Handwritten Notes

On

Matrices

MATRICES

* multiplication

(1) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}_{2\times 2} \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 6 & 1 \\ -10 & -9 \end{bmatrix}_{2\times 2}$

Que:
(2) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$= \begin{bmatrix} -1 & 4 & 5 \\ 9 & 8 & -1 \end{bmatrix}_{2\times 3}$

* **Properties:**

if $AB = C$ (where A, B are sq. matrix)

det. A. det. B = det C or $|A||B| = |C|$

Q. Let $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$

$B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$

if $(A+B)^2 = A^2 + B^2$ find a, and b

$\therefore (A+B)^2 = A^2 + B^2$ $\qquad AB = -BA.$

$AB = \left[\cancel{\begin{matrix} 1 & -1 \\ 2 & -1 \end{matrix}}\right. = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$= \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix}.$

$= \cancel{a-b}.$

$BA = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} a+2 & -a+1 \\ b-2 & -b+1 \end{bmatrix}$

$\begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} = -\begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix}$

$= \begin{bmatrix} -a-2 & a+1 \\ -b+2 & b-1 \end{bmatrix}$

Q. if $X_{m\times n} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$

then find X.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

let $X = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times 3}$$

$= [$

let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$

$n = 2 \quad m = 2$

$$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$$

$a + 4b = -7$

$2a + 5b = -8$

$c + 4d = 2$

$2c + 1d = 4.$

AB+BA = Sym
AB−BA = Skew.

Q:1 1, 2, 3, 4, 5, 6, 7, 8, 9
10, 15, 16,
S−1 = 1, 2, 3, 0,

Q. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$f(x) = x^2 - 4x + 7$

then show that $f(A) = 0$

$= 4+3 = 7$

$= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$f(A) \Rightarrow A^2 - 4A + 7$

$\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7I$

$\begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\begin{bmatrix} \cancel{6} & \cancel{-12} \\ 0 & \cancel{-7} \end{bmatrix}\begin{bmatrix} \cancel{0} & \cancel{12} \\ \cancel{0} & \cancel{0} \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{null}.$

Q. Expressed as sum of two matrix. one symmetric and other one is skew sym.

$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} \quad A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$\frac{1}{2}(A + A') = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$

$\frac{1}{2}(A - A') = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$A = \frac{A + A'}{2} + \frac{A - A'}{2}$

H. J.M and J.A.

Q. If Matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$

is sym matrix then find x, y, z, t

$AA' = I$

$\frac{A+A'}{2} = A + A'$

$A = \begin{bmatrix} 5 & 2 & x \\ y & 2 & -3 \\ 4 & t & -7 \end{bmatrix} + \begin{bmatrix} 5 & y & 4 \\ 2 & z & t \\ x & -3 & -7 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 10 & 2+y & x+4 \\ y+2 & 2z & -3+t \\ 4+x & t-3 & -14 \end{bmatrix}$

$= x, y, z, t$

$4 \quad 1 \quad 1 \quad -3$

$\quad\;\; 2$

5

9

Q. if $|A| = 2$ where A is sq matrix of order 3. then find ~~adj~~ adj A

(i) $|adj\ A| = 2^{3-1} = 2^2 = 4$

(ii) $|adj\ adj\ A| = 2^{(3-1)^2} = 2^{(2)^2} = 2^4 = 16$

(iii) $|adj\ adj\ adj\ A| = 2^{(3-1)^3} = 2^{(2)^3} = 2^8 =$

Que! Construct 2×3 matrix

$a_{ij} = \left|\frac{i-2j}{3}\right| =$

$= \left| \begin{matrix} \cancel{a_{11} = \frac{-1}{3} \quad a_{12} = \frac{-3}{3} = -1 \quad a_{13} = \frac{-5}{3}} \\ a_{21} \end{matrix} \right.$

$= \begin{vmatrix} -\frac{1}{3} & -1 & -5/3 \\ 0 & -\cancel{\frac{2}{3}} & -4/3 \\ \frac{1}{3} & \frac{-1}{3} & \frac{-3}{3} = -1 \end{vmatrix} = \begin{pmatrix} \frac{1}{2} & 3/2 & 5/2 \\ 0 & 1 & 2 \end{pmatrix}$

