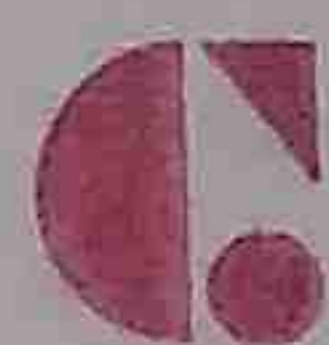




Handwritten Notes
on
Kinetic Theory of Gases



Kinetic Theory of Gases

Gas Laws

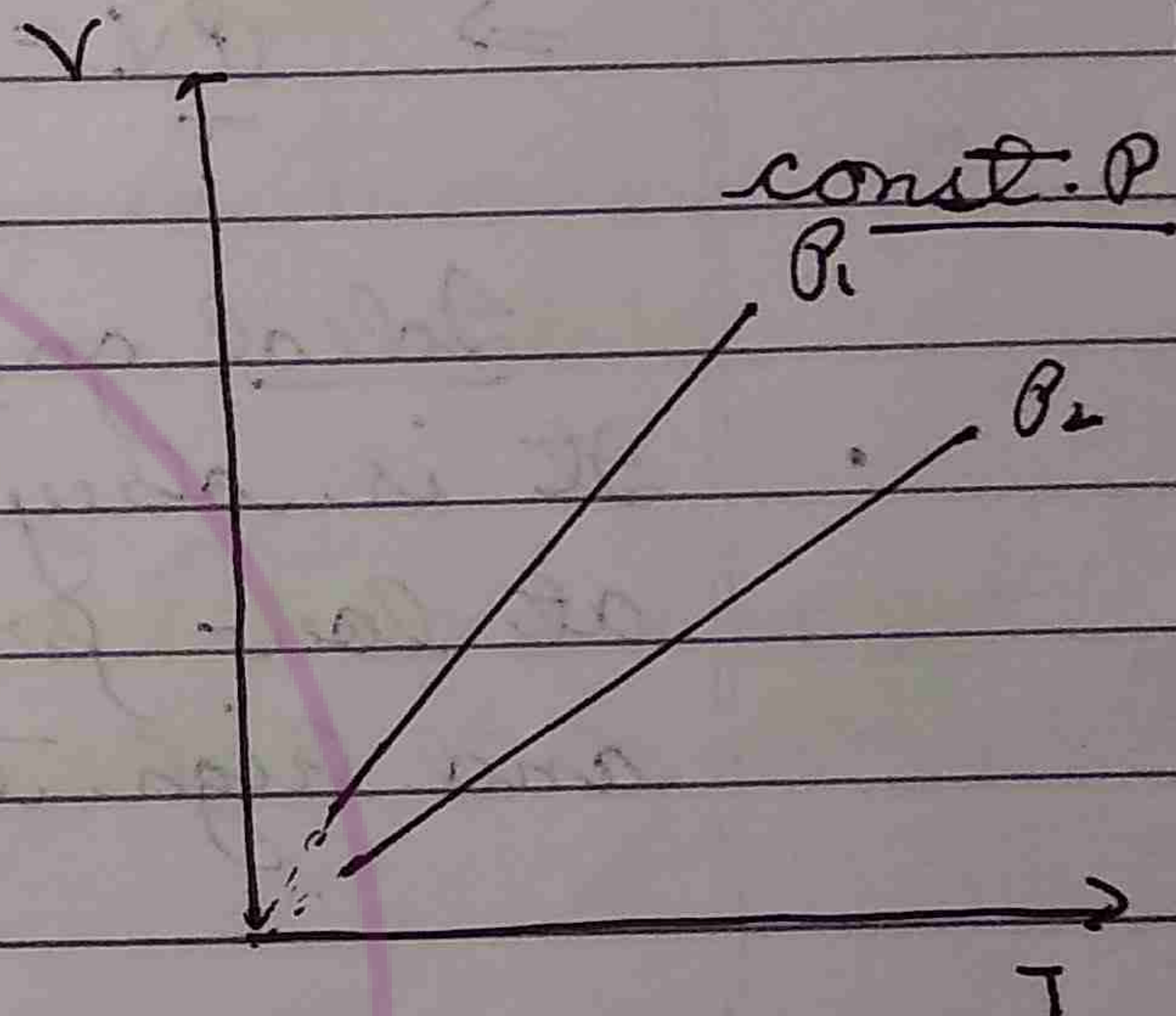
1 Charles's Law

At constant Pressure

$V \propto T$ ← absolute temperature

$$\left| \frac{V}{T} = \text{const.} \right|$$

$$\left| \frac{V_1}{T_1} = \frac{V_2}{T_2} \right|$$



2 Boyle's Law

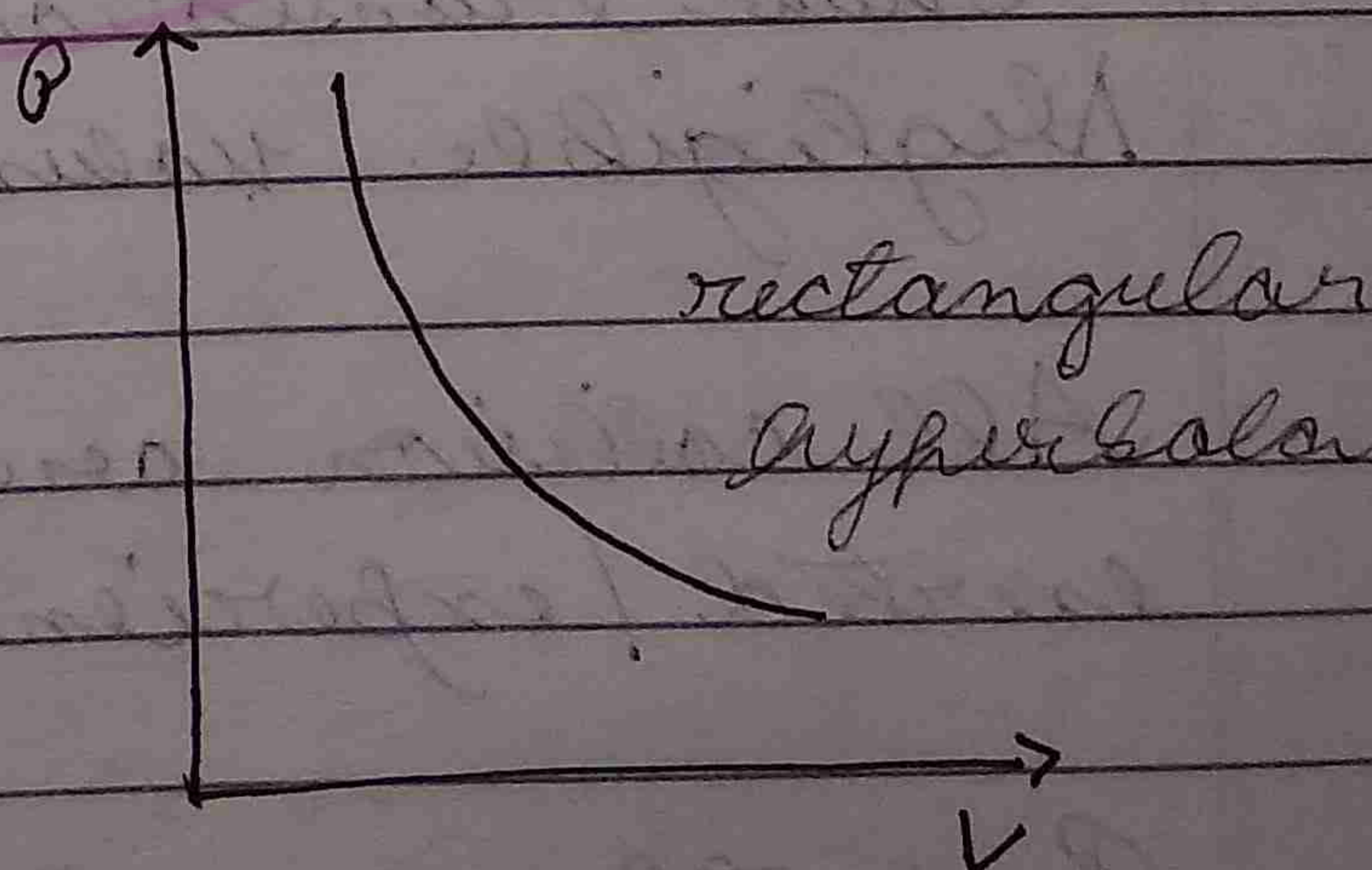
At constant T

$$P \propto \frac{1}{V}$$

or

$$V \propto \frac{1}{P}$$

$$PV = \text{const.}$$



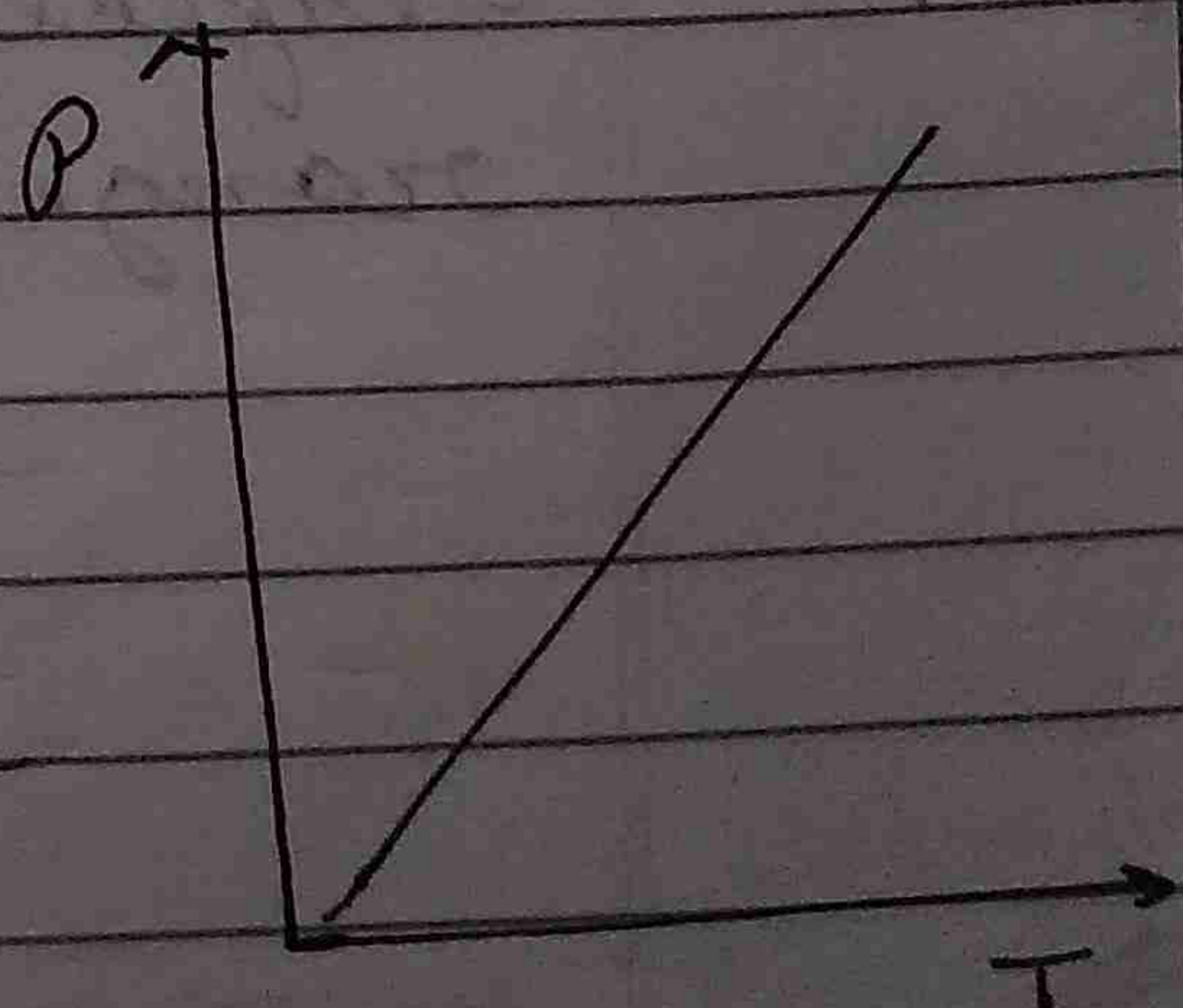
3 Gay-Lussac's Law

At const. V

$$P \propto T$$

$$\frac{P}{T} = \text{const}$$

$$\left| \frac{P_1}{T_1} = \frac{P_2}{T_2} \right|$$



★ Hence U_p (Potential energy = 0)



Date _____

Page _____

$$\left[\frac{P_1}{T_1} = \frac{P_2}{T_2} \right] \quad \left[\frac{V_1}{T_1} = \frac{V_2}{T_2} \right] \quad \left[P_1 V_1 = P_2 V_2 \right]$$

$$\left[\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \right]$$

$$\Rightarrow \frac{PV}{T} = \text{const} \Rightarrow \boxed{PV = nRT}$$

Ideal gas equation no. of moles
It is obeyed by gases at low pressure and high temperature.
 $R =$ Universal gas constant
 $= 8.314 \text{ J/mol K}$

