

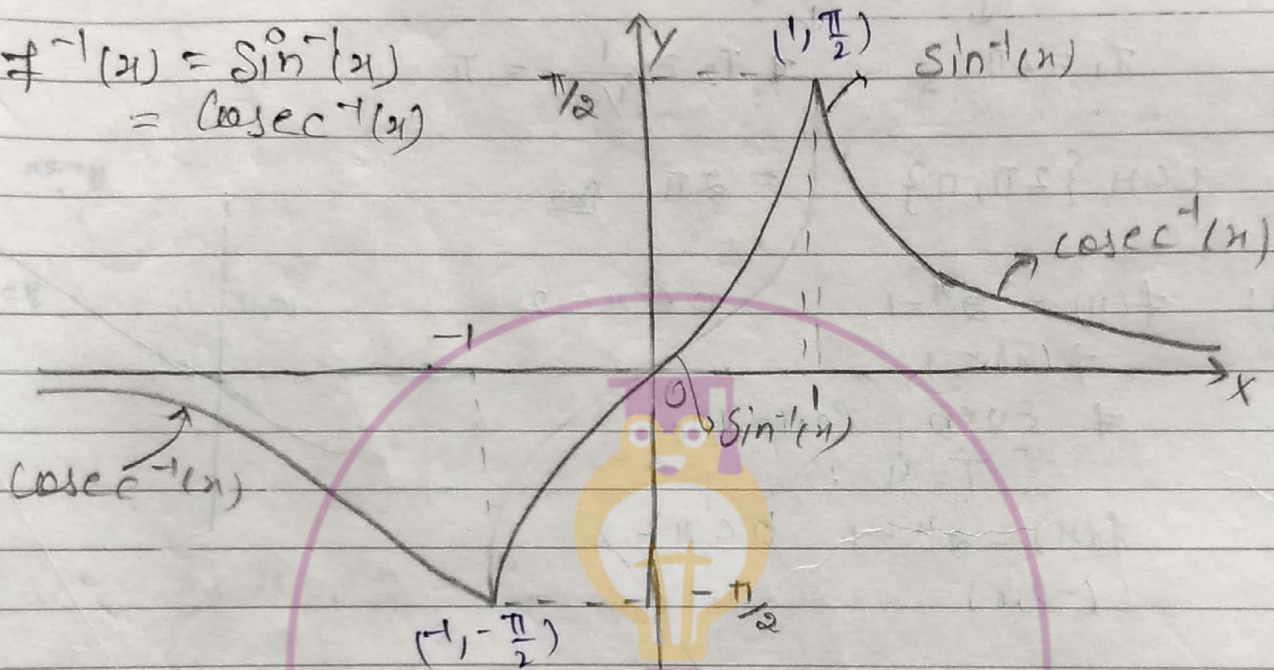


Handwritten Notes
On
Inverse Trigonometric Function

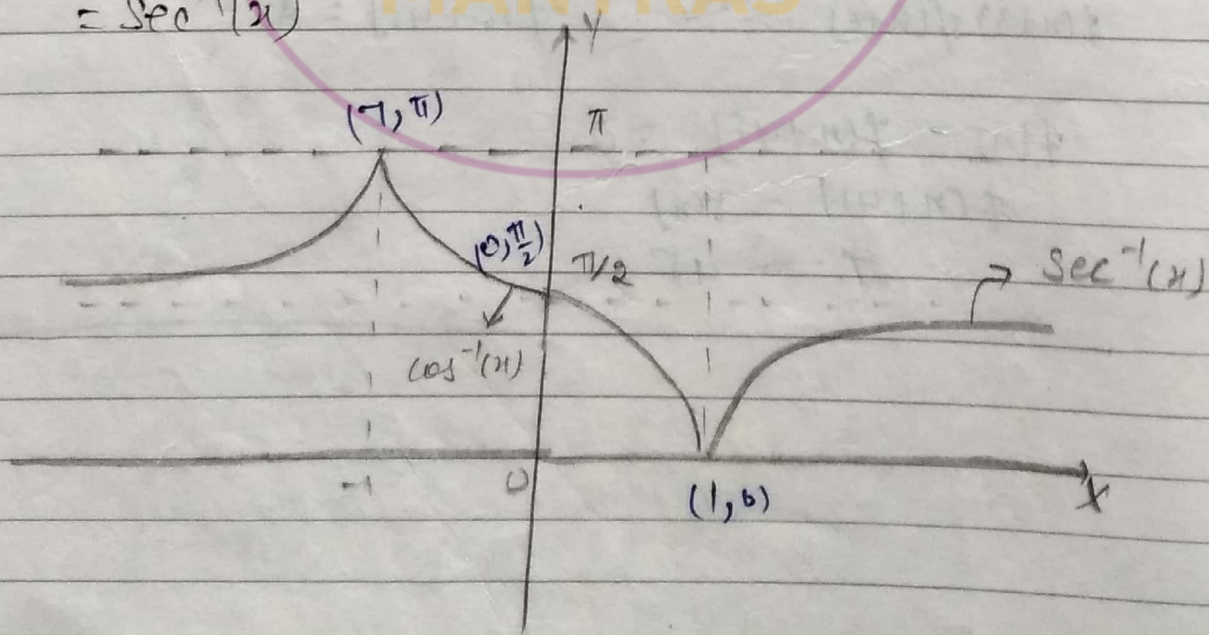
Inverse Trigonometric function

* Graph:

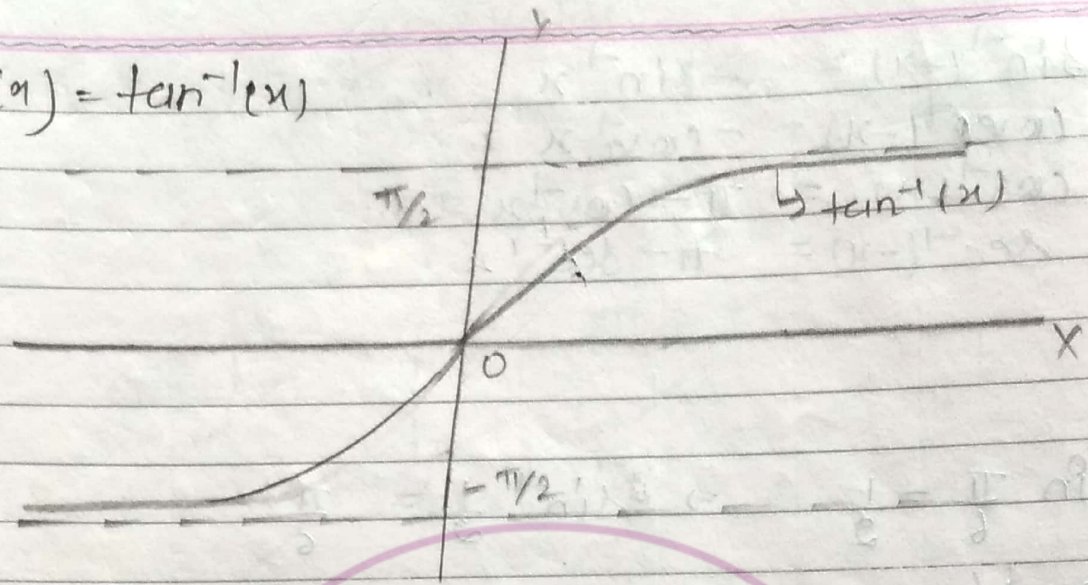
$$\begin{aligned} \sin^{-1}(x) &= \sin^{-1}(x) \\ &= \operatorname{cosec}^{-1}(x) \end{aligned}$$



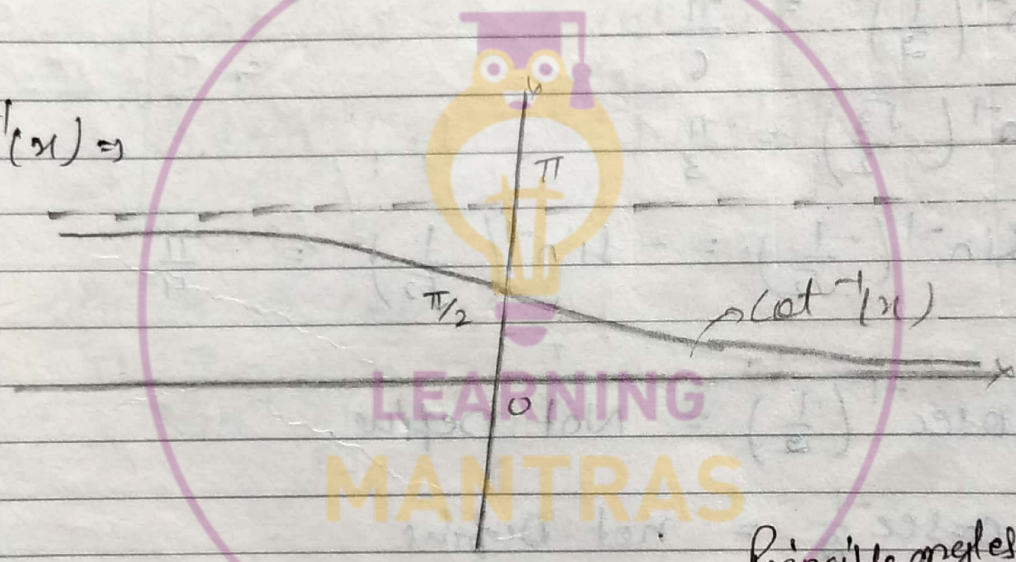
$$\begin{aligned} \cos^{-1}(x) &= \cos^{-1}(x) \\ &= \operatorname{sec}^{-1}(x) \end{aligned}$$



$f^{-1}(x) = \tan^{-1}(x)$



$\cot^{-1}(x) =$



Principle angles,

	Domain	Range	Asymptot / Nature
\sin^{-1}	$[-1, 1]$ $\begin{cases} -1 \leq x \leq 1 \\ \text{or} \\ x \leq 1 \end{cases}$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	x odd
$\csc^{-1}(x)$	$(-\infty, -1] \cup [1, \infty)$ $ x \geq 1$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$	$y=0$ odd
$\cos^{-1}(x)$	$[-1, 1]$	$[0, \pi]$	x Neither Even or Odd
$\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$ $ x \geq 1$	$[0, \pi] - \frac{\pi}{2}$	$y = \frac{\pi}{2}$ Neither
\tan^{-1}	R	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$y = \pm \frac{\pi}{2}$ odd
\cot^{-1}	R	$(0, \pi)$	$y=0, y=\pi$ Neither

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sin^{-1}(-x) = -\sin^{-1} x$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x$$

$$\sin \frac{\pi}{6} = \frac{1}{2} \Rightarrow \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

$$\sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{6}$$

$$\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = \frac{\pi}{3}$$

$$\sin^{-1} \left(-\frac{1}{\sqrt{2}} \right) = -\sin^{-1} \left(\frac{1}{\sqrt{2}} \right) = -\frac{\pi}{4}$$

$$\operatorname{cosec}^{-1} \left(\frac{1}{2} \right) = \text{Not Def'ne.}$$

$$\operatorname{cosec}^{-1} \frac{1}{2} = \text{not Def'ne}$$

$$= \sin^{-1} \frac{1}{2} = \frac{\pi}{6} \text{ not def'ne.}$$

$$\sin \frac{\pi}{6} = \frac{1}{2} \Rightarrow \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

$$\boxed{\sin \frac{5\pi}{6} = \frac{1}{2} \Rightarrow \sin^{-1} \frac{1}{2} = \frac{5\pi}{6} \quad \times \quad \times}$$

$$\boxed{\operatorname{arc} \tan x = \tan^{-1} x}$$

$\frac{\pi}{2}$
 π - (cos, sec, cot)

\times $\sin^{-1} x$ ——— max value $\frac{\pi}{2}$
 ——— min value $-\frac{\pi}{2}$

\times $\cos^{-1} x$ ——— max π
 ——— min 0

$$\boxed{\begin{aligned} \cot^{-1}(-x) &= \pi - \cot^{-1} x \\ \tan^{-1}(-x) &= -\tan^{-1} x \end{aligned}}$$

$$\sin^{-1} x = \operatorname{cosec}^{-1} \frac{1}{x}$$

$$\tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$\cot^{-1}(1) = \frac{\pi}{4}$$

$$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3})$$

$$= \pi - \tan^{-1} \frac{1}{\sqrt{3}}$$

$$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Note:

$$\sin x = \frac{1}{\operatorname{cosec} x}$$

$$\sin^{-1} x = \frac{1}{\operatorname{cosec}^{-1} x}$$

$$\times \operatorname{cosec}^{-1} \frac{1}{x}$$

Que! find $\left[\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) + \cot^{-1}\sqrt{3} - \tan^{-1}(1) \right]$
 $+ \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sec^{-1}\frac{2}{\sqrt{3}} + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

Ans:

$$\left[\cancel{\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)} + \left(\cot^{-1}\sqrt{3} \right) - \left(\tan^{-1}(1) \right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sec^{-1}\frac{2}{\sqrt{3}} + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) \right]$$

Ans:

$$\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \tan^{-1}\frac{1}{\sqrt{3}} - \frac{\pi}{4} + \frac{\pi}{3} + \cos^{-1}\frac{1}{2} - \sin^{-1}\frac{\sqrt{3}}{2} + \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \pi - \frac{\pi}{4} + \frac{\pi}{6} - \frac{\pi}{4} + \frac{\pi}{3} + \frac{\pi}{3} - \frac{\pi}{3} + \pi - \frac{\pi}{3}$$

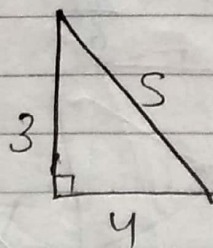
$$= \frac{3\pi}{2} + \frac{\pi}{6}$$

$$= \frac{10\pi}{6} = \frac{5\pi}{3}$$

Que: 2: Solve $(2 \sin^{-1} \frac{3}{5}) = 2 \sin \cos \theta$

$$\sin^{-1} \frac{3}{5} = \theta$$

$$\sin \theta = \frac{3}{5}$$

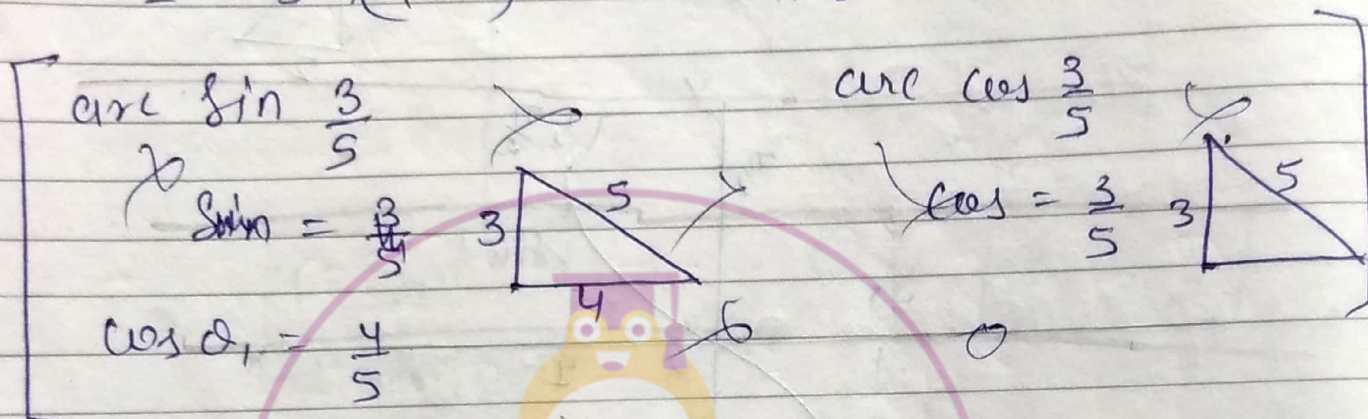


$$= 2 \times \frac{3}{5} \cdot \frac{4}{5}$$

$$= \frac{24}{25}$$

Ques: $\sin(\underbrace{\arcsin \frac{3}{5}}_{\theta_1} - \underbrace{\arccos \frac{3}{5}}_{\theta_2})$

