Handwritten Notes

On

Matter Wave

'MATTER WAVE'

Wave Associated With Moving particle.

Wave or, De-brogaly Wave or, *probable Wave.

De-brogali Hypothesis → Every particle present in nature represent dual nature (wave & particle)

⇒ Wavelength of light $\lambda = \frac{h}{p} = \frac{h}{m_{eff}(c)}$

⇒ Wavelength of Moving particle

$\boxed{\lambda = \frac{h}{p} = \frac{h}{m_e v}}$ * $\boxed{m_R = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$ → Rest mass; v → velocity of particle; c → velo of light

$\boxed{\lambda = \frac{h}{m_0 v}\sqrt{1-\frac{c^2}{v^2}}}$ $\boxed{\begin{matrix} v \neq 0 \\ v \neq 0 \end{matrix}}$

* $v = 0$ (Rest) ⇒ $\boxed{\lambda = \infty}$ (not define) * $v = c \Rightarrow \lambda = 0$
* $v <<< c \Rightarrow \frac{v}{c} <<< 1$ (neglect) ⇒ $\boxed{\lambda = \frac{h}{m_0 v}}$
* 'V' in range of 'c' → $\boxed{\lambda = \frac{h}{m_R v}}$

K.E Transfer

$= \frac{1}{2} m_0 v^2 = \frac{p}{2m_0} \Rightarrow p = \sqrt{2 m_0 K.E_{Transfer}}$

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2 m_0 K.E_{Transfer}}}}$$

Work done by pot. on charge particle →

* $\boxed{K.E = q\Delta V}$*

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{m_R v} = \frac{h}{\sqrt{2 m_0 K.E}} = \frac{h}{\sqrt{2 m_0 q \Delta V}}}$$

*** # * If Wavelength of Wave (λ) = size of obstracle ⇒ Differaction take place.
* If Wavelength of Wave (λ) > size of obstracle ⇒ Reflection [In case of light shadow formed]
* If Wavelength of Wave(λ) < size of obstracle ⇒ Rectilinear propagation.

NOTE → * De-brogali principle is applicable on micro as well as macro particle but practically it is proved only for micro particle.

2016 AMU * Wave nature of particle practically proved by diffraction** principle in a devission Germer & G.B Thomson Exp.

*** # Standard Result : → $\lambda = \frac{h}{p} = \frac{h}{m_R v} = \frac{h}{\sqrt{2 m_0 K.E}} = \frac{h}{\sqrt{2 m_0 q \Delta V}}$

|I| → Electron → $\boxed{\lambda_e = \frac{12.27}{\sqrt{\Delta V (volt)}} A^\circ = \frac{12.27}{\sqrt{K.E(ev)}}}$

$\boxed{\Delta V = \frac{(12.27)^2}{\lambda_e^2} = \frac{150}{\lambda_e^2}}$ $\boxed{\Delta V = \frac{150}{\lambda_e^2} \text{ volt}}$ ($A^\circ \rightarrow \lambda_e$), $\boxed{K.E_e = \frac{150}{\lambda_e^2} ev}$ ($\uparrow A^\circ$)

|II| → Proton → $\boxed{\lambda_p = \frac{0.286}{\sqrt{\Delta V (volt)}} A^\circ = \frac{0.286}{\sqrt{K.E (ev)}} A^\circ}$

|III| → Deutron → $\boxed{\lambda_D = \frac{0.202}{\sqrt{\Delta V (volt)}} A^\circ = \frac{0.202}{\sqrt{K.E (ev)}} A^\circ}$

|IV| → α-particle → $\boxed{\lambda_\alpha = \frac{0.101}{\sqrt{\Delta V(\text{volt})}} A^\circ = \frac{0.101}{\sqrt{\frac{K.E}{2}(eV)}}}$

particle	charge	Mass
e^-	$-e$	$m_e = \frac{m_p}{1836}$
p	+e	m_p
d	+e	$2m_p$
α	+2e	$4m_p$

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Ratio of de-brogali wavelength associated with e^-, H^+, deutron & α-particle when it is move with.

|i| → same Momentum |ii| → same velocity |iii| → same K.E |iv| → same pot.

|i| → $\lambda = \frac{h}{p}$ = same ⇒ $\lambda_e : \lambda_p : \lambda_d : \lambda_\alpha = \boxed{1:1:1:1}$

|ii| → $\lambda = \frac{h}{mv} \propto \frac{1}{m}$ ⇒ $\lambda_e : \lambda_p : \lambda_d : \lambda_\alpha = \boxed{\frac{1}{m_e} : \frac{1}{m_p} : \frac{1}{m_d} : \frac{1}{m_\alpha}}$

