Handwritten Notes

on

Laws of Motion

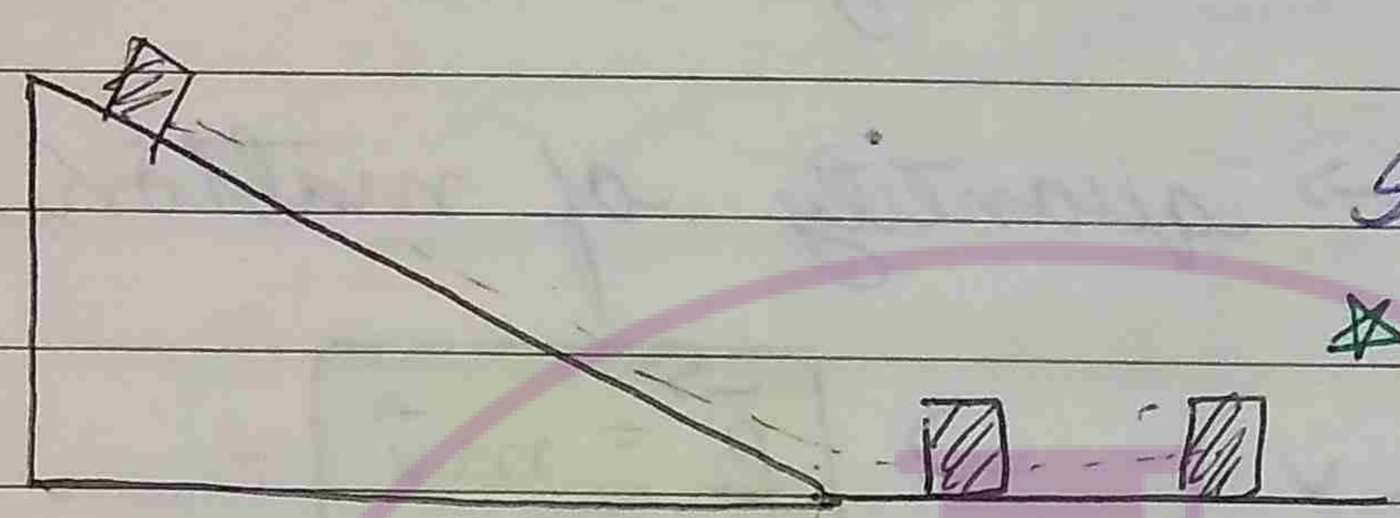
⇒ Pulling of a rope is due to electromagnetic force.

- Earth is assumed inertial frame.
- Three concepts from first law : force, inertia, frame of reference

Force and laws of motion

Newton's laws of motion

1st law

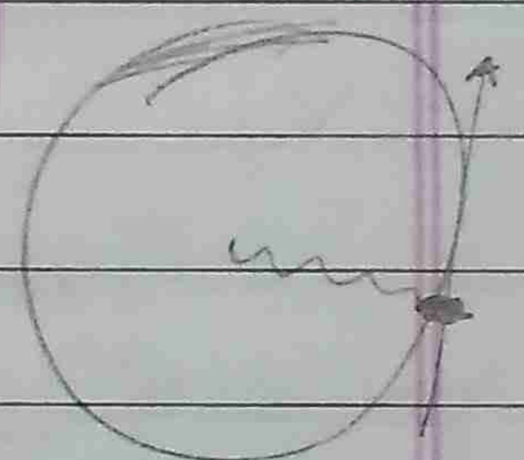


Inertial Mass

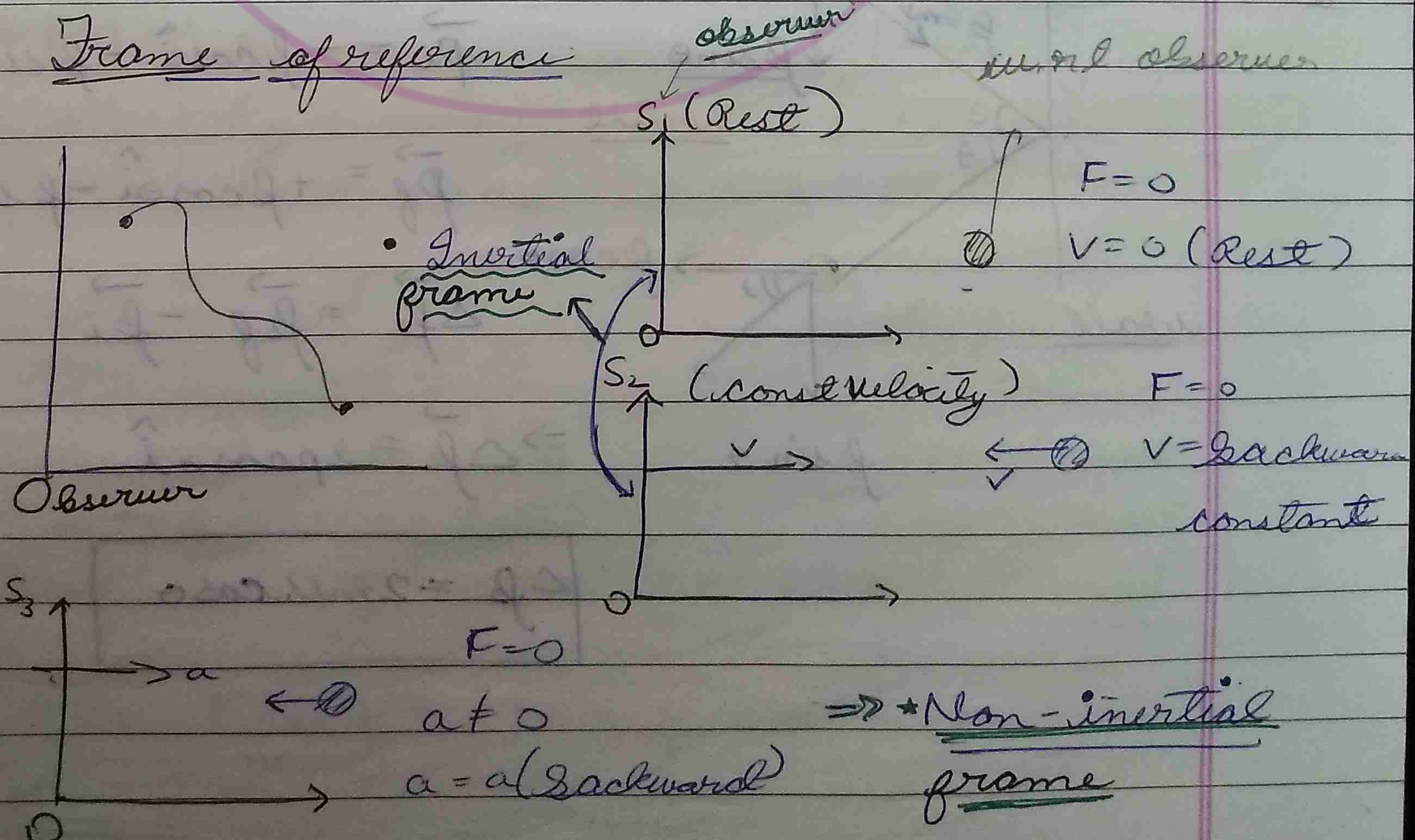
* Inertia ∝ Mass

- A body at rest remains at rest and if in motion, it remains in motion on a straight path with constant velocity until an external force is applied on the body.

Inertia → Inability
- of rest
- of motion
  - of speed
  - of direction



Frame of reference



★ direction of momentum is same as direction of velocity.

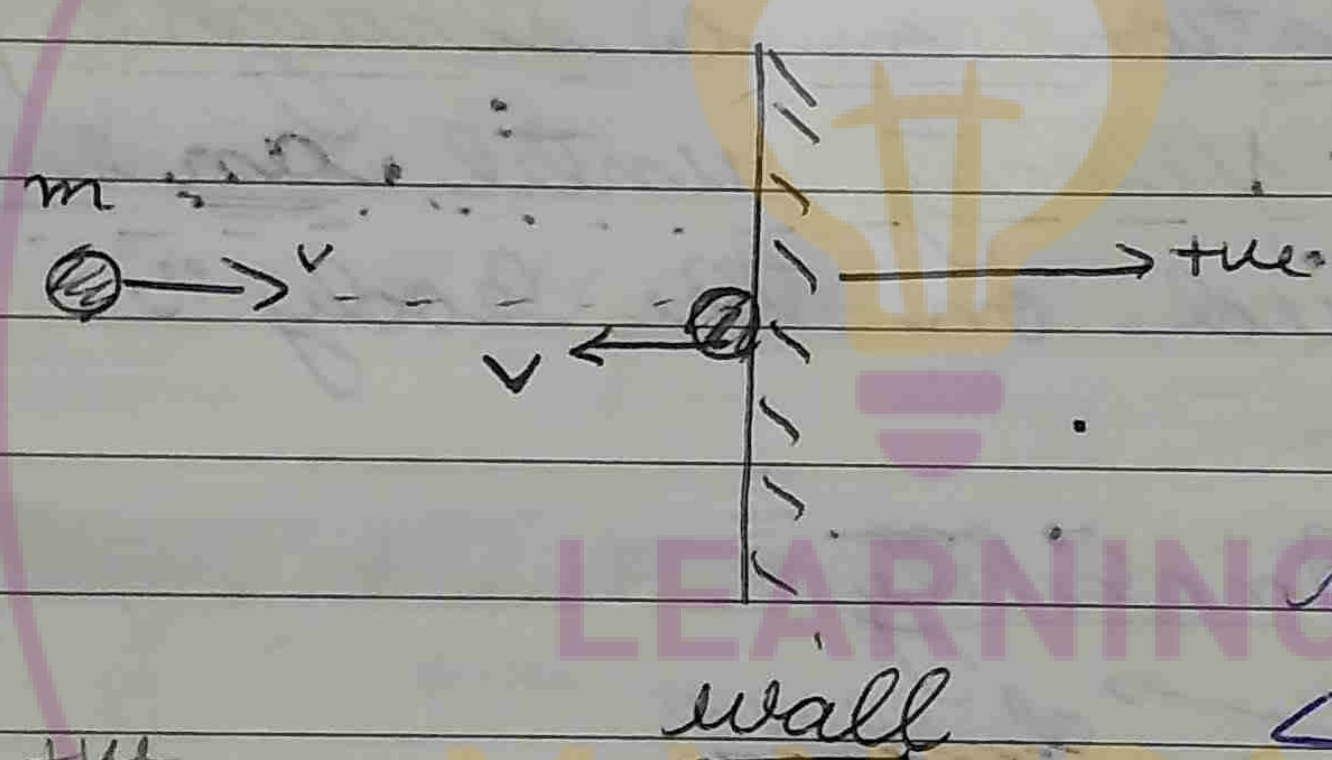
2nd law

- Force ∝ Rate of change in momentum
- Direction of force → in the direction of change in momentum

Momentum (p) → quantity of motion.

$p \propto m$ $p \propto v$ ⟹ $\vec{p} = m\vec{v}$

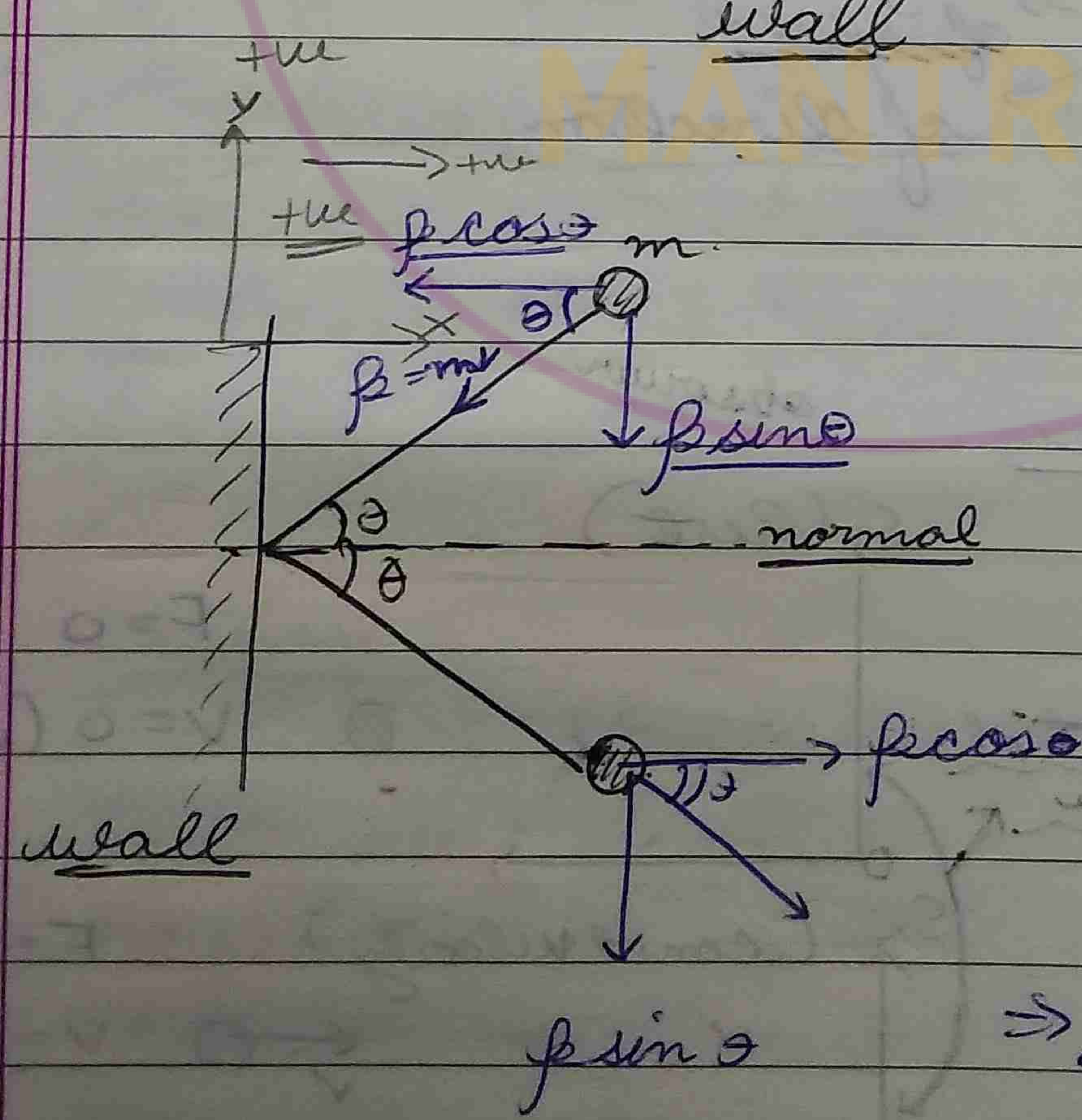
Ex



$p_i = mv$
$p_f = -mv$

change in momentum
$\Delta p = p_f - p_i$
$= -mv - (+mv)$
$= -2mv$

Ex



$\vec{p_i} = -p\cos\theta\,\hat{i} - p\sin\theta\,\hat{j}$

$\vec{p_f} = +p\cos\theta\,\hat{i} - p\sin\theta\,\hat{j}$

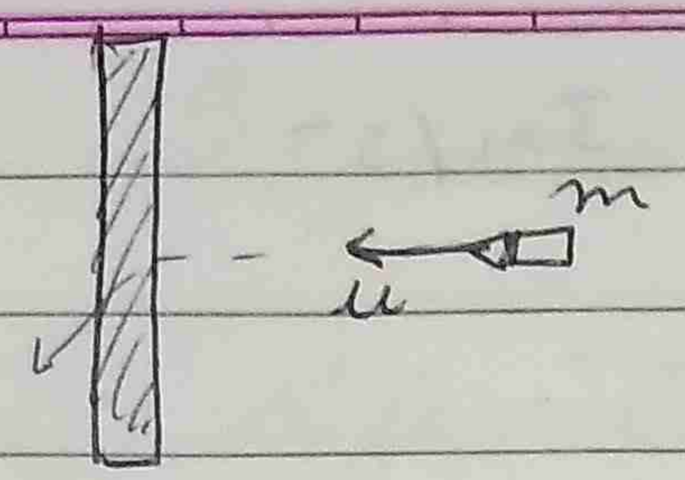
$\vec{\Delta p} = \vec{p_f} - \vec{p_i}$

$\Rightarrow \vec{\Delta p} = 2p\cos\theta\,\hat{i}$

$\Delta p = 2mv\cos\theta$

- Force = rate of change of momentum. ($K = 1$)
- Force is in the direction of acceleration.

Ex



change in momentum per sec; if n bullets fired per sec.

Single bullet $p_i = -mu$, $p_f = 0$

$\Rightarrow \Delta p = mu$ $\quad$ $\boxed{\Delta p_{net} = nmu}$

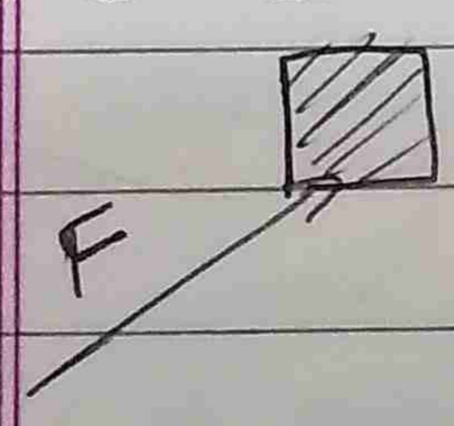
$\bar{F} \propto \frac{dp}{dt} \Rightarrow \vec{F} = K\frac{d\vec{p}}{dt}$ ($K = 1$) $\Rightarrow \boxed{\vec{F} = \frac{d\vec{p}}{dt}}$

$\because \bar{p} = m\vec{v} \Rightarrow \vec{F} = \frac{d}{dt}(m\vec{v}) \Rightarrow \vec{F} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$

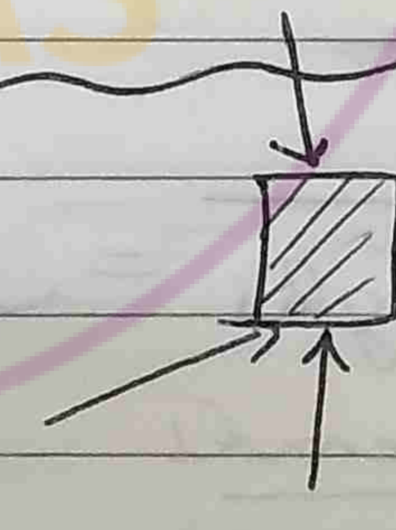
- If mass is constant

$\Rightarrow \vec{F} = m\frac{d\vec{v}}{dt} \Rightarrow \boxed{\vec{F} = m\vec{a}}$: equation of motion of body.

☆



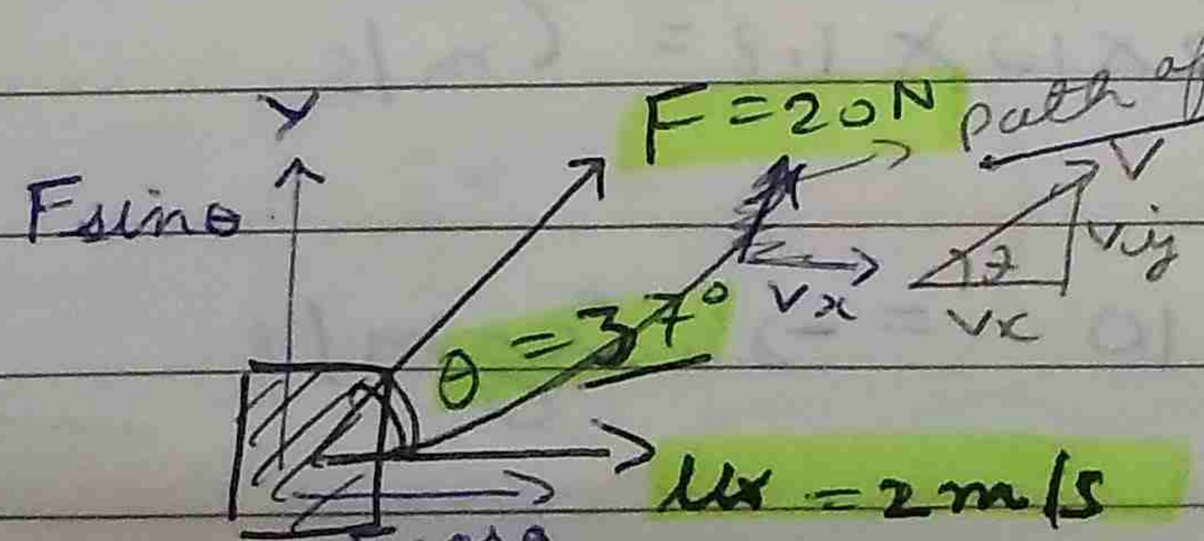
$F_x = ma_x$
$F_y = ma_y$
$F_z = ma_z$



Use superposition. Net force acting on the body is vector sum of all the forces:

$\Sigma \vec{F} = m\vec{a}$
$\Sigma F_x = ma_x$
$\Sigma F_y = ma_y$
$\Sigma F_z = ma_z$

Ex



$m = 4\,kg$ $\quad$ $\Delta t = 10$ seconds

- Final velocity after Δt?

$\Rightarrow a_x = \frac{F\cos\theta}{m} = \frac{20 \times \frac{4}{5}}{4} = 4\,m/s^2$

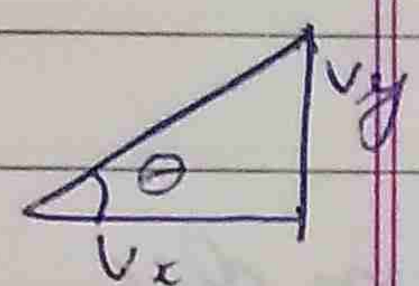
☆ when force and velocity are not in same direction then path is curved and if the force is constant it is parabolic.

$$a_y = \frac{F \sin\theta}{m} = \frac{20 \times \frac{3}{5}}{4} = 3 \, m/s^2$$

$$v_x = u_x + a_x \Delta t \Rightarrow v_x = 2 + 4 \times 10 = 42 \, m/s$$

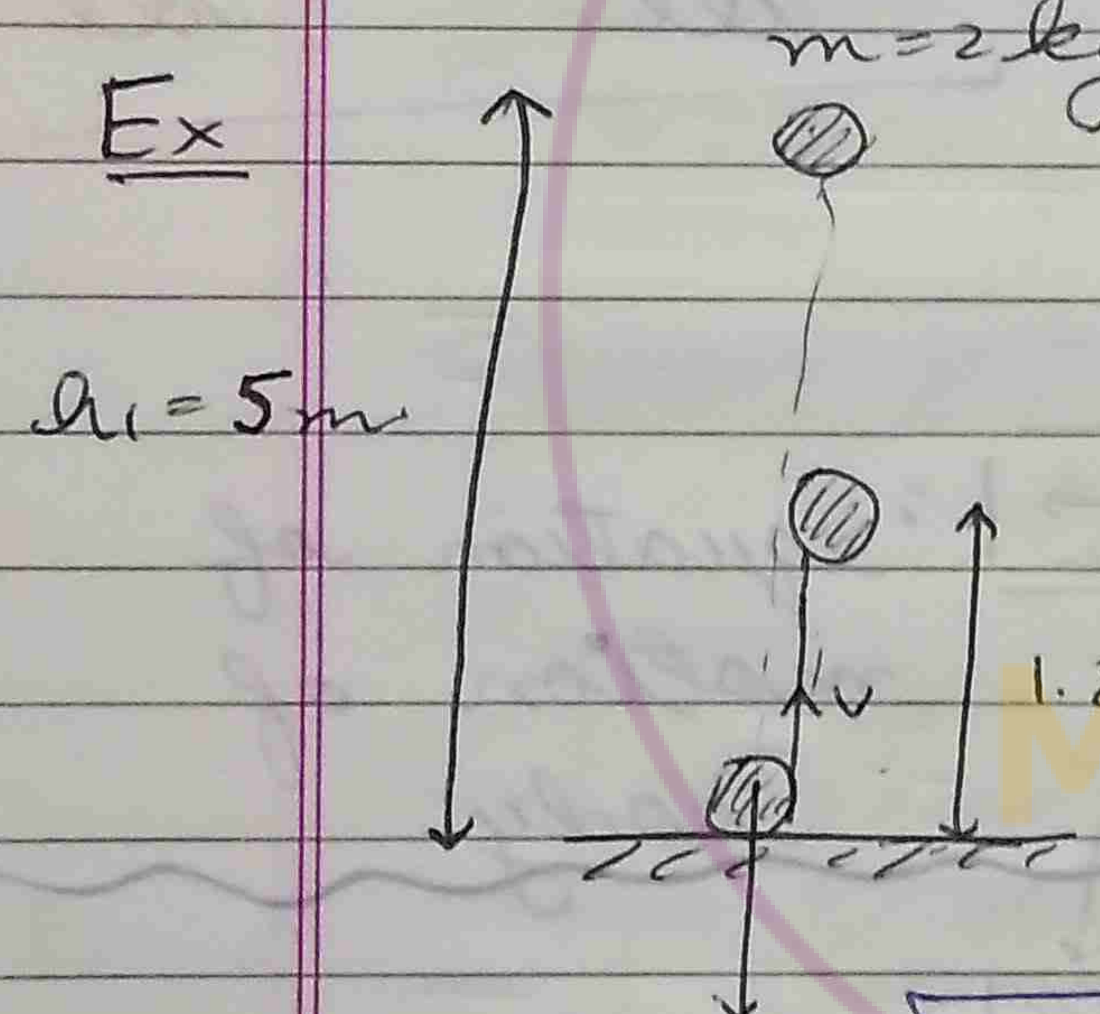
$$v_y = u_y^{\to 0} + a_y \Delta t = 3 \times 10 = 30 \, m/s$$

$$V = \sqrt{v_x^2 + v_y^2} \Rightarrow \boxed{V = \sqrt{42^2 + 30^2}}$$



$$\tan\theta = \frac{30}{42} = \frac{5}{7}$$ angle in which body moves i.e path is parabolic.

Ex



Force exerted by the floor on the body?

$$F = \frac{\Delta p}{\Delta t}$$

$$u = \sqrt{2gh_1} = \sqrt{2 \times 10 \times 5} = 10 \, m/s$$

$\Delta t = 0.01$ seconds

$$V = \sqrt{2gh_2} = \sqrt{2 \times 10 \times 1.8} = 6 \, m/s$$

$$p_i = -mu = -2 \times 10 = -20 \, kg\,m/s$$

$$p_f = +mv = 2 \times 6 = +12 \, kg\,m/s$$

$$\Delta p = p_f - p_i = +32 \, kg\,m/s$$

$$\Rightarrow F_{av} = \frac{\Delta p}{\Delta t} = \frac{32}{0.01} \, \underset{N}{kg\,m/s^2} = \underline{\underline{3200 \, N}}$$

Impulse

- By definition, Impulse, $\vec{J} = \vec{F} \times t$

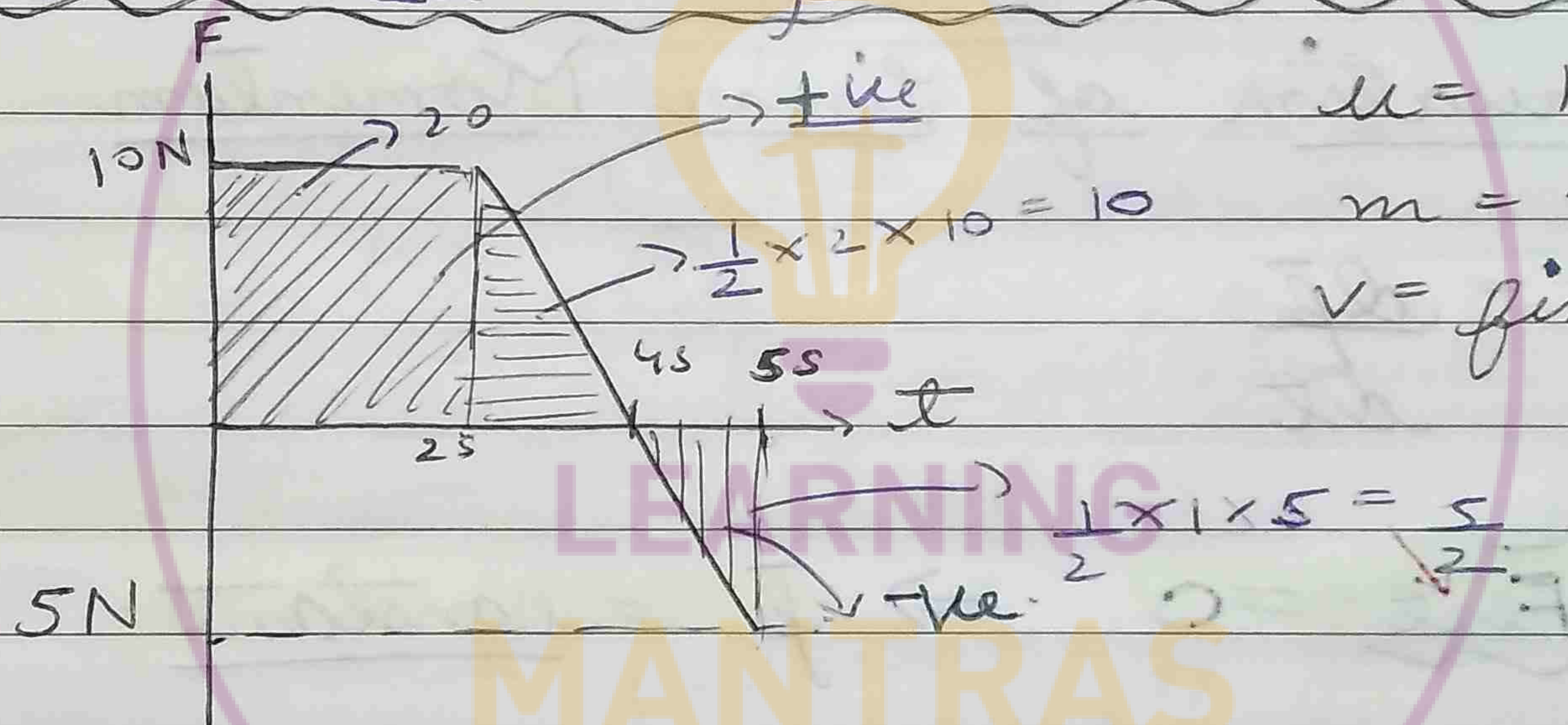
$d\vec{J} = \vec{F} \cdot dt \Rightarrow \boxed{\vec{J} = \int \vec{F} \cdot dt}$ (If force is not constant)

$\Rightarrow \vec{J} = \int \frac{d\vec{p}}{dt} \cdot dt \Rightarrow \vec{J} = \int_{p_i}^{p_f} d\vec{p} = \vec{p_f} - \vec{p_i}$

or $\boxed{\vec{J} = \Delta \vec{p}}$ Impulse momentum theorem

$\vec{J} = \vec{F} \times \Delta t = \Delta \vec{p}$

Ex



$u = 10 \text{ m/s}$

$m = 2 \text{ kg}$

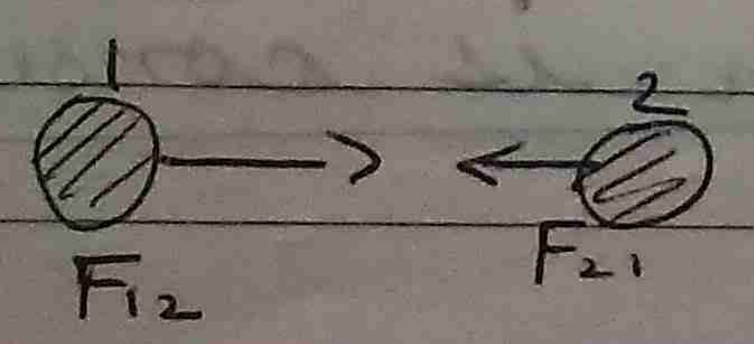
v = final velocity = ?

$J = \int F dt$ = Area under F–t graph

$= 20 + 10 - \frac{5}{2} = \frac{55}{2} \text{ kg m/s} = mv - mu$

$\Rightarrow v = \left(\frac{55}{2} + 2 \times 10\right) / 2 = \frac{95}{4} \text{ m/s}$

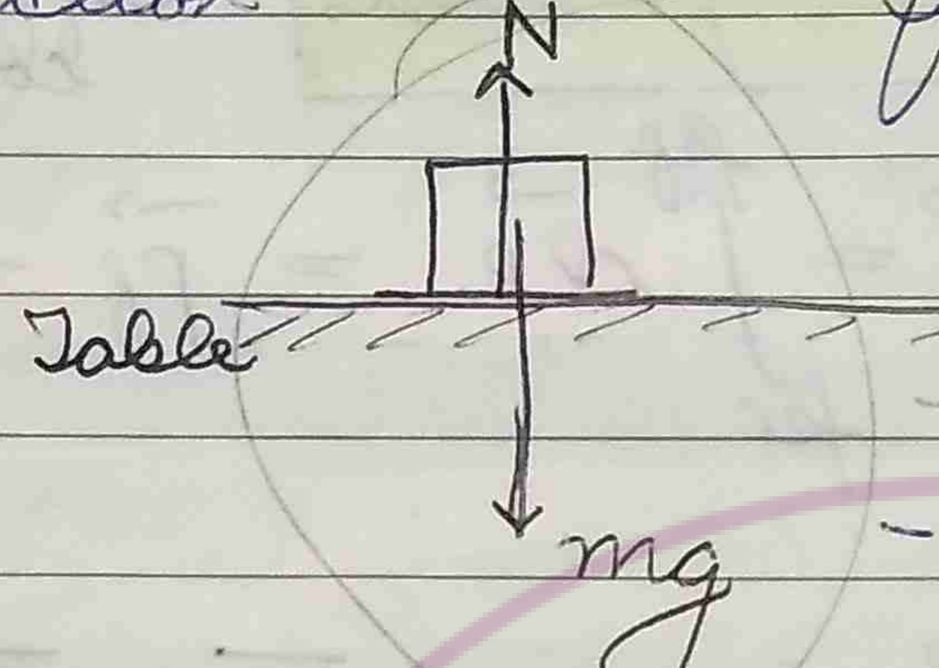
- Newtons III law
- Every action has an equal and opposite reaction



$\boxed{\vec{F_{12}} = -\vec{F_{21}}}$

- Properties of action reaction pair
(i) act simultaneously
(ii) acts on two different bodies
(iii) must come from same interaction.
(iv) Action - reaction forces never cancel eachother

Ex



$N = mg$ | because they act on two different bodies

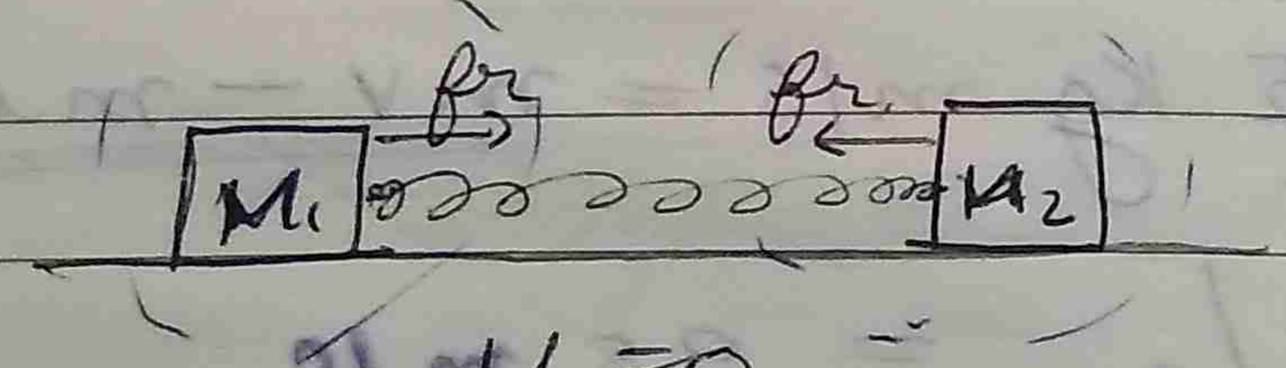
→ Not an action reaction pair because N and mg act on same body and violate conditions (ii) and (iii)

- Conservation of Linear Momentum

- $\bar{F}_{ext} = \frac{d\bar{p}}{dt}$

- If $\bar{F}_{ext} = 0 \Rightarrow \bar{p} = \text{constant}$

- $\vec{p}_{i_1} + \vec{p}_{i_2} = \vec{p}_{f_1} + \vec{p}_{f_2}$ (check vector sign)

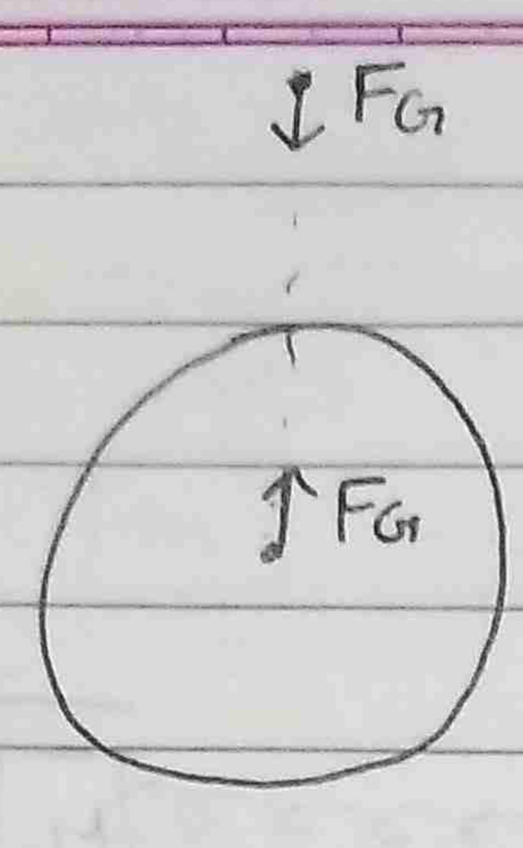


- Individually momentum of M_1 and M_2 is not conserved because restoring force is present but together

$m_1 + m_2 \Rightarrow$ momentum is conserved.
(system)

• If momentum is conserved, that does not mean energy is conserved or vice-versa.

(i)

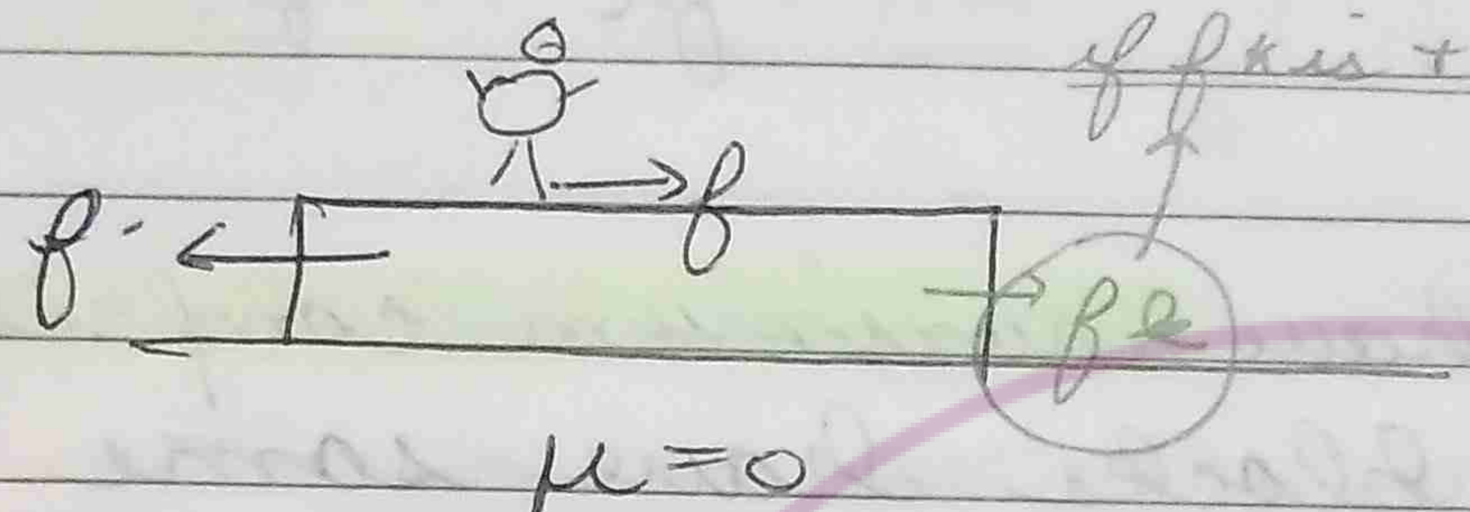


Block + Earth (system)

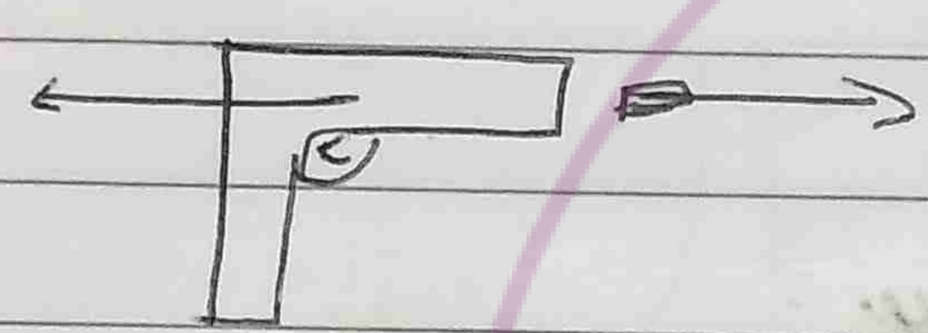
momentum is constant
Because F_G and F_G become internal forces.

(ii)

If f_{ext} is +nt $\bar{p}$ is not conserved for any arrangement

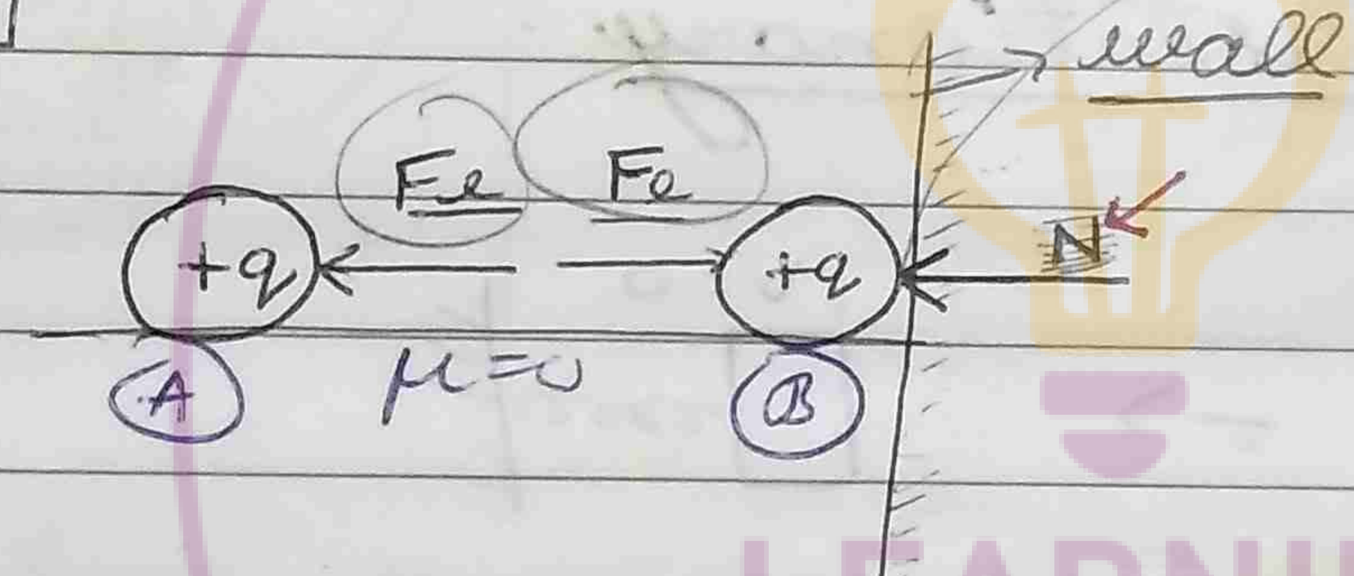


man + plank $\bar{p}$ = constant
(system)



displacement of this point is zero.

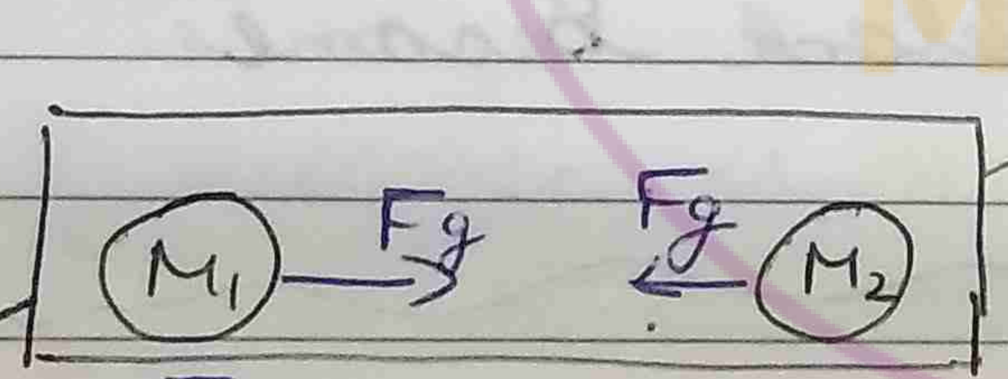
Ex



A + B ⇒ momentum is
→ not conserved because
external force normal reaction is present.

$W_{ext} + W_{N.C} = \Delta E_i \Rightarrow 0 = \Delta E$
⇒ Mechanical Energy is conserved.

Ex



$\bar{p}_{1+2}$ = constant, Mechanical Energy = constant

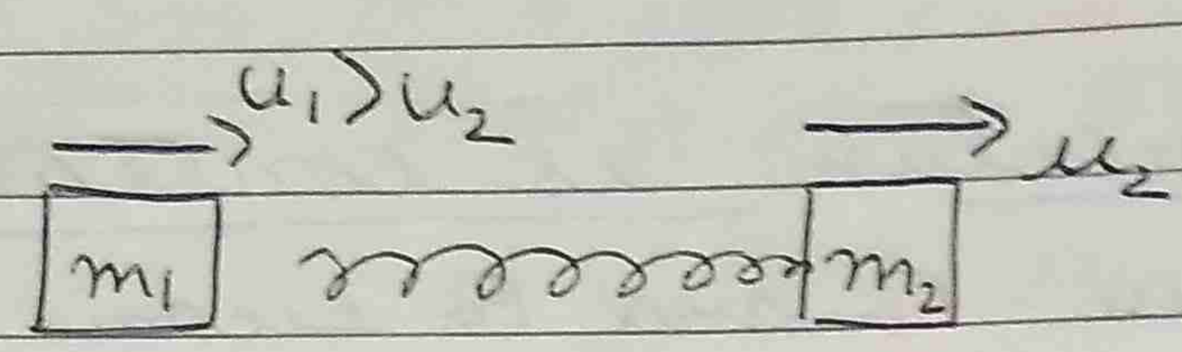
Work done by all forces = $\Delta K.E$

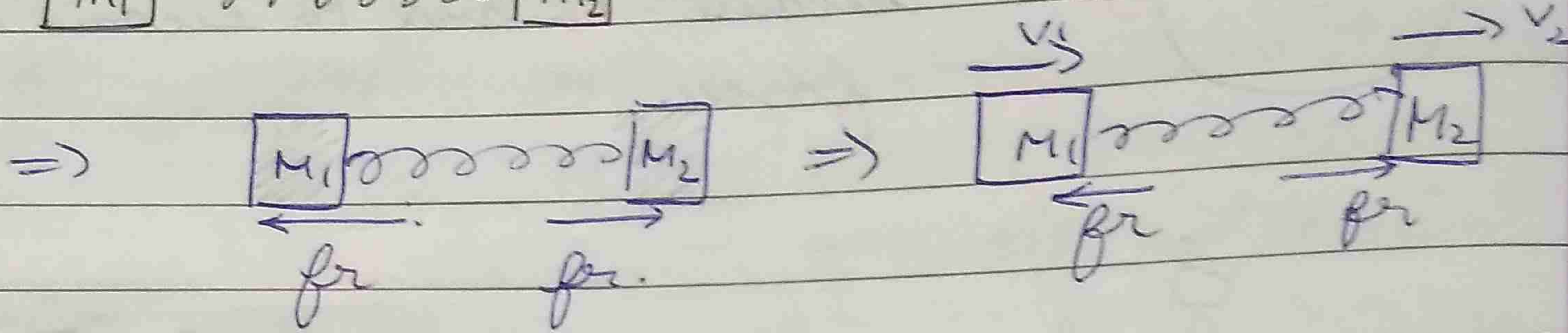
☆ Q Internal forces can't change momentum but can change K.E

Because
→ Momentum → vector : $\bar{F}_1 + \bar{F}_2 = 0$ can cancel each other
→ Energy → scalar : $\vec{F}_1 \cdot \vec{S}_1 + \vec{F}_2 \cdot \vec{S}_2$ cannot cancel each other

- Spring will be compressed if backward velocity is greater than forward velocity.

- If both block are moving

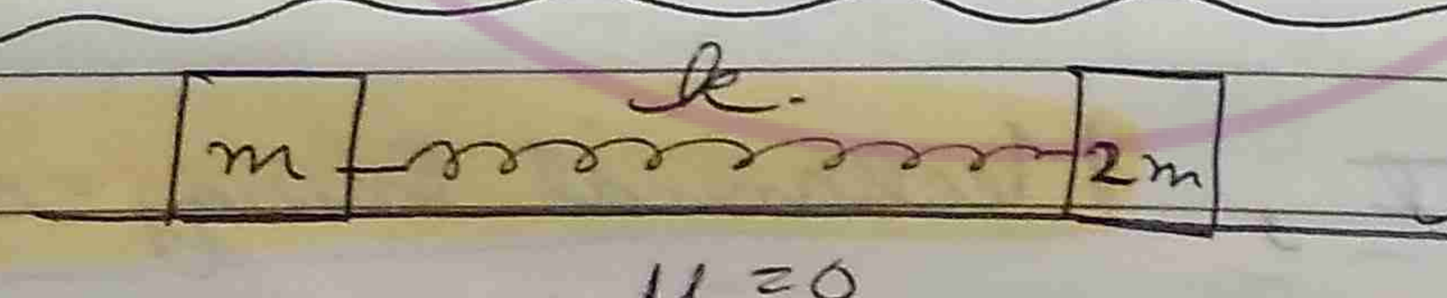




☆ The spring will have maximum compression when both the blocks have same speed (i.e) $(v_1 = v_2)$

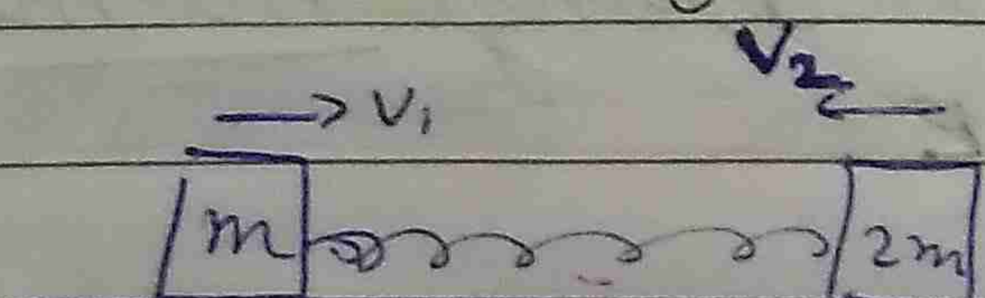
- If a spring is stationary:

☆ In this case spring will have max compression if speed of block becomes zero.

Ex: 

let elongation = x

- find the speeds of blocks at natural length?



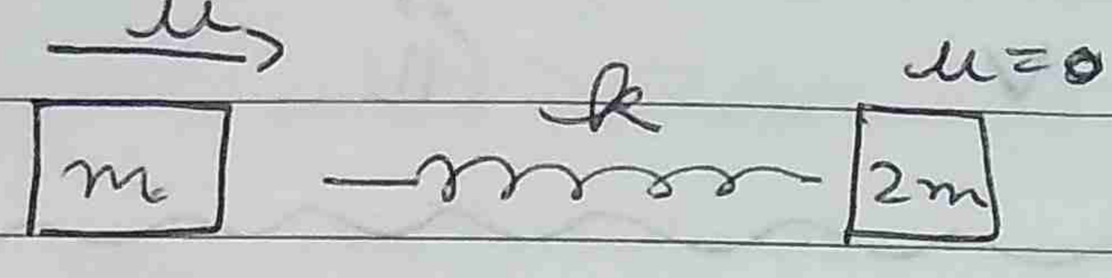
$E_1 = E_2$

$\frac{1}{2} m v_1^2 + \frac{2}{2} m v_2^2 = \frac{1}{2} k x^2$ --- (i)

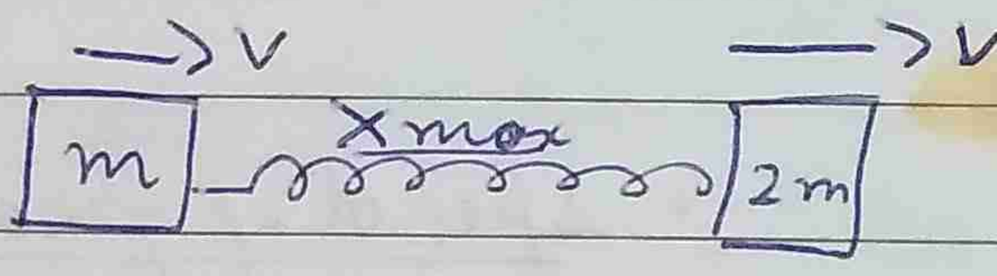
$\vec{p}_i = \vec{p}_f \Rightarrow 0 = m v_1 - 2 m v_2$ --- (ii)

- when block reaches the maximum height the speed of block becomes equal to speed of wedge if the wedge is movable.

$\Rightarrow V_1 = 2V_2$

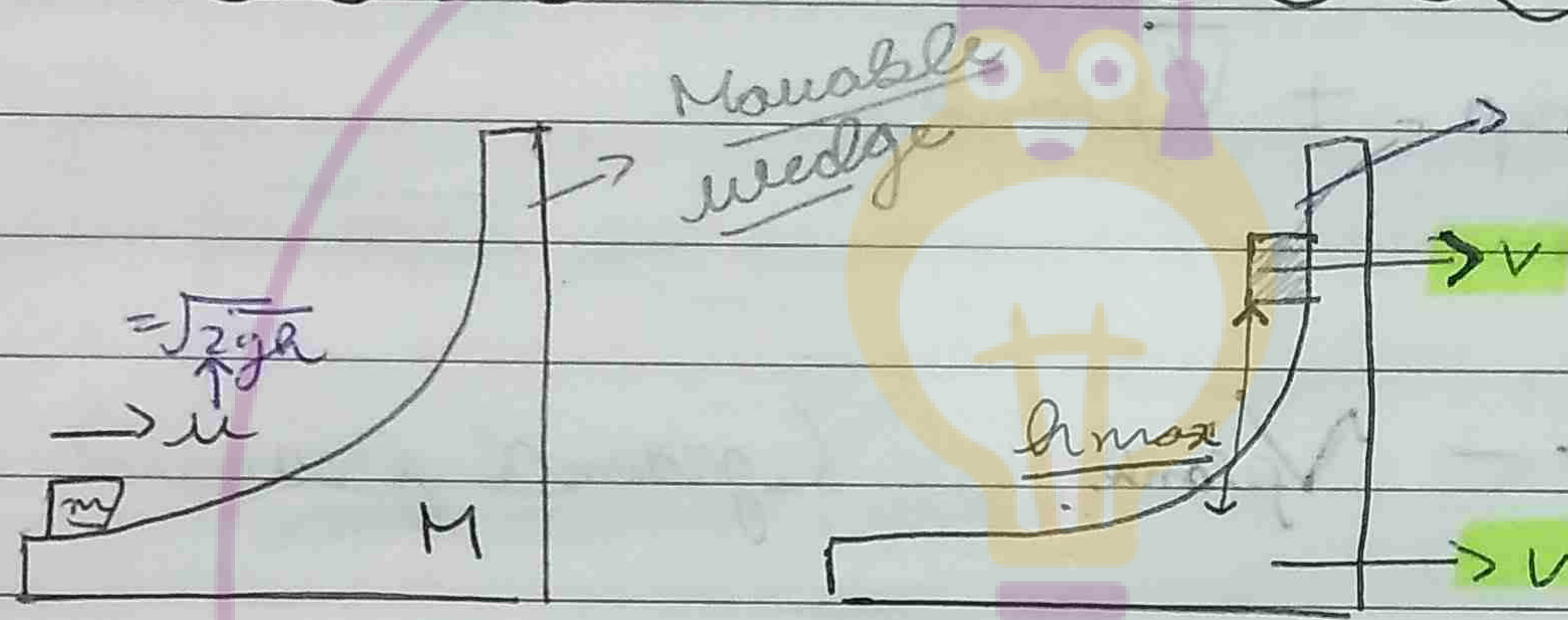
Ex 

find maximum compression?



$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}2mv^2 + \frac{1}{2}kx_{max}^2$ (i)

$mu = mv + 2mv \Rightarrow v = \frac{u}{3}$ (ii)

Ex 

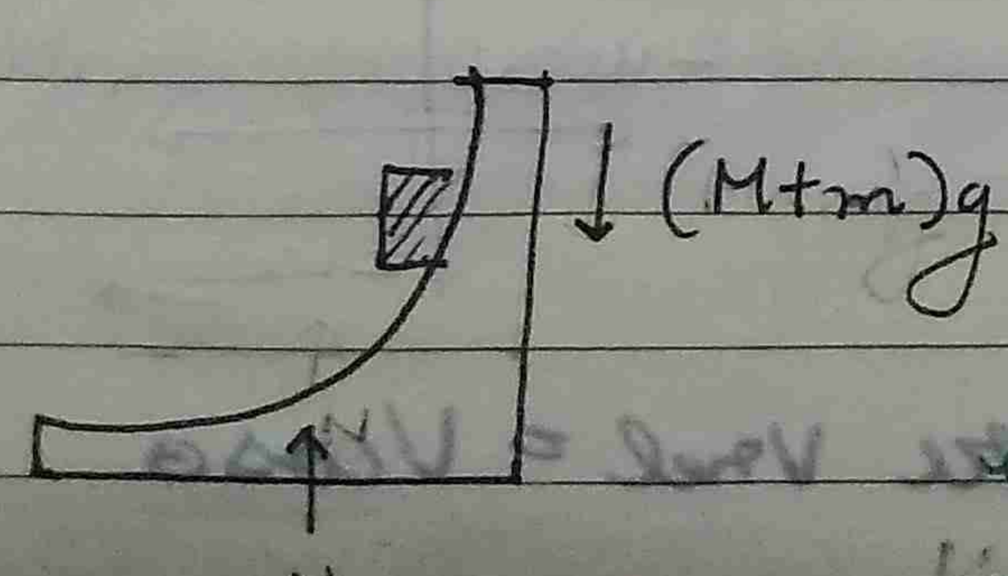
At hmax y component of velocity should be zero.

- If ~~block~~ is wedge was fixed then (block) KE → block P.E

- If wedge is movable
(Block) K.E ⟶ (~~block~~ wedge) K.E + (block) P.E

$\frac{1}{2}mu^2 = \frac{1}{2}(m+M)V^2 + mgh_{max}$

$\frac{1}{2}m(2gh) = \frac{1}{2}(m+M)V^2 + mgh_{max}$ --- (i)



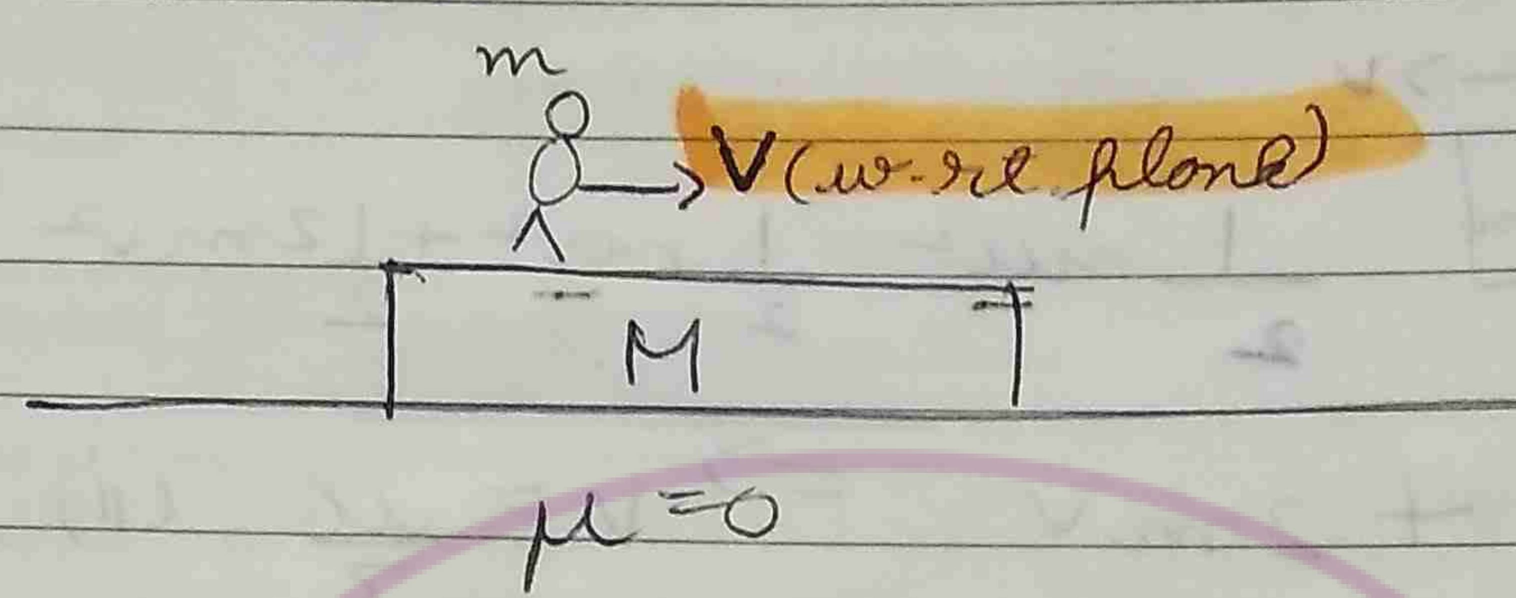
- If C.M ~~is~~ is moving up then upward force is larger than downward force, hence momentum is not conserved along y, axis.

∴ motion is conserved along x-axis.

along x → momentum is conserved.

$m\sqrt{2gh} = (m+M)V$ --- (ii)

Ex



· speed of plank = ?

$\bar{V}_m = \bar{V}_{mp} + \bar{V}_{plank}$

(↓ of man; man w.r.t plank)

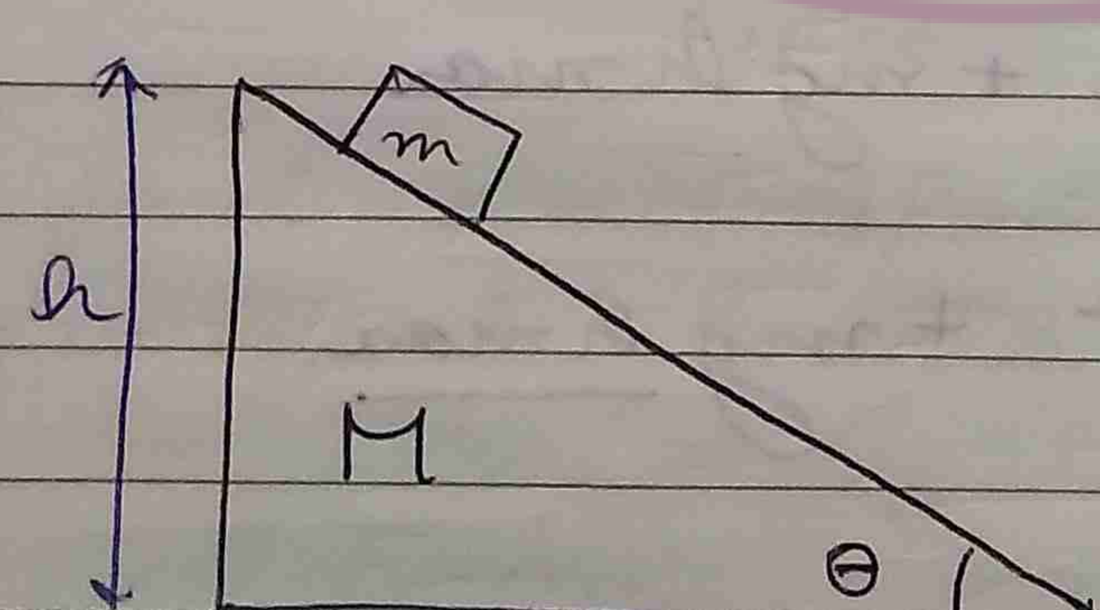
$\bar{V}_m = v - V_{plank}$ (ground frame)

$\bar{p}_i = \bar{p}_f \Rightarrow 0 = mv_m - MV$

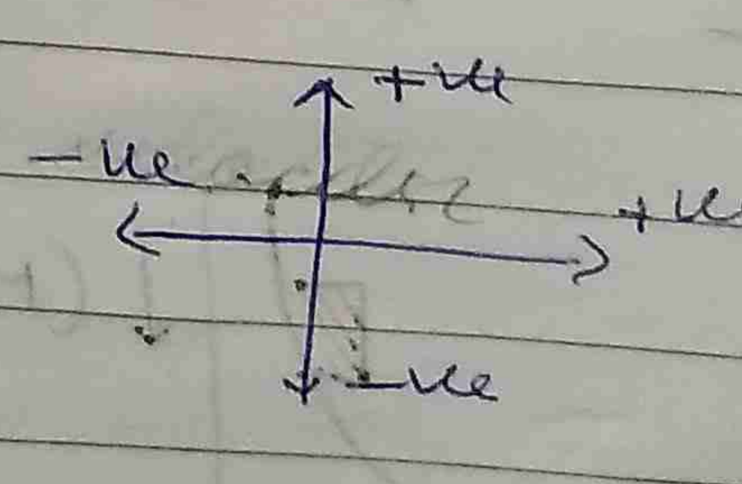
$0 = m(v - V) - MV$

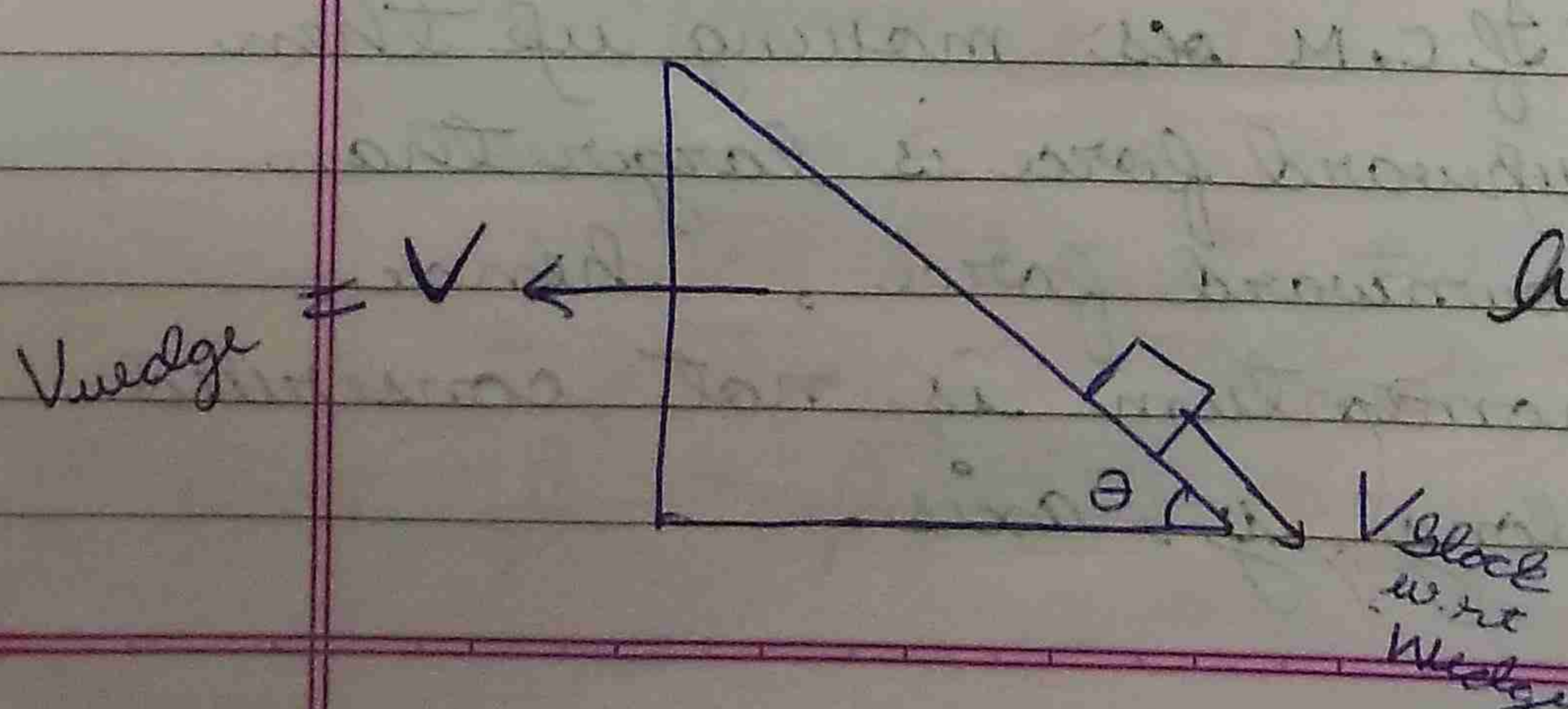
$$\Rightarrow V = \frac{mv}{m+M}$$

Ex

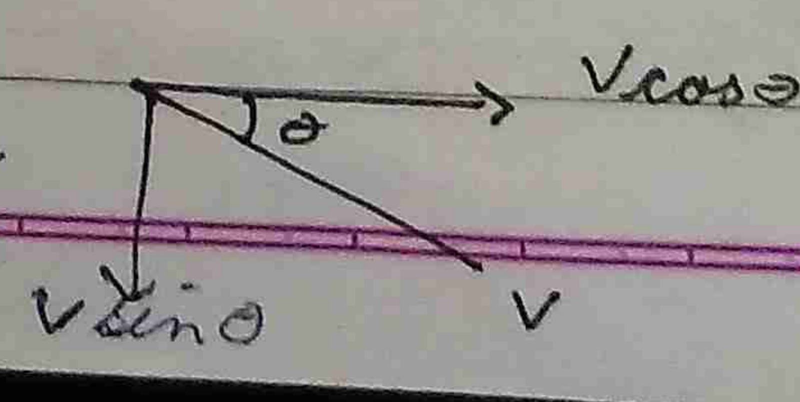


· Find the speed of wedge when block finally reaches bottom.





here $V_{rel} = V\cos\theta$

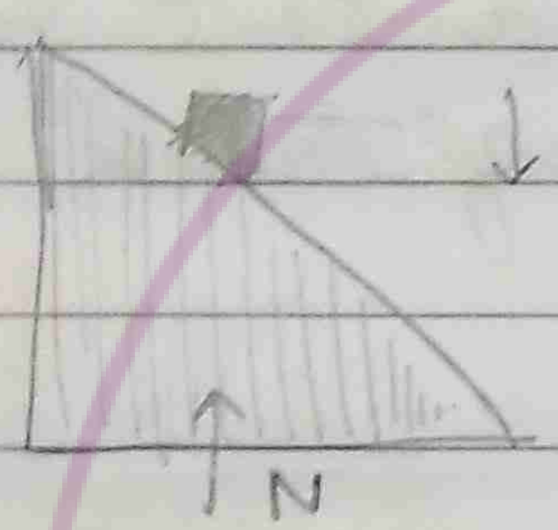


$\bar{V}_{block(B)} = \bar{V}_{BW} + \bar{V}_W$

$= (v\cos\theta\,\hat{i} - v\sin\theta\,\hat{j}) + (-V\hat{i})$

$\Rightarrow \bar{V}_B = \underbrace{(v\cos\theta - V)}_{V_{Bx}}\hat{i} - \underbrace{v\sin\theta}_{V_{By}}\hat{j}$

$mgh = \frac{1}{2}MV^2 + \frac{1}{2}m(V_{Bx}^2 + V_{By}^2)$ - - - (i)



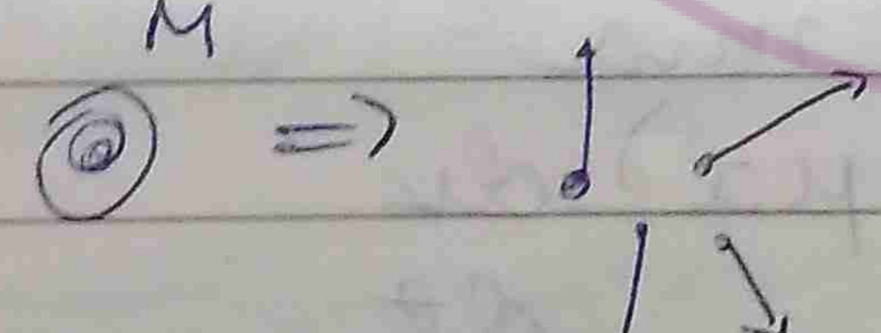
∵ block is moving down (C.M) is moving down hence (M+m) g is greater than N, hence momentum is not conserved along y axis.

but momentum is conserved

$p_{ix} = p_{fx}$ along x.