Q. for $\theta = \frac{3\pi}{5}$

let $B = [b_{ij}]$ B is a sq-matrix of order 2 such that

$$b_{ij} = \begin{cases} \cos\theta & i = j \\ \cos\, j\left(\frac{\pi}{2} + \theta\right) & i > j \\ \sin\left(\frac{j\pi}{2} - \theta\right) & i < j \end{cases}$$

$1 > 2$

a_{12}

$1 < 2$

then find $Tr(B)^8$

$A = $

$$B = \begin{bmatrix} \cos\theta & & \\ & \cos\theta & \\ & & \cos\theta \end{bmatrix}$$

$$B = \begin{bmatrix} \cos\theta & \cancel{\cos\left(\frac{3\pi}{2}+\theta\right)} \ \sin\left(\frac{\pi}{2}-\theta\right) \\ \cos\left(\frac{\pi}{2}+\theta\right) & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\frac{3\pi}{5} & \sin\left(\frac{\cancel{2}\pi}{\cancel{2}}-\theta\right) \\ \cos\left(\frac{\pi}{2}+\theta\right) & \cos\frac{3\pi}{5} \end{bmatrix}$$

$\cancel{\cos}$
$\pi - \frac{\cancel{2}2\pi}{5}$
$\pi - \frac{2\pi}{5}$

$$= \begin{bmatrix} \cos\frac{2\pi}{5} & \sin \frac{3\pi}{5} \\ -\sin\frac{3\pi}{5} & \cos\frac{3\pi}{5} \end{bmatrix}$$

$$= \cos\frac{2\pi}{5} \times \cos\frac{3\pi}{5}$$

$$B^2 = B\cdot B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$B^5 = \begin{bmatrix} \cos 3\theta & \sin 5\theta \\ -\sin 5\theta & \cos 5\theta \end{bmatrix}$$

$$Tr(B^5) = 2\cos 5\theta$$
$$= 2\cdot\cos 5\,\frac{3\pi}{5}$$
$$= 2\cos(3\pi)$$
$$= -2.$$

Que: Solve

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & -1 \\ 0 & 4 \end{bmatrix}$$

$$2A - B + 3I =$$

$$\begin{bmatrix} 2 & 4 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 7 & 5 \\ -2 & 5 \end{bmatrix} =$$

Que: Solve the eqn. $[x \quad 2y \quad 3z)$

$[x \quad 2y$

$[x \quad 2y \quad 3z] - 2[y \quad z \quad -x] + 3[-z \quad x \quad y]$

$= [-12 \quad 1 \quad 17]$

$[x \quad 2y \quad 3z] - [2y \quad 2z \quad +2x] + [-3z \quad 3x \quad 3y]$

$$\left[\begin{array}{l} x - 2y - 3z = -12 \quad -i, \\ 2y - 2z + 3x = 1 \\ 3z - 2x + 3y = 17. \end{array}\right]$$

$x - 2y - 3z = -12$

$3x + 2y - 2z = 1$

$= 4x - 5z = -11 \quad -ii$

$-2x + 3y + 3z = 17$

$D_1 =$

D, D_1, D_2, D_3

$x = \frac{D_1}{D}.$

Q. A matrix has 12 elements. find no. of possible order it can have

1×12
2×6
3×4
$\cancel{6\times2}$
4×3
6×2
12×1

$= 6$

Q. if $A+2B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

and $2A - B = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 5 & 7 \\ 4 & -1 & 1 \end{bmatrix} \times 2$

then find trace A ~~and~~

$3\,Tr(A) + 5(Tr(B)) \cdot 2$

$A + 2B = \quad A + 2B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

$4A - 2B = \begin{bmatrix} 4 & 0 & 2 \\ 6 & 10 & 14 \\ 8 & -2 & 2 \end{bmatrix}$

$= 5A = \begin{bmatrix} 5 & 2 & 5 \\ 5 & 14 & 19 \\ 8 & -2 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2/5 & 1 \\ 1 & 14/5 & 19/5 \\ 8/5 & -2/5 & 3/5 \end{bmatrix}$