$= \sin(\theta_1 - \theta_2)$ ~~to~~ ~~sin~~



$= \sin \theta_1 \cos \theta_2 - \cos \theta_1 \sin \theta_2$
 $\frac{3}{5} \cdot \frac{3}{5} - \frac{4}{5} \cdot \frac{4}{5}$
 $= \frac{-7}{25}$

Ques: $\sin(2 \sin^{-1} \frac{3}{5})$

$= \sin(\tan^{-1} \cos \cot^{-1}(-\frac{1}{\sqrt{3}}))$

~~$\tan^{-1} \cot \frac{\pi}{3}$~~ $-\cot(-)$

$\cot^{-1}(-\frac{1}{\sqrt{3}}) = \pi - \cot^{-1} \frac{1}{\sqrt{3}}$
 $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\sin(\tan^{-1}(\cos \frac{2\pi}{3}))$

$\sin \tan^{-1}(-\frac{1}{2}) = \sin(-(\tan^{-1} \frac{1}{2}))$

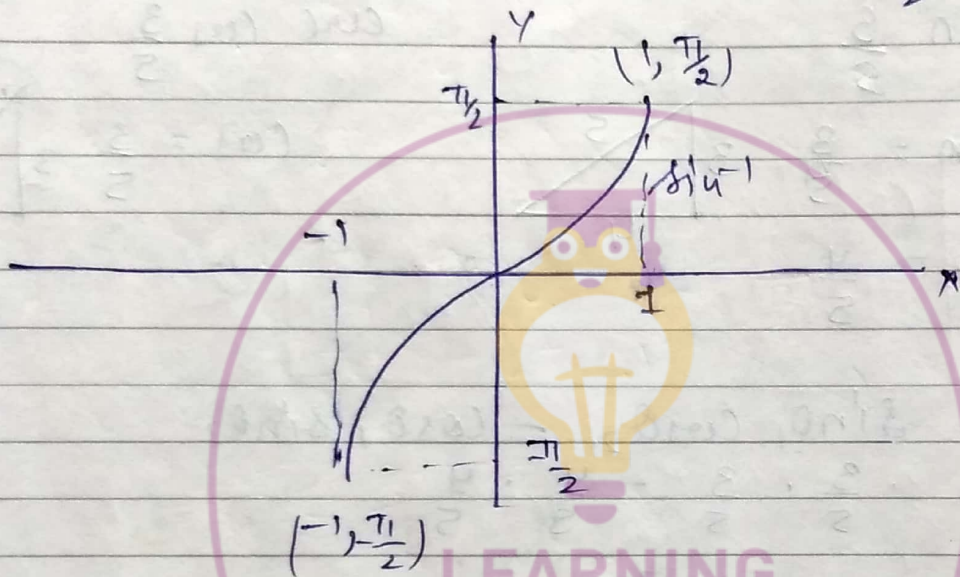
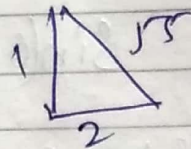
[1, 2)

h
p

$$\begin{aligned}
 & -\sin\left(\tan^{-1}\frac{1}{2}\right) \\
 & = -\sin\theta \\
 & = -\frac{1}{\sqrt{5}}
 \end{aligned}$$

$$\tan^{-1}\frac{1}{2} = \theta$$

$$\tan\theta = \frac{1}{2}$$



Que: find domain of $\sin^{-1}(x^2)$

$$\begin{aligned}
 & a^2 \leq x^2 \leq b^2 \\
 & x \in (-b, -a) \cup (a, b)
 \end{aligned}$$

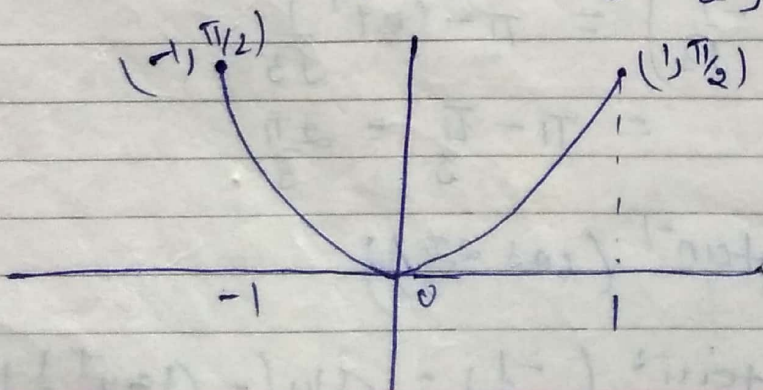
$$|x^2| \leq 1$$

$$\text{or } -1 \leq x^2 \leq 1$$

$$0 \leq x^2 \leq 1 \Rightarrow x \in (-1, 0) \cup (0, 1)$$

$$\text{Domain} \in [-1, 1]$$

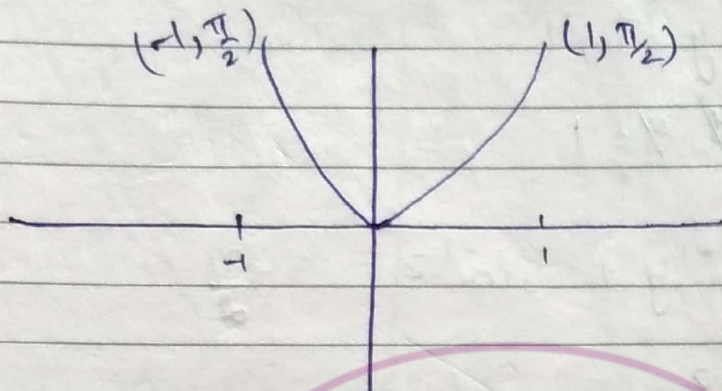
$$\text{Range} \in \left[0, \frac{\pi}{2}\right]$$



$$\{0\} = [0, 1]$$

Que: find D/R $y = \sin^{-1}[x]$

~~$y = \sin^{-1}[x]$~~



$$y = \sin^{-1}[x] \quad \text{Range: } \left[-\frac{\pi}{2}, 0, \frac{\pi}{2}\right]$$

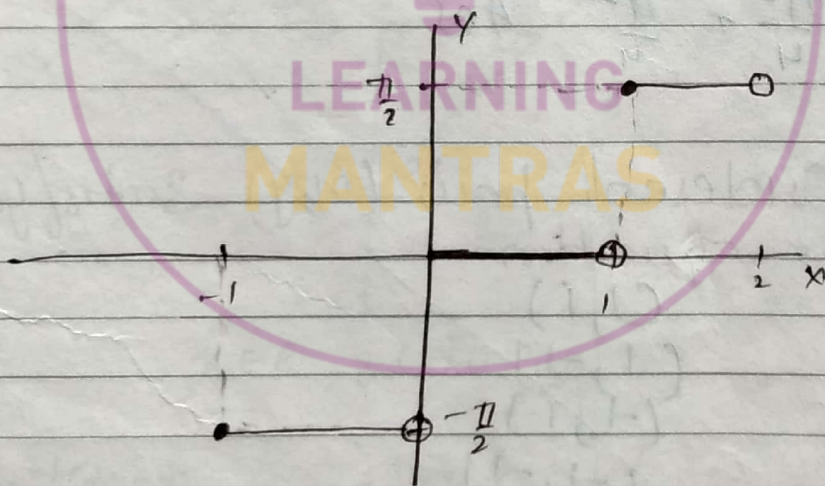
$$-1 \leq [x] \leq 1$$

$$[x] = -1, 0, 1$$

Domain -

$$-1 \leq x \leq 1$$

Que:



Que:

$$\sin^{-1}\{x\}$$

Domain - R. $[0 \in R]$

Boundary value solve

* Boundary value Problem:

* If $\sin^{-1}x + \sin^{-1}y = \pi$
 $\Rightarrow x = 1 \quad \& \quad y = 1$

* If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = -\frac{3\pi}{2}$
 $x = -1 \quad y = z$

* If $(\sin^{-1}x)^2 + (\sin^{-1}y)^2 = \frac{\pi^2}{4}$
 $x = \pm 1 \quad , \quad y = \pm 1$

$\frac{\pi^2}{4} + \frac{\pi^2}{4}$ Ans

* Find Ordered pair (x, y) satisfying above equation.

(x, y)

$(1, 1)$

$(1, -1)$

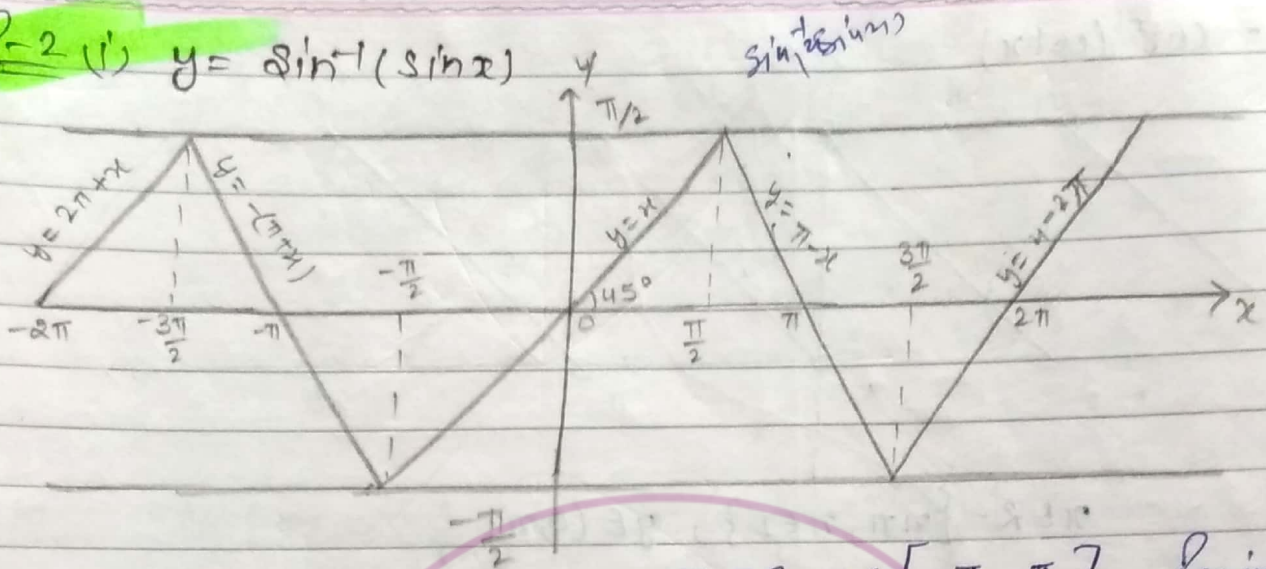
$(-1, 1)$

$(-1, -1)$

* If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$
 $x = y = z = -1$

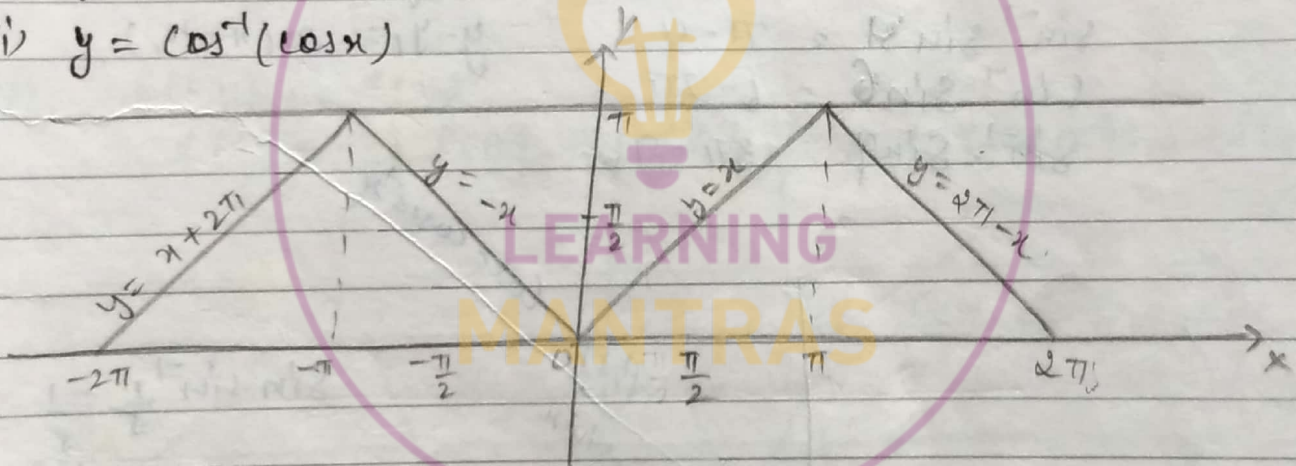
$$y - y_1 = m(x - x_1) \quad y - 0 = 1(x - 2\pi)$$

Q-2 (i) $y = \sin^{-1}(\sin x)$



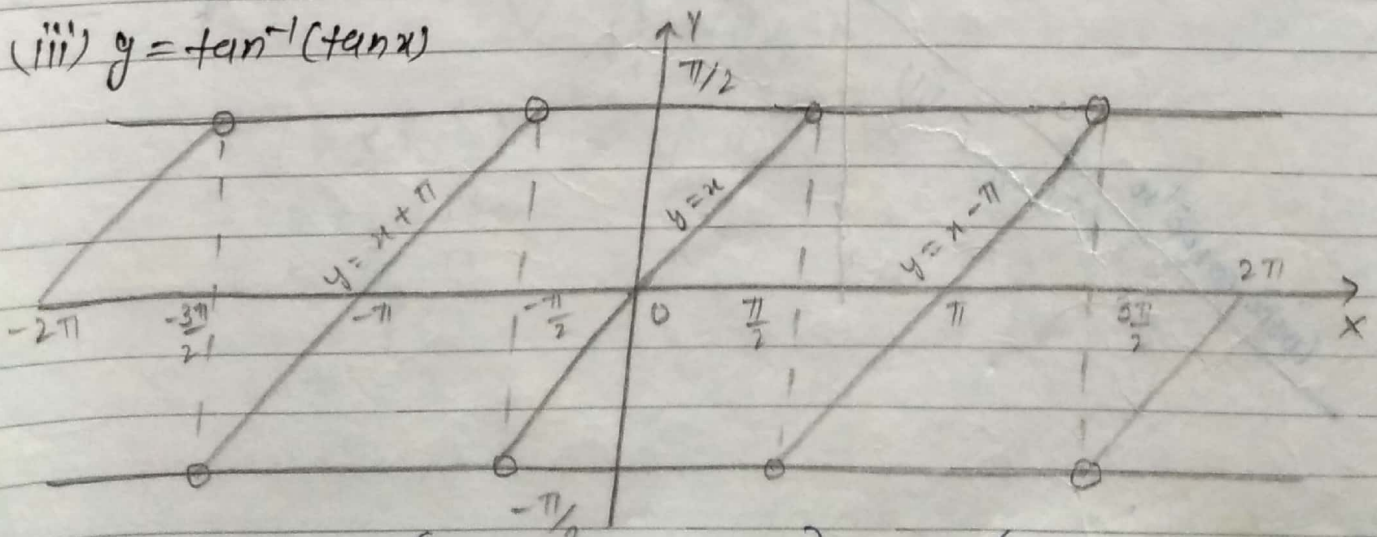
$$x \in \mathbb{R}, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ Periodic.}$$