$0 = m(v\cos\theta - V) - MV \Rightarrow V = \frac{mv\cos\theta}{M+m}$ — (ii)

Explosion : $\bar{p}$ = constant energy ≠ constant

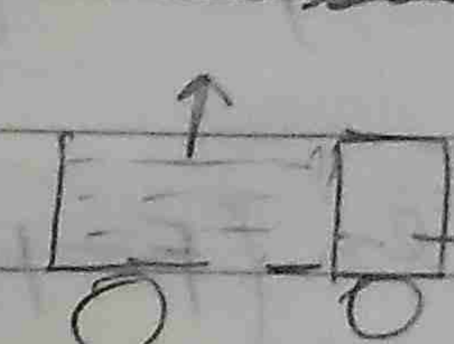


$0 = \bar{p}_1 + \bar{p}_2 + \bar{p}_3 + \bar{p}_4$

- Variable Mass

$$\bar{F}_{thrust} = \vec{v}_{relative}\frac{dM}{dt}$$

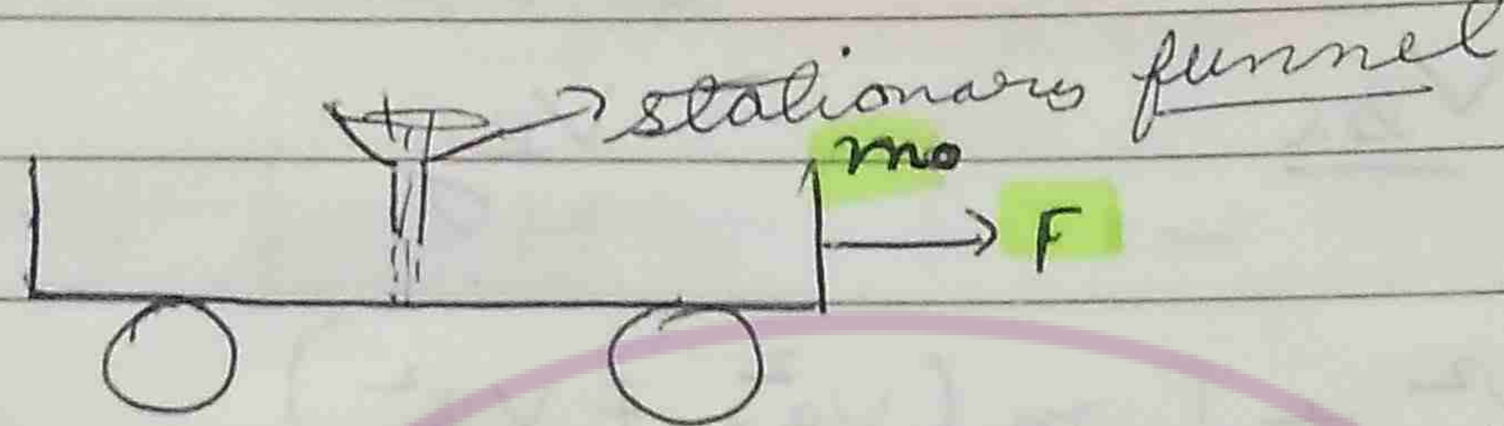
added or subtracted part



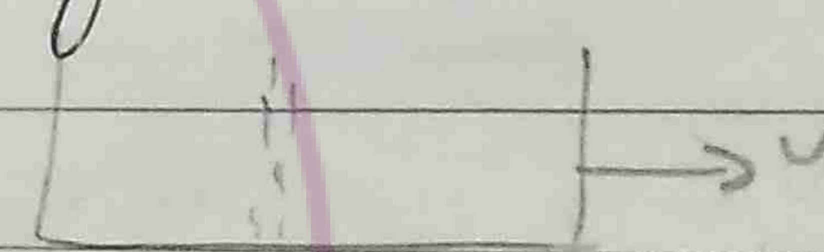
- $\bar{V}_{rel}$ ⇒ Relative velocity of added or subtracted part w.r.t main body

$\frac{dm}{dt}$ = rate of change of mass
↘ (+ve if mass is added)
↘ (−ve if mass is subtracted)

Ex



Q If sand is poured at constant rate μ kg/s; find the speed of truck as function of time.

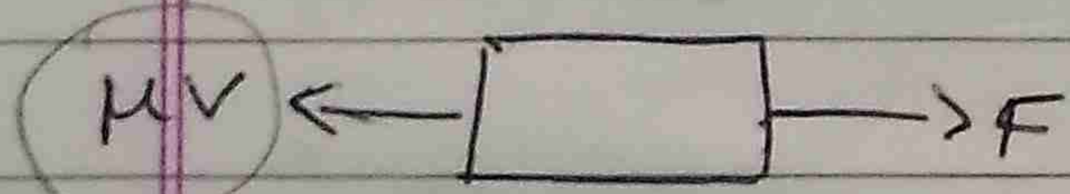


→ $\vec{F}_{thrust} = \vec{V}_{rel} \frac{dM}{dt}$ v = velocity of truck at any instant

velocity of added part ← main body

$= (0 - \vec{V}) \frac{dm}{dt}$ along x axis

$= (0 - v)\mu = -\mu v$



μv is thrust force which is added

$F - \mu v = ma$

$F - \mu v = (m_0 + \mu t) \frac{dv}{dt}$

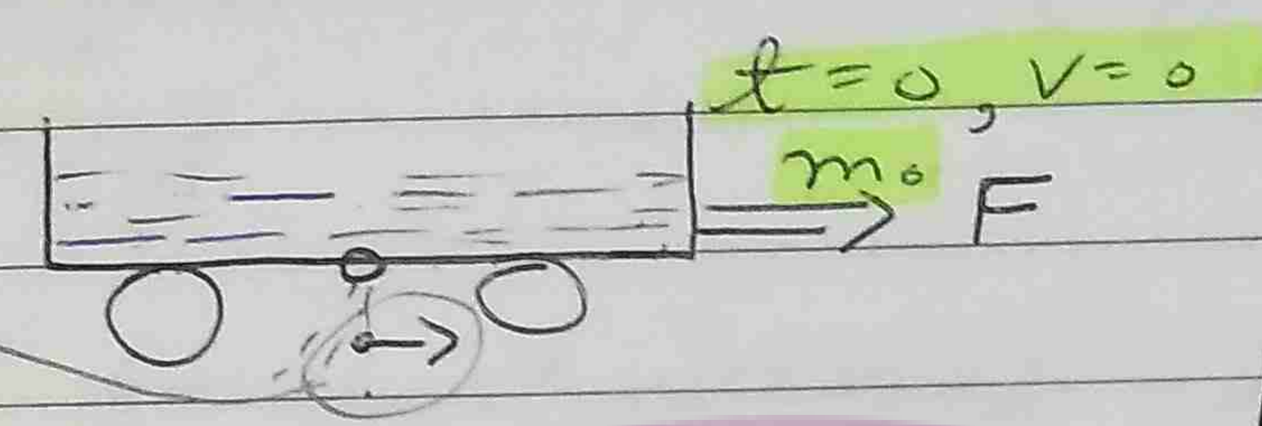
$\Rightarrow \int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{m_0 + \mu t}$ $\left[\int \frac{dx}{A + Bx} = \frac{1}{B} \ln(A + Bx) \right]$

$\frac{1}{-\mu} \left[\ln(F - \mu v) \right]_0^v = \frac{1}{\mu} \left[\ln(m_0 + \mu t) \right]_0^t$

$$-\ln\left(\frac{F-\mu v}{F}\right) = \ln\left(\frac{m_0+\mu t}{m_0}\right)$$

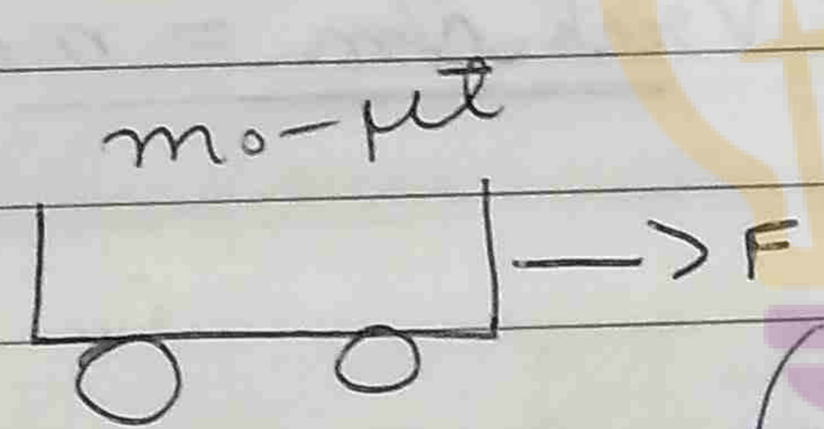
$$\Rightarrow \frac{F}{F-\mu v} = \frac{m_0+\mu t}{m_0} \Rightarrow v = \ldots$$

Q Ex – If sand leaks at a constant rate in μ kg/s; find the speed of truck as $f(t)$. (Here m_0 is the mass of truck + sand).



just as the sand comes out it has the same speed as that of the truck due to inertia

$$F_{th} = \vec{V}_{rel}\frac{dm}{dt} = 0$$



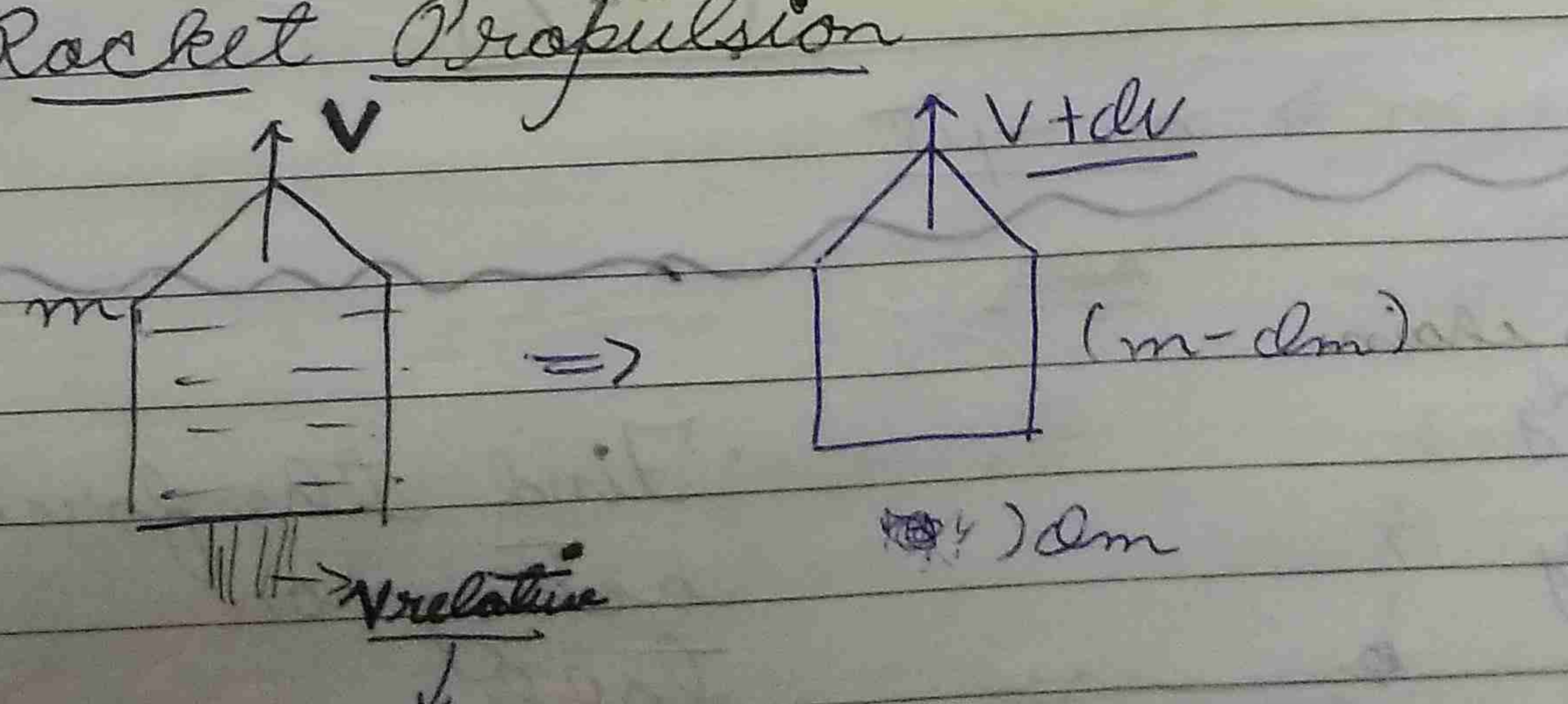
$$F = (m_0 - \mu t)\frac{dv}{dt}$$

$$\int_0^t \frac{dt}{m_0-\mu t} = \int_0^v \frac{dv}{F}$$

$$\frac{1}{-\mu}\Big[\ln(m_0-\mu t)\Big]_0^t = \frac{1}{F}\Big[v\Big]_0^v$$

$$\Rightarrow \ln\left(\frac{m_0-\mu t}{m_0}\right) = \frac{-\mu v}{F}$$

- Rocket Propulsion



relative velocity of gas w.r.t rocket.

Method 1

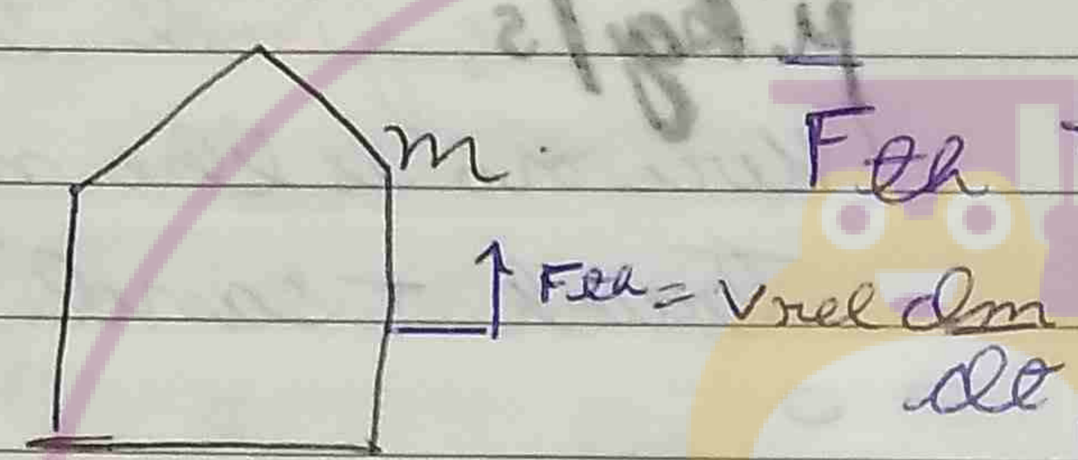
$\bar{V}_{gas} = \bar{V}_{gas, rocket} + \bar{V}_{rocket}$

$= -V_{rel} + v$ ↗ velocity of dm with respect to ground frame.

$mv = (m - dm)(v + dv) + dm(-V_{rel} + v)$

Here g is neglected otherwise P would not be conserved

Method 2



$F_{th} = V_{rel} \frac{dm}{dt} = -V_{rel}\left(-\frac{dm}{dt}\right)$ (upwards)

$V_{rel} \frac{dm}{dt} = m\frac{dv}{dt} \Rightarrow V_{rel}\, dm = m\, dv$

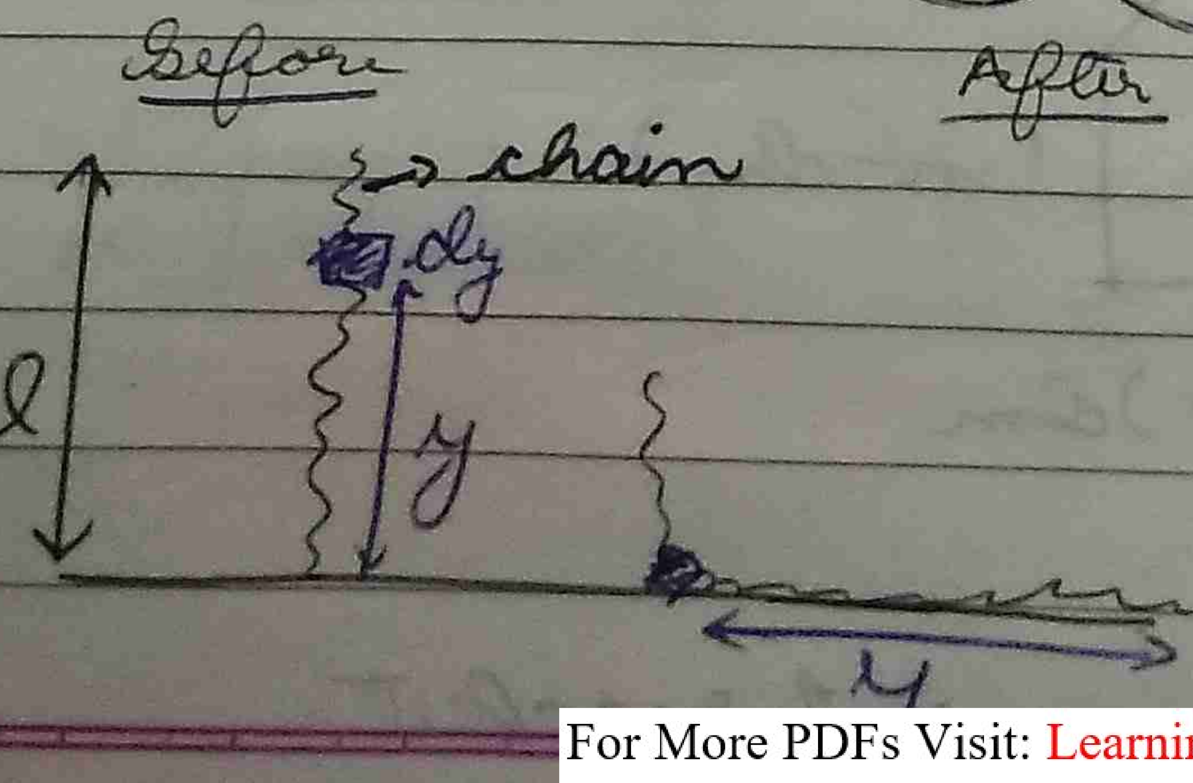
$\int_{m_0}^{m} \frac{dm}{m} = \frac{-1}{V_{rel}} \int_{0}^{v} dv$ → constant

$\ln\left(\frac{m}{m_0}\right) = \frac{-1}{V_{rel}} v \Rightarrow v = V_{rel} \ln\left(\frac{m_0}{m}\right)$

$$v = u - gt + V_{rel} \ln\left(\frac{m_0}{m}\right)$$ → If g is considered and initial velocity is u.

where $m \Rightarrow m_0 - \mu t$

☆ Ex



Find the force acting on table.

- N is perpendicular to surface in contact.
- Tension always acts away from system.
- Friction acts in tangential direction.

$$F_{th} = v_{rel}\frac{dm}{dt} = v_{rel}\frac{dm}{dy}\frac{dy}{dt}$$

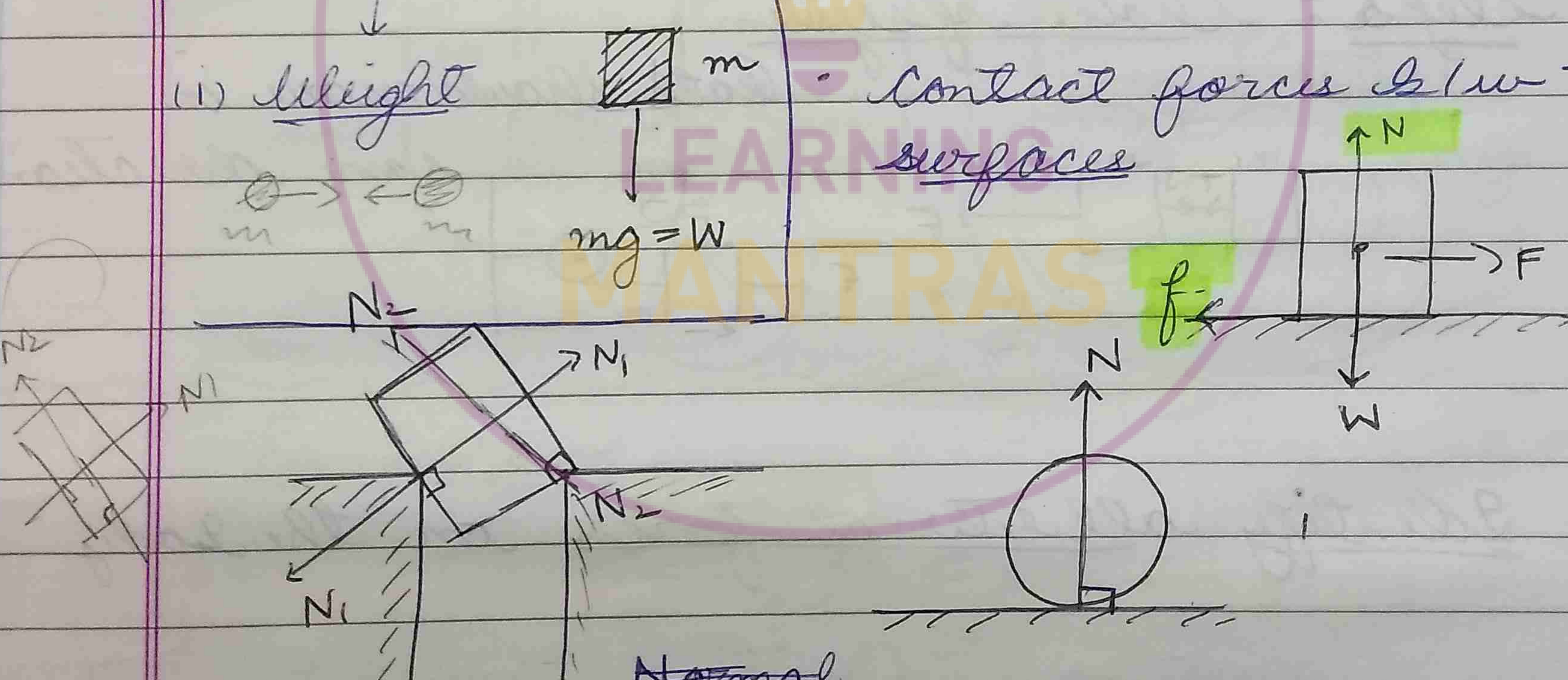
$$= (-\sqrt{2gy})\frac{M}{L} \times \sqrt{2gy} = F_{th} = -\frac{M}{L}2gy$$ (− for direction)

Force due to chain already on ground i.e. weight

$= \frac{M}{L}yg$

Total force $= \frac{3Mgy}{L}$

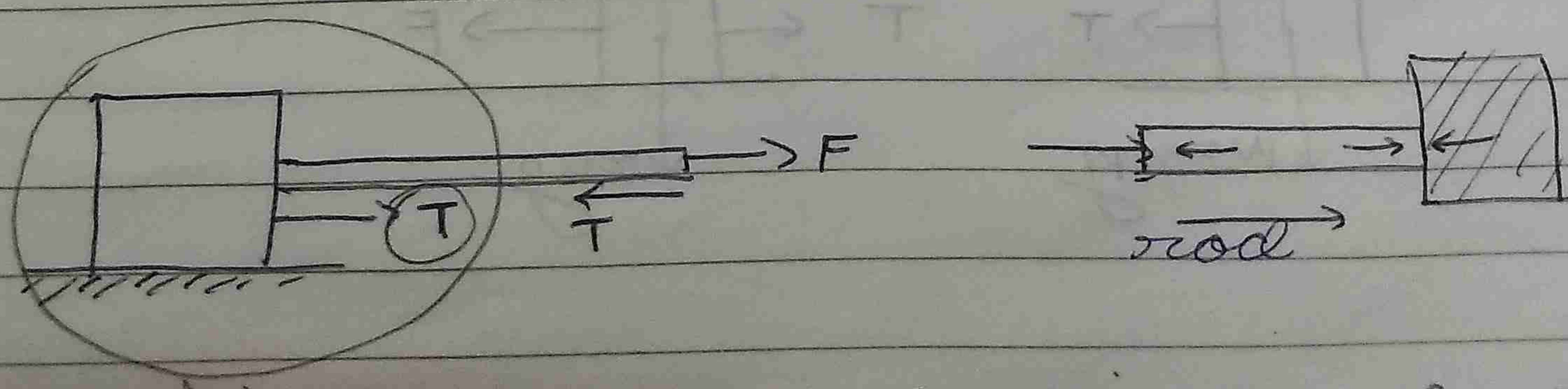
- Common Forces
  - Field force
    - (1) Weight
  - Contact force
    - Contact forces b/w two surfaces



~~Normal~~

- Tension

Compression



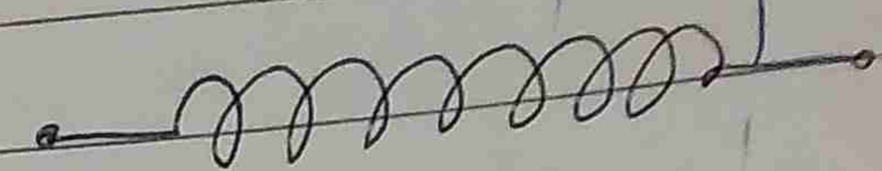
→ force acting on block is T and not F.

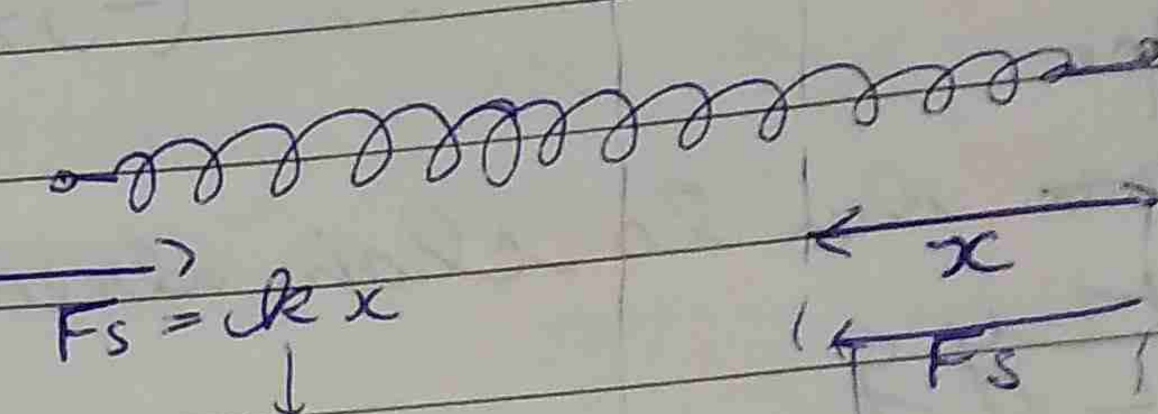
T may be equal to F.

- For massless string tension is same at all points.
- For a system the motion of bodies should be same i.e they should move in same direction.

- ## Spring force

Natural length

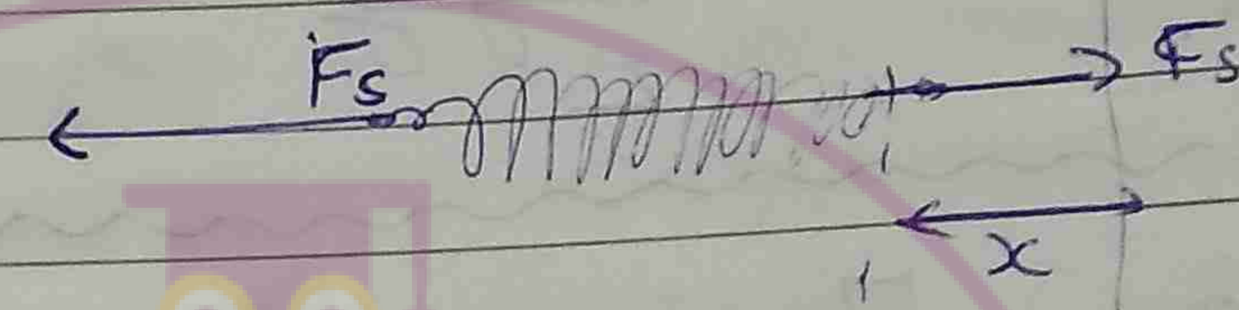




☆ Spring force Fs is same at both the ends.

$F_s = kx$ → spring constant

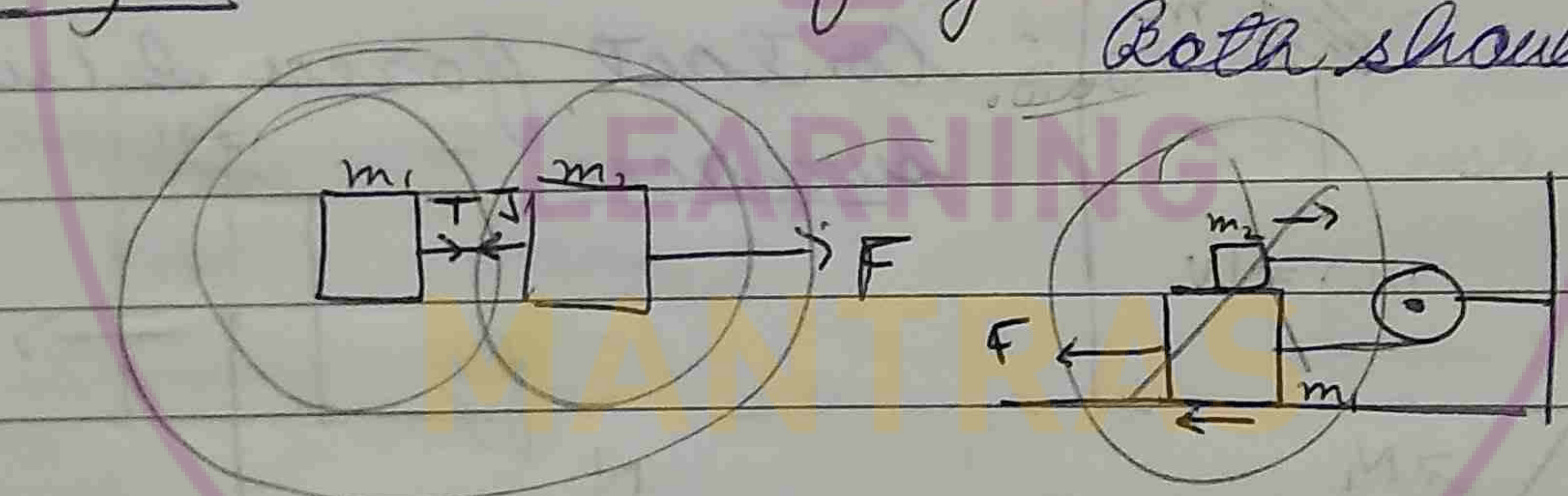
$F_s = kx$



- ## Applications of laws of Motion

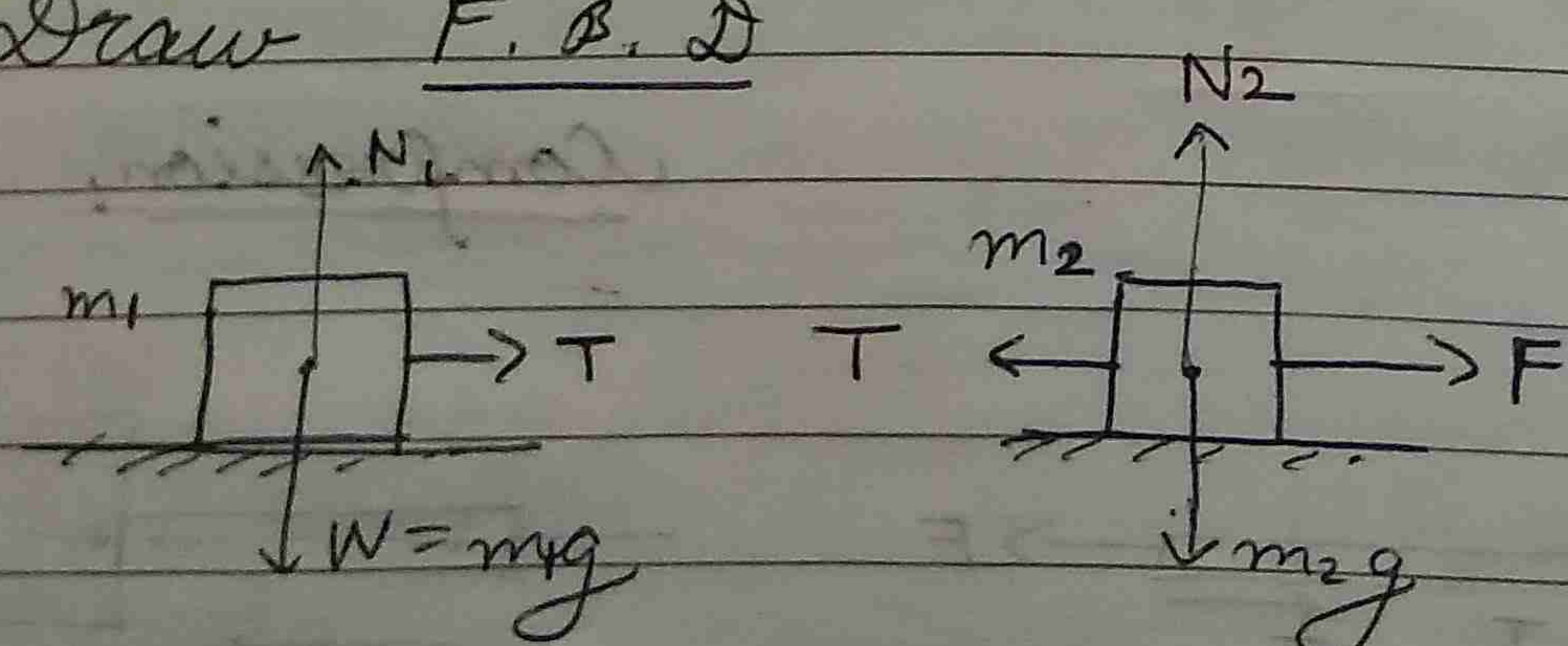
1 Steps : Choice of system

Both should move in same direction

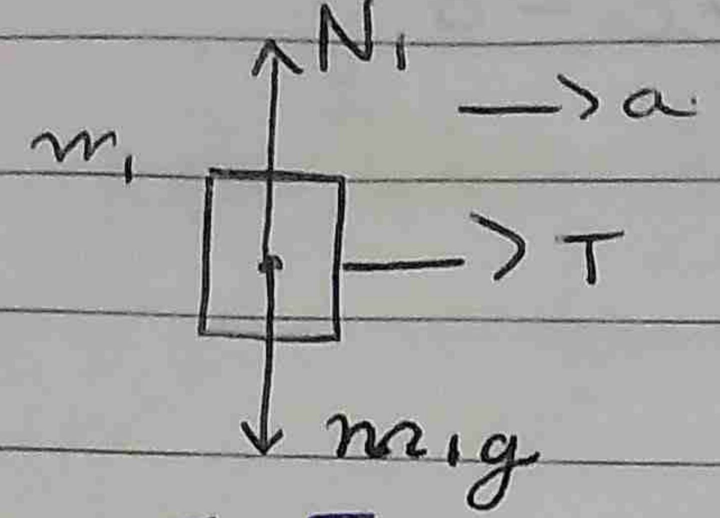


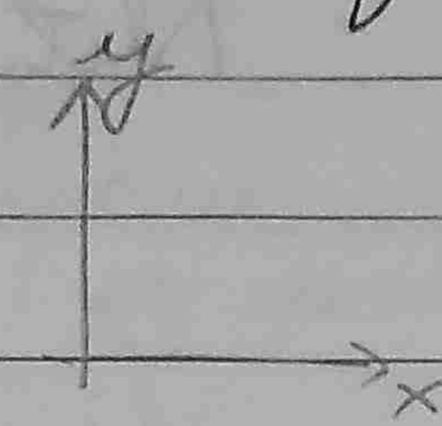
2 Identify all external forces on the body

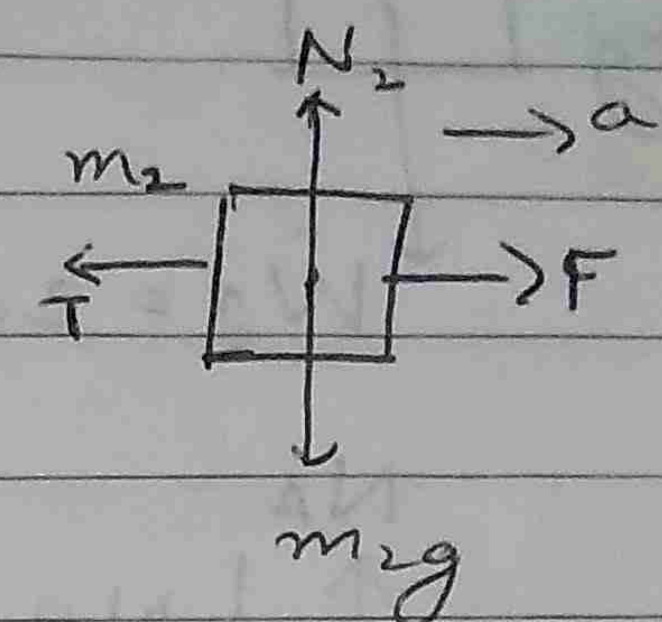
3 Draw F.B.D



4 Chose a reference frame





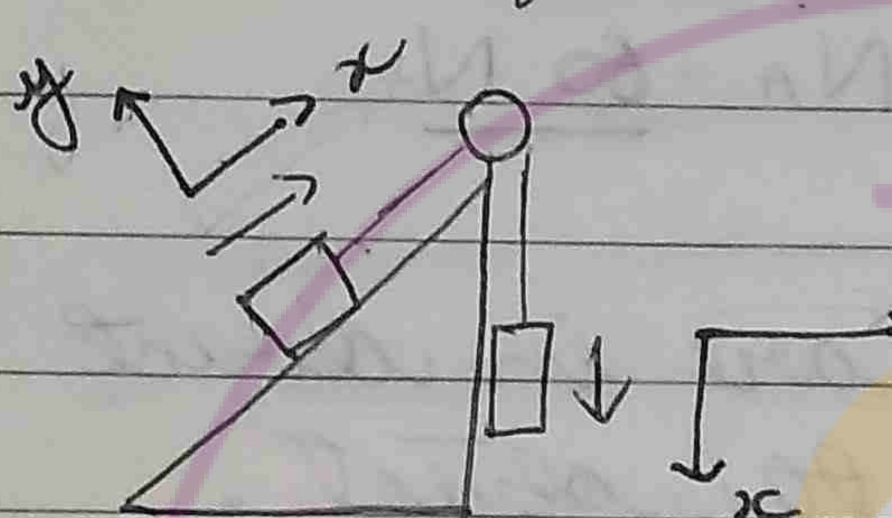


$\Rightarrow F - T = m_2 a$

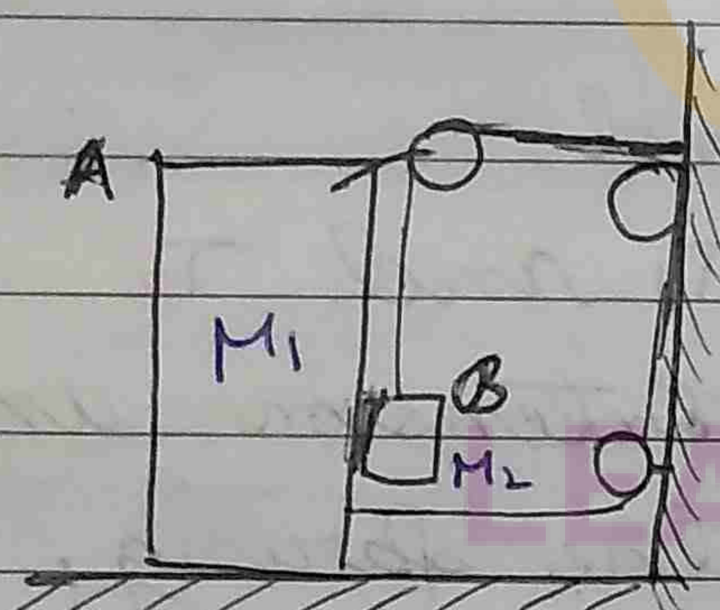
$\Sigma F_x = m a_x$

$\Rightarrow T = m_1 a \; (i)$

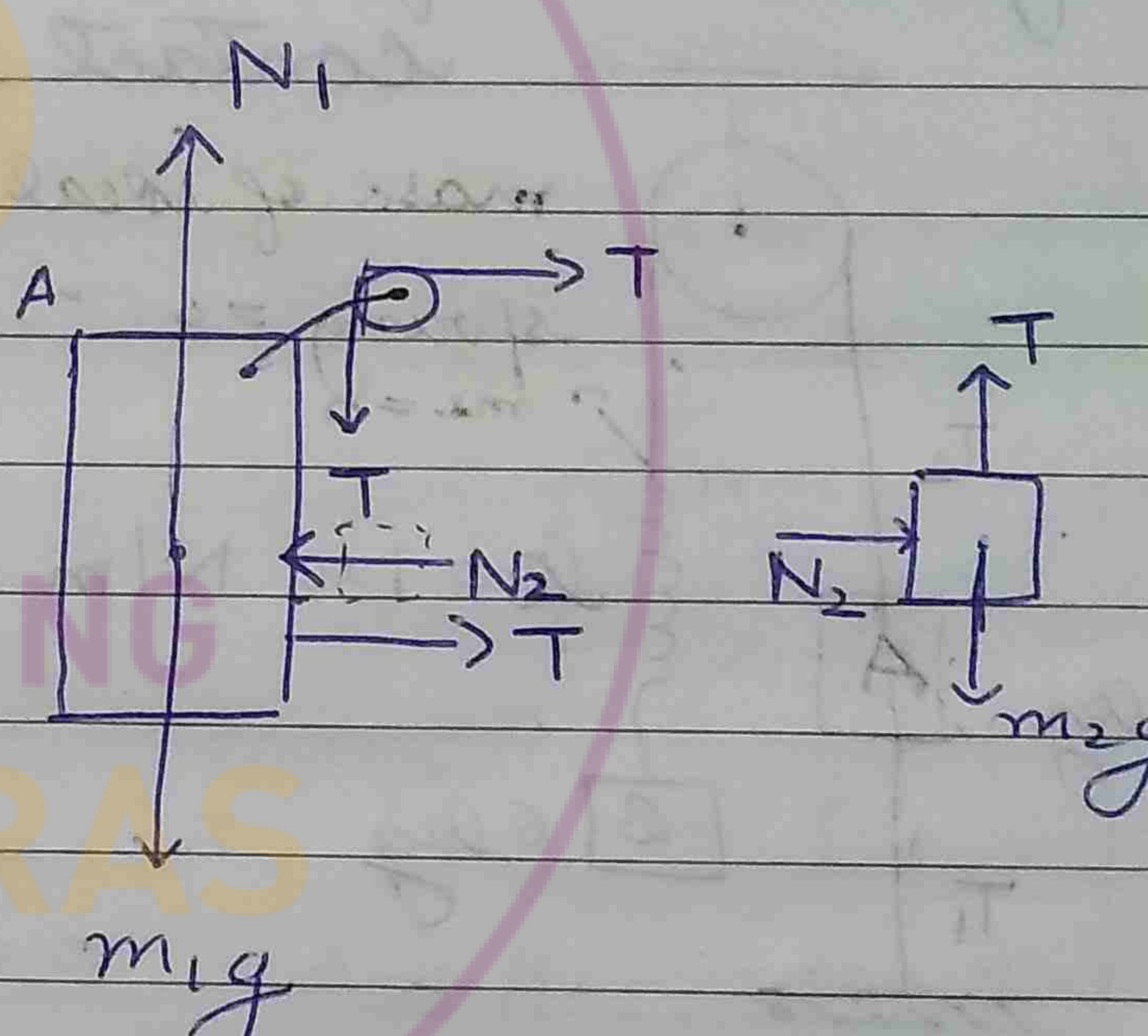
5 Write equations of motion and solve them.



Ex



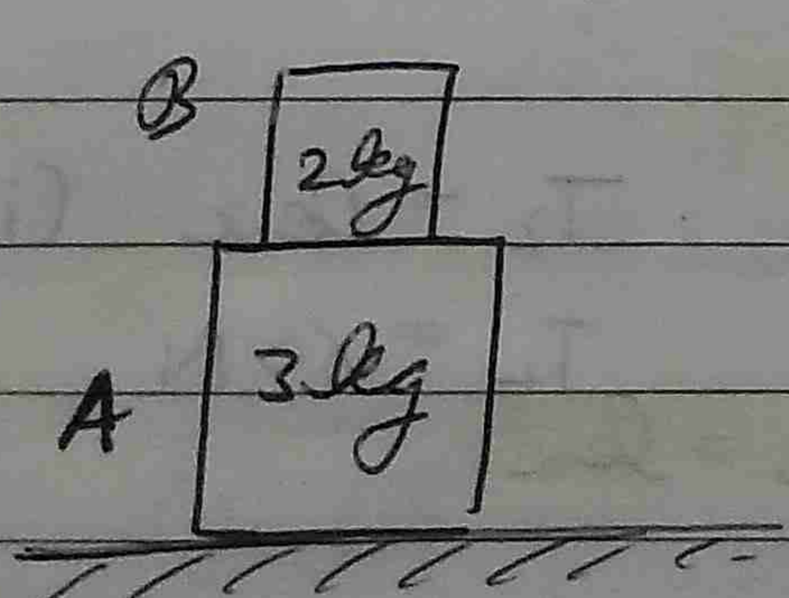
$\mu = 0$



- Equilibrium

$F_{net} = 0$

Ex:

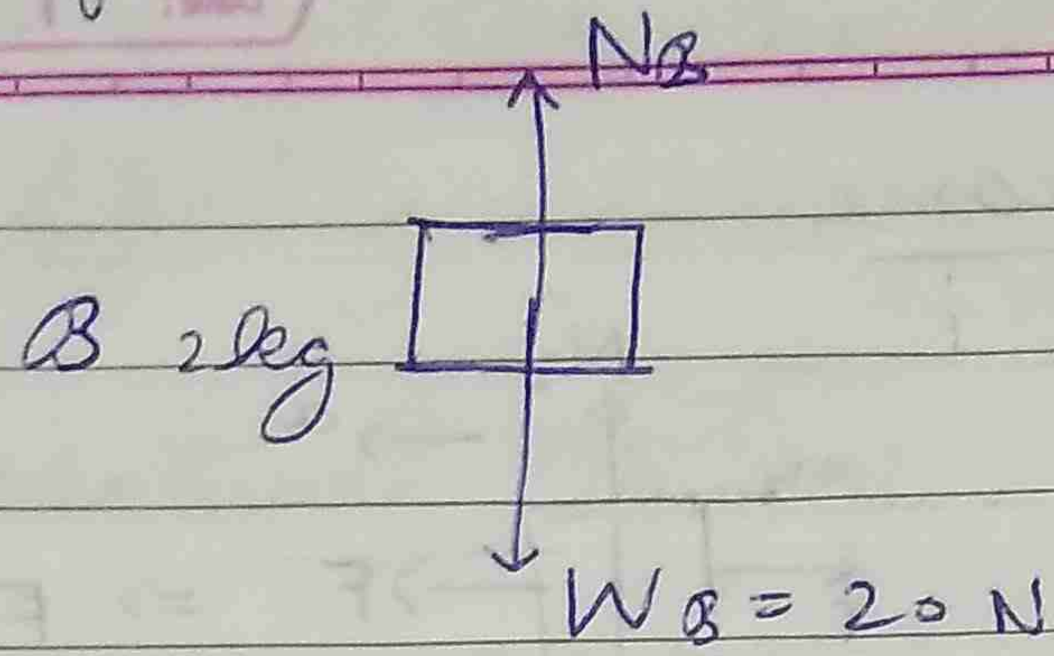


Find:

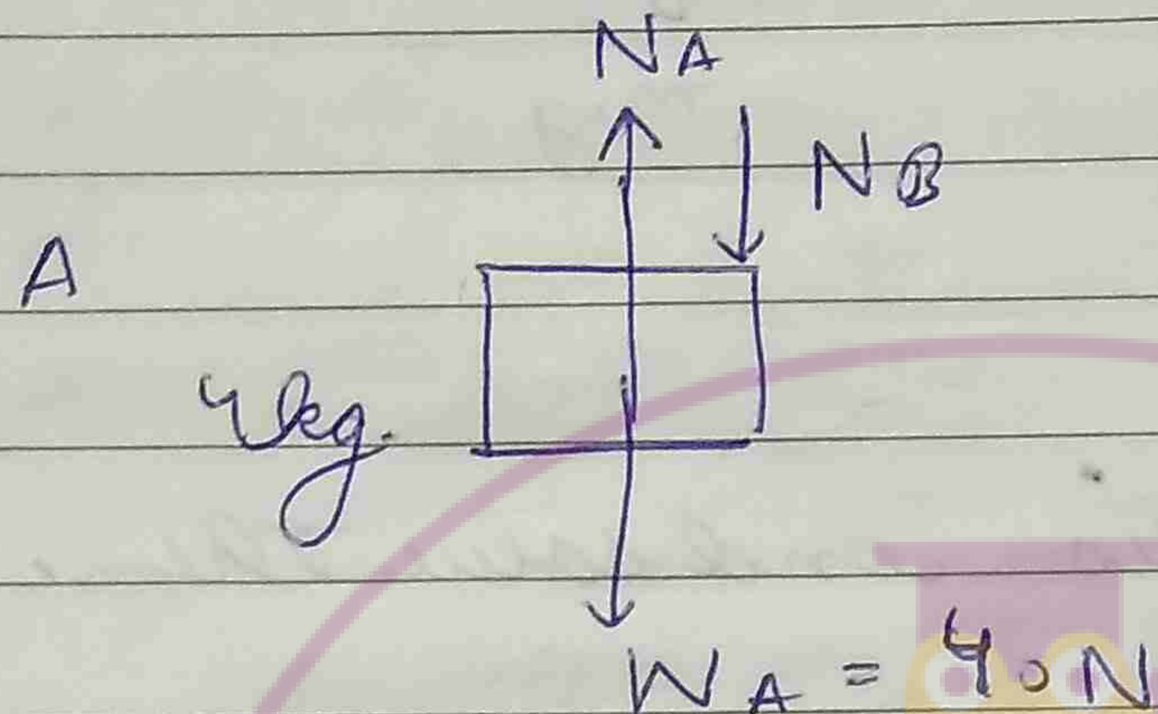
a) Force on B by A

b) Force on A by the table

- Here net force = 0 is from first law of motion.
- mass of ideal spring is zero.



∵ Both bodies are rest
∴ Net force = 0
which is possible only if



B: $W_B = N_B = 20N$

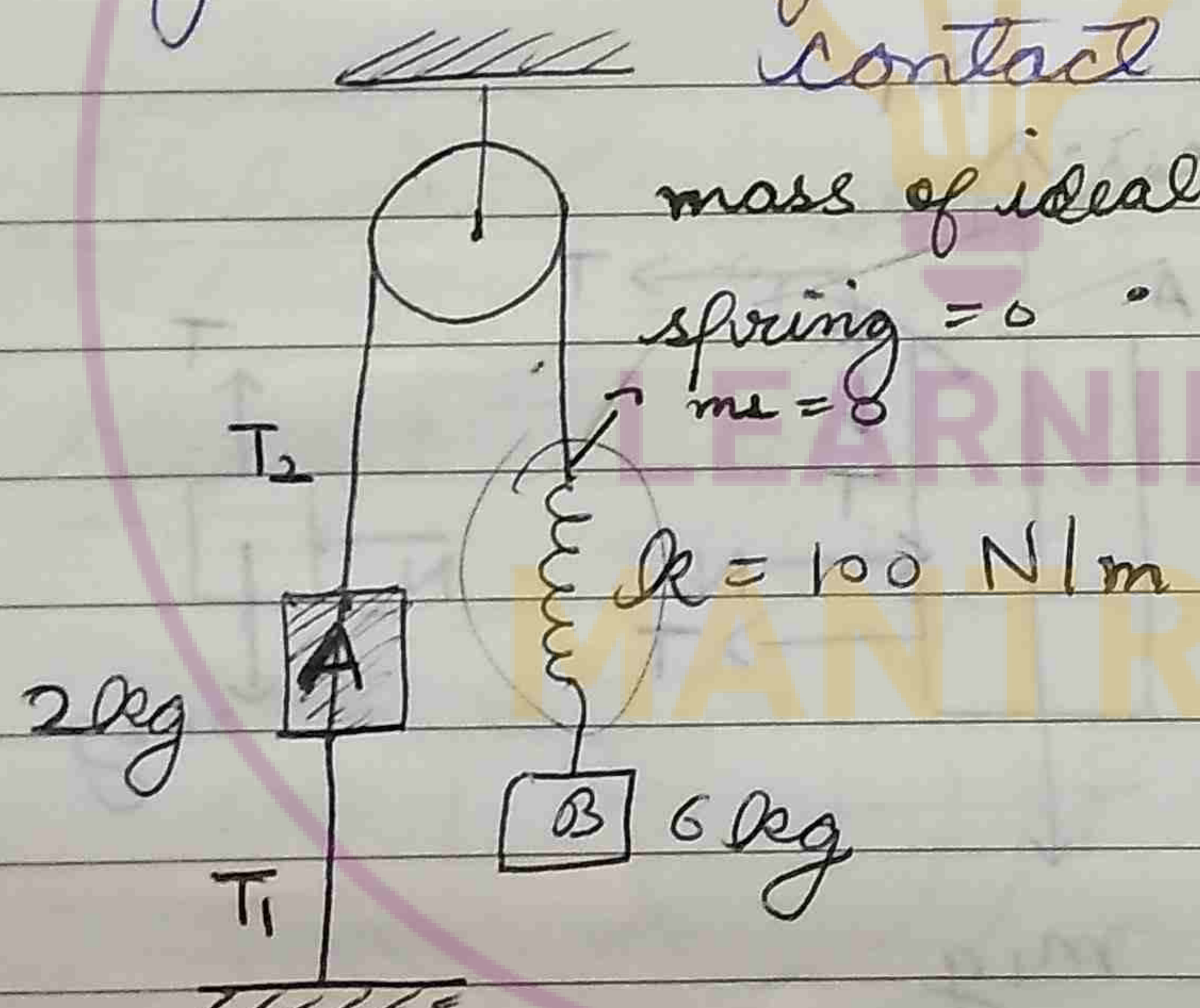
A: $W_A + N_B = N_A$

$\Rightarrow 40 + 20 = N_A$

$\Rightarrow N_A = \underline{60\ N}$

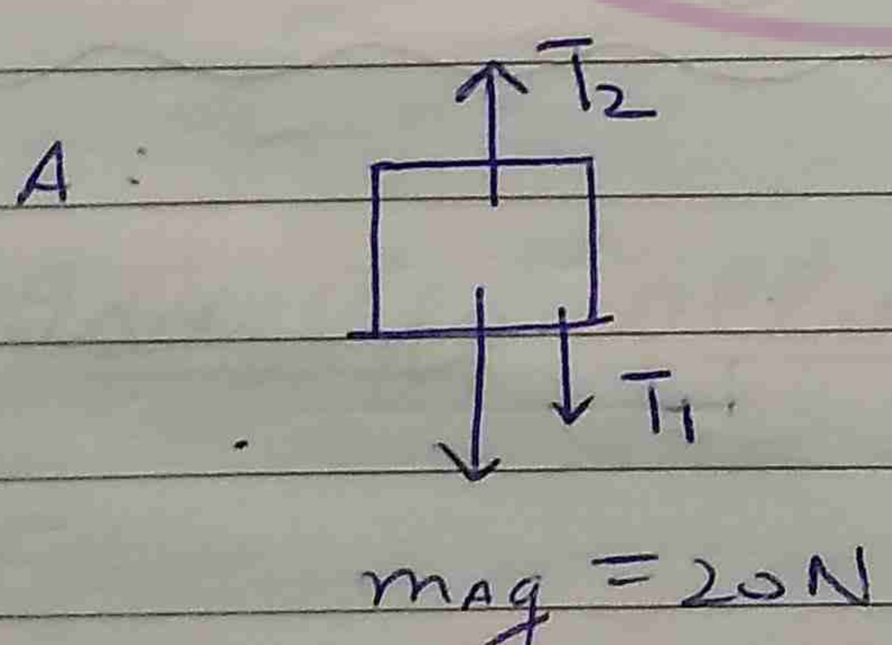
☆ Only take those forces which are in direct contact with the object.

★ Ex



Find
(a) T_1 and T_2
(b) Extension in the spring.