$2A - B = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 5 & 7 \\ 4 & -1 & 1 \end{bmatrix}$

$\frac{17}{5}$

Ans: $Tr(A+2B) = Tr\begin{bmatrix} 1 & 2 & 2 \\ -1 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

$Tr A + 2\, Tr B = 6.$ — (i)

$Tr(2A-B) = Tr\begin{pmatrix} 2 & 0 & 1 \\ 3 & 5 & 7 \\ 4 & -1 & 1 \end{pmatrix}$

$2x - y = 8$ — (ii)

$x + 2y = 6 \qquad 4x - 2y = 16$

$5x = 22$

$x = 22/5$

$y = 2x - 8 = 2 \cdot \frac{22}{5} - 8 = \frac{44-40}{5} = \frac{4}{5}$

Q. $\alpha, \beta, \gamma \in R$ and

$A = \begin{bmatrix} \alpha^2 & 6 & 8 \\ 3 & \beta^2 & 9 \\ 4 & 5 & \gamma^2 \end{bmatrix} \qquad B = \begin{bmatrix} 2\alpha & 3 & 5 \\ 2 & 2\beta & 6 \\ 1 & 4 & 2\gamma-3 \end{bmatrix}$

if $Tr(A) = Tr(B)$ then find $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$Tr(A) = \alpha^2 + \beta^2 + \gamma^2 \qquad Tr(B) = 2\alpha + 2\beta + 2\gamma - 3$

$\alpha^2 + \beta^2 + \gamma^2 = 2\alpha + 2\beta + 2\gamma - 3$

m > order

$\alpha^2+\beta^2+\gamma^2 = 2\alpha+2\beta+2\gamma-3$

$\alpha^2-2\alpha+\beta^2-2\beta+\gamma^2-2\gamma = -3$

$\alpha^2-2\alpha+1+\beta^2-2\beta+1+\gamma^2-2\gamma+1 = 0$

$(\alpha-1)^2+(\beta-1)^2+(\gamma-1)^2 = 0$

$\alpha = 1,\ \beta-1=0,\ \gamma-1=0$

$\alpha = 1 = \beta = \gamma$

$A^2 = A$

Q. if A is an idempotent non zero matrix and I is an identity matrix of same order and find $n \in N$ such that

$(A+I)^n = I+127A$

$(A+I)^n = {}^nC_0 A^n + {}^nC_1 A^{n-1} + \dots {}^nC_n A^n$

$(A+I)^n = I+127$

$I+(2^n-1)A =$

$2^n-1 = 127$

$2^n = 128$

$= 2^7$

$n = 7$ Ans

$2^n-1 = 127$

$^{n}C_0 = 1$

$^{n}C_1 = n.$

Q. (i) Show that $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ can be decomposed as sum of a unit and a nilpotent matrix.

(ii) and hence find the value of $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{2007}$ (1)

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} =$ $A + A' =$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} =$

$A = I + M$

$M^2 = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = I.$

M is nilpotent.

$= \cancel{\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{2007}} = \cancel{(1)^{2007}} = \cancel{A}$ $\cancel{\begin{bmatrix} 1^{2007} & 0^{2007} \\ 2^{2007} & 1^{2007} \end{bmatrix}} = \cancel{\begin{bmatrix} 1 & 0 \\ 2^{2007} & 1 \end{bmatrix}}$

$A^{2007} = (I + m)^{2007}$

$= {}^{2007}C_0\, I^{2007} + {}^{2007}C_1\, I^{2006} M + {}^{2007}C_2\, I^{2005} M^2 \ldots$ — high power 0/m

$= 1 \cdot I + 2007$

$A^{207} = I + 2007M.$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 4014 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4014 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q. A, B, C are given sq. matrix such that

$AB = 0$ & $BC = I$

then Proff that $(A+B)^2 (A+C)^2 = I$

$AB = 0$

$C(AB) =$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$ABC = OC = 0$

$A(BC) = 0$ (Associative Property).