Assumption

- 1 No intermolecular attraction or repulsion.★
- 2 Molecules are tiny spheres
(have translational motion only).
Negligible volume occupied by gas molecules.
- 3 All collision are perfectly elastic, force exerted / experienced only during collision.
- 4 Perfectly random motion.
range of velocity \rightarrow zero to very large values

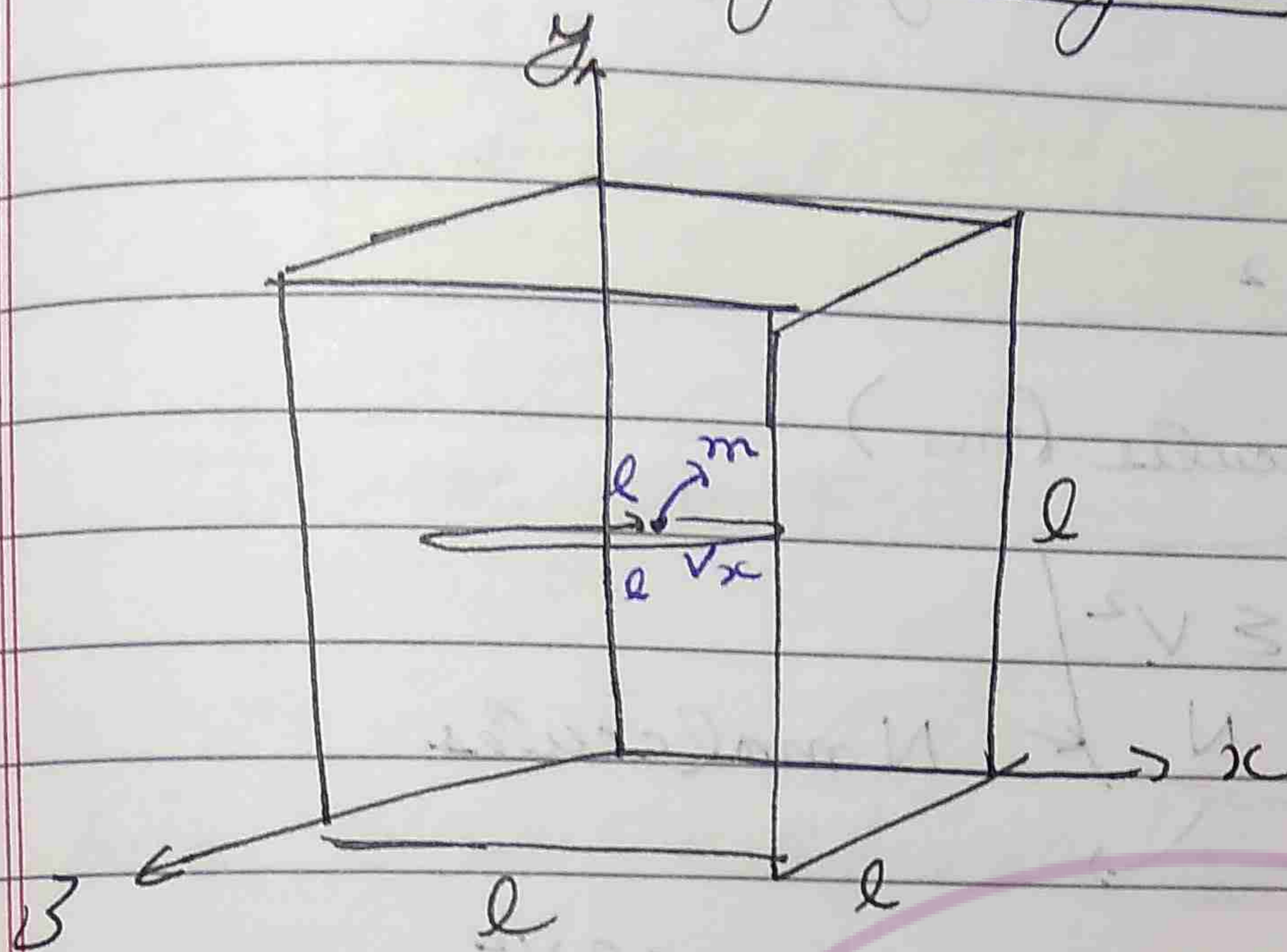
Average velocity of gas in equilibrium is 0



Date ___/___/___

Page _____

Pressure of a gas



Time b/w two collisions = $\frac{2l}{v_x}$

Collision frequency = $\frac{v_x}{2l}$ (per second)

change in momentum/collision = $2mv_x$

change in momentum/sec = $2mv_x \times \frac{v_x}{2l} = \frac{mv_x^2}{l}$

If there are N molecules

Force in x direction (for a wall || to $y-z$ plane)

$$F_x = \sum_l m v_x^2 \Rightarrow F_x = \frac{m}{l} \sum v_x^2$$

Pressure on the wall

$$P_x = \frac{F_x}{l^2} = \frac{M}{l^3} \sum v_x^2$$

$$P_y = \frac{m}{l^3} \sum v_y^2$$

$$P_z = \frac{m}{l^3} \sum v_z^2$$

$$\star P_x = P_y = P_z = P$$

$$\Rightarrow P = \frac{P_x + P_y + P_z}{3}$$



$$P = \frac{m}{3L^3} \sum (v_{x0}^2 + v_{y0}^2 + v_{z0}^2)$$

$$= \frac{m}{3L^3} \sum v^2$$

Total mass of gas molecules (m_0)

$$P = \frac{mN}{3V} \cdot \frac{\sum v^2}{N}$$

volume
N molecules

average of v^2
(Mean sq. velocity)

$$= \frac{\rho}{3} (V_{rms})^2$$

$$P = \frac{1}{3} \rho V_{rms}^2$$

$$V_{rms} = \sqrt{\frac{3P}{\rho}}$$

Kinetic interpretation of temperature

$$P = \frac{1}{3} \rho V_{rms}^2$$

$$= \frac{1}{3} \frac{m_0}{V} V_{rms}^2$$

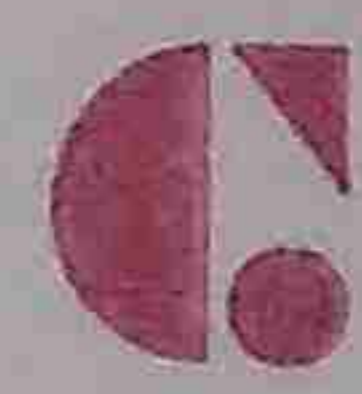
$$\Rightarrow PV = \frac{1}{3} m_0 V_{rms}^2$$

$$= \frac{2}{3} \times \frac{1}{2} m_0 V_{rms}^2$$

K.E of the gas molecules in the vessel.