A:

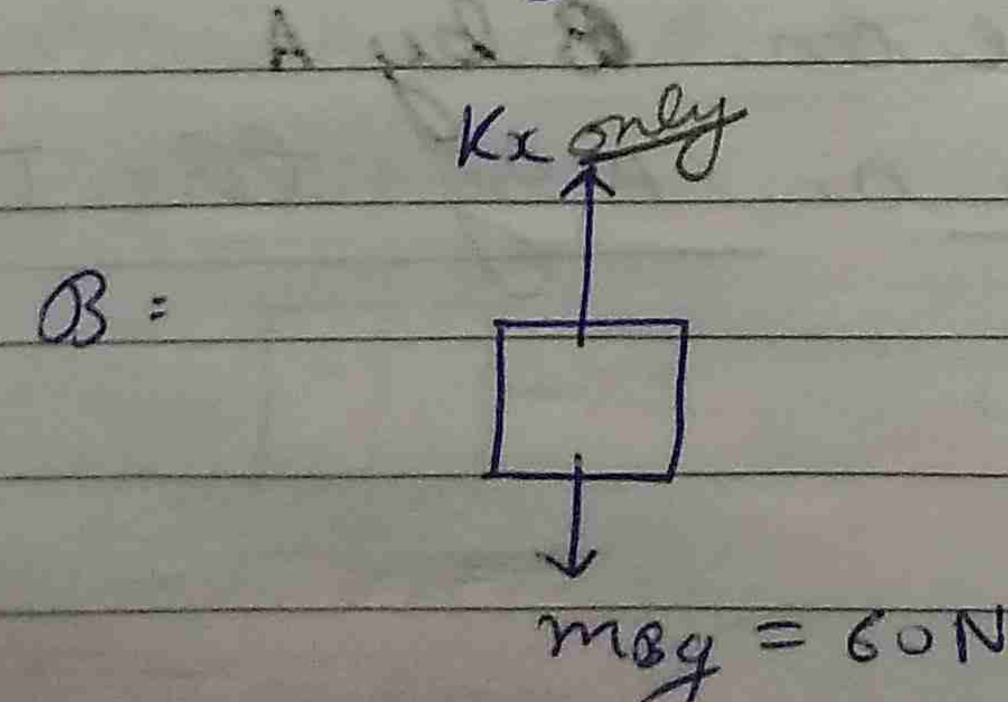


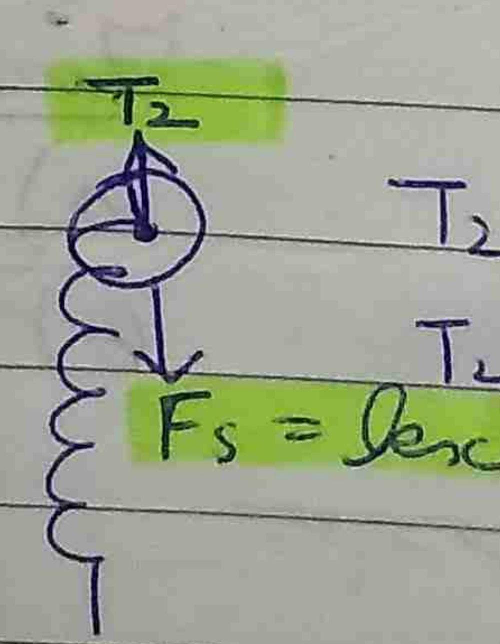
$\Rightarrow 20 + T_1 = T_2$ (i)

$\Rightarrow T_1 = \underline{40\ N}$

$60 = kx$ (ii)

B:





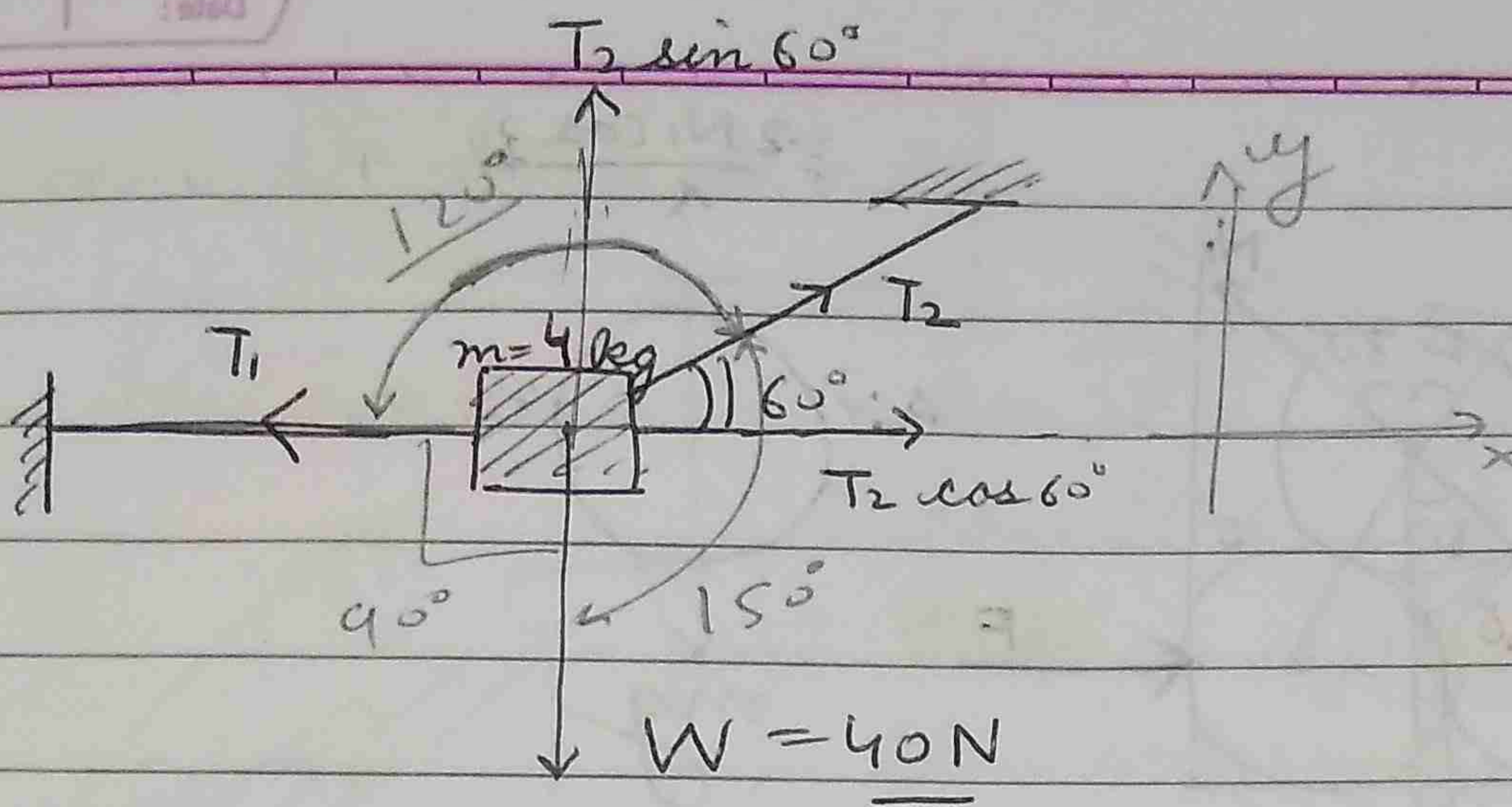
$T_2 = kx$ (iii)

$T_2 = 60N$

$kx = 60n$

$\Rightarrow x = \frac{60}{100} = 0.6\ m$

Ex



$\Sigma F_x = 0$

$\Sigma F_y = 0$

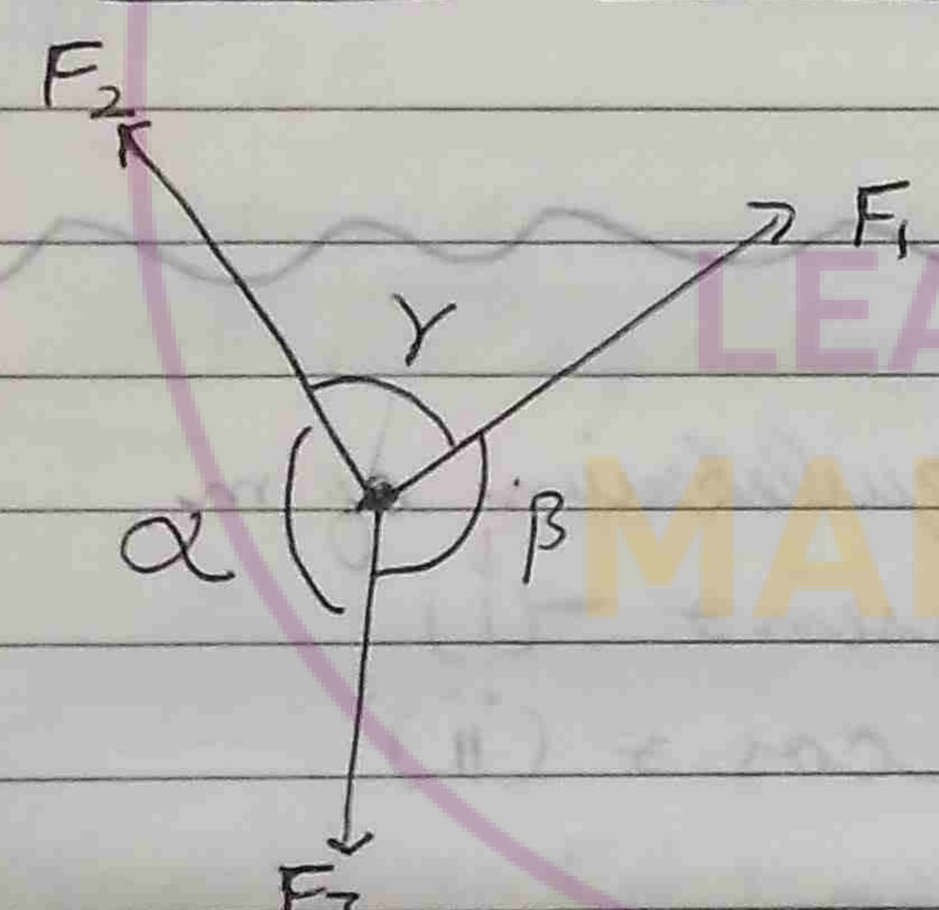
X: $T_2 \cos 60° = T_1$ --- (i) $\Rightarrow T_1 = \frac{80}{\sqrt{3}} \times \frac{1}{2} = \frac{40}{\sqrt{3}}$

Y: $T_2 \sin 60° = 40$ --- (ii)

$\Rightarrow T_2 = \frac{40}{\sin 60°} = \frac{80}{\sqrt{3}}$

$\sin 60° \rightarrow \frac{\sqrt{3}}{2}$

- **Lami's Theorem**: (applicable for upto 3 forces only)

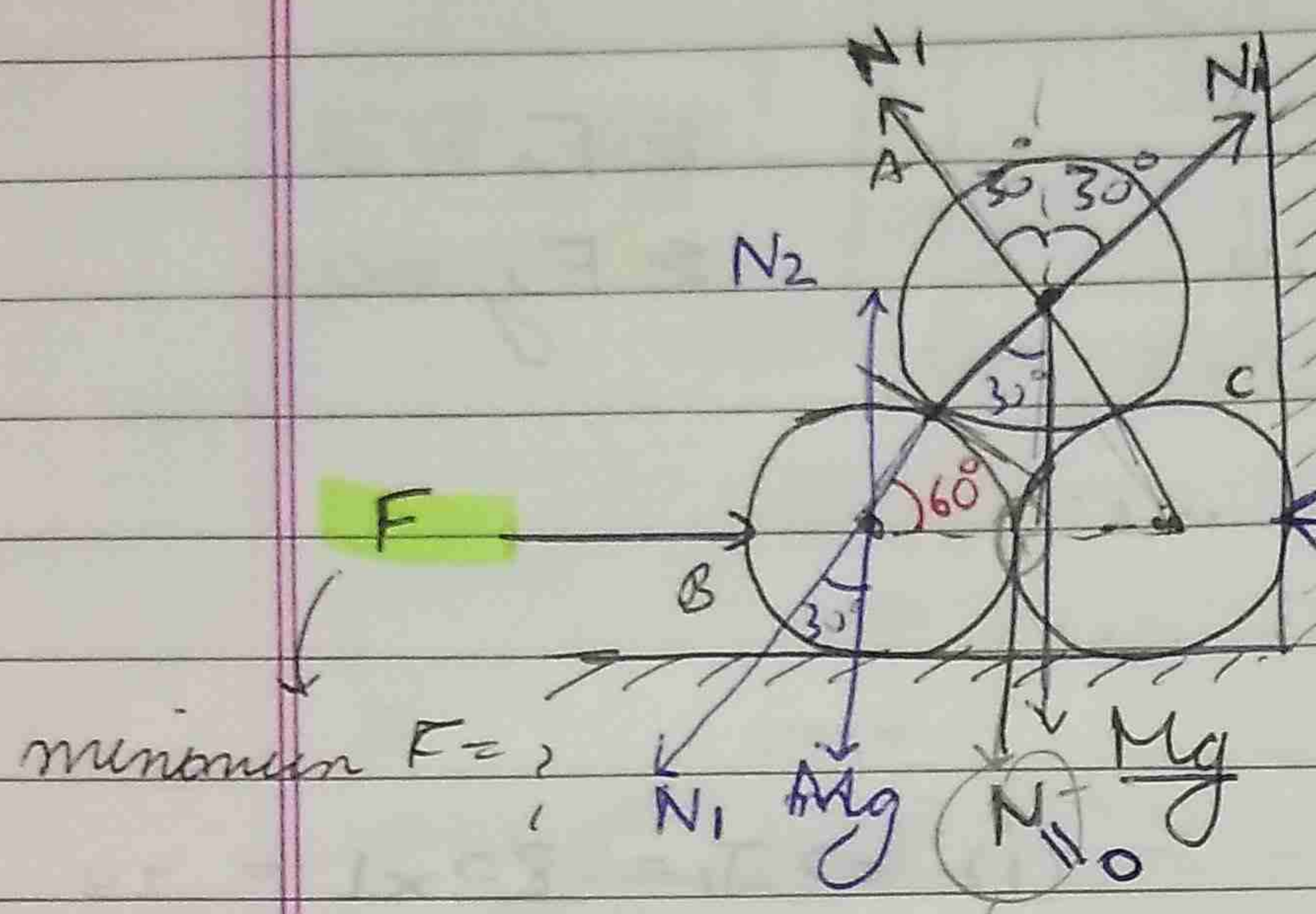


$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

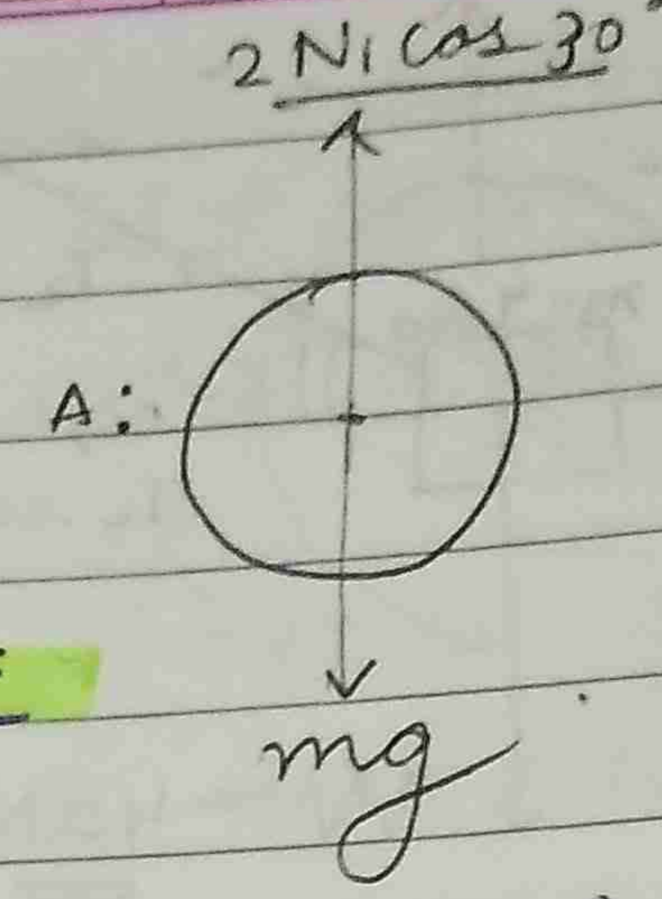
$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 90°} = \frac{W}{120°} \Rightarrow \frac{80}{\sqrt{3}}$$

($\sin 150° \rightarrow \frac{1}{2}$, $\sin 90° \rightarrow 1$, $120° \rightarrow \frac{\sqrt{3}}{2}$, $W \rightarrow 40$)

Ex The 3 identical cylinders are arranged as shown in figure having mass m each.
Find the minimum force F required to keep these bodies at rest.

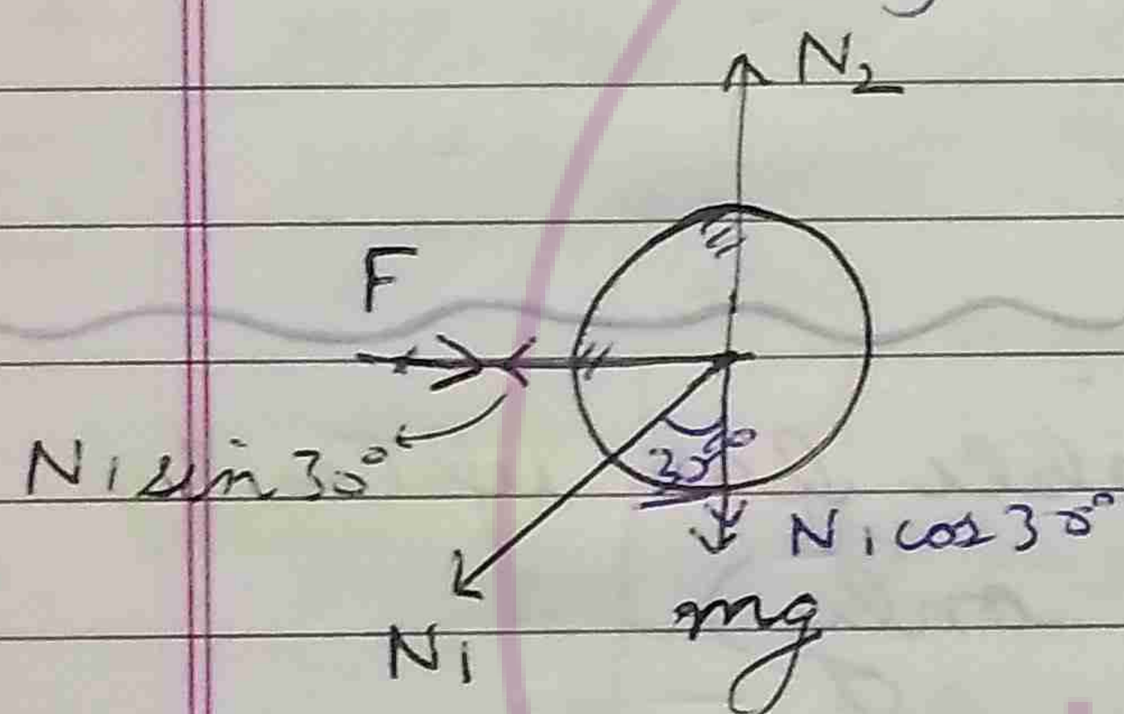


because they are on the verge of separation



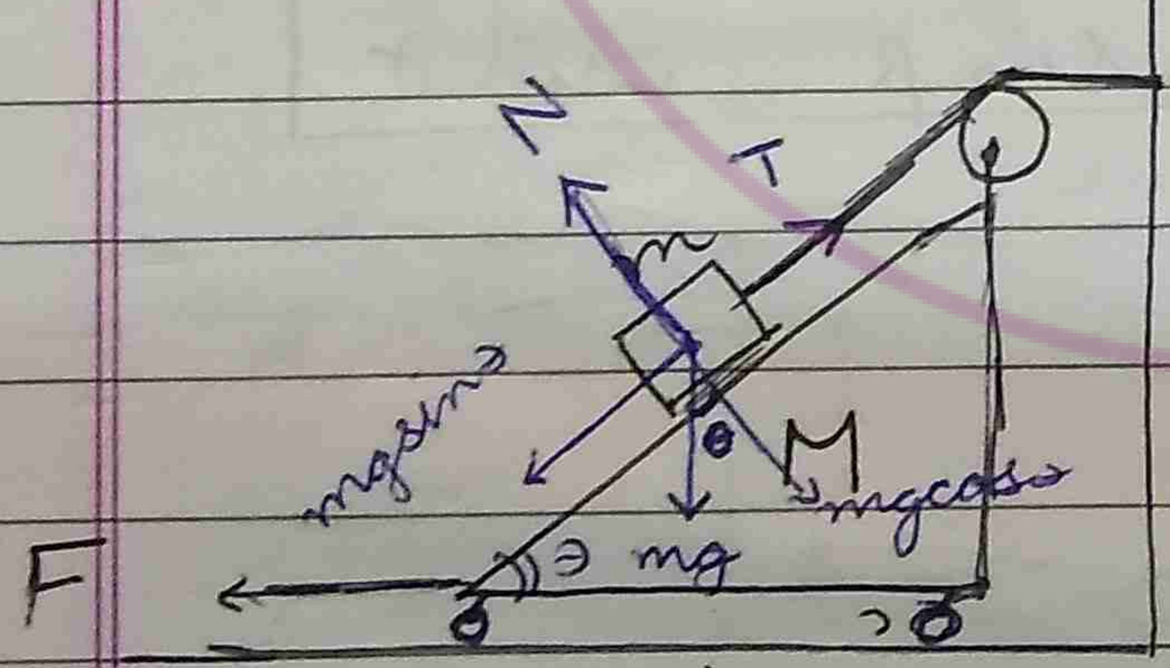
$2 N_1 \cos 30° = mg$

$\Rightarrow N_1 = \frac{mg}{\sqrt{3}}$



$F = N_1 \sin 30° = \frac{mg}{2\sqrt{3}}$

★ Ex

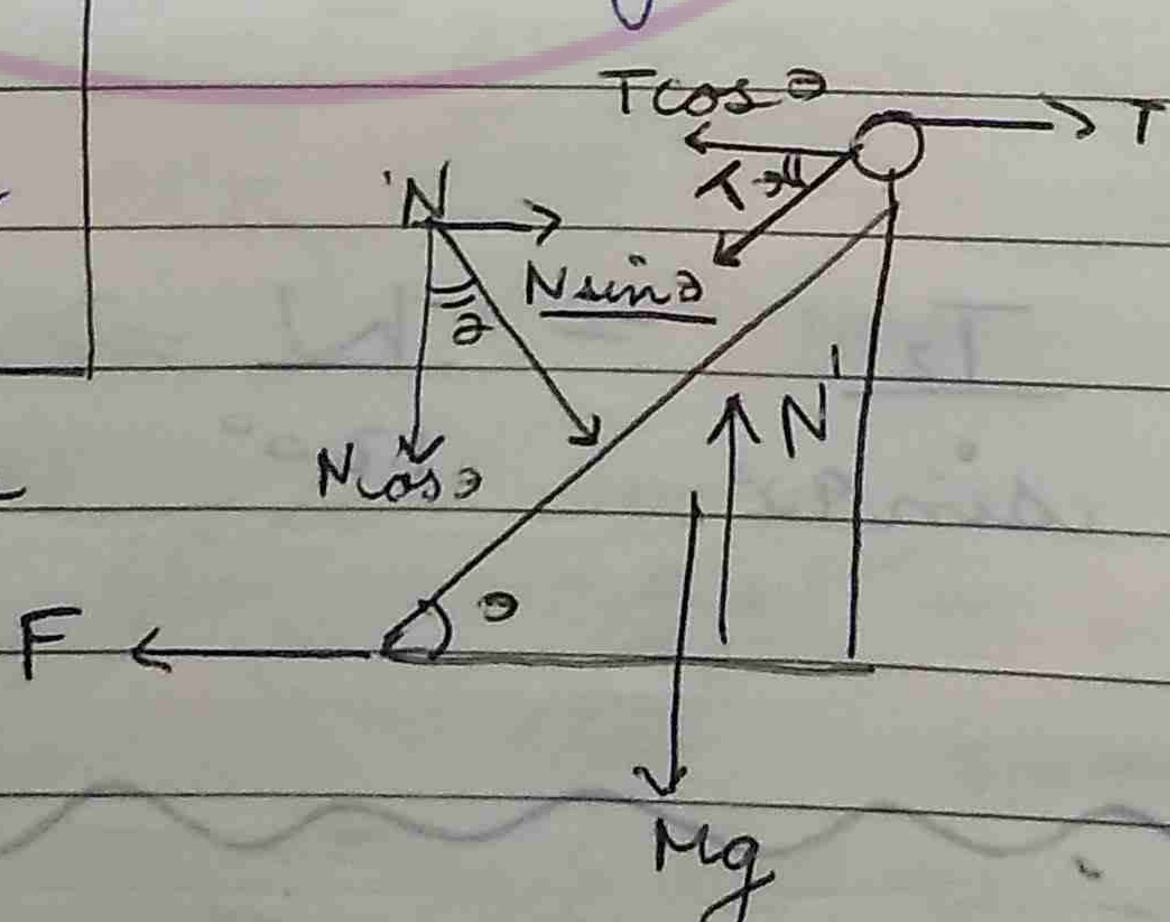


F required to keep the the system at rest.

For equilibrium of m

$T = mg \sin\theta \quad -(i)$

$N = mg \cos\theta \quad (ii)$



$\Sigma F_x = 0$

$T + N \sin\theta = T\cos\theta + F$

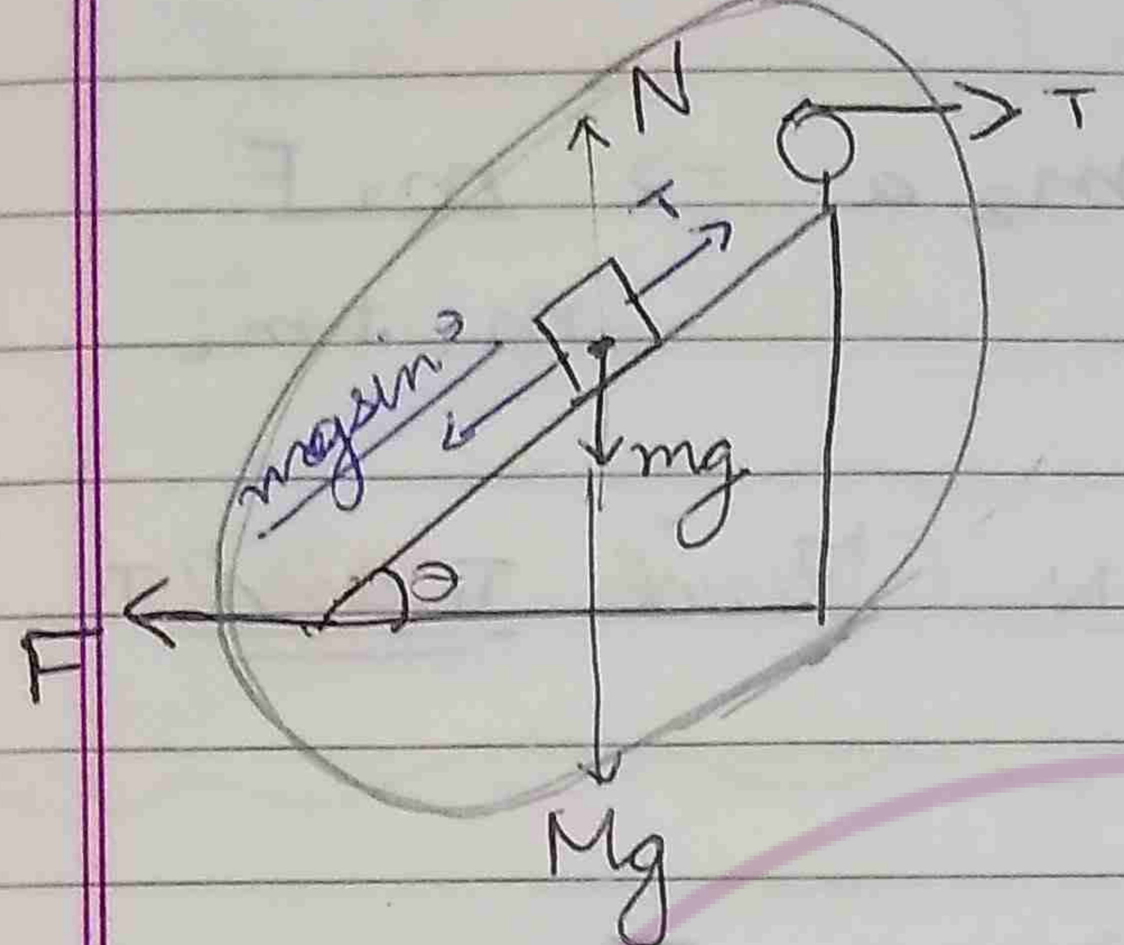
$\Rightarrow F = T - T\cos\theta + N \sin\theta$

$= mg\sin\theta - mg\sin\theta\cos\theta + mg\cos\theta\sin\theta$

- N is a pushing force and acts towards a body.

$\Rightarrow$ $\boxed{F = mg\sin\theta}$

Method



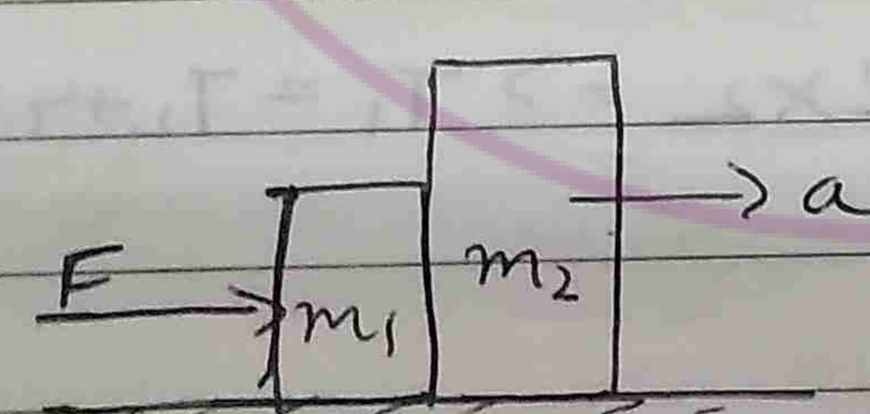
when it is completely taken as system then internal forces are neglected

$F = T \Rightarrow \boxed{F = mg\sin\theta}$

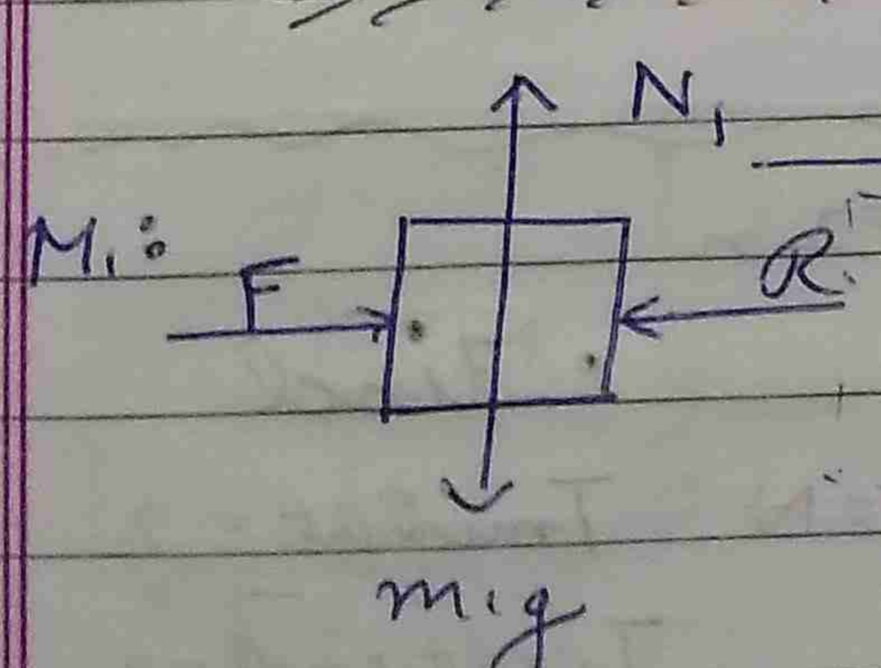
- Applications of laws of motion for moving bodies.

(i) system
(ii) forces.
(iii) FBD
(iv) Frame of reference.
(v) Write equation of motion.

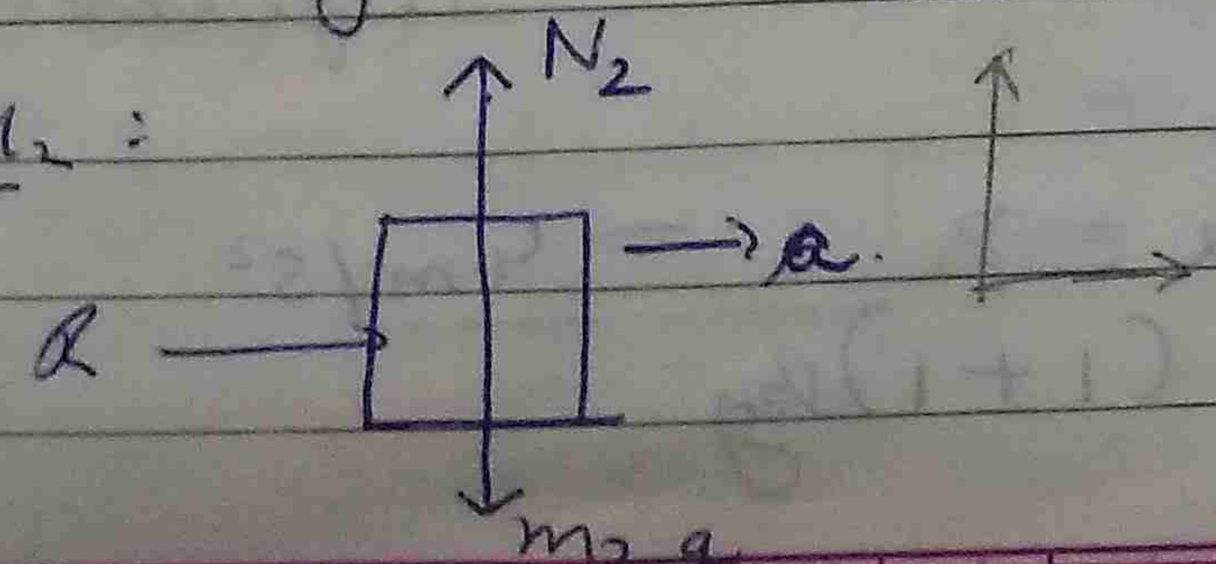
Ex



- Find:
(a) a : acceleration
(b) Normal reaction b/w m_1 and m_2



$F - R = m_1 a \quad (i)$

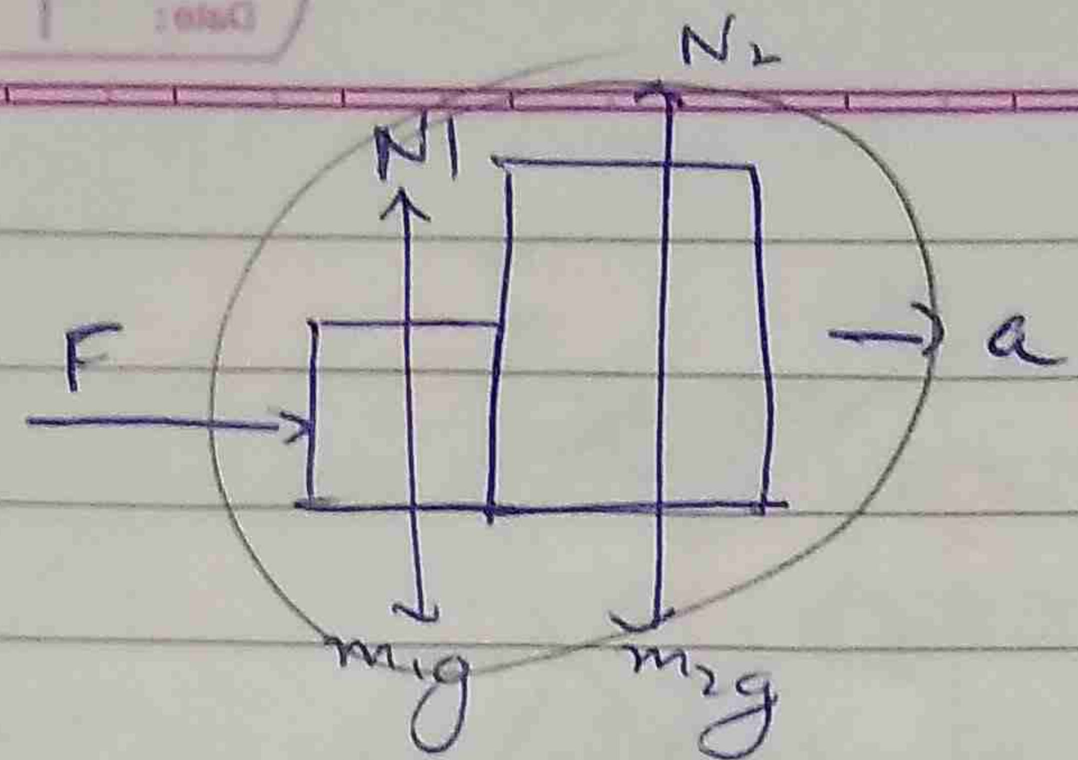


$R = m_2 a \quad (ii)$

adding (i) and (ii)

$\boxed{a = \frac{F}{m_1 + m_2}}$

• In rope of negligible mass, tension at all points is same.



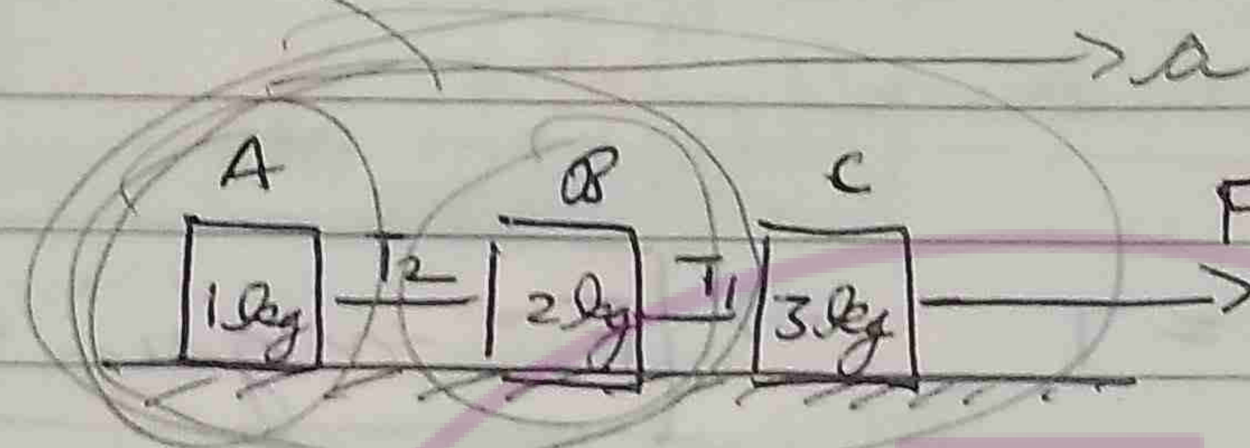
$\Rightarrow F = (m_1 + m_2)\, a$

$\Rightarrow \boxed{a = \dfrac{F}{m_1 + m_2}}$

$a = m_2 a \Rightarrow \dfrac{m_2 F}{m_1 + m_2}$

$T_1 = (1+2) \times 2 = \underline{\underline{6N}}$

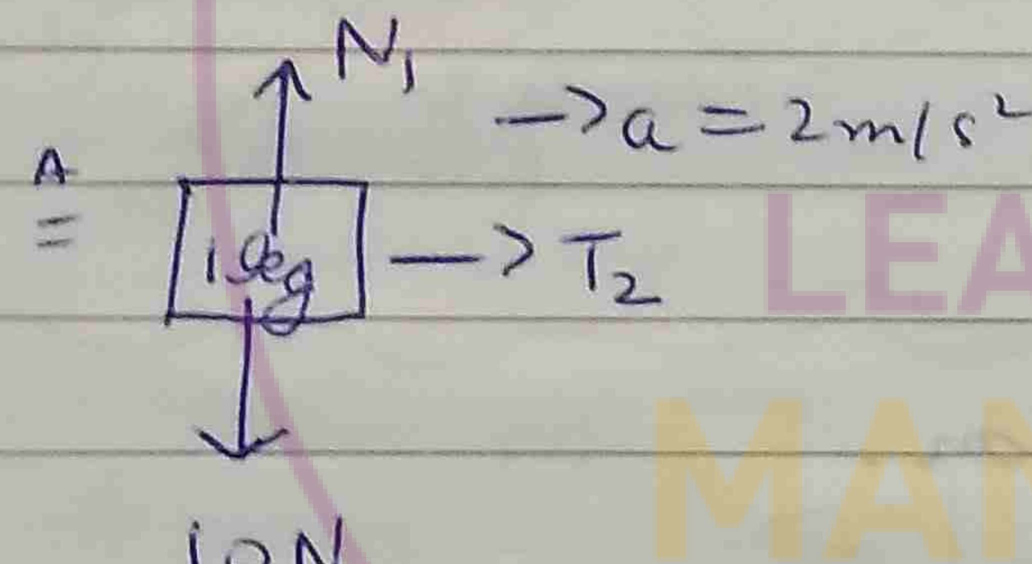
Ex



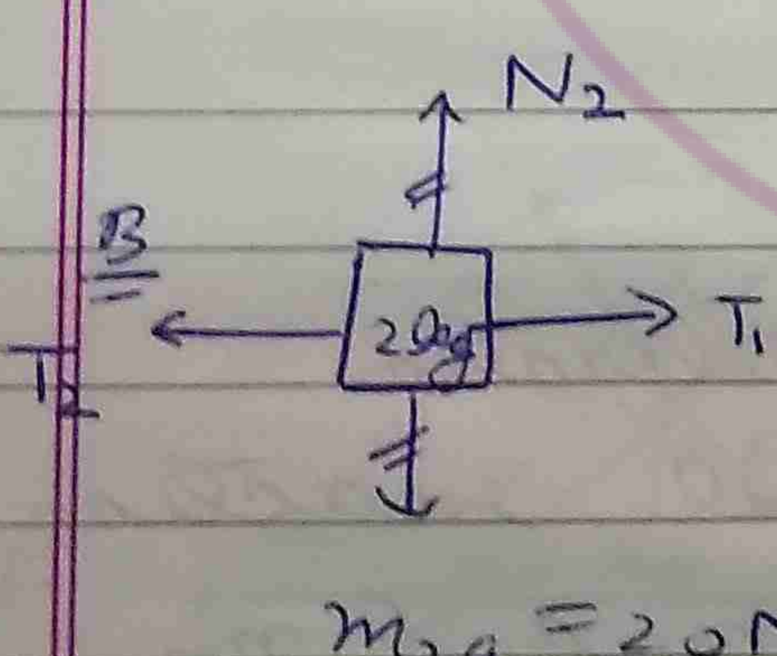
$F = 12N$ Find T_1 and T_2

$F = (m_1 + m_2 + m_3)\, a \Rightarrow a = \dfrac{F}{m_1 + m_2 + m_3}$

$\Rightarrow \dfrac{12}{6} = 2\ m/s^2$



$\Rightarrow T_2 = 1 \times 2 = \underline{\underline{2\ N}}$

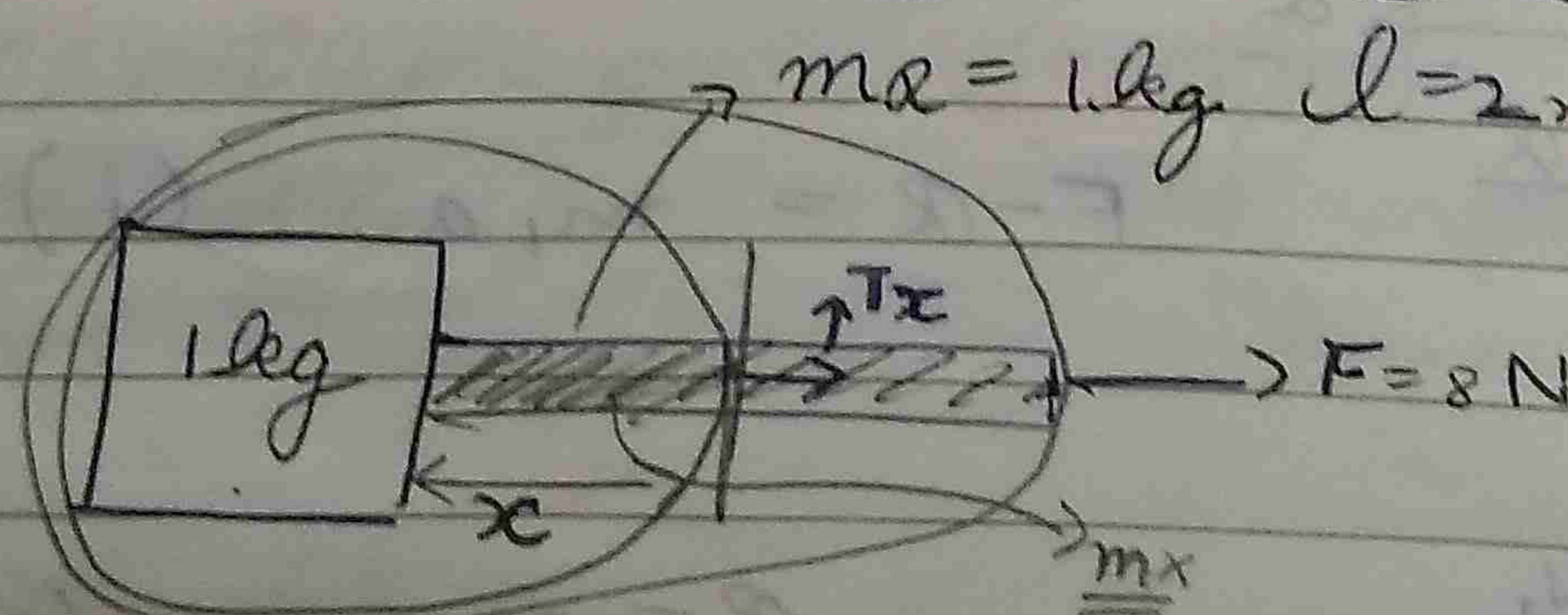


$\Rightarrow T_1 - T_2 = 2 \times 2 \Rightarrow T_1 = T_2 + 4 = \underline{\underline{6\ N}}$

$m_2 g = 20N$

Ex

$m_x = 1kg \quad l = 2m$



• Find
$T_{middle} = ?$
$T_{left\ end} = ?$

$F = (m + m_x)\, a = a = \dfrac{8}{(1+1)kg} = 4\ m/s^2$

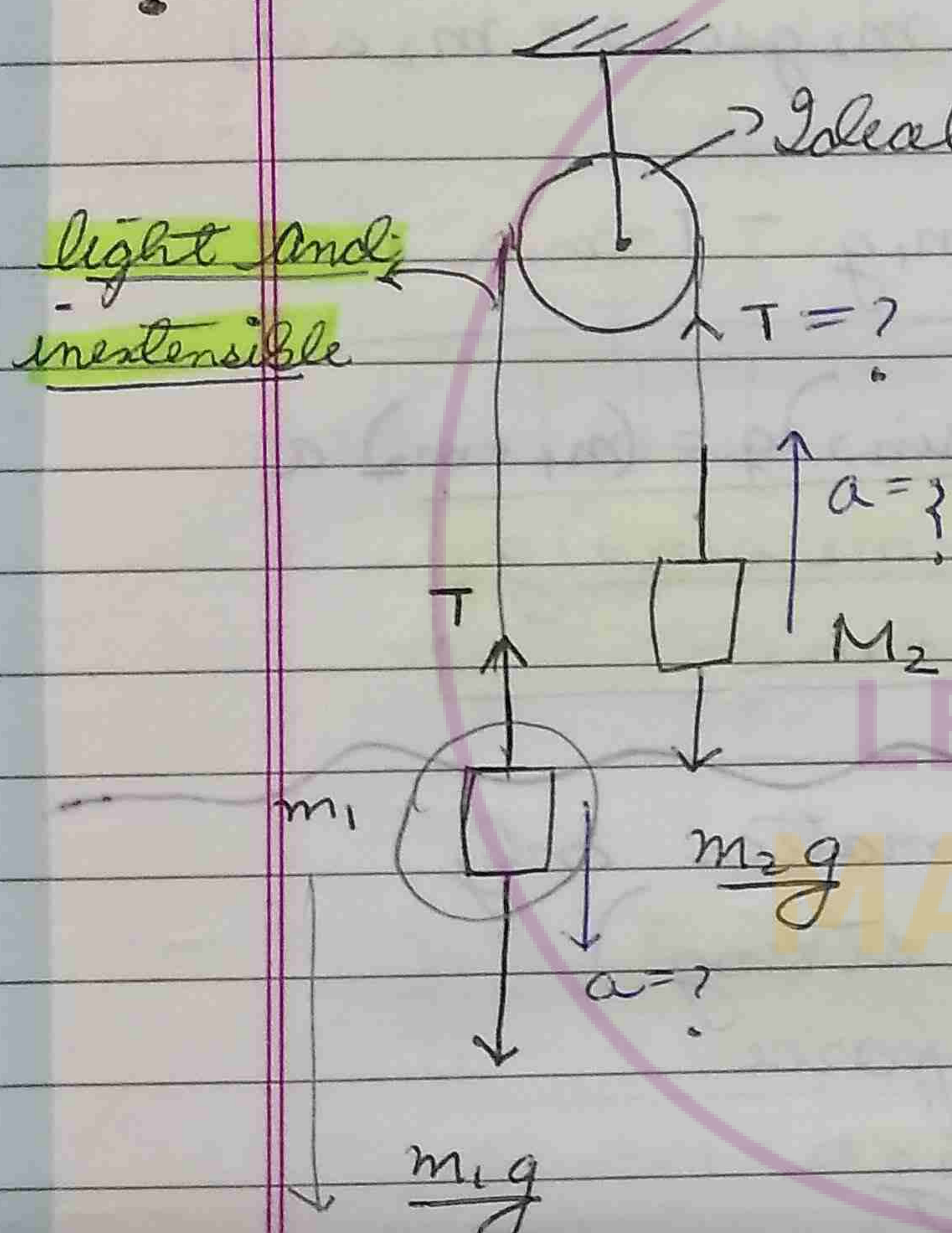
$$m_x = \frac{m_R}{l}\cdot x = \frac{x}{2}$$

$$T_x = (m + m_x)\,a \Rightarrow T_x = \left(1 + \frac{x}{2}\right)\times 4$$

$$T_{\text{mid point}} = T_x = \frac{3}{2}\times 4 = 6\,N \quad \because \left(x = \frac{l}{2} = 1\,m\right)$$

$$T_{\text{left end}} = T_{x=0} = (1+0)\times 4 = \underline{4\,N}$$

•



→ Ideal pulley → smooth and frictionless and massless

$$m_1 : m_1 g - T = m_1 a \quad (i)$$

$$m_2 : T - m_2 g = m_2 a \quad (ii)$$

$$(m_1 - m_2)g = (m_1 + m_2)a$$

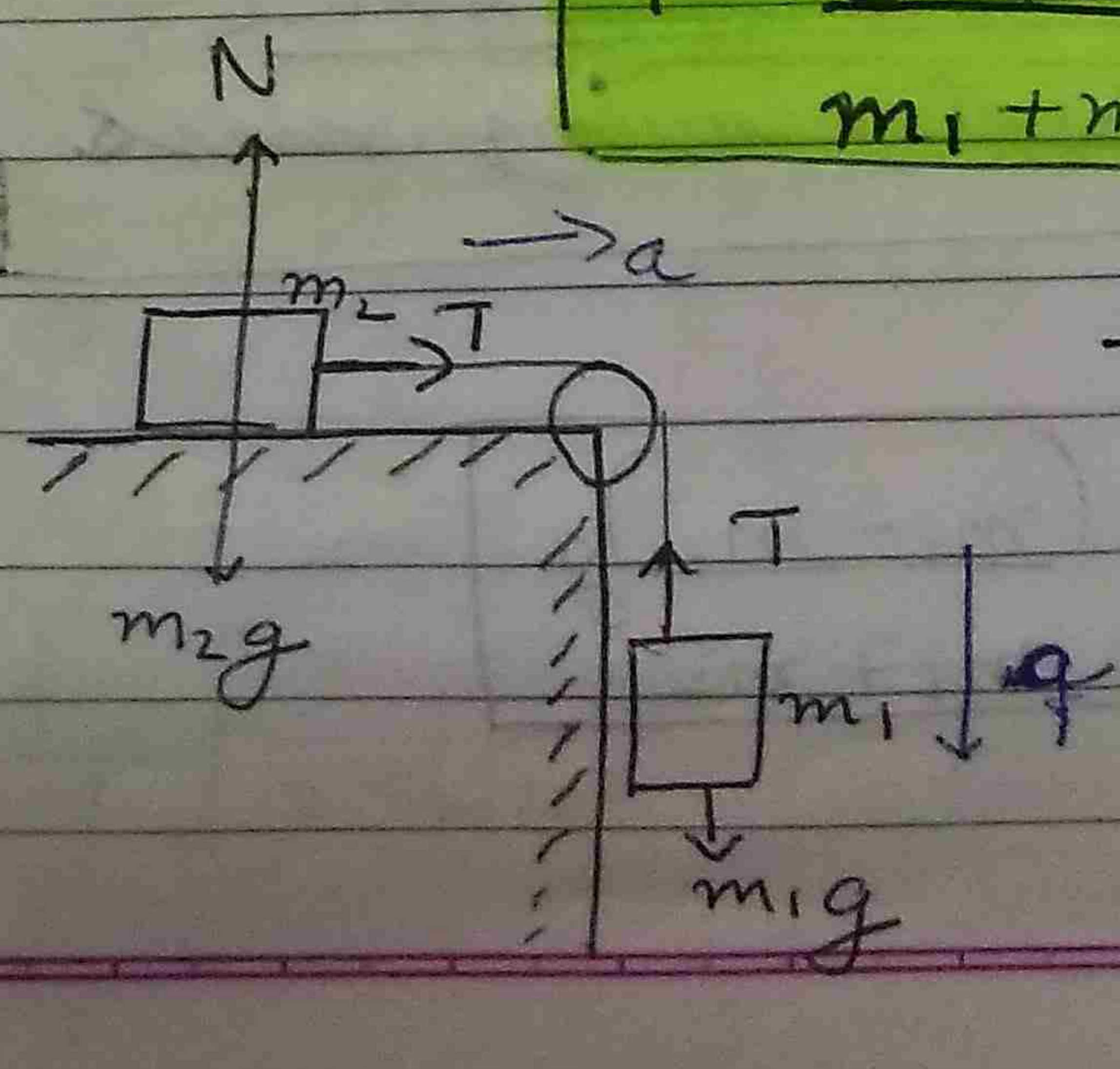
$$\Rightarrow \boxed{a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g}$$

$$m_1 g - T = m_1 a \Rightarrow T = m_1(g - a)$$

$$T = m_1\left(g - \frac{(m_1 - m_2)}{(m_1 + m_2)}g\right)$$

$$\boxed{T = \frac{2 m_1 m_2}{m_1 + m_2}\,g}$$

•



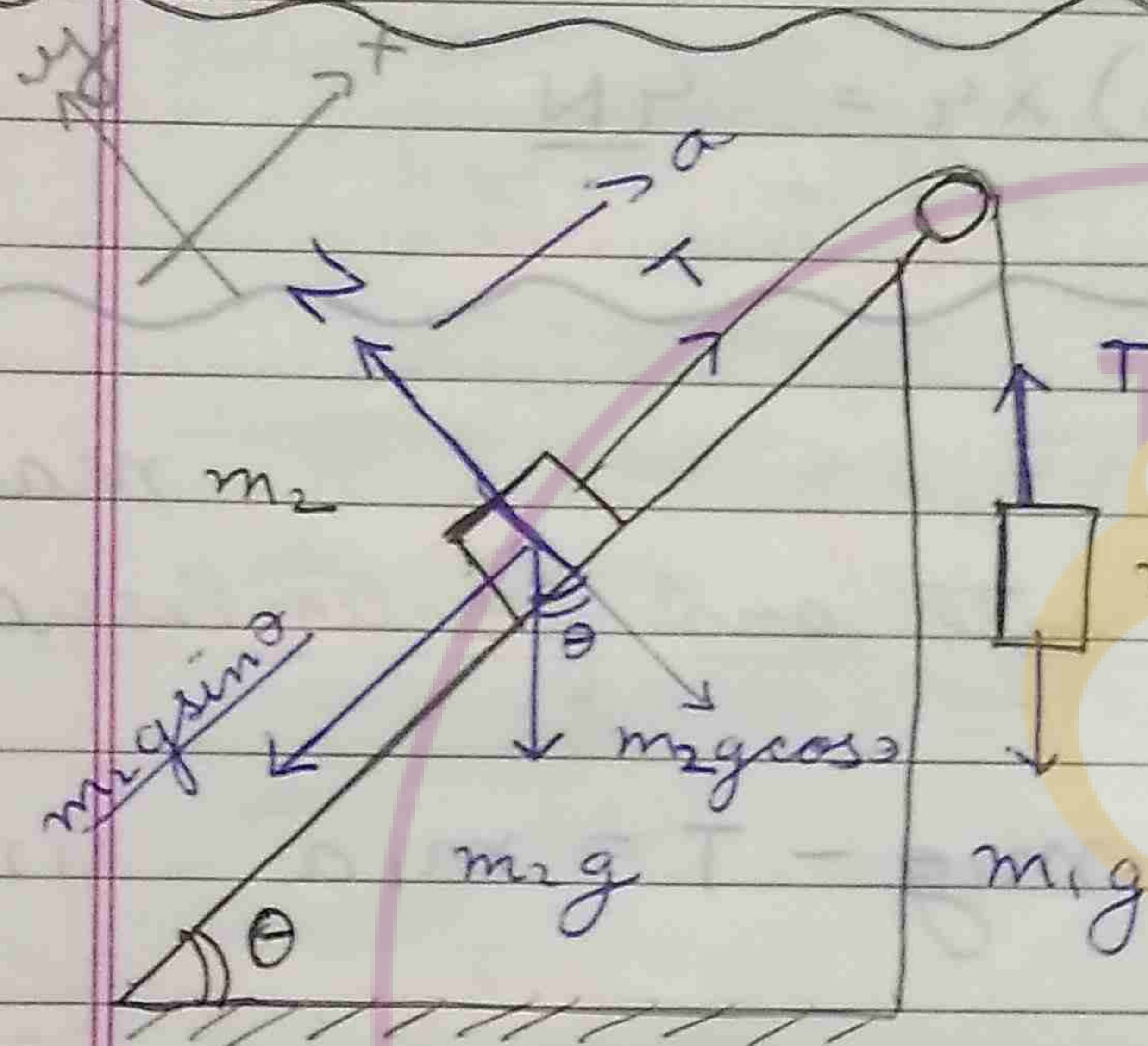
T = ? a = ?

$$m_1 : m_1 g - T = m_1 a \quad (i)$$

$$m_2 : T = m_2 a$$

$$a = \frac{m_1 g}{m_1 + m_2}$$

$$T = m_2 \cdot \frac{m_1 g}{m_1 + m_2} \Rightarrow T = \frac{m_1 m_2 g}{m_1 + m_2}$$



Find a, T

m_2 : $T - m_2 g\sin\theta = m_2 a$ (i)

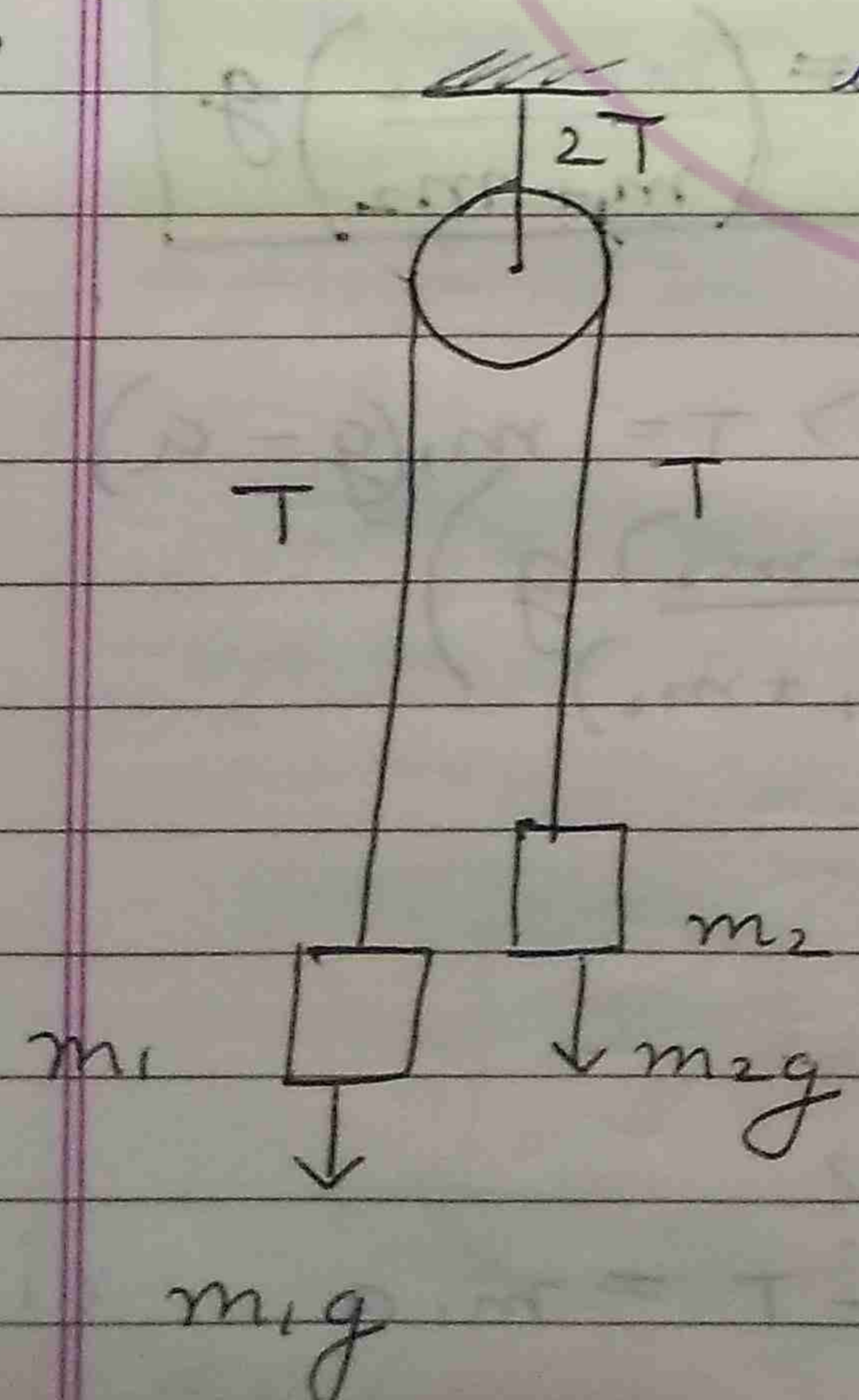
m_1 : $m_1 g - T = m_1 a$ (ii)

$(m_1 - m_2\sin\theta)g = (m_1 + m_2)a$

$$\Rightarrow a = \frac{(m_1 - m_2\sin\theta)g}{m_1 + m_2}$$

T =

Method-2 : (applicable for motion only along a single string)

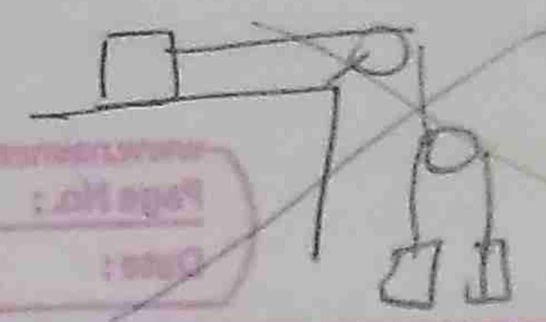


- Net driving force $= m_1 g - m_2 g$

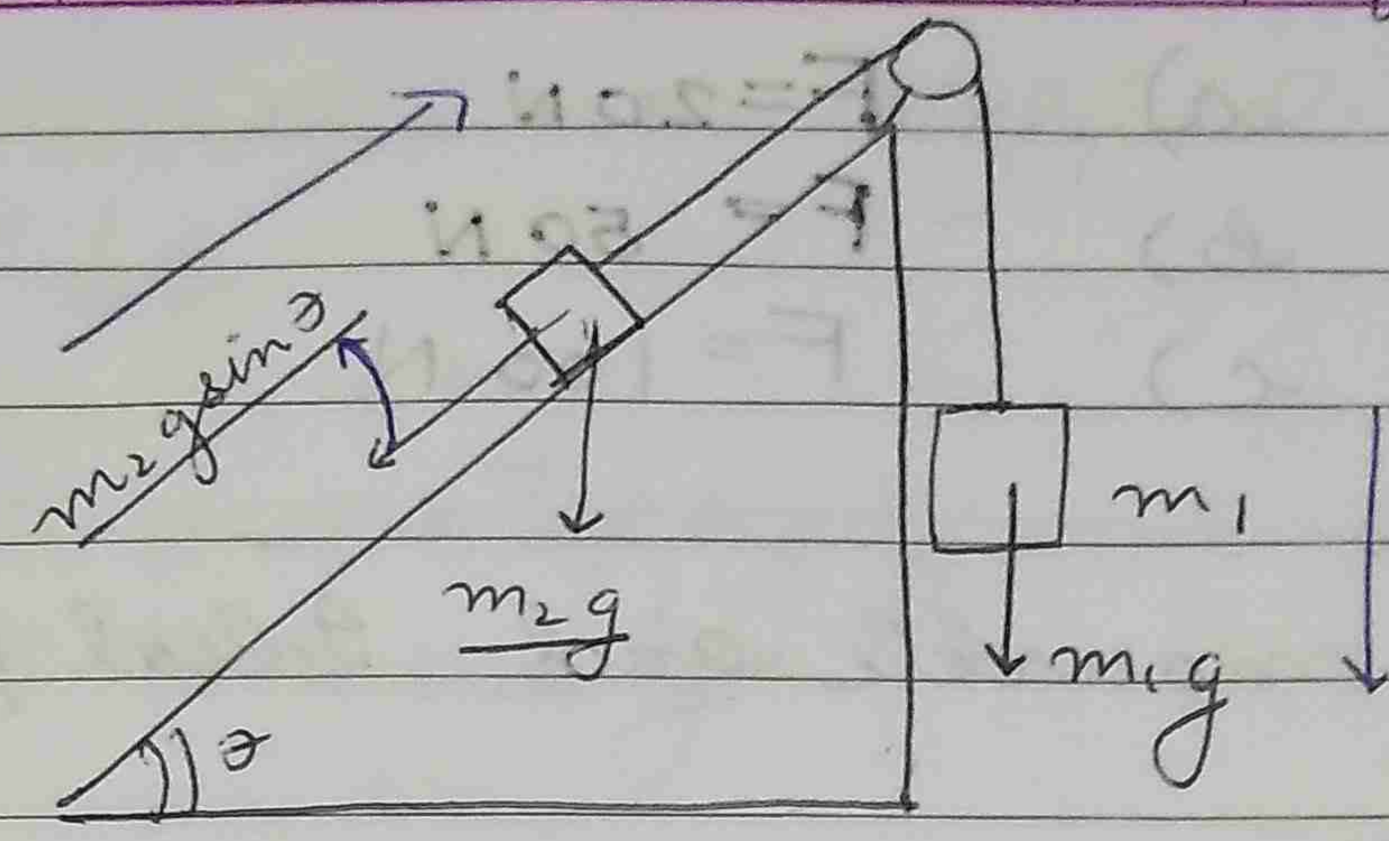
Net moving mass $= m_1 + m_2$

$$a = \frac{\text{Net driving force}}{\text{Net moving mass}}$$

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$



~~Not applicable because its not single string system~~ so here concept of net ~~driving force not applicable~~

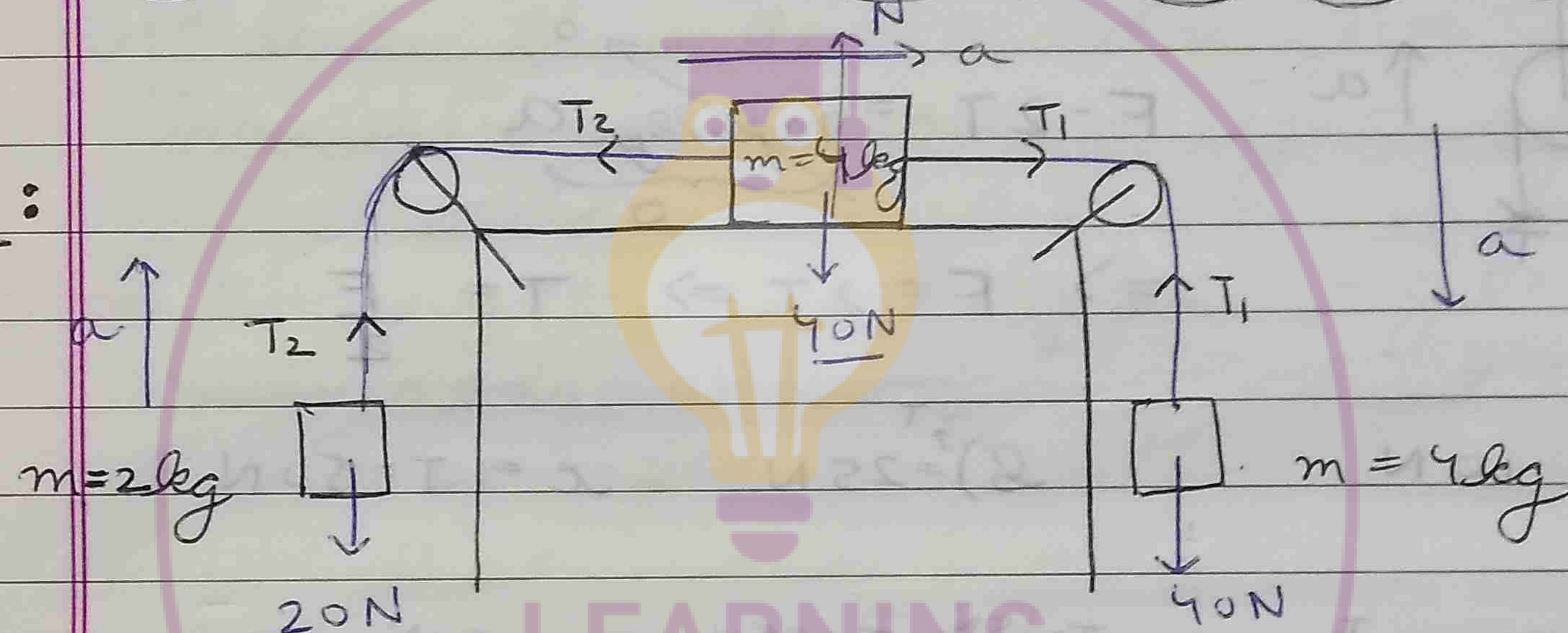


Net driving force
$= m_1 g - m_2 g\sin\theta$

Total moving mass
$= m_1 + m_2$

$\Rightarrow$ acceleration $\boxed{a = \dfrac{(m_1 - m_2\sin\theta)\,g}{m_1 + m_2}}$

Ex :



Find a, T_1, T_2

$40 - T_1 = 4a$ (i)

$T_1 - T_2 = 4a$ (ii)

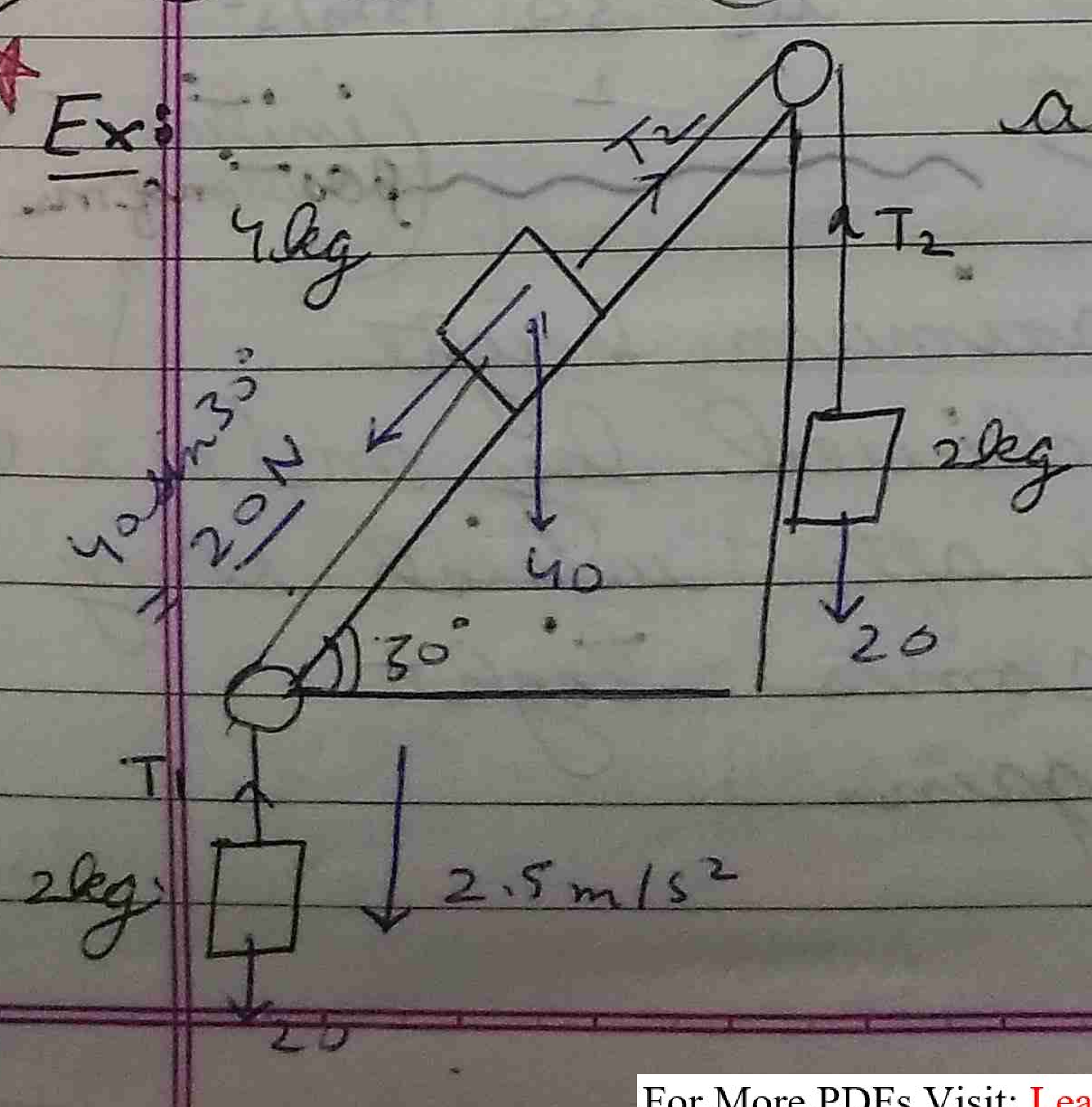
$T_2 - 20 = 2a$ (iii)

$20 = 10a$

$\Rightarrow a = 2\,m/s^2$

or $\dfrac{40-20}{4+4+2} = 2\,m/s^2$

Ex :



$a = ?$ $\quad a = \dfrac{20 + 20 - 20}{2+4+2} = 2.5\,m/s^2$

$20 - T_1 = 2 \times 2.5$

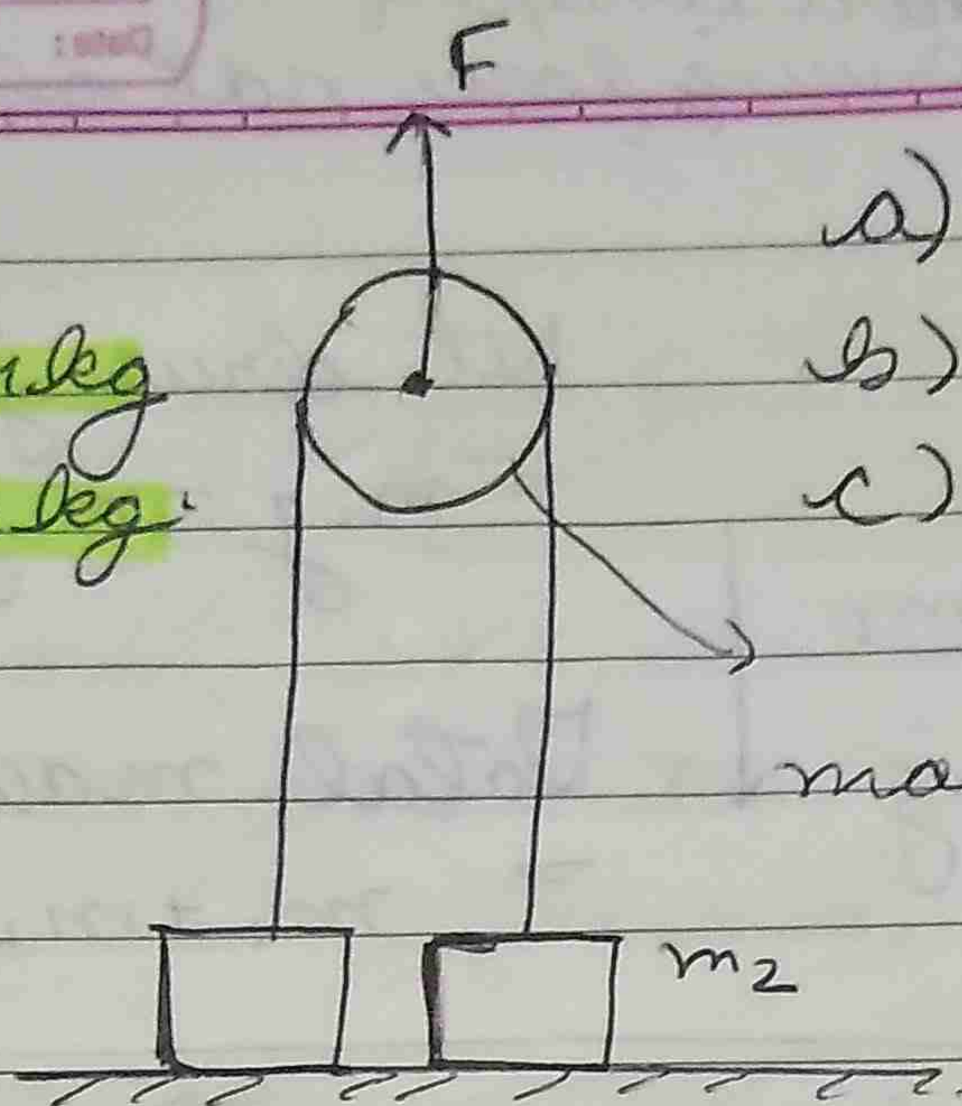
$\Rightarrow T_1 = 15$ N

$T_2 - 20 = 2 \times 2.5 \rightarrow T_2 = 25$ N

Ideal pulley has zero mass

☆ Ex

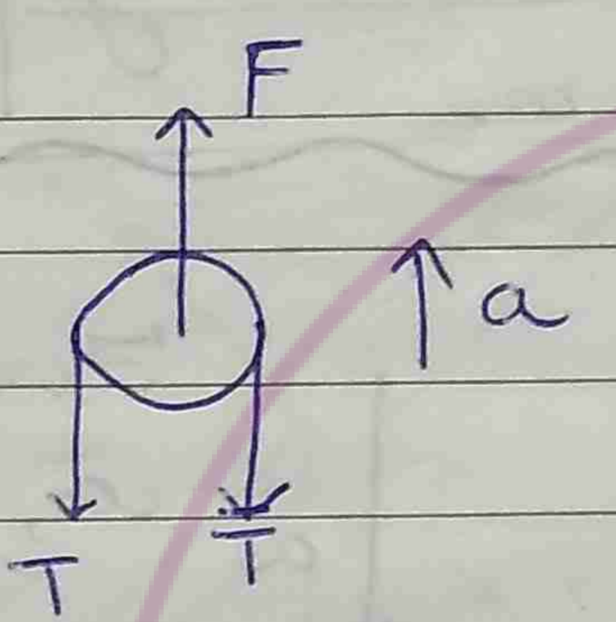
$m_1 = 4\,kg$
$m_2 = 2\,kg$



a) F = 20 N
b) F = 50 N
c) F = 100 N

movable and Ideal pulley

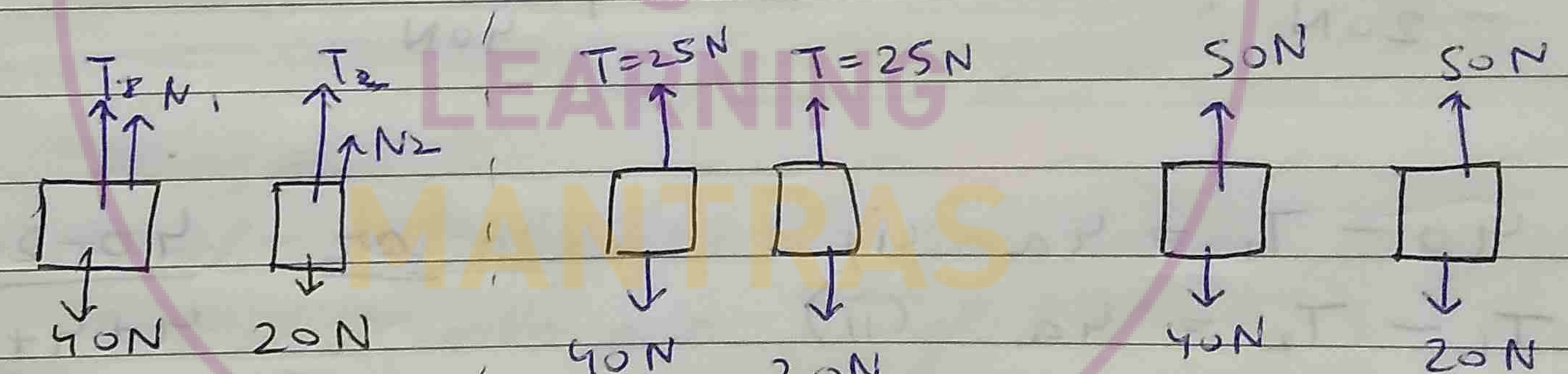
. Find acceleration and tension in each case.