$AI = 0$

$\boxed{A = 0}$

LHS: $(0+B)^2 (0+C)^2$

$= (BC)^2 = I^2 = I$ Wrong

$= BBCC$

$= B(BC)C$

$= BIC$

$= (BI)C$

$= BC = I$ An

~~D-1~~ : 11, 12, 13, 14, 17, 18, 19, 20

Q-1 : 4, 5, 6, 9, 11, 12, 13, 15, 17,

J. M : 1, 2, 3, 4, 5, 6.

Que: A is sq. matrix of order 3. then value of

$|(A - A')^{2017}| = 0$ T/F.

True.

order is odd $= 0$.

$(A - A')^{2018} = 0$ $|A - A'| |(A - A')| \cdots |(A - A')|$

$= 0$

② Use : $|A| \neq 0$

Que: if $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ then P.T $5A^{-1} = A^2 + A - 5I$.

$|A - \lambda I| = 0$

$= (A - \lambda I) = 0$

$= \left(\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right) = 0$

$1 - \lambda = 0$
$\lambda = 1$

$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & -1-\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix}$

$= \cancel{1 - \lambda ((-1-\lambda) - (-1-\lambda))}$

$= 1 - \lambda(\cancel{-1 - \lambda + 1 + \lambda})$

$= \lambda^3 + \lambda^2 - 5\lambda - 5 = 0$ satisfied by matrix A^3

$$(A^{-1}A)A + (A^{-1}A) - 5(A^{-1}A) - 5(A^{-1}I) = 0$$

$$A^2 + A - 5I - 5A^{-1} = 0$$

* $\alpha, \beta > 0$

$$AM = \frac{\alpha + \beta}{2}$$

$$GM = (\alpha\beta)^{1/2}$$

$$\frac{2}{H} = \frac{1}{\alpha} + \frac{1}{\beta}$$

$$A \geq G \geq H$$

* $\alpha, \beta, \gamma > 0$

$$A = \frac{\alpha + \beta + \gamma}{3}$$

$$G = (\alpha\beta\gamma)^{1/3}$$

$$\frac{3}{H} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

$$A \geq G \geq H$$

If any two mean (it mean $\alpha = \beta (= \gamma)$) same

Q. if A and B are sq. matrix of order 3 such that $|A| = -1$

$|B| = 4$

then find $|3AB|$

$= |3AB| = \cancel{3|AB|} \cdot 3|A| \times |B| = 3 \times (-1) \times 4$ $= \cancel{-12}$

$= 3|AB|$

$|3A| \cdot |B|$

$= 3^3 |A| |B|$

$= 3^3 (-1) 4$

$= -27 \times 4$

$$\boxed{|kA| = k^n |A|}$$

A, B, and C are 3rd order de.

$|A| = -1$, $|B| = 4$, $|C| = 2$

then find $|3AB^2C^{-1}| =$

$|3A| \, |B^2| \, |C^{-1}| =$

$3^3 |A| \, |B^2| \, |C^{-1}|$

$= 27 |A| \, |B|^2 \, |\frac{1}{C}|$

$= 27 \times -1 \times 4^2 \times \frac{1}{2}$

$$\boxed{C^{-1} = \frac{1}{|C|}}$$

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Qu: if Diag. element Diag(α, β, γ) of a non singular diagm. of order 3 are root of the eq.

$$x^3 - 9x^2 + kx - 27 = 0 \qquad k \in R.$$

Where

$\alpha, \beta, \gamma > 0$. then find such Matrix.

Ans

$= (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 9$

$\alpha\beta + \beta\gamma + \gamma\alpha = k$

$\alpha\beta\gamma = 27$

$A = \frac{\alpha + \beta + \gamma}{3} = 3$

$G = (\alpha\beta\gamma)^{1/3}$

$= (27)^{1/3} = 3$

$\therefore A = G$

$\alpha = \beta = \gamma = 3$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\begin{vmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{vmatrix} \neq 0.$

$x^3 - 9x^2 + kx - 27.$

$x^3 + 9x^2 - kx = 27$

$x(\underline{x^2 - 9x} + k) = 27.$

$x(x^2 - 3x - 6x + k$

$AM = \frac{\alpha + \beta}{2},$

$A = \frac{\alpha + \beta + \gamma}{3}$

Q. A is sq. mat order ≤ 4.

Q. A is sq. matrix of order ≤ 4
Such that $|A - A'| \neq 0$
and $B = \text{adj } A$.
if $|A| = 3$ then find $Tr(\text{adj}(AB))$

$B = \text{adj } A$. $\qquad |A| = 3$.

