Handwritten Notes

On

Method of Differentiation

LearningMantrasOfficial LearningMantras

Method of Differentiation:

* Method of differentiation:

$$\frac{d}{dx}x^2 = 2x \quad , \quad \int 2x\,dx = x^2 + c$$

Q. diff $y = \sin x$. using first principle

$$f(x) = y = \sin x$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} =$$

$$\lim_{h\to 0} \frac{2\cos(x+\frac{h}{2})\sin\frac{h}{2}}{\frac{h}{2}\times 2} = \cos x \cdot 1 = \cos x.$$

*

$$y = f(x)$$

$$\frac{dy}{dx} = y_1 = y' = f'(x) = Dy$$

* Basic theorems:

1) Term by Term Differ-

If $y = u(x) \pm v(x)$

then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$

OR

$$y' = u' \pm v' \pm w'$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

ii) If $y = kf(x)$
then $\frac{dy}{dx} = k f'(x)$

iii) **Product Rule:**

$y = f(x) \cdot g(x) = f \cdot g$
(I, II)

$\frac{dy}{dx} = f \cdot \frac{d}{dx} g + g \frac{df}{dx}$

$y' = fg' + gf'$

$y = e^x \cdot \sin x$
(I, II)

$= e^x \cdot \frac{d}{dx} e^x \cdot e^x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} e^x$

$y' = e^x \cos x + \sin x e^x$.

$y = \sin x - 3e^x + x \ln x$

$y' = \frac{d}{dx} \sin x - 3 \cdot \frac{d}{dx} e^x + x \cdot \frac{d}{dx} \ln x$.

$y = fgh$

$y' = \frac{d}{dx} f \cdot \frac{d}{dx} g \cdot \frac{d}{dx} h$.

$y' = f'gh + fg'h + fgh' \Rightarrow \boxed{y' = (fg)'}$

OR

$y' = \frac{(fg)'h + (fh)'g + (gh)'f}{2}$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Co → -ve

Quotient

* ~~Cosent~~ Rule!

$$y = \frac{f(x)}{g(x)}$$

$$f \Rightarrow \cancel{f(x)} - \cancel{f\, g(x)} = \frac{g\frac{d}{dx}f - f\cdot\frac{d}{dx}g}{g^2}$$

$$= \frac{gf' - fg'}{g^2}$$

Q. If ~~f(x)~~ = $f(x) = \frac{\sin x}{x}$

$$f'(x) = ? \quad \frac{\sin x}{x} = \frac{x\frac{d}{dx}\cancel{\sin x}\ \cancel{\cos x}}{x^2}$$

$$= \frac{x\frac{d}{dx}\sin x - \sin\frac{d}{dx}x}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

* Note: before Applying ~~Cosent~~ Quotient Rule Try to simplyfy ~~fied~~ if Possible

* $y = \sin x \Rightarrow \frac{dy}{dx} = \cos x$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{6}} = \cos\frac{\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

* $\left.\begin{array}{l}\text{Cos}\\ \text{Cosec}\\ \text{Cot}\end{array}\right\}$ -ve

For More PDFs Visit: LearningMantras.com

$\frac{2\sin x}{2\sin x \cos x}$

Que!

(i) $D\left(\frac{1}{\sin x}\right)$

Ans. $-\operatorname{cosec} \cdot \cot x$

(ii) $D(\sin^{-1}x + \cos^{-1}x) = \frac{\pi}{2}$

$= 0$

(iii) $D(\tan \tan^{-1} x) = 1$

$D(\tan^{-1} \tan) = 1$

(iv) $D\left(\frac{1}{\log_x e}\right)$

Diff not possible.

(v) $D\left(\frac{1-\cos 2x}{\sin 2x}\right)$

$= D\left(\frac{1-1+2\sin^2 x}{2\sin x \cos x}\right)$

$= D(\tan x) = \sec^2 x$

(vi) If $De^x = e^x$

then $De^\pi = 0$

$\log_a b = \frac{1}{\log_b a}$ formula

$= D(\log_e e^x) = \frac{1}{x}$

(viii) $D\left(\cos^4\frac{x}{2} - \sin^4\frac{x}{2}\right)$

$= D\left(\cos^2\frac{x}{2}\right)^2 - \left(\sin^2\frac{x}{2}\right)^2$

$= D\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right)$

$D(\cos x) = -\sin x.$

$1 - \sin^2 x$

$1 - 2\sin^2 x$

(ix) $D(e^{-\ln_e \cot^{-1} x})$

$= D(\cot^{-1} x) = -\frac{1}{1+x^2}$ $\rightarrow \left[\log_a N = N\right]$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Q. If $y = 1 + x^2 + x^3 - \dots x^{100}$

then $\left(\frac{dy}{dx}\right)_{x=1} = 1 + 2x + 3x^2 - \dots 100x^{99}$

$\left(\frac{dy}{dx}\right)_{x=1} = 1 + 2 + 3 + \dots - 100$

Here $= \frac{n(n+1)}{2} \quad n = 100$

$\frac{100(101)}{2} =$

$= 5050$ $\longleftarrow$ $50^2 \times 101$

Qu: let $f(x) = f(x) \cdot (g(x) \cdot h(x))$ for some $x = \alpha$

$f'(\alpha) = 21 f(\alpha)$

$f'(\alpha) = 4 f(\alpha)$

$g'(\alpha) = -7 g(\alpha)$ $f(\alpha) = f(\alpha) \cdot g(\alpha) \cdot h(\alpha)$

$h'(\alpha) = k h \alpha$ $(4 - 7 + k) = 21$

then find k: $10 = 10$

$f'(x) = f'(x) g(x) h(x) + f(x) g'(x) h(x) + f(x) g(x) h'x$

$f'(\alpha) = f'(\alpha) g(\alpha) (h(\alpha) + f(\alpha) g'(\alpha) h(\alpha) + f(\alpha) g(\alpha) h'\alpha$

$21 f(\alpha) = 4 \cdot f(\alpha) g(\alpha) h(\alpha) - 7 f(\alpha) g(\alpha) h(\alpha) + k$

$(f(\alpha) g(\alpha) h(\alpha))$

$21 f(\alpha) (g(\alpha) h(\alpha)) = f(\alpha) g(\alpha) h(\alpha) (4 - 7 + k)$

$21 = 4 - 7 + k$

$k = 21 - 4 + 7 = 24$

Qu: Diff. fn fgh are given such that

$f(0) = 1$, $g(0) = 2$, $h(0) = 3$

fg $(fg)'(0) = 6$, $(gh)'(0) = 4$, $(hf)'(0) = 1$

then find $(fgh)'(0) =$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Ans $(fg)'(0)\cdot h(0) + (fh)'(0) + (gh)'\cdot f(0)$