$F - 2T = m_{pulley}\, a$ (with $m_{pulley} = 0$)

$\Rightarrow F = 2T \Rightarrow T = \frac{F}{2}$

a) T = 10 N   b) T = 25 N   c) T = 50 N



$a_1 = a_2 = 0$ $a_1 = 0$ $a_2 = \frac{25-20}{2} = 2.5\,m/s^2$ $a_1 = \frac{10}{4} = 2.5\,m/s^2$

$T + N_1 = 40 \Rightarrow N_1 = 30\,N$ $a_2 = \frac{30}{2} = 15\,m/s^2$

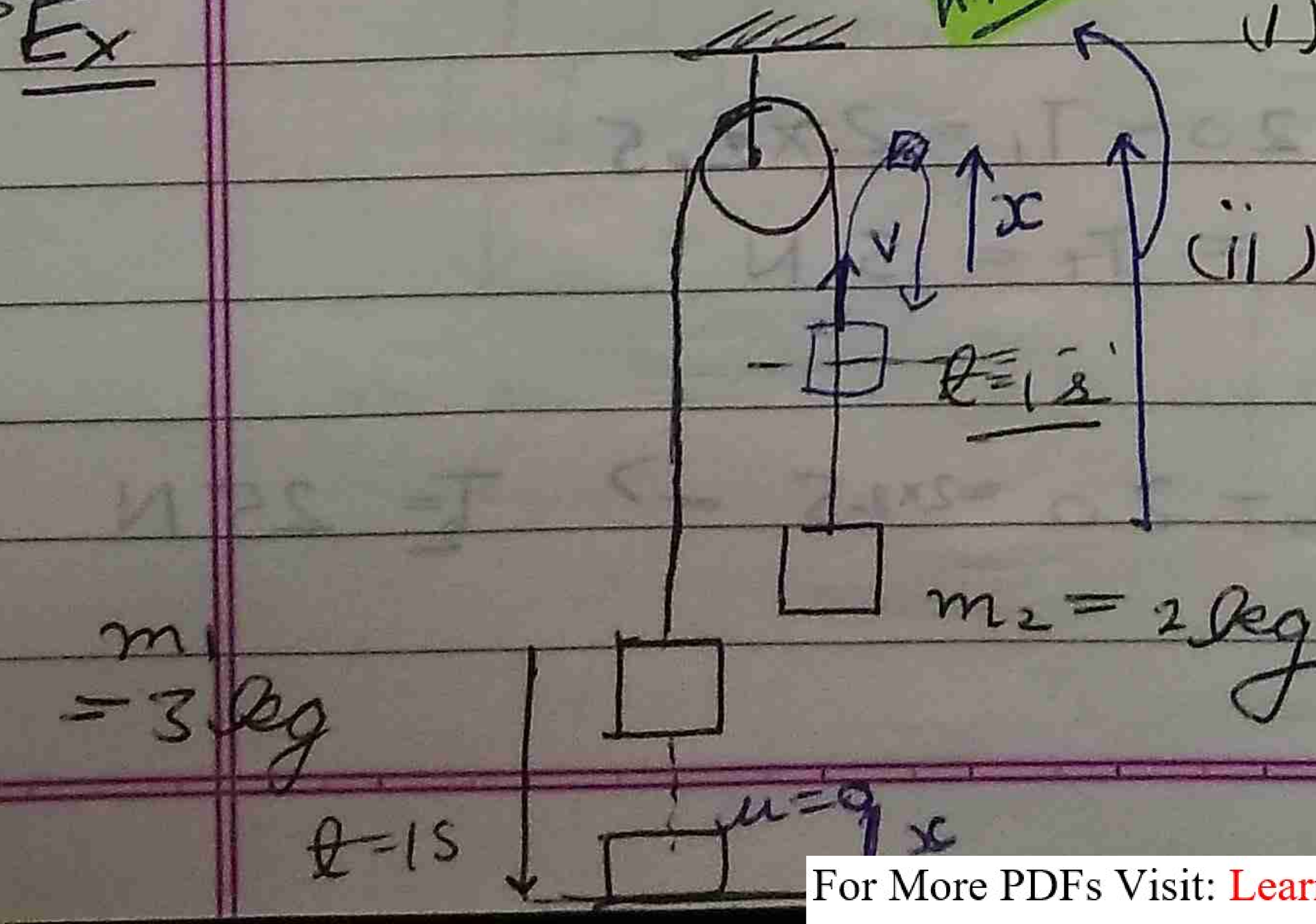
$T + N_2 = 20 \Rightarrow N_2 = 10\,N$

$H_{max} = 1 + 0.2 = 1.2\,m$

☆ Ex

(i) Maximum height achieved by m_2 (w.r.t initial position of m_2)

(ii) Time after which string becomes tight again.



- distance travelled before spring becomes tight = 1 + 0.2 m ⇒ 1.2 m
- ~~Spring force action both the ends and is equal in~~ magnitude ↑.

$u = 0$, m_1 was stopped for a moment at $t = 1\,s$

~~m_1 was stopped for a moment~~

$a = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{1}{5} \times 10 = 2\,m/s^2$

$v = 0 + 2 \times 1 = 2\,m/s$

- At time 't' when S_1 (down) = S_2 (up) [displacement, displacement]
Then string becomes tight again.

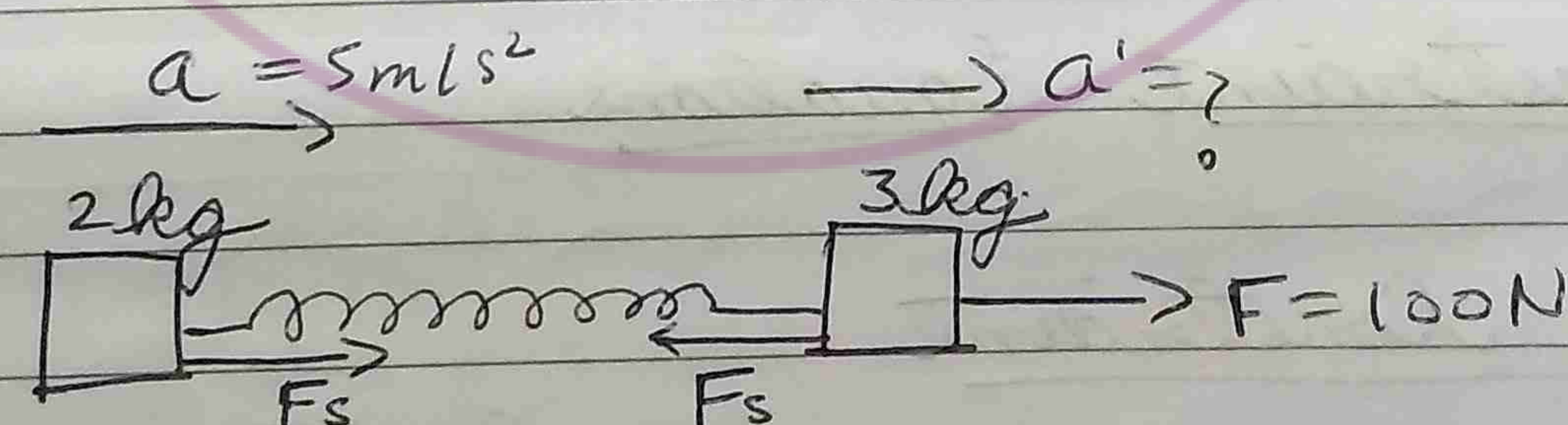
$\underbrace{0\,t + \frac{1}{2} g t^2}_{S_1} = \underbrace{2 \times t - \frac{1}{2} g t^2}_{S_2}$ $= g t^2 = 2t$
$\Rightarrow t = \frac{2}{g} = 0.2\,s$

$v = 2\,m/s$ $\frac{v}{g} = T_{up} = 0.2\,s$ (luckily)

Max height from initial position: $\frac{v^2}{2g} = \frac{2^2}{2 \times 10} = 0.2\,m$

$\frac{1}{2} \times a \times t^2 = \frac{1}{2} \times 2 \times 1^2 = 1\,m$

☆ **Ex**



- At some instant $a_{2kg} = 5\,m/s^2$
$a_{3kg} = ?$

2kg : $F_s = 2 \times 5 = 10\,N$

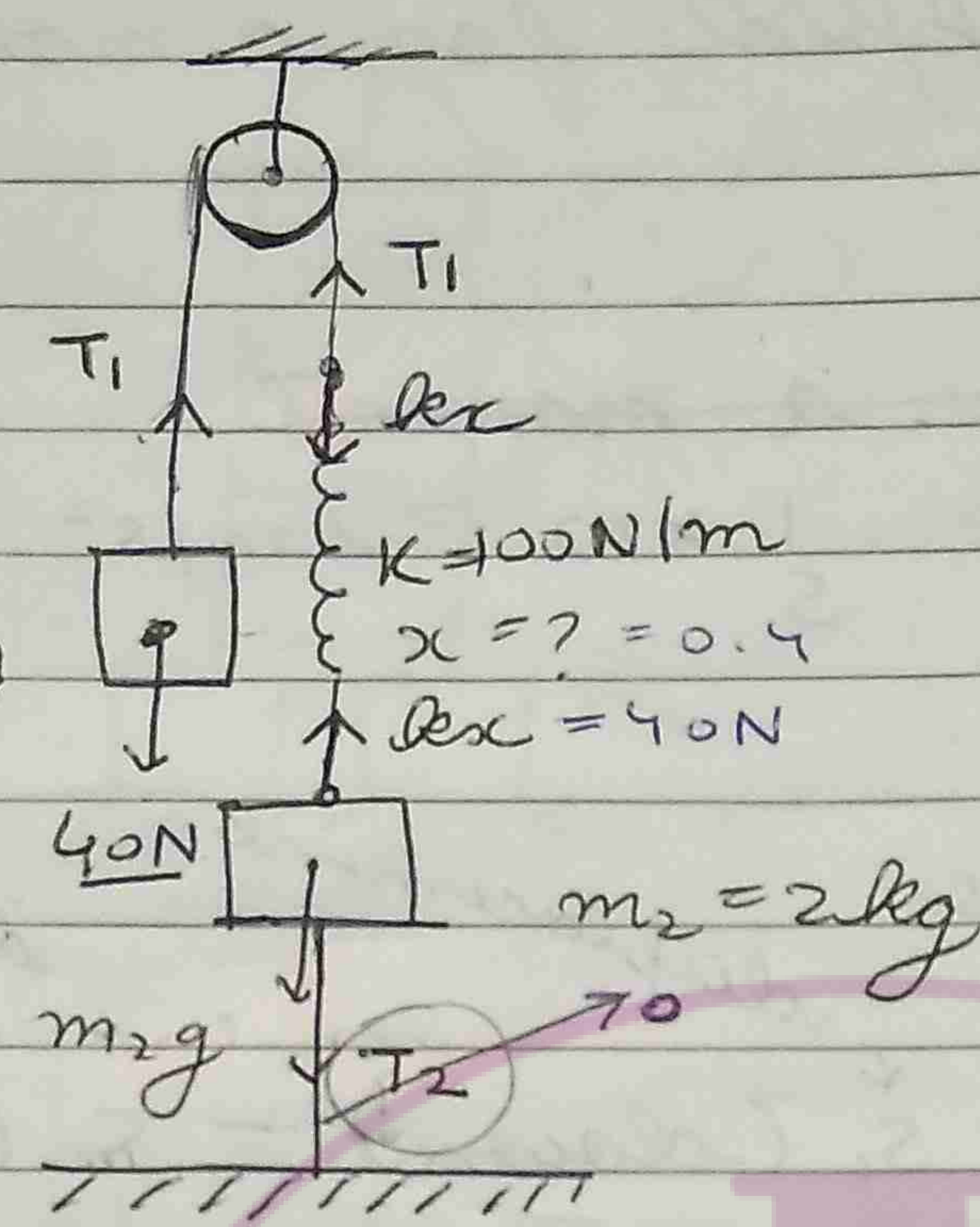
∵ ~~singe~~ spring force is equal.

3kg : $100 - 10 = 3 \times a' \Rightarrow a' = 30\,m/s^2$

★ Hence kx is still acting hence $kx = 40N = T_1$ still.

★ We have to equate the velocities along the string

Ex



$T_1 = 40N$

$Kx = T_1 = 40 \Rightarrow x = 0.4m$

$kx = T_2 + m_2 g$

$40 = 20 + T_2$

$\Rightarrow T_2 = 20N$

lower string breaks suddenly acceleration of each block (just after) at that instant.

$a_1 = 0$ ($\because$ spring remains stretched for a short moment) ★

a_2: $kx - m_2 g = m_2 a \Rightarrow 40 - 20 = 2a_2$

$\Rightarrow a_2 = 10 m/s^2$

• acceleration of each block after a long time.

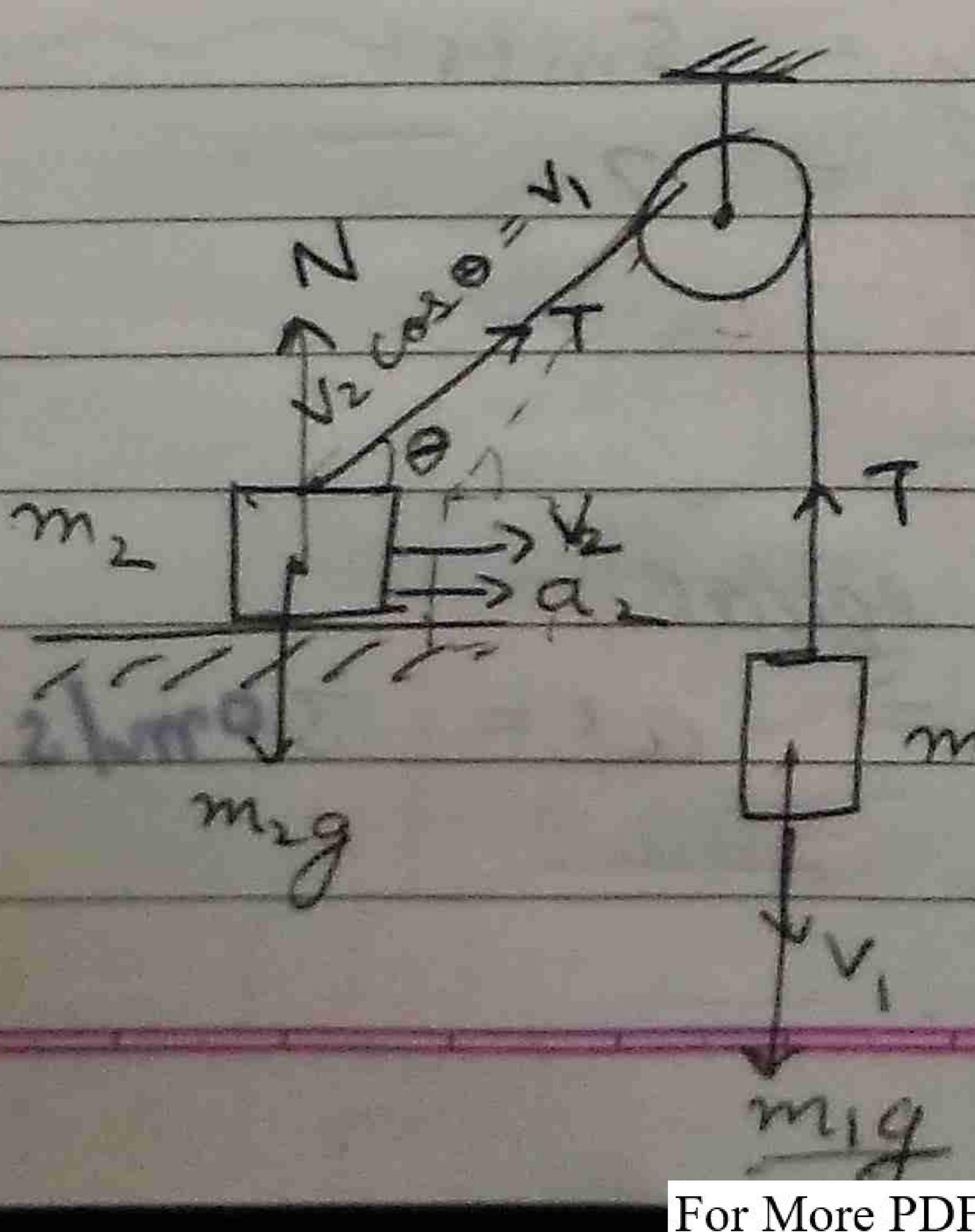
$$a_{final} = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{g}{3}$$

Constraint Equations

(i) String Constraint

Acceleration of blocks?

• Velocity of the ends of string, along string are equal.



$$v_1 = v_2 \cos\theta$$

$$\frac{dv_1}{dt} = \frac{dv_2}{dt} + v_2(-\sin\theta)\frac{d\theta}{dt}$$

$$\Rightarrow a_1 = a_2\cos\theta - \cancel{v_2\sin\theta\frac{d\theta}{dt}}$$ 0 (initially)

$$\boxed{a_1 = a_2\cos\theta}$$

$m_1 g - T = m_1 a_1$ (i)

$T\cos\theta = m_2 a_2$ (ii)

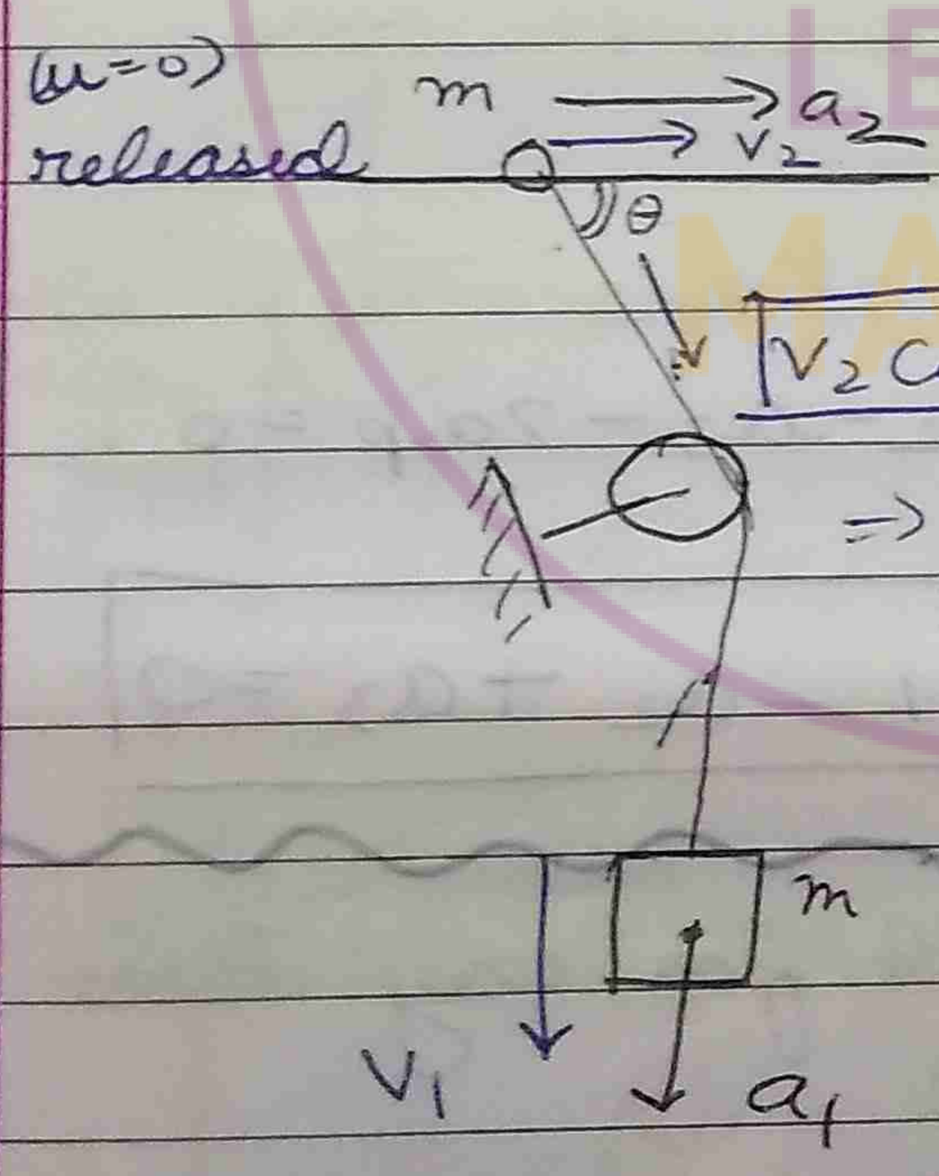
$= m_2 \cdot \frac{a_1}{\cos\theta}$

[$T\sin\theta$ is not considered because there is no motion along vertical direction, motion is only along horizontal direction]

$$\Rightarrow m_1 g = \left(m_1 + \frac{m_2}{\cos^2\theta}\right)a_1$$

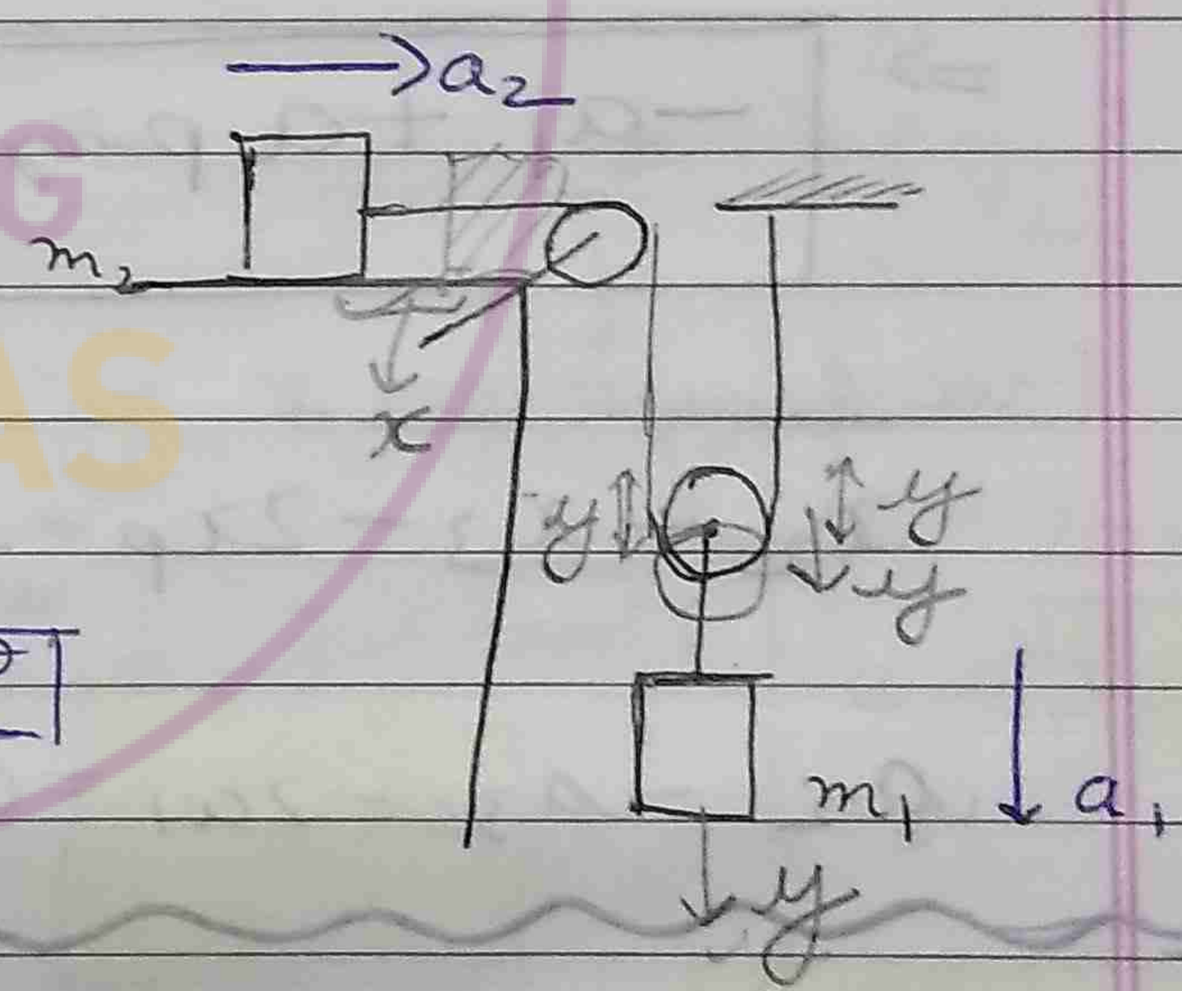
$\Rightarrow a_1 = \ldots$

Ex



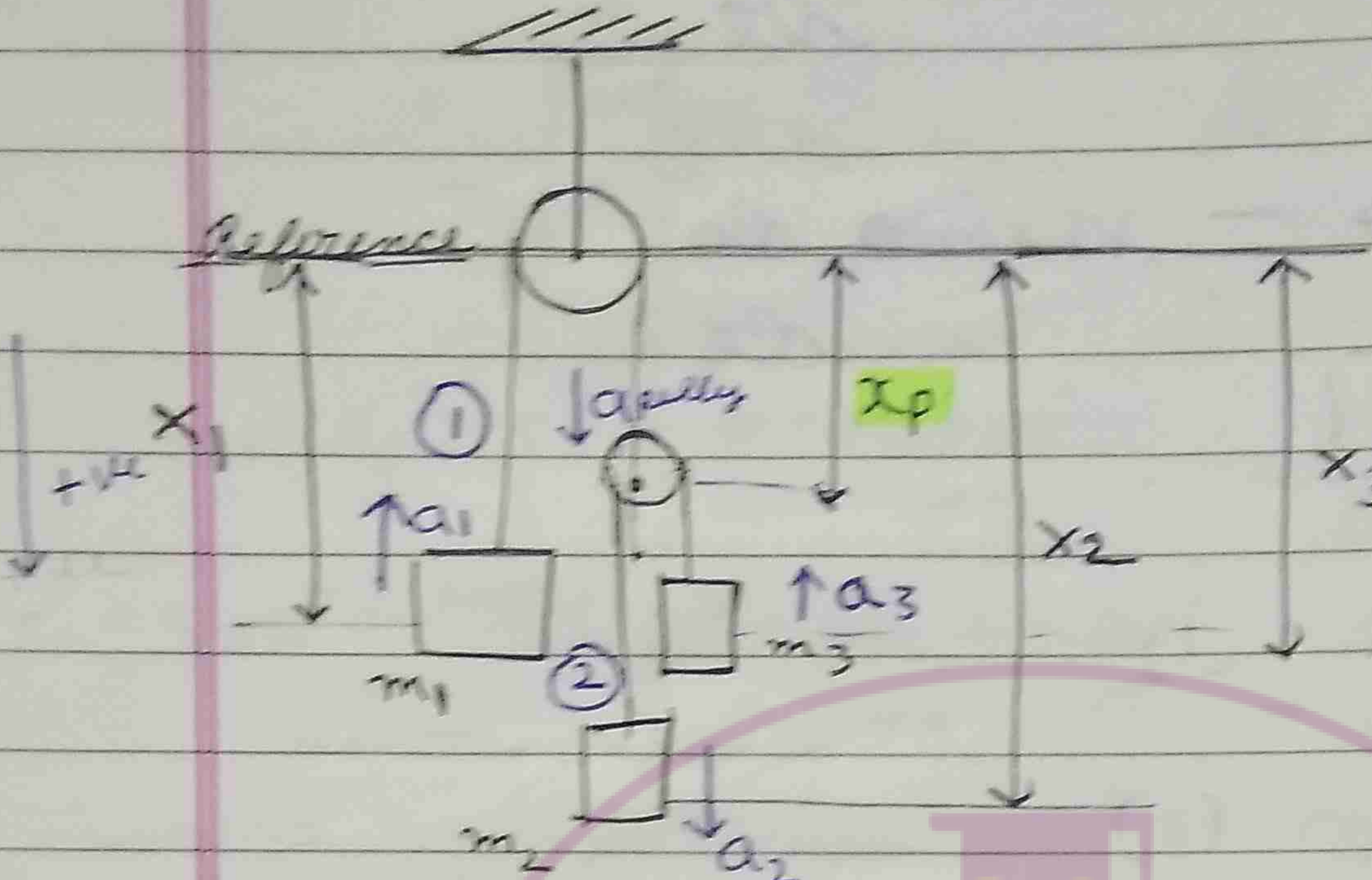
$\boxed{V_2\cos\theta = V_1}$

$\Rightarrow \boxed{a_1 = a_2\cos\theta}$



$x = 2y \Rightarrow \boxed{a_2 = 2a_1}$

acceleration at initial moment.



(we have to assume direction of acceleration)

- string ① $x_1 + x_p = l_1 = \text{constant}$ (i)

string ② $(x_2 - x_p) + (x_3 - x_p) = l_2 = \text{constant}$ (ii)

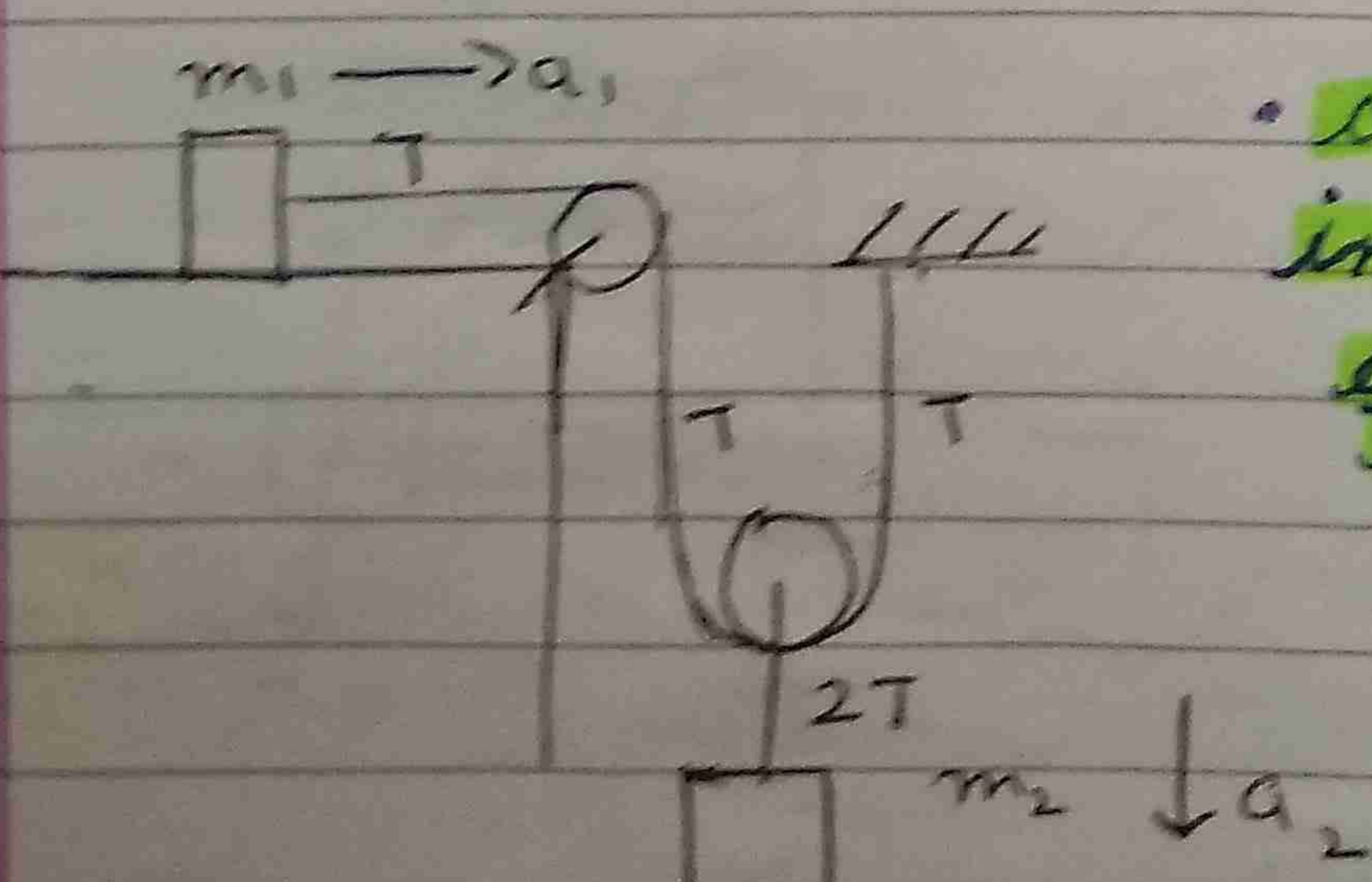
$\Rightarrow$ $\boxed{-a_1 + a_{pulley} = 0}$ ← double differentiating equation(i)

(-ve because w.r.t reference point it is moving in opposite direction)

$x_2 + x_3 - 2x_p = \text{const} \Rightarrow a_2 - a_3 - 2a_p = 0$

$a_2 - a_3 - 2a_1 = 0$ or $\boxed{2a_1 - a_2 + a_3 = 0}$

- Tension Method : (applicable for only two blocks)



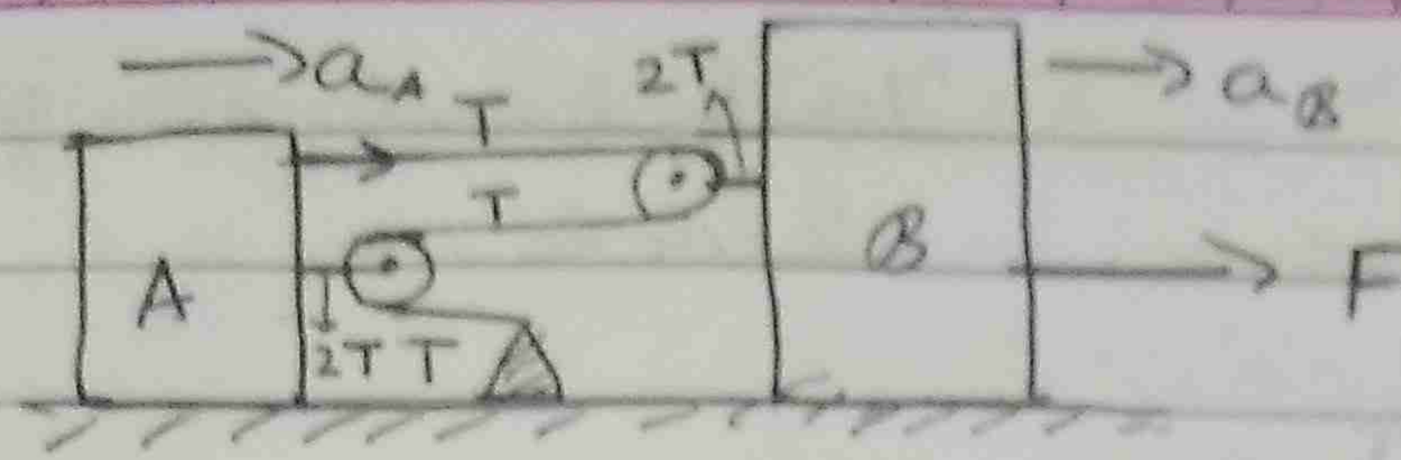
- acceleration are in inverse ratio of tensions

$\frac{a_1}{a_2} = \frac{T_2}{T_1}$

$\frac{a_1}{a_2} = \frac{2T}{T} \Rightarrow \boxed{a_1 = 2a_2}$

• Higher the tension lesser the acceleration and vice versa

Ex

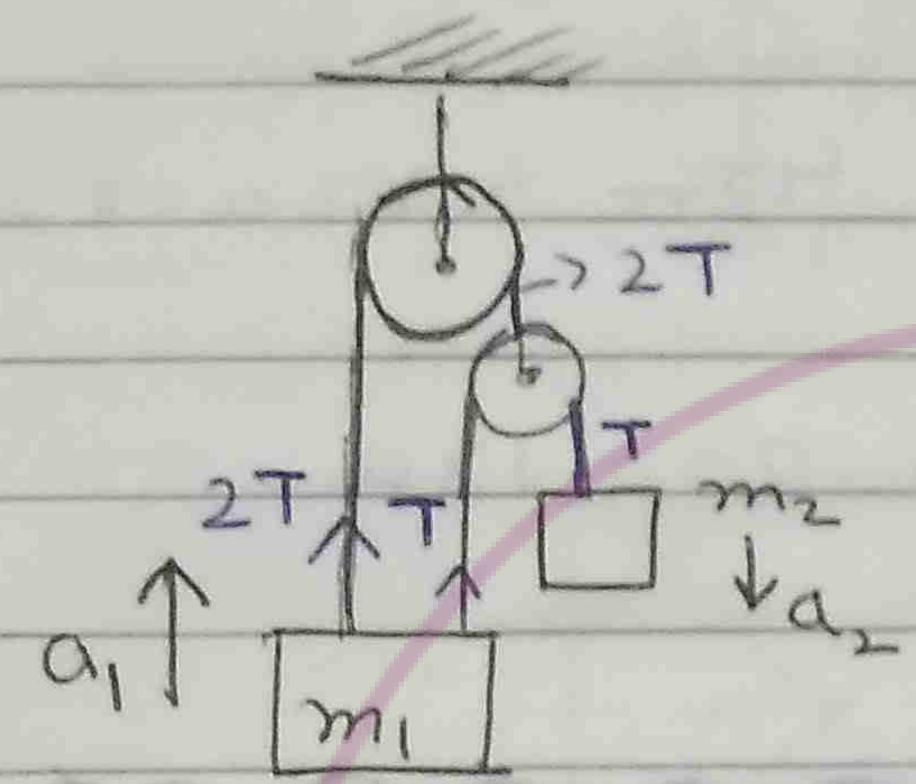


$T_B = 2T \quad T_A = 3T$

$\frac{a_B}{a_A} = \frac{T_A}{T_B} = \frac{3T}{2T} = \frac{3}{2}$

$\Rightarrow \boxed{2a_B = 3a_A}$

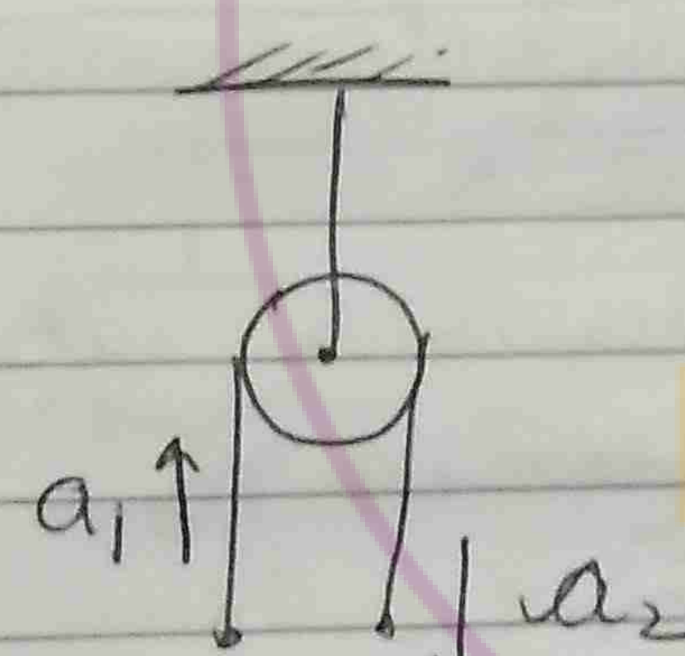
Ex



$T_1 = 3T \quad T_2 = T$

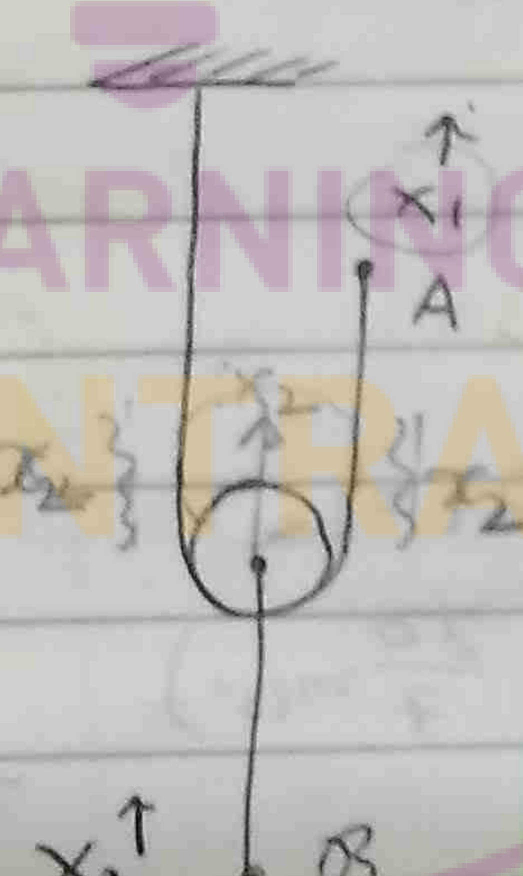
$\Rightarrow \frac{a_1}{a_2} = \frac{T}{3T} = \frac{1}{3} \Rightarrow \boxed{a_2 = 3a_1}$

• Case 1



$\boxed{a_1 = a_2}$

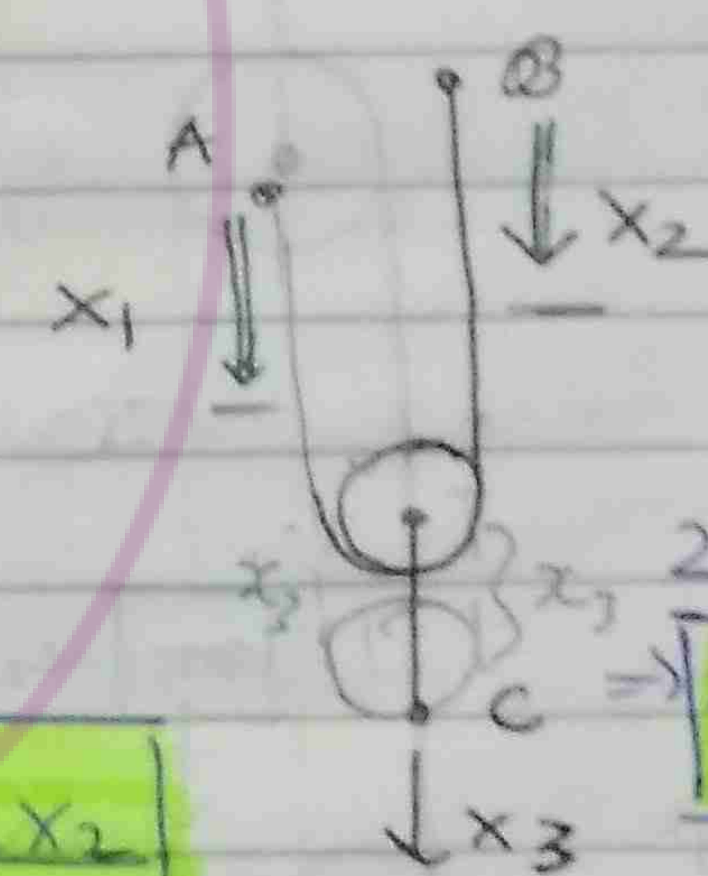
Case 2



$\boxed{x_1 = 2x_2}$

$\boxed{a_1 = 2a_2}$

Case 3

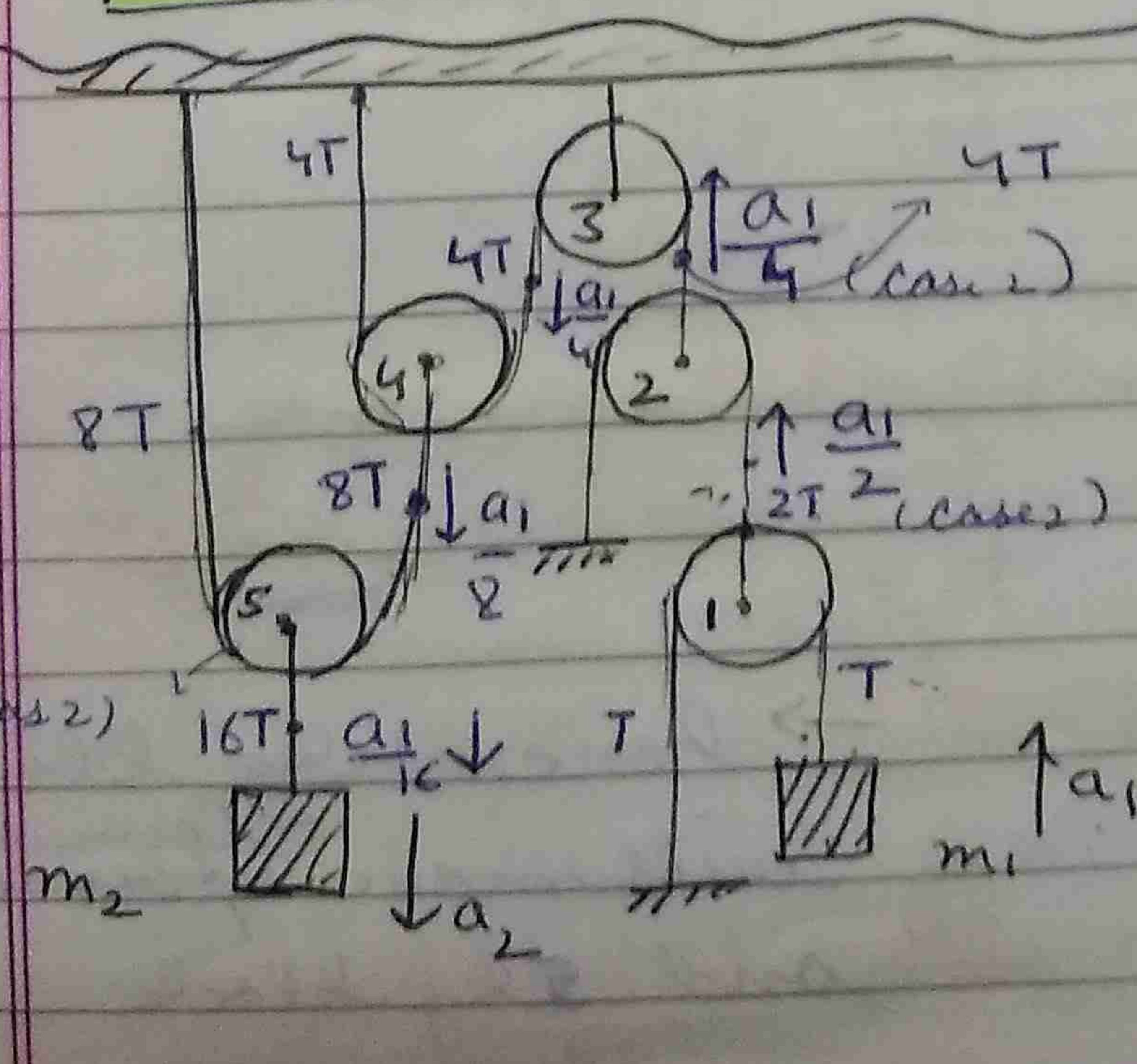


$2x_3 = x_1 + x_2$

$\Rightarrow \boxed{x_3 = \frac{x_1 + x_2}{2}}$

$\boxed{a_3 = \frac{a_1 + a_2}{2}}$

Q



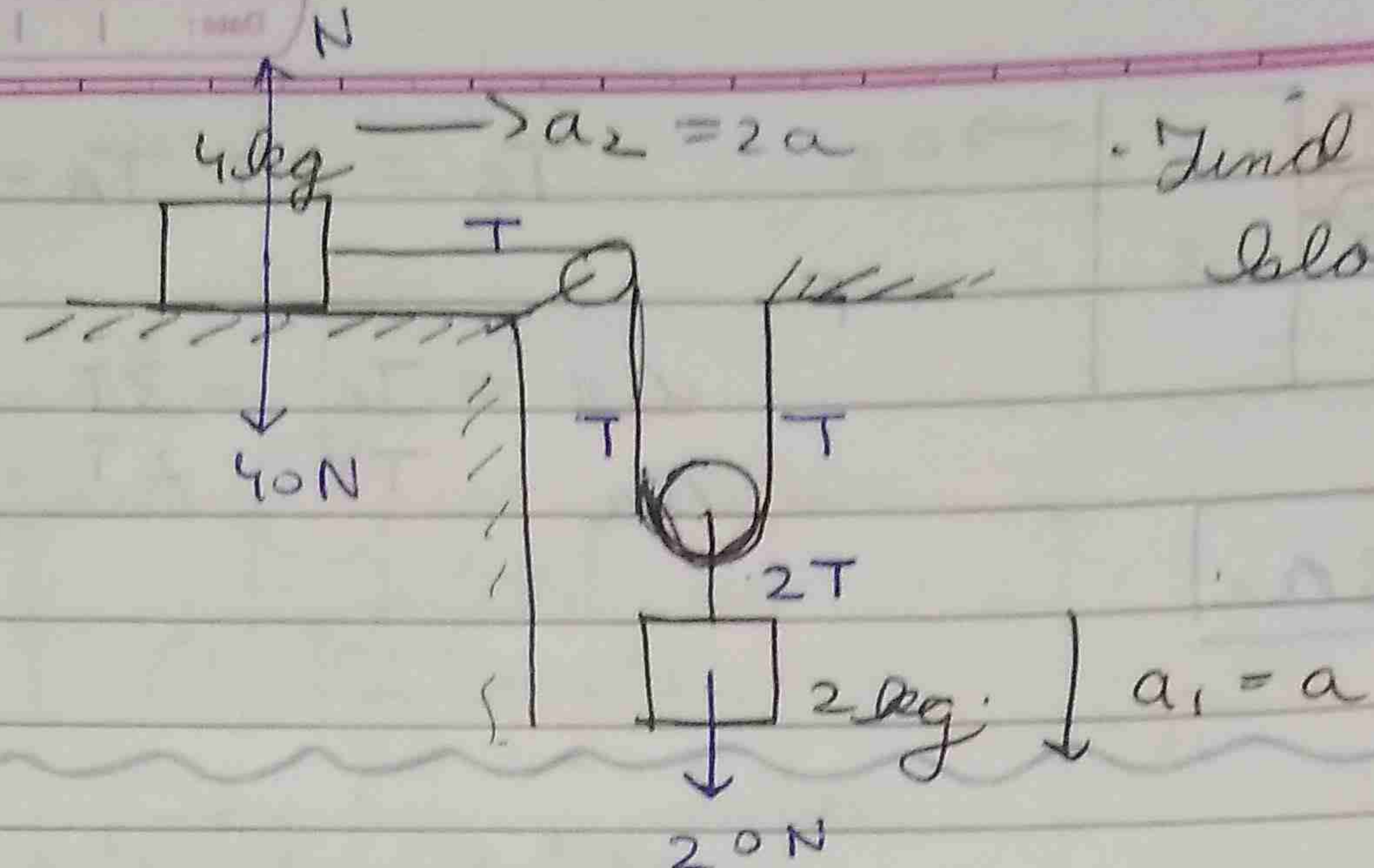
$\frac{a_1}{4}$ (case 2) $\Rightarrow a_2 = \frac{a_1}{16}$

$\frac{a_1}{2}$ (case 2)

• Tension method can be applied ∵ it is 2 block system

$\frac{T_1}{T_2} = \frac{a_2}{a_1} = \frac{1}{16} \Rightarrow \boxed{a_1 = 16a_2}$

Ex



· Find acceleration of blocks

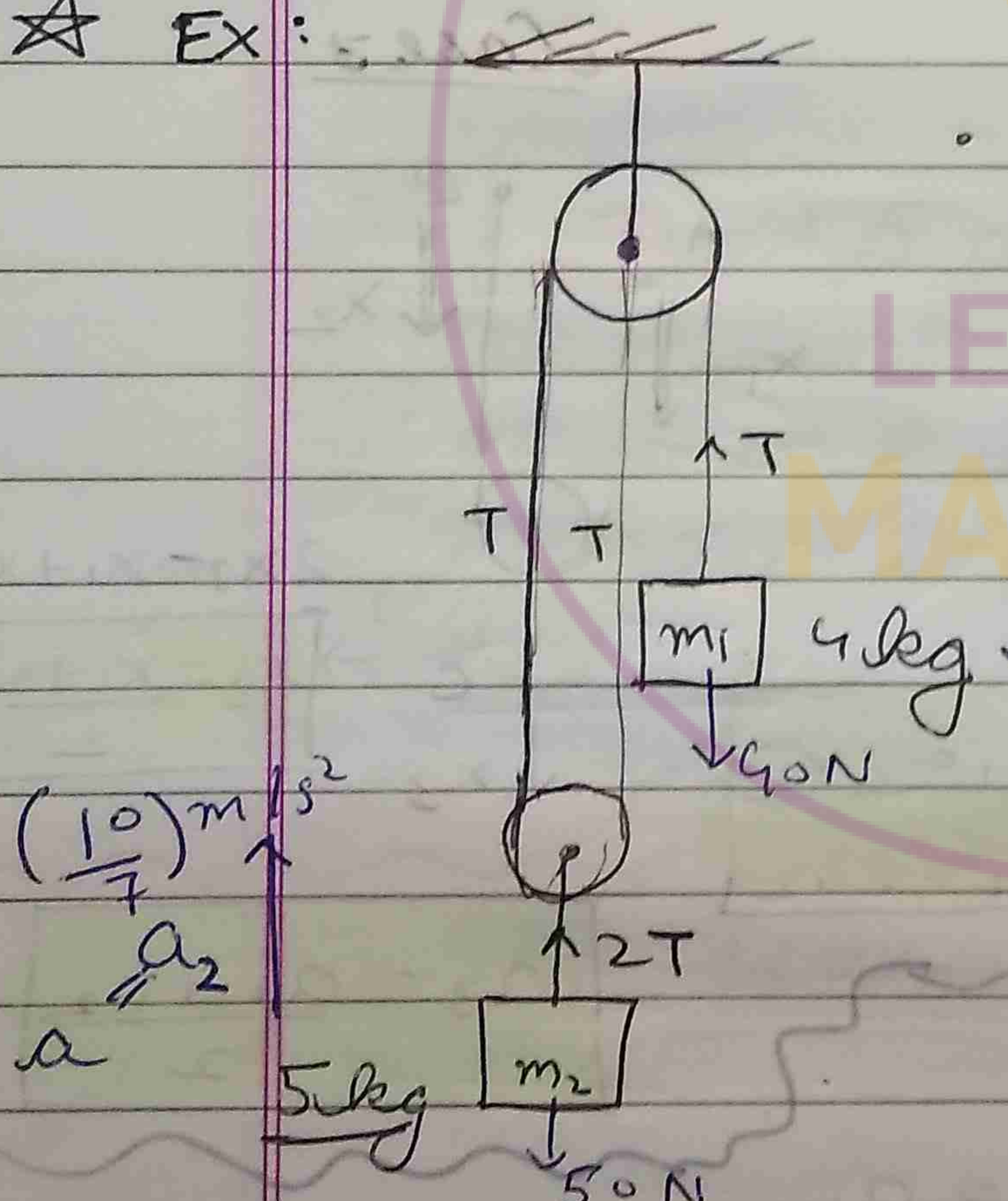
$20 - 2T = 2a$ (i) $\qquad T = 4 \times 2a$ (ii)

from equation i and (ii) $20 = 18a$

$\Rightarrow a_1 = \frac{10}{9} \quad a_2 = \frac{20}{9}$

☆ Ex :

· Find acceleration of blocks

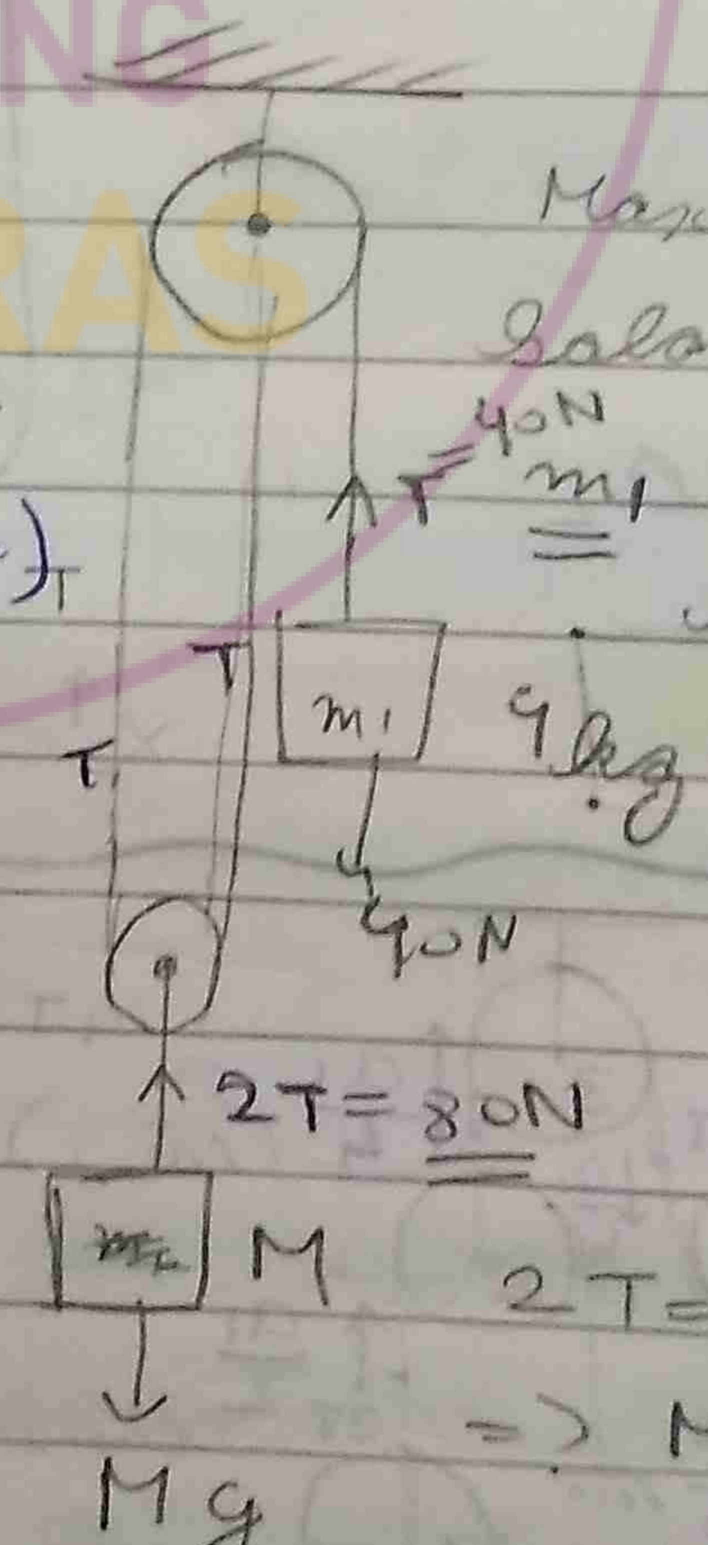


$40 - T = 4 \times 2a \; (\times 2)$

$2T - 50 = 5a$

$30 = 21a$

$\Rightarrow a = \frac{30}{21} = \frac{10}{7}$



Maximum net balanced by m_1 (equilibrium condition)

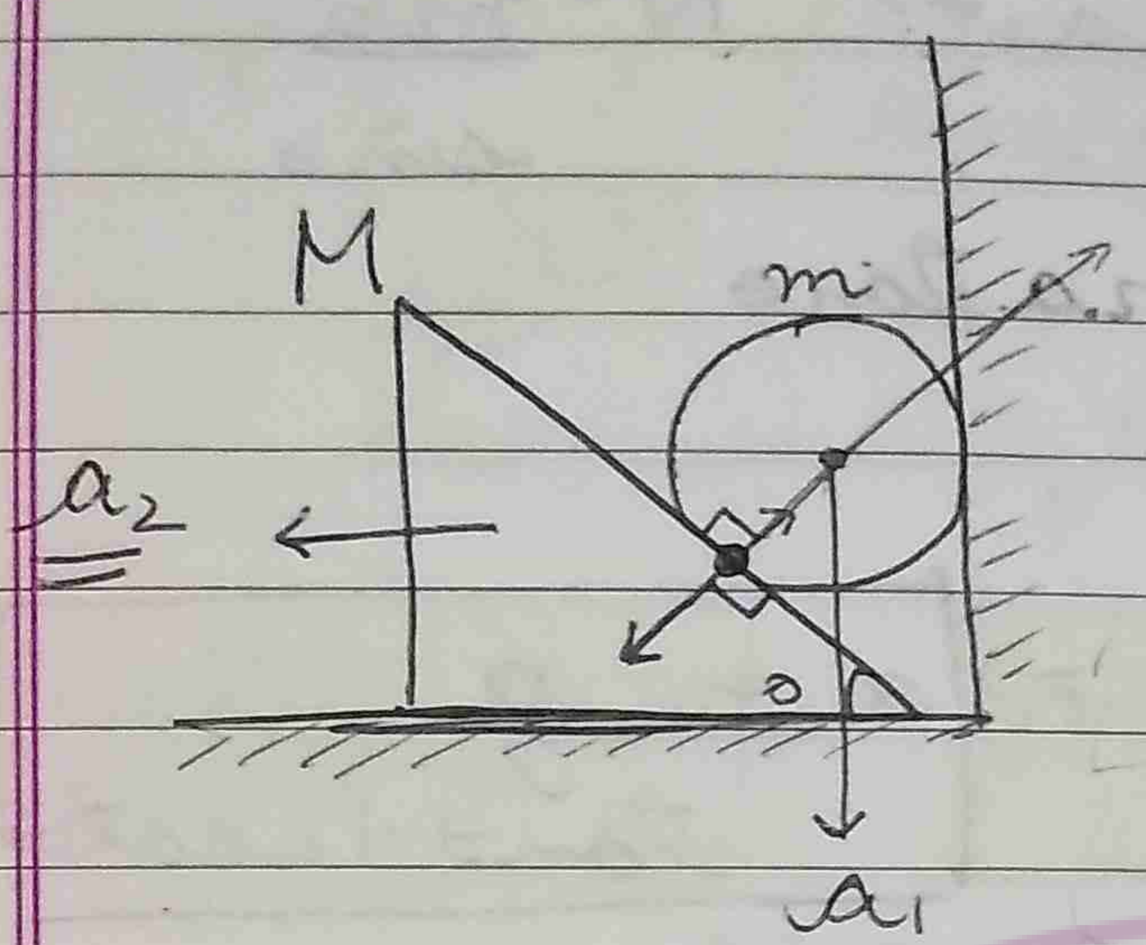
$2T = Mg$

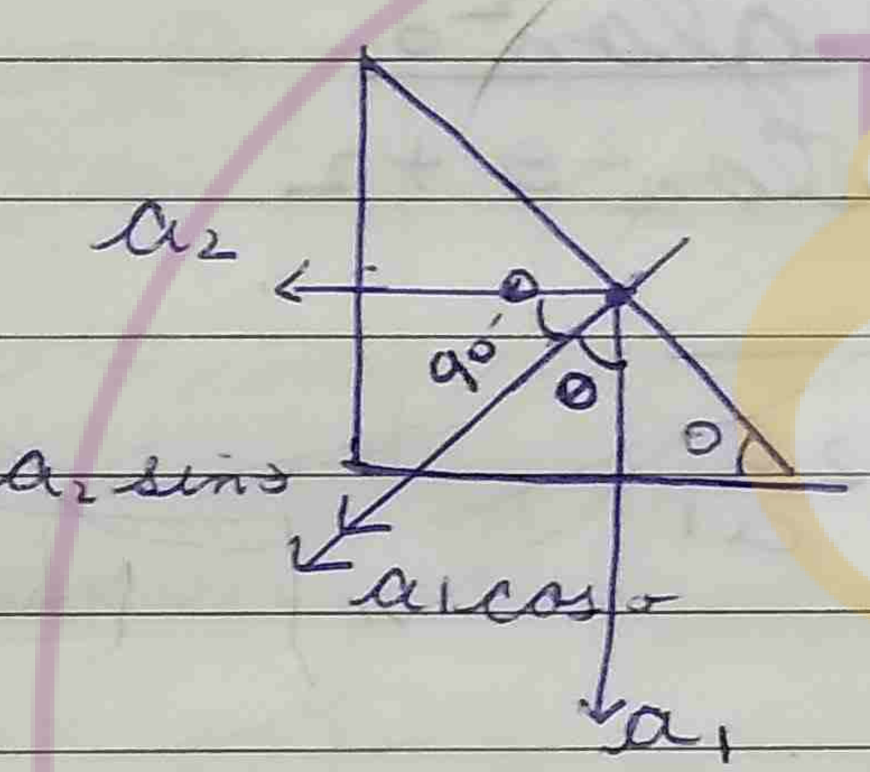
$\Rightarrow M = 8\,kg$

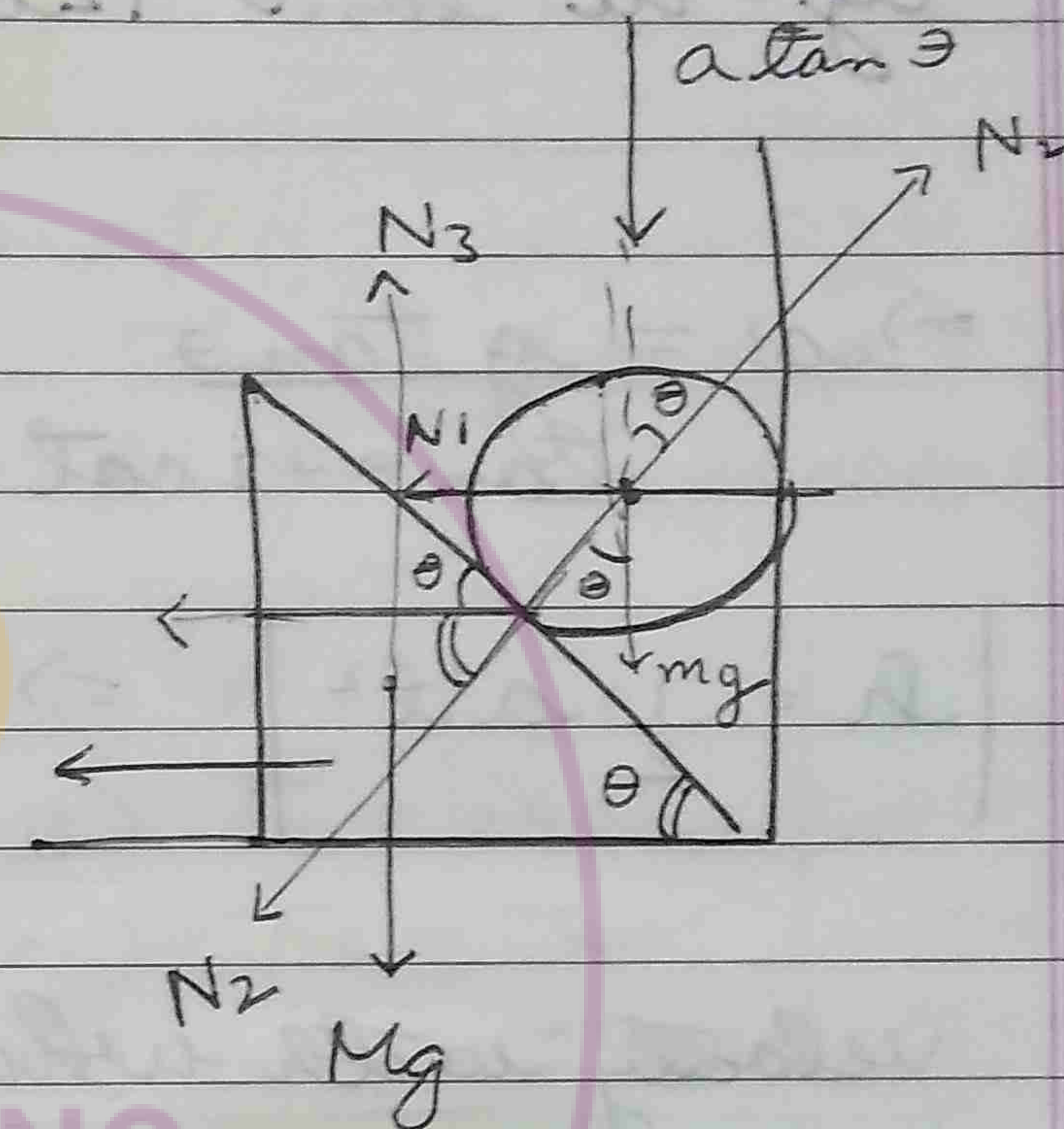
→ hence 4kg block will move ~~upwards~~ downwards and 5kg block will move upwards.