(ii) $y = \cos^{-1}(\cos x)$



$$x \in \mathbb{R}, y \in [0, \pi]$$

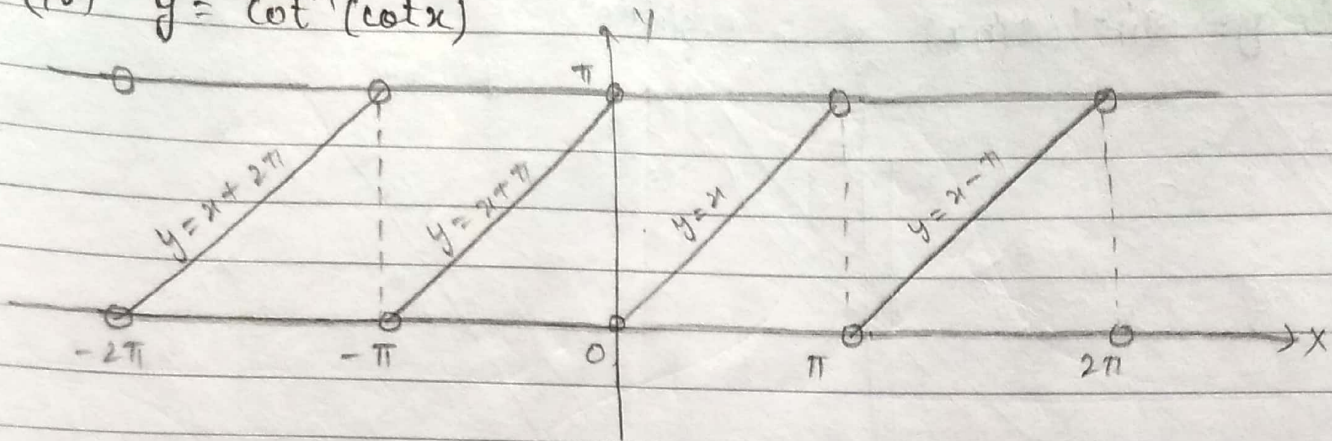
(iii) $y = \tan^{-1}(\tan x)$



$$x \in \mathbb{R} - \left\{ (2n-1)\frac{\pi}{2}, n \in \mathbb{I} \right\}; y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin^{-1} \sin \frac{1}{5} = \frac{1}{5}$$

(iv) $y = \cot^{-1}(\cot x)$



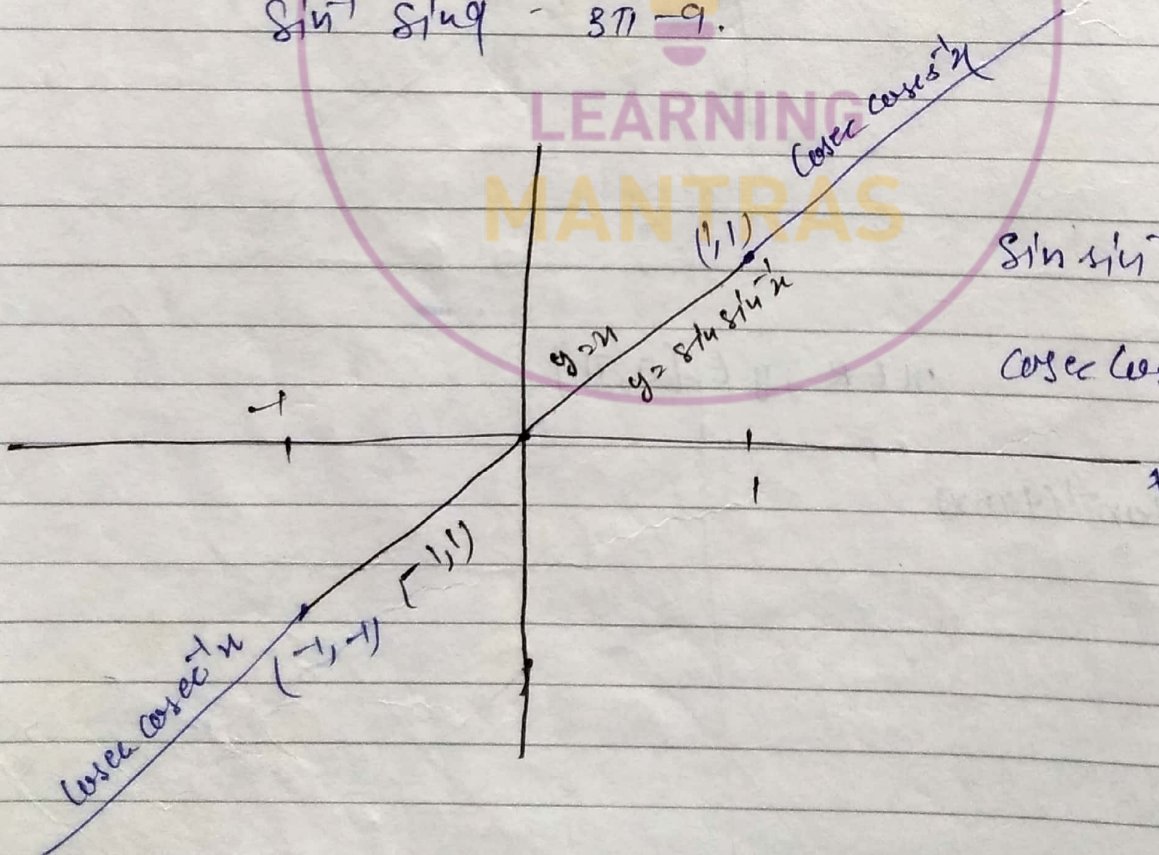
$$x \in \mathbb{R} - \{n\pi, n \in \mathbb{I}\}, y \in (0, \pi)$$

Properties -

$$\begin{aligned} \sin^{-1} \sin x &= x & \sin^{-1} \sin \frac{1}{5} &= \frac{1}{5} \\ \sin^{-1} \sin 2 &= 2 & \sin^{-1} \sin \pi &= \pi \\ \sin^{-1} \sin 3 &= 3 & \sin^{-1} \sin \pi &= \pi \\ \sin^{-1} \sin 6 &= 6 - 2\pi & \sin^{-1} \sin \pi &= \pi \\ \sin^{-1} \sin 9 &= 9 - 3\pi \end{aligned}$$

$$\begin{aligned} y = 0 &= 1(x - 2\pi) \\ y - 0 &= -1(x - \pi) \\ y - y_1 &= m(x - x_1) \end{aligned}$$

LEARNING MANTRAS



$$\begin{aligned} \sin \sin^{-1} \frac{1}{5} &= \frac{1}{5} \\ \operatorname{cosec} \operatorname{cosec}^{-1} (-5) &= -5 \end{aligned}$$

$$x + \frac{1}{x} \in (-\infty, -2) \cup (2, \infty)$$

19/05/17

Exercise 10-1

⑧ $\tan^{-1}\left(1 - x^2 - \frac{1}{x^2}\right) + \sin^{-1}\left(x^2 + \frac{1}{x^2} - 1\right)$

Domain = $\{1, -1\}$. $\frac{x = \pm 1}{x}$ $x + \frac{1}{x} \in (-\infty, -2] \cup (2, \infty)$

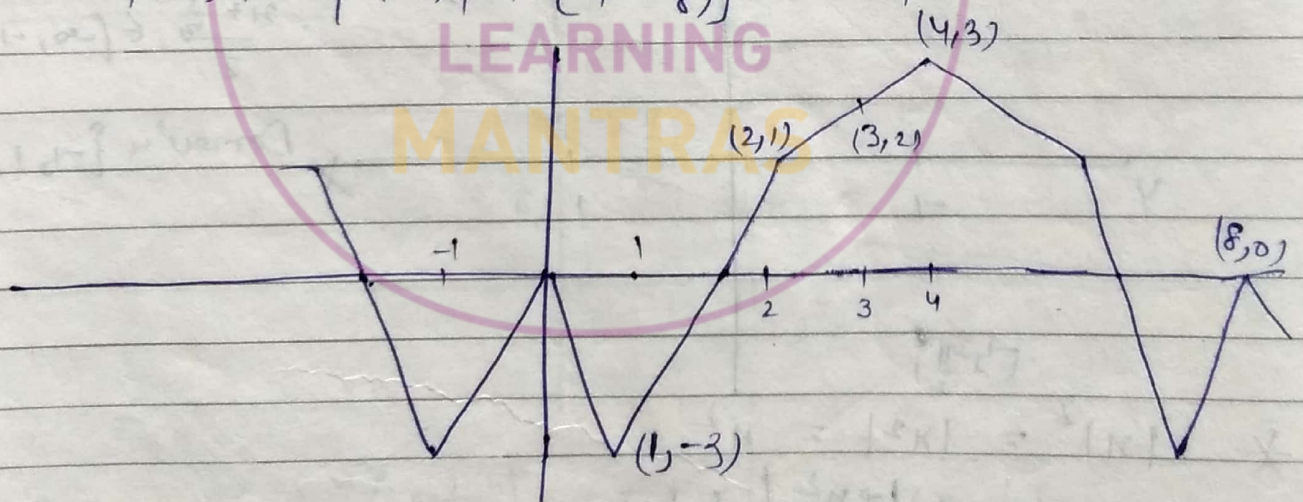
$$x^2 + \frac{1}{x^2} \in [2, \infty)$$

10) $\sin\left(\underbrace{2 \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)}_{\theta_1}\right) + \cos\left(\underbrace{2 \tan^{-1}\left(\frac{1}{3}\right)}_{\theta_2}\right) = \frac{p}{q}$

$$\frac{p}{q} = 1$$

11) $f(-x) = f(x)$

$$f(3) + 2|f(1)| + \left[f\left(\frac{7}{8}\right)\right] + \cos^{-1} f(2) + f(7) + f(20)$$

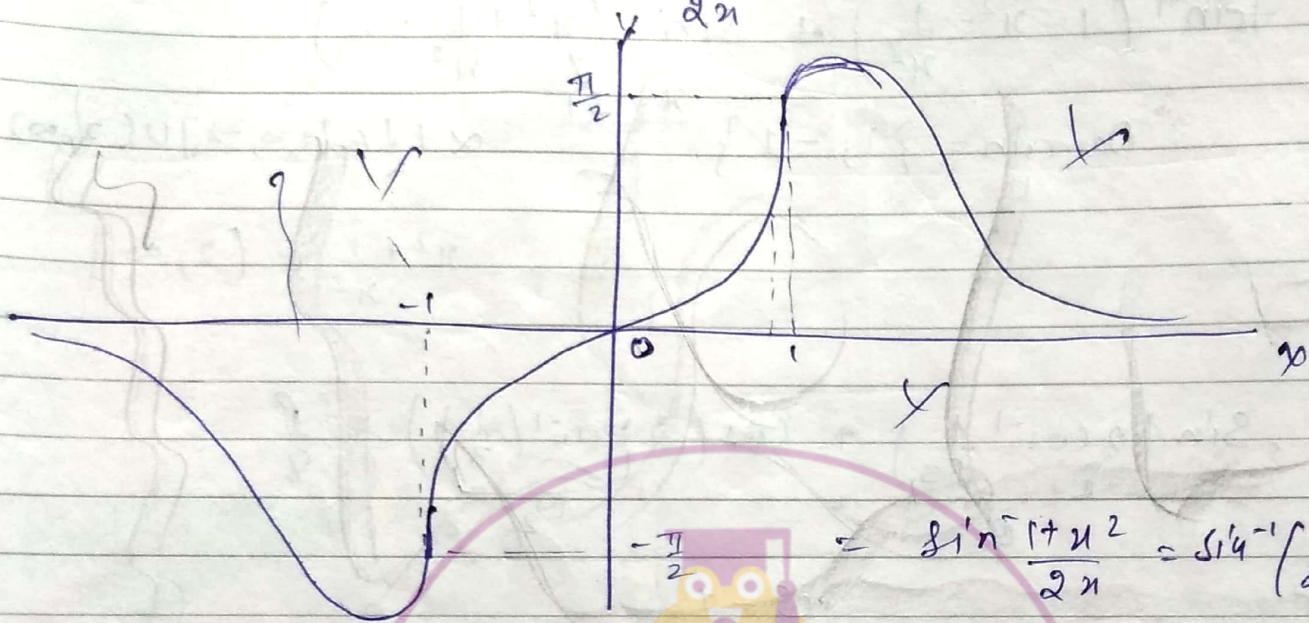


$$f(8 \times 2 + 4) = f(4) = 3$$

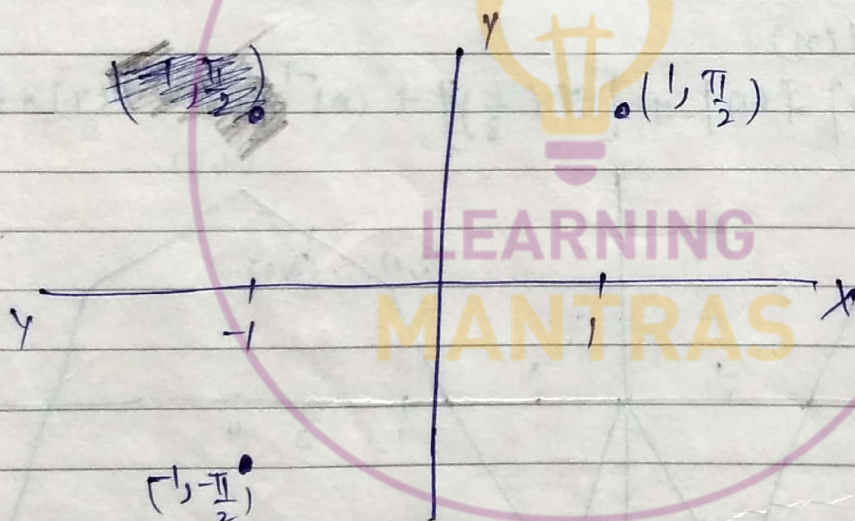
$$\begin{aligned} f(8 + (-1)) &= f(-1) \\ &= f(1) \\ &= -3 \end{aligned}$$

$$\begin{aligned} f(24 + \pi) &= f(\pi) \\ f(24 \times 2 + 4) &= f(4) \\ f(26 + \pi) &= f(\pi) \end{aligned}$$