$= \frac{6\times 3 + 5\times 2 + 4\times 1}{2}$

$= \frac{18+10+4}{2} = \frac{32}{2} = 16$

Que: $y = \frac{x^3 + 2^x}{e^x}$

Ans: $\frac{e^x\cdot[3x^2 + 2^x \log 2) - (x^3+2^x)\cdot e^x}{(e^x)^2}$

Q. $y = \frac{1+\sin x}{\tan x}$

Ans: $y' = \frac{(\tan x)\cdot(\cos x) - (1+\sin x)\cdot \sec^2 x}{\tan^2 x}$

Q. $y = \frac{1-\ln x}{1+\ln x}$

$y' = \frac{(1+\ln x)\left(-\frac{1}{x}\right) - (1-\ln x)\left(\frac{1}{x}\right)'}{(1+\ln x)^2}$

Que: Diff. if $y = \frac{x^4+x^2+1}{x^2+x+1}$

then find y'.

$x^4+x^2+1 = (x^2+x+1)(x^2-x+1)$

$= x^2-x+1$

$y' = 2x-1$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Que. $y = \frac{\sin^{-1}x - \cos^{-1}x}{\sin^{-1}x + \cos^{-1}x} = \frac{2}{\pi}\left[\sin^{-1} - \cos^{-1}x\right]$

Ans.

$$y' = \frac{2}{\pi}\left[\left(\frac{1}{\sqrt{1-x^2}}\right) - \left(-\frac{1}{\sqrt{1-x^2}}\right)\right]$$

Book

Que: $y = \frac{x^3+x^2+x}{1+x^2}$

Degree of $N^r \geq$ Deg of D^r $\frac{x^3}{x^2}$

$$y = \frac{x^3+x^2+x}{1+x}$$

if deg of $N^r \geq$ Deg. of D^r

then Devide first

$$y = \frac{x^3+x^2+x+1-1}{1+x^2}$$

$$\frac{x(x^2+1)+(x^2+1)-1}{x^2+1} = \frac{(x^2+1)(x-1)-1}{x^2+1}$$

$$= x+1-\frac{1}{x^2+1}$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Q: $y = \dfrac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$

then find $\dfrac{dy}{dx}$

Ans $(\sec x - \tan x)(\sec x + \tan x) = 1$

$$y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + (\sec x - \tan x)(\sec x + \tan x)}$$

$$= \frac{\sec x + \tan x - 1}{(\sec x - \tan x)[\sec x + \tan x - 1]}$$

see

$$= \sec x + \tan x.$$

Diff. using chain Rule:

if $y = f(u)$
& $u = g(x)$

then $\dfrac{dy}{du} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

$$= f'(u) \cdot g'(x)$$

Q: $y = \sin x$

$\dfrac{dy}{dx} = \cos x$

$y = \sin x^2$

$\dfrac{dy}{dx} = \dfrac{d}{dx}(\sin^2)$

$\dfrac{d}{dx^2}(\sin^2 x) \cdot \dfrac{d(x^2)}{dx}$

$\cos^2 x \cdot 2x$

For More PDFs Visit: LearningMantras.com

H.W :- Q-1 1, 2, 3, 4,
S-2 , 1,

$$\frac{dy}{dx} = \cos x^2 \cdot \frac{d}{dx} x^2$$
$$= 2x \cos x^2$$

Q: $y = \sqrt{\sin x^2}$

$$\frac{dy}{dx} = \frac{d}{dy}(\sin^2 x)^{1/2}$$
$$= \frac{1}{2}(\sin^2 x)^{-1/2} \frac{d}{dx} \sin x^2$$
$$= \frac{1}{2\sqrt{\sin x^2}} \cdot \cos^2 x \cdot \frac{d}{dx} x^2$$
$$= \frac{1}{2\sqrt{\sin x^2}} \cdot \cos^2 x \cdot 2x.$$

Que : $y = (\ln x^3)^4$

$$4(\ln x^3) \cdot \frac{d}{dx}(\ln x^3)$$
$$\frac{1}{x^3} \frac{d}{dx} \cdot x$$
$$y' = 4(\ln x^3)^3 \cdot \frac{1}{x^3} \cdot 3x^2$$

Que :

Note :- $\boxed{f(x) = \exp \cdot \sin x = e^{\sin x}}$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Q. $y = (\tan e^{(\sin x^2)})^3$

$$y = 3(\tan e^{(\sin x^2)})^2 \cdot (\sec^2 e^{\sin x^2}) \cdot e^{\sin x^2} \cdot \cos x^2 \cdot 2x$$

Q. $y = e^{\sqrt{\sin(\ln(\ln x^2+7))^5}}$

$$= \frac{(e^{\sqrt{\sin(\ln(\ln(x^2+7)}}) \cdot \cos \ln(x^2+7)^5 \quad 5(\ln x^2+7)^4 \cdot 2x}{(2)(\sin(\ln(x^2+7)^5)) \ln(x^2+7)^3}$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Logarithmic diff :

$$\boxed{a^{\log_a N} = N}$$ Prop.

i) A fn which is the Product or Quotient of a no. of functions

ii) fn $(f(x))^{g(x)}$ where f and g both are diff.

then in such situation log. diff is needed.

$$\Rightarrow f(x) = \frac{x^2 2^x \sin x}{\cos 2x}$$

$$\log \frac{abc}{de} = \log a + \log b + \log c - \log d - \log e$$

$$\ln f(x) = \ln x^2 + \ln 2^x + \ln \sin x - \ln(\cos 2x)$$

$$\frac{1}{f(x)} \cdot f'(x) = \frac{1}{x^2} \cdot 2x + \frac{1}{2^x} \cdot 2^x \ln 2 + \frac{1}{\sin x} \cos x - \frac{1}{\cos 2x}(-2\sin 2x)$$

$$f'(x) = \frac{x^2 2^x \sin x}{\cos 2x}\left[\frac{2}{x} + \ln 2 + \cot x + \frac{2\sin 2x}{\cos 2x}\right]$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Q. $y = x^{\sin x}$

$\log y = \log x^{(\sin x)}$

$\ln y = \underset{I}{\sin x} \; \underset{II}{\ln x}$

$\frac{1}{y}\frac{dy}{dx} = \sin x \frac{1}{x} + (\ln x)\cdot(\cos x)$

$\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x (\ln x)\right)$

$y = e^{\ln x^{(\sin x)}}$

$y = e^{\sin x \cdot \ln x}$

$y' = e^{\sin x \ln x} \cdot \frac{d}{dx}\{\underset{I}{\sin x}\cdot\underset{II}{\ln x}\}$