Wedge constraint :

- velocity (motion / displacement) $\perp^r$ to the plane must be the same for both bodies because they remain in contact.







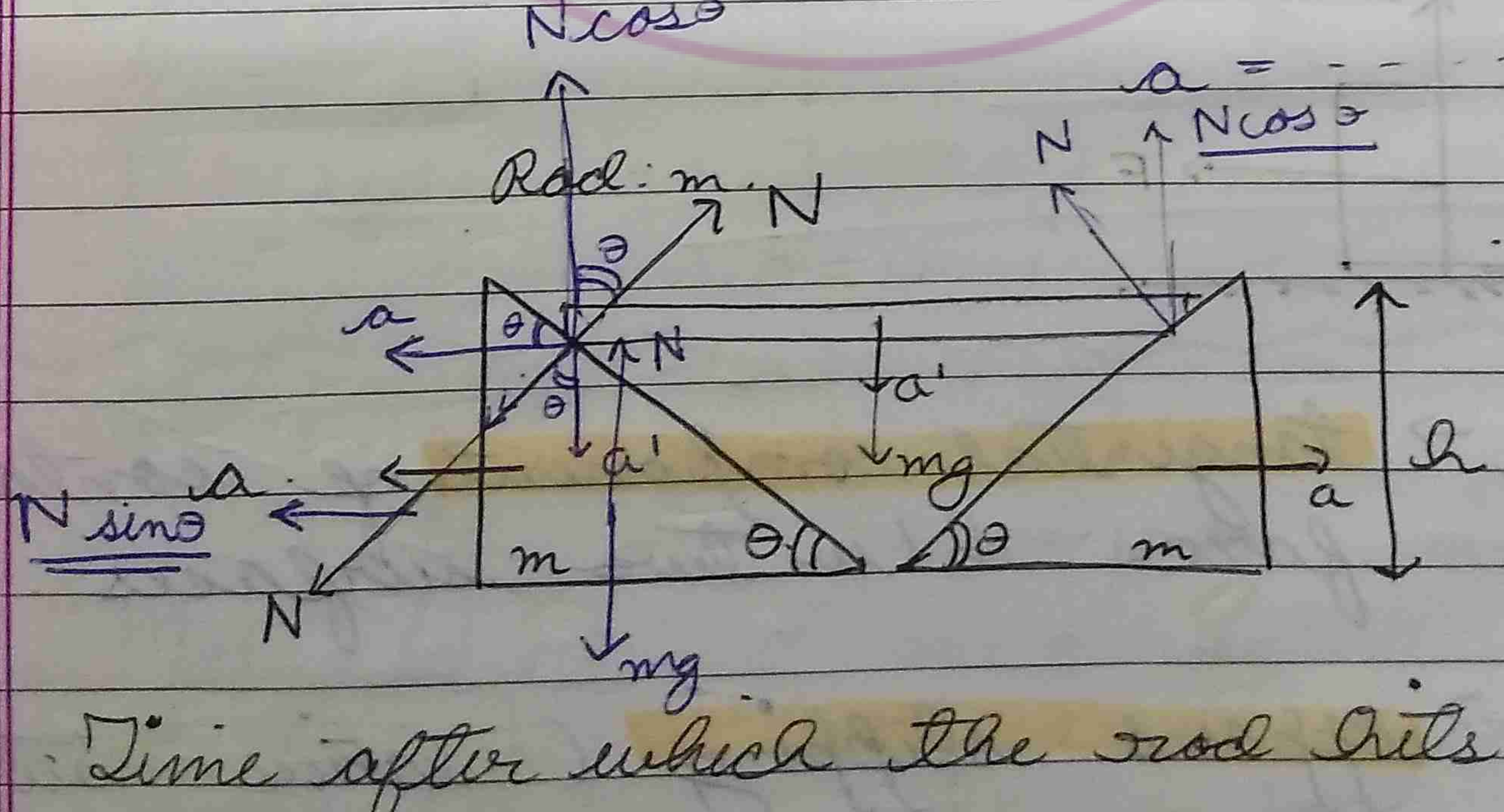
$a_2 \sin\theta = a_1 \cos\theta$

$\Rightarrow \boxed{a_1 = a_2 \tan\theta}$

Ball = $mg - N_2 \cos\theta = ma\tan\theta$ (i)

- If $a_2 = a$
  $\Rightarrow a_1 = a\tan\theta$

Wedge = $N_1 \sin\theta = ma$ (ii)



- Time after which the rod hits the ground,

$a' \cos\theta = a \sin\theta \Rightarrow \boxed{a' = a\tan\theta}$

Rod: $mg - 2N\cos\theta = ma\tan\theta$

Wedge: $N\sin\theta = ma \Rightarrow N = \frac{ma}{\sin\theta}$

$mg - 2\frac{ma}{\sin\theta}\cdot\cos\theta = ma\tan\theta$

$g = a(\tan\theta + 2\cot\theta) \Rightarrow \boxed{a = \frac{g}{\tan\theta + 2\cot\theta}}$

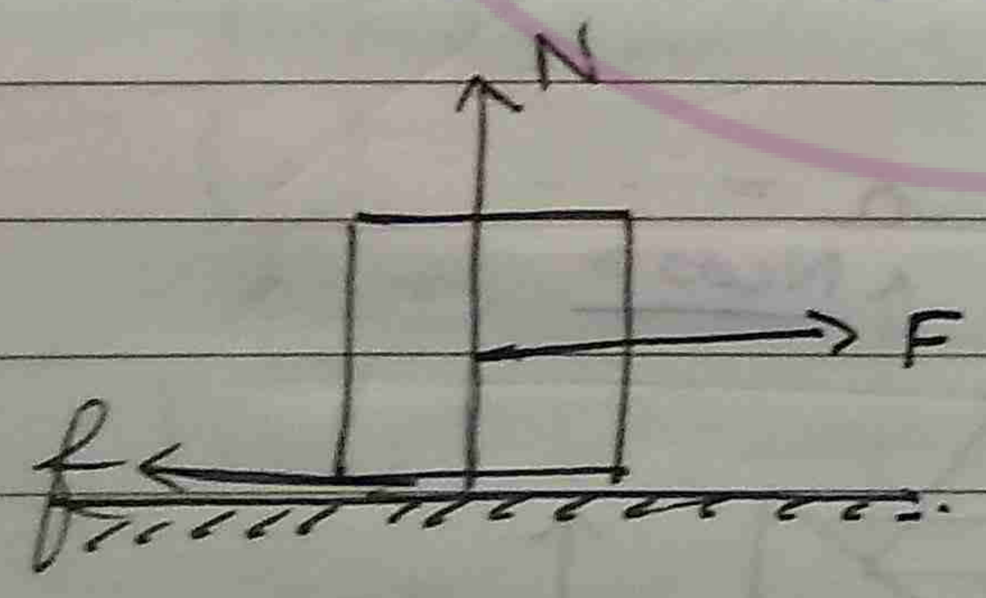
$\Rightarrow a' = \frac{g\tan\theta}{\tan\theta + 2\cot\theta} = \frac{g\tan^2\theta}{\tan^2\theta + 2}$

$\left| h = \frac{1}{2}a't^2 \right| \Rightarrow t = \sqrt{\frac{2h}{a'}}$

velocity with which rod hits the ground

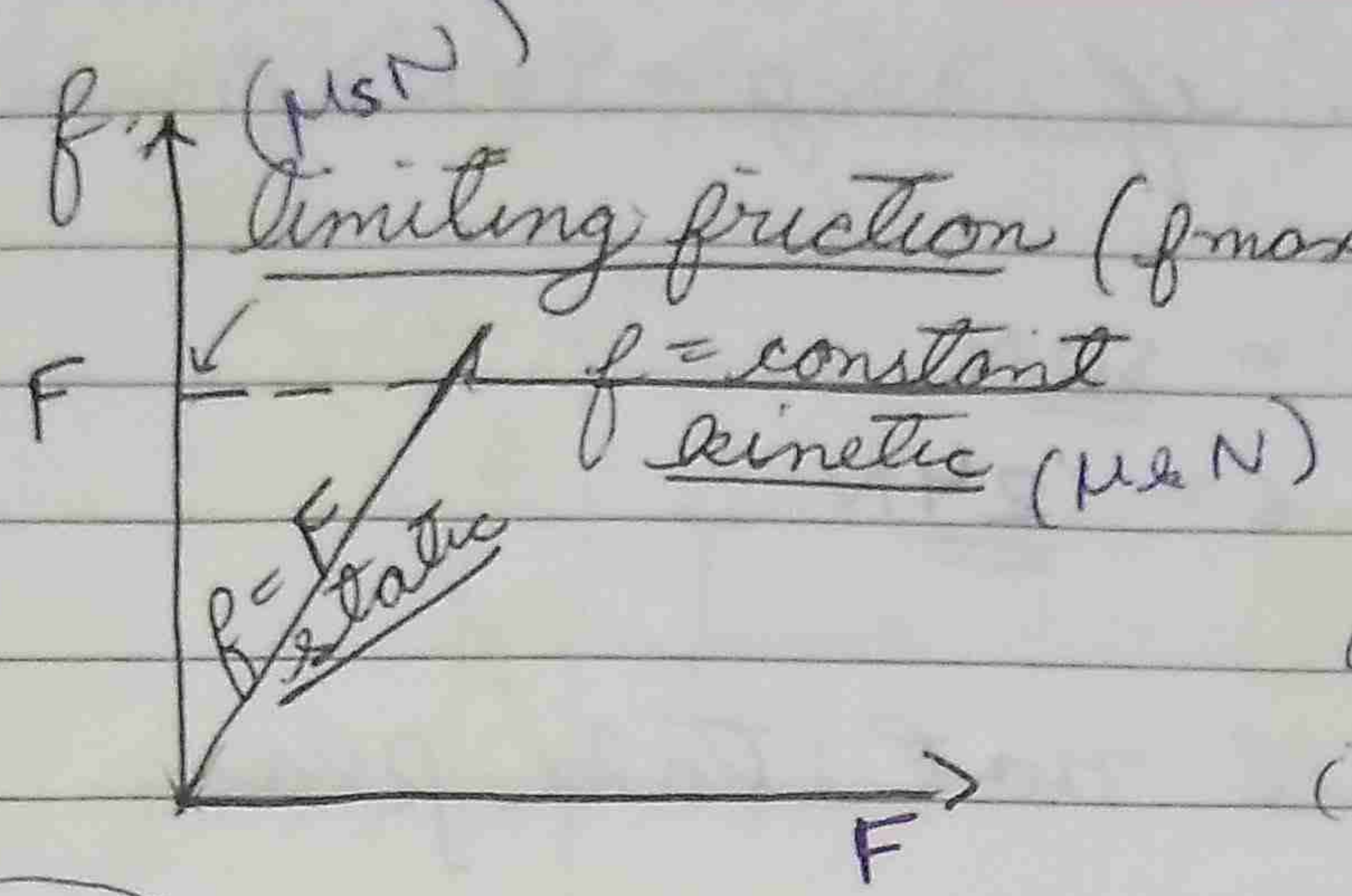
$\Rightarrow v = \sqrt{2a'h}$

- Friction



- friction: → tangential component of contact force b/w two surfaces.
  - → opposes slipping (relative motion b/w surfaces)
  - → self adjusting force.

- static friction is self adjusting
- kinetic friction is constant.
- Static friction can have value from 0 to max.



laws of friction (experimental)

(i) $f \propto N$

★ (ii) Independent of area of contact

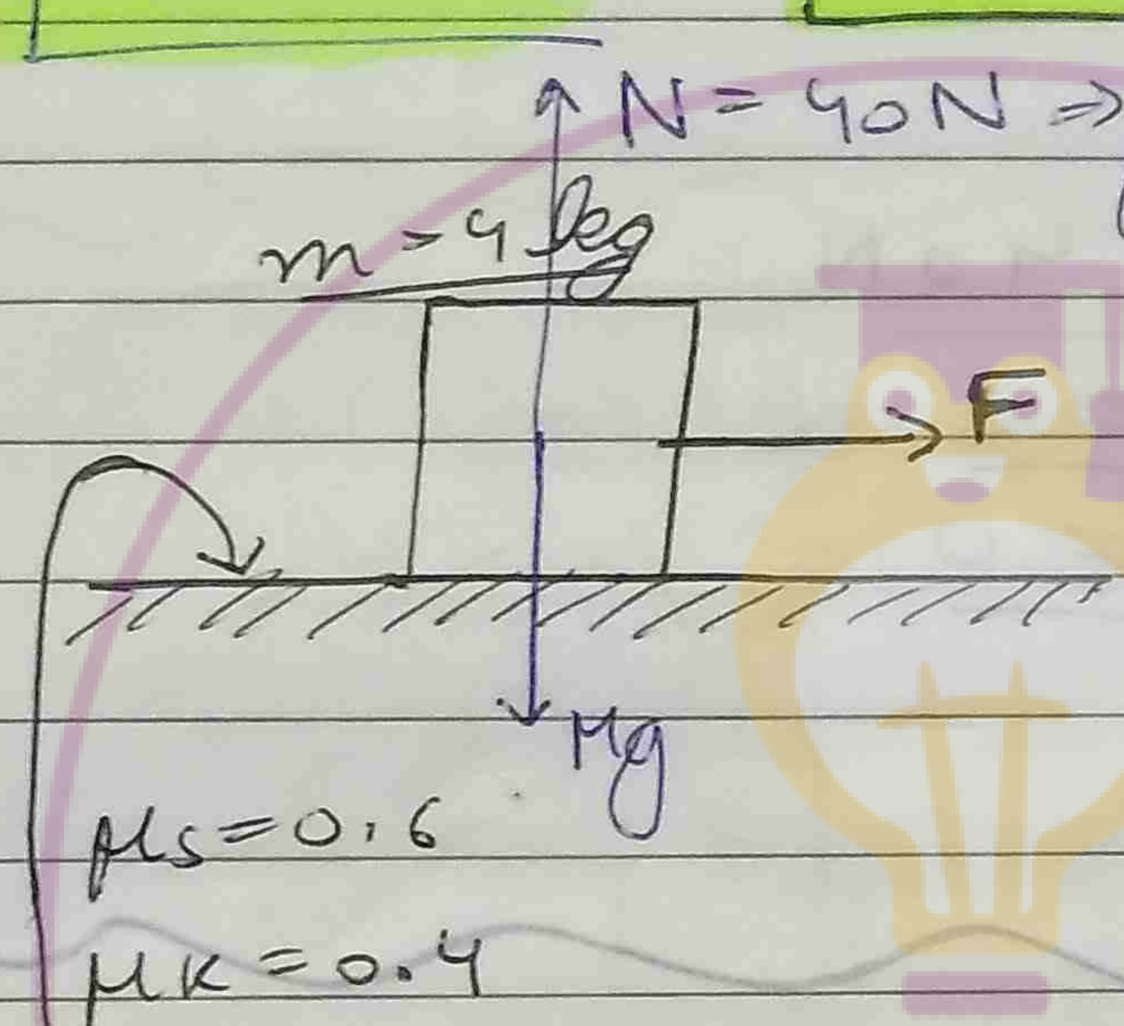
coefficient of friction

$$f = \mu N$$

$$\mu_s > \mu_k$$

$N = 40N \Rightarrow f_{s\,max} = \mu N = 0.6 \times 40 = 24N$

Ex



- Find f and a.
If force is increased sequentially

a) $F_1 = 12N$
b) $F_2 = 24N$
c) $F_3 = 30N$
d) $F_4 = 24N$

- Motion will take place if $F_{app} > f_{s,max}$ where $f_{s\,max} = \mu_s N$

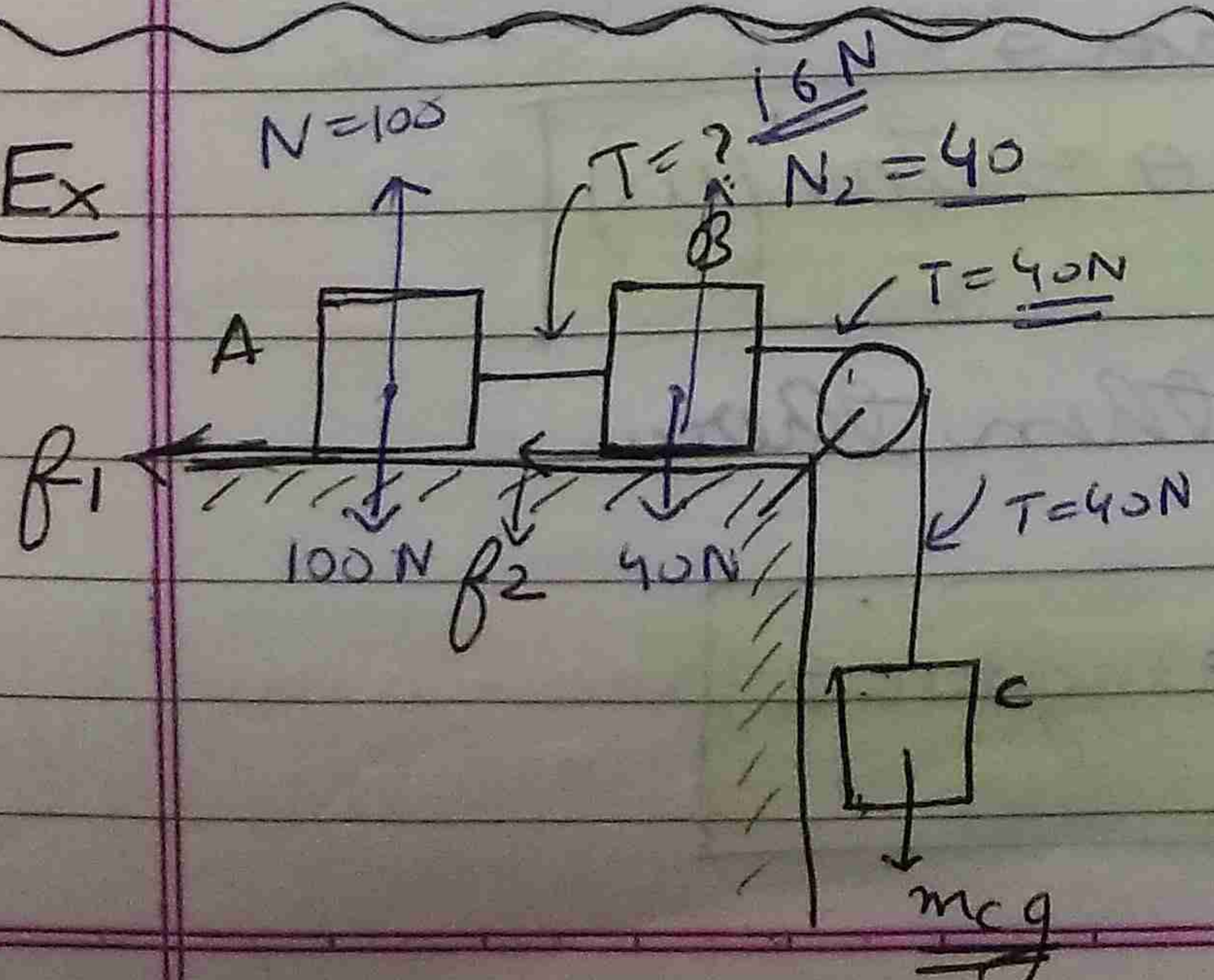
a) $F_1 = 12$
$a = 0$
$f = 12N$

b) $F_2 = 24N$
$f = 24N$
$a = 0$

c) $F_3 = 30N$
Motion starts ✓
$f_k = \mu_k N = 0.4 \times 40 = 16$
$\Rightarrow a = \frac{F - f}{m} = \frac{30 - 16}{4} = 3.5$

d) $F_4 = 20N$ $a = \frac{20 - 16}{4} = \frac{4}{4}$
$\Rightarrow a = 1 m/s^2$

Ex



$m_A = 10kg$ $m_B = 4kg$
$m_C = 4kg$
$\mu_A = 0.5$ $\mu_B = 0.6$

- Find T, f_1, and f_2

angle of repose → angle at which slipping just starts.
μ can be less or greater than 1 e.g. ice has μ...

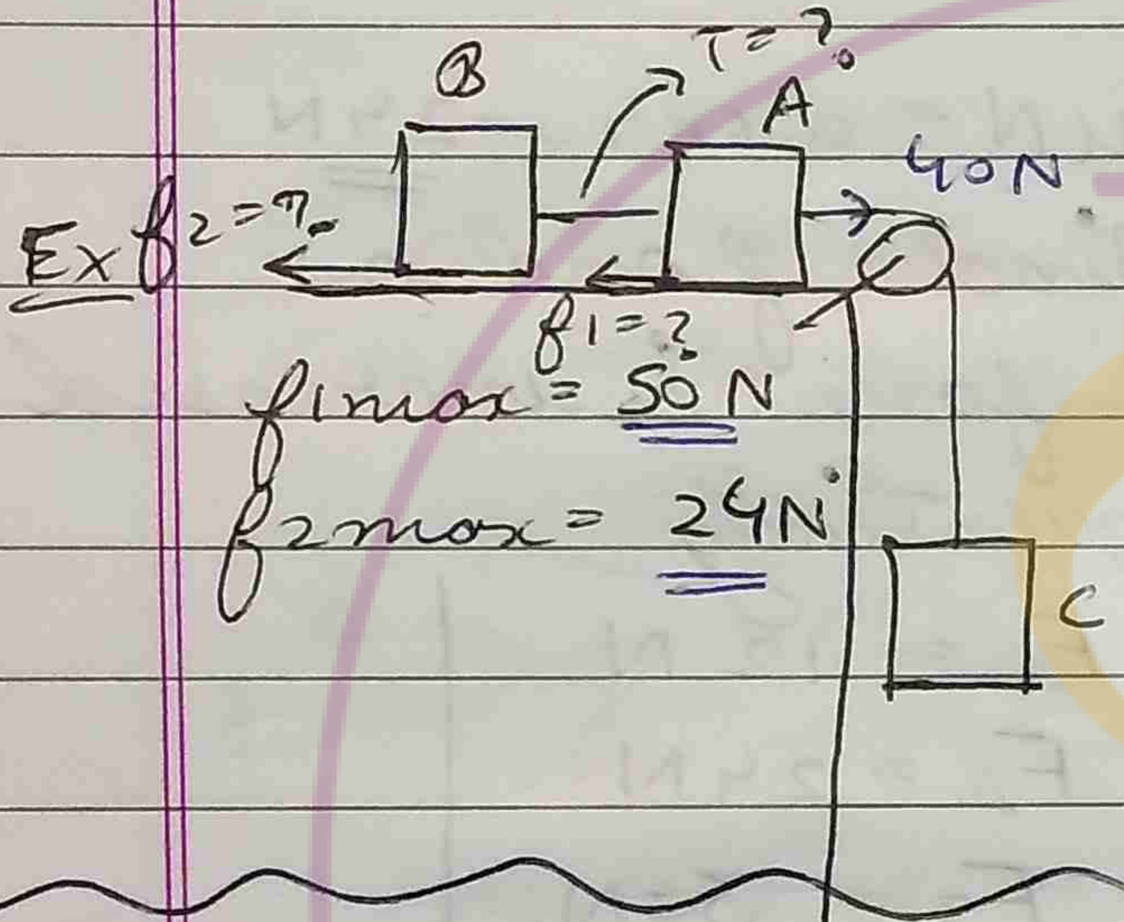
motion will occur if $m_c g = 40N > f_1 max + f_2 max$

$f_1 max = 100 \times 0.5 = \underline{50N}$
$f_2 max = 40 \times 0.6 = \underline{\underline{24N}}$

Hence motion will not take place.

⇒ $f_1 = \underline{16N}$ $f_2 = \underline{24N}$ $T = \underline{16N}$

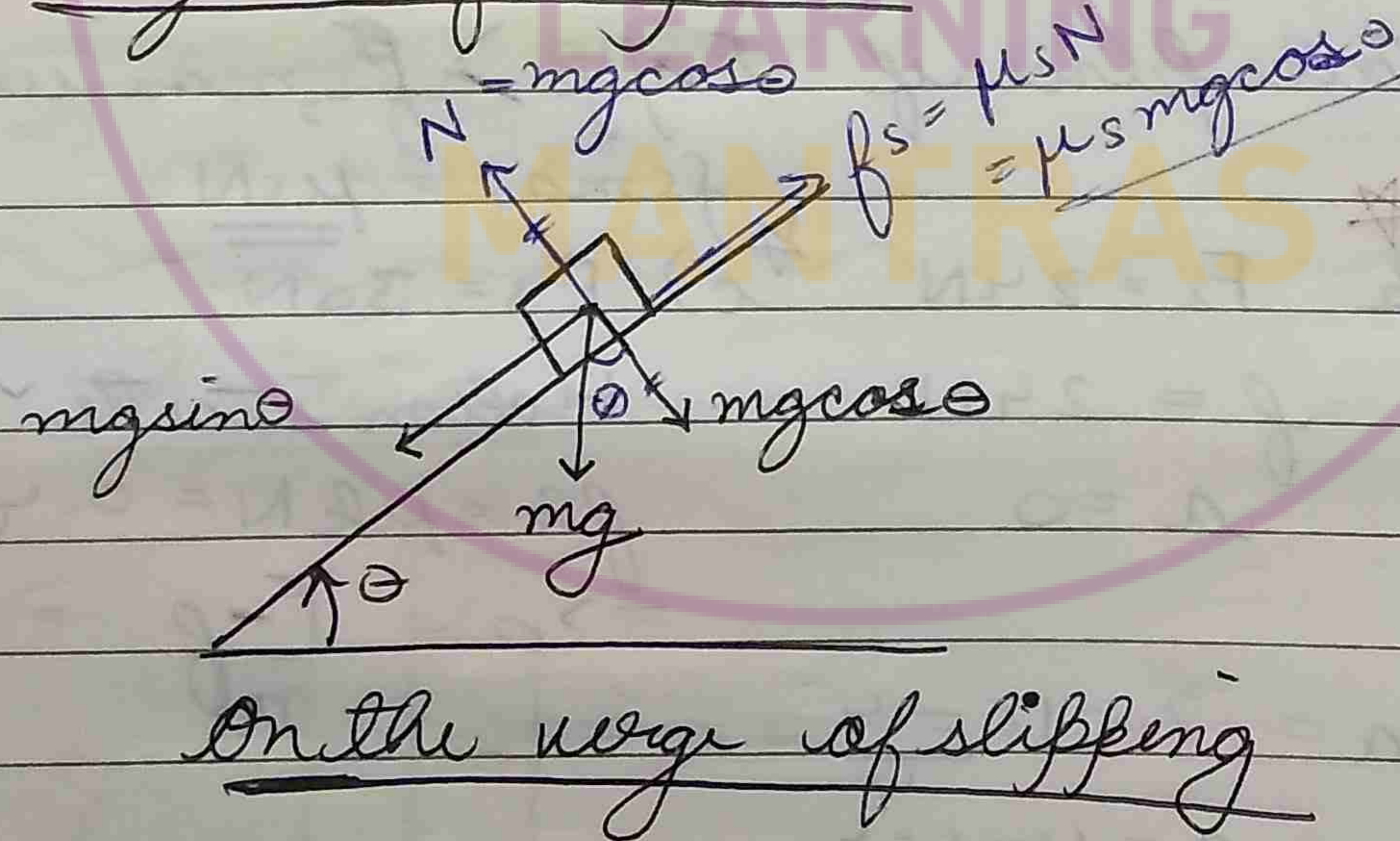
Ex



$f_1 max = \underline{50N}$
$f_2 max = \underline{24N}$

$f_1 = 40N$
$f_2 = 0$
$T = 0$

Angle of Repose



On the verge of slipping

⇒ $mg\sin\theta = \mu_s mg\cos\theta$
⇒ $\tan\theta = \mu_s$ or $\theta = \tan^{-1}(\mu_s)$

If θ < angle of repose then there is
⇒ No motion ⇒ $f = mg\sin\theta$

- Net contact force is the total force applied by the surface on the object.

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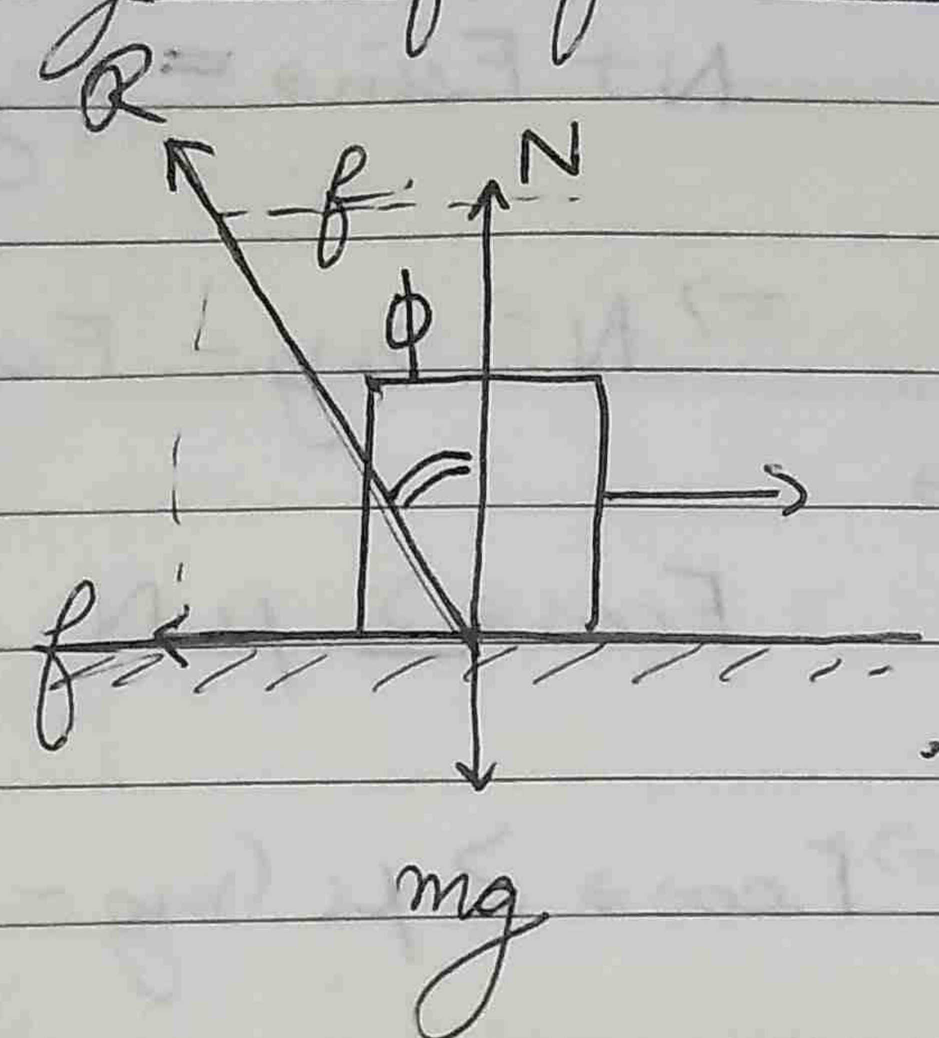
- Angle of friction, (ϕ) : Angle between N and R (Net contact force)

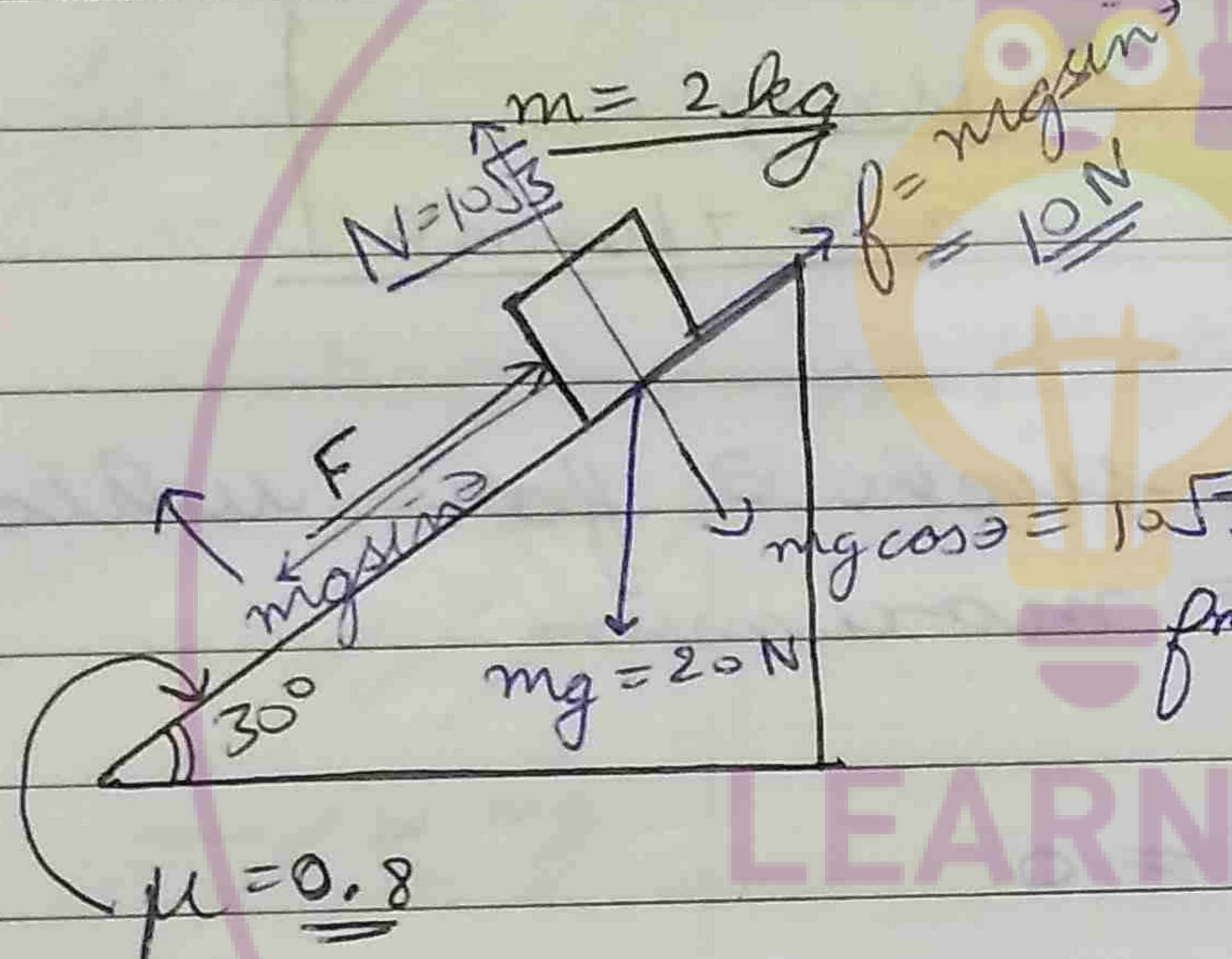


$$\tan\phi = \frac{f}{N}$$

- Angle of limiting friction $= \frac{\mu N}{N}$

$$\tan\phi = \mu$$

Ex



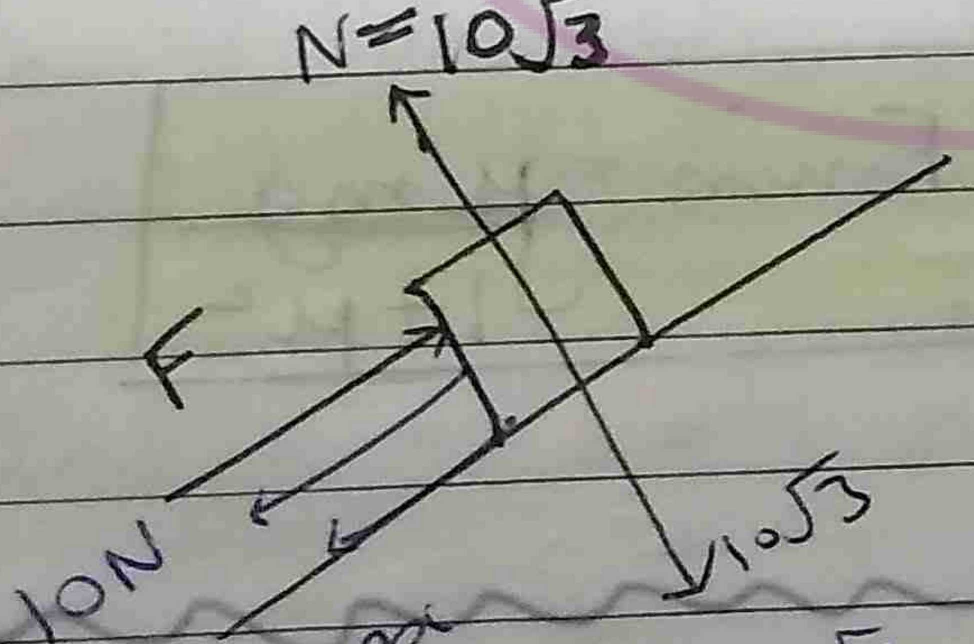
a) friction acting ?
b) minimum F to push the body up the plane?

~~$f_{max} = f = \mu N = 0.8 \times 10\sqrt{3} = 8\sqrt{3} = \underline{14N}$~~

a) = friction acting = 10N i.e. $mg\sin\theta$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \underline{0.57}$

$0.57 < 0.8$



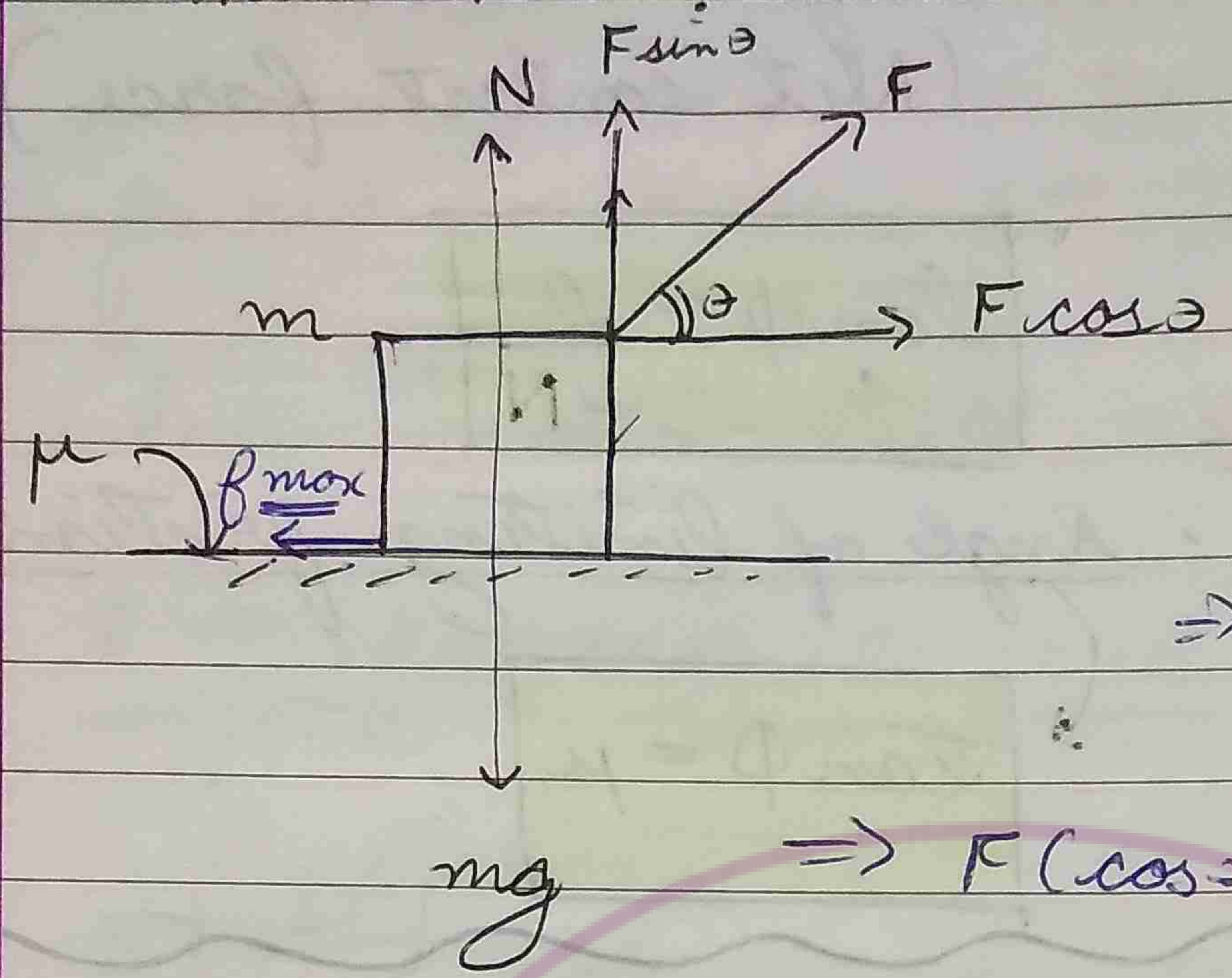
$\Rightarrow \mu N = 0.8 \times 10\sqrt{3} = \underline{14N}$

$\rightarrow F \geq mg\sin\theta + f_{smax}$

or $F \geq \underline{24N}$

Pulling

Minimum F = ?



$N + F\sin\theta = mg$

$\Rightarrow N = mg - F\sin\theta$

$\therefore$

$F\cos\theta \geq \mu N$

$\Rightarrow F\cos\theta \geq \mu(mg - F\sin\theta)$

$\therefore$

$\Rightarrow F(\cos\theta + \mu\sin\theta) \geq \mu mg$

$\Rightarrow \boxed{F \geq \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}}$

F is minimum for such θ for which $\cos\theta + \mu\sin\theta$ = maximum

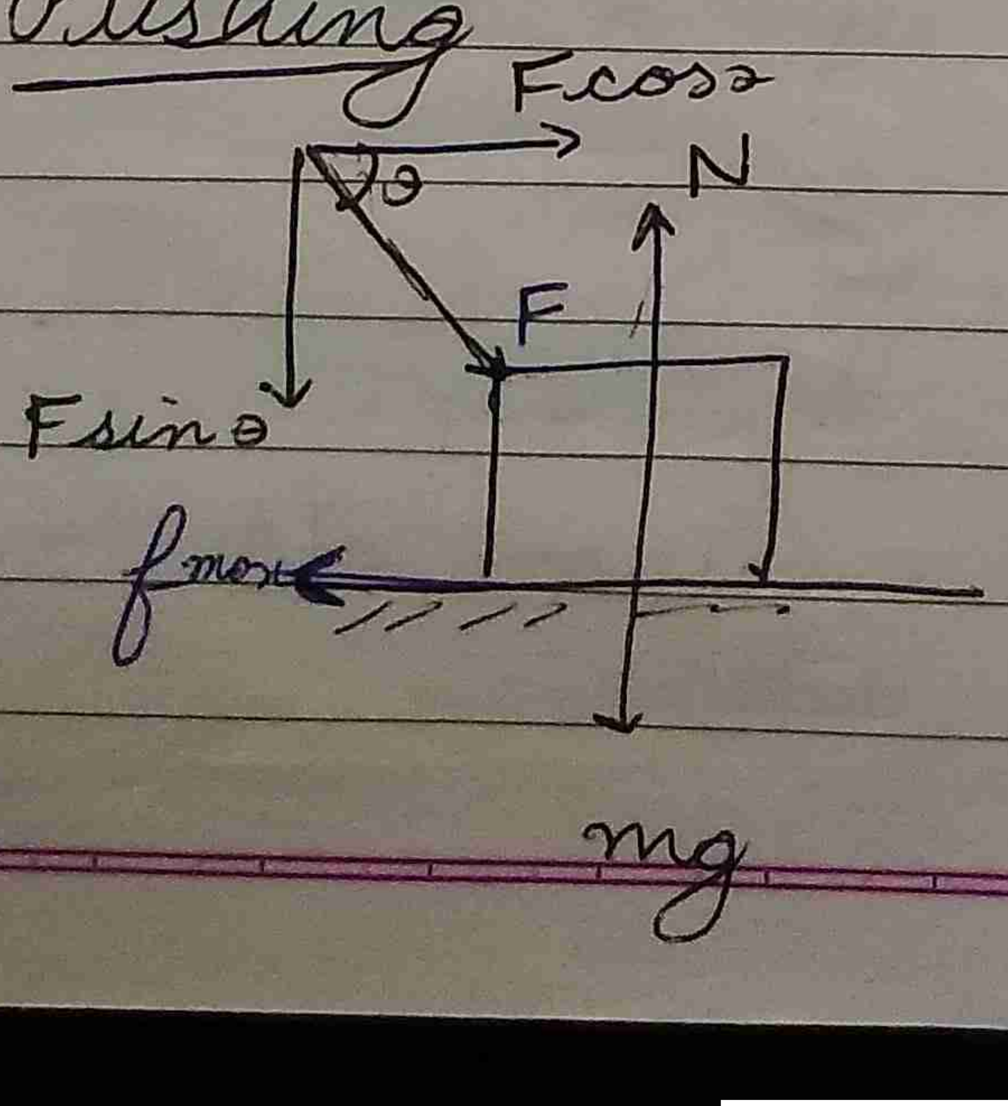
$-\sin\theta + \mu\cos\theta = 0$

$\Rightarrow \boxed{\tan\theta = \mu} \Rightarrow \sin\theta = \dfrac{\mu}{\sqrt{1+\mu^2}} \quad \cos\theta = \dfrac{1}{\sqrt{1+\mu^2}}$

$F_{min} = \dfrac{\mu mg}{\dfrac{1}{\sqrt{1+\mu^2}} + \dfrac{\mu^2}{\sqrt{1+\mu^2}}}$

$\boxed{F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}}$

Pushing



$N = mg + F\sin\theta$

$\Rightarrow F\cos\theta \geq \mu N$

$\Rightarrow F\cos\theta \geq \mu(mg + F\sin\theta)$

• Pushing requires greater force rather than pulling.

$$\Rightarrow F(\cos\theta - \mu\sin\theta) \geq \mu mg$$

$$\Rightarrow \boxed{F \geq \frac{\mu mg}{\cos\theta - \mu\sin\theta}}$$

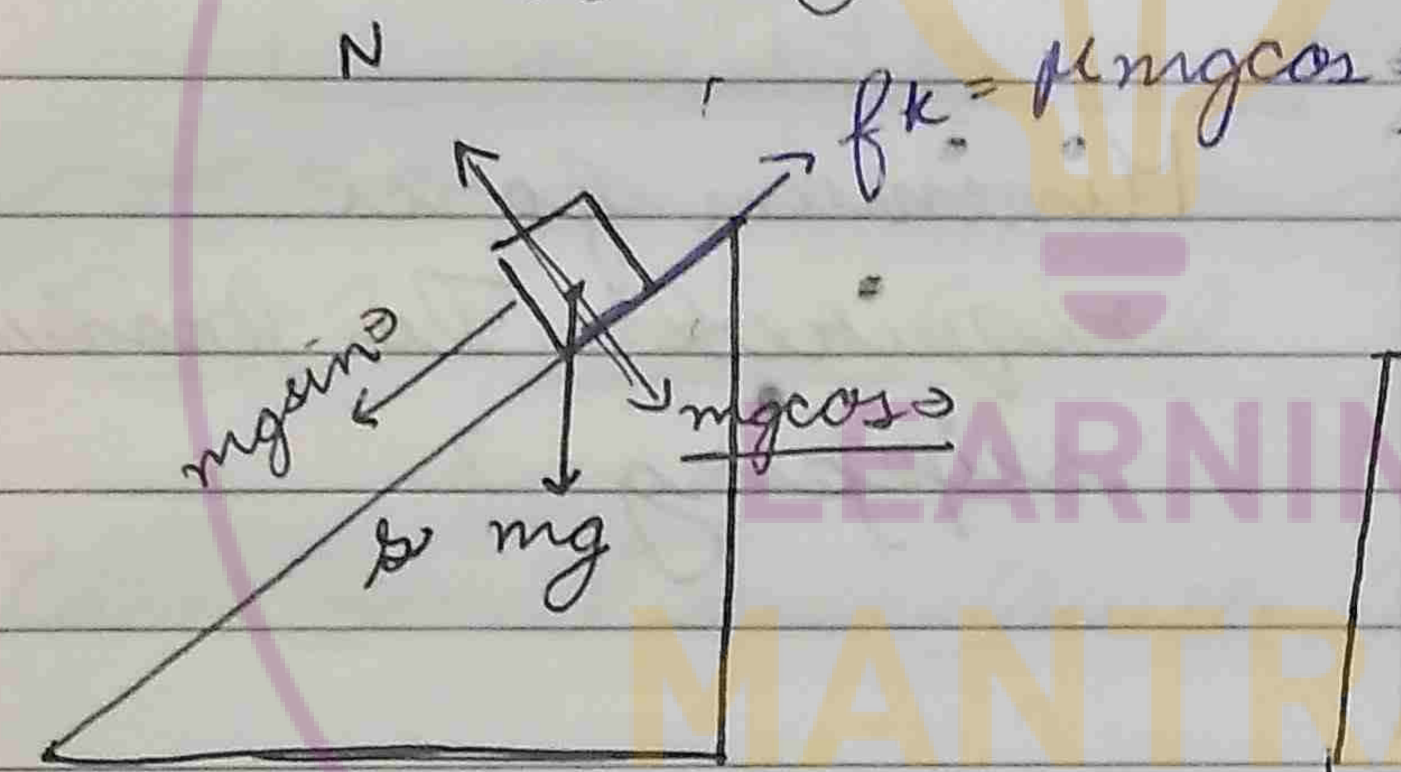
• If $\cos\theta = \mu\sin\theta$ or $\cot\theta = \mu$ or $\boxed{\theta = \cot^{-1}\mu}$

• Then $F_{required} = \infty$

Maximum angle upto which one can push a body

• Special Examples

• Block slipping on an incline.



$$a_{rough} = g(\sin\theta - \mu\cos\theta)$$

$$a_{smooth} = g\sin\theta$$

$t_1 \rightarrow$ time taken to slide down a rough plane.

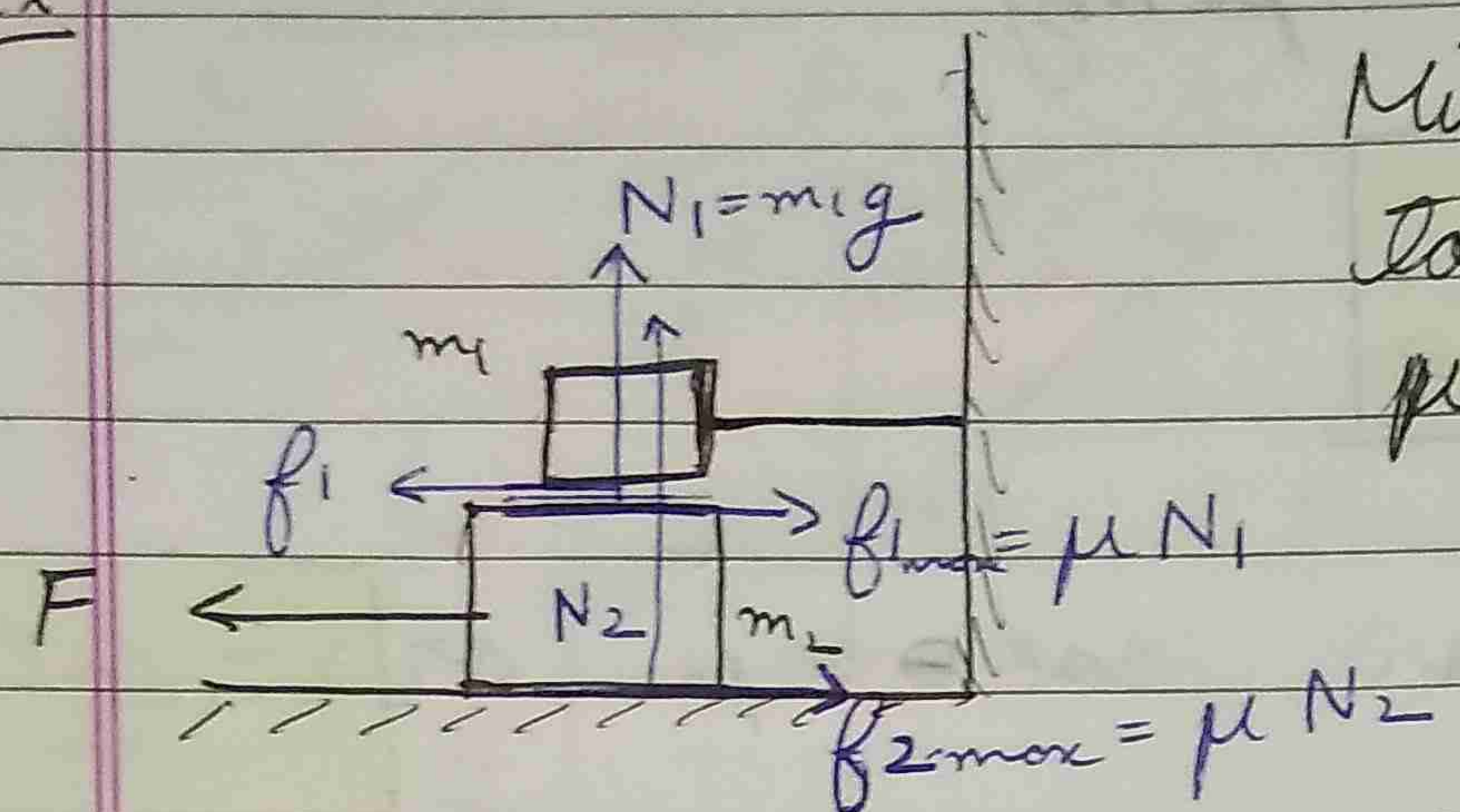
$t_2 \rightarrow$ smooth plane of same length and inclination

Rough = $s = \frac{1}{2} a_{roug} \cdot t_1^2$

Smooth = $s = \frac{1}{2} a_{smooth} \cdot t_2^2$ $\Rightarrow \frac{t_1^2}{t_2^2} = \frac{a_{smooth}}{a_{rough}}$

$$\Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{g\sin\theta}{g(\sin\theta - \mu\cos\theta)}} \Rightarrow \boxed{\frac{t_1}{t_2} = \sqrt{\frac{\sin\theta}{(\sin\theta - \mu\cos\theta)}}}$$

Ex



Minimum force required to pull m_2

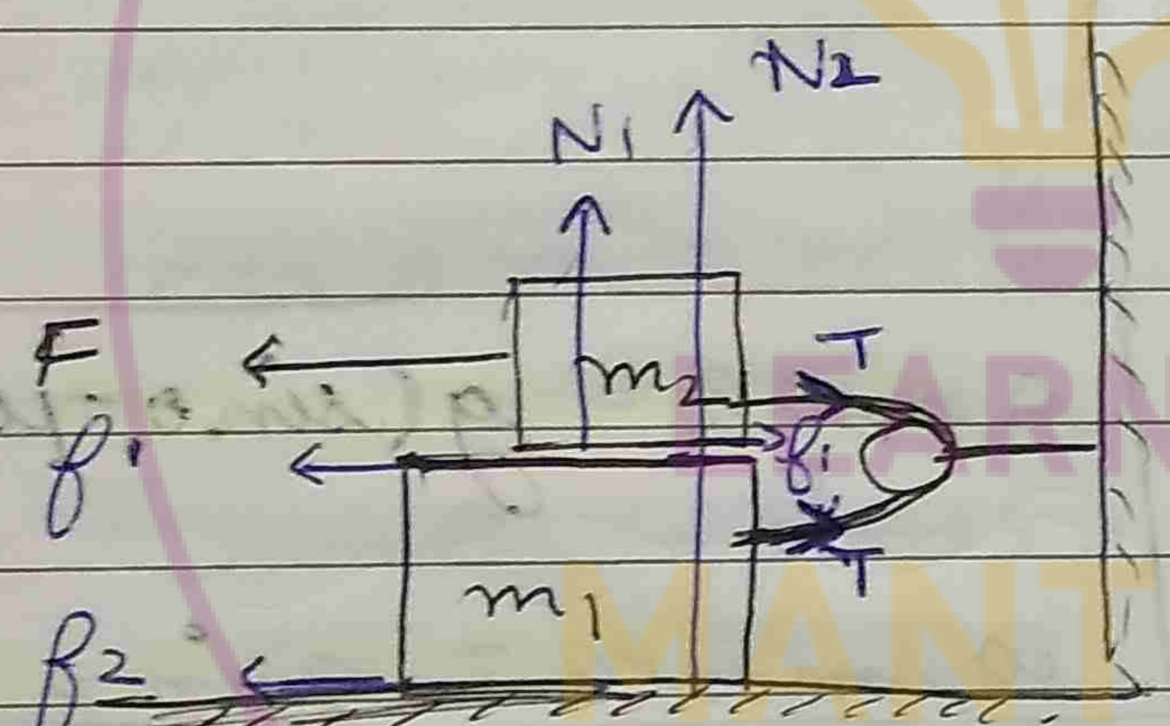
μ = all surfaces

$N_2 = (m_1 + m_2)g$

$\Rightarrow F \geq (f_1 + f_2)_{max}$

$\Rightarrow F \geq \mu(2m_1 g + m_2 g) \Rightarrow F \geq \mu g(2m_1 + m_2)$

Ex



Minimum force required to produce slipping?

For motion of m_2 → $F \geq T + f_{1max}$

for motion of m_1 $T \geq f_{1max} + f_{2max}$

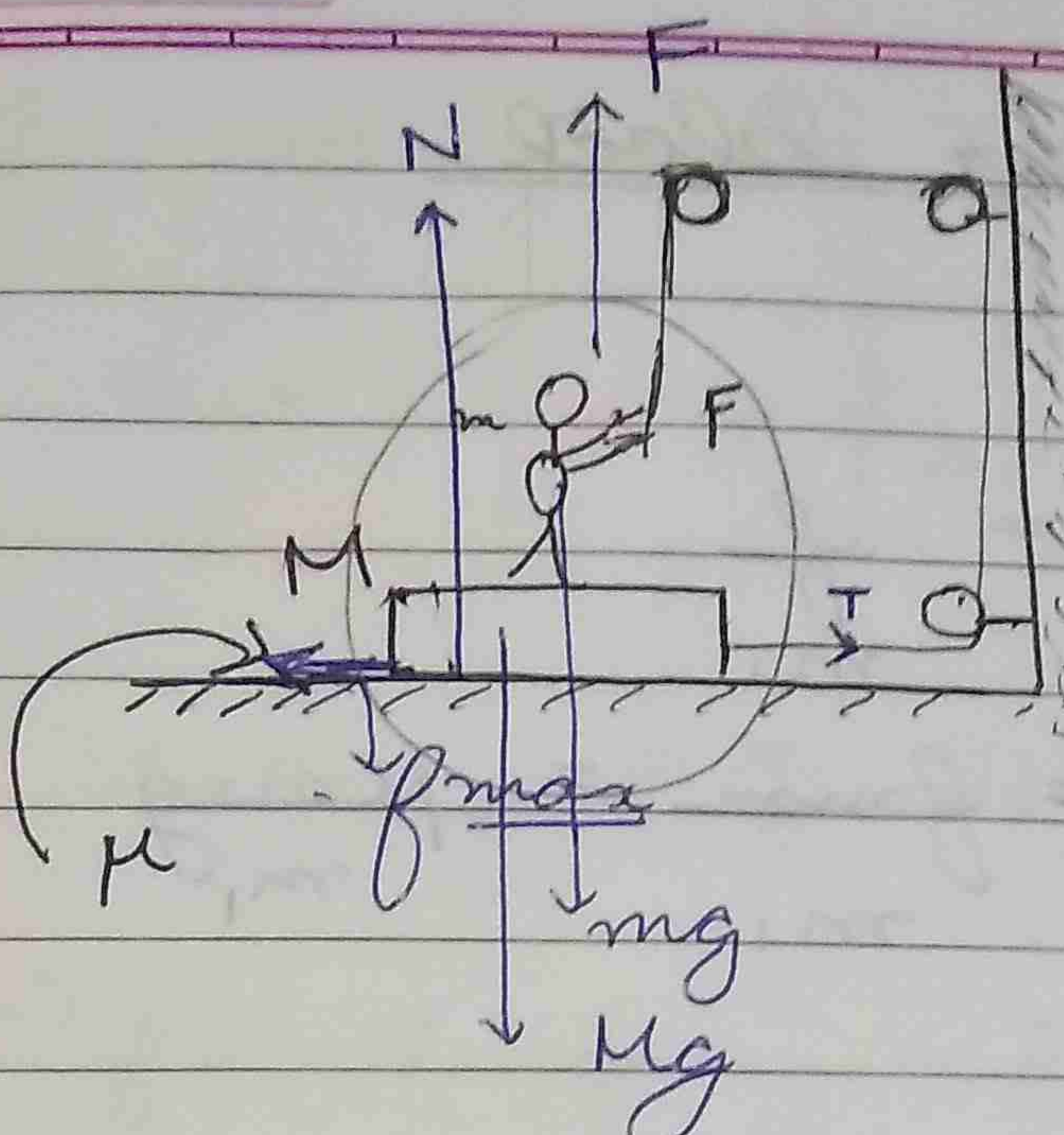
$\Rightarrow F \geq 2f_{1max} + f_{2max} \Rightarrow F \geq 2\mu m_2 g + \mu(m_1 + m_2)g$

$\Rightarrow$

$F = (3m_2 + m_1)\mu g$

• during slipping $f = f_{max}$.