Ques! $f(x) = \sin^{-1} \frac{1+x^2}{2x}$



$$-\frac{\pi}{2} = \sin^{-1} \frac{1+x^2}{2x} = \sin^{-1} \left(\frac{1}{2} \left(x + \frac{1}{x} \right) \right)$$



$$\frac{x+1}{x} \in (-\infty, -2) \cup (2, \infty)$$

$$\frac{x+\frac{1}{x}}{2} \in (-\infty, -1] \cup [1, \infty)$$

Domain $x \in \{-1, 1\}$

$$x \quad |x|^2 = |x^2| = x^2$$

$$\left| \frac{1+x^2}{2x} \right| \leq 1$$

$$|1+x^2| \leq 2|x|$$

$$1+x^2 \leq 2|x|$$

$$|x^2| + |1-2|x|| \leq 0$$

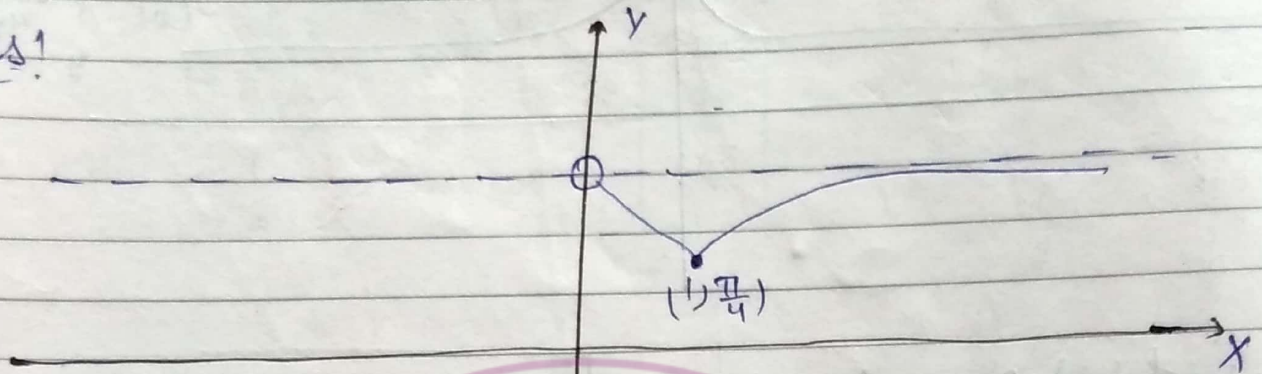
$$(|x|-1)^2 \leq 0$$

$$||x|-1| = 0$$

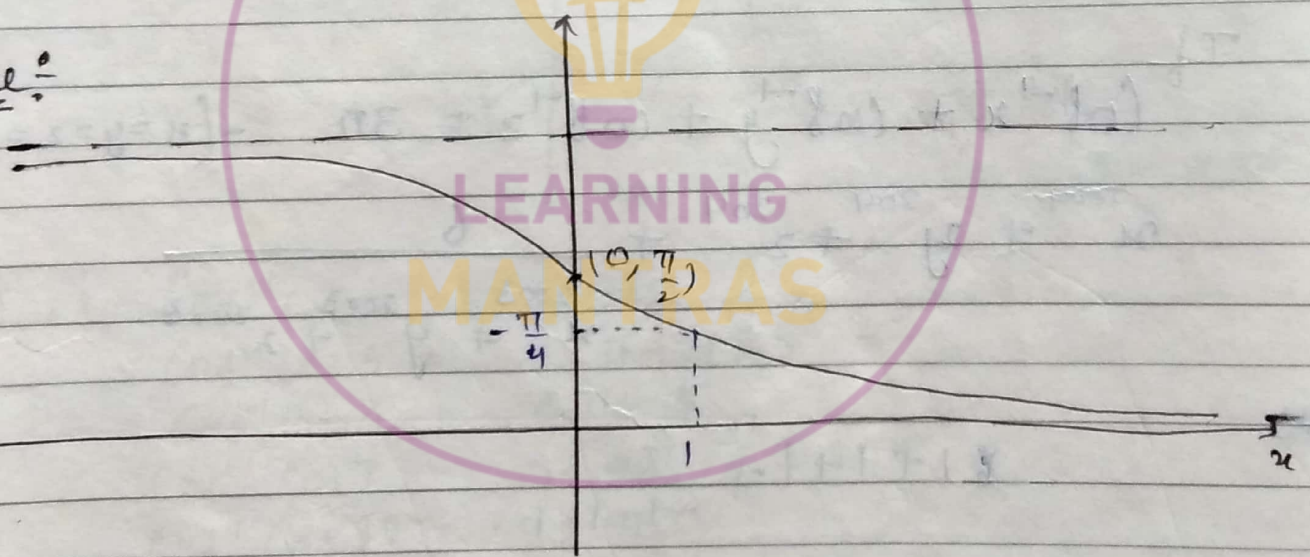
$$|x|=1 \Rightarrow x = \pm 1 \quad \text{A}$$

Que! $f(x) = \tan^{-1}\left(\frac{x^2+1}{2x}\right) = \tan^{-1}\left(\frac{1}{2}\left(x + \frac{1}{x}\right)\right)$

Ans!

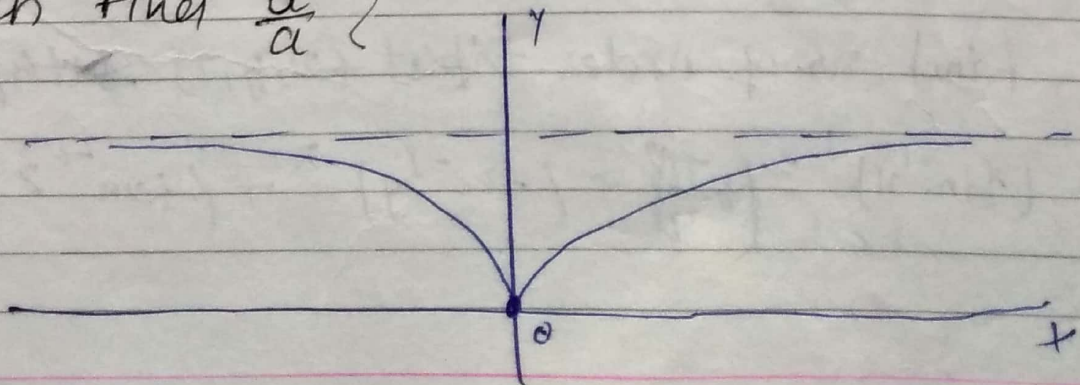


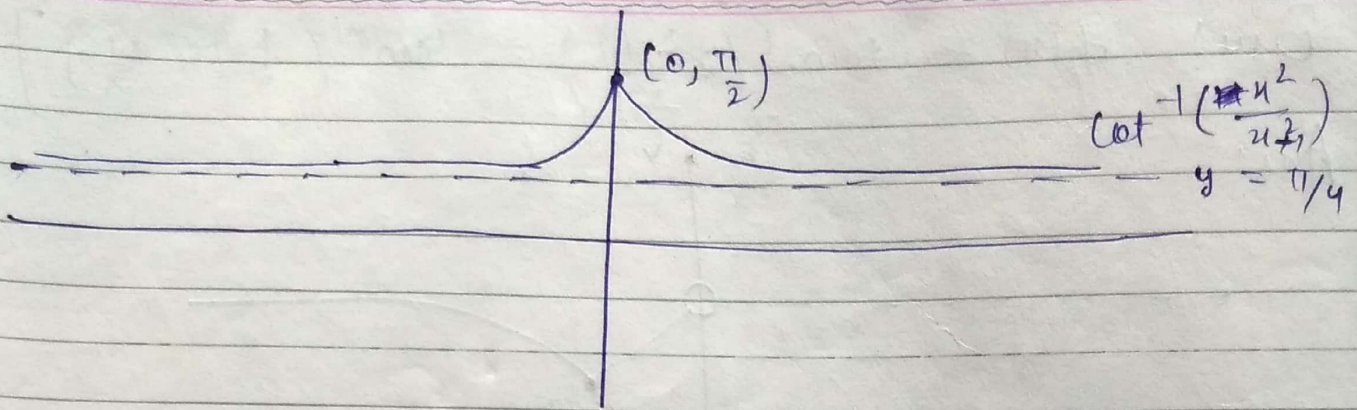
Que:



$f(x) = \cot^{-1}\left(\frac{x^2}{x^2+1}\right)$, If range (a, b)

then find $\frac{b}{a}$?





Que! find Domain $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$

Ans =

* If $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = 3\pi$ ⇒ $(x=y=z=1)$

$$\frac{\cot^{-1}x + \cot^{-1}y + \cot^{-1}z}{\cot^{-1}1 + \cot^{-1}1 + \cot^{-1}1} = \frac{3\pi}{\pi} = 3$$

Ans

$$1 + 1 + 1 = \frac{6}{-1 - 1 - 1}$$

$$= \frac{3 - 6}{3}$$

$$= 1 \text{ Ans}$$

Que! Find no. of order triplet (x, y, z) satisfying.

$$(\sin^{-1}x)^2 = \left(\frac{\pi}{4}\right)^2 + (\sec^{-1}y)^2 + (\tan^{-1}z)^2$$

$$y - y' = m(m - h, i) \quad \text{P.N.}$$

Stoke Value

Boundary Problem Value:

Ans! $\sin^{-1} x = \frac{\pi^2}{4} + (\sec^{-1} y)^2 + (\tan^{-1} z)^2$

minimum value $\rightarrow 0$
minimum value $\rightarrow 0$

$$\left(\pm \frac{\pi}{2}\right)^2$$

$$x = \pm 1$$

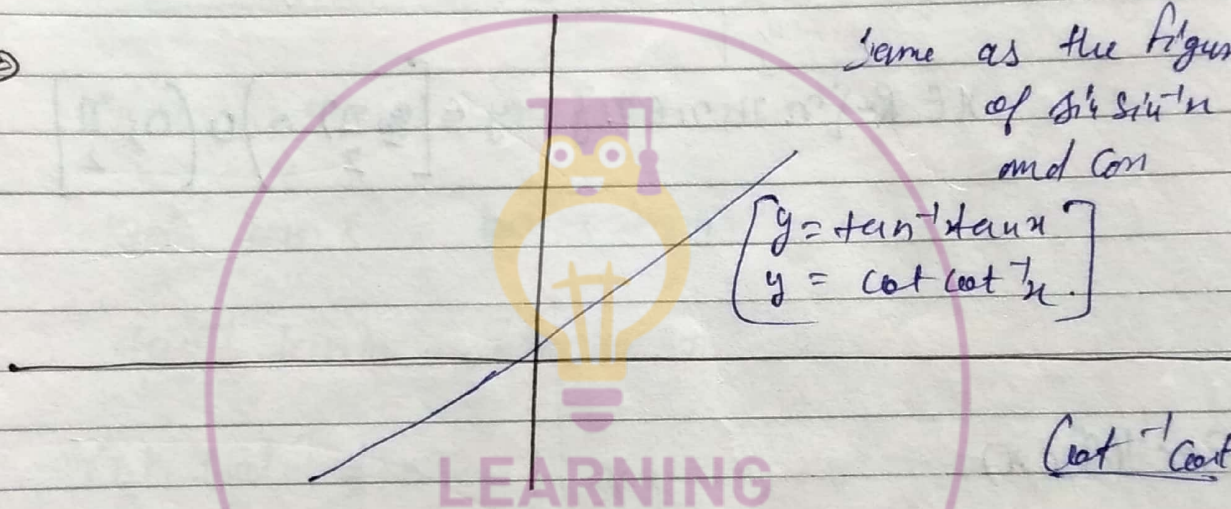
$$z = 0$$

x	y	z
1	1	0
-1	1	0

$$\text{at } z = 0$$

$$2 \times 1 \times 1 = 2$$

\rightarrow



Same as the figure of $\sin^{-1} x$ and $\cos^{-1} x$

$$\begin{cases} y = \tan^{-1} \tan x \\ y = \cot^{-1} \cot x \end{cases}$$

$$\cot^{-1} \cot x$$

Que! $\cos^{-1} \cos 2 = 2$

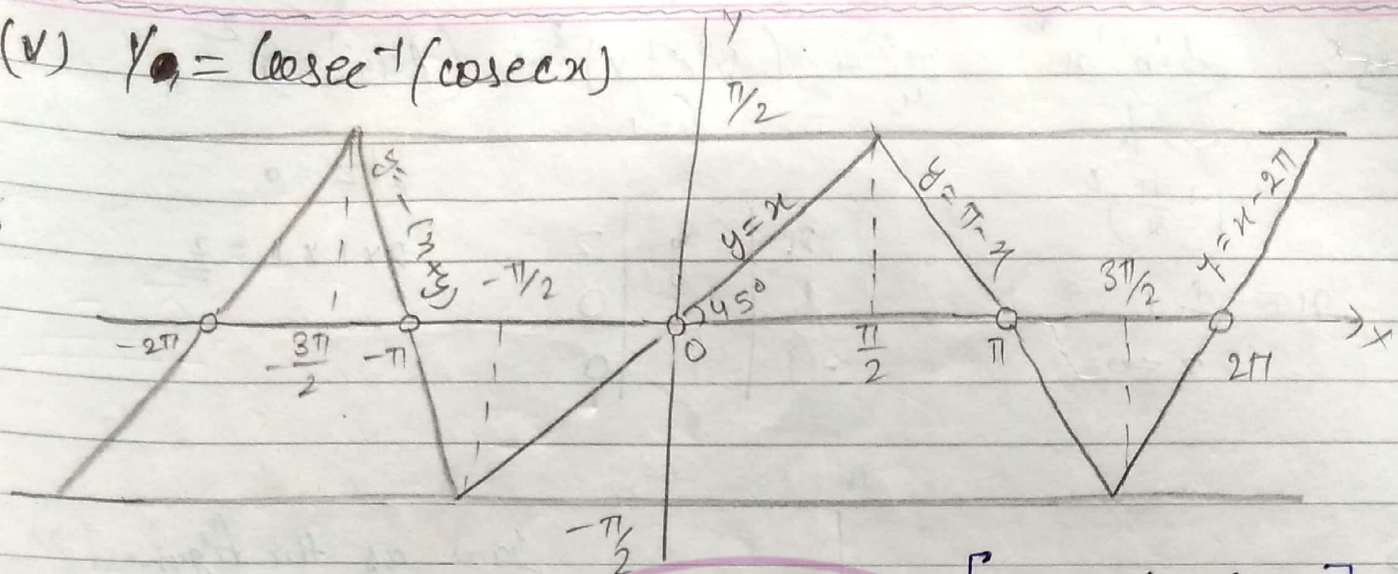
$\cos^{-1} \cos 5 = 2\pi - 5$

$\cos^{-1} \cos 6 = 2\pi - 6$

$\cos^{-1} \cos 7 = 7 - 2\pi$

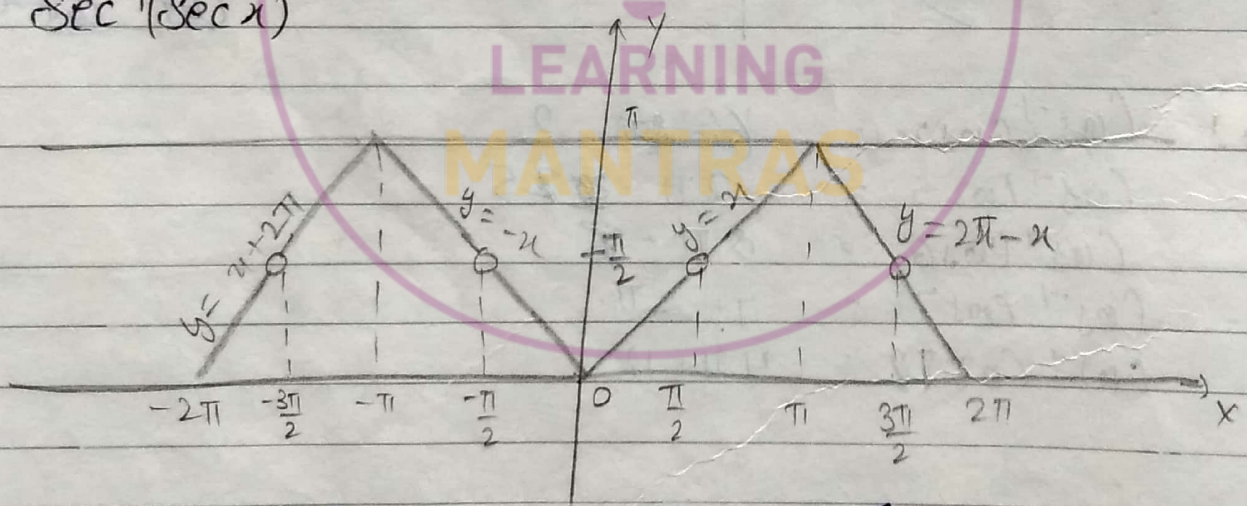
$\cos^{-1} \cos 12 = 4\pi - 12$

(v) $y = \operatorname{cosec}^{-1}(\operatorname{cosec} x)$



$x \in \mathbb{R} - \{n\pi, n \in \mathbb{I}\}, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \cup \left(0, \frac{\pi}{2}\right]$