Que: $y = f(x) = x^{x^x}$

$\log f(x) = \log x^{x}$

$\frac{1}{f} f' = x \cdot \log x$

$= x \cdot \frac{1}{x} \cdot \log x \cdot 1$

$\Rightarrow f' = x^x(1 + \ln x)$ Remember

Que: $y = x^{\sin x} + x^x$

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= x^{\sin x}\left(\frac{\sin x}{x} + \cos x \cdot \ln x\right) + x^x(1 + \ln x)$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Que: $y = x^{x^x} + \tan x^{(\sin x)}$

$\ln y = \ln x^{x^x} + \ln \tan x^{(\sin x)}$

$= x^x \ln x + \sin x \ln \tan x$

$x^x \cdot \frac{x}{x} + x^{x-1} \ln x (1 + \ln x) \cos x \cdot \ln \tan x + \sin x \cdot \frac{1}{\tan x} \sec^2 x$

$x^{x^x} \left(x^x \frac{1}{x} + \ln x (1 + \ln x) + \tan^{(\sin x)} \left(\quad , \right) \right)$

Sol: $y = x^{x^x} + \tan x^{(\sin x)}$

$u \qquad v$

$v = (\tan x)^{\sin x}$

$\log v = \sin x \ln(\tan x)$

$\frac{1}{v} \log v = (\cos x \cdot \ln(\tan x) + \sin x \frac{1}{\tan x} \sec^2 x)$

$\tan x \left(\cos x \cdot \ln(\tan x) + \sin x \frac{1}{\tan x} \sec^2 x \right)$

$u = x^{x^x}$

$\ln u = \ln x^{x^x}$

$= x^x \ln x$

$\frac{1}{u} \cdot u' = x^x \cdot \frac{1}{x} + \log(x^x)(1 + \ln x)$

$= x^{x^x} \left(x^x \cdot \frac{1}{x} + \log(x^x)(1 + \ln x) \right)$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

$\sqrt{x} = x^{1/2}$ $\quad \frac{1}{2}$

Que: $y = \sin x \cdot \sin 2x \cdot \sin 3x \cdots \sin nx$.

Ans $\ln y = \ln \sin x + \ln \sin 2x \cdots \ln \sin nx$

$$\frac{1}{y} y' = \frac{\cos x}{\sin x} + \frac{2\cos 2x}{\sin 2x} \cdots \frac{n(\cos nx)}{\sin nx}$$

$$y' = (\sin x \sin 2x \cdots \sin nx)\left[\cot x + 2\cot 2x + \cdots n(\cot nx)\right]$$

Sub Que: $y = \sqrt{x}^{\sqrt{x}} \cdot e^{x^2}$ then find $y'(1)$

Ans

$$\ln y = \ln \sqrt{x}^{\sqrt{x}} \cdot \ln e^{x^2}$$

$$\sqrt{x} \ln \sqrt{x} \cdot \quad x^2 \ln e$$

$$\sqrt{x} \cdot \frac{1}{\sqrt{x}} + \frac{1}{2\sqrt{x}} \ln \sqrt{x} \cdot \quad 2x \ln e + x^2 \cdot \frac{1}{e}$$

$$\frac{\sqrt{x}}{\sqrt{x}} + \frac{1}{2\sqrt{x}} \ln \sqrt{x}$$

Ans: $\ln y = \ln$

$$= \sqrt{x} \ln x^{1/2} + x^2$$

$$\frac{\sqrt{x}}{2} \cdot \frac{1}{x} + \frac{\ln x}{2} \times \frac{1}{2\sqrt{x}} + 2x$$

$$\frac{y'(1)}{e} = \left(\frac{1}{2} + 2\right) \Rightarrow y'(1) = \frac{5e}{2}$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

$$\prod_{r=1}^{3} \sin r$$

Imp

Que: if $f(x) = \prod_{n=1}^{100} (x-n)^{n(101-n)}$

find $\frac{f(101)}{f'(101)}$

Ans $\prod_{n=1}^{100} (x-1)^{1(101-1)} \times (x-2)^{2(101-2)} \cdots (x-n)^{n(101-n)}$

$(x-1)^{100} \times (x-2)^{2(99)} \times \cdots (x+100)^{100(101-100)}$

$= (x-1)^{100} \cdot (x-2)^{198} \times (x-3)^{294} \cdots (x-100)^{100(101-100)}$

$\ln f(x) = 1 \cdot 100 \ln(x-1) + 2 \cdot 99 \ln(x-2) + 3 \cdot 98 (\ln(x-3) + 100 \cdot 1 \ln|x-100|)$

$\frac{1}{f(x)} \cdot f'(x) = \frac{1 \cdot 100}{x-1} + \frac{2 \cdot 99}{x-2} + \frac{3 \cdot 98}{x-3} + \cdots \frac{100 \cdot 1}{x-100}$

$\frac{f'(101)}{f(101)} = 1+2+3+\cdots 100 = 5050$

find $\frac{f(101)}{f'(101)} = \frac{1}{5050}$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

* Parametric diff.

1) $y = f(x)$ x

$y = f(t)$ & $x = g(t)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{f'(t)}{g'(t)}$$

* Diff of one fu ~~with~~ $f(x)$ with respect to another fu $g(x)$.

$$\frac{d f(x)}{d g(x)} = \frac{\frac{d f(x)}{dx}}{\frac{d g(x)}{dx}} = \frac{f'(x)}{g'(x)}$$

Diff different $\sin x$ with respect to $\log x$

$$\frac{d f(\sin x)}{d(\log} \quad \frac{d(\sin x)}{d(\ln x)} =$$

$$= d \frac{\frac{d \sin u}{dx}}{\frac{d}{dx} \ln} = \frac{\cos u}{1/x} = x \cos x.$$

Q: find $\frac{dy}{dx}$ = if $x = a(\cos t + t \sin t)$

$y = a(\sin t - t \cos t)$.

$$\frac{\frac{dx}{dx}}{\frac{dx}{dt}} = \frac{d \cdot a(\cos ut + t \sin t)}{d(d(\sin ut + t \cos t))}$$

For More PDFs Visit: LearningMantras.com

Ans: $\frac{dx}{dt} = a(-\sin t + \sin t + t\cos t)$

$\frac{dy}{dt} = a(\cos t - \cos t + t\sin t)$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$

Que: $x = \frac{3t}{1+t^3}$; $y = \frac{3t^2}{1+t^3}$

$\frac{dx}{dx} = \frac{\frac{3t}{1+t^3}}{\frac{3t^2}{1+t^3}} = \frac{3t}{3t^2} = \frac{1}{t}$