☆ Ex



Minimum F to pull the plank?

$T = F$

$N + F = mg + Mg$ (i)

$\Rightarrow N = (m+M)g - F$

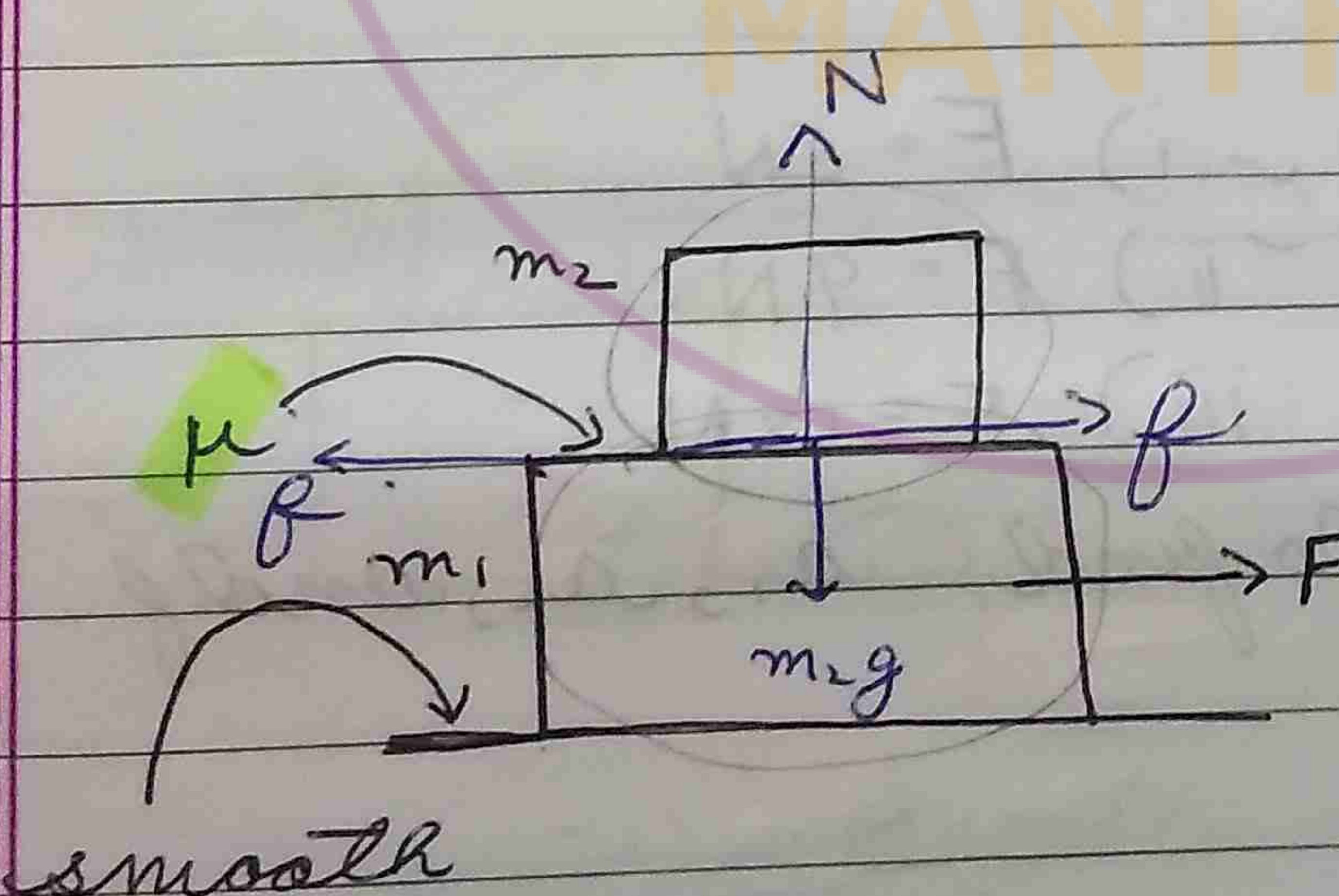
$T \geq f_{max}$ $T \geq \mu N$

$\Rightarrow F \geq \mu[(m+M)g - F] \Rightarrow F(1+\mu) = \mu(m+M)g$

$$\Rightarrow \boxed{F = \frac{\mu(m+M)g}{(1+\mu)}}$$

• Block-on-block systems

• Case 1 → F on lower block



• F_{max}, so that they blocks move together.

$a_2 = \frac{f}{m_2}$

$a_{2max} = \frac{f_{max}}{m_2} = \frac{\mu N}{m_2}$

$= \frac{\mu m_2 g}{m_2} = \boxed{\mu g}$

• $a_{common\ max} = a_{2max}$
$= \mu g$

• $F_{max} = (m_1 + m_2) \times a_{max}$

$$\Rightarrow \boxed{F_{max} = \mu g(m_1 + m_2)}$$

• If F is less than F_{max} the blocks move together

• If F is more than F_{max} the block will separate

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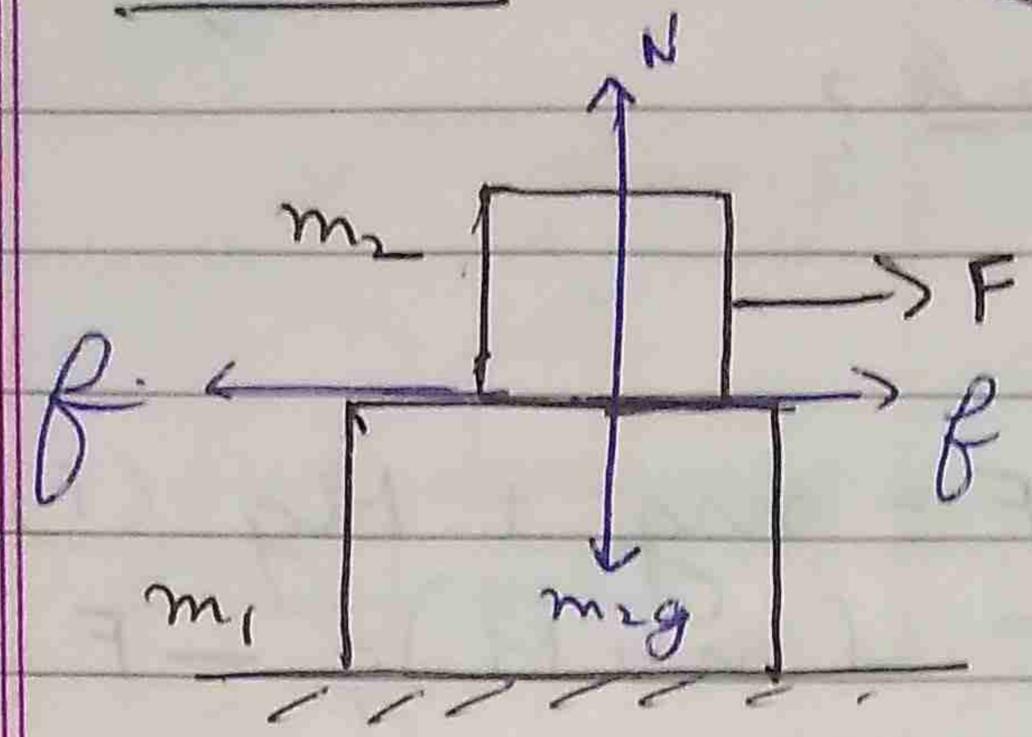
- Case ii → F on upper block



$$a_1 = \frac{f}{m_1}$$

$$a_{1\,max} = \frac{f_{max}}{m_1} = \frac{\mu m_2 g}{m_1}$$

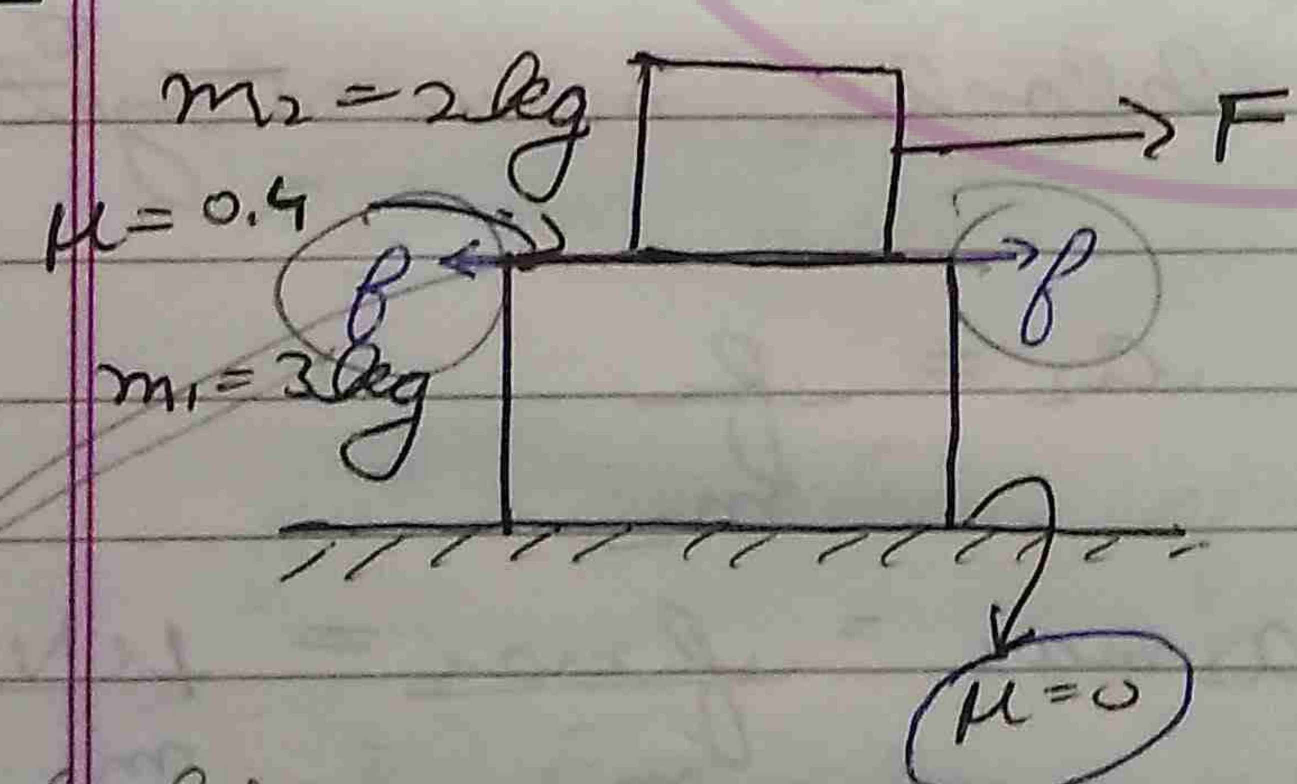
$$a_{common,\,max} = \frac{\mu m_2 g}{m_1}$$

$$F_{max} = \frac{\mu m_2 g}{m_1}(m_1 + m_2)$$

M + m

when we take the two block together then the friction between the block become internal forces and are not consider. Then only external forces are considered which act on the system.

Ex



i) F = 6N
ii) F = 9N
iii) F = 18N

To find a_1, a_2, and f.

Internal forces

$$F_{max} = \frac{\mu m_2 g}{m_1}(m_1 + m_2) = \frac{0.4 \times 20}{3}(5)$$

$$\Rightarrow \frac{40}{3} N = 13.3 N$$

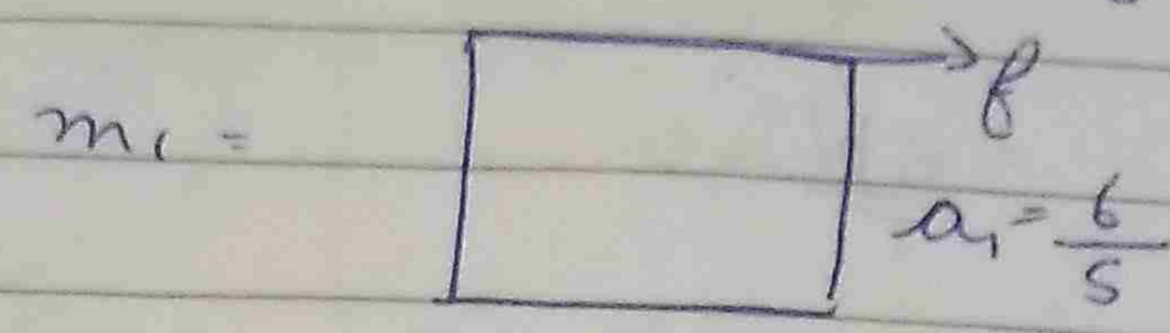
i) F = 6N

$$a_1 = a_2 = \frac{F}{m_1 + m_2} = \frac{6}{5} m/s^2$$

∵ two blocks move together

• Since in (iii) force applied is more than Fmax hence relative slipping will occur and friction will also be f_{max}.

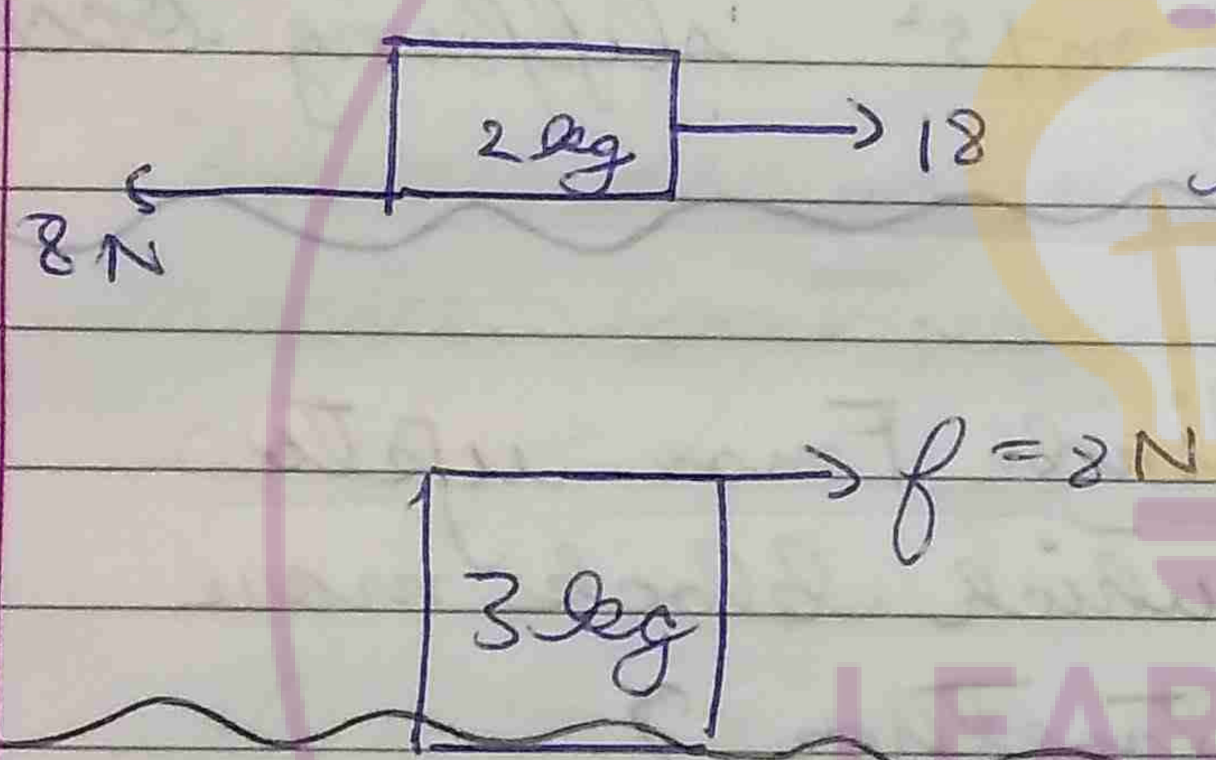
$f = m_1 a_1 \Rightarrow 3 \times \frac{6}{5} = 3.6\,N$



ii) $a_1 = a_2 = \frac{9}{5}\,m/s^2$

$\Rightarrow f = 3 \times \frac{9}{5} = 5.4\,N$

iii) $f_{max} = \mu m_2 g = 8\,N$



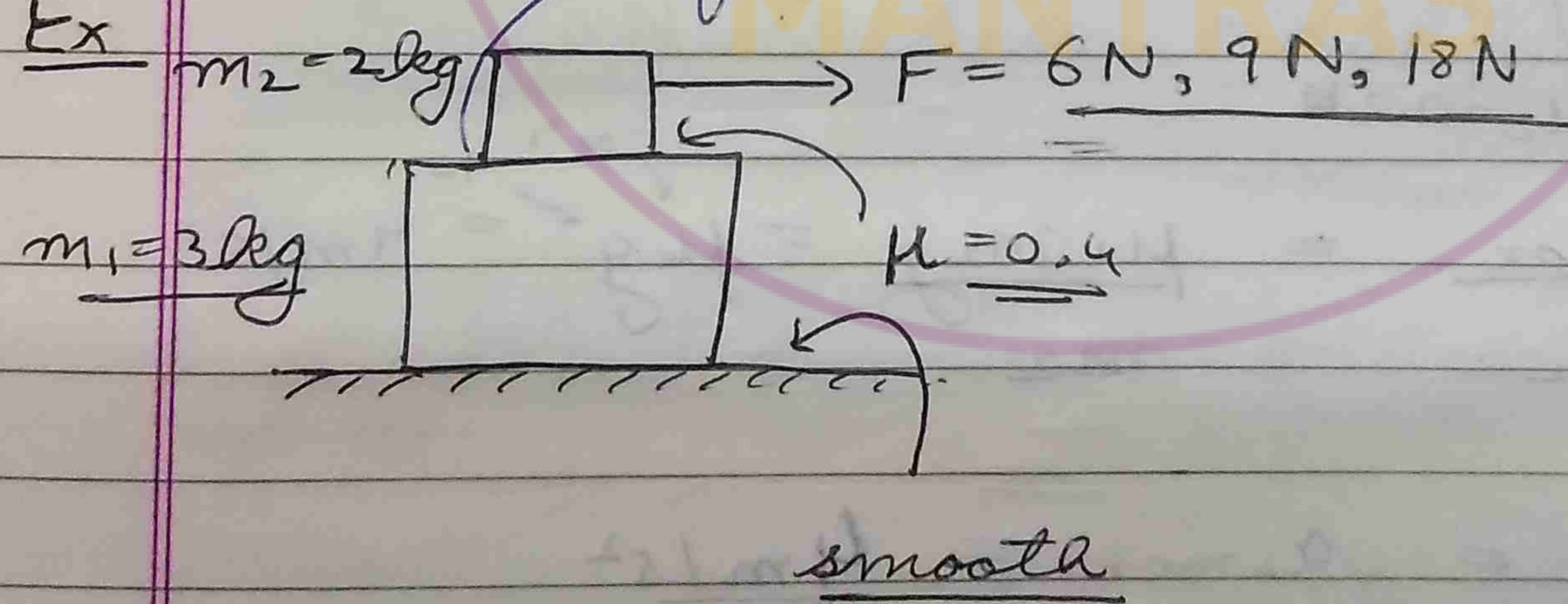
$a_2 = \frac{18-8}{2} = 5\,m/s^2$

$a_1 = \frac{8}{3}\,m/s^2$

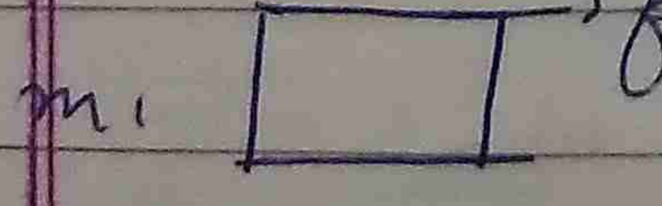
$\Rightarrow a_1 = 2.67\,m/s^2$

Ex

$f_{max} = 0.4 \times 20 = 8\,N$



(i) $a_1 = a_2 = \frac{6\,N}{(2+3)} = \frac{6}{5}\,m/s^2$



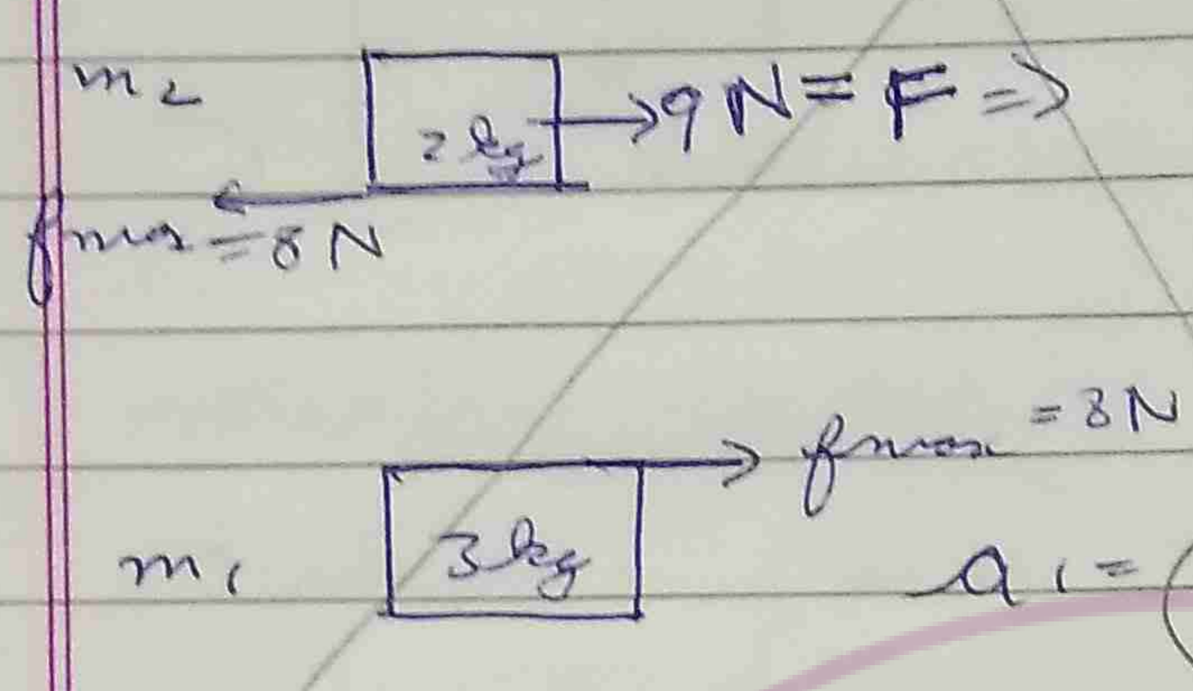
$f = m_1 \times a = \frac{3 \times 6}{5} = 3.6\,N$

Here f is less than f max hence possible.

ii) $F = 9N$

Assume slipping

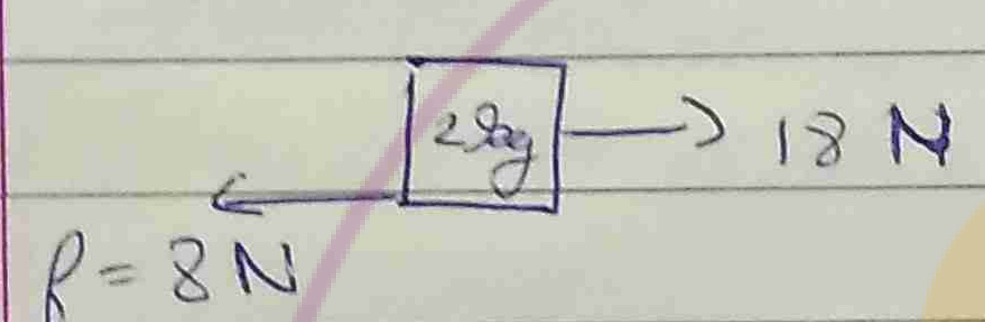
$\Rightarrow f = f_{max} = \mu m_2 g = 8N$



$\Rightarrow a_2 = \frac{1}{2} m/s^2$

$a_1 = \frac{8}{3} m/s^2$

iii)

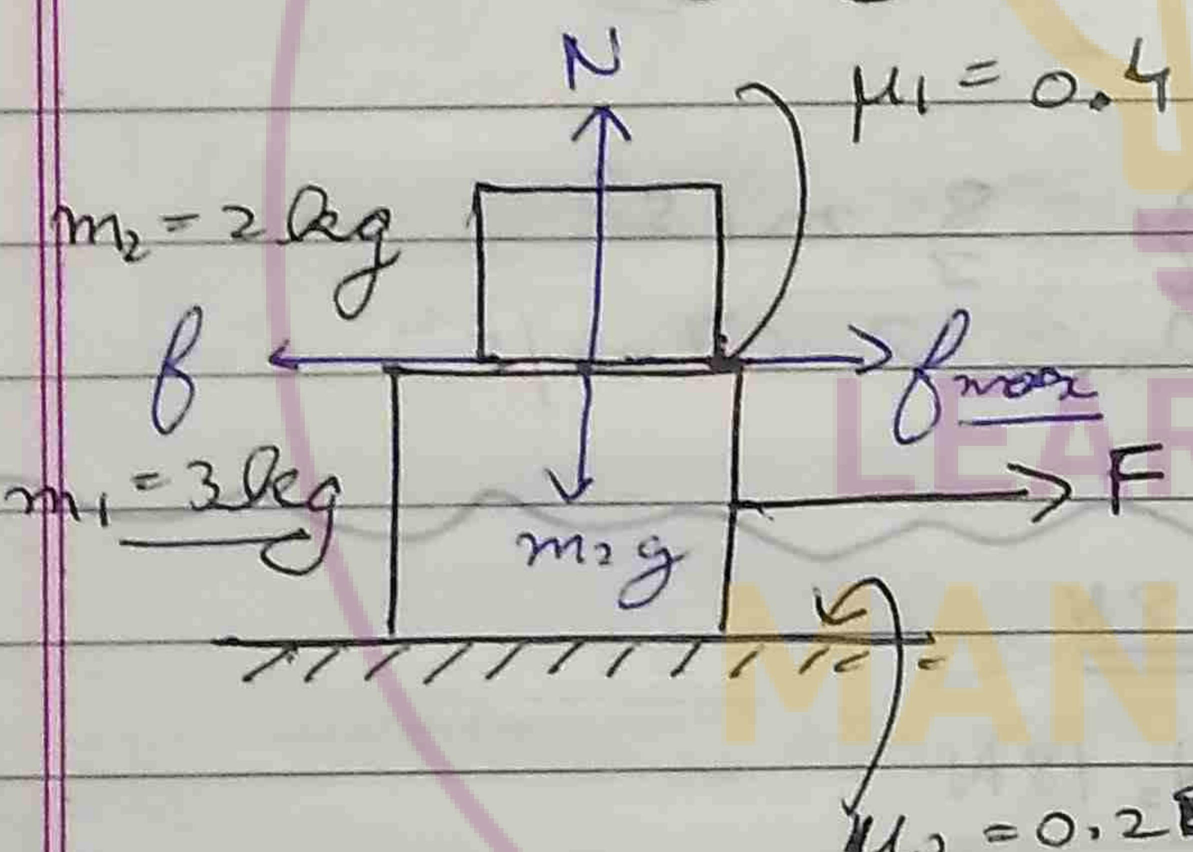


$a_2 = 5 m/s^2$

$a_1 = \frac{8}{3} m/s^2$

Hence in this case slipping occurs.

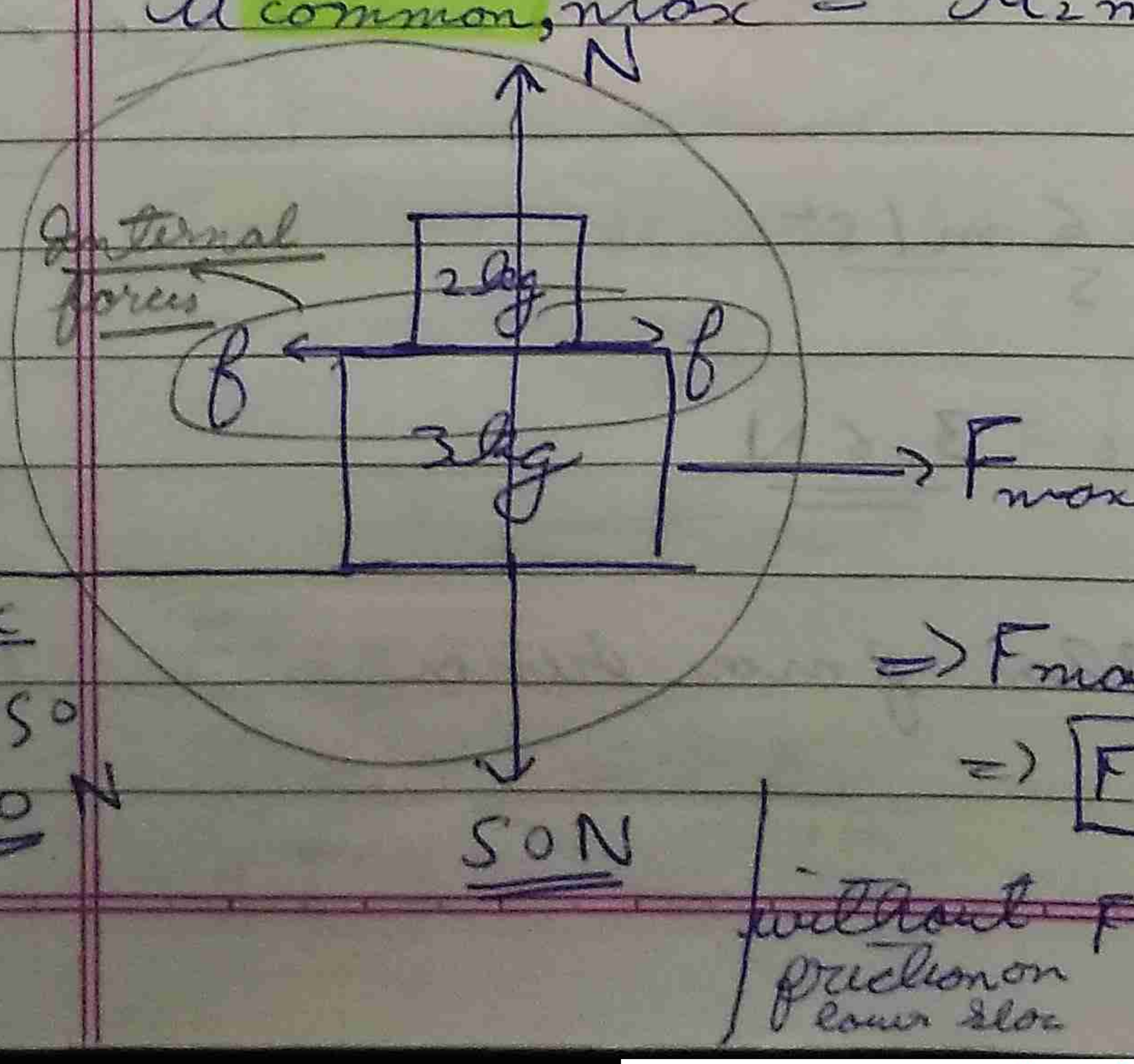
Ex



Find F_{max} upto which blocks move together?

$$a_{2\,max} = \frac{f_{max}}{m_2} = \frac{\mu_1 m_2 g}{m_2} = \mu_1 g = 4 m/s^2$$

$$a_{common, max} = a_{2\,max} = 4 m/s^2$$



f'_{max}

0.2×50

$\Rightarrow 10 N$

$$F_{max} - f'_{max} = (m_1 + m_2) \times a_{common, max}$$

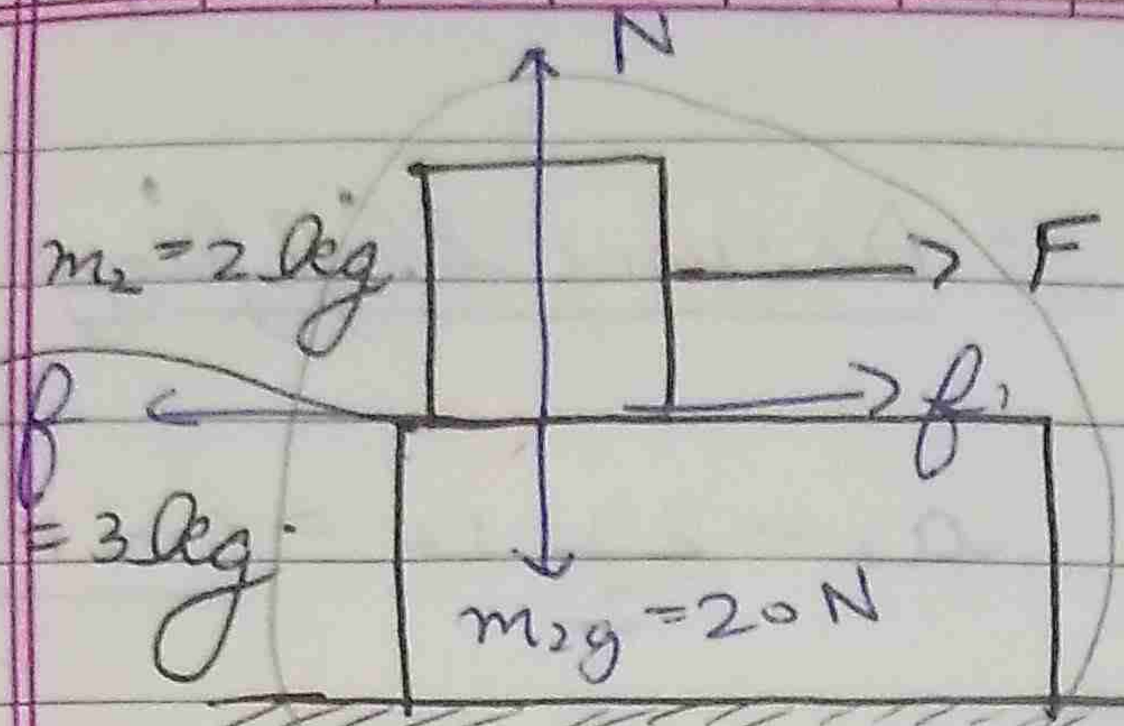
$\Rightarrow F_{max} - 10 = 5 \times 4$

$\Rightarrow \boxed{F_{max} = 30 N}$

without friction on lower block: $F_{max} = \mu g (m_1 + m_2) = 0.4 \times 10 (2+3) = 20 N$

$F_{max} = 0.6 \times \frac{2}{3} \times 10 \, (5)$ (20 N)

Ex



$\mu_s = 0.6$

$\mu_k = 0.4$

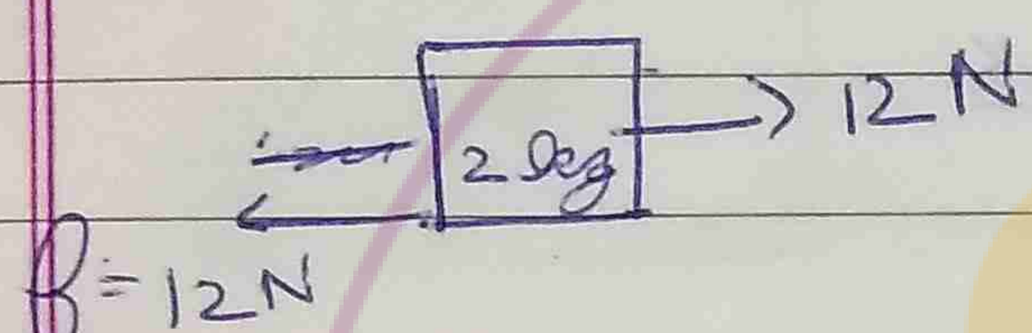
$f_{max} = 12N$

To find a_1, a_2 and f = ?

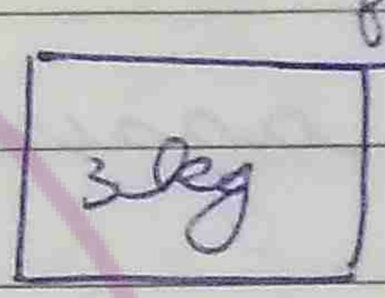
(i) $F = 12N$

(ii) $F = 20N$

i) $F = 12N \Rightarrow$ ~~Assume slipping~~



$a_2 = 0$



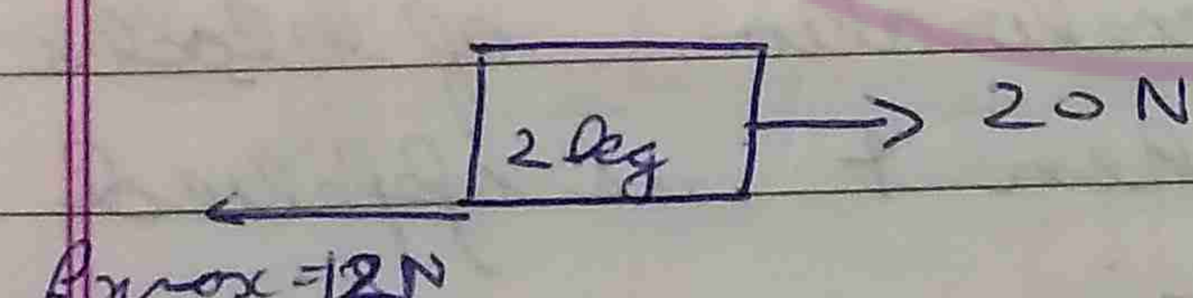
$\Rightarrow a_1 = \frac{12}{3} = 4 m/s^2$

There is no slipping hence two block will move together

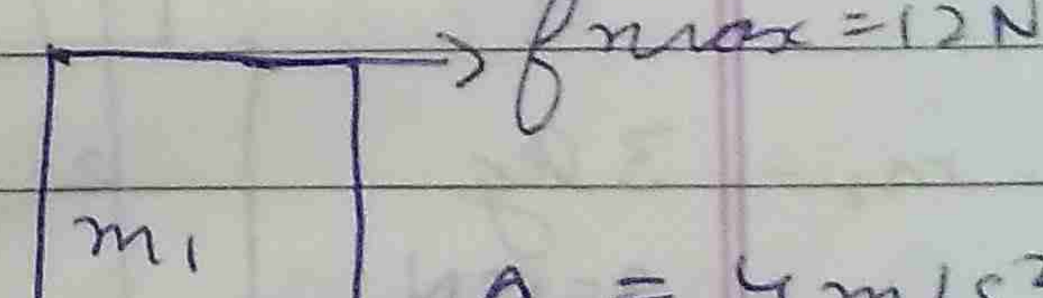
$a_1 = a_2 = \frac{12}{5} = 2.4 m/s^2$

$f = m_1 \times a_1 = 7.2N$

ii) $F = 20N \Rightarrow$ ~~Assume slipping~~



$a_2 = \frac{20-12}{2} = \frac{8}{2} = 4 m/s^2$

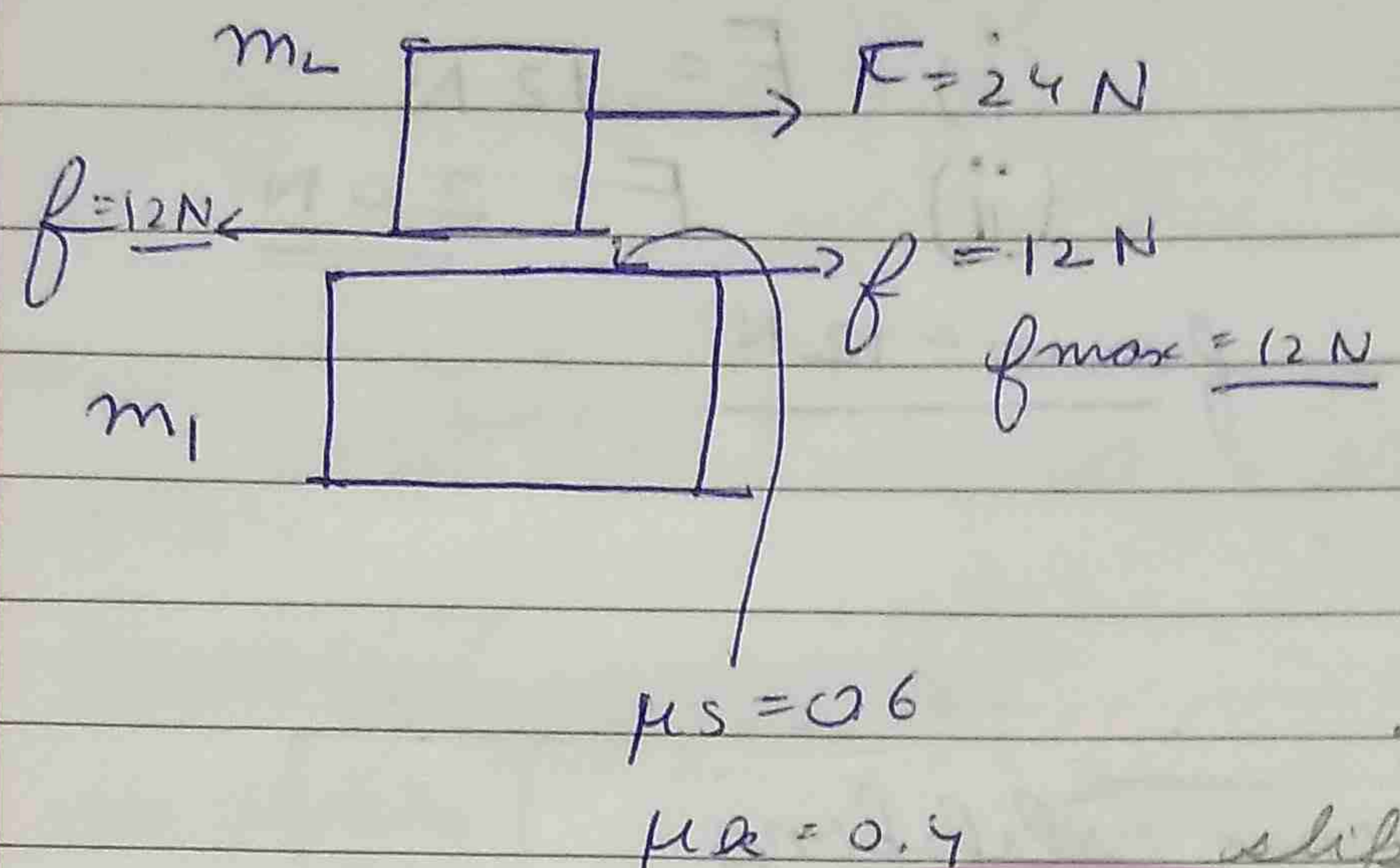


$a_1 = 4 m/s^2$

$\Rightarrow$ There will be no slipping, because both blocks having same acceleration is possible

$f = 12N$

iii) $F = 24\,N$



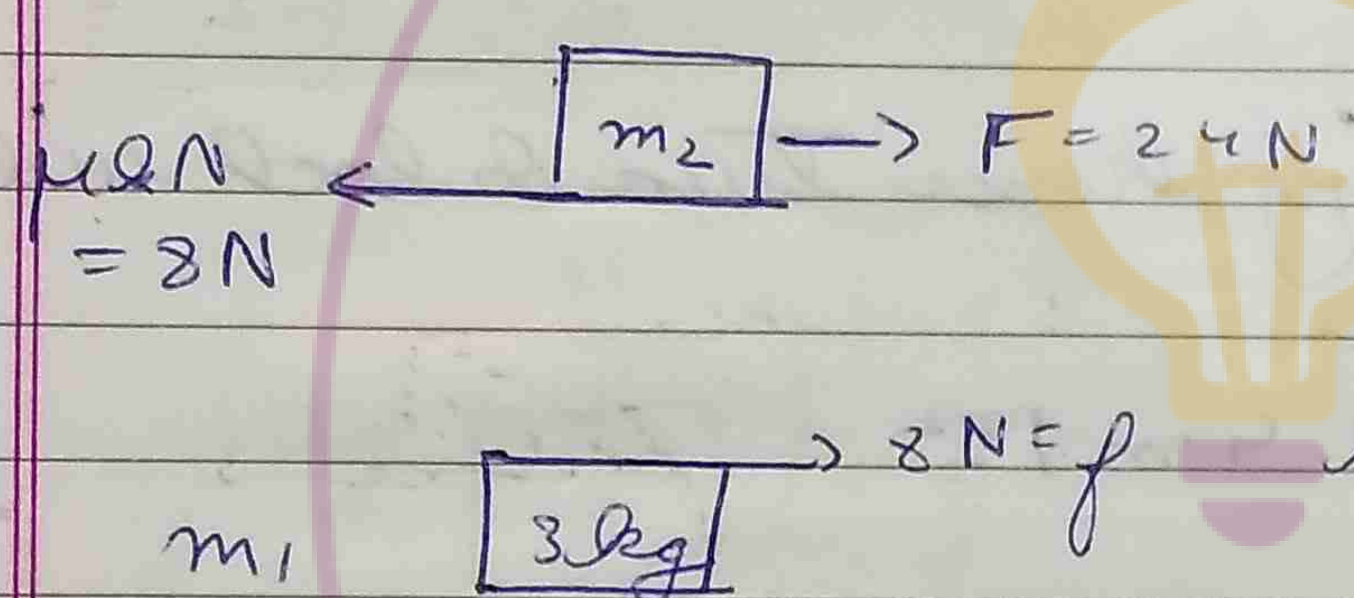
• Assume slipping

$a_2 = \frac{24-12}{2} = 6\,m/s^2$

$a_1 = \frac{12}{3} = 4\,m/s^2$

This results in slipping
slipping will occur.

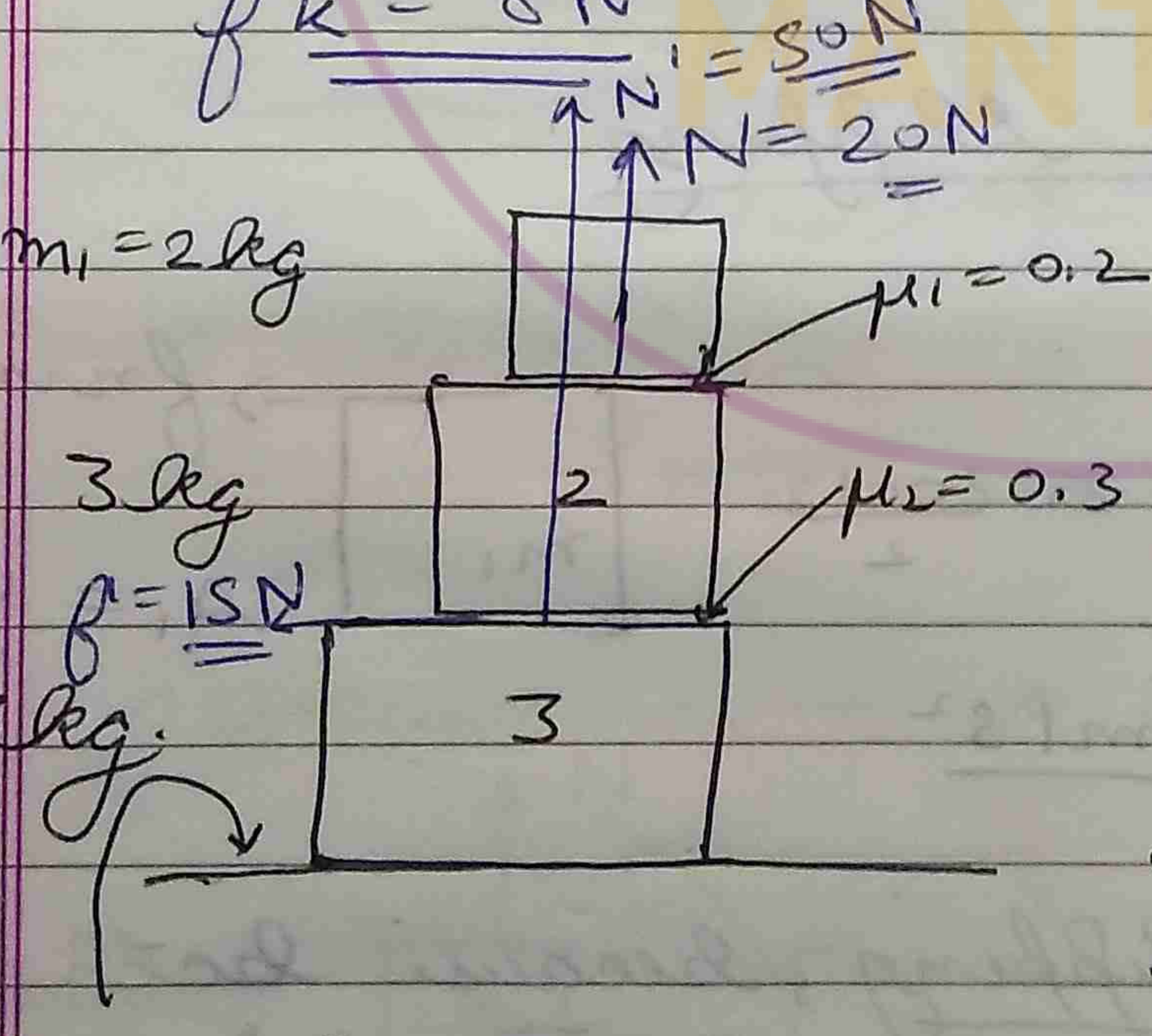
• Solve again with $\mu = \mu_k$



$a_2 = \frac{24-8}{2} = 8\,m/s^2$

$a_1 = \frac{8}{3}\,m/s^2$

$f_k = 8\,N$

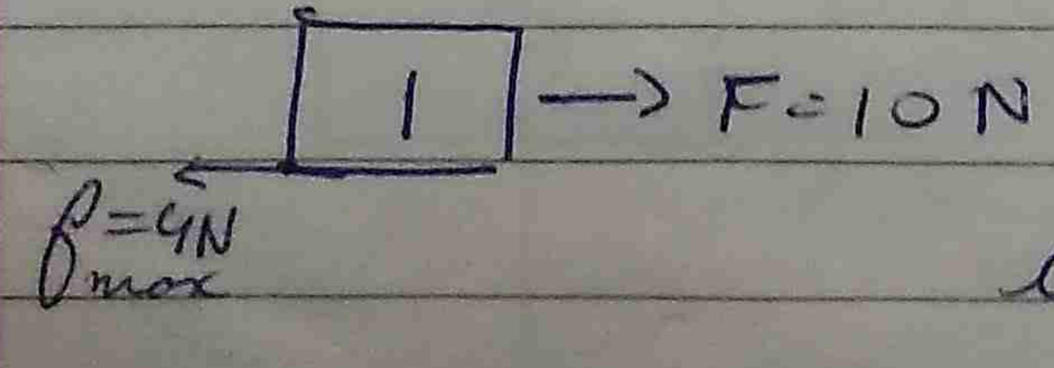


Ex $m_1 = 2kg$
$m_2 = 3kg$
$m_3 = 7kg$

$F = 10\,N$
acceleration of blocks when F is applied
To:
a) block 1
b) block 2
c) block 3
} one by one

Assume slipping ✓

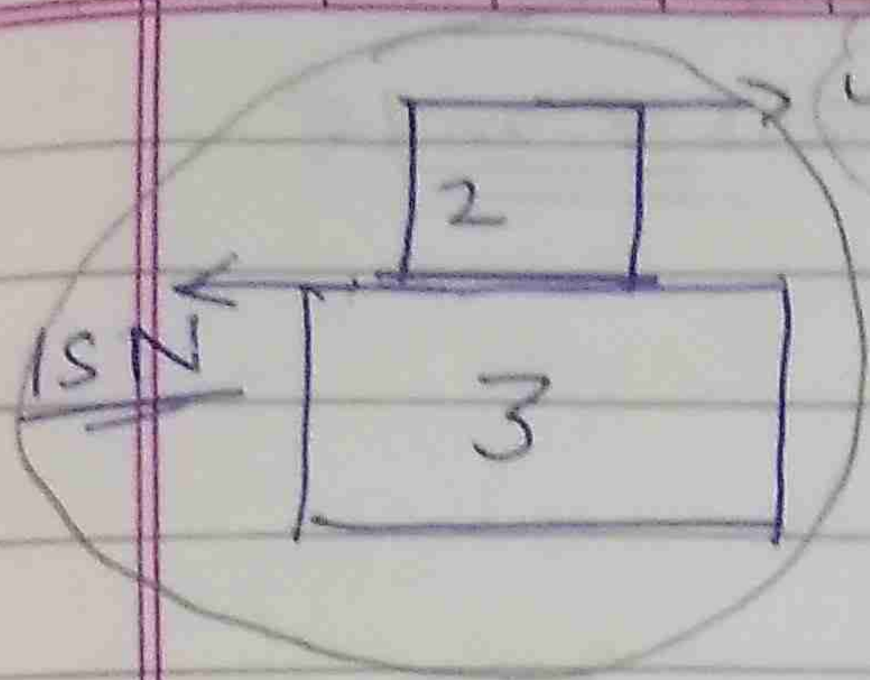
a



$a_1 = \frac{10-4}{2} = 3\,m/s^2$

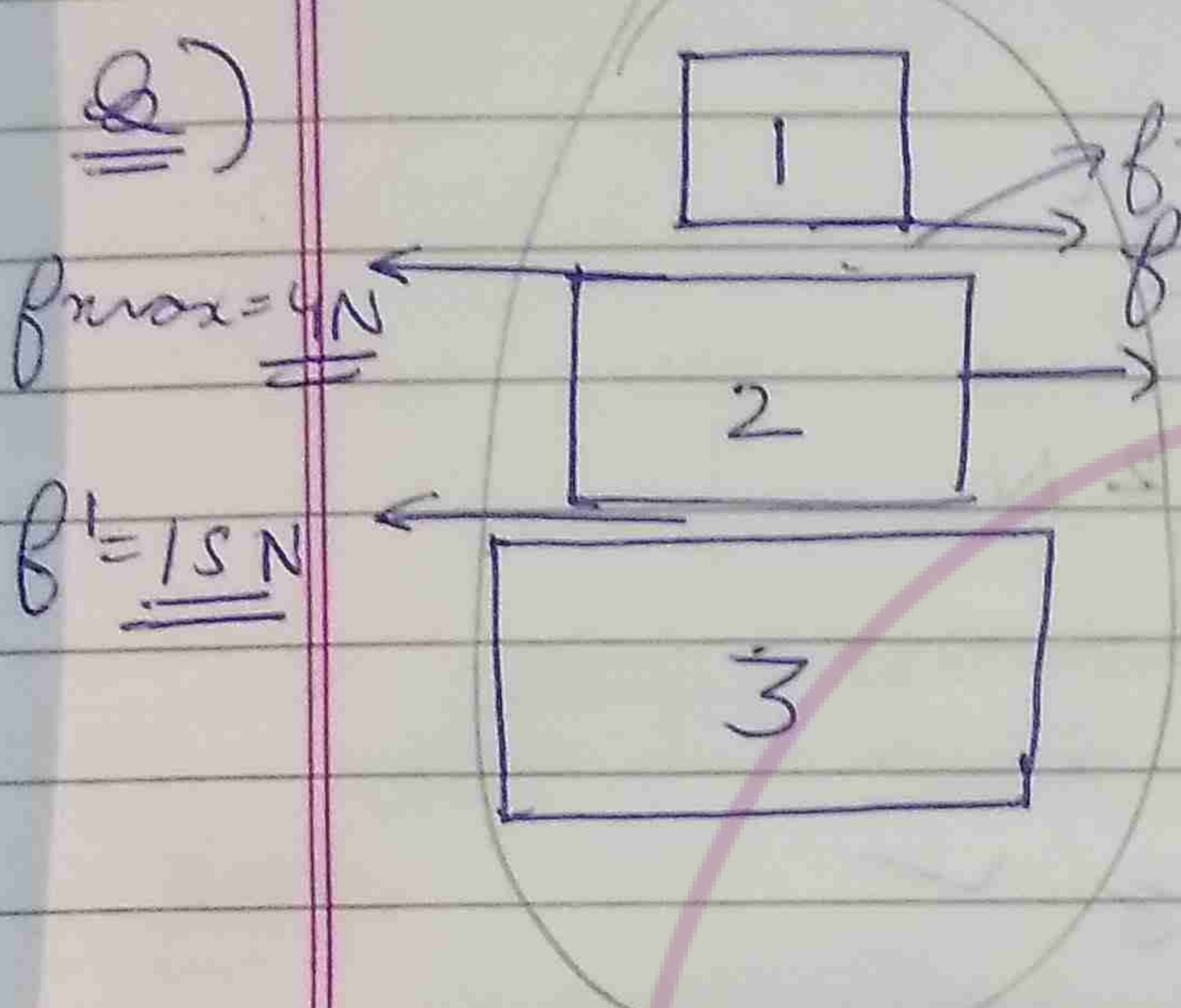
• The block on which force acts should have greater acceleration than the block that are indirectly attached.

This can't overcome the backward force



$a_2 = a_3 = \frac{4}{3+7} \Rightarrow$ ~~0.8~~ $0.4 m/s^2$

Q)



• ~~Assume slipping between 1 and 2~~

$a_1 = \frac{4N}{2kg} = 2m/s^2$

• Assume 2 and 3 move together

$a_2 = a_3 = \frac{10-4}{10} = 0.6 m/s^2$

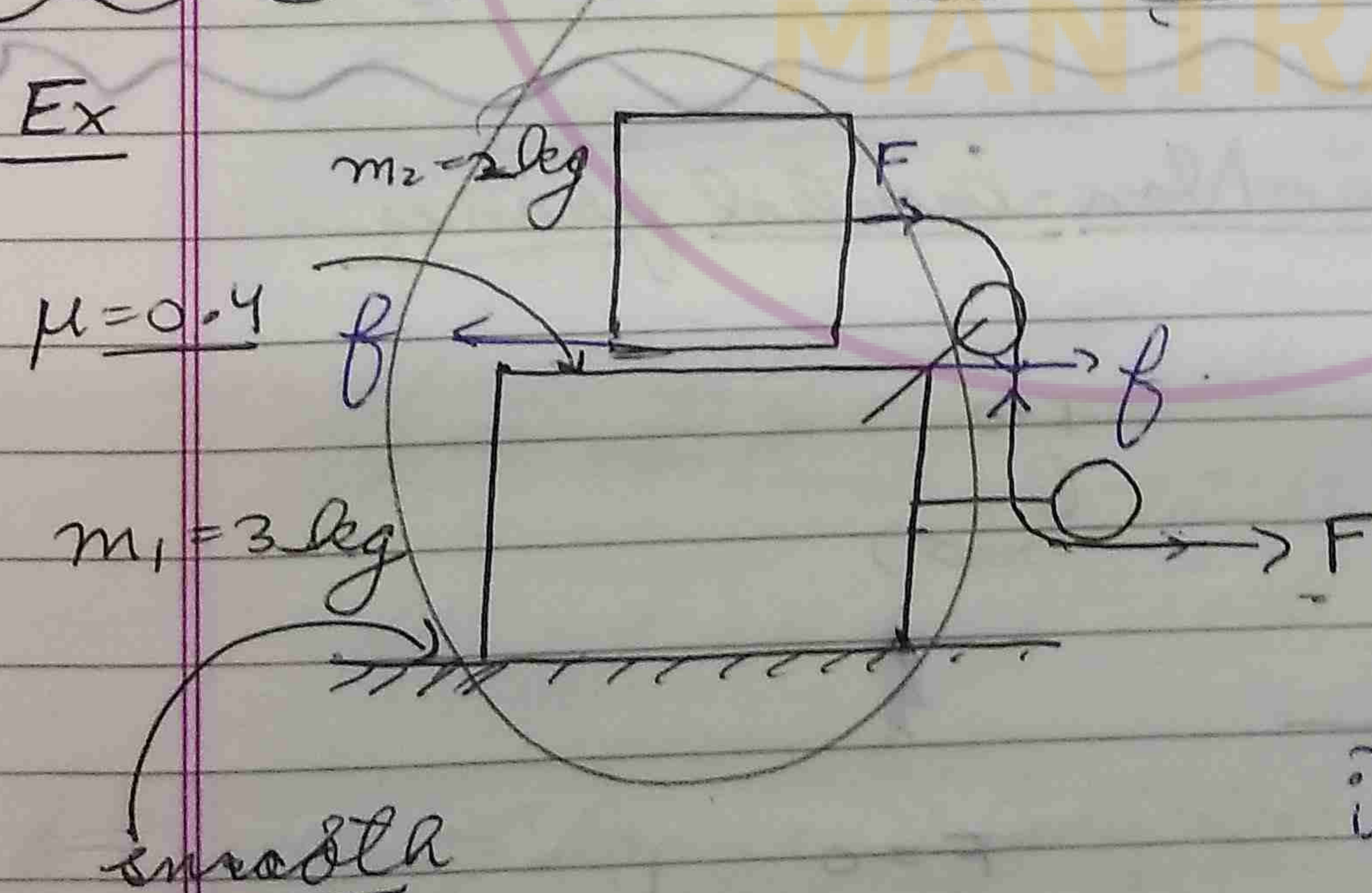
Absurd results (i) block is moving faster than (ii)

all blocks move together!