$Tr(\text{adj}(AB)) = Tr((\text{adj } B)(\text{adj } A))$.

$\cancel{Tr((B)(}\ Tr((\text{adj } B)(B))$.

$Tr((\text{adj } B)(\text{adj } B))$.

$Tr((\text{adj } B)^2) = |B| I_n^2$

$= 3^2 = 9$.

Sol: $A = 2$
$n = 2$.

$|A| = 3$
$Tr(\text{adj}(AB))$

$= Tr(\text{adj}(A \text{ adj } A))$
$= Tr \text{ adj}(|A| I)$
$= Tr(|A|^{n-1} \text{ adj } I)$

$= Tr(3I) = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 6$.

$$\boxed{\text{adj}(kA) = k^{n-1}(\text{adj } A)}$$

J.M = 7, 8, 10, 11, 12, 13, 14, 15, 16,

J.A ⇒ 1, 4(B), 8, 9, 10, 15, 16.

Q4 skew = C)'.

Q. C is skew symmetric matrix of order 3.
X is 3×1 column matrix
then Proof that

$X'CX$ is singular.

(1=0)

C = skew sym.

$X = \quad 3\times1$

$= 0$

$X'CX = 0.$

P.T $\quad X'CX = [K]_{1\times1}$

$1\times3 \quad 3\times3 \quad 3\times1$

$(X'CX)' = [K]$

$X'C'(X')' = [K]$

$X'(-C)X = [K]$

$X'CX = [K]$

$-[K] = [K]$

$[-K] = (K)$

$-K = K$

$K = 0$

Q. Sol

$x+2y+3z=2$
$2x+4y+5z=3$
$3x+5y+6z=4$

$\cancel{A+XB}$ $\quad AX=B.$

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix} \Big| = -1-2(-3)+3(-2)$$
$$-1+6-6 = -1$$

$$\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}2\\ 3\\ 4\end{bmatrix}$$

$$\boxed{\begin{array}{l} x+2y+3z=2\\ 2x+4y+5z=3\\ 3x+5y+6z=4\end{array}}$$

$$\boxed{X=A^{-1}B}$$

A (adj) =

$$Adj\,A=\begin{bmatrix}-1 & +3 & -2\\ +1 & -3 & +1\\ -2 & +1 & 0\end{bmatrix}$$

$$= A^{-1}=\begin{bmatrix}1 & 3 & 2\\ 3 & 3 & 1\\ 2 & 1 & 0\end{bmatrix}$$

$$X = -\begin{bmatrix}-1 & 3 & -2\\ 3 & -3 & 1\\ -2 & 1 & 0\end{bmatrix}\begin{bmatrix}2\\ 3\\ 4\end{bmatrix} = \begin{bmatrix}+1\\ -3\\ 1\end{bmatrix}$$

$x=-1$
$y=-3$
$z=1$

Determinant

J.M

Q.6

$$x + \alpha y + \alpha^2 z = 1$$
$$\alpha x + y + \alpha z = -1$$
$$\alpha^2 x + \alpha y + z = 1.$$

$$D = \begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$= x + \cancel{\alpha y} + \alpha^2 \quad D = (\alpha^2 - 1)^2 = 0 \qquad \alpha \Rightarrow \pm 1$

$$D_1 = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ -1 & 1 & \alpha \\ 1 & \alpha & 1 \end{vmatrix} = 1 - \alpha^2 - \alpha(-1$$
$$= \alpha^3 + \alpha^2 - \alpha - 1$$
$$\alpha = 1, \; -1 + 1 + 1 - 1 = 0$$

$$D_2 = \begin{vmatrix} 1 & 1 & \alpha^2 \\ \alpha & -1 & \alpha \\ \alpha^2 & 1 & 1 \end{vmatrix} = 2\alpha^3 + \alpha^2 - 2\alpha - 1$$
$$\alpha = -1, \quad -2 + 1 + 2 - 1 = 0$$

$$D_3 = \begin{vmatrix} 1 & \alpha & 1 \\ \alpha & 1 & -1 \\ \alpha^2 & \alpha & 1 \end{vmatrix} = (1+\alpha)^2(1-\alpha)$$