(vi) $\operatorname{Sec}^{-1}(\operatorname{Sec} x)$



$x \in \mathbb{R} - \left\{(2n-1)\frac{\pi}{2}, n \in \mathbb{I}\right\}, y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

Date: 20/05/17

Ques: $\sin^{-1} \sin(-2) = \pi + 2$

$\sin^{-1} \sin 4 = \pi - 4$

$\sin^{-1} \sin 10 = 3\pi - 10$

$\cos^{-1} \cos(-10) = 10 - 4\pi - 10$

$\cos^{-1} \cos 5 = 2\pi - 5$

$\tan^{-1} \tan 6 = \pi + 6 - 2\pi$

$\tan^{-1} \tan 10 = 10 - 3\pi$

$\cot \cot^{-1} \frac{1}{2} = \frac{1}{2}$

$\cot \cot^{-1} 20 = 20$

P 2.0

i) $\sin^{-1} x = \operatorname{cosec}^{-1} \frac{1}{x}, x > 0 \text{ \& } x \in \text{domain.}$

ii) $\cos^{-1} x = \operatorname{sec}^{-1} \frac{1}{x}$

(iii) $\cot^{-1} x = \begin{cases} \tan^{-1} \frac{1}{x}, & x > 0 \\ \pi + \tan^{-1} \frac{1}{x}, & x < 0 \end{cases}$

$$\operatorname{cosec}^{-1} x = \sin^{-1} \frac{1}{x}$$

P-3:

$$(i) \sin^{-1}(-x) = -\sin^{-1} x$$

$$(ii) \cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$(iii) \tan^{-1}(-x) = -\tan^{-1} x$$

$$(iv) \cot^{-1}(-x) = \pi - \cot^{-1} x$$

P-4:

$$(i) \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad |x| \leq 1 \text{ or } -1 \leq x \leq 1$$

$$(ii) \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \quad x \in \mathbb{R}$$

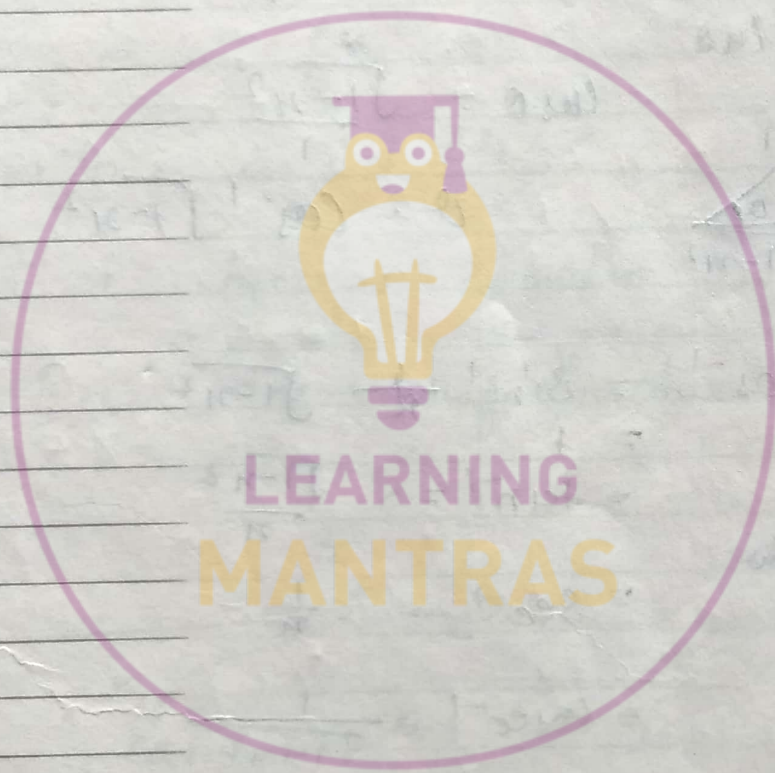
$$(iii) \operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2} \quad |x| \geq 1 \text{ or } x \in (-\infty, -1] \cup [1, \infty)$$

P-5:

$$(i) (a) \tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1} \left(\frac{x+y}{1-xy} \right) & xy < 1 \\ \pi + \tan^{-1} \frac{x+y}{1-xy} & xy > 1 \end{cases} \quad x > 0, y > 0$$

$$(b) \tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy} \quad x > 0, y > 0$$

$$(c) \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1} \frac{x+y+z-xyz}{1-(xy+yz+zx)}$$



Q	cos	+
P	B	P
H	H	B

Q1: * If $x > 0$, $\sin^{-1} x =$

$$\frac{\sin^{-1} x}{0} = \operatorname{cosec}^{-1} = \frac{1}{x}$$

$$= \operatorname{csc}^{-1} = \frac{1}{\sqrt{1-x^2}}$$

$$= \tan^{-1} = \frac{x}{\sqrt{1-x^2}}$$

$$\tan \theta = \frac{x}{\sqrt{1-x^2}}$$

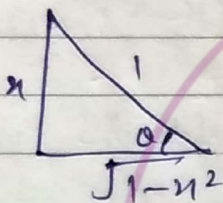
$$\sin^{-1} x = \theta$$

$$= \sec^{-1} = \frac{1}{\sqrt{1-x^2}}$$

$$\text{take sin with side} = \cot^{-1} \rightarrow \frac{\sqrt{1-x^2}}{x}$$

$$\sin \sin^{-1} x = \sin \theta$$

$$x = \sin \theta$$



$$\cos \theta = \frac{\sqrt{1-x^2}}{1}$$

$$\theta = \cos^{-1} \sqrt{1-x^2}$$

Q2) $x > 0$ $\cos^{-1} x = \sin^{-1} \frac{1}{x} \cdot \sqrt{1-x^2}$

$$\cos^{-1} x = \theta$$

$$\cos^{-1} \cos \theta = \cos^{-1} x$$

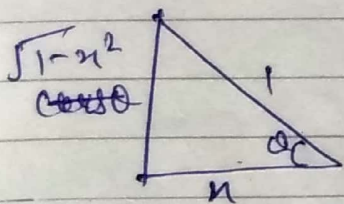
$$x = \cos \theta$$

$$= \tan^{-1} = \frac{\sqrt{1-x^2}}{x}$$

$$= \sec^{-1} = \frac{1}{x}$$

$$= \operatorname{cosec}^{-1} = \frac{1}{\sqrt{1-x^2}}$$

$$= \cot^{-1} = \frac{x}{\sqrt{1-x^2}}$$

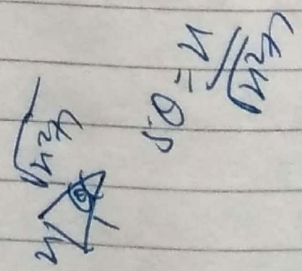


$$\tan = \frac{\sqrt{1-x^2}}{x}$$

$$\sec = \frac{1}{x}$$

Q3) $x > 0$ $\tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1+x^2}}$

$$\cos^{-1} = \frac{1}{\sqrt{1+x^2}}$$



$$\text{decreasing } u \Rightarrow \frac{dy}{dx} < 0 \text{ if } u$$

$$\text{increasing } u = \frac{dy}{dx} > 0 \text{ if } u$$

* Inequalities in case of inverse function.

\sin^{-1} is increasing function

$$y = \sin^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} > 0$$

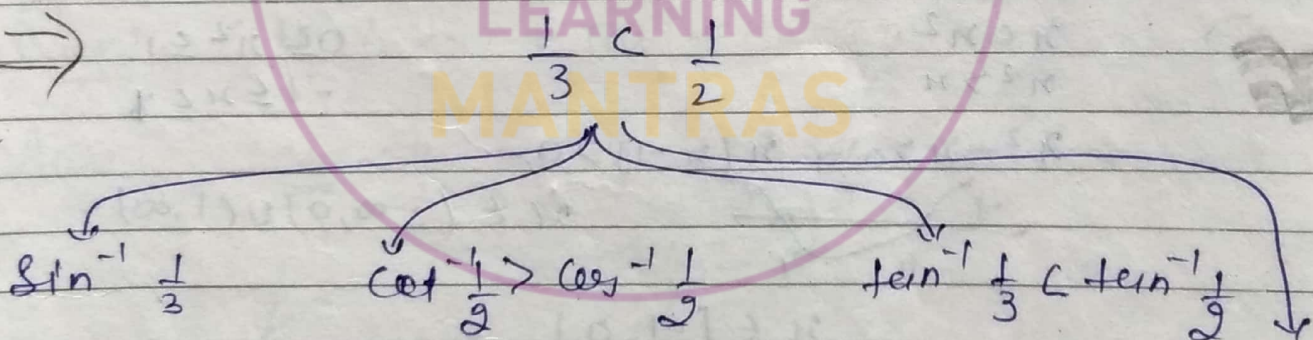
in : I fu.

$\cos^{-1} x$ is decreasing function

$\tan^{-1} x$ increasing function

$\cot^{-1} x$ decreasing function.

\Rightarrow



Ques!

Solve $\sin^{-1} x > \sin^{-1}(\frac{1}{2})$

$$x > \frac{1}{2} - x$$

$$2x > \frac{1}{2}$$

$$x > \frac{1}{4}$$

$$\frac{1}{4} < x \leq 1$$

Ans

$$-1 \leq x \leq 1 \text{ (i)}$$

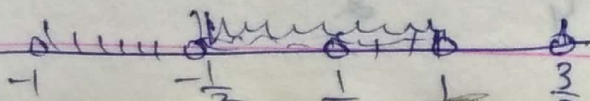
$$-1 \leq \frac{1}{2} - x \leq 1$$

$$-1 - \frac{1}{2} \leq -x \leq 1 - \frac{1}{2}$$

$$-\frac{3}{2} \leq -x \leq \frac{1}{2}$$

$$\frac{1}{2} \geq x \geq -\frac{1}{2}$$

$$-\frac{1}{2} \leq x \leq \frac{3}{2} \text{ (ii)}$$



$$\cos^{-1} x \geq \cos^{-1} \left(\frac{1}{2} - x \right)$$

$$\rightarrow -1 \leq x \leq 1 \quad \text{--- (i)}$$

$$-1 \leq \frac{1}{2} - x \leq 1 \quad \text{--- (ii)}$$

$$\rightarrow x \leq \frac{1}{2} - x \quad \text{--- (iii)}$$

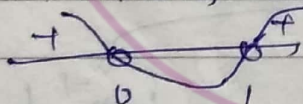
Que! Solve $\cos^{-1} x > \cos^{-1} x^2$

$$\left\{ \begin{array}{l} \text{A } -1 < x < 1 \\ -1 \leq x \leq 1 \\ -1 \leq x^2 \leq 1 \\ x \in (-1, 1) \\ x \in (-\infty, \infty) \end{array} \right. \left\{ \begin{array}{l} -\infty < x < \infty \\ x \in x^2 \\ x \in x^2 \\ x \in x^2 \\ (-\infty, \infty) \end{array} \right.$$

$$\begin{array}{l} -1 \leq x \leq 1 \\ x \in x^2 \\ x^2 > x \end{array}$$

$$\begin{array}{l} -1 \leq x^2 \leq 1 \\ 0 \leq x^2 \leq 1 \\ -1 \leq x \leq 1 \end{array}$$

$$x^2 - x > 0 \Rightarrow x(x-1) > 0$$



$$x \in (-\infty, 0) \cup (1, \infty)$$

$$x \in [-1, 0)$$

$$\text{Que! } \tan^{-1} 1 + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3}$$

$$xy = \frac{1}{2} \cdot \frac{1}{3} < 1$$

$$= \tan^{-1} 1 + \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} \right)$$

$$= \tan^{-1} 1 + \tan^{-1} 1$$

$$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

$$\boxed{\frac{\pi}{4} = \tan^{-1} 1}$$

$$\boxed{\cos^{-1} 1 + \cos^{-1} 2 + \cos^{-1} 3}$$

Ques = $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

$\underbrace{\qquad\qquad\qquad}_{x > 0} \qquad\qquad\qquad \underbrace{\qquad\qquad\qquad}_{x > 0}$

~~2 > 1~~
2.3 > 1

$$= \tan^{-1} \pi + \left(\frac{2+3}{1-6} \right)$$

$$= \tan^{-1} \pi + \left(\frac{5}{-5} \right)$$

$\tan^{-1} > -1$ $\tan^{-1} 1 + \pi + \tan^{-1} (-1)$

$$= \pi$$

Ques! $4 \sin^{-1} x + \cos^{-1} x = \frac{3\pi}{4}$

Ans! $\frac{4}{2} \cdot \frac{\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{4}$

$$= \frac{3\pi}{4} - \frac{2\pi}{2}$$

$$= \frac{3\pi - 4\pi}{4} = \frac{-\pi}{4}$$

Ques! $3 \sin^{-1} x + \sin^{-1} x + \cos^{-1} x = \frac{3\pi}{4}$

$$3 \sin^{-1} x + \frac{\pi}{2} = \frac{3\pi}{4}$$

$$3 \sin^{-1} x = \frac{\pi}{4}$$

$$\sin^{-1} x = \frac{\pi}{12} \longrightarrow x = \sin^{-1} \frac{\pi}{12}$$

→ Jarak sin baru ada. $\sin \sin^{-1} = \sin \frac{\pi}{12}$

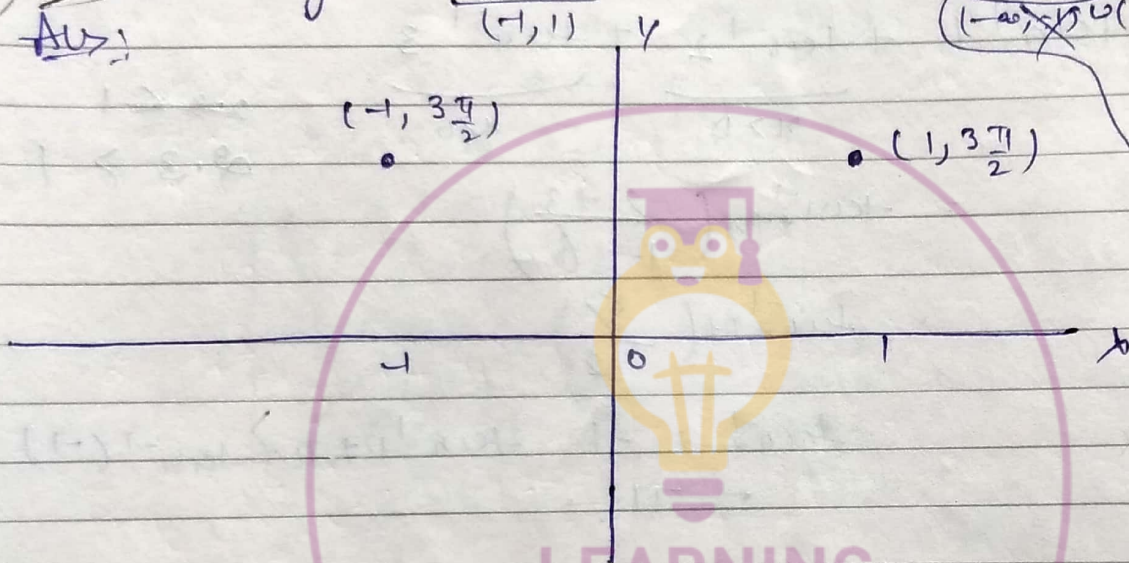
$$x = \sin \frac{\pi}{12}$$

\mathbb{R}

* 1) D: aw

A: s:

$$y = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x + \sec^{-1} x + \csc^{-1} x + \cot^{-1} x$$