$\Rightarrow a_1 = a_2 = a_3$

$\Rightarrow \frac{10}{12} = \frac{5}{6} m/s^2$

Ex

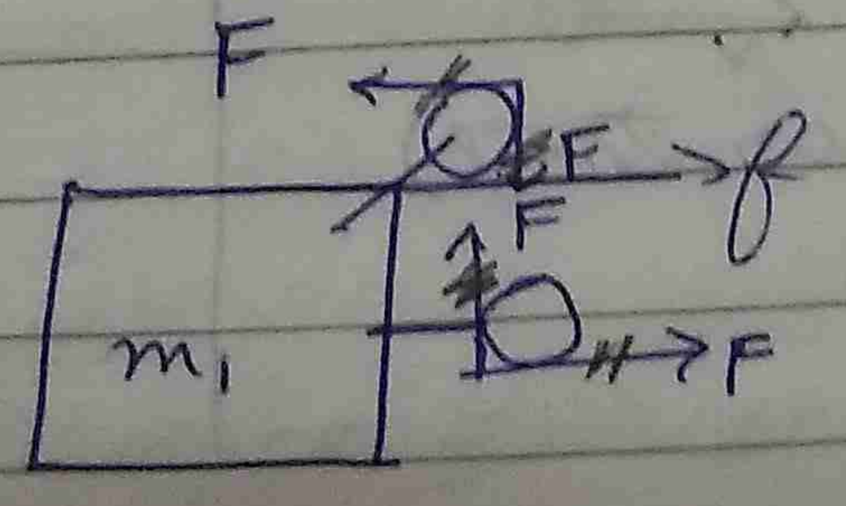


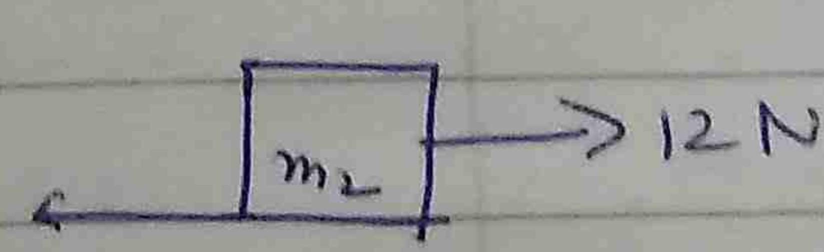
i) F = 12N

ii) F = 60N

• Find a_1, a_2 and f.

i) ~~Assume slipping~~ ✕





$f_{max} = 8N \quad a_2 = \frac{12-8}{2} = 2m/s^2$

m_1 $\rightarrow f_{max} = 8N \Rightarrow a_1 = \frac{8}{3} = 2.67 \ m/s^2$

Absurd result

Hence there is no slipping =

$a_1 = a_2 = \frac{12}{5} \Rightarrow 2.4 \ m/s^2$

$f = m_1 \times a = 3 \times 2.4 = 7.2 \ N$

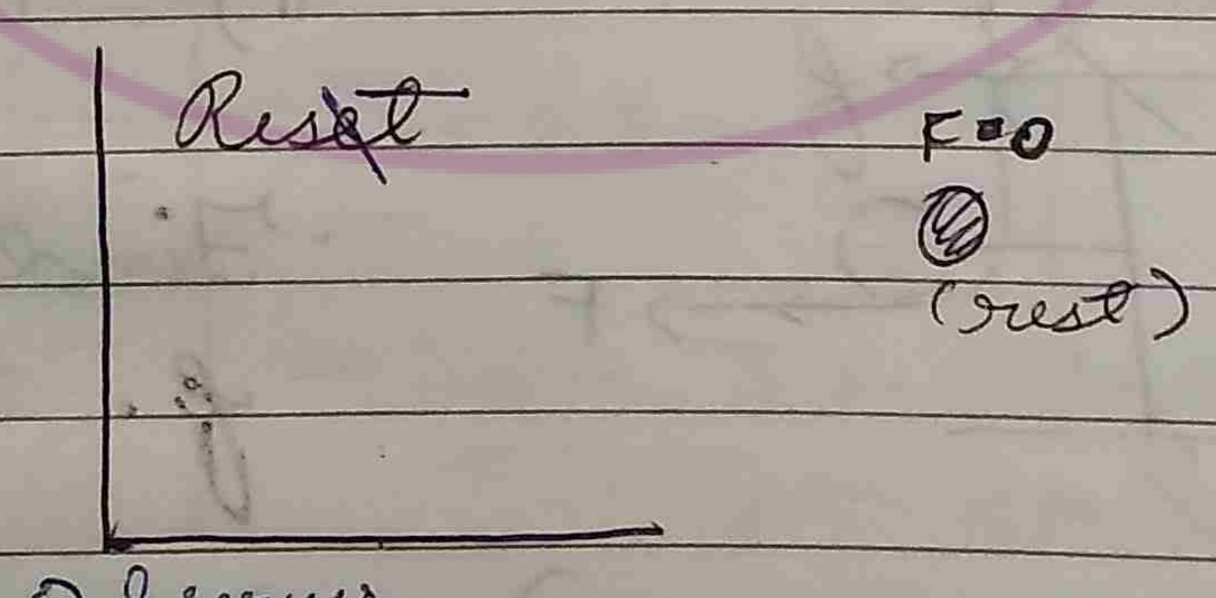
ii) $F = 60N$: Assume slipping ✓

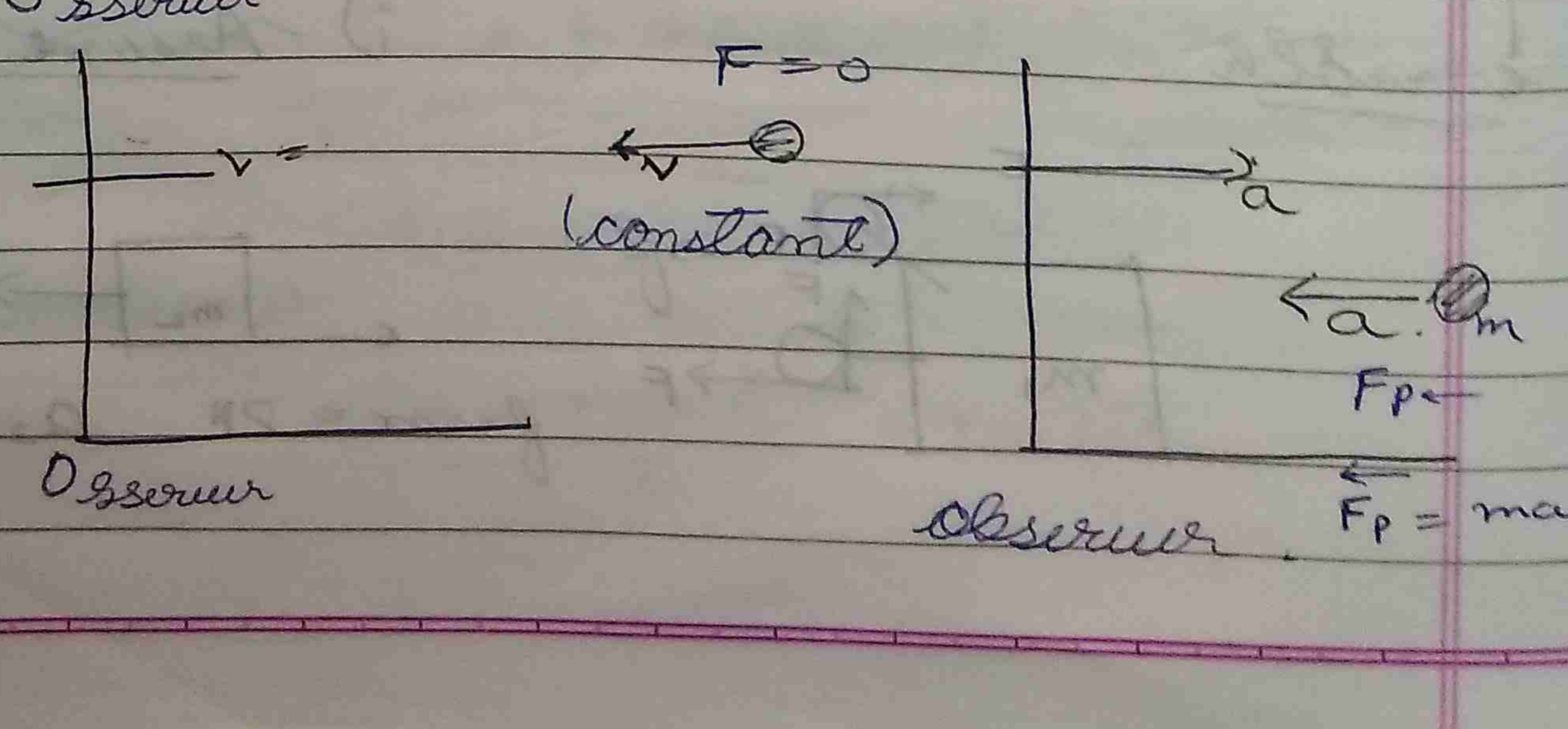
$a_2 = \frac{52}{2} = 26 \ m/s^2$

8N ← 2kg → 60 N

3kg → 8N $\quad a_1 = \frac{8}{3} \ m/s^2 \Rightarrow$ slipping ✓

Inertial and Non-inertial frames

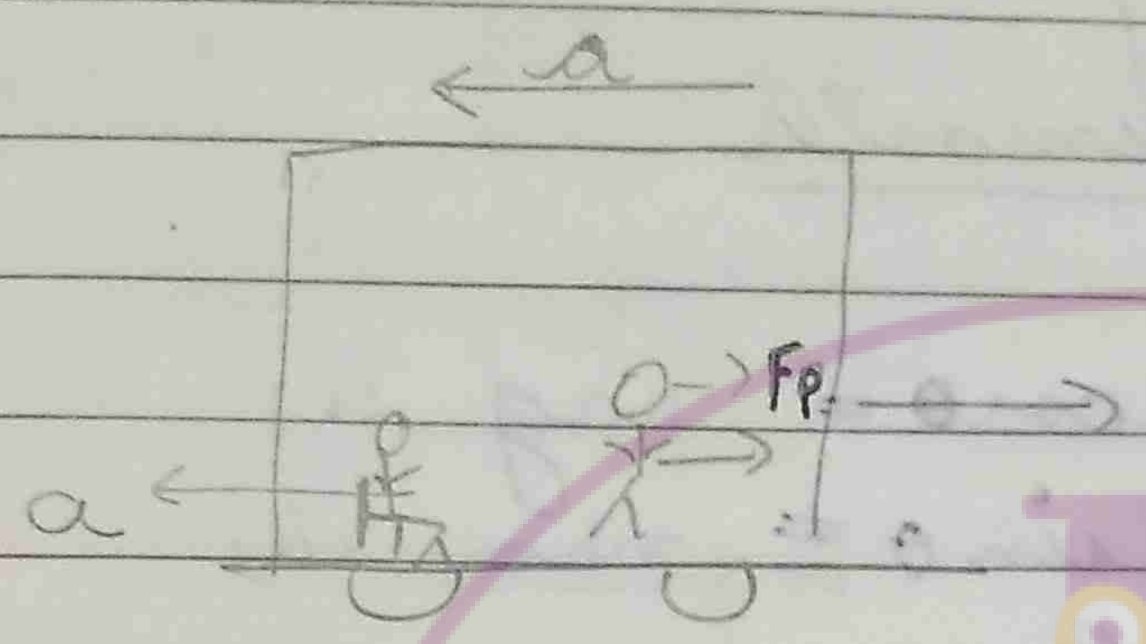




- In a non-inertial frame, the second law of motion is written as $F = ma - F_P$

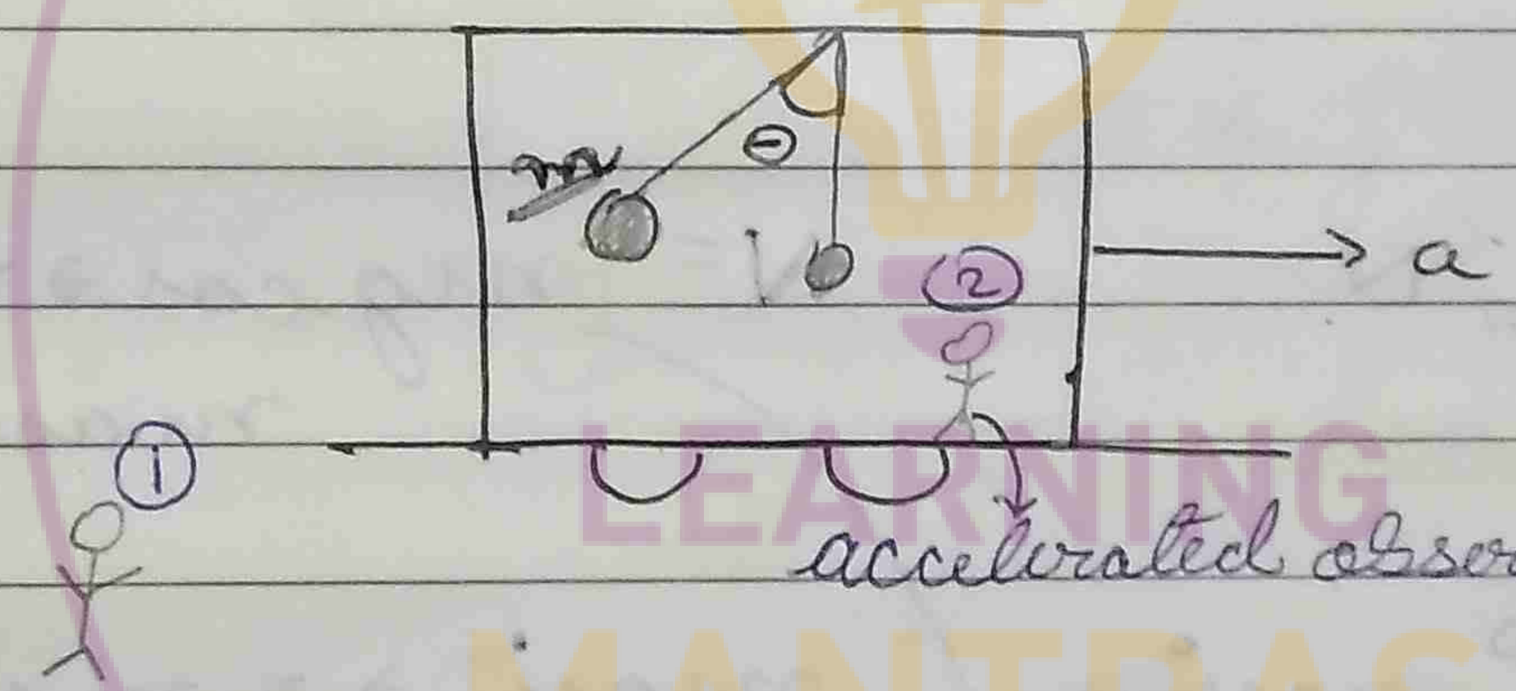
- Pseudo force: F_P = (mass under observation) × (acceleration of observer / frame)

- Direction of $\vec{F_P}$ → opposite to direction of acceleration of frame



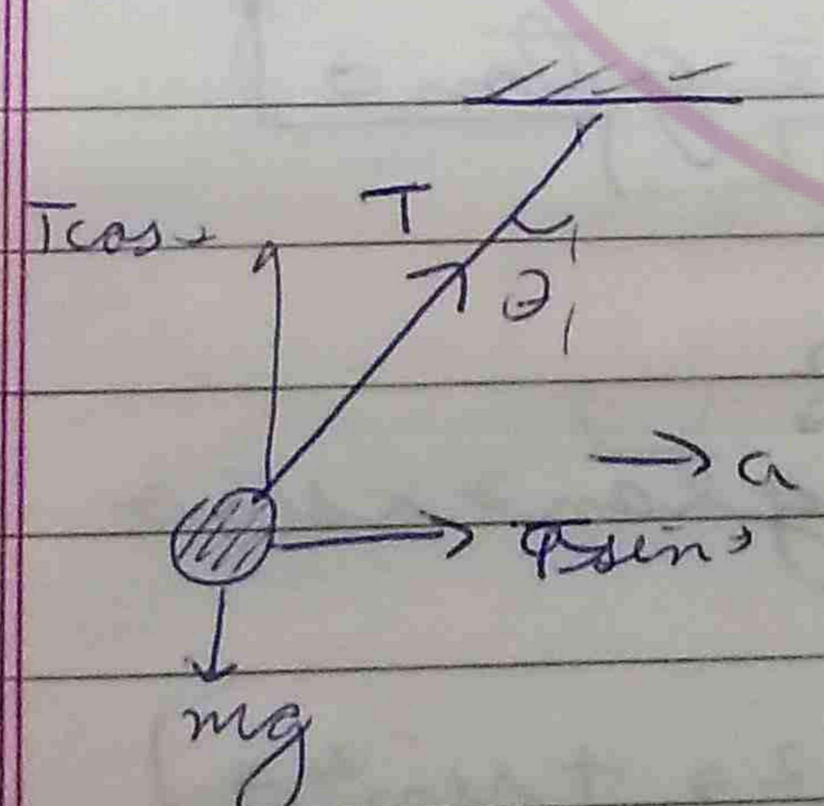
∴ pseudo force is not a real force hence it has no reaction of force.

Ex



Find angle θ.

- **Observer 1**



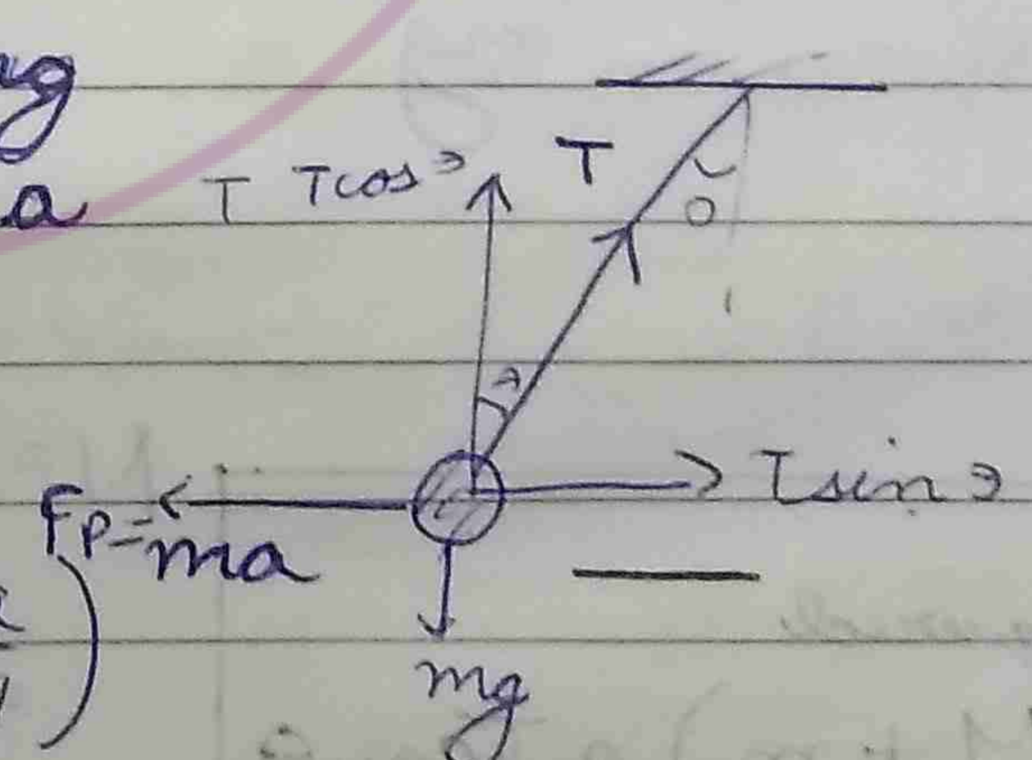
$T\cos\theta = mg$

$T\sin\theta = ma$

$\Rightarrow \tan\theta = \frac{a}{g}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$

Observer 2



$T\cos\theta = mg$ (i)

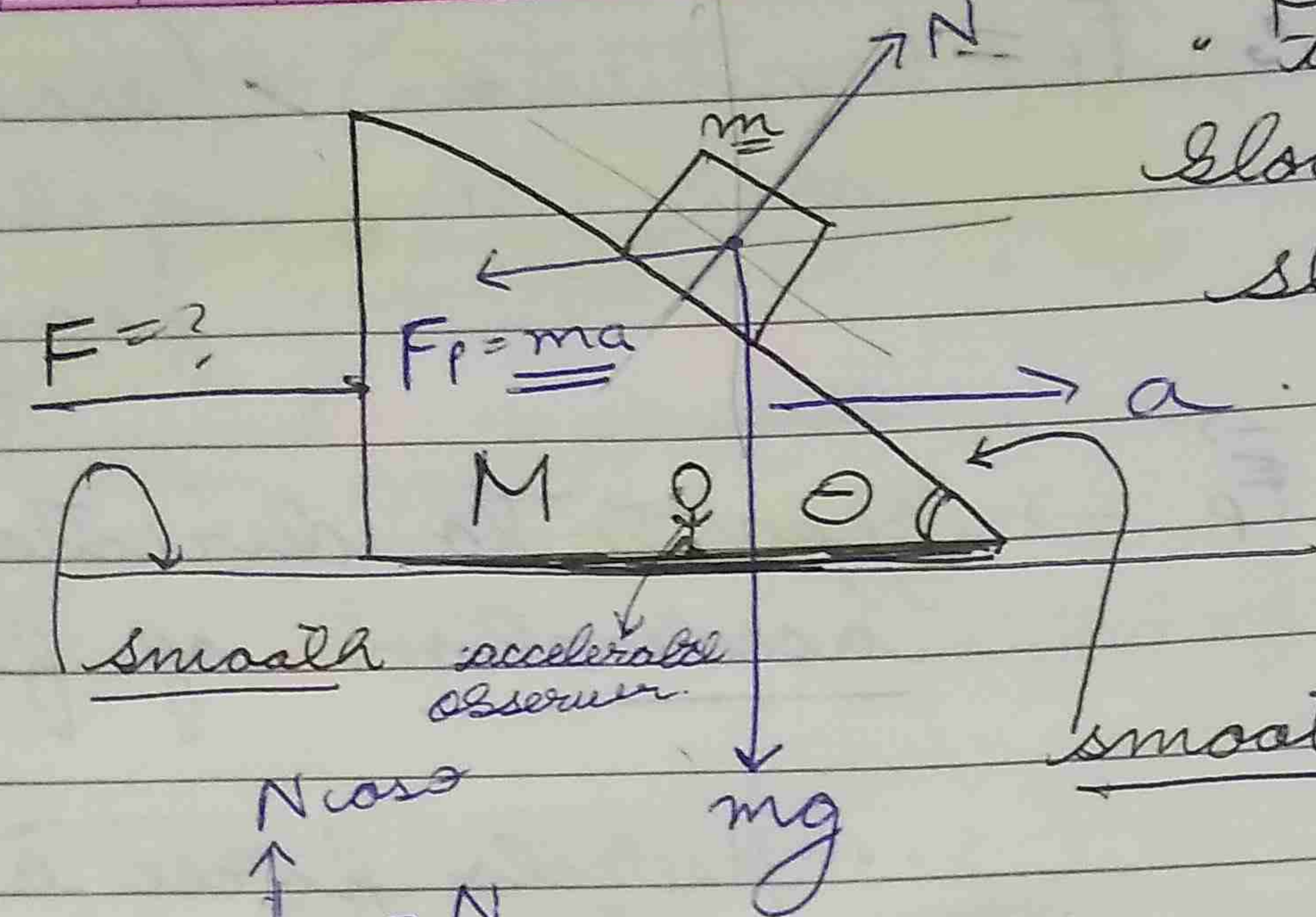
$T\sin\theta = ma$ (ii)

$\tan\theta = \frac{a}{g}$

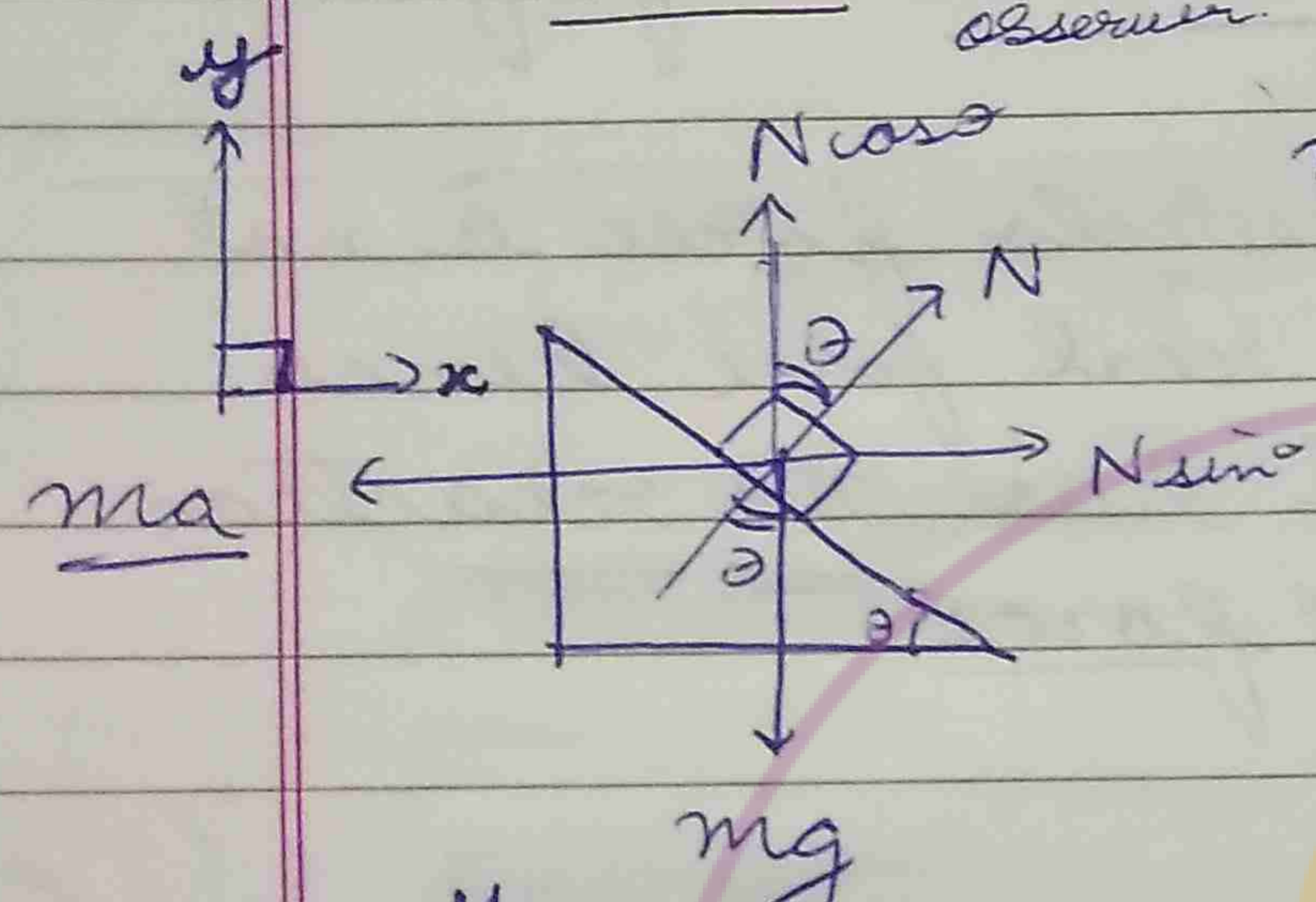
$\Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$

☆ Using non-inertial frame i.e. using Pseudo force will be case of statics and is easier as we just have to balance forces.

Ex



• Find F so that block does not slip on the wedge.

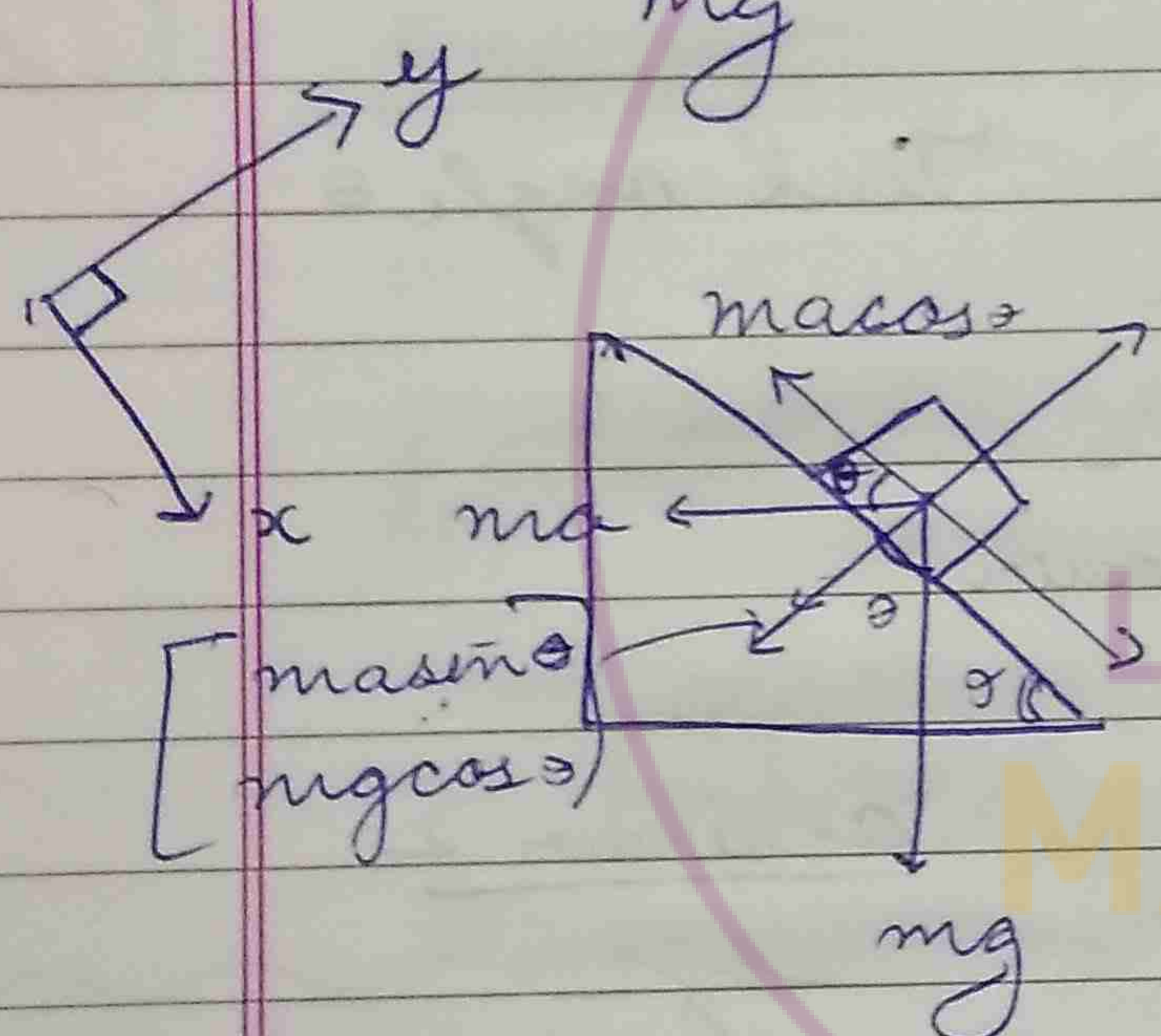


$N\cos\theta = mg$ (i)

$N\sin\theta = ma$ (ii)

$\Rightarrow \tan\theta = \frac{a}{g}$

$\Rightarrow \underline{a = g\tan\theta}$



$N = mg\cos\theta + ma\sin\theta$ (i)

$mg\sin\theta = ma\cos\theta$ (ii)

$\Rightarrow \boxed{a = g\tan\theta}$

$N = mg\cos\theta + mg\tan\theta \times \sin\theta$

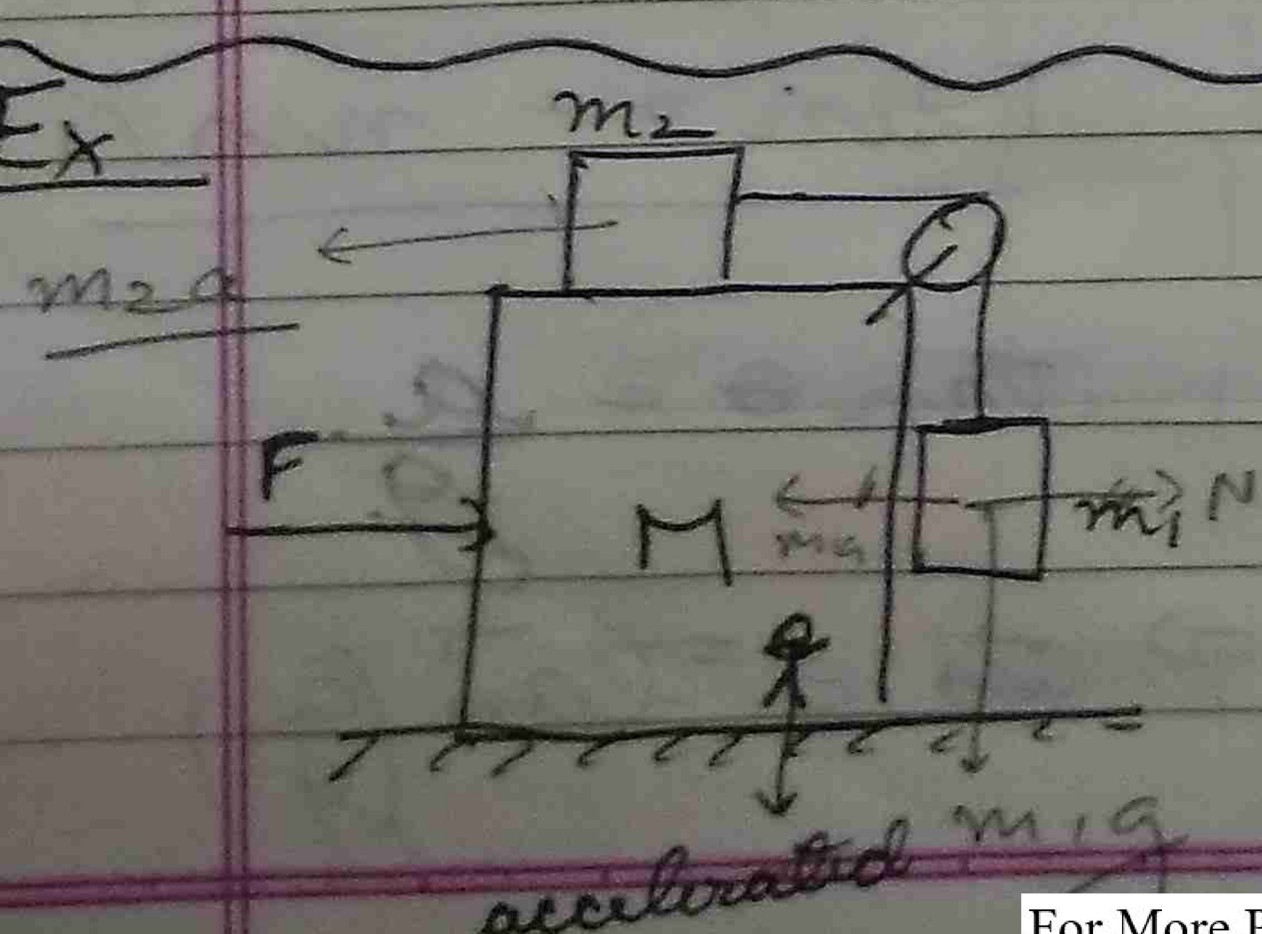
$= mg\frac{(\cos^2\theta + \sin^2\theta)}{\cos\theta}$

$\Rightarrow$ ~~$mg\cos\theta$~~

$\boxed{N = \frac{mg}{\cos\theta}}$

$\boxed{F_{required} = (M+m)g\tan\theta}$

Ex



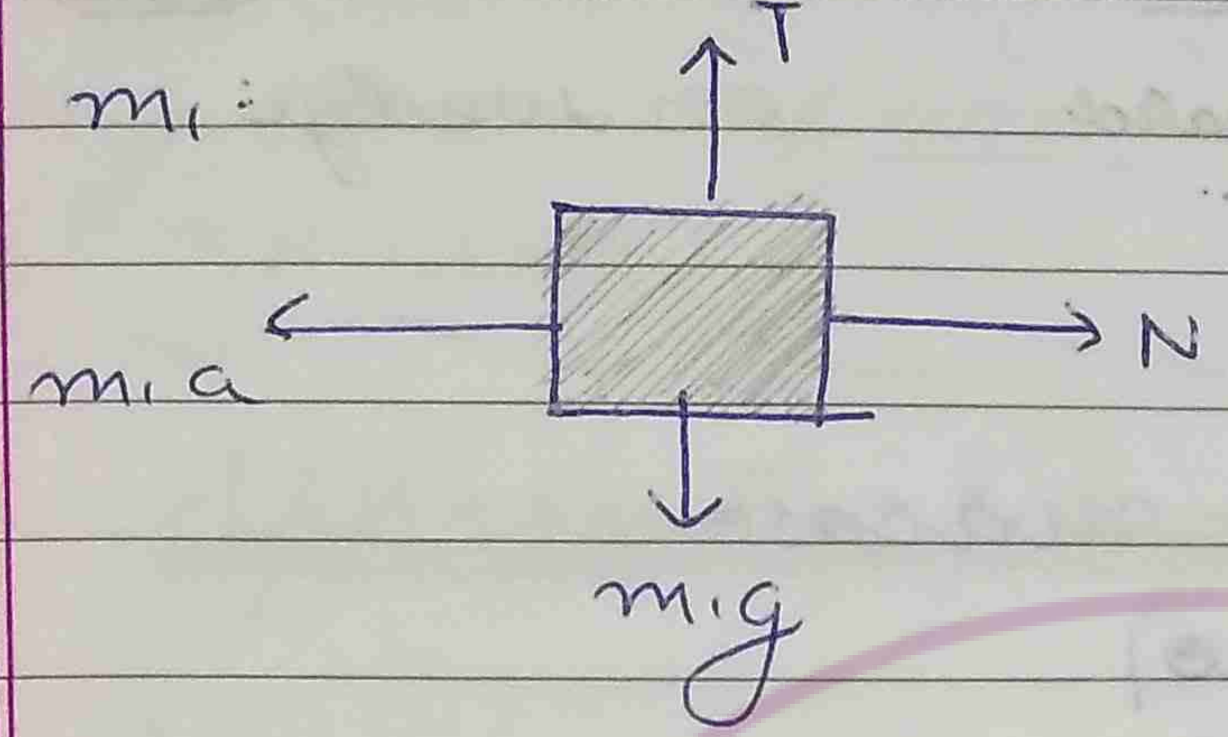
• Find F so that m_1 and m_2 are at rest w.r.t M.

- If driving forces are equal then the system will not move.
- If $a > g$ system will move upwards.

- **w.r.t M** m_1 and m_2 are at rest.

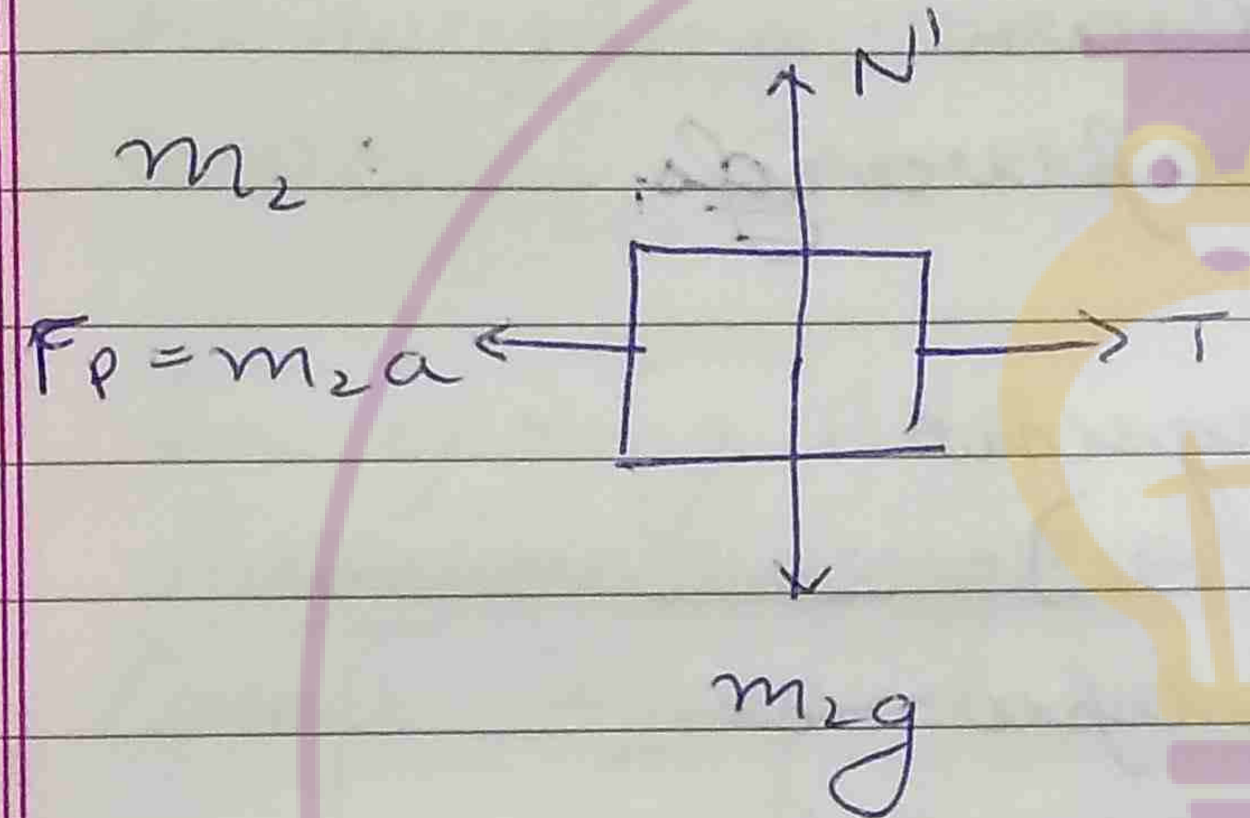
$\Rightarrow$ $F_{net} = 0$ on each

m_1:



$$T = m_1 g \quad (i)$$
$$N = m_1 a \quad (ii)$$

m_2:

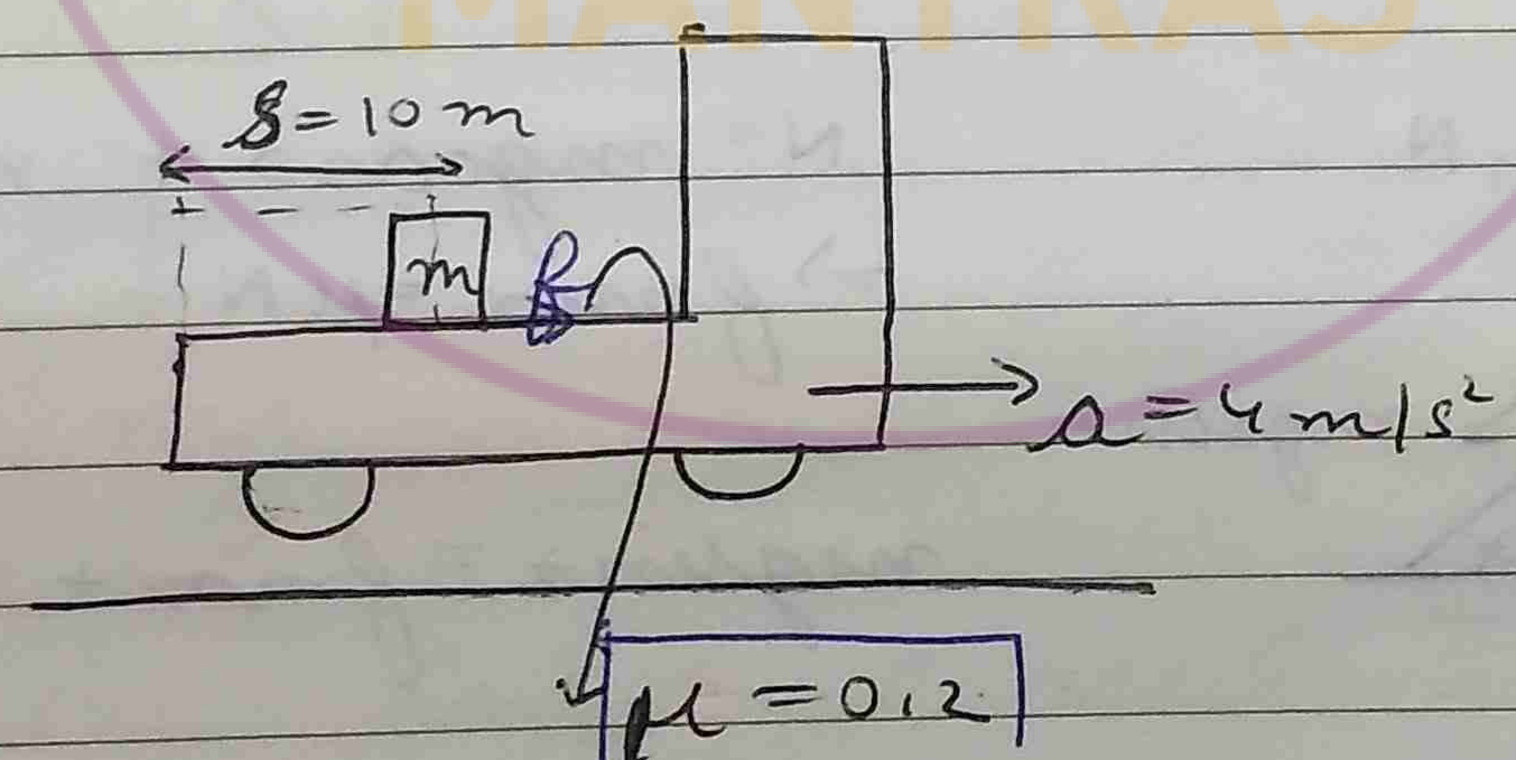


$$T = m_2 a \quad (iii)$$
$$N' = m_2 g \quad (iv)$$

$$m_1 g = m_2 a$$
$$\Rightarrow \boxed{a = \frac{m_1 g}{m_2}}$$

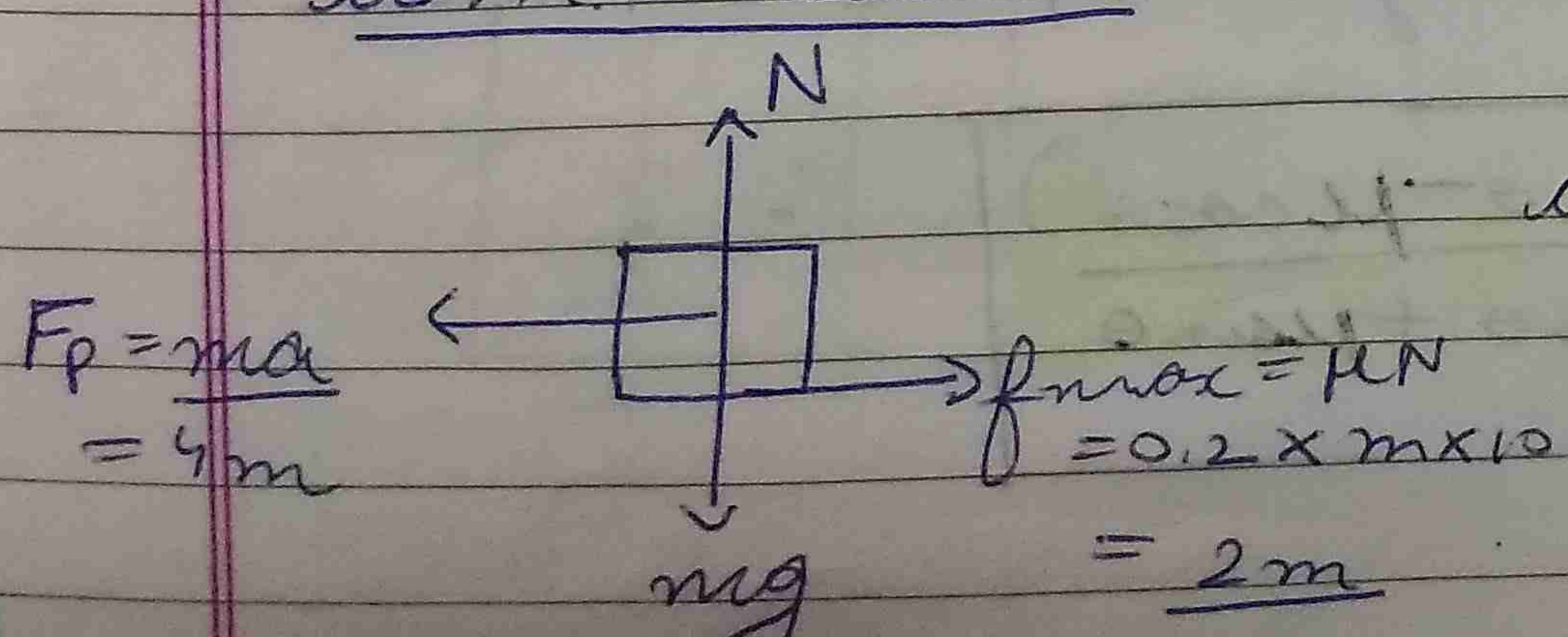
$$F = (M_1 + m_1 + m_2) a$$

★ **Ex**



$$a_{max} = \frac{\mu m g}{m} = \mu g$$
$$= 2\,m/s^2$$

- **w.r.t truck**

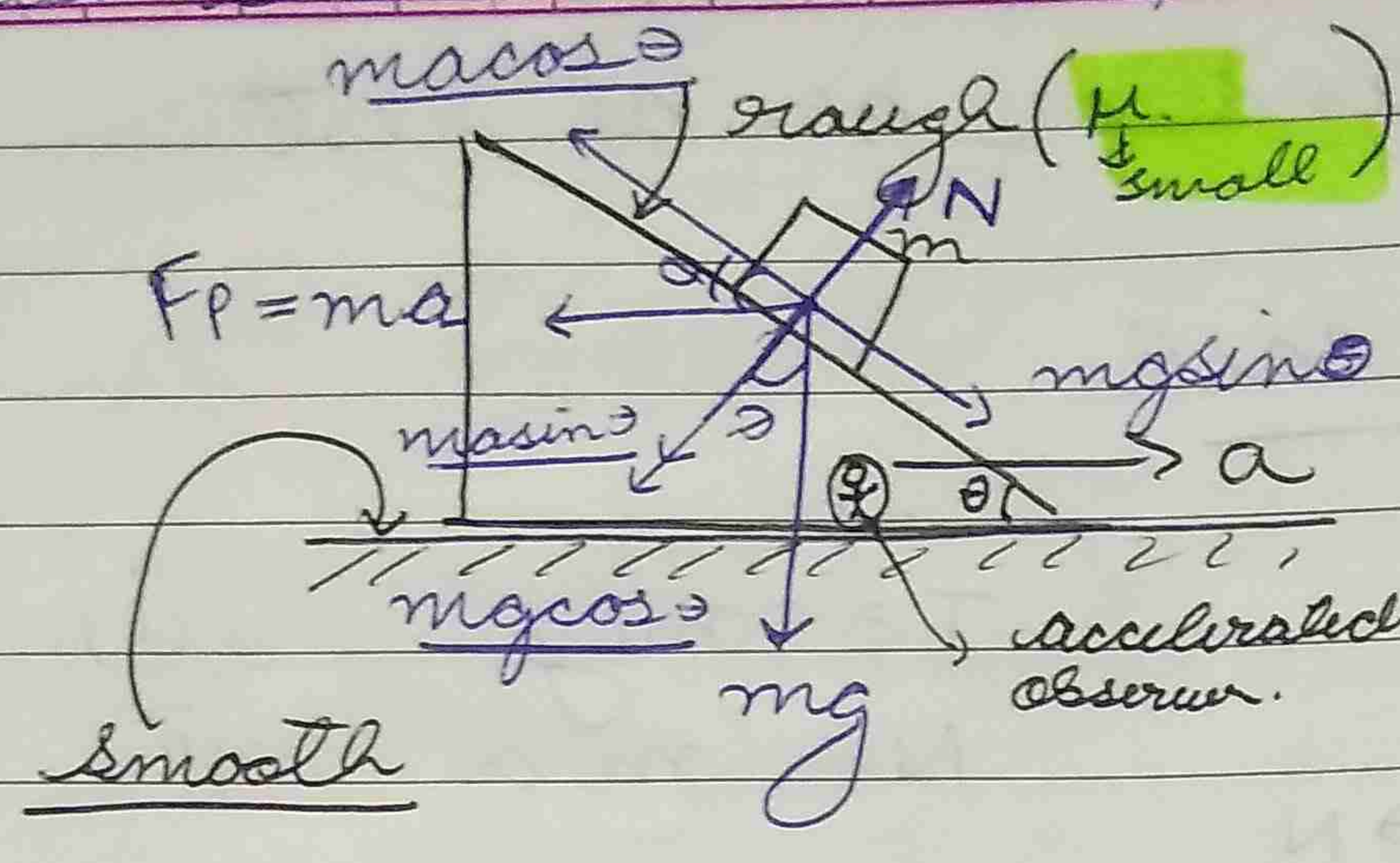


$$a_{block\ w.r.t\ truck} = \frac{4m - 2m}{m}$$
$$\Rightarrow 2\,m/s^2 \text{ (backward)}$$

$$s = ut + \frac{1}{2}at^2 = 10 = \frac{1}{2} \times 2\, t^2$$
$$\Rightarrow \boxed{t = 3.16 \text{ seconds}}$$

• Since motion is wrt accelerated observer or wedge hence w.r.t wedge net force on block should be zero.
• If a is high block tends to slip upward & if a is less block tends to slip downward.

Ex

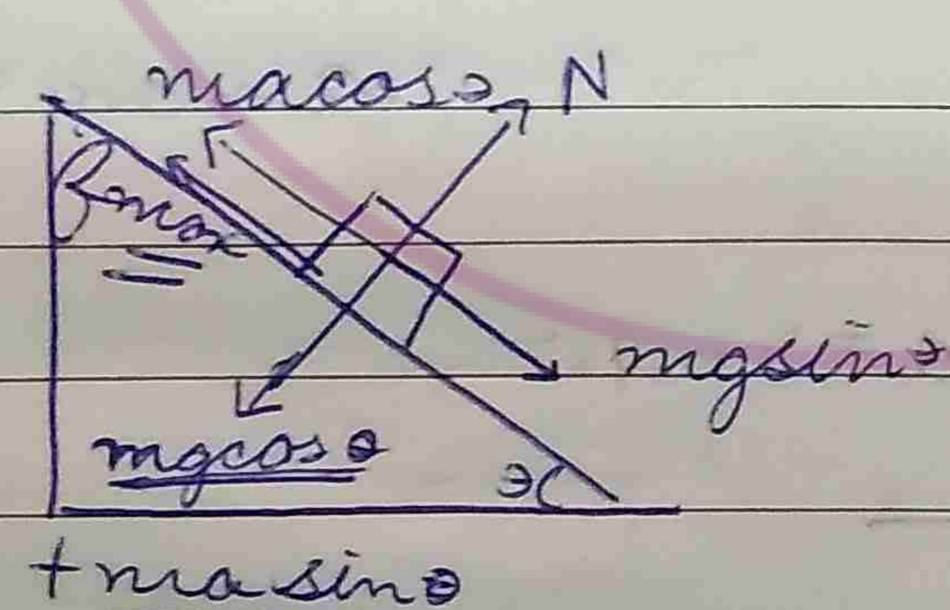


• Minimum and Maximum 'a' so that block does not slip on wedge.

Case 1 : $mg\sin\theta > ma\cos\theta$
or ($a < g\tan\theta$)
⇒ tendency to slip down.
⇒ friction will act upwards

Case 2 : $ma\cos\theta > mg\sin\theta$
(or $a > g\tan\theta$)
⇒ tendency to slip upward.
⇒ friction acts downward

For a_{min} ⇒ f up the plane.

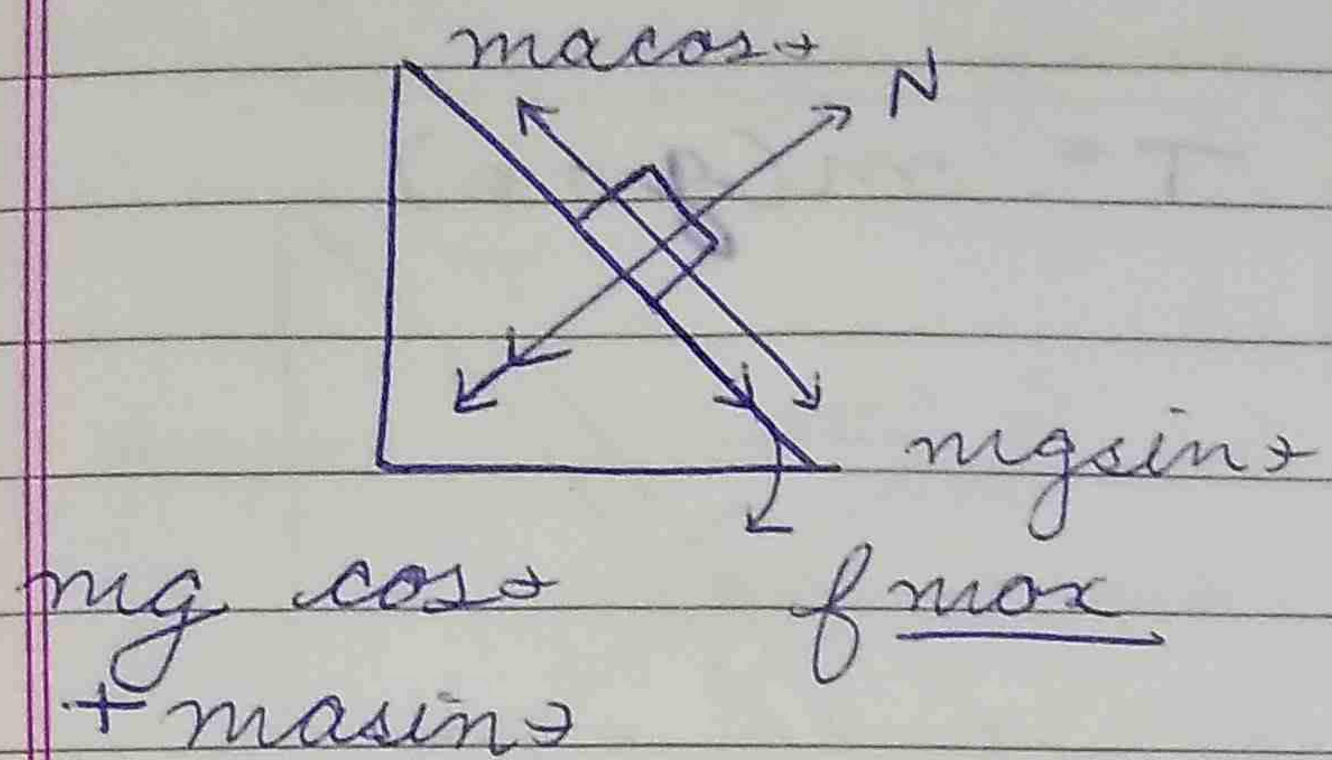


$N = mg\cos\theta + ma\sin\theta$
⇒ $f_{max} = \mu N$

$mg\sin\theta = f_{max} + ma\cos\theta$

⇒ $mg\sin\theta = \mu mg\cos\theta + \mu ma\sin\theta + ma\cos\theta$

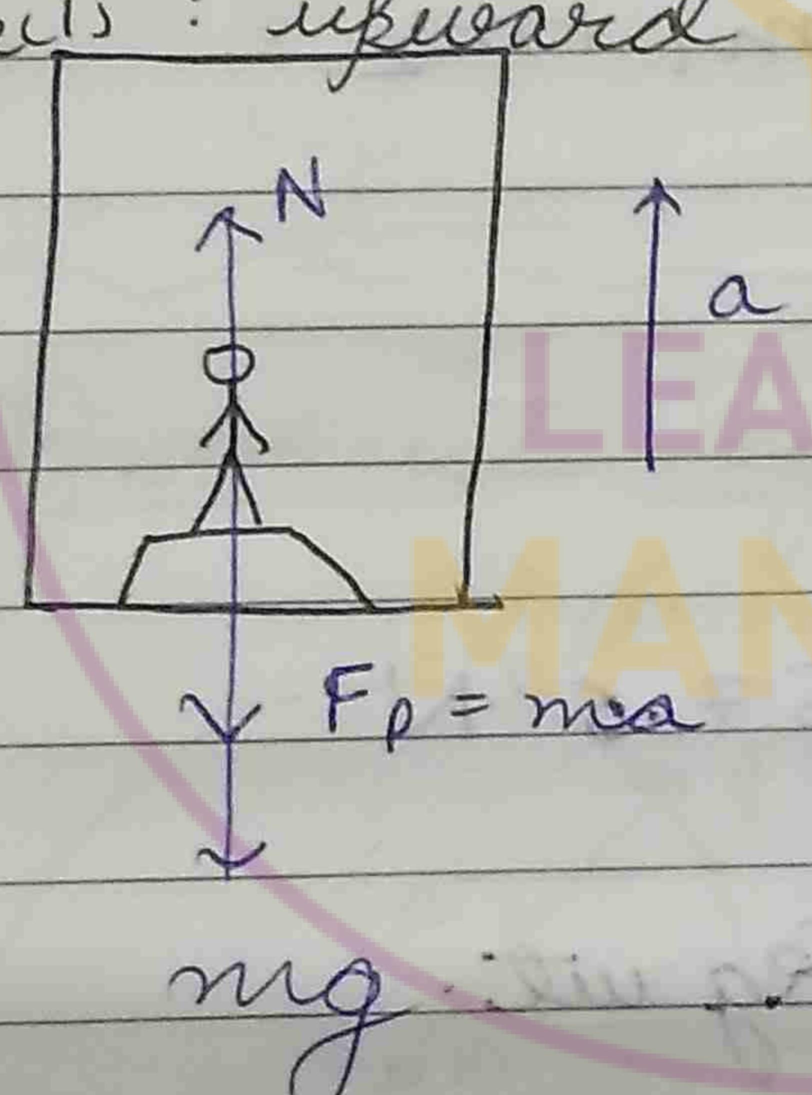
$$a_{min} = g\frac{(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta}$$



$$a_{max} = g\frac{(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$$

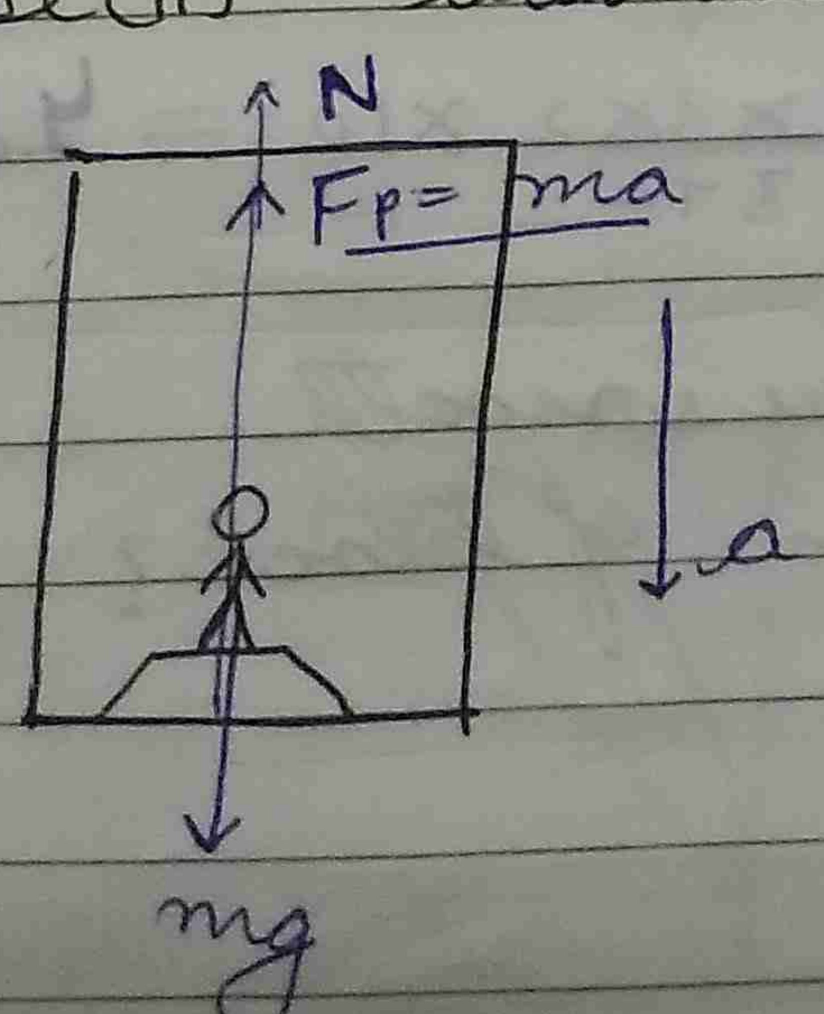
Apparent Weight

- In weighing machine we consider **normal reaction** to measure weight.
- In spring balance we use **tension** to measure weight.
- Case (i) : upward acceleration

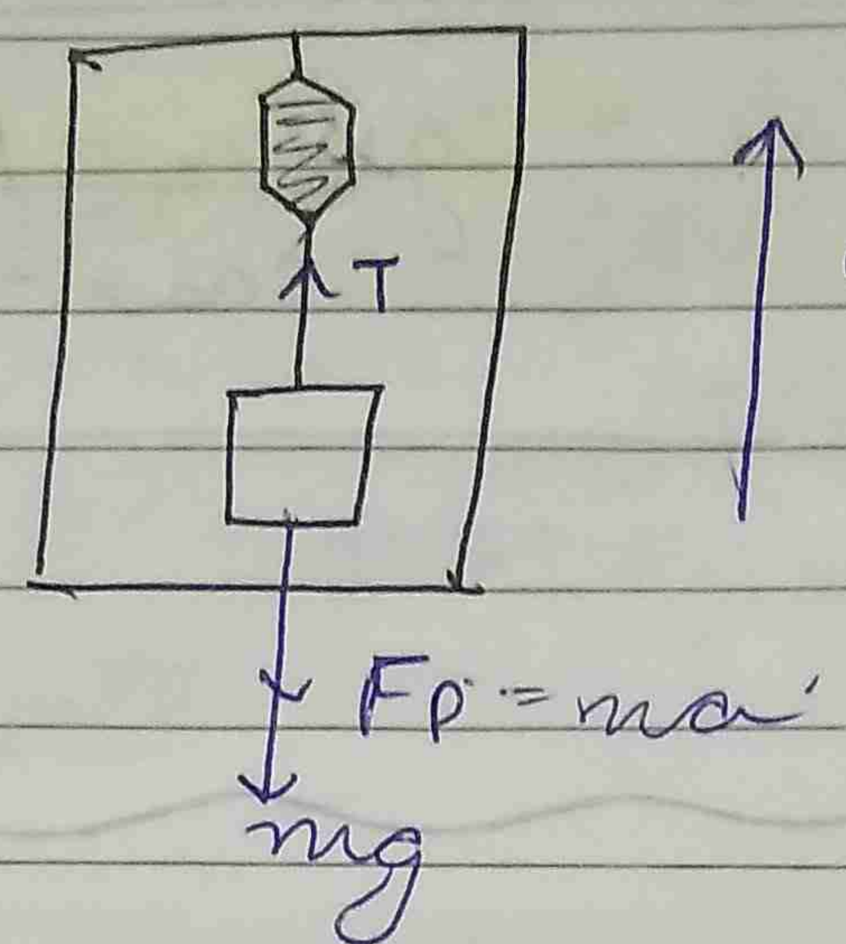


$N = mg + ma = m(g+a)$ → $(g+a)$ = $g_{effective}$

- Case (ii) : downward acceleration

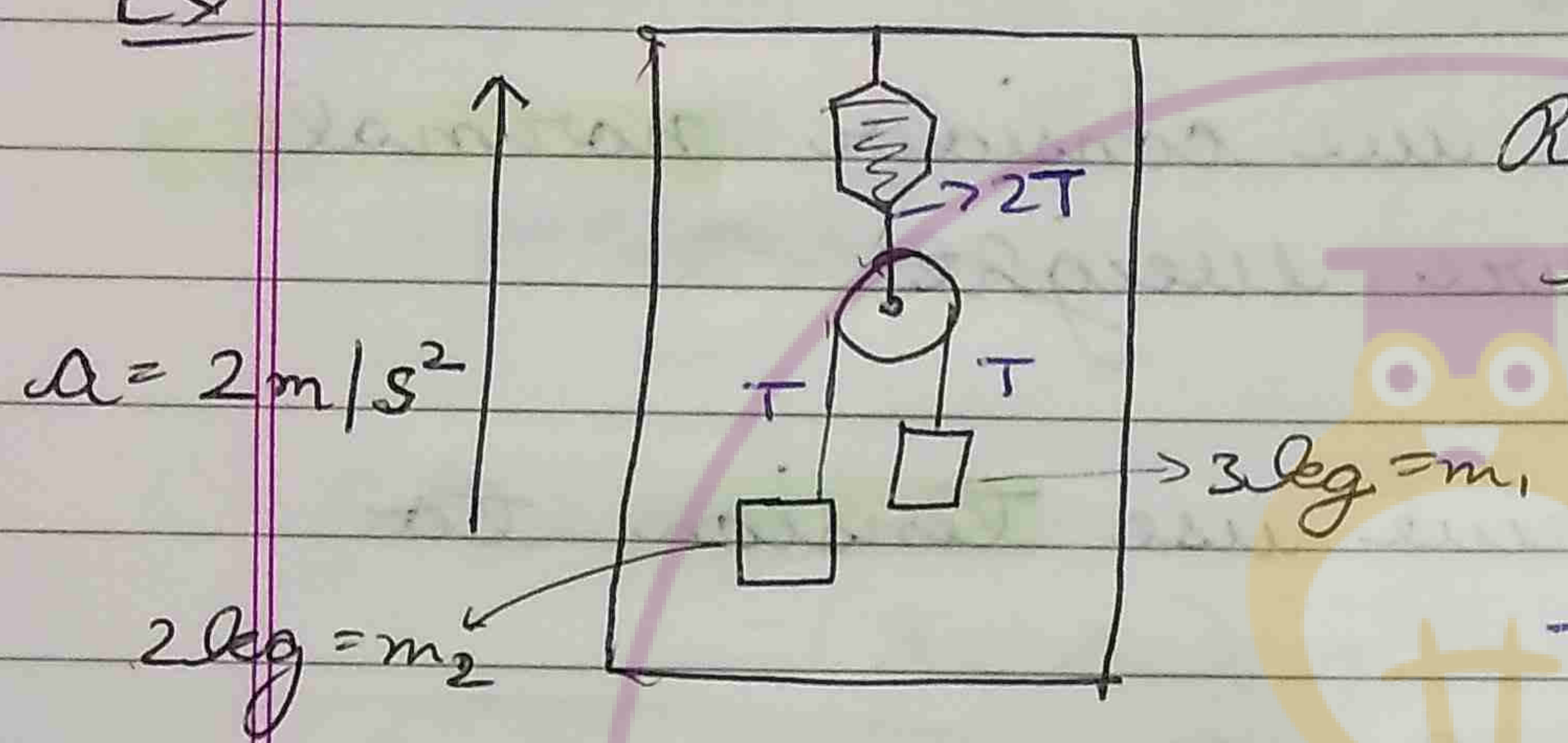


$N = mg - ma = m(g-a)$ → $(g-a)$ = $g_{effective}$



$T = m(g + a)$

Ex



Reading of the spring balance?

