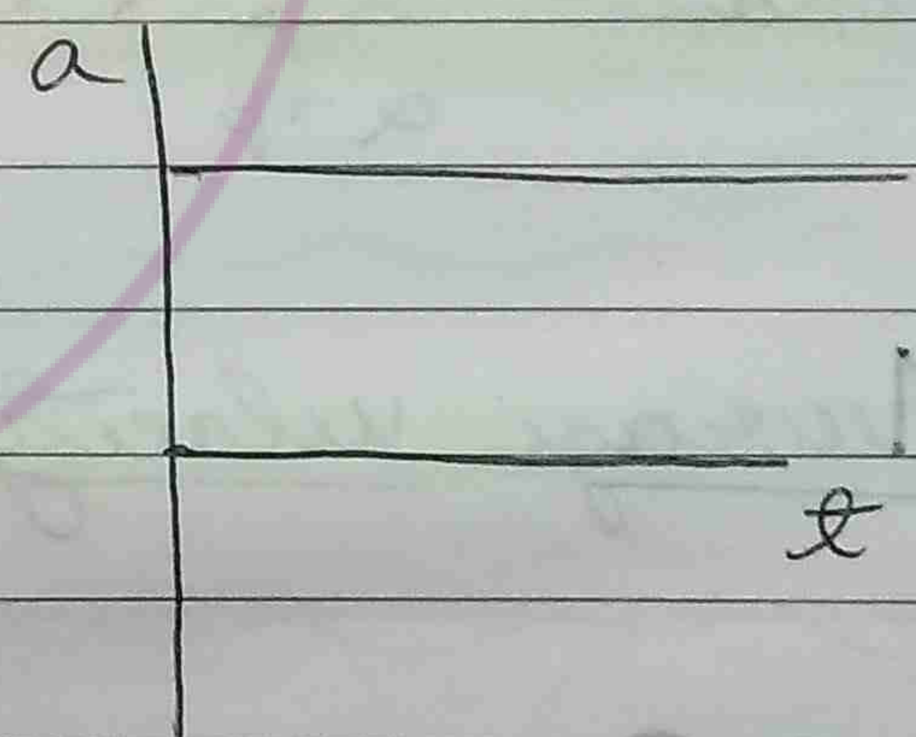
acceleration at these points are not define

Ex



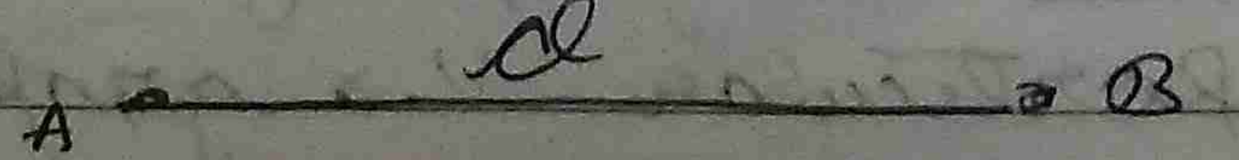






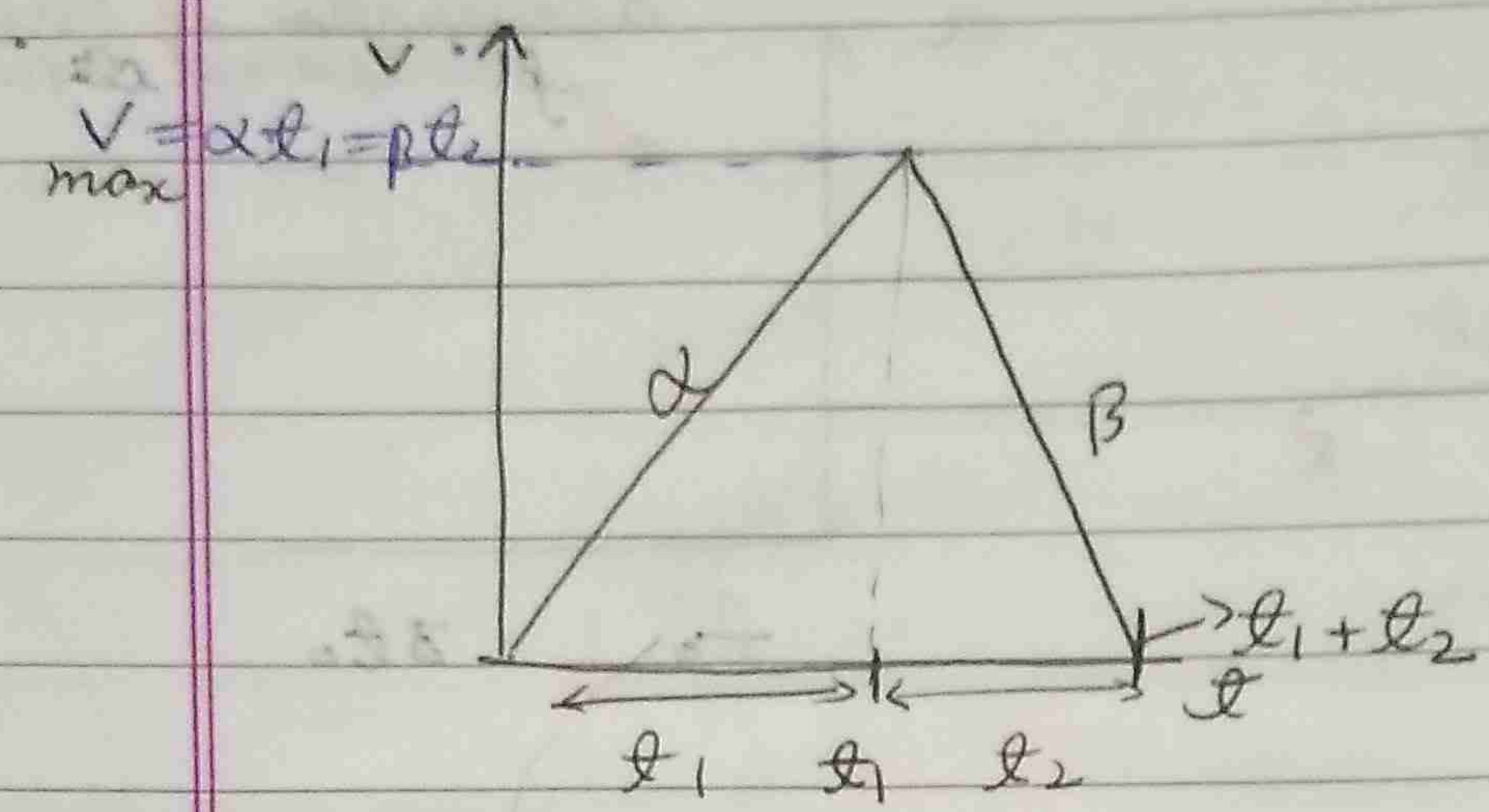
Q α = acceleration
β = retardation

Minimum time in which object moves from A to B?



↓ for that body should not move with constant velocity

V_{av} is half of V_{max} and they ...
β.