$$PV = \frac{2}{3} K.E$$

But $PV = nRT$



$$\Rightarrow nRT = \frac{2}{3} K.E$$

$$\Rightarrow \boxed{K.E / \text{mole} = \frac{K.E}{n} = \frac{3}{2} RT}$$

$$T \propto K.E$$

$$\bullet \text{ Aug. Trans } K.E / \text{mole} / \text{direction} = \frac{1}{3} \left(\frac{3}{2} RT \right)$$

$$= \frac{1}{2} RT$$

\Rightarrow equipartition of energy

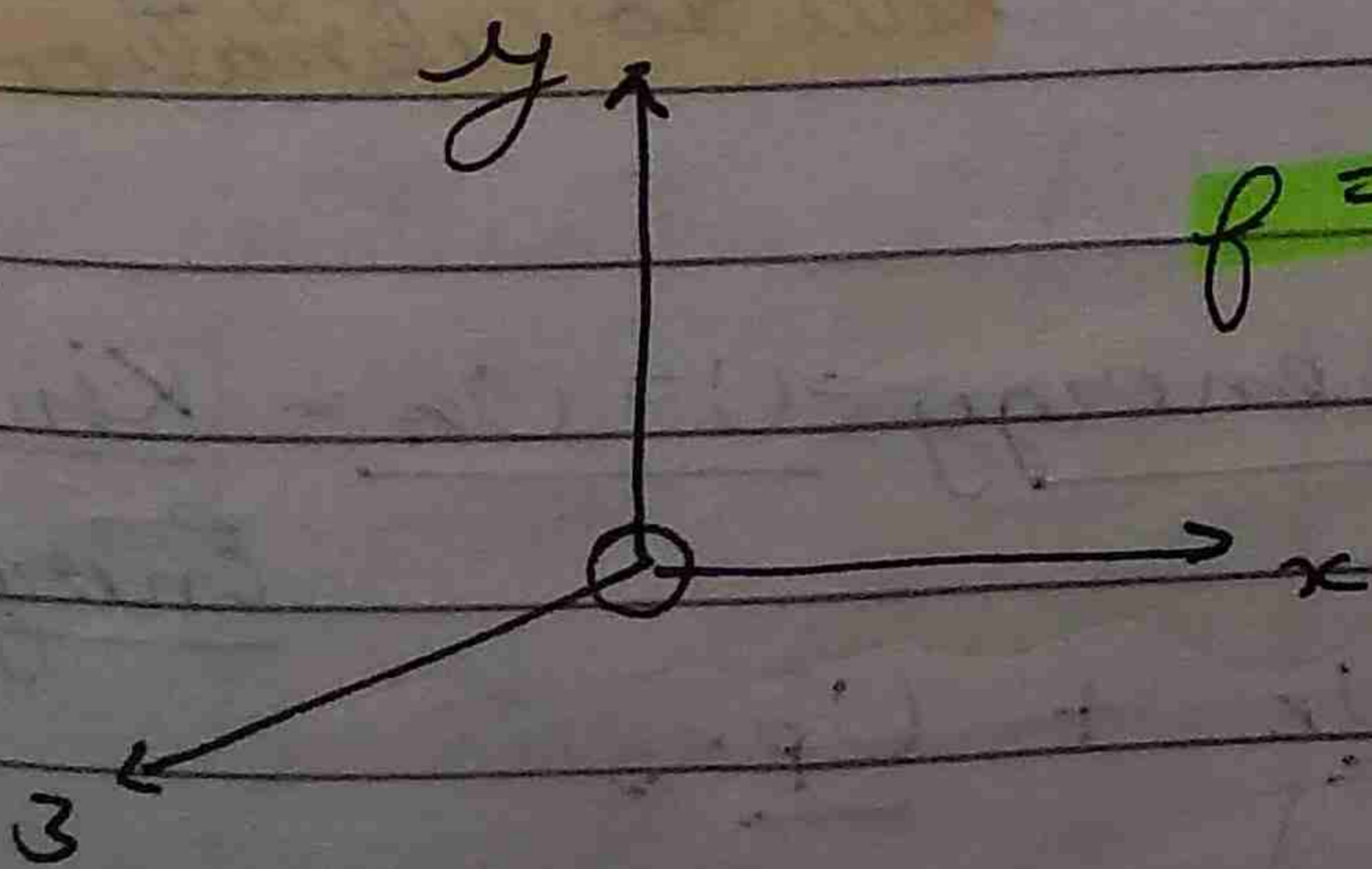
$$\bullet K.E / \text{molecule} = \frac{3}{2} RT / N_A = \frac{3}{2} k_B T$$

$$\bullet K.E / \text{molecule} / \text{direction} = \frac{1}{2} k_B T \quad \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J/K}$$

Degree of freedom (f)