$\frac{dy}{dx} \frac{(1+t^3)\cdot 6t - 3t^2 \cdot 3t^2}{(1+t^3)^2}$

Sol: $\frac{dx}{dt} = \frac{(1+t^3)3 - 3t \times 3t^2}{(1+t^3)^2}$

Que: Diff $(\ln x)^{\tan x}$ w.r.t x^x

$\frac{d}{dx}$

$y = (\ln x)^{\tan x}$

$\ln y = \tan x \cdot \ln(\ln x)$

$\frac{1}{y} \cdot y' = \sec^2 x \cdot \ln(\ln x) + \tan x \frac{1}{\ln x} \cdot \frac{1}{x}$

$= \frac{(\ln x)^{\tan x} \cdot \left[\sec^2 x \cdot \ln(\ln x) + \frac{\tan x}{x \ln x}\right]}{x^n (1 + \ln x)}$

$\frac{d}{dx}(\ln x)^{\tan x} / \frac{d}{dx} x^n$

$= \ln x + x \cdot \frac{1}{x}$

$x^4 \ln x + 1$

For More PDFs Visit: LearningMantras.com

Q.1 ⇒ 2(a), b, 3, 4, 5, 6, 7, 8, 10

Q.1 ⇒ 10

Que: $e^{\sin^{-1}x}$ w.r.t $e^{-\cos^{-1}x}$

$\ln e^{\sin^{-1}x}$

$\sin^{-1}x \ln e$

$\frac{1}{\sqrt{1-x^2}} \cdot \ln e + \sin^{-1}x \frac{1}{e}$

$\ln e^{-\cos^{-1}x}$

$-\cos^{-1}x \cdot \ln e$

$\frac{1}{\sqrt{1-x^2}} \cdot \ln\frac{1}{e} + -\cos^{-1}x \frac{1}{e}$

$$\Rightarrow \frac{\frac{d}{dx} e^{\sin^{-1}x}}{\frac{d}{dx} e^{-\cos^{-1}x}} = \frac{e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}}{e^{-\cos^{-1}x} \times + \frac{1}{\sqrt{1-x^2}}}$$

$$= e^{\sin^{-1}x} \cdot e^{\cos x^{-1}} = e^{\sin^{-1}x + \cos^{-1}x} = e^{\pi/2}$$

Que: $x = a\sqrt{\cos 2t}\cos t \quad y = a\sqrt{\cos 2t}\sin t$

then find $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{6}}$

$\frac{dy}{dx} = a\sqrt{\cos 2t}\cos t + \frac{1}{2\sqrt{\cos 2t}} \sin t$

Sol: $x = a\sqrt{\cos 2t}$

$= \frac{a}{2\sqrt{\cos 2t}} + a\sqrt{\cos 2t}$

$a\sqrt{\cos 2t} \quad -\sin t + \frac{1}{2\sqrt{\cos 2t}} \cot t$

(√3)

$= a\sqrt{\cos 2\frac{\pi}{3}} - \sin\pi/6 + \frac{1}{2\sqrt{\cos 2\frac{\pi}{3}}} \cot\frac{a\pi}{6}$

$a\sqrt{\cos\frac{\pi}{3}} - \sin\frac{\pi}{6} + \frac{\cot\pi/6}{\sqrt{\cos 2\pi}}$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Sol:- $\frac{dx}{dt} = a\cos t \frac{1}{2\sqrt{\cos 2t}} \times -2\sin 2t + a\sqrt{\cos 2t}(-\sin t)$

$= \frac{a}{\sqrt{\cos 2t}} \left[\cos t \sin 2t + \sin t \cos 2t \right]$

$= -\frac{a}{\sqrt{\cos 2t}} (\sin 3t)$

$y = a\sqrt{\cos 2t} \sin t$

$\frac{dy}{dt} = a\sqrt{\cos 2t} \cdot \cos t + a\sin t \frac{1}{2\sqrt{\cos 2t}} \times -2\sin 2t \cdot$

$= \frac{a}{\sqrt{\cos 2t}} \left[\cos 2t \cdot \cos t - \sin 2t \sin t \right)$

$= \frac{a}{\sqrt{\cos 2t}} \cdot \cos(2t + t)$

$\left(\frac{dy}{dx}\right) = \frac{\frac{a}{\sqrt{\cos 2t}} \cdot \cos 3t}{-\frac{a}{\sqrt{\cos 2t}} \cdot \sin 3t} = \cot 3t$

$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{6}} = \cot \frac{\pi}{2} = 0$

* Important Substitution:

Expression	Substitution.
$a^2 + x^2$	$x = a\tan\theta$
$a^2 - x^2$	$x = a\sin\theta$ or $x = a\cos\theta$
$x^2 - a^2$	$x = a\sec\theta$ or $x = a\operatorname{cosec}\theta$
$\sqrt{\frac{a+x}{a-x}}$ or $\frac{a-x}{a+x}$	$x = a\cos\theta$
$\frac{2x}{1+x^2}, \frac{2x}{1-x^2}, \frac{1-x^2}{1+x^2}$	$x = \tan\theta$ or $\cos 2\theta$

For More PDFs Visit: LearningMantras.com

$$\boxed{\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}}$$

Q.1 (5) $y = \sqrt{\frac{1-x}{1+x}}$ $\quad x = \cos\theta \quad \frac{dx}{d\theta} = -\sin\theta$ (1)

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad x = 2\cos^2\frac{\theta}{2} - 1$$

$$= \sqrt{\frac{1-(1-2\sin^2\theta/2}{1+(2\cos^2\frac{\theta}{2}-1)}} \qquad \frac{(x+1)}{2} = \cos^2\frac{\theta}{2}$$

$$= \sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\theta/2}}$$

$$\therefore y = \tan\frac{\theta}{2}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{d}{d\theta}\left(\tan\frac{\theta}{2}\right)\frac{d\theta}{dx}$$

$$= \sec^2\frac{\theta}{2}\cdot\left(-\frac{1}{\sin\theta}\right)$$ from (1)

$$= \frac{1}{\cos^2\frac{\theta}{2}}\left(-\frac{1}{\sin\theta}\right)$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-x^2}$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

6. $y = \ln\left\{\frac{x + \sqrt{(a^2+x^2)}}{a}\right\} = \ln(\sec\theta + a)$

$$y = \sin^{-1}\frac{2x}{1+x^2}$$

$$y = \cos^{-1}\frac{1-x^2}{1+x^2}$$

$$y = \tan^{-1}\frac{2x}{1-x^2}$$

Que: $y = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$ Diff.