$\alpha t_1 = \beta t_2$ --- (i)

$t_1 + t_2 = t$

$\Rightarrow t_1 = \frac{t\beta}{\alpha+\beta}$

$t_2 = \frac{\alpha t}{\alpha+\beta}$

$\Rightarrow v_{max} = \alpha t_1 = \frac{\alpha \beta t}{\alpha+\beta}$

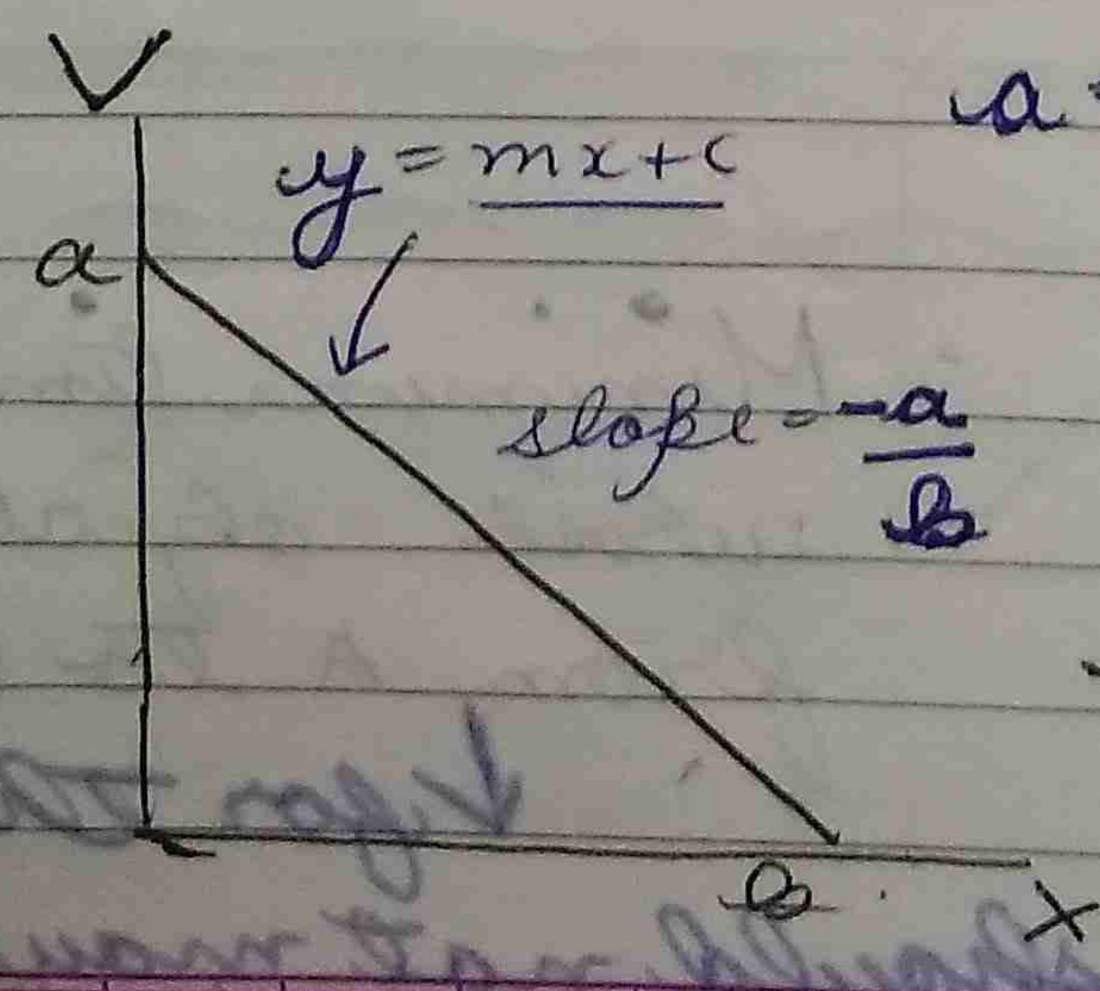
$Area = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta} = d = \frac{\alpha\beta t^2}{2(\alpha+\beta)} = d$

$$\Rightarrow t = \sqrt{\frac{2(\alpha+\beta) d}{(\alpha * \beta)}}$$

$$v_{max} = \frac{\alpha\beta}{\alpha+\beta} \times t = \sqrt{\frac{2\alpha\beta\, d}{(\alpha+\beta)}} = v_{max}$$

- Average velocity = $\frac{\frac{1}{2} \times t \times v_{max}}{t} = \frac{v_{max}}{2}$

- V-x graph



$a = \frac{dv}{dt} = v \frac{dv}{dx}$ (slope of v-x graph)

draw a-x graph for this particular v.x graph.

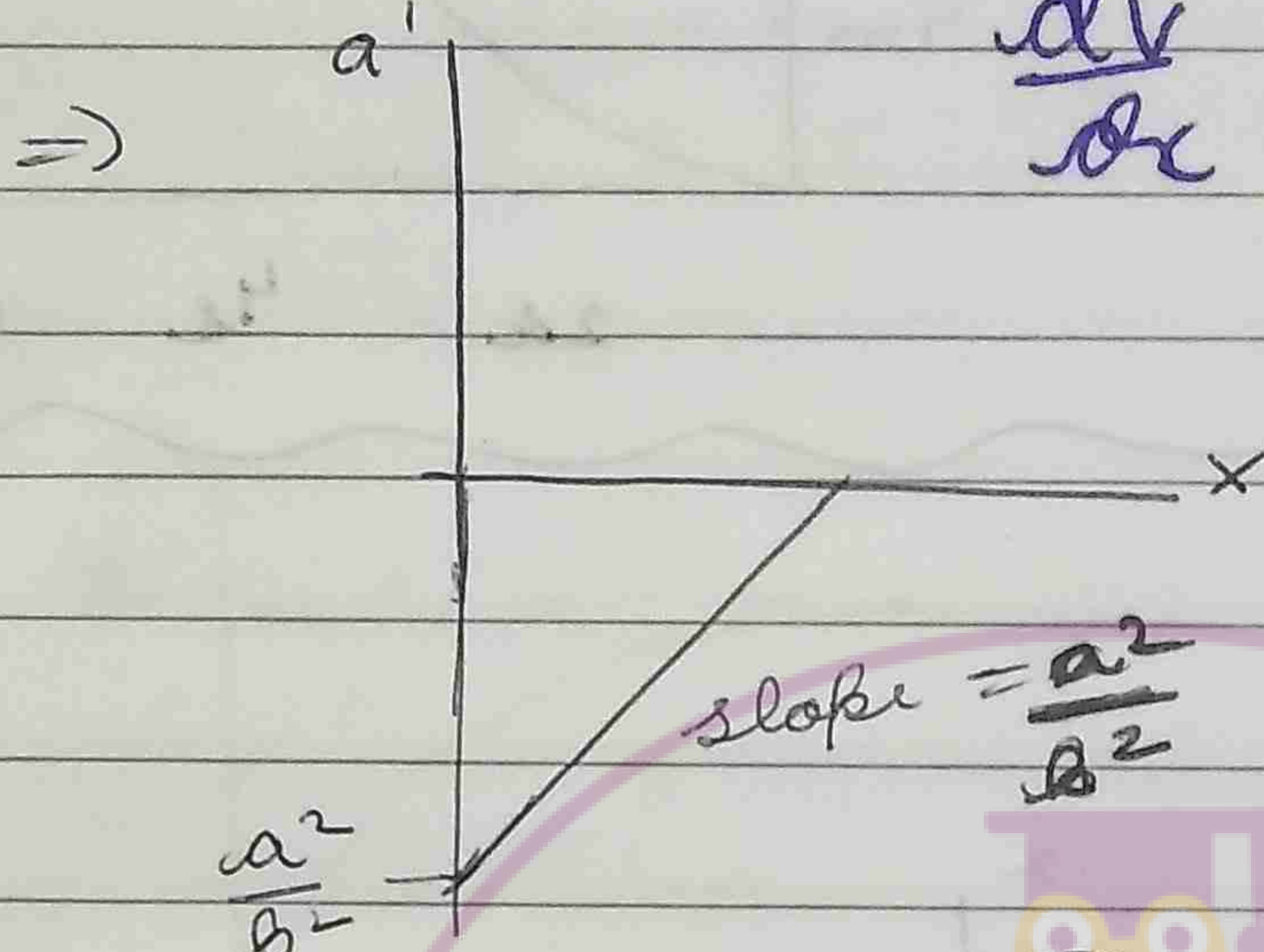
$V = -\frac{a}{b}x + a \Rightarrow \frac{dv}{dx} = -\frac{a}{b}$

acceleration $a' = \underbrace{-\frac{a}{b}}_{\frac{dv}{dx}}\underbrace{\left(-\frac{a}{b}x + a\right)}_{v} \Rightarrow a' = \frac{a^2}{b^2}x - \frac{a}{b}$