$= 1840 : 1 : \frac{1}{2} : \frac{1}{4}$

$\boxed{\lambda_e > \lambda_p > \lambda_d > \lambda_\alpha}$ $= \boxed{1800 \times 4 : 4 : 2 : 1}$

|iii| → K.E ⇒ same ⇒ $\lambda = \frac{h}{\sqrt{2mK.E}} \propto \frac{1}{\sqrt{m}}$

$\lambda_e : \lambda_p : \lambda_d : \lambda_\alpha = \boxed{\sqrt{1840 \times 4} : \sqrt{4} : \sqrt{2} : \sqrt{2}}$

|iv| → ΔV ⇒ same ⇒ $\lambda = \frac{h}{\sqrt{2m_0 q \Delta V}} \propto \frac{1}{m}$

de-brogali wavelength of moving e^- is max.

NOTE → In same condition de-brogali wavelength max for photon. & in a neutral particle

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Standard Result for Neutral particle

|I| → Neutron: → $\boxed{\lambda_n = \frac{h}{p} = \frac{h}{m_R v} = \frac{h}{\sqrt{2m_0 K.E}} = \frac{0.286}{\sqrt{K.E(eV)}} A^\circ}$

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|II| → Thermal Neutron: → In a fussion Rkn energy of the neutron is very high that's why Moderator is required for fission, Moderated neutron.

* It behave as a gas molecule. → $\boxed{K.E = \frac{f}{2} KT}$

Boltzmann const.

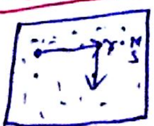
$* K = \frac{R}{N_A} = 1.38 \times 10^{-23} J/K$

f=3 $\boxed{K.E = \frac{3}{2}KT \Rightarrow \lambda = \frac{h}{\sqrt{2m(\frac{3}{2}KT)}}}$

$\lambda = \frac{h}{\sqrt{3mKT}} = \boxed{\frac{28.2}{\sqrt{T(K)}} A^\circ}$

*
|III| → Photon → $\boxed{\lambda = \frac{h}{p} = \frac{h}{2m_e v(c)} = \frac{hc}{E_{ph}} = \frac{12400}{E_{ph}(eV)} A^\circ}$

|IV| → Gas molecule → $\boxed{\lambda = \frac{h}{3\sqrt{\frac{3KT}{m}}}}$ $\boxed{\lambda = \sqrt{\frac{h}{\sqrt{3mKT}}}}$



$\propto \frac{1}{\sqrt{MOT}}$

$\boxed{\lambda = \frac{h}{\sqrt{3\left(\frac{M_W}{N_A}\right)K.T}}}$ $\boxed{V_{R.M.S} = \sqrt{\frac{3KT}{m}}}$

comparison of Electron & photon →

	Electron	photon
[I] → Rest mass →	$m_0 = 9.1\times10^{-31} kg = \frac{m_p}{1840}$	$(m_0)_{ph} = 0$
[II] → R.M.S →	$E_0 = m_0c^2 = 0.51 mev$	$(E_0)_{ph} = 0$
[III] → K.E →	$K.E = \frac{1}{2}mv^2$ $(v<<c)$; $\boxed{K.E = \frac{hv}{2\lambda e}}$	
[IV] → T.E →	$\boxed{T.E = \frac{hc^2}{\lambda e v}}$	$T.E_{ph} = K.E_{ph} = \boxed{\frac{hc}{\lambda}}$

[I] → condition Ist → Electron & photon move with same de-brogli wavelength. *

$\boxed{\lambda e = \lambda ph = \lambda}$ $\boxed{K.E_{ph} = \frac{hc}{\lambda ph}}$ $\boxed{K.E_{e^-} = \frac{hc}{2\lambda e^-}}$ $\boxed{\frac{K.E_{e^-}}{K.E_{ph}} = \frac{hv/2\lambda}{hc/\lambda} = \frac{v}{2c}}$

$v << c \Rightarrow \boxed{K.E_{e^-} << K.E_{ph}}$

[II] → condition 2nd →

$\lambda e^- = \lambda ph \Rightarrow$ compair its total energy.

$\boxed{T.E_{ph} = \frac{hc}{\lambda ph}}$, $\boxed{T.E_e = \frac{hc^2}{\lambda e^2}}$ * $\boxed{\frac{T.E_{e^-}}{T.E_{ph}} = \frac{hc^2/hv}{hc/\lambda} = \frac{c}{v}} > 1$

$\boxed{T.E_{e^-} > T.E_{ph}}$ *

III → condition 3rd ⇒ Eelectron & photon move with same K.E compair its de-brogali wavelength.