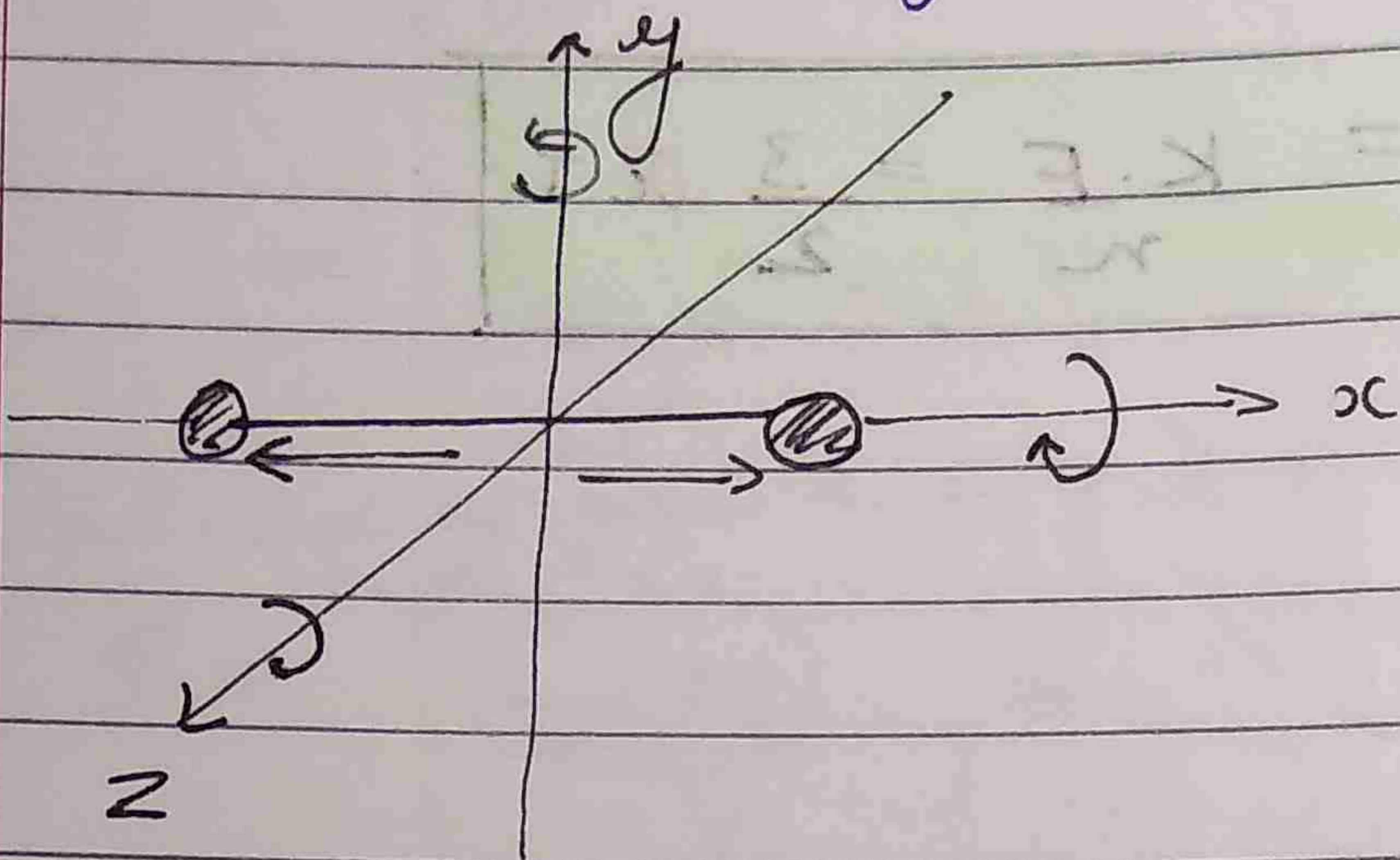
No. of ways in which energy can be associated with a gas molecule.

Monatomic



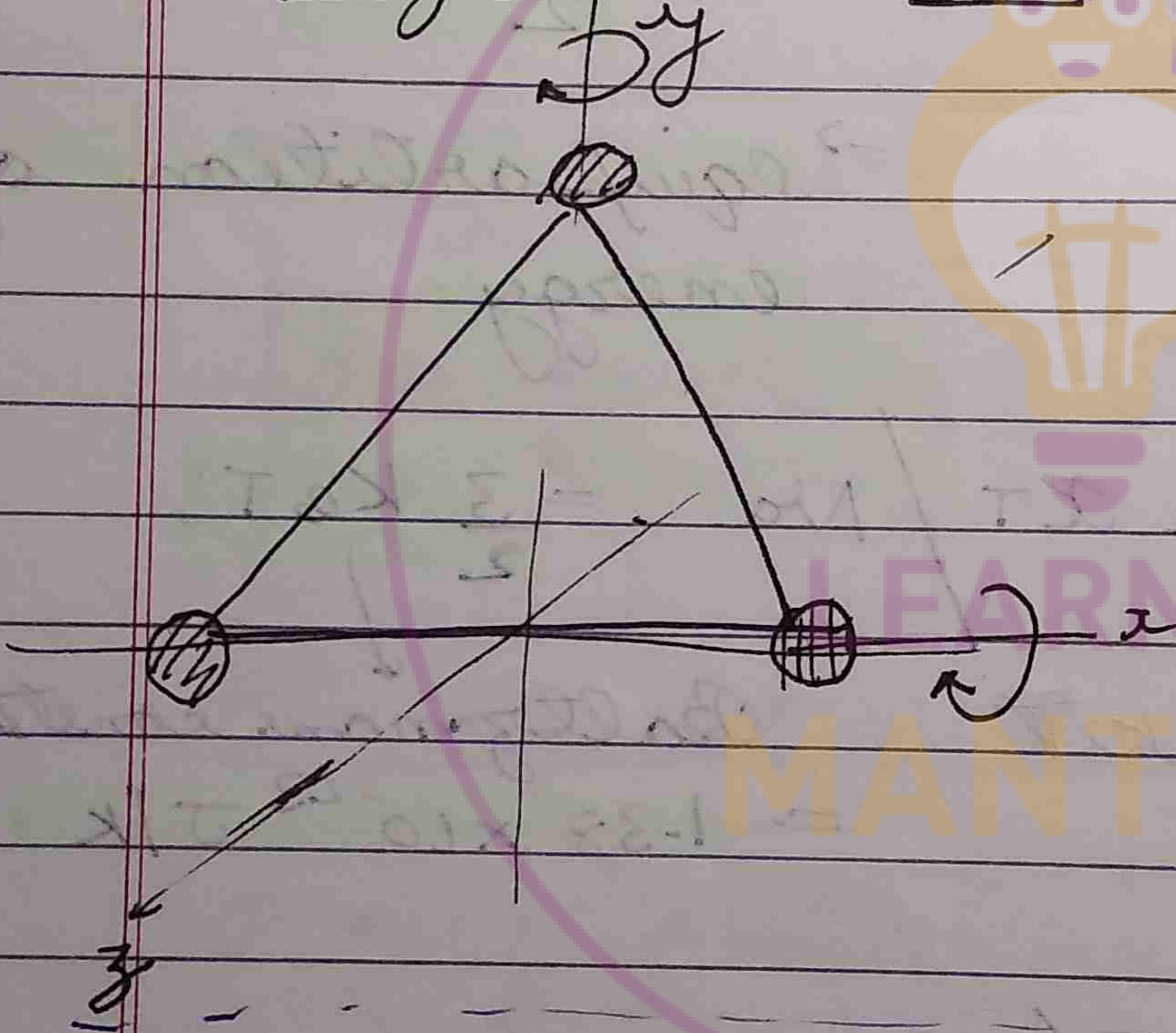
Diatomic / Polyatomic linear

$$f = 3 \text{ trans} + 2 \text{ rot} = \underline{\underline{5}}$$



Polyatomic non-linear molecule

$$f = 3 \text{ trans} + 3 \text{ rot} = \underline{\underline{6}}$$



Diatomic gas

< 70 K (3)

250 - 750 K (5)