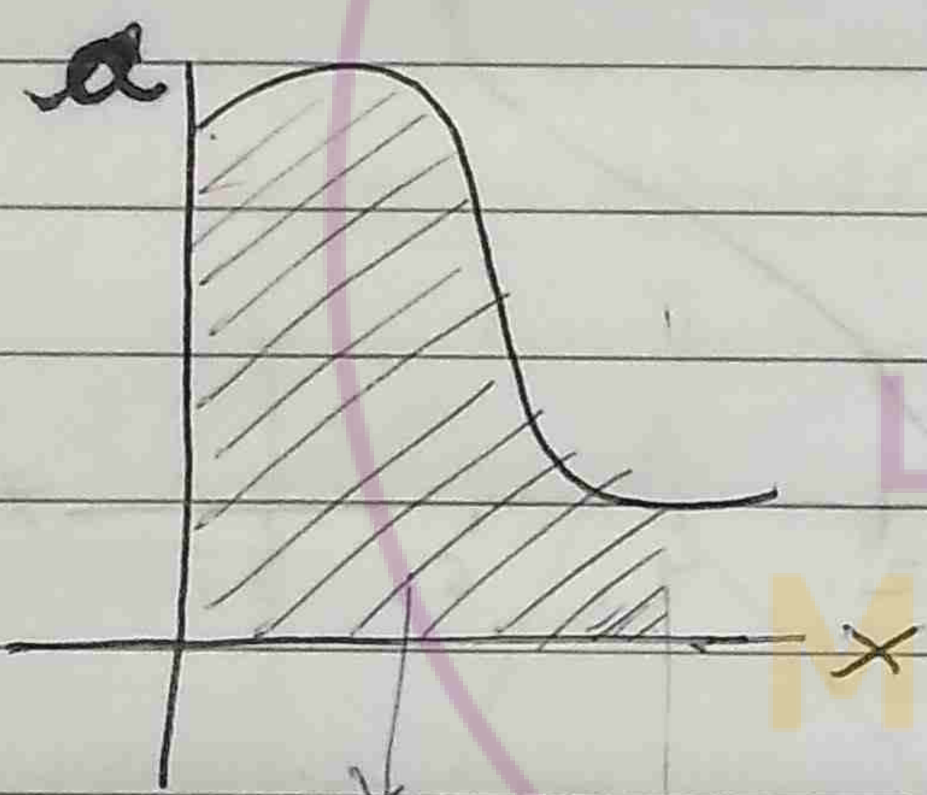
$(y = mx - c)$

$(\because a = v\frac{dv}{dx})$

$\Rightarrow$



• $a - x$



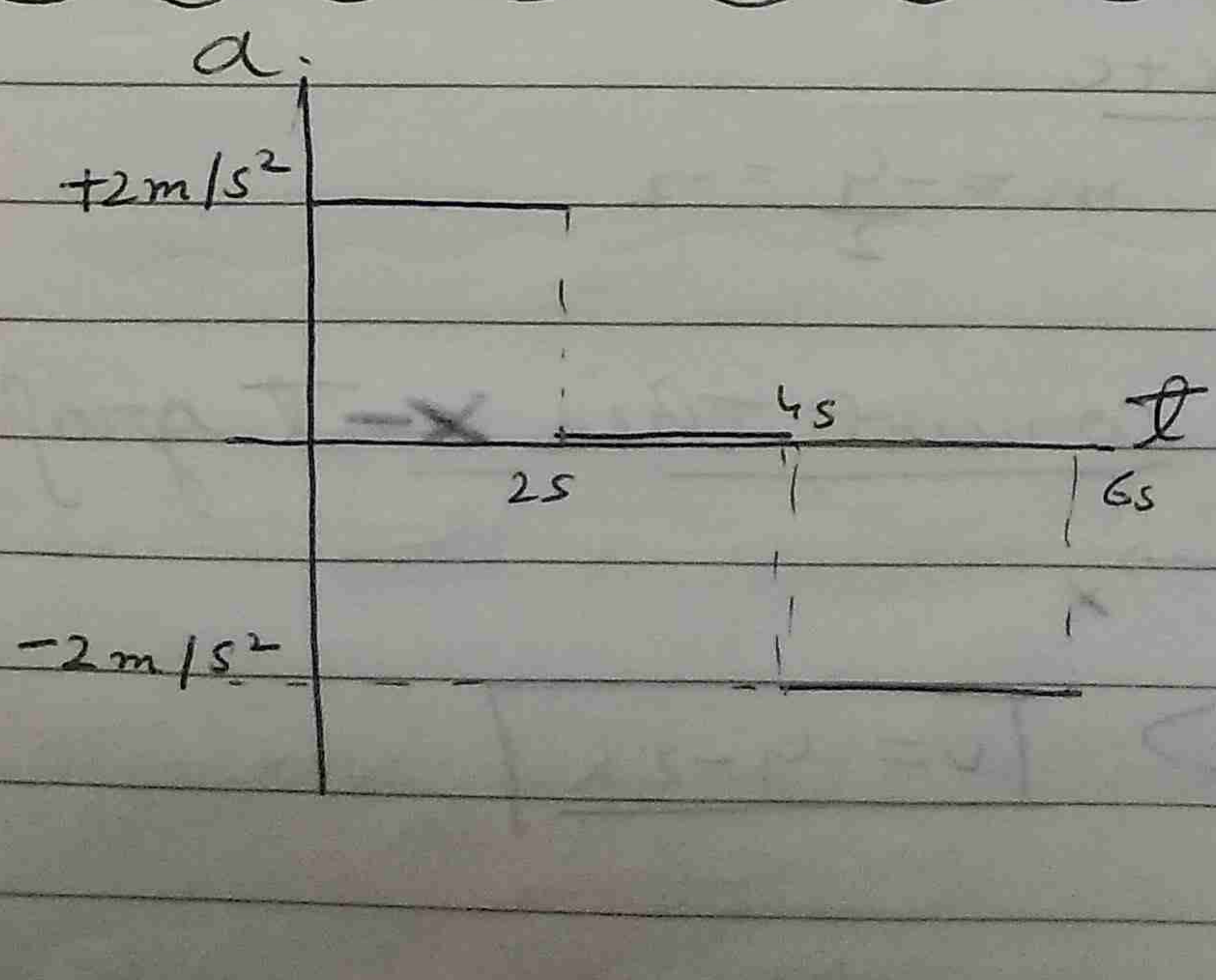
$a = v\frac{dv}{dx} \Rightarrow \int_u^v v\,dv = \int_{x_1}^{x_2} a\,dx$

$\Rightarrow \frac{v^2}{2} - \frac{u^2}{2}$ = area under a-x graph.