$(-\infty, -1) \cup (1, \infty)$

$$\frac{\pi}{2} \pm \frac{\pi}{2} = \pi$$

$$\frac{\pi}{2} \pm \frac{\pi}{2} = \pi$$

$$\frac{\pi}{2} \pm \frac{\pi}{2} = \pi$$

$$= [-1, 1] \quad [-\infty, -1] \cup (1, \infty), \mathbb{R}$$

$$\text{Domain} = \{1, -1\}$$

$$y = \frac{3\pi}{2} \quad \left(\frac{\pi}{2}, \frac{3\pi}{2} \right)$$

Que: $y = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$

$x \in \mathbb{R}$

$$= \text{Domain} = [-1, 1]$$

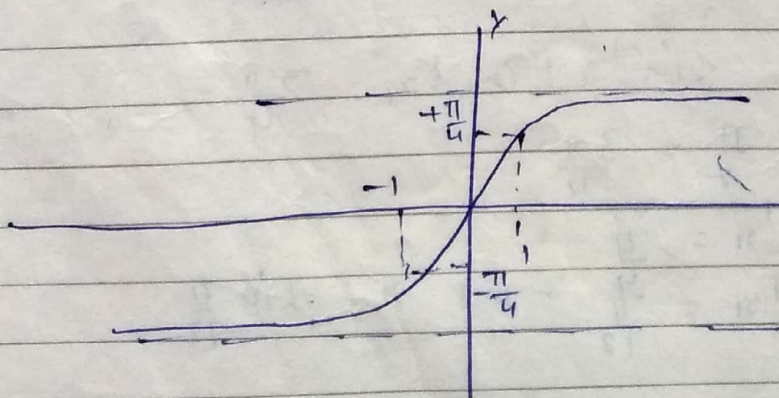
$$-1 \leq x \leq 1$$

$$-\frac{\pi}{4} \leq \tan^{-1} x \leq \frac{\pi}{4}$$

$$-\frac{\pi}{4} + \frac{\pi}{2} \leq \frac{\pi}{2} + \tan^{-1} x$$

$$\leq \frac{\pi}{2} + \frac{\pi}{4}$$

$$\frac{\pi}{4} \leq y \leq \frac{3\pi}{4}$$



Hiw: 0-1, 6, 8, 10, 12, 13, 14, 15,
 8-1: 8, 9, 10,
 Jy 4 →

Ques: Solve $\tan^{-1} x > 2$

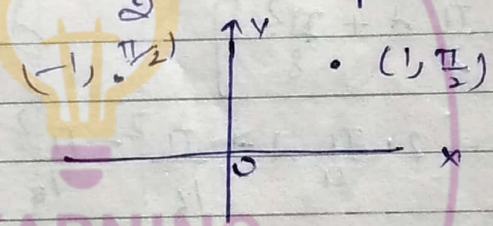
Ans: ~~Let~~ ~~$x > 2$~~ ~~$\tan^{-1} x$~~ $\tan^{-1} x$ is not greater than 2.
~~or~~ ϕ

Date: 22/05/17

Ex: 0-1

12) $f(x) = \sin^{-1}(\tan x) = \cos^{-1}(\cot x)$
 $x = +\frac{\pi}{4}$ $-1 \leq \cot x \leq 1$ or $-1 < \tan x \leq 1$
 $\tan = \cot x = 1$ or -1

$f(x) = \sin^{-1} + \cos^{-1} = \frac{\pi}{2}$
 $x = \frac{\pi}{4}$ $f(x) = \frac{\pi}{2}$ for $\forall y = f(x)$



(8) $1 + b + b^2 + \dots + \infty = \frac{a^3}{b} + \dots + \infty$
 $\frac{1}{1-b} = \frac{a}{1+\frac{a}{b}}$
 $\frac{1}{1-b} = \frac{3a}{3+a}$

(10)

$\frac{4}{5} + \frac{4}{5} = \frac{p}{q}$ $\frac{p}{q} = \frac{8}{5}$
 $p = 8, q = 5$ $\frac{(p-q)}{3^{2k+1}}$

$3^1 = 3$
 $3^3 = 27$
 $3^5 =$

22/05/17



Ques: 1) $f(x) = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$, $f(x) = (\sin^{-1}x)^3 + (\cos^{-1}x)^3$,

$f(x) = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$
 Formula
 $a^2 + b^2 = (a+b)^2 - 2ab$

Formula
 $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$= (\sin^{-1}x + \cos^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$ | $(\sin^{-1}x + \cos^{-1}x)^3$
 $= \frac{\pi^2}{4} - 2\sin^{-1}x \left(\frac{\pi}{2} - \sin^{-1}x \right)$

$= 2(\sin^{-1}x) - \pi\sin^{-1}x + \frac{\pi^2}{4}$

$= 2 \left[x^2 - \frac{\pi}{2}x + \frac{\pi^2}{8} \right]$ | $x = \sin^{-1}x$

$= 2 \left[x^2 - 2 \cdot \frac{\pi}{4}x + \frac{\pi^2}{16} - \frac{\pi^2}{16} + \frac{\pi^2}{8} \right]$

$= 2 \left[\left(x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{16} \right]$

$= 2 \left(\sin^{-1}x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{8}$

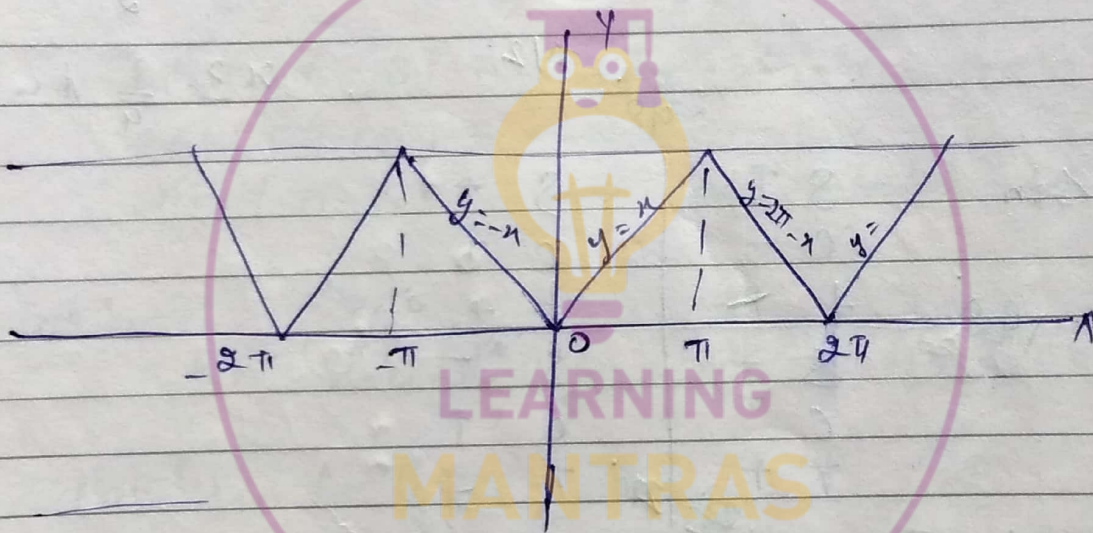
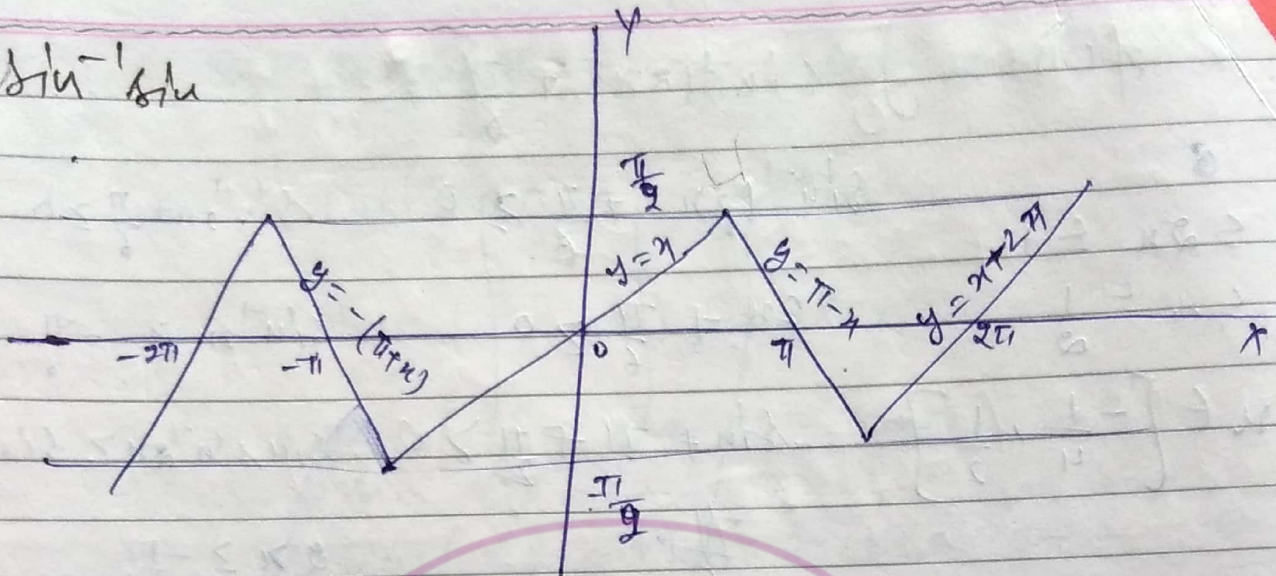
$= 2 \left(\sin^{-1}x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{8}$

$-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$

$f_{\max} = 2 \left[2 \cdot \frac{9\pi^2}{16} + \frac{\pi^2}{8} \right] = \frac{10\pi^2}{8} = \frac{5\pi^2}{4}$

$f_{\min} = \frac{\pi^2}{8}$ | $\sin^{-1}x = \frac{\pi}{4}$

$\sin^{-1} \sin$



Ques! Find domain
 $f(x) = \int \sin^{-1}(2x) + \frac{\pi}{6}$

③
 $-1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$

$x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$\sin^{-1}(2x) + \frac{\pi}{6} \geq 0$

$2x + \frac{\pi}{6} \geq 0$

$\sin \left(\frac{\pi}{6} + \frac{\pi}{2} \right) \geq 0$

$\sin^{-1} 2x + \frac{\pi}{6} \geq 0$

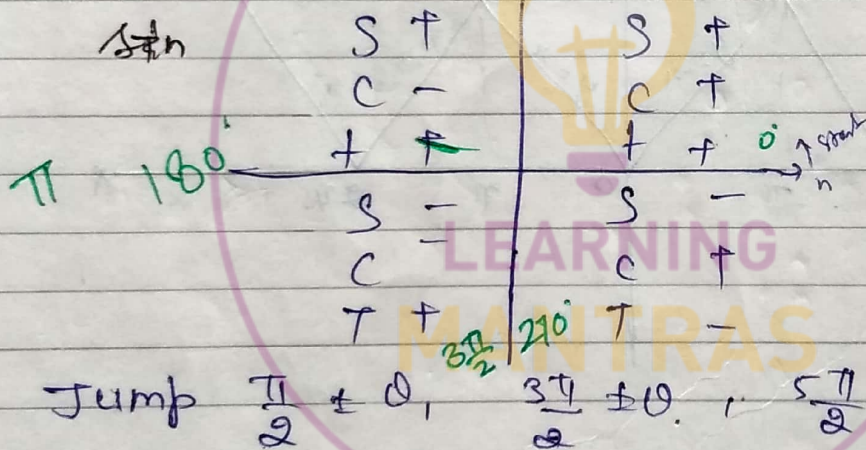
$\sin^{-1} 2x \geq -\frac{\pi}{6}$

$\sin \sin^{-1} 2x \geq \sin \left(\frac{\pi}{6} \right)$

$2x \geq -\frac{1}{2}$

$x \geq -\frac{1}{4}$

Ques! Sketch



S → C
 C → S
 T → cot

③ Jump $\pi \pm 0, 2\pi \pm 0, 3\pi \pm 0, \dots$

S → S
 C → C
 + → +

$$\sin^2\left(\frac{\pi}{2} + \theta\right) = + \cos^2 \theta$$

$$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot \theta$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = -\cot \theta$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = + \cot \theta$$

$$\sin(2\pi - \theta) = + \sin \theta$$

$$\cos(5\pi + \theta) = -\cos \theta$$

$$\tan(\pi - \theta) = -\tan \theta$$

Que:

$$\sin^{-1} \sin\left(\frac{10\pi}{7}\right) = \sin^{-1} \left(-\sin \frac{3\pi}{7}\right)$$

$$\cos^{-1} \sin\left(-\frac{\pi}{4}\right)$$

$$\sin^{-1} \left(\cos \frac{93\pi}{10}\right)$$

$$\textcircled{i} \quad \sin 10\pi = \sin\left(\pi + \frac{3\pi}{7}\right) = -\sin \frac{3\pi}{7}$$

$$\sin^{-1} \left(-\sin \frac{3\pi}{7}\right) = -\sin^{-1} \sin\left(\frac{3\pi}{7}\right) = -\frac{3\pi}{7} \text{ A}$$

$$\textcircled{ii} \quad \cos^{-1} \left(\cos\left(\frac{\pi}{2} - \left(-\frac{\pi}{4}\right)\right)\right) = \cos^{-1} \left(\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right)$$

$$= \frac{\pi}{2} + \frac{\pi}{4}$$

(iii)

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$(iii) \sin^{-1}\left(\cos \frac{33\pi}{10}\right) = \sin^{-1}\left(\cos\left(3\pi + \frac{3\pi}{10}\right)\right)$$

$$= \cos\left(\frac{3\pi + 3\pi}{10}\right) = -\cos \frac{3\pi}{10}$$

$$\sin^{-1}\left(\cos \frac{33\pi}{10}\right) = \sin^{-1}\left(-\cos \frac{3\pi}{10}\right)$$

$$= -\sin^{-1}\left(\cos \frac{3\pi}{10}\right) = -\sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)\right)$$

$$= -\sin^{-1}\left(\sin \frac{2\pi}{10}\right) = -\sin^{-1}\left(\sin \frac{\pi}{5}\right)$$

$$= -\frac{\pi}{5} \text{ Ans}$$

Que! Let, $a < b < c$ be three integers

$$\text{CO} \left(\frac{x^2(x-1)}{(x-3)(x^2-2x+5)} \right) < 0$$

then find value of $\cos^{-1}(a) + \sin^{-1}(b) + \sin^{-1}(c) + \text{cosec}^{-1}(b + \tan a)$

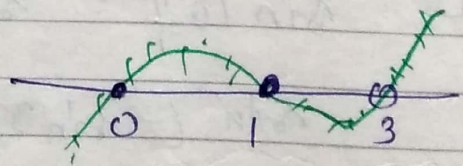
Ans!

$$\frac{x^2(x-1)(x-0)}{(x-3)}$$

$$x \in [1, 3) \cup \{0\}$$

\therefore Integral value =

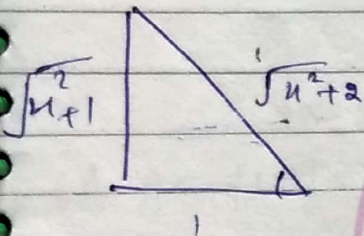
a	b	c
↓	↓	↓
0	1	2



$$\begin{aligned}
 &= \cos^{-1}(0) + \sin^{-1}(1) + \sec^{-1}(2) + \operatorname{cosec}^{-1} 1 + \tan^{-1} 0 \\
 &= \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{3} + \frac{\pi}{2} \\
 &= \frac{3\pi}{2} + \frac{\pi}{3} = \frac{11\pi}{6} \text{ Ans}
 \end{aligned}$$

Que! Prove that $\cos \tan^{-1} \sin \cot^{-1} x = \sqrt{\frac{x^2+1}{x^2+2}}$

$$\sin \alpha = \frac{x^2+1}{x^2+2}$$



$$\cot^{-1} x = \alpha$$

$$\cot \alpha = x$$

$$\tan \beta = \frac{1}{\sqrt{1+x^2}}$$

$$= \cos \tan^{-1} \sin \alpha$$

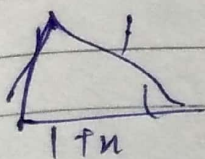
$$= \cos \tan^{-1} \frac{1}{\sqrt{1+x^2}}$$

$$= \cos \alpha$$

$$= \sqrt{\frac{1+x^2}{2+x^2}}$$

Que! $\sin \cot^{-1}(1+x) = \cos \tan^{-1} x$

Ans!