> 1000 K

(7) (+2 degree of freedom due to vibration)

$$U = f(T)$$

Ideal gas → Internal energy = U = U_k = Kinetic Energy

Real gas → $U = U_k + U_p$

only companion $U = f(T, v)$

depends on
depends on volume



$$\text{K.E of mole / degree of freedom} = \frac{1}{2} RT$$

Internal Energy of an ideal gas per mole :-

$$U = f \times \frac{1}{2} RT$$

$$U = n C_v T \Rightarrow C_v = \frac{U}{T}$$

$$\Rightarrow C_v = \frac{1}{2} f R$$

$$C_p = C_v + R = \frac{(f+2) \cdot R}{2}$$

$$C_p = \frac{(f+2) \cdot R}{2}$$

$$\gamma = \frac{C_p}{C_v} = \frac{f+2}{f}$$

$$\Rightarrow \gamma = 1 + \frac{2}{f}$$

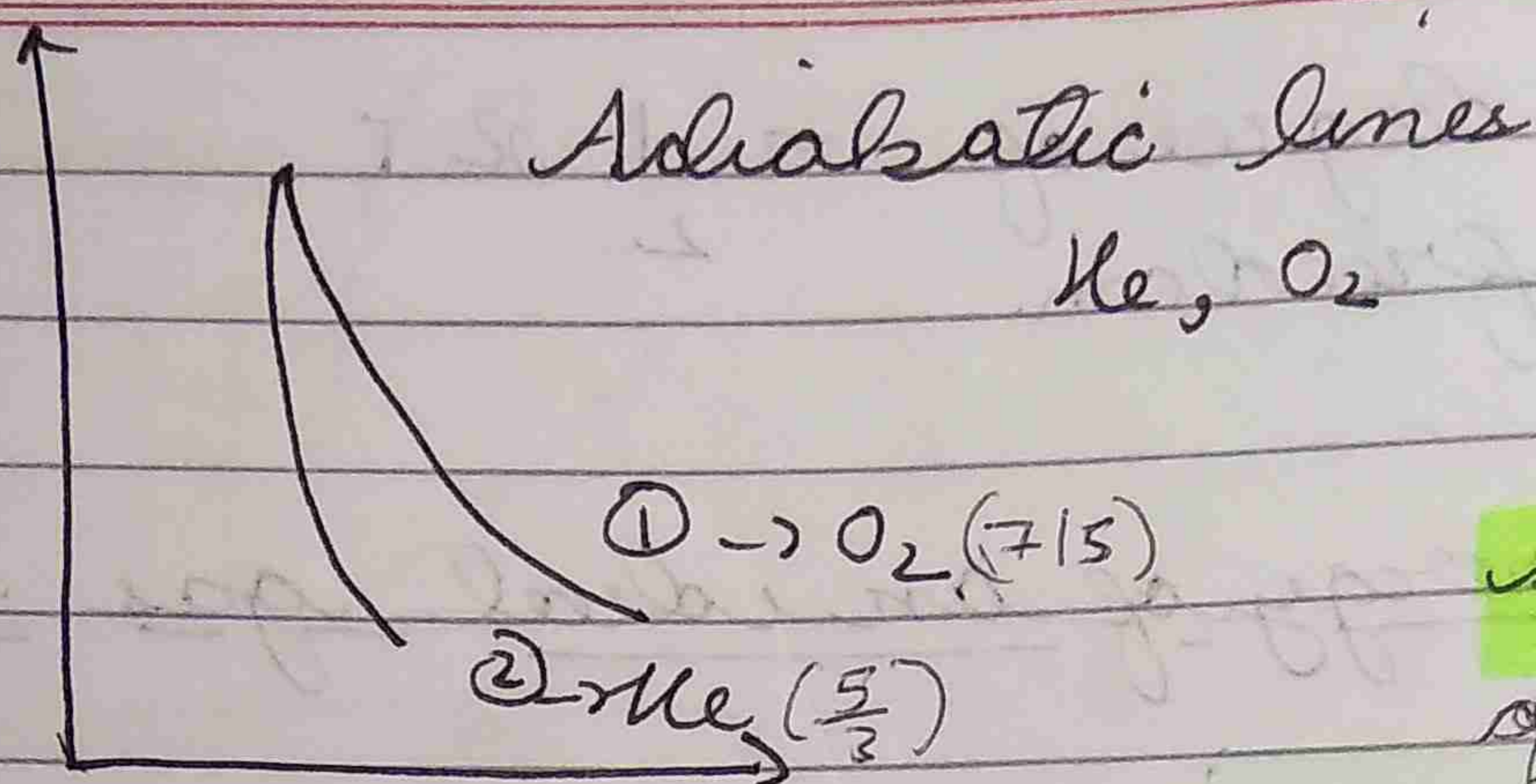
$$\gamma = 1 + \frac{2}{f}$$

C_p, C_v and γ for diff. gases

Type of gases	C _v	C _p	γ
1 Monatomic	$\frac{3}{2} R$	$\frac{5}{2} R$	$\frac{5}{3}$
2 Diatomic / Poly linear	$\frac{5}{2} R$	$\frac{7}{2} R$	$\frac{7}{5}$
3 Poly non-linear	$3R$	$4R$	$\frac{4}{3}$