Area $= \frac{v^2}{2} - \frac{u^2}{2}$

$V = - - - - -$

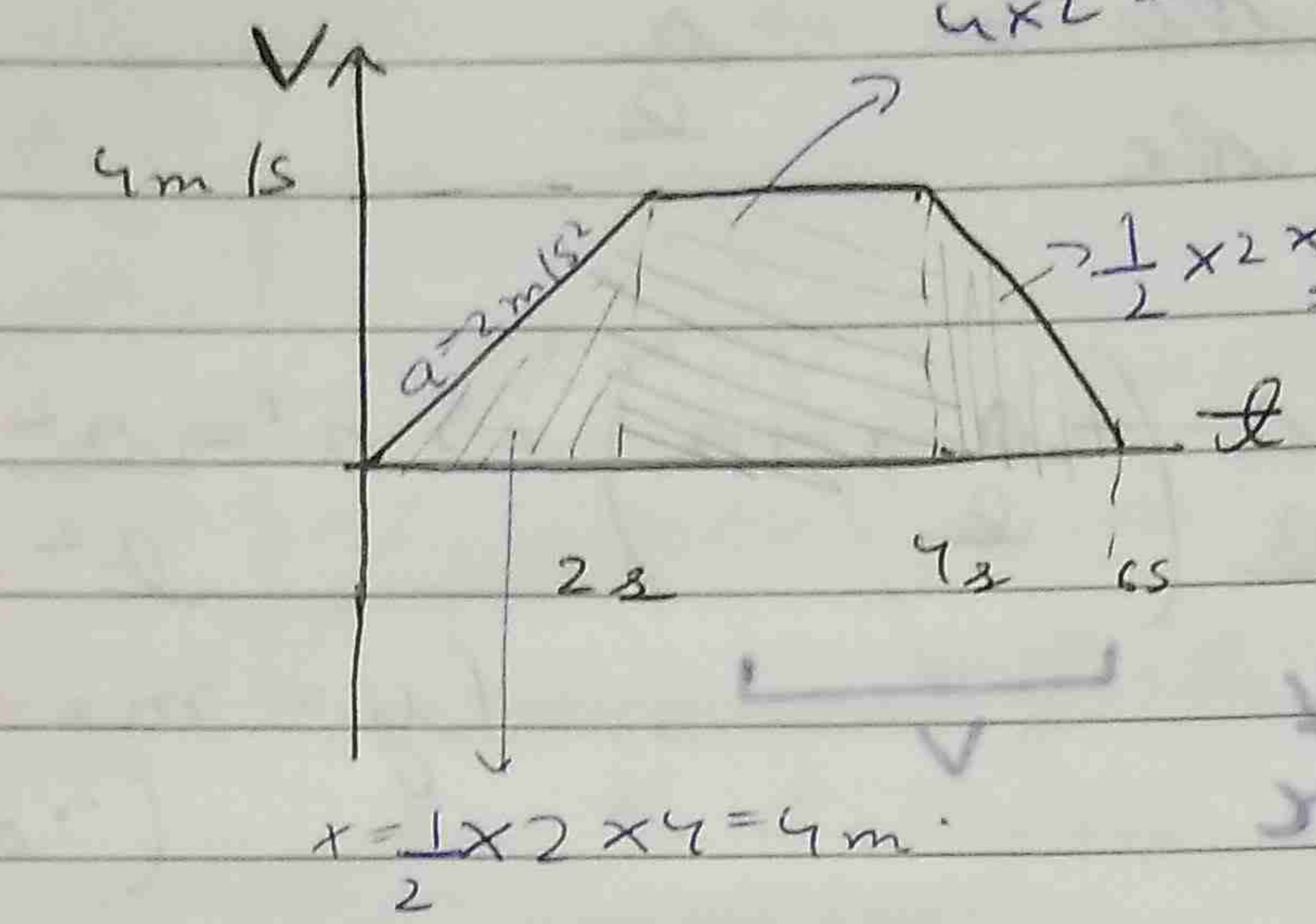
Ex

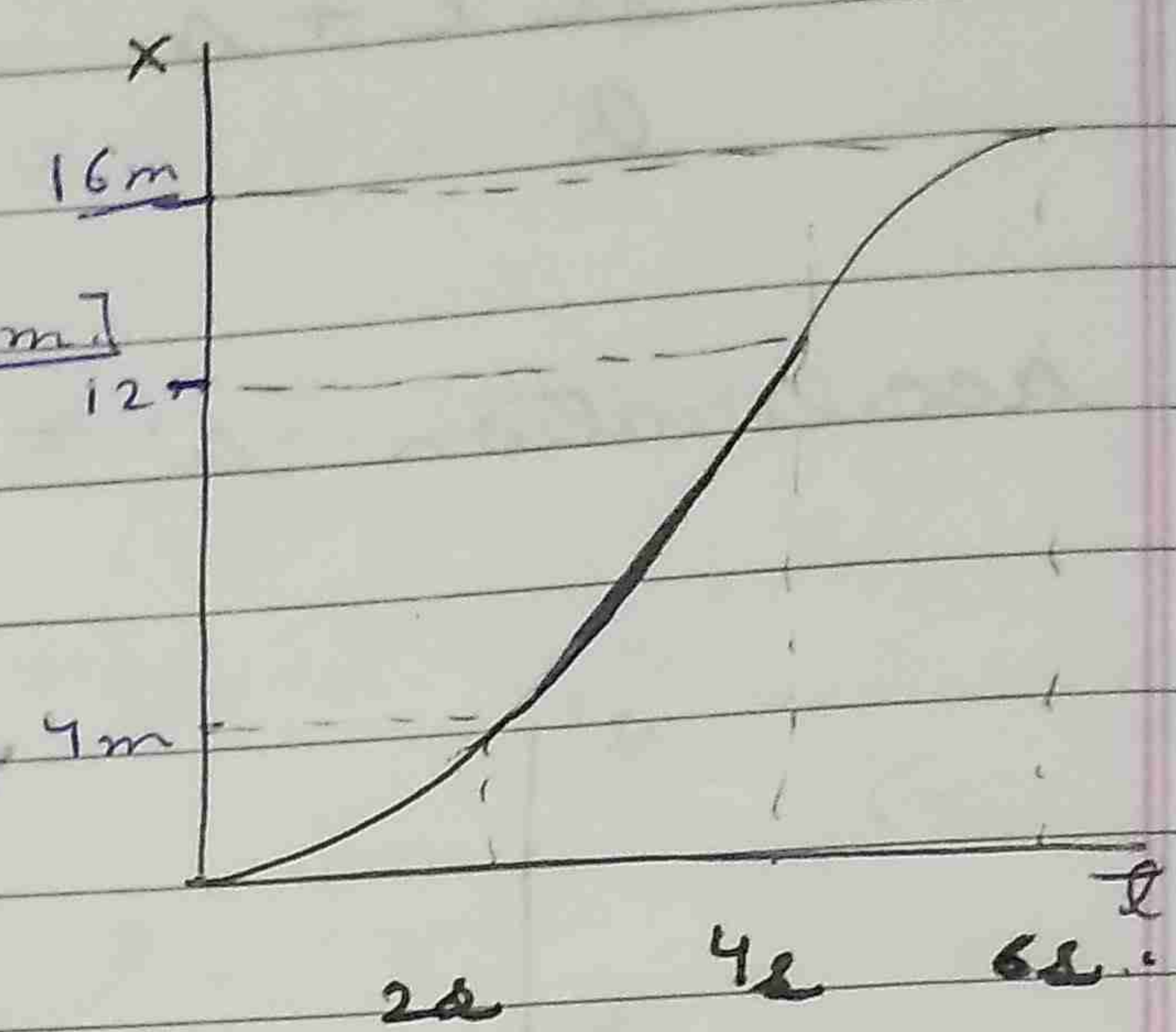


• At $t = 0$
$u = 0$
$x = 0$ } given

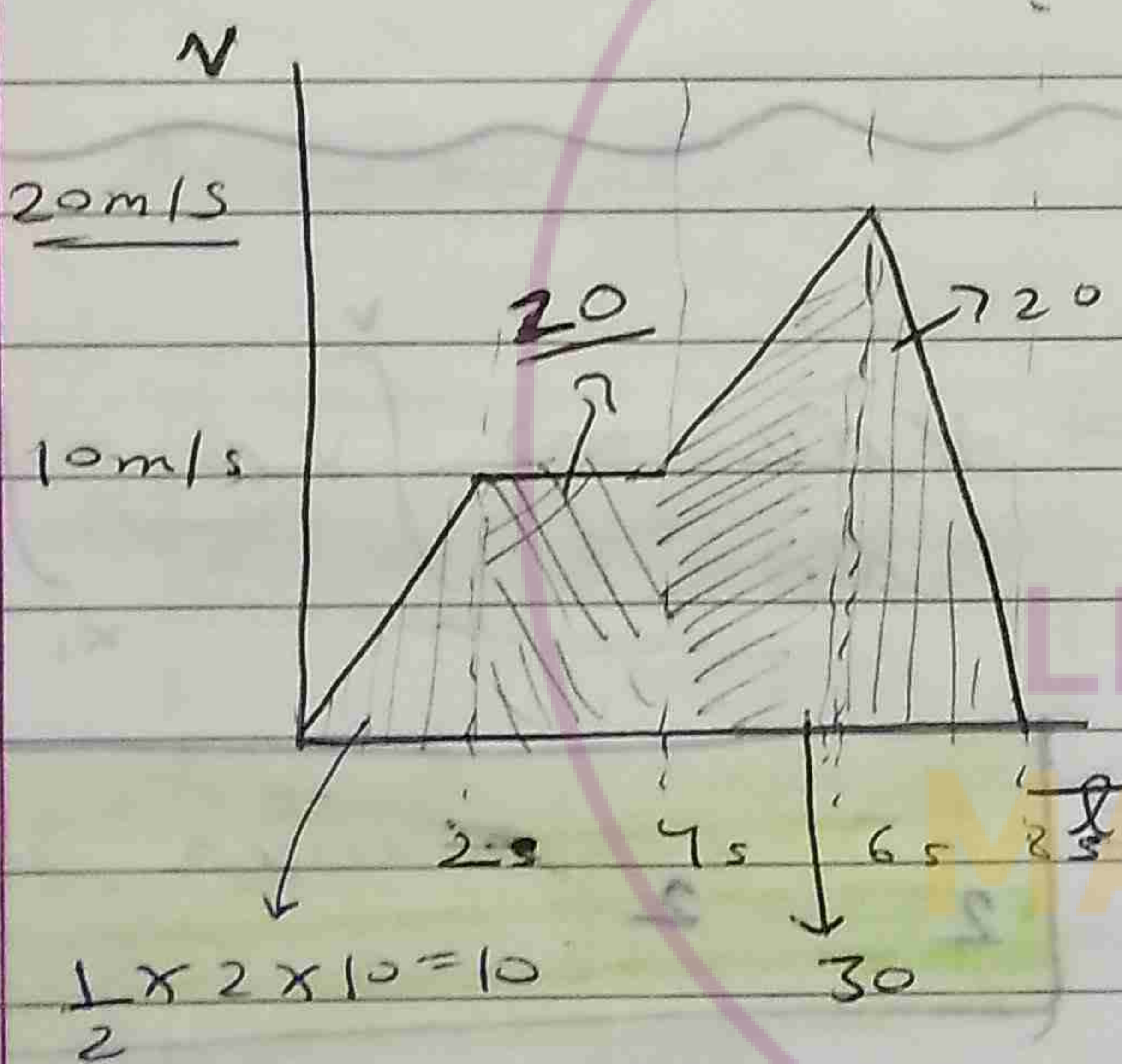
draw v-t and x-t graph.