Q. A Sq. matrix of order 4. such that $|A| = 2$ then find $|Adj\ A|$

(i) $|Adj\ A| = 2^{4-1} = 2^3$

(ii) $|adj(adj\ A)| = 2^{(4-1)^2}$

(iii) $|adj\ adj\ (adj\ A)| = 2^{(4-1)^3}$

Q. find $|adj(K I_n)|$ $= |K^{n-1} adj\ I|$

$= \cancel{|K^{n-1}| |adj\ I_n|}$

$=$

$\cancel{|K^{n-1}|} |I|^{n-1}$

$|K^{n-1} I_n| = \begin{vmatrix} \lambda & 0 & 0 & - & - \\ 0 & \lambda & - & - \\ 0 & 0 & \cdots & \lambda \end{vmatrix}$

$= \lambda^n |I_n|$

$= (K^{n-1})^n$

$K^{n(n-1)}$

$\cancel{|K^{-1}|}$

$|K^{-1}| |adj\ I_n|$

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Q. If P is orthogonal matrix and A is Involutory ($A^2 = I$). if Q is PAP' and $X = P'Q^3P$ then find X inverse.

$PP' = P'P = I$

$A^2 = I$

$Q = PAP'$

$X = I^3$ $X = P'P^3A^3(P')^3P$ ~~X~~

$X =$

$= P'QQQP$

$= P'PAP'.PAPQ\ PAP'P$

$= (IA)(IA)(IA)I = A^3 = A^2A = IA = A$.

$X = P'Q^3P$.

$X = Q^3 I$.

$X = Q$.

$X = PAP'$

$X = AI$

$X = A$.

$X^{-1} = A^{-1}$

$(A+B)^2 = (A+B)(A+B)$

$A^2 + B^2 + \cancel{2}AB - \quad X = A$

Q. $A = \begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix}$ $= 2$

$\begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix} = \begin{vmatrix} 36+22 & 66+44 \\ 12+8 & 22+16 \end{vmatrix} =$

then find $A = \cancel{20}$

$|A^{2005} - 6A^{2004}|$

$A^{2004}(A' - 6A)$

$A^{2004} \cdot |A^{-1}$

$= A^{2004} |A - 6|$

$A^{2004} |A - 6|$

$A^{2004} |A - 6|$

$|A|^{2004} \, |A - 6I|$

$= 2^{2004}$

$A - 6I = \begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

$= \begin{vmatrix} 0 & 11 \\ 2 & -2 \end{vmatrix}$

$2^{2004} \begin{vmatrix} 0 & 11 \\ 2 & -2 \end{vmatrix} = -22 \times 2^{2004}.$

Q. A and B are sq. matrix of order ~~three~~ 3.

$|A| = 2 \quad , |B| = 1$

then find $|A^{-1} (\operatorname{adj} B^{-1})(\operatorname{adj} 2A^{-1})|$

$$|kA| = k^n |A|$$

$$|(adj\ A^{-1})| = |(adj)^{-1}|$$

Ans $|A| = 2 \quad |B| = 1 \quad AA^{-1} = I$

$$|A^{-1}(adj\ B^{-1})(adj\ 2A^{-1})|$$

$$= |A|^{-1} |adj\ B^{-1}| \, |adj\ 2A^{-1}|$$

$$\frac{1}{|A|} |adj(adj\ B)| \, |2^2 (adj\ A^{-1})|$$

$$= \frac{1}{|A|} \cdot |B|^{(n-1)^2} \cdot 4^3 |adj\ A^{-1}|$$

$$= \frac{1}{|A|} \cdot |B|^{(n-1)^2} \cdot 4^3 |(adj\ A)^{-1}|$$

$$\frac{1}{|A|} \cdot |B|^{(n-1)^2} \cdot 4^3 \frac{1}{|adj\ A|}$$

$$= \frac{1}{|A|^3} \cdot |B|^4 \cdot 64 = \frac{1}{8} \times 1 \times 64 = 8 \text{ Ans}$$