Q



slope = $-\gamma \frac{P}{V}$

of adiabatic line

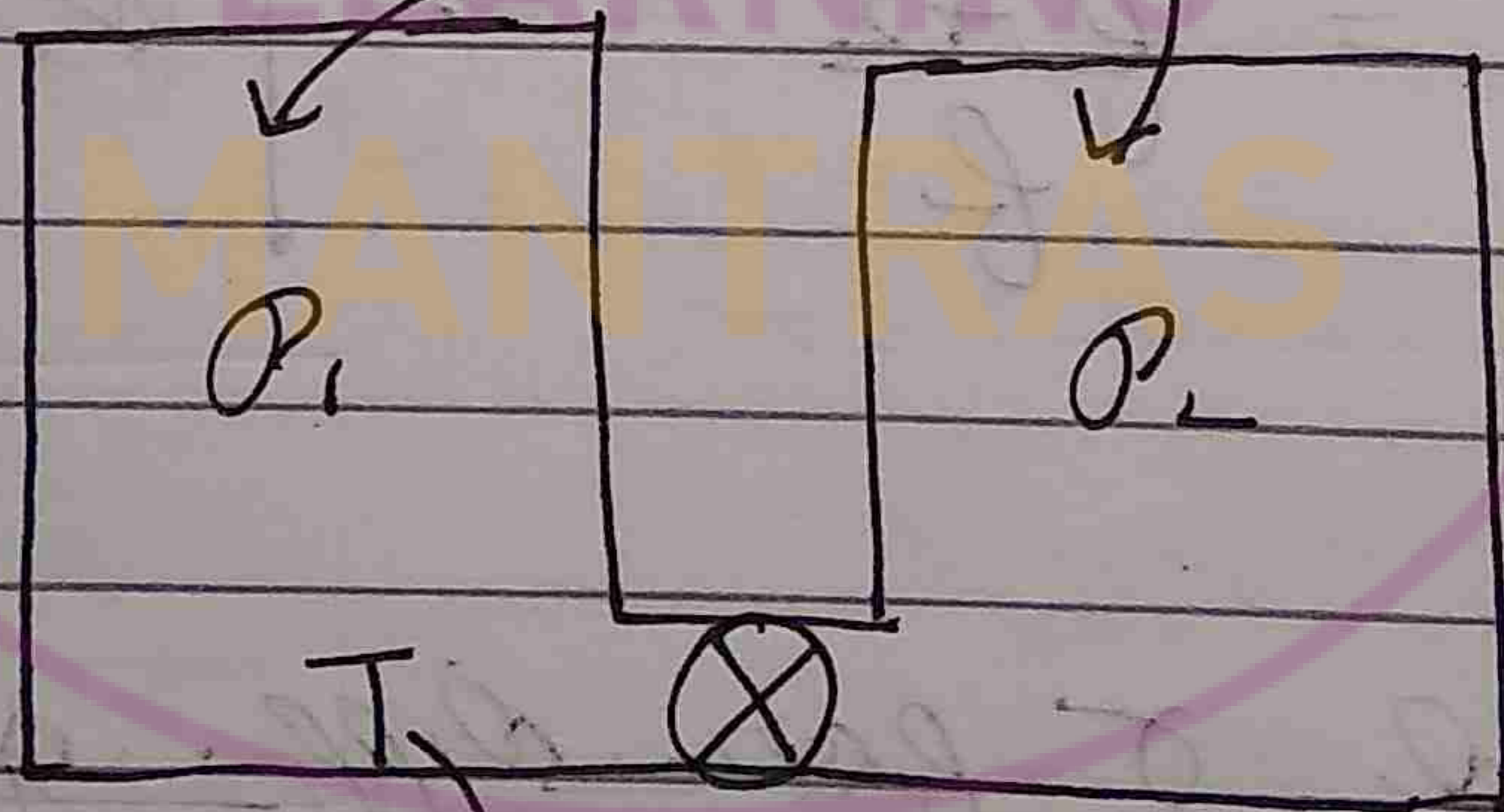
Q

Mixture ^{of gas} $PV^{3/2}$ = constant

Mixture of gases

adiabatic vessel

n_1 moles of 1 at T_1
 n_2 moles of 2 at T_2



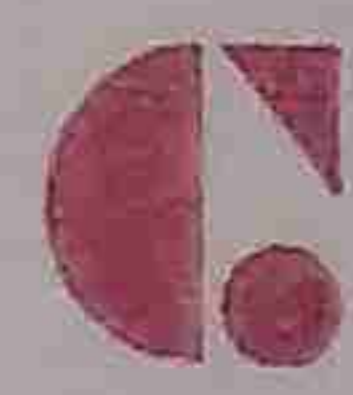
$U_i = U_f$

Temperature of mixture

$U_i = n_1 C_{v1} T_1 + n_2 C_{v2} T_2$

$U_f = (n_1 C_{v1} + n_2 C_{v2}) \cdot T$

$T = \frac{n_1 C_{v1} T_1 + n_2 C_{v2} T_2}{n_1 C_{v1} + n_2 C_{v2}}$



$$C_{v \text{ mix}} = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$$

$$C_{p \text{ mix}} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 + n_2} \text{ or } C_{v \text{ mix}} + R$$

$$T_{\text{mix}} = \frac{C_{p \text{ mix}}}{C_{v \text{ mix}}}$$

Q. $\xrightarrow{300 \text{ K}}$ 1 mole of O_2 + $\xrightarrow{400 \text{ K}}$ 2 mole He

$$C_{v \text{ mix}} = \frac{1 \times \frac{5}{2} R + 2 \times \frac{3}{2} R}{1+2} = \frac{11R}{6}$$

$$C_{p \text{ mix}} = \frac{11R + R}{6} = \frac{12R}{6} = 2R$$

$$T_{\text{mix}} = \frac{12R}{11R} = \frac{12}{11}$$

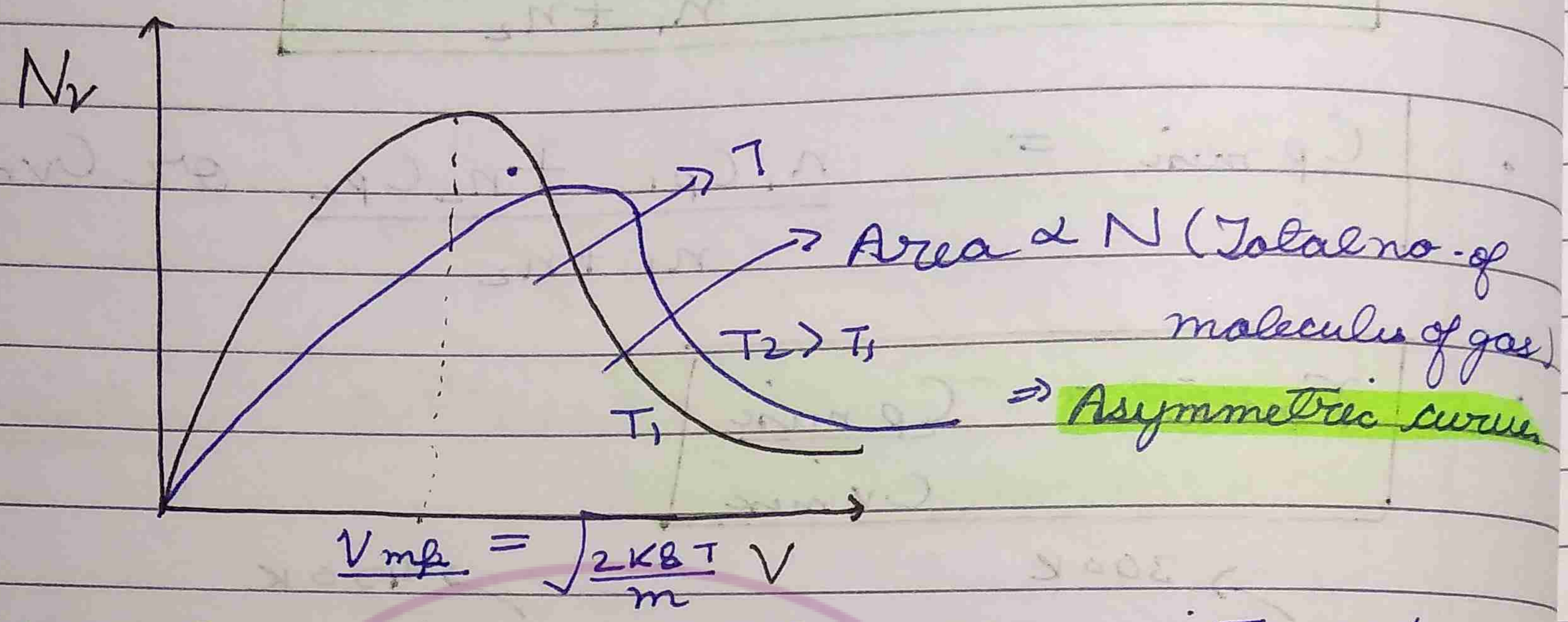
$$T_{\text{mix}} = \frac{1 \times \frac{5}{2} R \times 300 + 2 \times \frac{3}{2} R \times 400}{\frac{5}{2} R + \frac{6}{2} R} = 354.5 \text{ K}$$