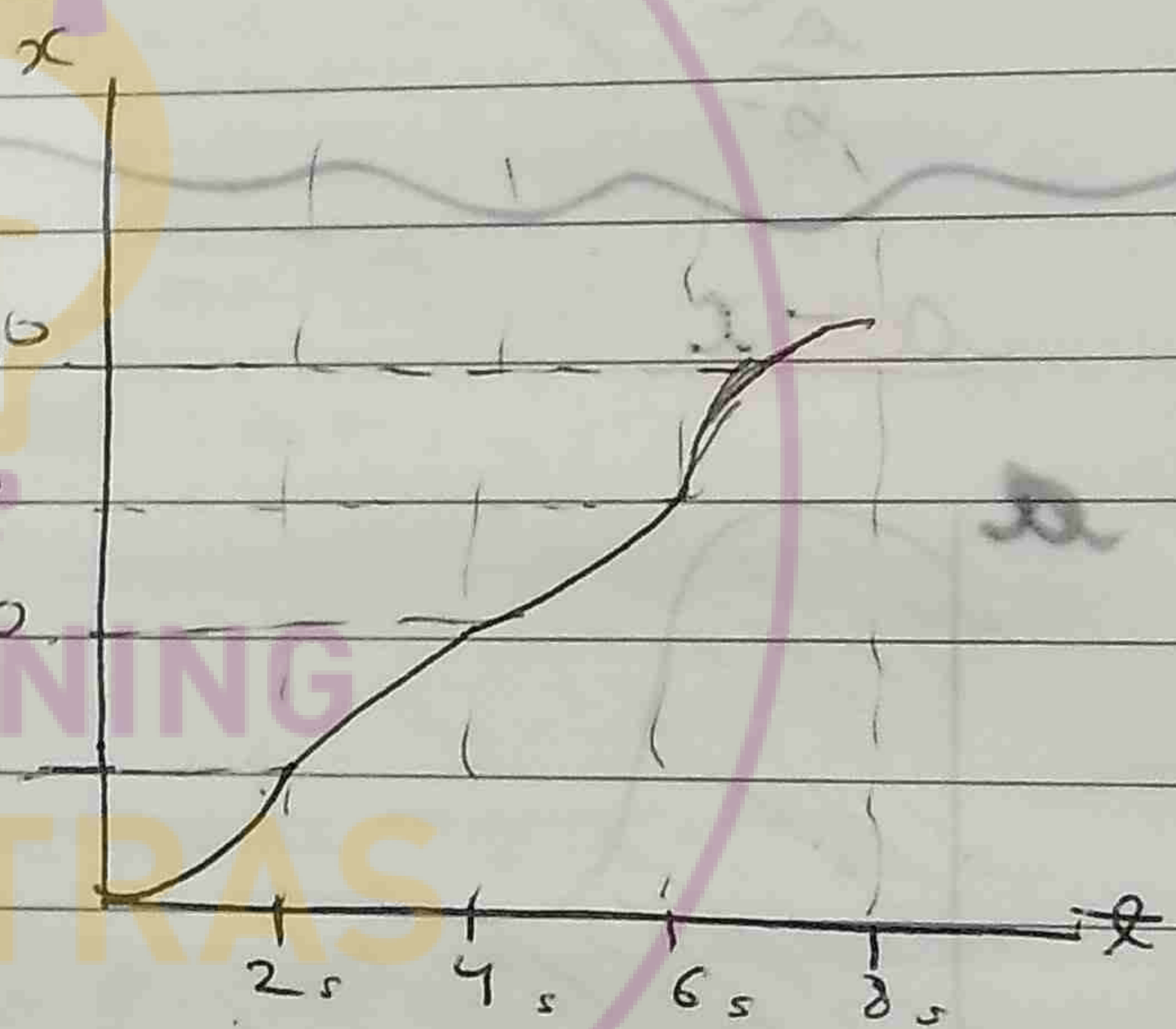
• In v–t graph velocity above the axis is positive and below is –ve.