AIPMT 2016 * $\boxed{K.E_{ph} = \frac{hc}{Ph}}$ $= \frac{h^2}{2m\lambda^2}$

$* \lambda e = \frac{h}{\sqrt{2mK.E_{e^-}}}$

* $\boxed{\frac{\lambda e^-}{\lambda ph} = \frac{h/\sqrt{2K.E}}{hc/E}}$

* $\boxed{\frac{\lambda e}{\lambda ph} = \frac{1}{c}\sqrt{\frac{E}{2m_0}}}$

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* $\sqrt{E} = c\sqrt{2m_0} \Rightarrow \lambda e^- = \lambda Ph$
* $\sqrt{E} = c\sqrt{2m_0} \Rightarrow \lambda e > \lambda ph$
* $\sqrt{E} = c\sqrt{2m_0} \Rightarrow \lambda e < \lambda Ph$

$\sqrt{E} = c\sqrt{2m_0}$

$E = 2(m_0c^2) = 2\times0.51$

$\boxed{E = 1.02 MeV}$

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* $E = 1.02 MeV \Rightarrow \lambda e^- = \lambda ph$
* $E > 1.02 MeV \Rightarrow \lambda e^- > \lambda ph$
* $E < 1.02 MeV \Rightarrow \lambda e^- < \lambda Ph$

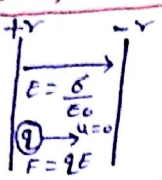
NOTE → * If e^- & photon move $\bar{c}$ same de-brogali wavelength then $K.E_{ph} > K.E_{e^-}$ · but Total energy of photon is less than from e^-

* If e^- & photon move $\bar{c}$ same K.E then de-brogali wavelength depend on magnitude of energy.

*** # Special case →

Case [I] → Motion of charged particle in presence of electric field.

[A] → Uniform Electric field →

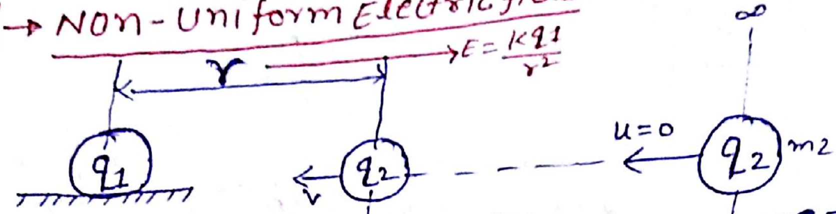


$$\boxed{V = \frac{qEt}{m}}$$

$$* \boxed{P = mv = qEt}$$

$$* \boxed{\lambda = \frac{h}{P} = \frac{h}{qET}}$$

[B] → Non-Uniform Electric field: →



$K \cdot E_r = \frac{1}{2}mv^2$

$P \cdot E_r = \frac{K(q_1)(q_2)}{r}$

$K \cdot E_\infty = 0$

$P \cdot E_\infty = 0$

$$* \boxed{K \cdot E_r = \frac{Kq_1q_2}{r}}$$

$$** \boxed{\lambda = \frac{h}{\sqrt{2m\left(\frac{Kq_1q_2}{r}\right)}}}$$

NOTE → In presence of electric field, de-broglie wavelength of charged particle is change at every instant.

Case [II] → Motion of charge particle in presence of Mag. field →

Lorentz force → $\boxed{F = qVB\sin\theta}$

$* \; V = 0 \Rightarrow F = 0 \Rightarrow \text{Rest} \Rightarrow \lambda = \infty$

[A] → $\theta = 0°$

q →V →B

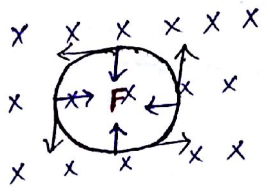
$$\boxed{F = 0 \Rightarrow a = 0 \Rightarrow V = \text{const} \Rightarrow \lambda = \frac{h}{mv} = \text{const.}}$$

[B] → $\theta = 180°$

V ← q →B

$$\boxed{F = 0,\ a = 0,\ V = \text{const},\ \lambda = \frac{h}{mv} = \text{const}}$$

[C] $\theta = 90°$



$$\boxed{\vec{F} \perp \vec{V} \Rightarrow W = 0 = \Delta K \cdot E}$$

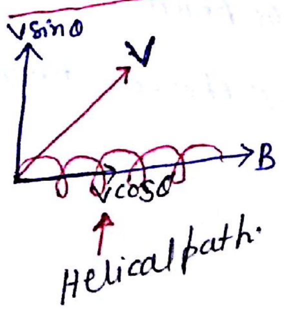
$* K \cdot E = \text{const}$, $*$ Speed = const.