$\tan^{-1}\frac{\sqrt{x}-x}{1+x\cdot x^{1/2}}$

$y = \tan^{-1}\sqrt{x} - \tan^{-1}x$,

$y' = \frac{1}{1+x}\frac{d}{dx}\sqrt{x} - \frac{1}{1+x^2}$ A

Que: Diff $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ $\begin{matrix}\text{b com}\\ \text{b com}\end{matrix}$ divide both are b cos x.

Ans: $\tan^{-1}\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}$ ($\frac{a}{b} = x$, $\tan x = y$)

$y = \tan^{-1}\frac{a}{b} - \tan^{-1}\tan x$

For More PDFs Visit: LearningMantras.com

Q. $\tan^{-1}\left(\frac{a\cos x - b\sin x}{a\cos x + b\sin x}\right)$ Both divided by $a\cos x$

$= \tan^{-1}\left(\frac{1 - \frac{b}{a}\tan x}{1 + \frac{b}{a}\tan x}\right)$

$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right)$

$= \tan^{-1}\tan\frac{\pi}{4} - \tan^{-1}\tan\theta$

$= \frac{\pi}{4} - \tan^{-1}\tan\theta.$

$$\sqrt{x^2} = |x|$$

Derivative of Implisit fu :

$x + y = 1$ Implisit

$y = 1 - x$ Explisit

* Diff. term by term and Collect dy togethr of one side one equal side find $\frac{dy}{dx}$ its value.

Q. Diff. $x^y = e^{x-y}$ P.T $\neq \frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$

$y \ln x = x - y$

$\frac{y}{x} + \ln x \cdot y' = 1 - y'$

$\rightarrow y'[1 + \ln x] = \left(1 - \frac{y}{x}\right)$ $y' = \frac{1 - \frac{y}{x}}{(1+\ln x)}$

For More PDFs Visit: LearningMantras.com

$$y \ln x = x - y$$

$$y(1+\ln x) = x$$

$$y' = \frac{1 - \frac{y}{x}}{(1+\ln x)}$$

$$= \frac{1 - \frac{1}{1+\ln x}}{1+\ln x} = \frac{\ln x}{(1+\ln x)^2}$$

Que: $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x} \cdots \infty}}$

$$y = \sqrt{\sin x + y}$$

$$\frac{dy}{dx} = \frac{\cos x + y'}{2\sqrt{\sin x + y}}$$

Sol: $y^2 = \sin x + y$

$$2yy' = \cos x + y'$$

$$y' = \frac{\cos x}{2y - 1}$$

Q: $y = x^{x^{x^{x^{\cdots \infty}}}} \rightarrow y$

$$y = x^y$$

$$\ln y = y \ln x$$

$$\frac{1}{y} y' = \frac{y}{x} + \ln y'$$

$$y' = \cdots$$

For More PDFs Visit: LearningMantras.com

Q. $y = (\ln x)^{(\ln x)^{(\ln x)^{\cdots\infty}}}$

$y = (\ln x)^y$

$\ln y = y \ln(\ln x)$

$\frac{1}{y} \cdot y' = y'(\ln(\ln x)) + y \cdot \frac{1}{x \ln x}$

$= y'\left(\frac{1}{y} \cdot y'\right) = \theta$

$y'\left(\frac{1}{y} \cdot y'\right) = (\ln x)^y ($ " $)$

Que: $y = \cfrac{x}{1 + \cfrac{x}{2 + \cfrac{x}{1 + \cfrac{x}{2 + \cdots \infty}}}}$

$= \cfrac{x}{1 + \cfrac{x}{2 + \cfrac{x}{(1 + \cfrac{x}{2 + \cdots \infty})}}}$

$y = \cfrac{x}{1 + \cfrac{x}{2 + y}}$

$y = \frac{x(2+y)}{x+y+2} \rightarrow (x+y+2)y = x(2+y)$

$xy + y^2 + 2y = 2x + xy'$

$= 2yy' + xy' = 2$

$y' =$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Que: if $\sin y = x \sin(a+y)$

then Prof that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

Sol: $x = \frac{\sin y}{\sin(a+y)}$

$$\frac{dx}{dy} = \frac{\sin(a+y)\cdot\cos y - \sin y \cdot \cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

Que: If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ find $\frac{dy}{dx} = ?$

$x = \sin\theta, y = \sin\phi$

$\cos\theta + \cos\phi = a(\sin\theta + \sin\theta)$

$$2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = a\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$$

$$\cos\left(\frac{\theta+\phi}{2}\right)\cdot\left[2\cos\frac{\theta-\phi}{2} - a\sin\frac{\theta-\phi}{2}\right]$$

$\cos\frac{\theta+\phi}{2} = 0$

$\cos\frac{\theta+\phi}{2} = 0$

$\frac{\theta+\phi}{2} = (2n+1)\frac{\pi}{2}$

$2\cos\frac{\theta-\phi}{2} = a\frac{\theta-\phi}{2}$

$\tan\left(\frac{\theta-\phi}{2}\right) = \frac{2}{a} = \cot\alpha$

$\theta - \phi = \cot$

$\sin^{-1}x - \sin^{-1}y = \cot$

$\sin^{-1}x - \sin^{-1}y = \cot$

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

For More PDFs Visit: LearningMantras.com

ITF = curve learning.

H.W : S-I 11, 12, 13, 15, 19, 25
BJ-M : 1, 2, 3, 5,
J.A : 5

Que: If $y^5 + xy^2 + x^3 = 4x + 3$ find $\left(\frac{dy}{dx}\right)_{(2,1)} = ?$

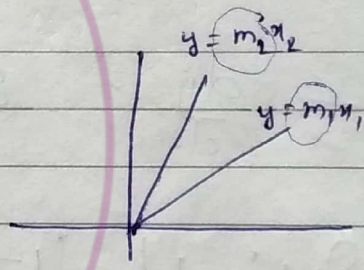
$$5y^4 y' + y^2 + 2xyy' + 3x^2 = 4$$

$$y'(5y^4 + 2xy) = 4 - 3x^2 - y^2$$

$$(y')_{(2,1)} = \frac{4 - 3\cdot 4 - 1}{5\cdot 1 + 2\cdot 2\cdot 1} = \frac{-9}{9} = -1$$

Que: if $ax^2 + 2hxy + by^2 = 0$
P.T $\frac{dy}{dx} = \frac{ax + hy}{hx + by} = \left(\frac{y}{x}\right)$

$$2ax + 2h(xy' + y) + 2byy' = 0$$
$$(ax + hy) + y'(hx + by) = 0$$



Que: diff. $y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1+x}}\right)$ $x \in [-1, 1]$