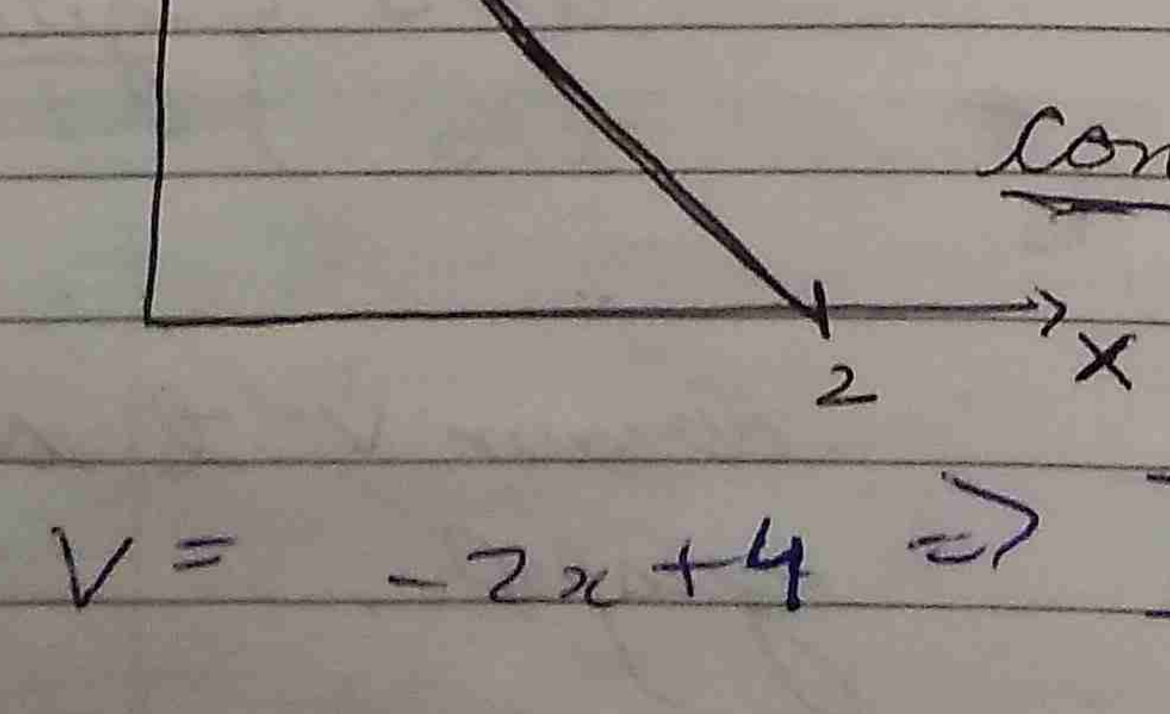


• **V–t graph**





Ex



straight line

$y = mx + c$

$c = 4 \quad m = -\frac{4}{2} = -2$

At $t = 0$, $x = 0$

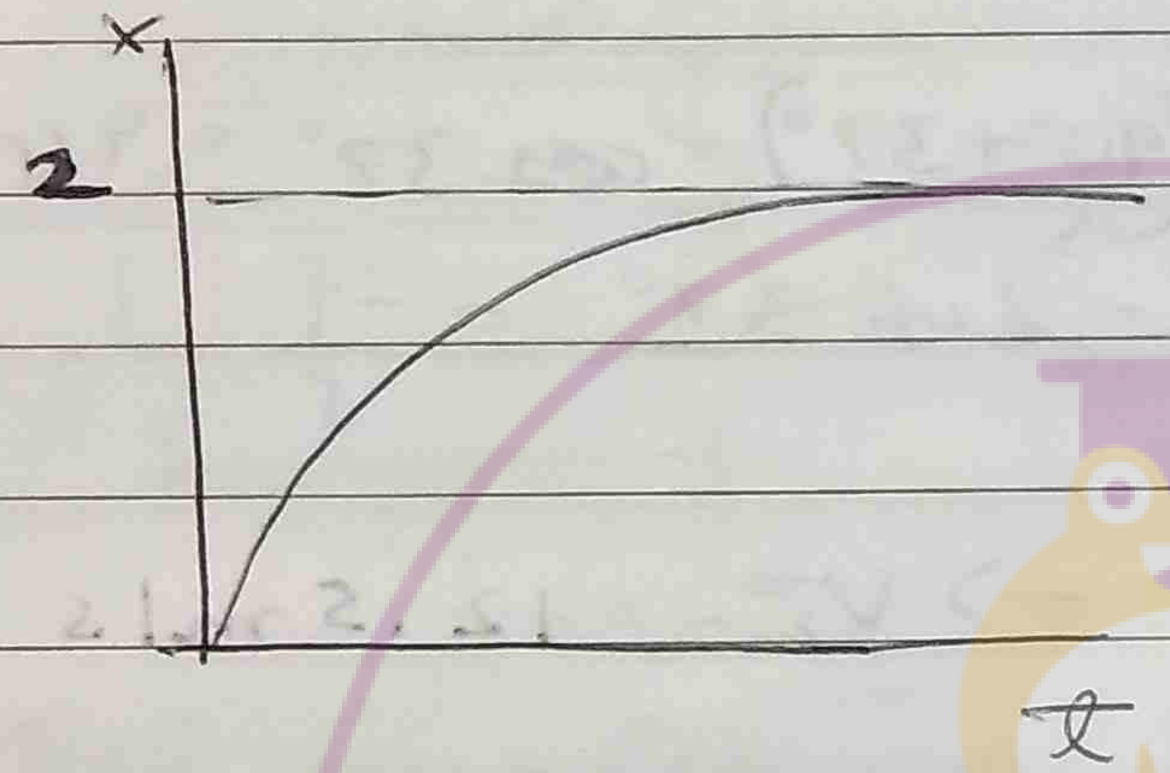
Convert this x–t graph

$V = -2x + 4 \Rightarrow \boxed{v = 4 - 2x}$

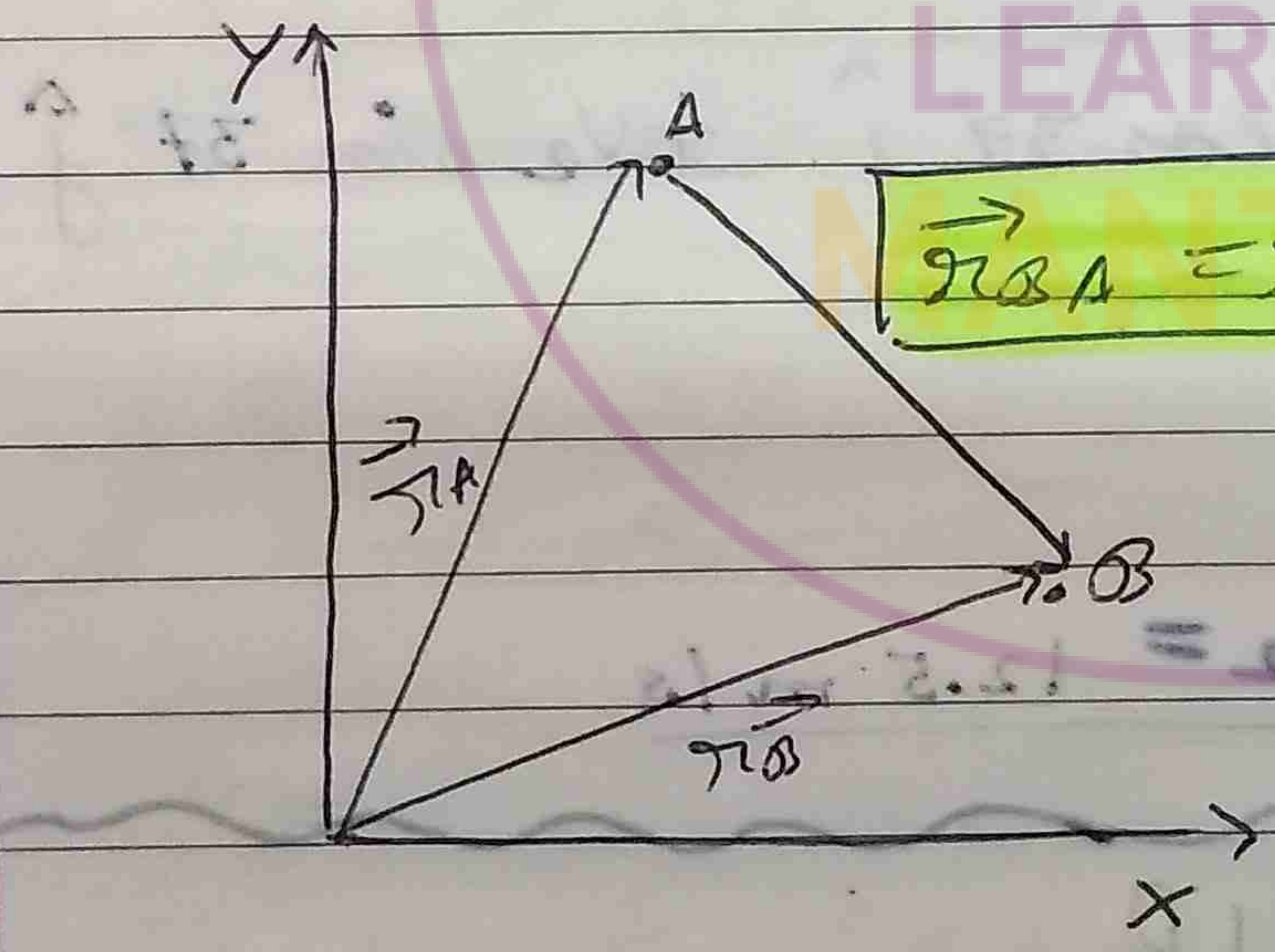
$$\frac{dx}{dt} = 4 - 2x \;\Rightarrow\; \int_0^x \frac{dx}{4-2x} = \int_0^t dt$$

$$\Rightarrow \left[\frac{\ln(4-2x)}{-2}\right]_0^x = [t]_0^t \Rightarrow \ln\left(\frac{4-2x}{4}\right) = -2t$$

$$\Rightarrow \frac{4-2x}{4} = e^{-2t} \Rightarrow \boxed{x = 2(1-e^{-2t})}$$



- Relative Velocity



$$\boxed{\vec{r}_{BA} = \vec{r}_B - \vec{r}_A}$$ Position of B w.r.t. to A

$$\vec{V}_{BA} = \frac{d\vec{r}_{BA}}{dt} = \frac{d\vec{r}_B}{dt} - \frac{d\vec{r}_A}{dt}$$

$$\Rightarrow \vec{V}_{BA} = \vec{V}_{B,\text{ground}} - \vec{V}_{A,\text{ground}}$$