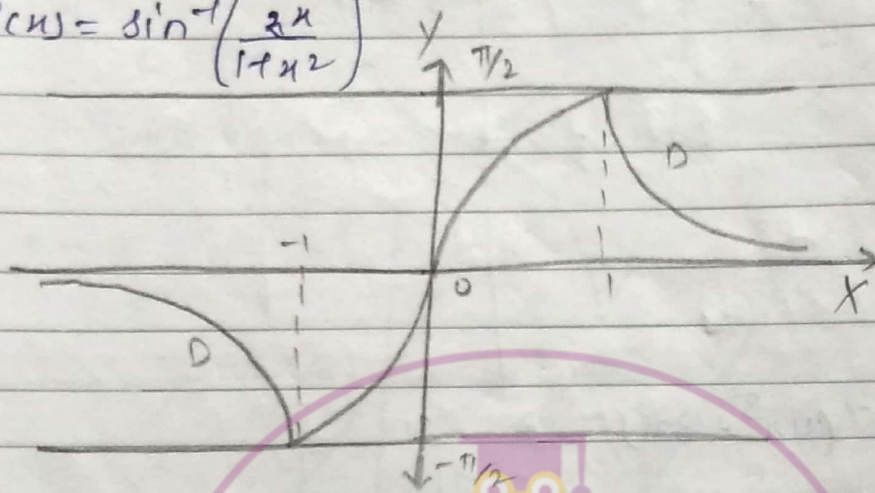
$$\cot \alpha = (1+x)$$

$$\tan \beta = (1+x)$$

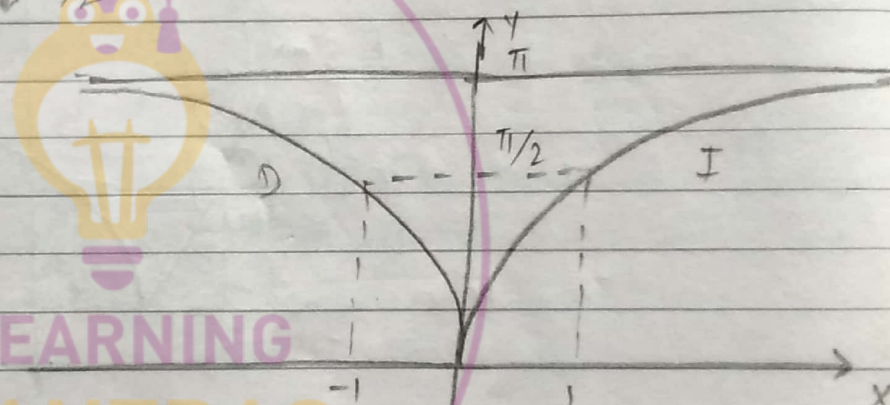
$$\cot \alpha = (1+x)$$

* Simplified inverse trigonometric function :

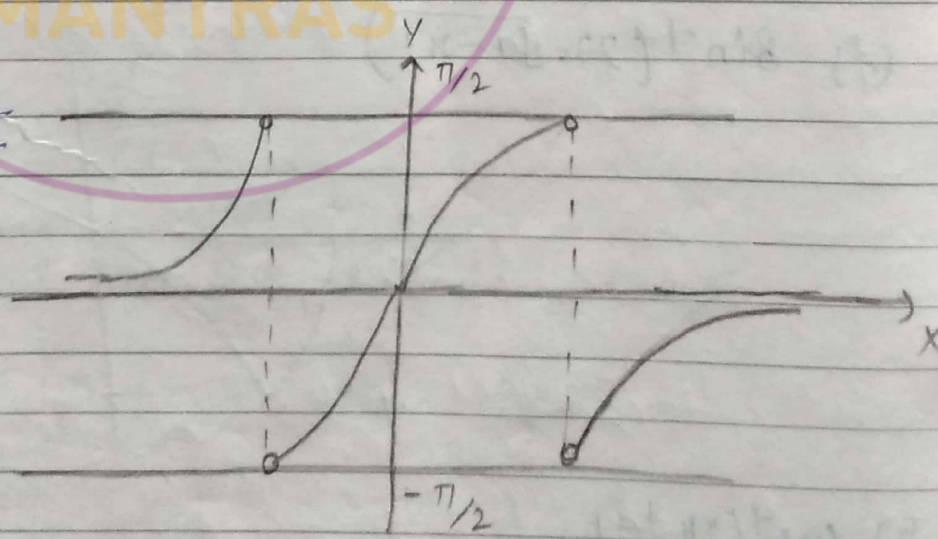
a) $y = f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$



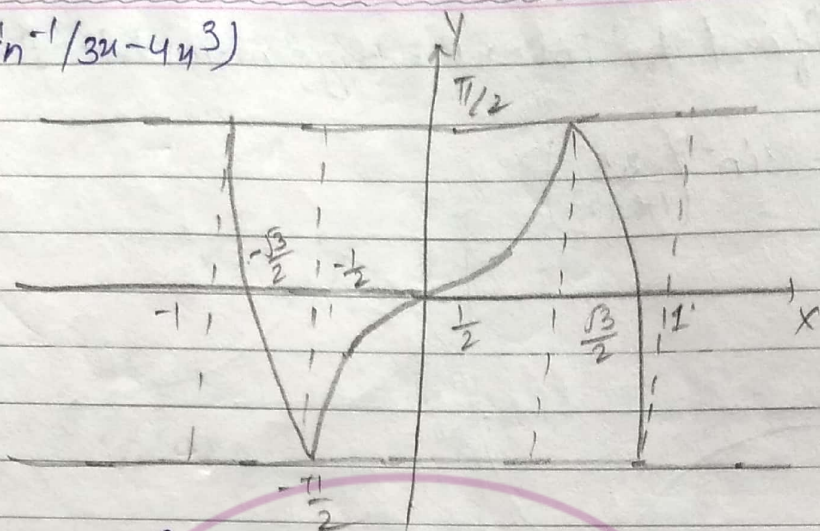
(b) $y = f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$



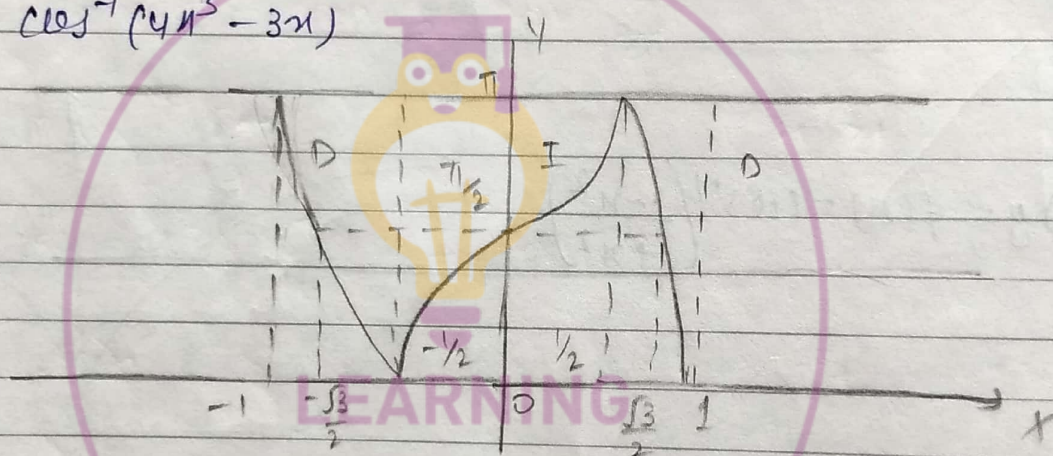
c) $y = f(x) = \tan^{-1}\frac{2x}{1-x^2}$



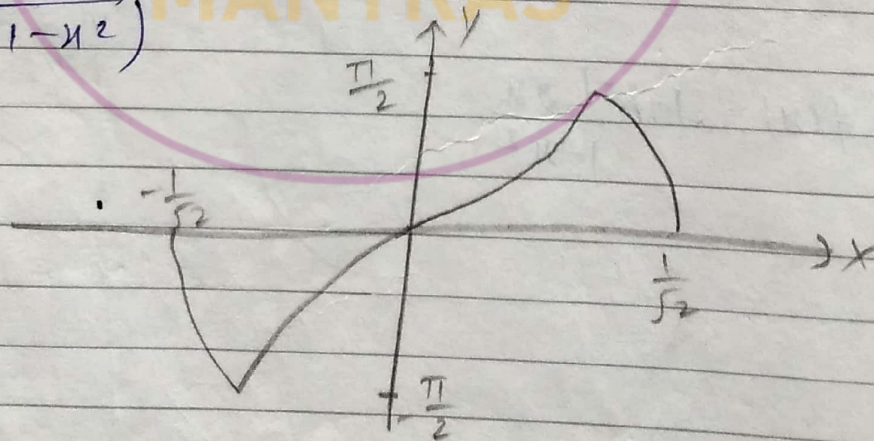
d) $y = f(x) = \sin^{-1}(3x - 4x^3)$



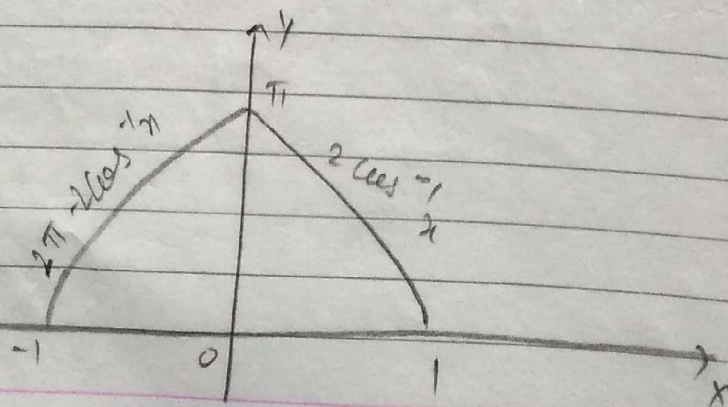
e) $y = f(x) = \cos^{-1}(4x^3 - 3x)$



f) $\sin^{-1}(2x\sqrt{1-x^2})$



g) $\cos^{-1}(2x^2 - 1)$



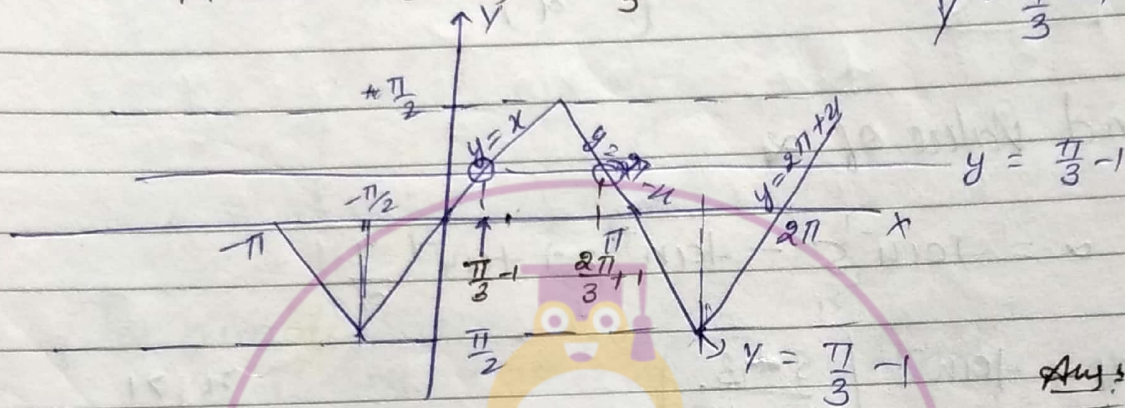
Date: 23/05/17

Que: find no. of points find no. of x . Satisfying

$$x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2} \right]$$

$$1 + \sin^{-1}(\sin x) = \frac{\pi}{3}$$

$$y = \frac{\pi}{3} - 1 = \text{small + we No.}$$



Ans: 2

Method - 2:

Case - 1: $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ $x = \frac{\pi}{3} - 1$

Case - 2: $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

$$\pi - x = \frac{\pi}{3} - 1$$

$$x = \frac{2\pi}{3} + 1$$

Que: 2. If $\tan^{-1} \frac{4}{x} + \tan^{-1} \frac{5}{y} = \cot^{-1} \lambda$.

Ans:

$$\pi + \tan^{-1} \left[\frac{4+5}{1-20} \right]$$

$xy > 1$

$$\pi + \tan^{-1} \left[\frac{9}{-19} \right]$$

$$\pi + \tan^{-1} \left[-\frac{9}{19} \right]$$

$$\begin{aligned}\cot^{-1} 2 &= \pi - \tan^{-1} \frac{9}{19} \\ &= \pi - \cot^{-1} \left(\frac{19}{9} \right) \\ &= \cot^{-1} \left(-\frac{19}{9} \right)\end{aligned}$$

Ques: find value of α

$$\alpha = \tan^{-1} \frac{5}{8} - \tan^{-1} \frac{3}{5} + \tan^{-1} \frac{7}{9}$$

Ans:

$$\tan^{-1} \left(\frac{5+3}{1+15} \right)$$

$$xy > 1$$

$$\pi - \tan^{-1} \left(\frac{8}{16} \right) + \tan^{-1} \frac{7}{9}$$

$$= \tan^{-1} \frac{5-3}{1+5 \cdot 3} + \tan^{-1} \frac{7}{9}$$

$$\pi - \frac{5}{14} + \frac{7}{9}$$

$$\frac{98}{126} + \frac{7}{9}$$

$$= \tan^{-1} \frac{1}{8} + \tan^{-1} \frac{7}{9}$$

$$\frac{1}{8} \cdot \frac{7}{9} < 1$$

$$= \tan^{-1} \left(\frac{\frac{1}{8} + \frac{7}{9}}{1 - \frac{1}{8} \cdot \frac{7}{9}} \right)$$

$$= \tan^{-1} \frac{65}{65} = \tan^{-1} 1$$

$$= \frac{\pi}{4} \text{ Ans}$$

Ques: $\tan^{-1} \frac{n+1}{n-1} + \tan^{-1} \frac{n-1}{n} = \tan^{-1} (-7)$

Ans:

$$\tan^{-1} \left(\frac{\frac{n+1}{n-1} + \frac{n-1}{n}}{1 - \frac{n+1}{n-1} \cdot \frac{n-1}{n}} \right) = \tan^{-1} (-7)$$

$$\tan^{-1} 1 = \frac{\pi}{4}$$

Take \tan both side.

$$\tan \tan^{-1} \left(\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \frac{x+1}{x-1} \cdot \frac{x-1}{x}} \right) = \tan \tan^{-1} (7)$$

$$\left(\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \frac{x+1}{x-1} \cdot \frac{x-1}{x}} \right) = (-7)$$

After simplification.

$$x^2 - 4x + 4 = 0$$

$$x = 2$$

R.H.S

is -ve.