★ ★ no of moles remains constant: $n = \frac{Pv}{RT}$
(In a closed system)

$$\star \frac{P(V_1 + V_2)}{RT} = n_1 + n_2$$

$$= \frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}$$

Maxwell distribution of speeds.



N_v = Probability of molecule velocity b/w v and $v + dv$

dN = No. of molecules having speed b/w v and $v + dv$
 $= N_v dv$

$$dN = 4\pi N \left(\frac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}} dv$$

★ $v_{av} = \sqrt{\frac{8k_B T}{\pi m}}$

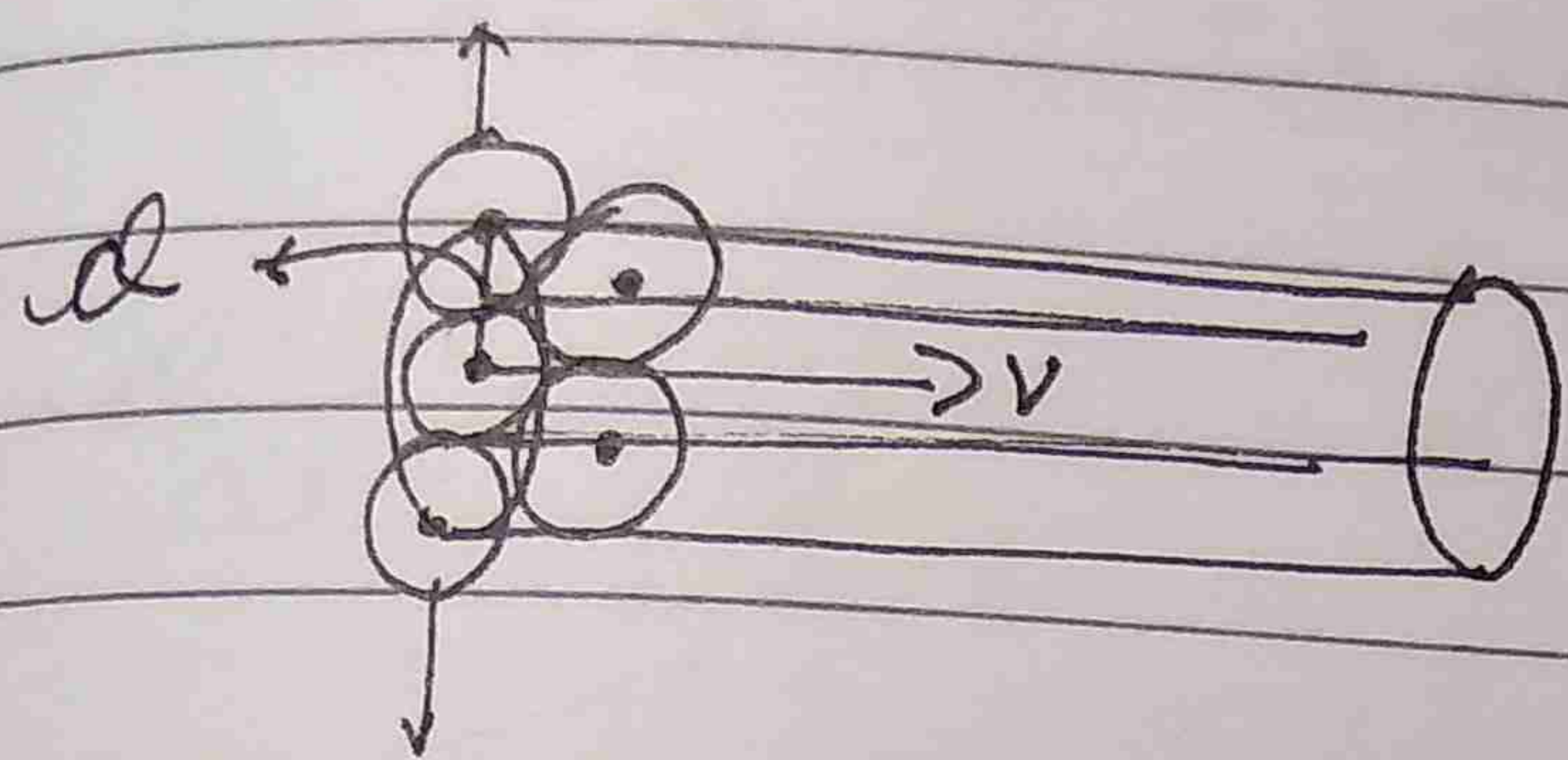
$v_{most\ prob} = \sqrt{\frac{2k_B T}{m}}$

★ $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{m}}$

$v_{mp} < v_{av} < v_{rms}$

★ v of a gas is independent of pressure change it changes only when temperature changes.

Mean free path λ



$$n = \text{molecules / volume}$$

\bar{v} = average velocity of gas molecule

$\pi d^2 \cdot \bar{v} l$ = volume of the cylinder in which gas molecule collides with other molecules.

$$\text{Total no. of collisions} = n \times \pi d^2 \bar{v} l$$

τ = Average time b/w two collisions

$$= \frac{l}{n \pi d^2 \bar{v} l} = \frac{1}{n \pi d^2 \bar{v}}$$

$$\lambda = \bar{v} \times \tau$$

$$\lambda = \frac{1}{\sqrt{2} n \pi d^2}$$



Characteristics of angular S.H.M

$$\tau = -c\theta$$

$$\tau = I\alpha = I \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = \frac{-c\theta}{I} \quad \text{ie } \frac{d^2\theta}{dt^2} = -\omega^2\theta \quad \text{with } \omega^2 = \frac{c}{I}$$

$$\frac{d\theta}{dt} = \omega \sqrt{\theta_0^2 - \theta^2}$$

$$\theta = \theta_0 \sin(\omega t + \phi)$$

LEARNING
MANTRAS