Ans:

$$\sqrt{1 - \cos\theta} = \sqrt{1 - 1 + 2\sin^2\frac{\theta}{2}} \qquad x \in (-1, 1)$$

$$\sqrt{1 + \cos\theta} = \sqrt{1 + 2\cos^2\frac{\theta}{2} - 1} \qquad \cos\theta \in [-1, 1]$$

$$\theta \in [0, \pi)$$

$$\frac{\theta}{2} \in \left(0, \frac{\pi}{2}\right)$$

$$\tan^{-1}\frac{\left|\cos\frac{\theta}{2}\right| - \left|\sin\frac{\theta}{2}\right|}{\left|\cos\frac{\theta}{2}\right| + \left|\sin\frac{\theta}{2}\right|}$$

$$\tan^{-1}\left(\frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}\right)$$

$$\tan^{-1}\left(\frac{1 - \tan\theta/2}{1 + \tan\theta/2}\right) = \tan^{-1}\left(\frac{\tan\pi/4 - \tan\theta/2}{1 + \tan\pi/4\tan\theta/2}\right)$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

$0 \le \frac{\theta}{2} \le \frac{\pi}{2}$

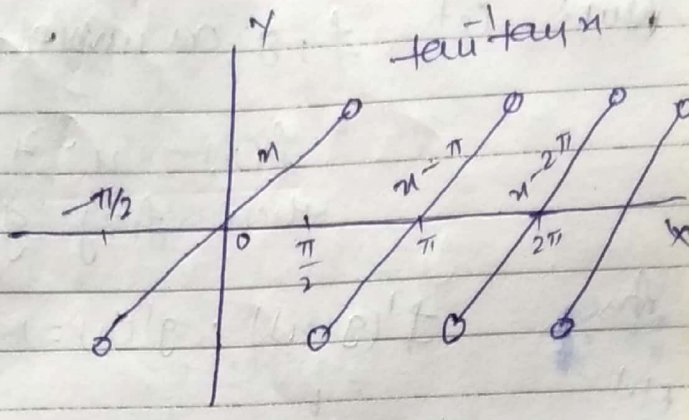
$\frac{\pi}{4} + 0 \ge \frac{\pi}{4} - \frac{\theta}{2} \ge \frac{\pi}{4} - \frac{\pi}{2}$

$-\frac{\pi}{4} \le \frac{\pi}{4} - \frac{\theta}{2} \le \frac{\pi}{4}$

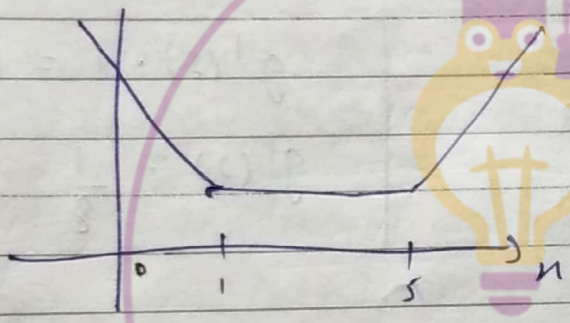
$= \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \frac{\pi}{4} - \frac{\theta}{2} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

$\tan^{-1}\tan x$:



Que: $y = |x-1| + |x-5|$ $\left(\frac{dy}{dx}\right)_{x=2}$



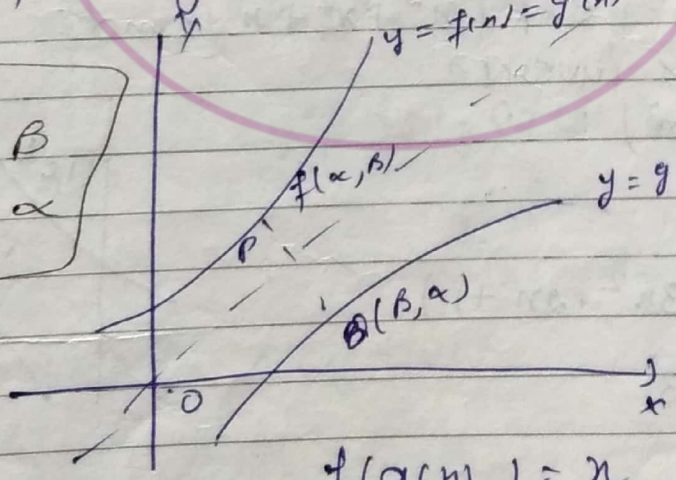
$= 0$

* Derivative of an inverse function:

If f and g are inverse fn of each other

$f(\alpha) = \beta$

$f(\beta) = \alpha$



$f(g(x)) = x$ & $gf(x) = x$

$f'(g(x)) \cdot g'(x) = 1$ | $g'(f(x)) \cdot f'(x) = 1$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Qu: Let f, g are inverse of each other

$$f(x) = x^3 + x^5 \qquad f(1) = 2$$

then find $g'(2)$

Ans $f'(g(x)) \cdot g'(x) = 1$

H.1

$x = 2$ $\quad g'(f(2)) \cdot g'(2) = 1$

$f'(1) \cdot g'(2) = 1$

$f'(1) = 3 + 5 = 8$

$g'(2) = \frac{1}{8}$

H.2

$g'(f(x)) \cdot f'(x) = 1$

$x = 1$ $\quad g'(f(1)) \cdot f'(1) = 1$

$g'(2) \cdot 8 = 1$

$g'(2) = \frac{1}{8}$

Qu: $f(x) = \exp(\sin x) = e^{\sin x}$.

Qu Let $f(x) = \exp(x^3 + x^2 + x)$ and $g(b)$ its inverse then $g'(e^3) = ?$

Ans:

$= e^{x^3+x^2+x} \cdot (3x^2 + 2x + 1)$

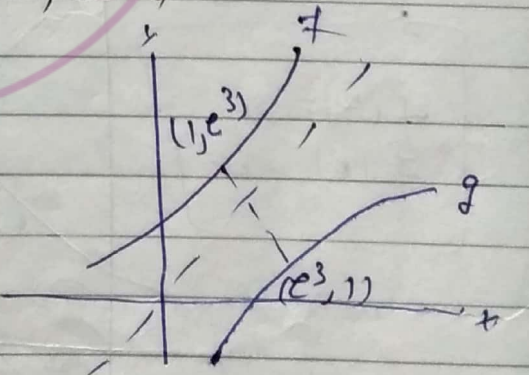
$x = 1, \quad f(1) = e^3$

$f'(g(e^3)) \cdot g'(e^3) = 1$

$f'(1) \cdot g'(e^3) = 1$

$6e^3 \cdot g'(e^3) = 1$

$$\boxed{g'(e^3) = \frac{1}{6e^3}}$$



For More PDFs Visit: LearningMantras.com

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Ques

$$y = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \cdots n$$

$$\frac{dy}{dx} \cdot T_1 = \tan^{-1}\frac{(x+1)-x}{1+x(x+1)} \qquad T_2 \tan^{-1}\frac{1}{1+(x^2+3x+2)}$$

$$T_2 = \frac{(x+2)-(x+1)}{1+(x+1)(x+2)}$$

$$T_1 = \tan^{-1}(x+1) - \tan^{-1}x \qquad t_2 = \tan^{-1}(x+2) - \tan^{-1}(x+1)$$

$$T_1 = \tan^{-1}(x+1) - \tan^{-1}x$$
$$T_2 = \tan^{-1}(x+2) - \tan^{-1}(x+1)$$
$$T_3 = \tan^{-1}(x+3) - \tan^{-1}(x+2)$$

$$T_n = \tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$$
$$y = \tan^{-1}(x+n) - \tan^{-1}x.$$

X **Successive diff.** : $y = f(x)$

$$\frac{dy}{dx} = f'(x) = y' = Dy$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x) = y'' = y_2 = D^2y$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = f'''(x) = y''' = y_3 = D^3$$

$$y = \sec^2 x$$

$$\frac{dy}{dx} = 2\sec^2 x \cdot \sec x \tan x$$

$$= 2\sec^2 x \tan x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(2\sec^2 x \tan x).$$

For More PDFs Visit: LearningMantras.com

O-1 ⇒ ap- 18th Qu.
O-2 ⇒ 3,
S-1 ⇒ 14,15,16,17
J.M = comb.

Que: $y = \sin(\sin x)$

P.T $y_2 + y_1(\tan x) + y\cos^2 x = 0$

Ans: $y_1 = \cos(\sin x)\cdot\cos x$

$$\cos x\left[-\sin(\sin x)\right] + \cos(\sin x)(-\sin x)$$

$$= -\cos^2 x \sin(\sin x) - \cos(\sin x)\cdot\sin x$$

$$y = -\cos^2 x \sin(\sin x) - \cos(\sin x)\cdot\sin x + \cos(\sin x)\cdot\frac{\sin x}{\cos x}\cdot\sin x$$

$$= 0$$

Que: $e^{x+y} = y^2$ P.T $y'' = \frac{2y}{(2-y)^3}$

$$e^{x+y}\cdot\left[1+\frac{dy}{dx}\right] = 2y\frac{dy}{dx}$$

$$y^2\left[1+\frac{dy}{dx}\right] = 2y\frac{dy}{dx}$$

$$y' = \frac{y}{2-y}$$

$$y'' = \frac{d}{dx}(y') = \frac{d}{dx}\left(\frac{y}{2-y}\right) = \frac{dy}{dx}\left(\frac{y}{2-y}\right)\frac{dy}{dx}$$

$$\frac{(2-y)\cdot 1 - y(-1)}{(2-y)^2}\times\frac{y}{2-y}$$

$$= \frac{2y}{(2-y)^3}$$

For More PDFs Visit: LearningMantras.com

Que: $x = a(t + \sin t)$

$y = a(1 - \cos t)$

$\frac{d^2y}{dx^2}$

$\frac{dx}{dt} = a(1 + \cos t)$

$\frac{dy}{dt} = a \sin t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t}{1 + \cos t}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{\sin t}{1+\cos t}\right)$

$\frac{d}{dt}\left(\frac{\sin t}{1+\cos t}\right) \cdot \frac{dt}{dx}$

$= \frac{(1+\cos t)\cos t - \sin t(-\sin t)}{(1+\cos t)^2} \times \frac{1}{a(1+\cos t)}$

$= \frac{(1+\cos t)}{a(1+\cos t)^3} = \frac{1}{a(1+\cos t)^2}$

$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$

$\frac{d}{dt}\left(\frac{1}{a(1+\cos t)^3}\right) \cdot \frac{dt}{dx}$

$= \frac{1}{a} \cdot \frac{-2}{(1+\cos t)^2} \times (-\sin t) \times \frac{1}{a(1+\cos t)}$

$= \frac{2 \sin t}{a^2(1+\cos t)^4}$

$x = a(t + \sin t)$

$y = a(1 - \cos t)$

$\frac{d^2y}{dx^2}$

For More PDFs Visit: LearningMantras.com

$e^{xy}=1$

Qu: $e^{xy} + y\cos x = 2$

(i) find $y'(0)$

$$e^{xy}(xy' + y) + y'\cos x - y\sin x = 0$$

$$y'(xe^{xy} + \cos x) = y(\sin x - e^{xy})$$

$$y' = \frac{y(\sin x - e^{xy})}{xe^{xy} + \cos x}$$

at $x = 0$

$e^{0} + y\cos x = 2$

$1 + y = 2$

$\therefore y = 1$

$(0, 1)$

$$y'(0) = (y')_{(0,1)} = \frac{1(0-1)}{0+1} = -1$$

21/9

(ii) find $y''(0)$

Imp

$$y'' = \frac{(xe^{xy} + \cos x)\cdot[y'(\sin x - e^{xy}) + y(\cos x - e^{xy}\cdot(y + xy'))] - y(\sin x - e^{xy})\cdot[e^{xy} + xe^{xy}[y + xy'] - \sin x]}{(xe^{2y} + \cos x)^2}$$

$$= \frac{(0+1)[-1(0-1) + 1(1 - 1(1+0)] - 1(0-1)\cdot[1+0-0]}{(0+1)^2}$$

$$= \frac{1(1+0] + 1}{1} = 2$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Que: find value of expression on the ellipse

$y^3 \frac{d^2y}{dx^2}$ $\qquad 3x^2 + 4y^2 = 12$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$x = 2\cos\theta, \quad y = \sqrt{3}\sin\theta$

$\frac{dx}{d\theta} = -2\sin\theta, \quad \frac{dy}{d\theta} = \sqrt{3}\cos\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\sqrt{3}}{2}\cot\theta$$

$$\frac{d^2y}{dx^2} = -\frac{d}{d\theta}\left(\frac{\sqrt{3}}{2}\cot\theta\right)\cdot\frac{d\theta}{dx} = -\frac{\sqrt{3}}{2}(-\operatorname{cosec}^2\theta) \times \frac{1}{-2\sin\theta}$$

$$x^2\, y^3 \cdot \frac{d^2y}{dx^2} = -3\sqrt{3}\sin^3\theta \times -\frac{\sqrt{3}}{2\cdot 2}\frac{1}{\sin^3\theta} = \frac{9}{4}$$