$$~~A~~ \; |A^{-1}(adj\ B^{-1})(adj$$

$$|A|^{-1} |(adj\ B^{-1})(adj\ 2A^{-1})|$$

$$|A|^{-1} \; adj \frac{1}{|B|} \quad adj \frac{1}{2A}$$

$$\frac{1}{2} \times 1 \times \frac{1}{4} = \frac{1}{8}.$$

$$B^{-1} = \frac{adj\ B}{|B|} = adj\ B.$$

Que: Find matrix A. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times 2} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}_{2\times 2} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

$$\times \quad \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix}_{2\times 2} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & 7 \end{bmatrix}.$$

$$A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 1 & -1/2 \end{bmatrix} \qquad A^{-1} = \times$$

$$A = \begin{bmatrix} 1/3 & 2/4 \\ 5 & 6 \end{bmatrix}$$

Revise Vector

H.W $\left\{ \frac{J - A}{8-1} \right\}$

$X A Y = M$

$|X| = 1$

$|Y| = -1$

$(X^{-1}X) A (Y Y^{-1}) = X^{-1} M Y^{-1}$

$A = \underbrace{\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}}_{1} \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \underbrace{\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}}_{-1}$

Q. Cofactor of elements of Diagonal matrix A of order 3 are Roots of eq.

$$x^9 + kx^8 - 16x^6 = 0 \qquad k \in R.$$

$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ Find $|A| = 2$

$a \to bc$

$b \to ac$

$c \to ab.$

$\begin{vmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{vmatrix}$

$bc(ab - ac)$

$acb^2 - abc^2.$

$x^6(x^3 + kx^2 - 16) = 0$

$x^9 + kx^8 - 16x^6 = 0,$

$x^6(x^3 + kx^2 - 16) = 0$

root 6 = 0

$\begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix}$

$x^3 + kx^2 - 16 = 0$ — ab, bc, ca.

$ab \cdot bc \cdot ca = 16$

$abc = 4.$

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$Adj(adj A) = |A|^{n-2} A.$

formula.

Q.

(1) Work A is done by m ways.
" B " " n ways.
" C " " r ways.

Work is finished only when work A, B, C or all of them is completed. then it can be done by
$= m \times n \times r$ ways

Que: A is sq. matrix of order 3 whose element are Real no. and

$$adj(adj(adj A)) = \begin{bmatrix} 16 & 0 & -3 \\ 0 & 4 & 0 \\ 0 & 3 & 4 \end{bmatrix} = 16(16)-0$$

$=$

16, 16, 96, 16x, 256, 256

then find Adjoint A. or A^{-1}

$adj(adj(adj A)) = x$

$A \longrightarrow adj A.$

$adj(adj(adj(adj A)) \to x$

$adj(adj\,\underbrace{(|A|^{n-2} A)}_{A}) = x$

$\left| |A|^{n-2} A \right|^{n-2} A^{n-2} A. \quad = |A^2|^2 \cdot A^2 =$

$A^4 = A \qquad A^4 = |$

=

J- Advanced.

$$| \quad | = \begin{vmatrix} 16 & 0 & -3 \\ 0 & 4 & 0 \\ 0 & 3 & 4 \end{vmatrix}$$

$$|A|^{(n-1)3} = 16 \cdot 4 \cdot 4$$

$$|A|^{8} = 2^{4} \cdot 2^{2} \cdot 2^{2} = 2^{8}.$$

$$|A| = 2.$$

$$adj(adj\,A) = |A|^{n-2} A.$$

$$adj\,adj\,adj\,A = |adj\,A|^{1} (adj\,A)$$

$$= |A|^{2}\, adj\,A$$

(Given) $= 4\, adj\,A$

$$adj = \begin{bmatrix} 4 & 0 & -3/4 \\ 0 & 1 & 0 \\ 0 & \frac{2}{4} & 1 \end{bmatrix}$$