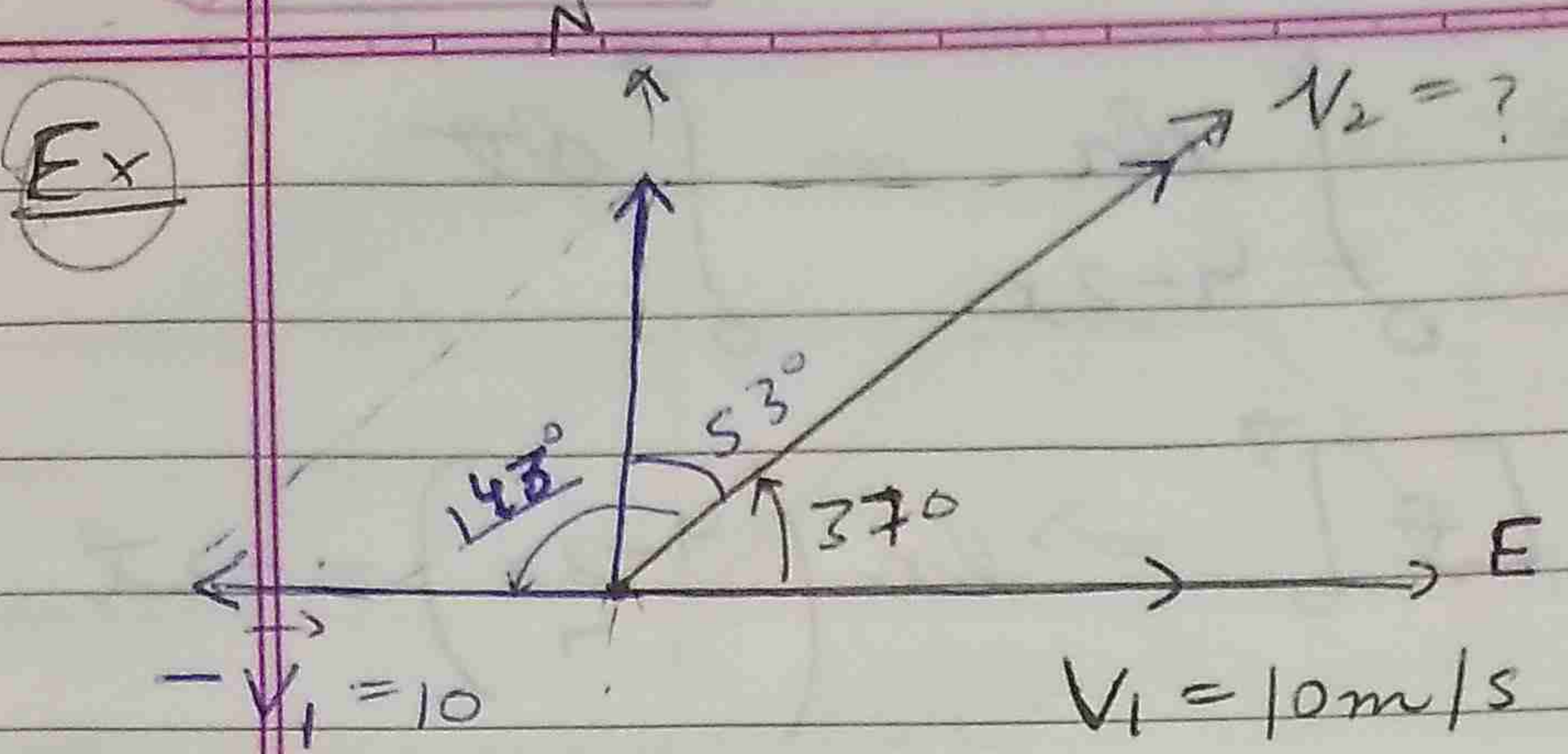
$$\Rightarrow \boxed{\vec{V}_{BA} = \vec{V}_B - \vec{V}_A}$$

$$\Rightarrow \boxed{\vec{a}_{BA} = \vec{a}_B - \vec{a}_A}$$

- all the above relations are vector relations.

★ So here subtractions will be done vectorially.

Ex



$\vec{V_2} = ?$ so that train 2 appears to be moving towards north to train 1.

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = \vec{V}_B + (-\vec{V}_A)$

$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta} \Rightarrow \tan 53^\circ = \dfrac{10 \sin 90 +}{}$

$\tan 53^\circ = \dfrac{B \sin 127^\circ}{A + B\cos\theta}$ $\quad \sin 127^\circ = (90^\circ + 37^\circ) = \cos 37^\circ = 4/5$, $\quad \cos\theta \rightarrow -\sin 37^\circ = \dfrac{-3}{5}$

$\Rightarrow \dfrac{4}{3} = \dfrac{10 \times 4/5}{V_2 + 10 \times \left(\frac{-3}{5}\right)} \Rightarrow V_2 = 12.5 m/s$

Method 2

$V_1 = 10\hat{i}$ $\quad V_2 = V_2 \cos 37^\circ \hat{i} + V_2 \sin 37^\circ \hat{j}$

ATQ $\quad (\vec{V_2} - \vec{V_1})_x = 0$

$\Rightarrow 10 = V_2 \times \dfrac{4}{5} \Rightarrow V_2 = 12.5 m/s$

Relative Motion in 1D

A $\bullet \longrightarrow V_1$

B $\bullet \longrightarrow V_2$

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

$= V_1\hat{i} - V_2\hat{i}$

$\boxed{V_{relative} = V_1 - V_2}$

A $\bullet \xrightarrow{V_1}$ $\quad \xleftarrow{V_2} \bullet$ B

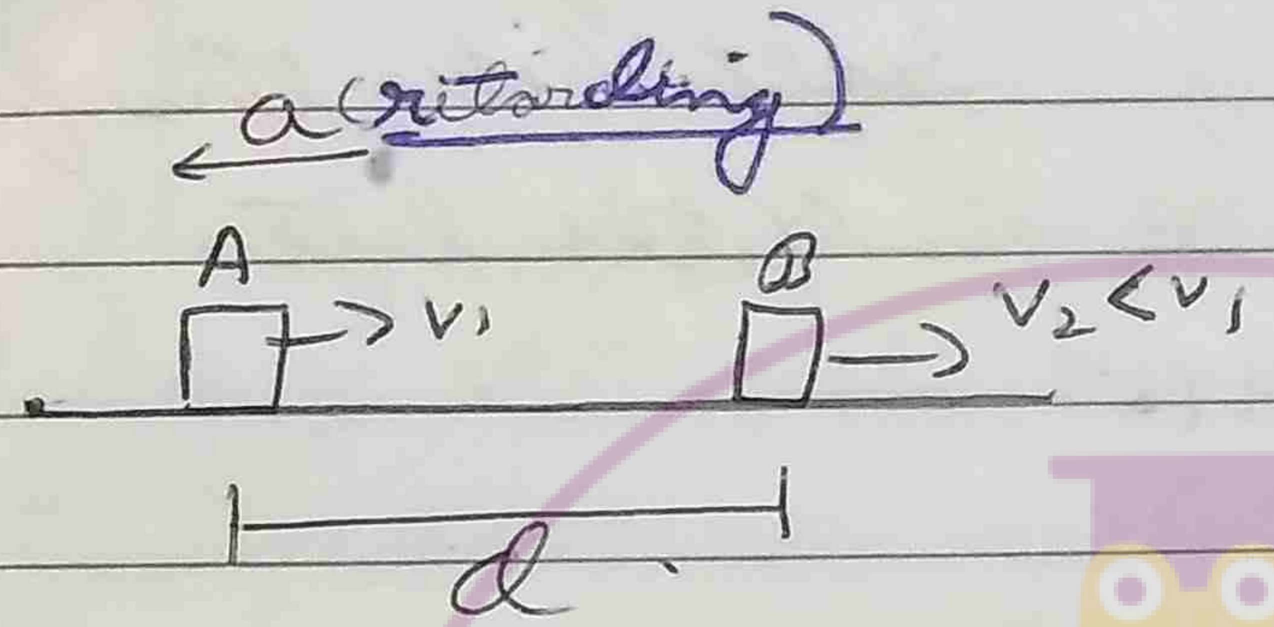
$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B \Rightarrow V_1\hat{i} - (-V_2\hat{i})$