Que: $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$

Starting with here

P.T $\frac{d^2x}{dy^2} = -\frac{d^2y/dx^2}{\left(\frac{dy}{dx}\right)^3}$

$$\frac{d^2x}{dy^2}\,\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right)$$

$$\frac{d^2x}{dy^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}\frac{dx}{dy}$$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

$$= -1\left(\frac{dy}{dx}\right)^{-2} \times \frac{d^2y}{dx^2} \times \frac{1}{\frac{dy}{dx}}$$

$$= \frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3} \quad \text{Ans}$$

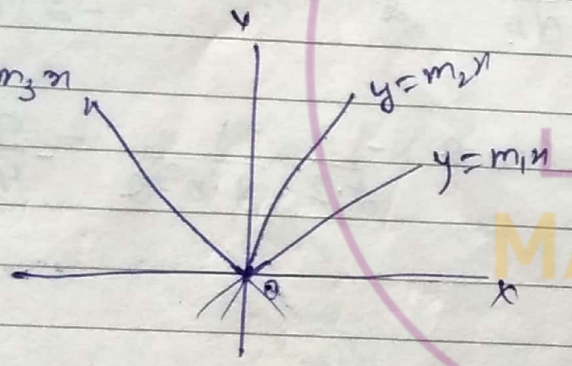
Que: $x^3 + 3x^2y - 6xy^2 + 2y^3 = 0$

find $\left(\frac{d^2y}{dx^2}\right)_{(1,1)}$

$\because \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}$ (slope)

$= 0$



$\frac{dy}{dx} = m_1$

LEARNING MANTRAS

L Hospital Rule:

Let f and g are diff^{le} for n times and $\lim_{x \to a} \frac{f(x)}{g(x)}$ $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ form.

then according to L Hospital Rule it will be equal to $\frac{f'(x)}{g'(x)}$ ← $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$

→ Indeterminant form is removed Put $x = a$

$\lim_{x \to a} \frac{f''(x)}{g''(x)}$

For More PDFs Visit: LearningMantras.com

cos 0 = ∞

Q. $\lim_{x\to 0} \frac{x - \sin x}{x^3}$ keep on above steps

So

$$\frac{x - \left(x - \frac{x^3}{\lfloor 3} + \dots\right)}{x^3} = \frac{1}{\lfloor 3}$$

(L.H) Rule

$$\lim_{x\to 0} \frac{1-\cos x}{3x^2} \quad \text{(L.H Rule)}$$

LH

$$\lim_{x\to 0} \frac{1-\cos x}{3x^2} \left(\frac{0}{0}\right)$$

$$\frac{1-\cos x}{3x^2} = \frac{1}{2\cdot 3} = \frac{1}{6}$$

$$\lim_{x\to 0} \frac{\sin x}{6x}$$

(L.H Rule)

$$\lim_{x\to 0} \frac{\sin x}{6x} = \frac{1}{6}$$

$$\lim_{x\to 0} \frac{\sin x}{\cos x} \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\cos x}{6} = \frac{1}{6}$$

* Method to handel to determine 0^0 and ∞^0 indeterminant:

$$A = \lim_{x\to 0} (\operatorname{cosec} x)^x$$

$$\ln A = \lim_{x\to 0} x \ln(\operatorname{cosec} x)$$

$$\lim_{x\to 0} \frac{\ln(\operatorname{cosec} x)}{\frac{1}{x}} \left(\frac{\infty}{\infty}\right)$$

$$= \lim \frac{\frac{1}{\operatorname{cosec} x} \times -\operatorname{cosec} x \cdot \cot x}{-\frac{1}{x^2}} = \lim_{x\to 0} \left(\frac{\cos x}{\frac{\sin x}{x}}\right) \cdot x = 0$$

For More PDFs Visit: LearningMantras.com

Q. $\lim_{x\to 0} \sin x^{\tan x}$

Ans $\ln \sin x^{\tan x}$

$\lim_{x\to 0} \tan x (\ln \sin x)$

$\lim_{x\to 0} \sec^2 x \ln \sin x + \tan x \left(\frac{1}{\sin x}\right) \cdot \cos x$

$\lim_{x\to 0} \sec x \tan x \ln \sin x$

$\lim_{x\to 0} \frac{1}{\sin x} \cdot \cos x$ ← $\lim_{x\to 0} (-\sin x \cos x) = 0$

$\frac{\sin x}{-\cot x \operatorname{cosec}^2 x}$

$\lim_{x\to 0} (-\sin x \cdot \cos x) = 0$

$A = e^0 = 1$

Que: $\lim_{x\to 0} \left(\frac{1}{x}\right)^{\tan x}$

$\ln A = \lim_{x\to 0} \ln \left(\frac{1}{x}\right)^{\tan x}$

$\lim_{x\to 0} \tan x \ln \left(\frac{1}{x}\right) =$

$\sec^2 x \cdot \ln \frac{1}{x} + \tan x \cdot \frac{x}{1}$

$\sec 0 \ln \infty + \tan 0$

$= 0$

$\lim_{x\to 0} \frac{-\ln x}{\cot x}$

$\lim_{x\to 0} \frac{-1/x}{-\operatorname{cosec}^2 x} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right) \cdot \sin x = 1 \cdot 0 = 0$

LEARNING MANTRAS

For More PDFs Visit: LearningMantras.com

Que: $\lim_{x\to 0} \frac{e^x \sin x - x - x^2}{x^2 + x - \ln(1-x)}$ $\left(\frac{0}{0}\right)$

$\lim_{x\to 0} \frac{e^x \sin x - x - x^2}{x^2 + x - \ln(1-x)}$

$\lim_{x\to 0} \frac{e^x \sin x + e^x \cos x - 1 - 2x}{2x + 1 + \frac{1}{1-x}}$ Ans $\frac{0}{2} = 0$ Ans

~~$\lim_{x\to 0}$ $\frac{e^x - \sin x - 2}{2}$~~

* Diff. of Determinant:

$$f(x) = \begin{vmatrix} f & g & h \\ u & v & w \\ l & m & n \end{vmatrix}$$

Here f, g, h, u, v, w, l, m, n are either f(x) or x are constant

Rule: $f'(x) = \begin{vmatrix} f' & g' & h' \\ u & v & w \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u' & v' & w' \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u & v & w \\ l' & m' & n' \end{vmatrix}$

Same diff Aplicable to column wise also.

Learning Mantras
Our Guidance, Your Success

For More PDFs Visit: LearningMantras.com