$T = \frac{2m_1 m_2}{m_1 + m_2} g_{effective}$

$T = \frac{2 \times 3 \times 2}{3 + 2} \times 12$

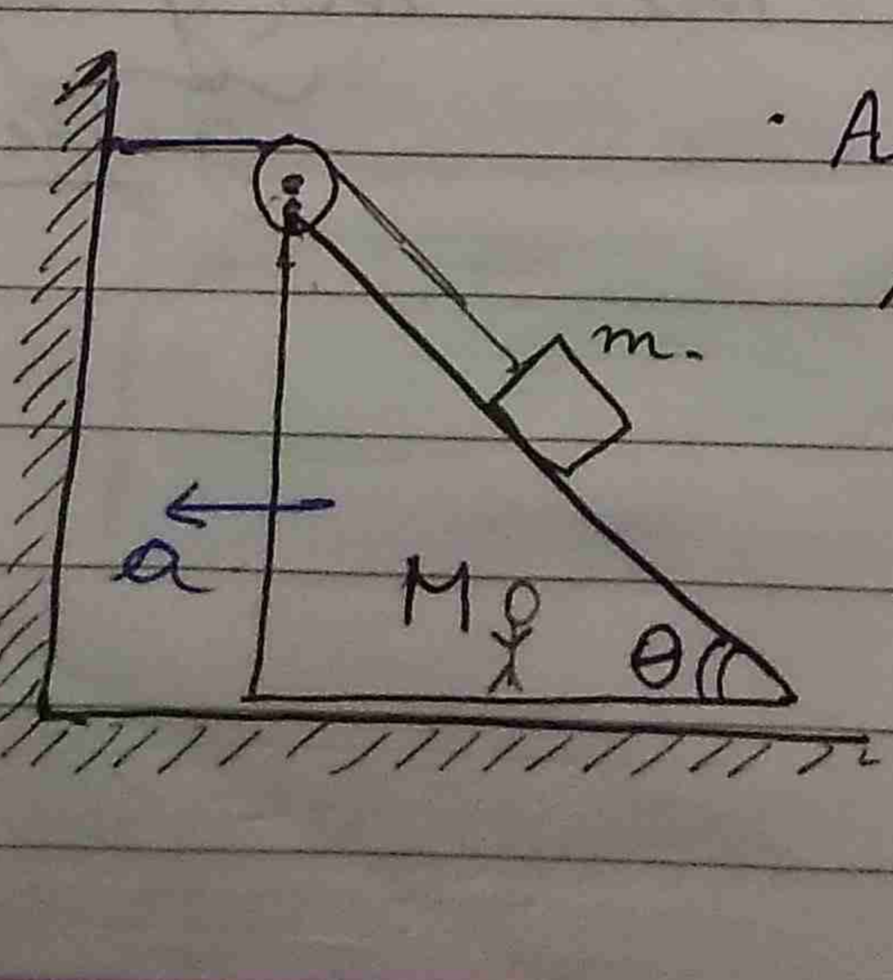
$= \frac{144}{5} = \underline{\underline{28.8\,N}}$

Reading $= 2 \times 28.8 = 57.6\,N$

kg wt $= \frac{57.6\,N}{10} = \underline{5.76}$ kg wt

If lift was at rest $\Rightarrow \frac{4 \times 3 \times 2}{3 + 2} \times \frac{10}{10} = 4.8$ kg wt

Ex

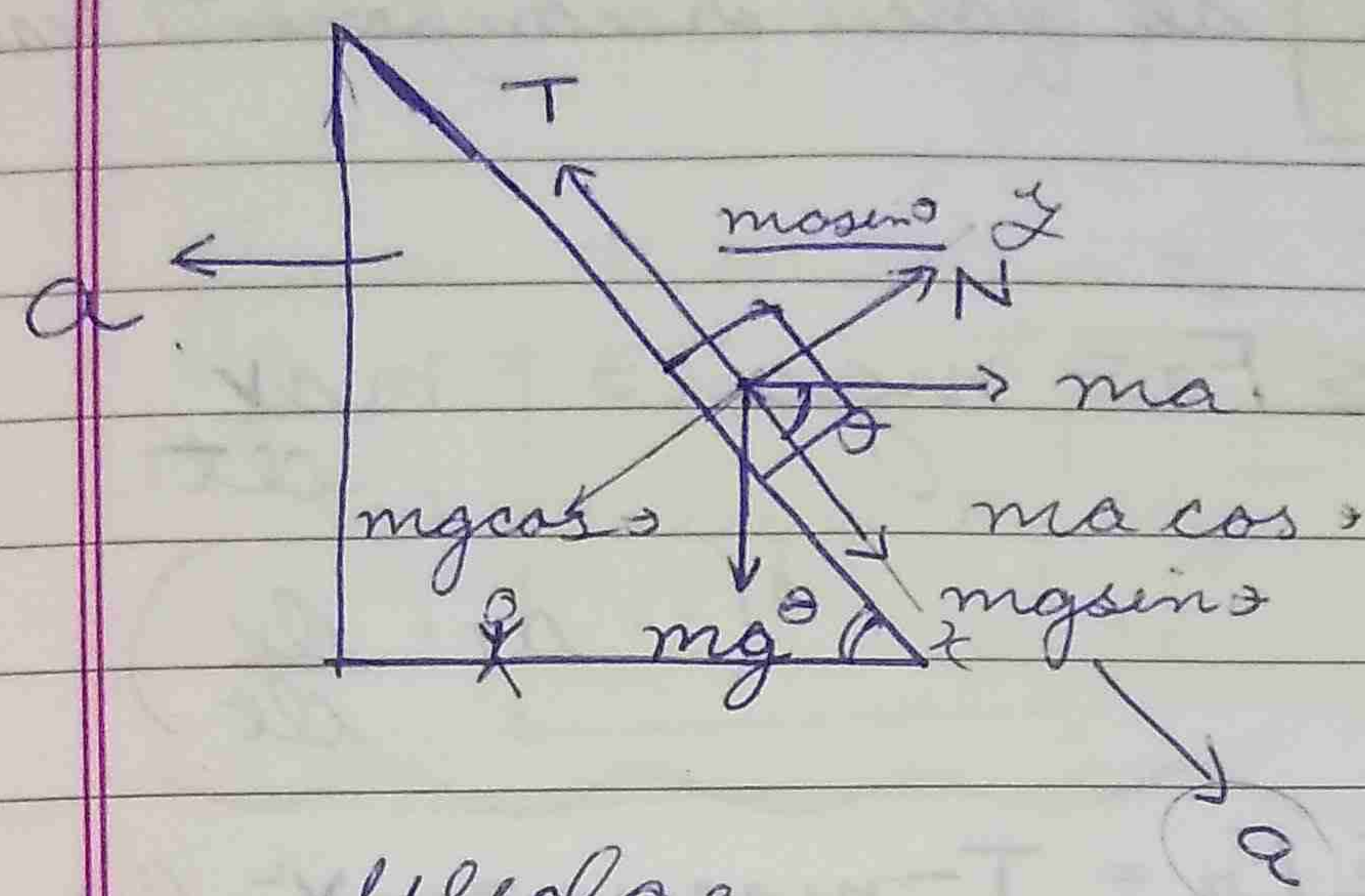


- All surfaces are smooth.

Acceleration of block?

∵ distance moved by wedge is same as that moved by block, hence both have same acceleration.

- In wedge frame

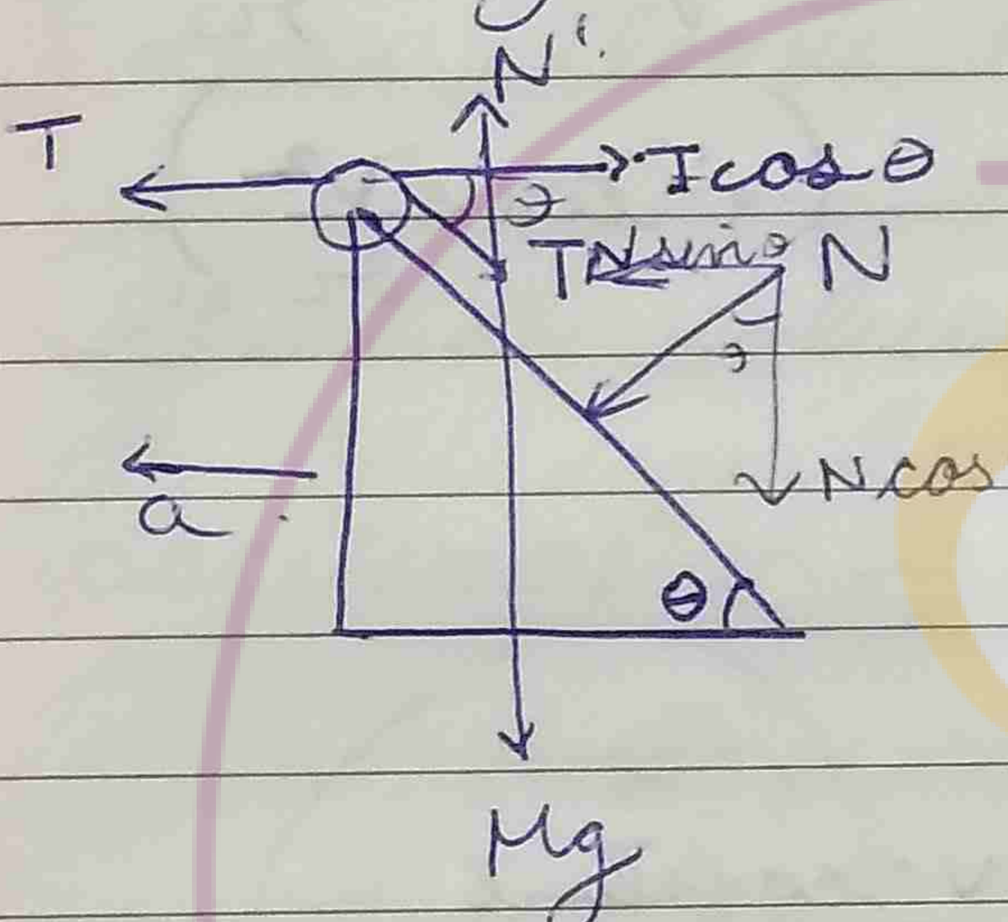


Block

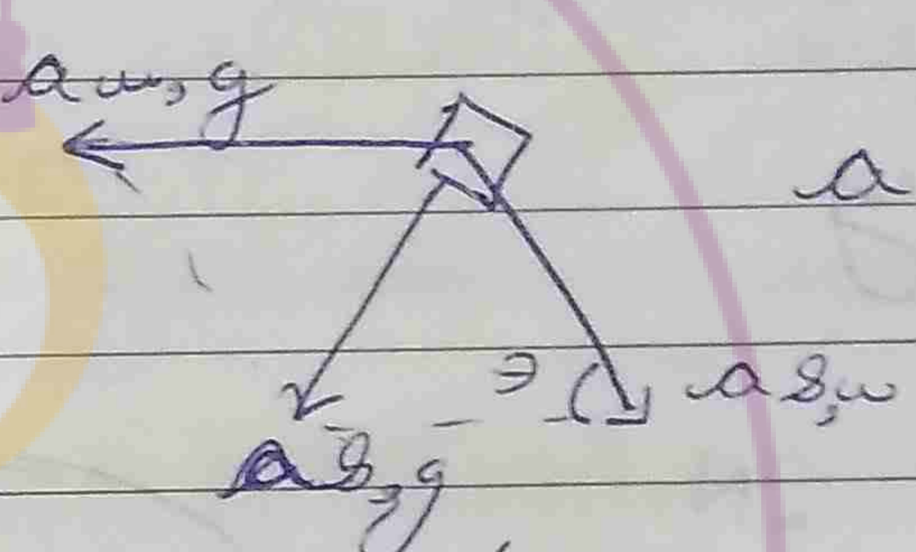
$mg\sin\theta + ma\cos\theta - T = m.a$ (i)

$N + ma\sin\theta = mg\cos\theta$ (ii)

Wedge



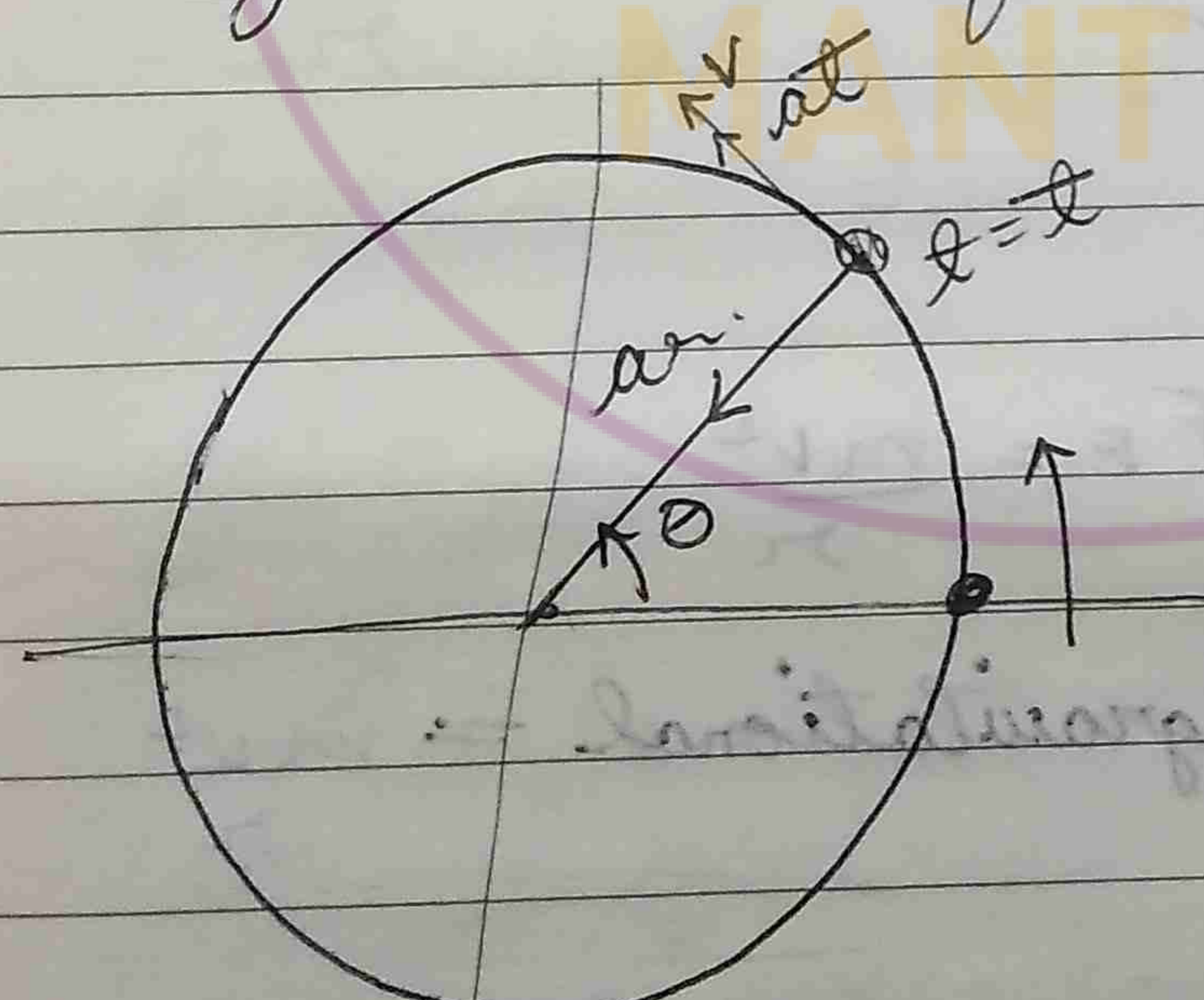
$T + N\sin\theta - T\cos\theta = ma$ (iii)



$a = - \div - -$

$(\because a_{bg} = a_{bw} + a_{wg})$

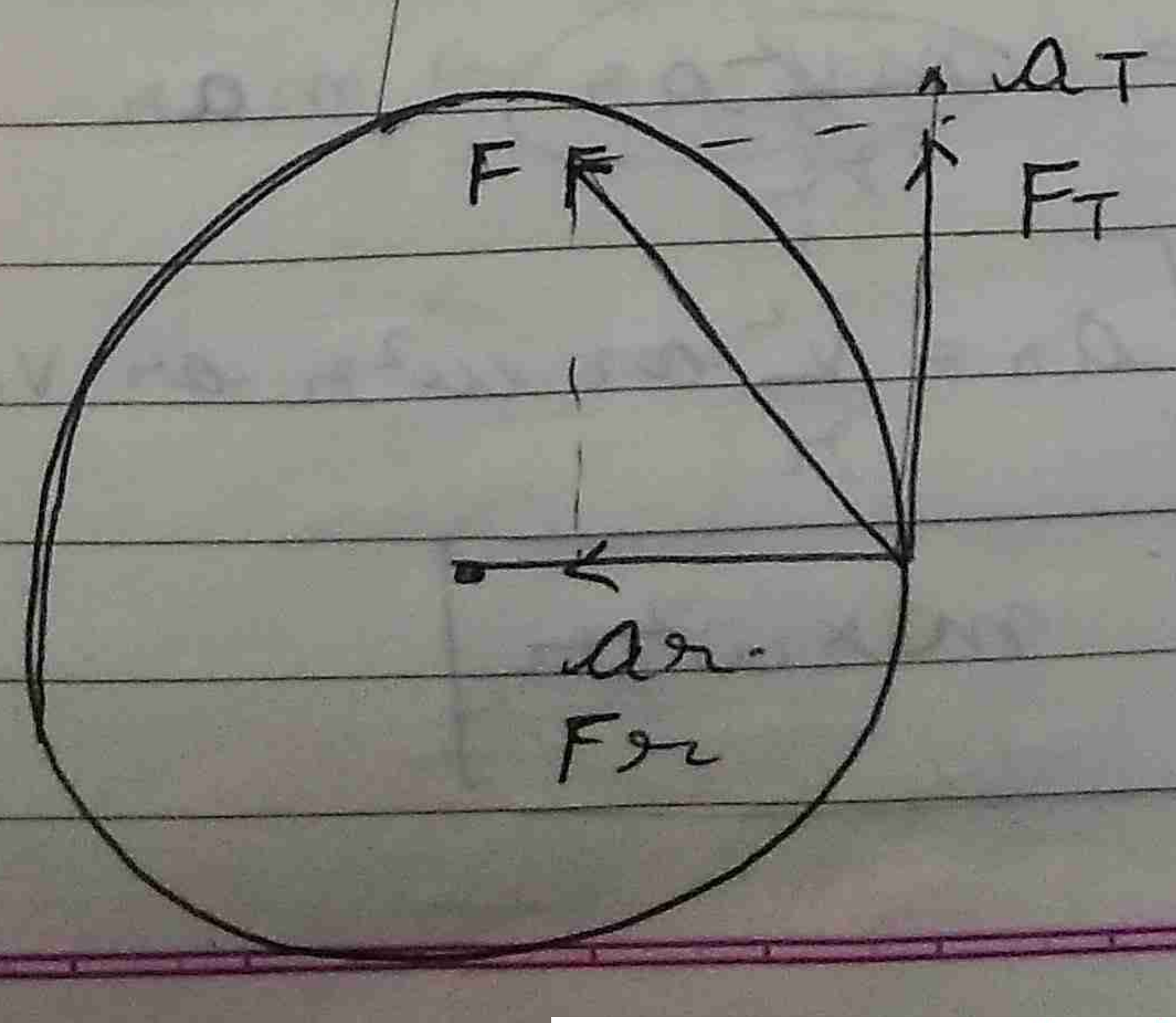
- Dynamics of circular motion



UCM (uniform circular motion)
⇒ v = constant

NCM → v = variable

a_T → changes speed
a_r → changes direction



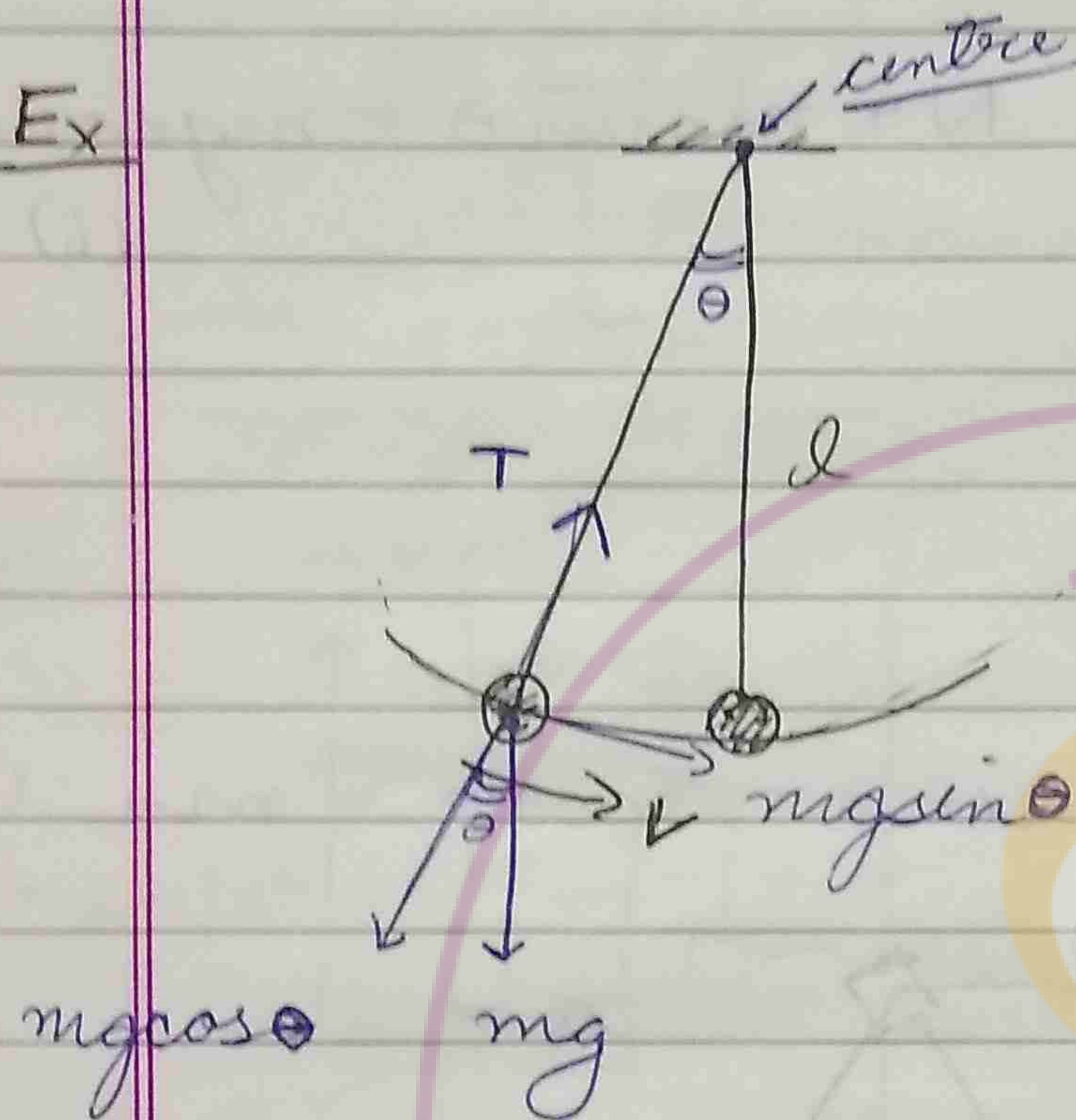
F_{net} → $F_T = ma_T$
F_{net} → $F_r = ma_r$

• radial direction towards the centre is +ve. and away from centre is −ve.

$\Sigma F_T = ma_T$ — all forces or components tangentially

$\Sigma F_r = ma_r$ — all forces or components radially

Ex



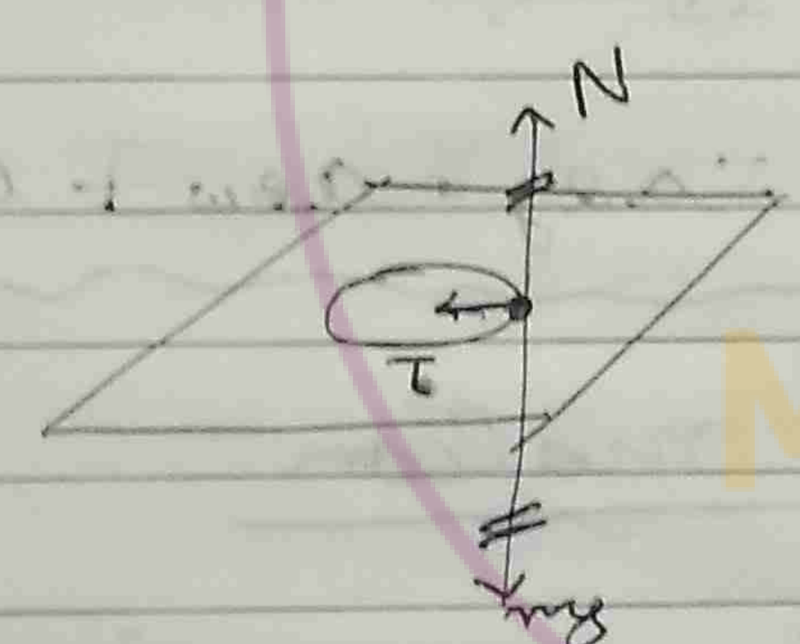
$\Sigma F_T = mg\sin\theta = m\frac{dv}{dt}$

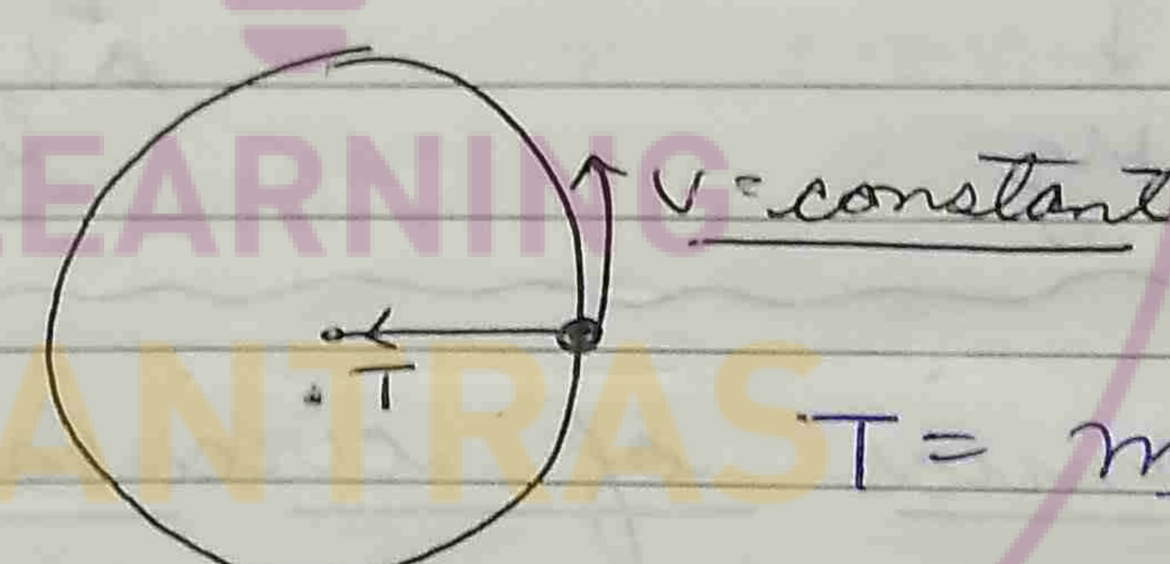
$\left(\because a_T = \frac{dv}{dt}\right)$

$\Sigma F_r = T - mg\cos\theta = \frac{mv^2}{r}$ (ii)

$\left(\because a_r = \frac{v^2}{r}\right)$

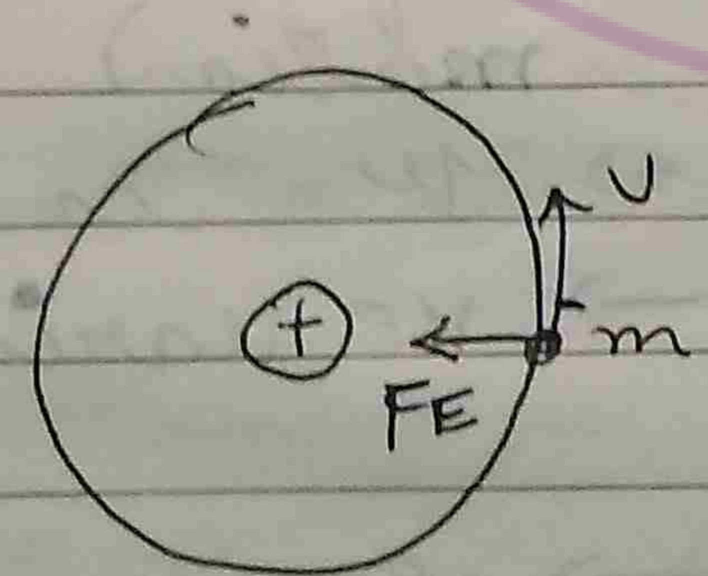
Ex





$T = \frac{mv^2}{r}$

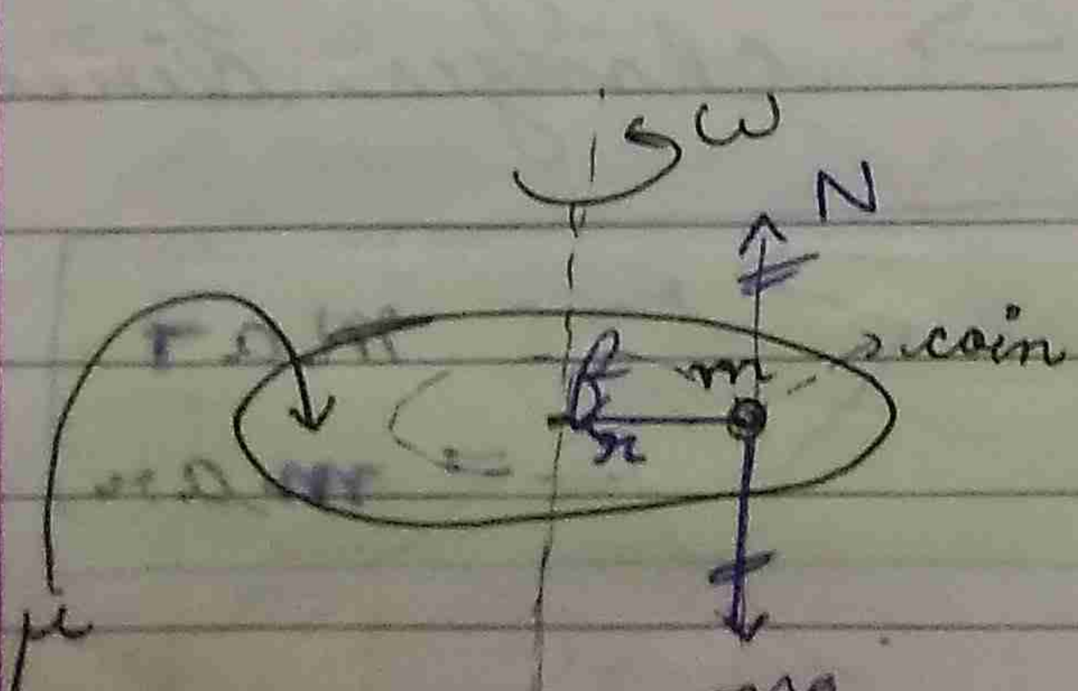
Ex



$F_E = \frac{mv^2}{r}$

$F_{gravitational} = \frac{mv^2}{r}$

Ex
Disc



$f = \frac{mv^2}{r}$ $a_r \Rightarrow ma_r$

$\left[a_r = \frac{v^2}{r} \text{ or } \omega^2 r \text{ or } v.\omega\right]$

$\boxed{f = m \times \omega^2 r}$

$\boxed{N = mg}$

• Centripetal force is not a special force. It is just summation of components of all other forces towards the centre.

• What can be ω_{max} so that the coin does not slip?

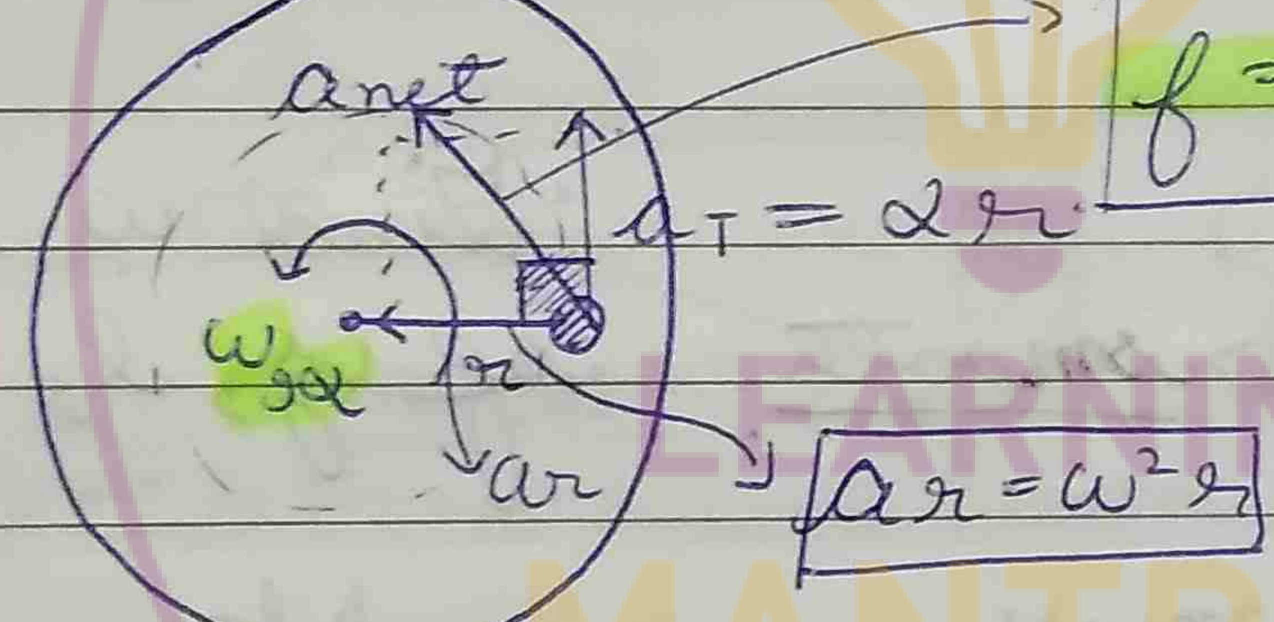
$f_{max} = m\omega_{max}^2 r = \mu mg = m\omega_{max}^2 r$

$\Rightarrow \boxed{\omega_{max} = \sqrt{\frac{\mu g}{r}}}$

Q ☆ Disc starts rotating from rest with angular acceleration α.

At what value of ω, slipping of coin begins?

In this case both radial and tangential acceleration is provided by friction.



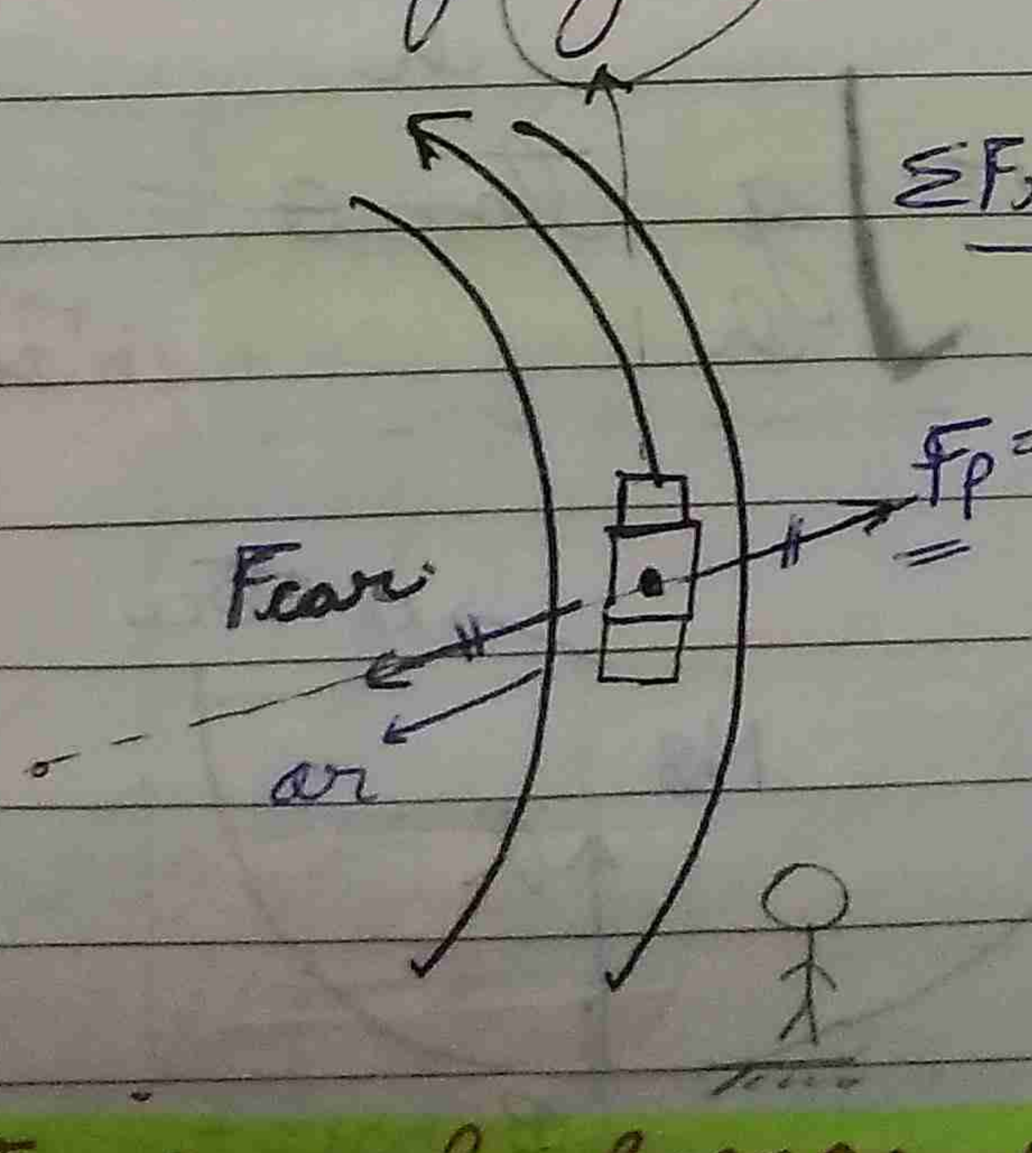
$\boxed{f = ma_{net}}$

$f = m\sqrt{(\omega^2 r)^2 + (\alpha r)^2}$

$f_{max} = ma_{max} \Rightarrow \mu mg = m\sqrt{\omega^4 r^2 + \alpha^2 r^2}$

$\omega = \omega_{max} = \text{-----}$

• Centrifugal (away) Forces



Earth observer

$\Sigma F_r = F_{car} = \frac{mv^2}{r}$

$F_p = \frac{mv^2}{r}$ Car observer

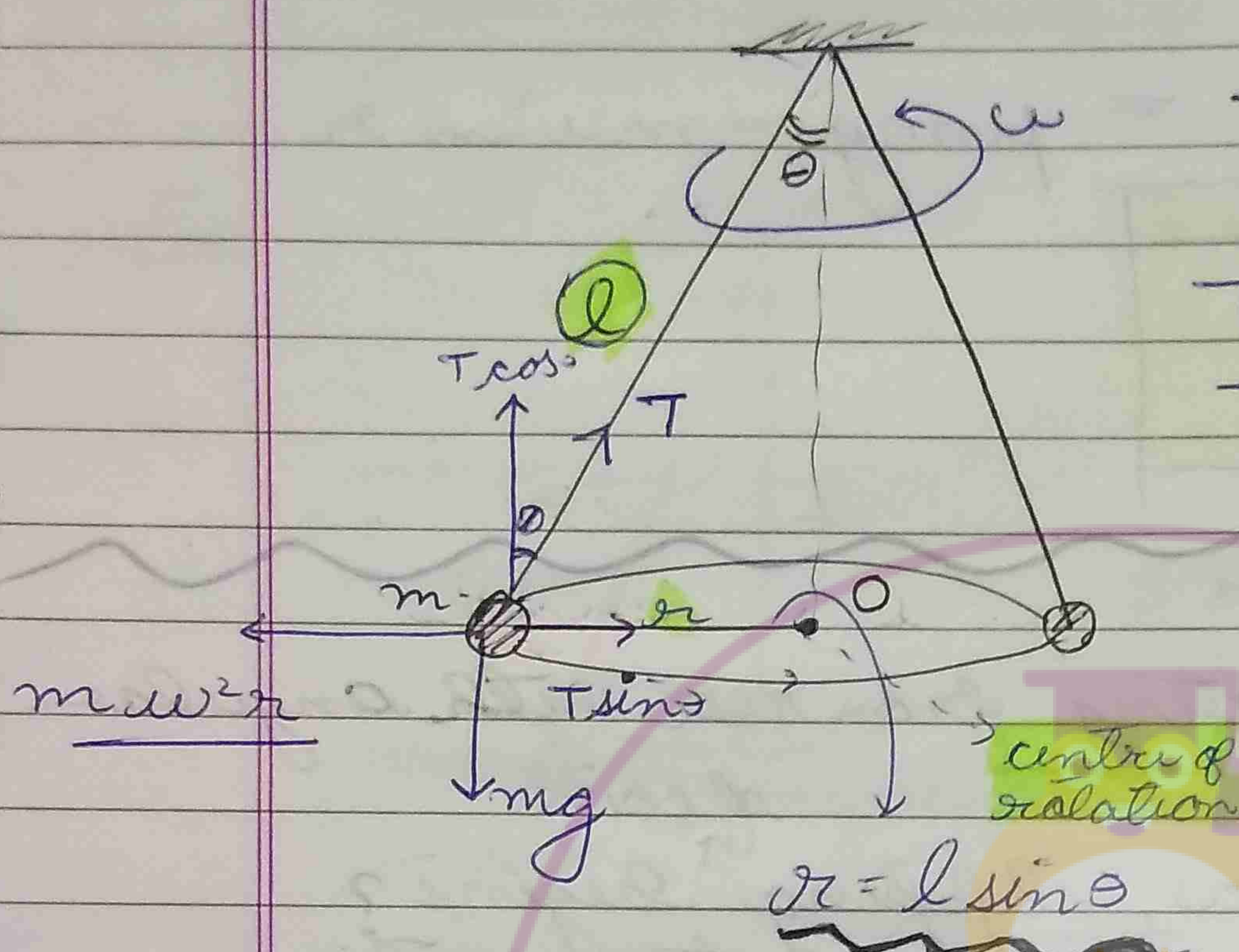
$\boxed{F_{car} = F_p = \frac{mv^2}{r}}$

pseudo force

☆ Centrifugal force is shown away from centre

- In case of centrifugal force ; first show the true force and then show centrifugal force away from centre

- ## Conical pendulum



$\omega = ?$

$T = ?$

$T\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta}$

$T\sin\theta = m\omega^2 r = m\omega^2 l \sin\theta$

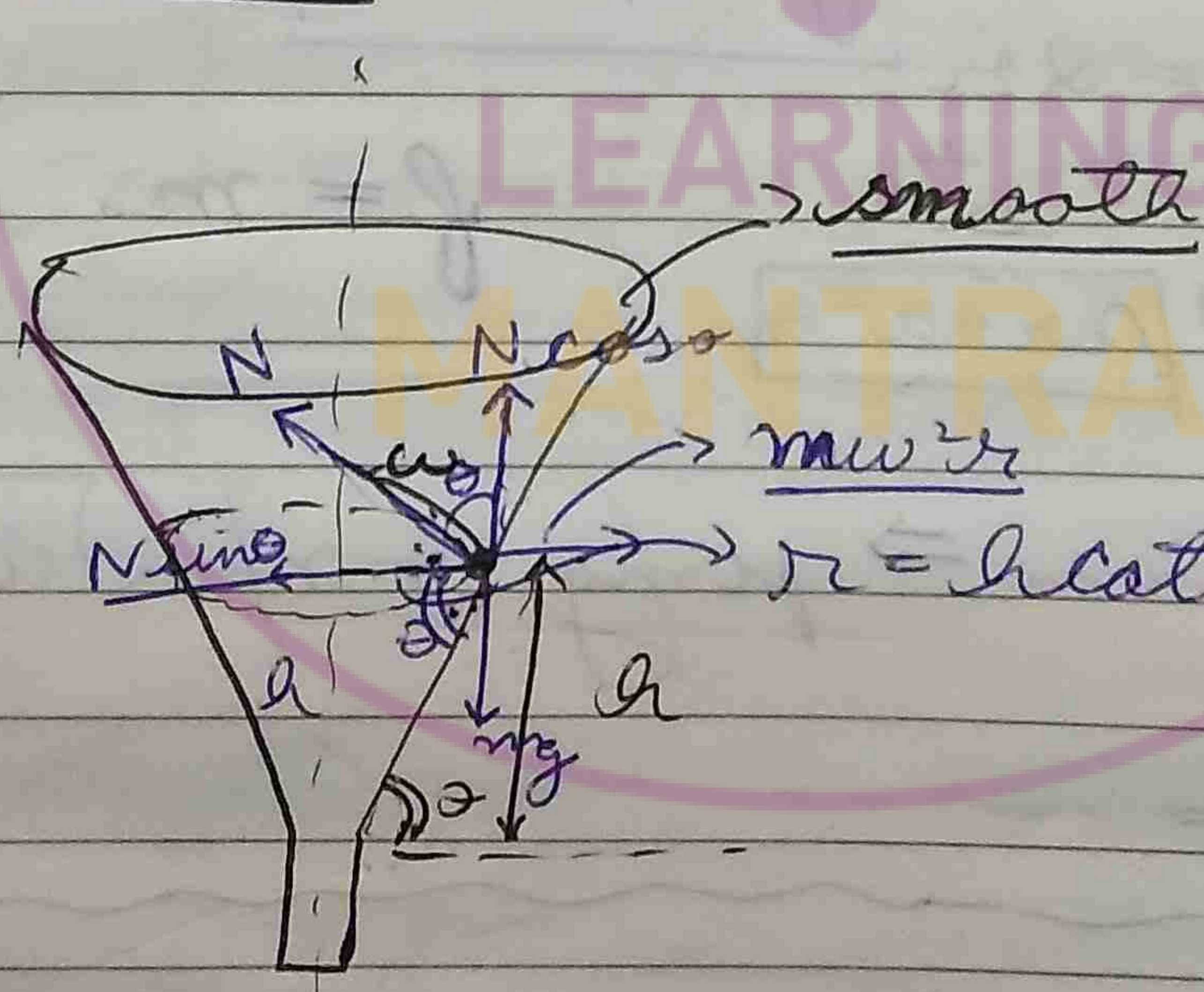
$\Rightarrow \frac{mg}{\cos\theta} = m\omega^2 l \Rightarrow \omega = \sqrt{\frac{g}{l\cos\theta}}$

$r = l\sin\theta$

∴ as ω increases θ will increase

- ## Funnel

Find ω in terms of g, l, θ.



$N\cos\theta = mg$

$N\sin\theta = m\omega^2 r$

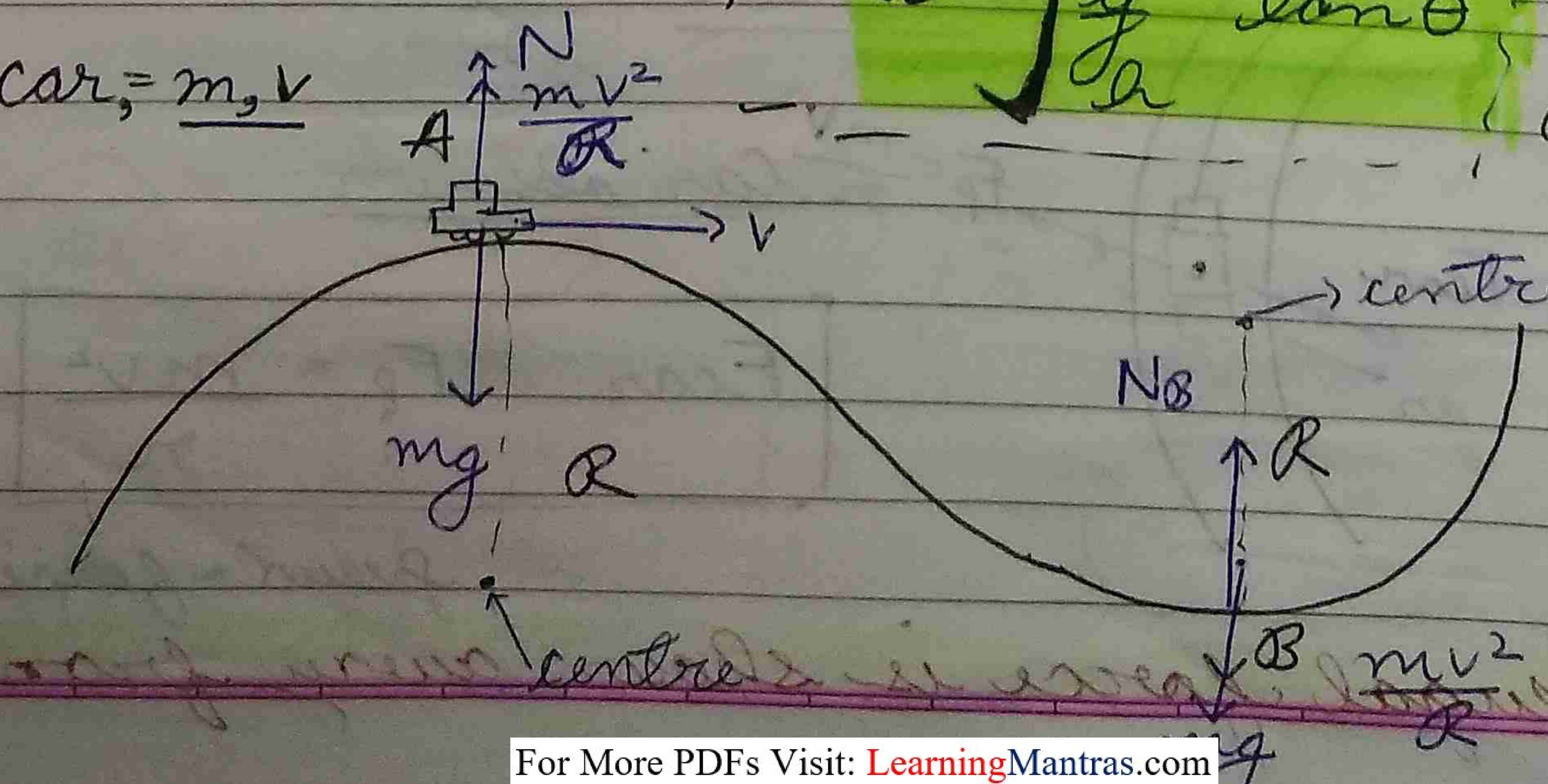
$\Rightarrow \tan\theta = \frac{\omega^2 r}{g} = \frac{\omega^2 h\cot\theta}{g}$

$\Rightarrow \omega^2 = \frac{g}{h}\tan^2\theta$

$\Rightarrow \omega = \sqrt{\frac{g}{h}}\tan\theta$

Ex car, = m, v

(i) Find N_A, N_B



☆ • at A smaller the radius of curvature (R) lesser the speed required to loose contact.

• On a plain road, the car will never loose contact. ∵ $r = \infty$ (infinite)

(vi) Which of the two can become zero and when?

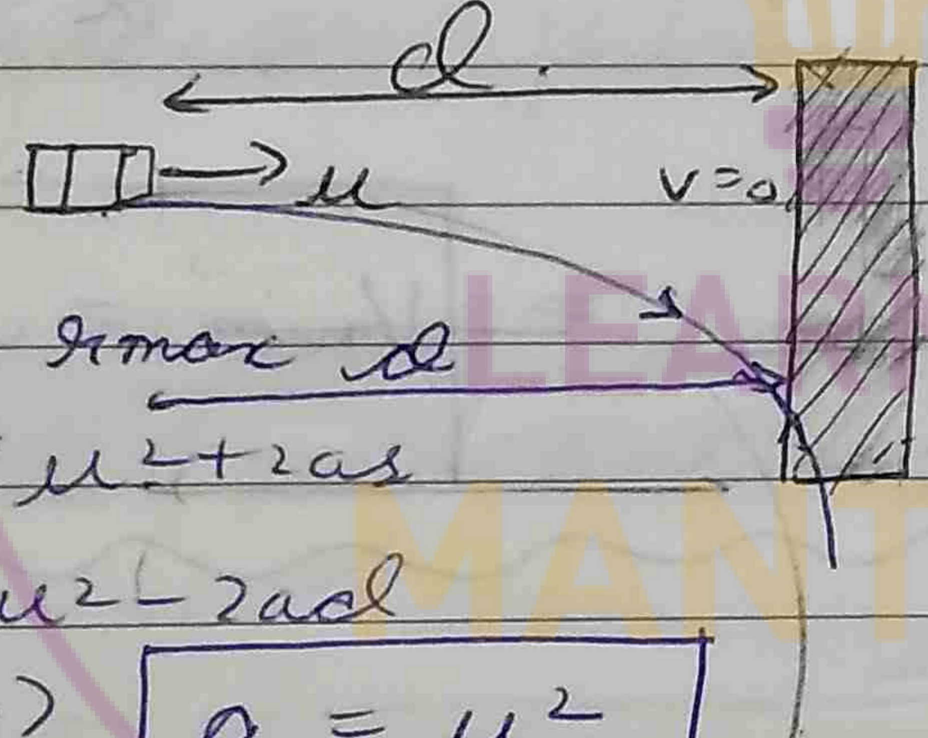
$N_A + \frac{mv^2}{R} = mg$ ⟹ $N_A = mg - \frac{mv^2}{R}$

$N_B = \frac{mv^2}{R} + mg$ The greater the normal reaction greater will be the weight

(ii) $N_A = 0 = mg - \frac{mv^2}{R}$ or ☆ $v = \sqrt{gR}$ — speed at which body looses contact.

$v \geq \sqrt{gR}$

Ex



Which is safer.
(i) Apply brakes
(ii) take turn

(i) $v^2 = u^2 + 2as$

$0 = u^2 - 2ad$

⟹ $a = \frac{u^2}{2d}$ retardation required to stop.

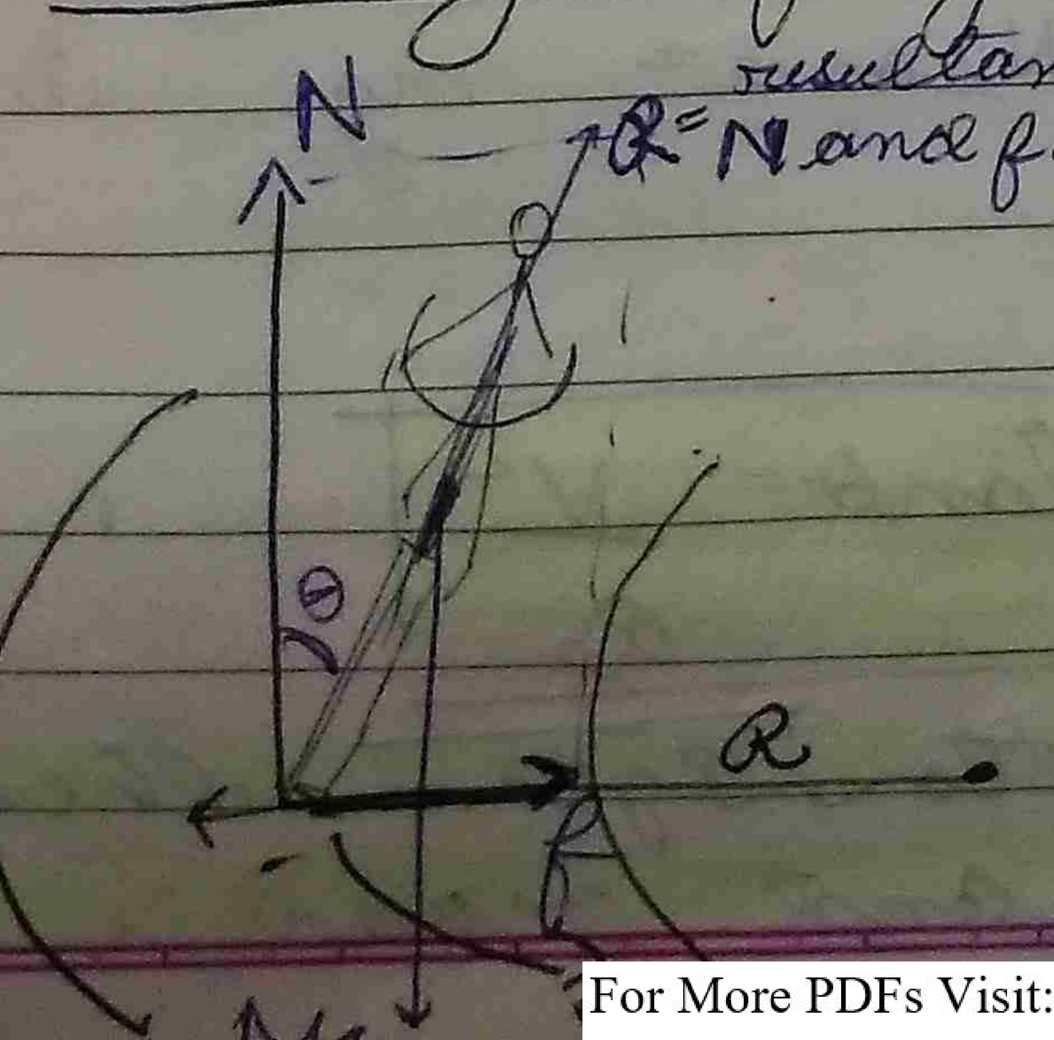
(ii) $r_{max} = d$

$a = \frac{u^2}{r}$ ⟹ $a_{min} = \frac{u^2}{d}$

→ This requires less friction hence safer.

(iii) Brakes + turn = worst case scenario

• Bending of cyclist



$R \sin\theta = f = \frac{mv^2}{R}$

$R \cos\theta = N = mg$

⟹ $\tan\theta = \frac{v^2}{Rg}$ ⟹ $\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$

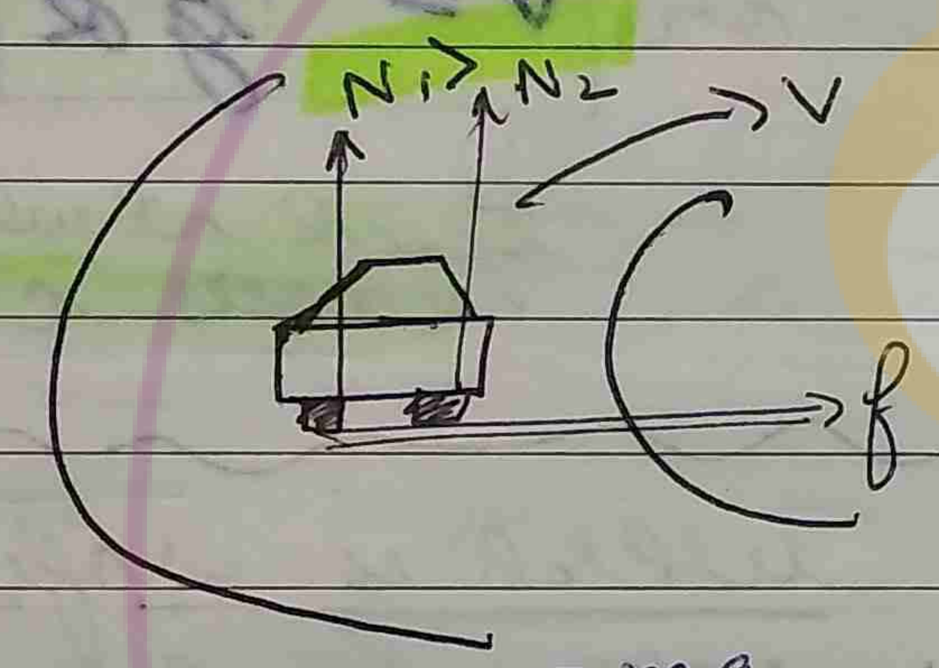
maximum safe velocity

$$f_{max} = \frac{m V_{max}^2}{R}$$

$$\mu N = \frac{m V_{max}^2}{R} \quad (N = mg)$$

$$\rightarrow \boxed{V_{max} = \sqrt{\mu R g}}$$

- ## Taking a turn by a car

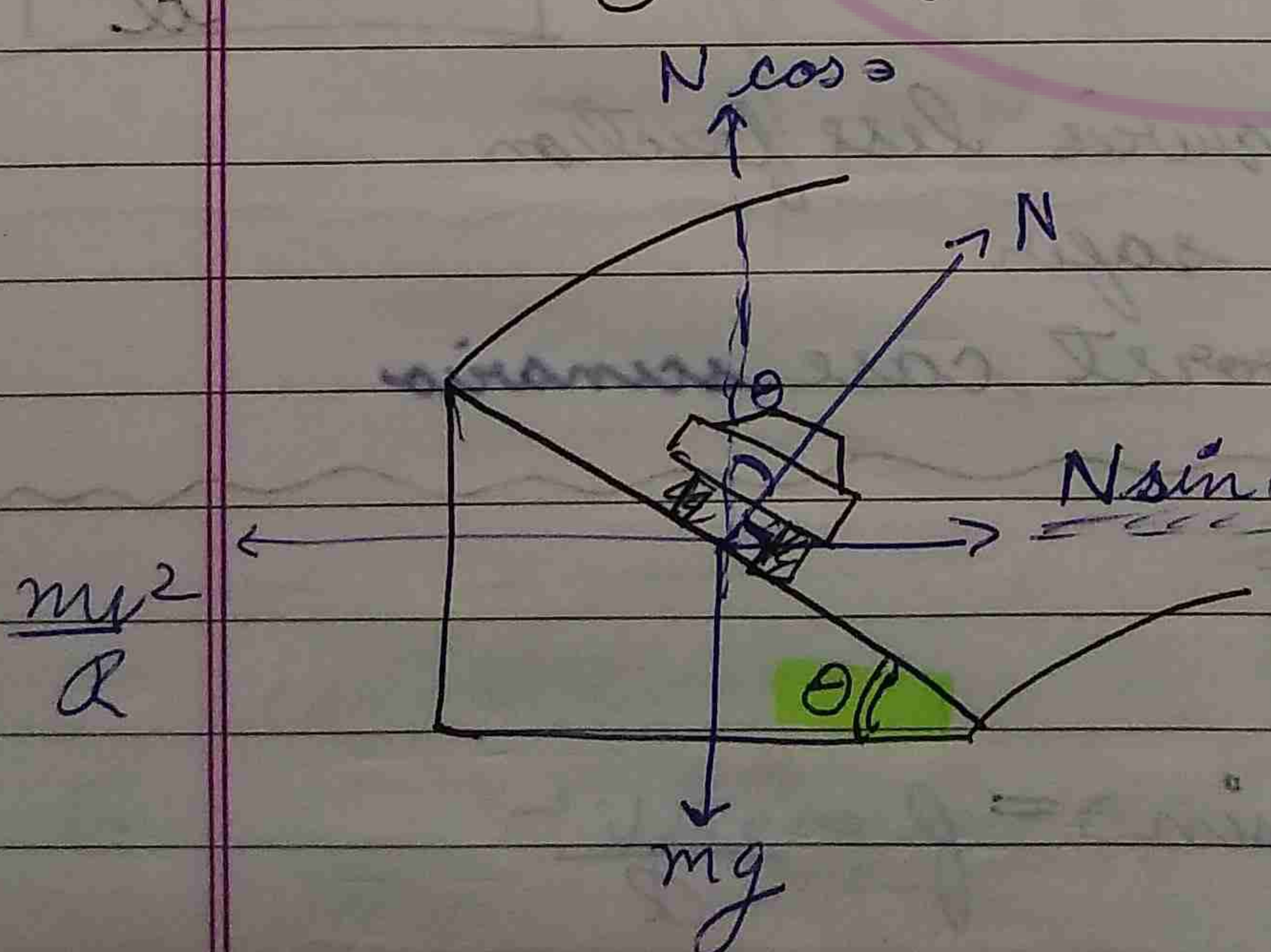


$$f = \frac{mV^2}{R}$$

$$f_{max} = \frac{m V_{max}^2}{R}$$

$$\mu (N_1 + N_2) = \frac{m V_{max}^2}{R} \quad (N_1 + N_2 = mg) \quad \Rightarrow \boxed{V_{max} = \sqrt{\mu g R}}$$

- ## Banking of Road



- friction is not required if travelling at design velocity = V

$$N \sin\theta = \frac{mV^2}{R} \quad (i)$$

$$N \cos\theta = mg \quad (ii)$$

$$\boxed{\tan\theta = \frac{V^2}{Rg}}$$

☆ If speed of car is equal to $\sqrt{Rg \tan\theta}$ then force of friction b/w tyre and road is zero.

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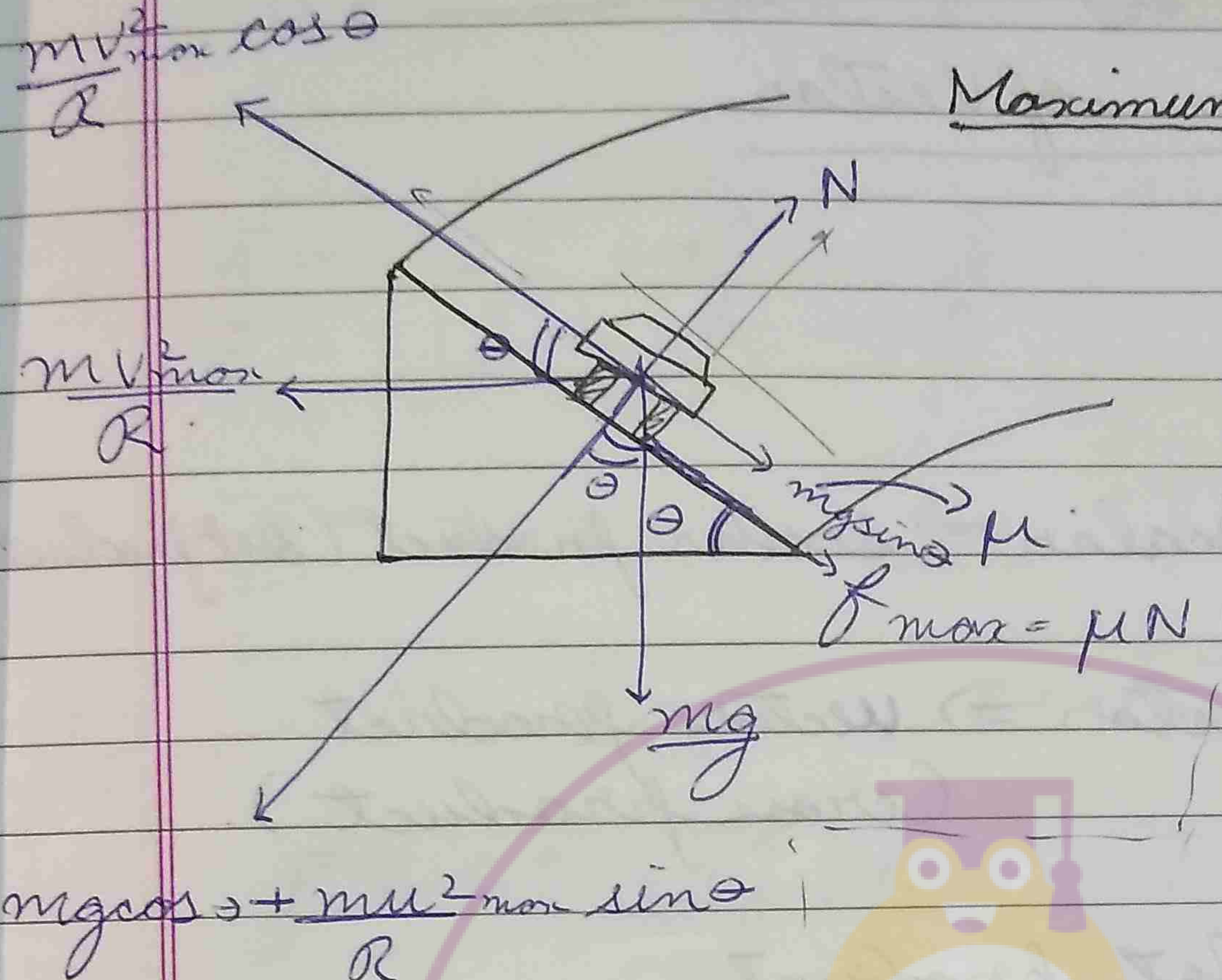
Maximum and Minimum safe velocity



Maximum safe velocity

$N = mg\cos\theta + \frac{mv^2}{R}\sin\theta$ (i)

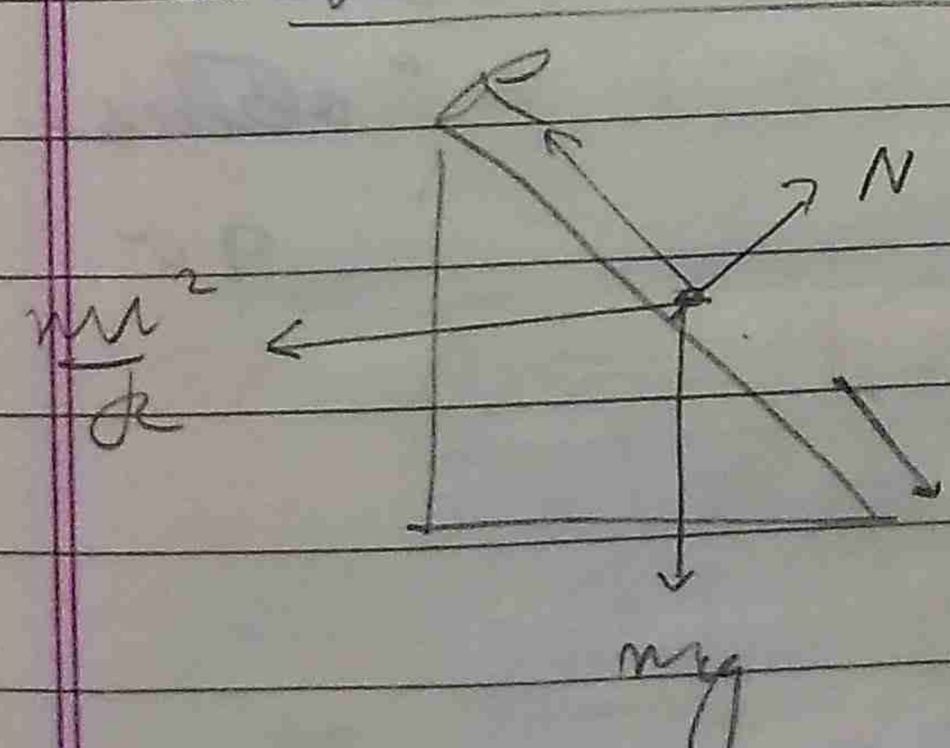
$mg\sin\theta + \mu N = \frac{mv^2_{max}}{R}\cos\theta$

$\Rightarrow mg\sin\theta + \mu\left(mg\cos\theta + \frac{mv^2_{max}}{R}\sin\theta\right) = \frac{mv^2_{max}}{R}\cos\theta$

$\frac{V^2_{max}}{R}(\mu\sin\theta - \cos\theta) = -g(\sin\theta + \mu\cos\theta)$

$$\Rightarrow V_{max} = \sqrt{\frac{Rg(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}}$$

Minimum velocity



$$V_{min} = \sqrt{\frac{Rg(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta}}$$

→ Minimum safe velocity
↓
so that it doesn't slip inside.

- In case of train tracks $\frac{h}{d} = \frac{v^2}{rg}$