$\Rightarrow (V_1 + V_2)\hat{i} \Rightarrow \boxed{V_{relative} = V_1 + V_2}$

Relative motion in 1D accelerated motion

$$\begin{cases} V_{rel} = u_{rel} + a_{rel}\, t \\ S_{rel} = u_{rel}\, t + \frac{1}{2} a_{rel}\, t^2 \\ V^2_{rel} = u^2_{rel} + 2 a_{rel} \cdot S_{rel} \end{cases}$$

Ex:



Minimum a so as to avoid collision

Rel. motion of A w.r.t. B

$u_{rel} = v_1 - v_2$

$V_{rel} = 0$

$a_{rel} = a_A - a_B = (-a) - 0$

$S_{rel} = d$

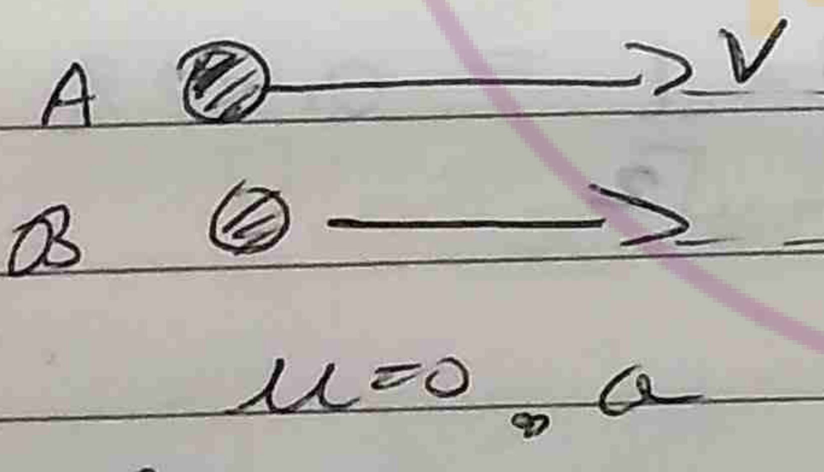
$V^2 = u^2 + 2as$

$0^2 = (v_1 - v_2)^2 - 2ad$

$\Rightarrow a = \boxed{\dfrac{(v_1 - v_2)^2}{2d}}$

$a \geq \dfrac{(v_1 - v_2)^2}{2d}$

Ex



Find maximum distance b/w A and B before B overtakes A

Method:

$S_A = v \cdot t \qquad S_B = 0t + \frac{1}{2} a t^2$

At max S

$\text{distance} = S = S_A - S_B = vt - \frac{1}{2} a t^2$

$\frac{ds}{dt} = 0 \Rightarrow V - at = 0 \Rightarrow \boxed{t = \frac{V}{a}}$ ★

★ **S is maximum when $V_A = V_B$**

★ $S_{max} = V \cdot \frac{V}{a} - \frac{1}{2} a \frac{V^2}{a^2} = \frac{V^2}{2a}$

when Distance b/w A and B is maximum Vrel should be zero.

A w.r.t. B : $V_{rel} = 0$

$u_{rel} = v$

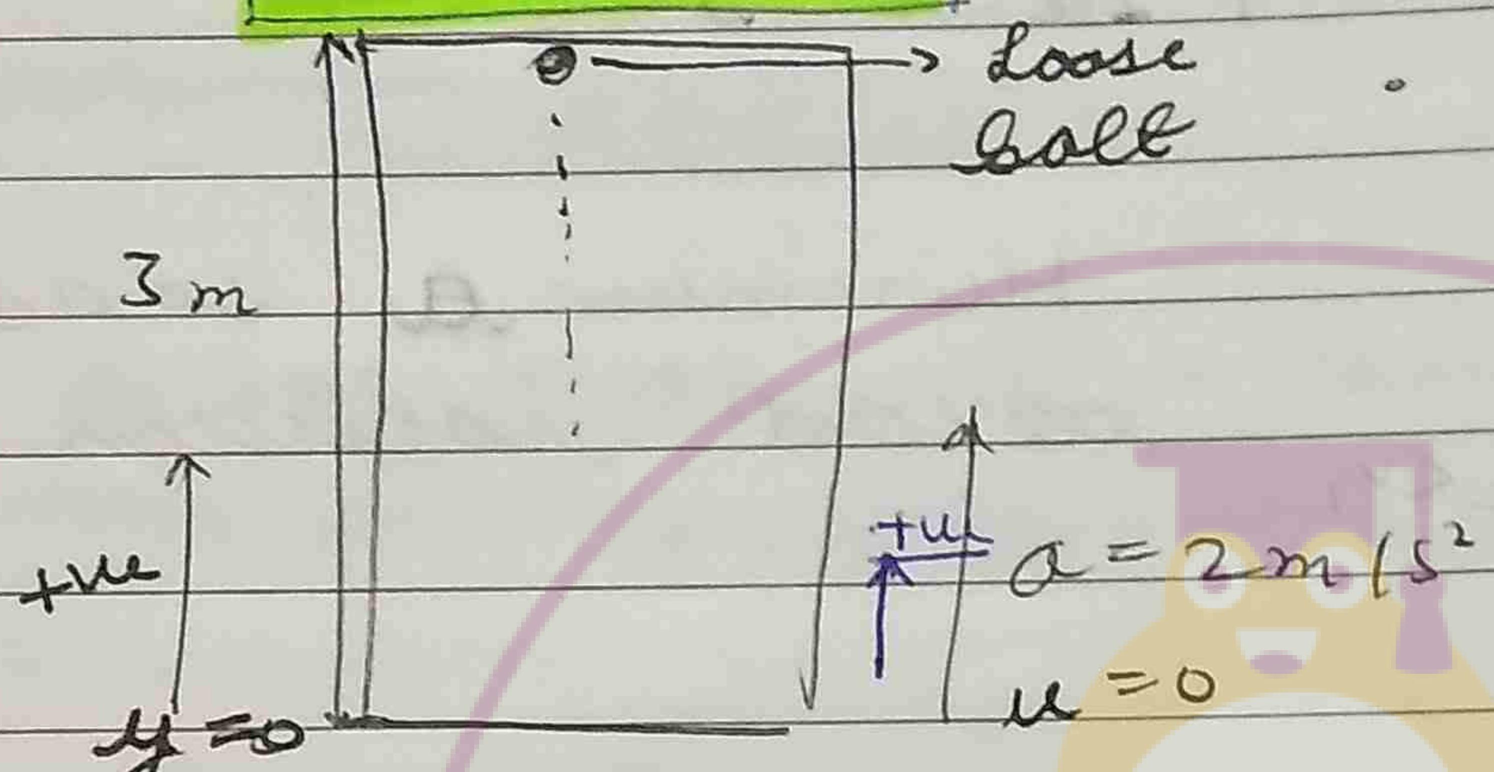
$a_{rel} = 0 - a = -a$

$s_{rel} = ?$

$\Rightarrow v^2 = u^2 + 2as \Rightarrow 0 = v^2 - 2a s_{rel}$

$\Rightarrow \boxed{s_{rel\,(max)} = \frac{v^2}{2a}}$

Ex



• Time after which bolt hits the floor of the lift.

Method 1

• At time t $y_{floor} = y_{bolt}$

$y = y_0 + ut + \frac{1}{2}at^2$

$\Rightarrow 0 + 0.t + \frac{1}{2}\times 2t^2 = 3 + 0.t - \frac{1}{2}10t^2$

$\frac{1}{2}(2+10)t^2 = 3 \Rightarrow 6t^2 = 3 \Rightarrow t = \frac{1}{\sqrt{2}} = 0.707s$

• Method 2

Bolt wrt floor

$u_{rel} = 0$

$s_{relative} = -3m$

$a_{relative} = (-10) - (2) = -12 m/s^2$ $[-g - (+a)]$

$s = ut + \frac{1}{2}at^2 = -3 = -\frac{1}{2}\times 12 \times t^2$

$\Rightarrow 3 = 6t^2 \Rightarrow \boxed{t = \frac{1}{\sqrt{2}}}$