$F = qVB\sin 90° = \frac{mv^2}{r}$

$$\boxed{P = mv = qBr}$$

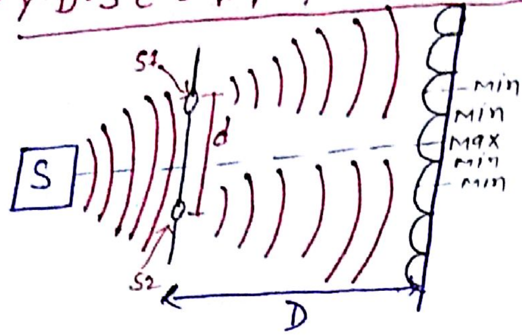
$$* \boxed{\lambda = \frac{h}{P} = \frac{h}{qBr} = \text{const.}}$$

[D] → $\theta \neq 90°, 0°, 180°$



$$* \boxed{\begin{aligned} \lambda_{\parallel} &= \frac{h}{mv\cos\theta} = \text{const.} \\ \lambda_{\perp} &= \frac{h}{mv\sin\theta} = \text{const.} \\ \lambda &= \frac{h}{mv} = \text{const.} \end{aligned}}$$

Case (III) → Y·D·S·C Exp perform with Matter Wave



* $\boxed{\beta = \frac{\lambda D}{d} = \frac{h}{\sqrt{2m_0 q \Delta V}}\frac{D}{d} \propto \frac{1}{\sqrt{\Delta V}}}$

* $\Delta V \uparrow \Rightarrow \beta \downarrow$
  $\Delta V \downarrow \Rightarrow \beta \uparrow$

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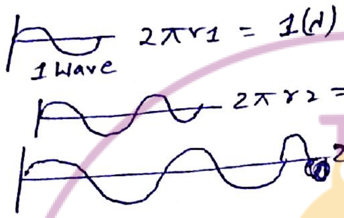
Explanation of Rutherford Drawback & Bohr Quantisation condn →

* (i) → Electron Rotate around the nucleus in a stable path that's why its Matter Waves is also move in a same circular orbit. When Wave are bounded it produce standing wave that's why energy of Rotating e^- remain same & e^- Rotate in stable circular orbit. AIR

(ii) → Length of its path is complete multiple of Wavelength. $\boxed{2\pi r = n\lambda}$

* $n = 1 \Rightarrow$ 1 Wave $2\pi r_1 = 1(\lambda)$
* $n = 2 \Rightarrow$ $2\pi r_2 = 2(\lambda)$
* $n = 3 \Rightarrow$ $2\pi r_3 = 3(\lambda)$.



$2\pi r = n\lambda$
$2\pi r = n\left(\frac{h}{mv}\right)$
* $\boxed{J = mvr = \frac{nh}{2\pi}}$

Devison – Germer Exp → practically prove Wave nature of particle.

*Construction & Working → Electron gun based on thermeonic emmison principle. Electron beam incident on 'Ni' crystal & scatered it is collected by ionising chamber & practically calculate Max. Intensity condition.

* Thermionic emission → Emission of e^- takes place by the thermal energy.
* Photoelectric emmision → Electron emission takes place with energy of photon.
* Field emission → Electron emission take place with external electrifield. $(10^8 \, Vm^{-1})$

Exp. Result → Max intensity is formed at 50° deviation & at 54 volt electron gun pot.

(i) → $\boxed{\Delta V = 54 \text{ Volt}}$*
$\theta = 50° \Rightarrow \phi = 90° - \frac{50}{2} = \boxed{65}$*

AIEEE/AIPMT
(ii) → * $\boxed{\begin{aligned} D\sin\theta &= n\lambda \\ 2d\sin\theta &= n\lambda \end{aligned}}$

λ = Wavelength, n = order of differaction = 1, 2, 3 - - -

$D\sin\theta = n\lambda$
$D_{Ni} = 2.15 A°$
$\lambda_{max} = \frac{D\sin\theta}{n_{min}} = 1$
$\lambda_{max} = D\sin\theta$*
$= \boxed{2.15\sin 50}$

* $\lambda_{max} = 1.66 A°$
* $\boxed{\lambda_{DB} = \frac{12.27}{\sqrt{\Delta V(\text{volt})}} A°}$
$= \frac{12.27}{\sqrt{54}}$
* $\boxed{\lambda_{DB} = 1.65 A°}$

$\boxed{\lambda \text{ practical value} \simeq \lambda_{DB} \text{ value}}$