Now check question.

$$\tan^{-1} 3 + \tan^{-1} \frac{1}{2} \quad \times$$

\Rightarrow L.H.S = +ve.

Que: $\sin^{-1} \frac{1}{\sqrt{5}} + \cos^{-1} x = \frac{\pi}{4}$

Ans

$$\tan^{-1} \frac{1}{2} + \dots = \tan^{-1} 1$$

$$\cos^{-1} x = \tan^{-1} 1 - \sin^{-1} \frac{1}{\sqrt{5}}$$

$$= \tan^{-1} 1 - \tan^{-1} \frac{1}{2}$$

$$= \tan^{-1} \frac{1 - \frac{1}{2}}{1 + 1 \cdot \frac{1}{2}}$$

$$= \tan^{-1} \frac{1}{3}$$

$$= \cos^{-1} \frac{3}{\sqrt{10}}$$

$$x = \frac{3}{\sqrt{10}}$$

$$\frac{3}{\sqrt{10}}$$

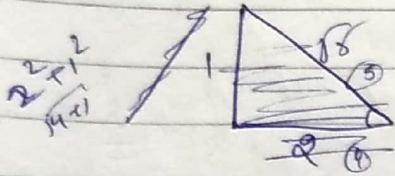
Que! Find x

$$2 \cot^{-1} 2 - \cos^{-1} \frac{4}{5} = \operatorname{cosec}^{-1} x$$

Ans $\cot \alpha = -4$

~~$\alpha = \cot^{-1} 4$~~

~~$$\alpha = \frac{2}{3} + \cot^{-1} \frac{4}{5}$$~~



$$\text{L.H.S} = 2 \tan^{-1} \frac{1}{2} = \cos^{-1} \frac{4}{5}$$

$$\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{2} = \tan^{-1} \frac{3}{4}$$

$$= \tan^{-1} \frac{\frac{1}{2} + \frac{1}{2}}{1 - \frac{1}{2} \cdot \frac{1}{2}}$$

$$= \tan^{-1} \frac{4}{3} - \tan^{-1} \frac{3}{4} = \tan^{-1} \left(\frac{\frac{4}{3} - \frac{3}{4}}{1 + \frac{4}{3} \cdot \frac{3}{4}} \right)$$

$$= \tan^{-1} \frac{7}{24} = \operatorname{cosec}^{-1} \frac{25}{7}$$

$$x = \frac{25}{7} \quad \text{Ans}$$

Method 2

$$\Rightarrow \tan^{-1} \frac{1}{2} + \cot^{-1} x = \frac{\pi}{4}$$

take tan both side

$$\tan \left(\tan^{-1} \frac{1}{2} + \cot^{-1} x \right) = \tan \frac{\pi}{4}$$

$$\frac{\tan \theta_1 + \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2} = 1$$

$$\cos^{-1} x \sin^{-1} x = 2$$

$$\frac{\tan^{-1} \frac{1}{2} + \tan^{-1} \cos^{-1} x}{1 - \left(\tan^{-1} \frac{1}{2}\right) \left(\tan^{-1} \cos^{-1} x\right)} = 1$$

$$\frac{\frac{1}{2} + \frac{\sqrt{1-x^2}}{x}}{1 - \frac{1}{2} \cdot \frac{\sqrt{1-x^2}}{x}} = 1$$

$$\frac{\frac{1}{2} + \frac{\sqrt{1-x^2}}{x}}{1 - \frac{1}{2} \cdot \frac{\sqrt{1-x^2}}{x}} = \frac{-1-x}{2} \cdot \frac{\sqrt{1-x^2}}{x}$$

=

Que: $\cos^{-1} x - \sin^{-1} x = \cos^{-1} x \sqrt{3}$

Ans:

take tan cos both side

$$\cos(\cos^{-1} x - \sin^{-1} x) = \cos(\cos^{-1} x \sqrt{3})$$

$$\cos\left(\frac{\cos^{-1} x}{\alpha} - \frac{\sin^{-1} x}{\beta}\right) = x \sqrt{3}$$

$$\cos(\alpha_1 - \alpha_2) = x \sqrt{3}$$

$$\cos \alpha_1 \cos \alpha_2 + \sin \alpha_1 \sin \alpha_2 = x \sqrt{3}$$

$$\cos(\cos^{-1} x) \cos(\sin^{-1} x) + \sin(\cos^{-1} x) \cdot \sin(\sin^{-1} x) = x \sqrt{3}$$

$$x \sqrt{1-x^2} + \sqrt{1-x^2} \cdot x = x \sqrt{3}$$

$$2x \sqrt{1-x^2} = x \sqrt{3}$$

$$x(2\sqrt{1-x^2} - \sqrt{3}) = 0$$

$$x=0 \quad \text{or} \quad 2\sqrt{1-x^2} = \sqrt{3}$$

Ans

$$4(1-x^2) = 3$$

$$4 - 4x^2 = 3$$

$$1 = 4x^2$$

$$x = \pm \frac{1}{2} \quad \text{Ans}$$

check :

$x = 0$ satisfy.

$$x = \frac{1}{2} \Rightarrow \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \quad \text{satisfy}$$

$$x = -\frac{1}{2} \quad \left(\pi - \frac{\pi}{3} \right) + \frac{\pi}{6} = \pi - \frac{\pi}{6} \quad \text{satisfy}$$

$$\left\{ 0, \frac{1}{2}, -\frac{1}{2} \right\}$$

Que: $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

Ans: ~~$\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$~~
take sin both side

$$\sin(\sin^{-1} x + \sin^{-1} 2x) = \sin \frac{\pi}{3}$$

$$\sin(\sin^{-1} x) \cos(\sin^{-1} 2x) + \cos(\sin^{-1} x) \cdot \sin(\sin^{-1} 2x)$$

$$x \cdot \sqrt{1-4x^2} + \sqrt{1-x^2} \cdot 2x = \frac{\sqrt{3}}{2} \quad \bullet (2x = \frac{\sqrt{3}}{2})$$

Method 2

$$\Rightarrow x \sin^{-1} 2x = \frac{\pi}{3} - \sin^{-1} x$$

$$\sin(\sin^{-1} 2x) = \sin\left(\frac{\pi}{3} - \sin^{-1} x\right)$$

$$\textcircled{1} \sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$H.W = \tan(M+A)$$

$$2x = \sin \frac{\pi}{3} \cos(\pi^{-1}x) - \cos \frac{\pi}{2} \sin(\pi^{-1}x)$$

$$= \frac{\sqrt{3}}{2} \cdot \sqrt{1-x^2} - \frac{1}{2}x$$

$$\frac{5x}{2} = \frac{\sqrt{3}}{2} \sqrt{1-x^2}$$

$$25x^2 = 3(1-x^2)$$

$$x = \pm \sqrt{\frac{3}{28}} \quad \rightarrow \text{ve } \wedge$$

$$x = \sqrt{\frac{3}{28}} \quad \text{Ans}$$

Que : Find all positive integral solution

$$\tan^{-1}x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$$

Ans : $\tan^{-1}x + \tan^{-1} \frac{1}{y} = \sin^{-1} 3$

$$\tan^{-1} \frac{1}{y} = \tan^{-1} 3 - \tan^{-1}x$$

$$\tan^{-1} \frac{1}{y} = \tan^{-1} \frac{3-x}{1+3x}$$

$$y = \frac{1+3x}{3-x}$$

$$x = 1, x = 2$$

$$x = \frac{3y-1}{3+y}$$

$$(x, y)$$

$$(2, 2)$$

$$(1, 2)$$

$0 \rightarrow 2\pi$

formula

(Q) $y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$

$y = \cos^{-1} \cos 2u$

$x = \tan u$

$0 \leq 2u < 2\pi$

$0 \leq u < \frac{\pi}{2}$

$x > 0$

$y = 2u$

$-\pi \leq 2u < 0$

$-\frac{\pi}{2} < u < 0$

$x < 0$

$y = -2u$

$x = d$

$y = \tan^{-1} \left(\frac{1-x^2}{1+x^2} \right) = \begin{cases} 2 \tan^{-1} x & x \geq 0 \\ -2 \tan^{-1} x & x < 0 \end{cases}$

Que!

$y = \sin^{-1} \left(\frac{2x}{1+x^2} \right) = \begin{cases} 2\lambda & -1 \leq x \leq 1 \\ \pi - 2\lambda & x \geq 1 \\ -\pi - 2\lambda & x \leq -1 \end{cases}$

$y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) = \begin{cases} 2\lambda & x \geq 0 \\ -2\lambda & x < 0 \end{cases}$

$y = f(x) = \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \begin{cases} 2\lambda - \pi & x > 1 \\ 2\lambda & -1 < x < 1 \\ \pi + 2\lambda & x < -1 \end{cases}$

$$f(x) + g(x) + h(x) \begin{cases} 2\lambda & x > 0 \\ 6\lambda & 0 < x < 1 \\ 2\lambda & -1 < x < 0 \\ -2\lambda & x < -1 \end{cases}$$

$$f(x) + g(x) = \begin{cases} 0 & x > 1 \\ 4\lambda & 0 < x < 1 \\ 0 & -1 < x < 0 \\ -\pi - 4\lambda & x < -1 \end{cases}$$

Que: $f(x) = \sin^{-1} \frac{2x}{1+x^2}$, $g(x) = \cos^{-1}$, $h(x) = \lambda^{-1}$

$x \in (-1, 1)$

$f(x) + g(x) + h(x) = \frac{\pi}{2}$

Ans:

$-1 < x \leq 0$

$2\lambda = 2\lambda^{-1} = \frac{\pi}{2}$

$\tan^{-1} x = \frac{\pi}{4}$

$x = \tan \frac{\pi}{4} = 1$

Reject X

$0 < x < 1$

$6\lambda = \frac{\pi}{2}$

$\tan^{-1} x = \frac{\pi}{12}$

$x = \tan \frac{\pi}{12}$ ✓

$\left\{ \tan \frac{\pi}{12} \right\}$ Ans

Date: 24/05/17

Ex J.A

Main

5) $x - 2 = 2 - x > d$ (Common difference)

$$\tan^{-1} y - \tan^{-1} x = \tan^{-1} 2 - \tan^{-1} y$$

$$\frac{y-x}{1+yx} = \frac{2-1}{1+2y}$$

$$1 + yx = 1 + 2y$$

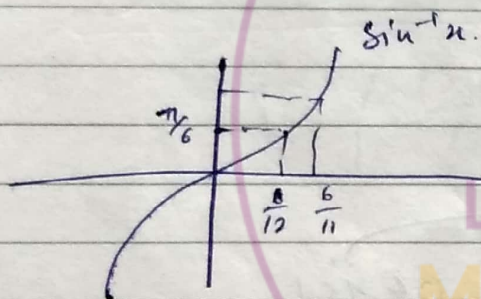
$$y(x-2) = 0$$

$$y = 0$$

$$x - 2 = 0$$

$$x = 2 = y = 2$$

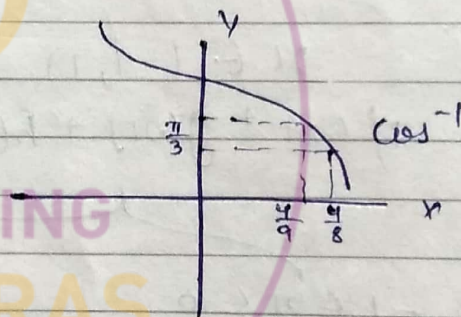
6



$$\frac{6}{11} > \frac{6}{12}$$

$$\sin^{-1} \frac{6}{11} > \sin^{-1} \frac{6}{12}$$

$$= \frac{\pi}{6} = 30^\circ$$



$$\frac{4}{8} > \frac{4}{9}$$

$$\cos^{-1} \frac{4}{8} < \cos^{-1} \frac{4}{9}$$

$$\cos^{-1} \frac{4}{9} > \cos^{-1} \frac{4}{8} = \frac{\pi}{3}$$

$$\alpha > 90^\circ$$

$$\alpha = 3 \sin^{-1} \frac{6}{11}$$

$$\alpha = 3 \cos^{-1} \frac{4}{9}$$

$$\Rightarrow \beta > 180^\circ$$

1. Create + Product difference = 1

Summation of series:

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y$$

Que 1. Solve = $\tan^{-1}\frac{1}{1+x+x^2} + \tan^{-1}\frac{1}{x^2+3x+3} + \dots$

T_1 T_2 n term.

$$\tan^{-1}\frac{1}{x^2+5x+7}$$

Ans!

$$T_1 = \tan^{-1}\frac{1}{1+(x+x^2)} = \tan^{-1}\frac{1}{1 \cdot 1 \cdot (x+1)^2}$$

$$= \tan^{-1}\frac{(x+1)-x}{1+x(x+1)} = \tan^{-1}\frac{(x+1)-x}{1+x(x+1)}$$

\downarrow \downarrow
 x y

$$T_2 = \tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)}$$

~~$$T_1 = \tan^{-1}(x+1) - \tan^{-1}x$$~~

~~$$T_2 = \tan^{-1}(x+2) - \tan^{-1}(x+1)$$~~

~~$$T_3 = \tan^{-1}(x+3) - \tan^{-1}(x+2)$$~~

~~$$T_n = \tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$$~~

~~$$S = \tan^{-1}(x+n) - \tan^{-1}x$$~~

Similar que. in different form.

$$S = \sum_{n=1}^n \tan^{-1}\frac{1}{1+n+n^2}$$

This que. is written same as first Que.

$$S = \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{13} + \dots \infty$$

$$\text{or } \sum_{r=1}^{\infty} \tan^{-1} \frac{1}{1+r+r^2}$$

$$T_r = \tan^{-1} \frac{1}{1+r+r^2} = \tan^{-1} \frac{(r+1) - r}{1+r(r+1)}$$

$$T_r = \tan^{-1}(r+1) - \tan^{-1} r$$

$$T_1 = \tan^{-1} 2 - \tan^{-1} 1$$

$$T_2 = \tan^{-1} 3 - \tan^{-1} 2$$

$$T_n = \tan^{-1}(n+1) - \tan^{-1} n$$

$$S = \tan^{-1}(n+1) - \tan^{-1} 1 \quad \text{Ans}$$



$$S = \sum_{n=1}^{\infty} \tan^{-1} \frac{1}{1+n+n^2} \quad \text{or} \quad \sum_{r=1}^{\infty} \tan^{-1} \frac{1}{1+r+r^2}$$

$(\tan^{-1}(n+1) - \tan^{-1} 1)$ as n is very very large no.

$$\lim_{n \rightarrow \infty} (\tan^{-1}(n+1) - \tan^{-1} 1)$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4} \quad \text{Ans}$$



Ques! $S = \sum_{n=1}^m \tan^{-1}(1+n+n^2)$

$$\tan^{-1} d = \frac{\pi}{2} - \cot^{-1} d$$

$$= \frac{\pi}{2} - \tan^{-1} \frac{1}{d}$$

$$\sum_{n=1}^m \left(\frac{\pi}{2} - \tan^{-1} \frac{1}{1+n+n^2} \right)$$

$$= \frac{\pi}{2} \sum_{n=1}^m 1 - \sum_{n=1}^m \tan^{-1} \left(\frac{1}{1+n+n^2} \right)$$

$$= \frac{\pi \cdot m}{2}$$

$$\cot^{-1} d = \tan^{-1} \frac{1}{d}, \quad d \neq 0$$

Ques! $S = \sum_{n=1}^m \cot^{-1}(2n^2)$

$$T_n = \cot^{-1}(2n^2) = \tan^{-1} \frac{1}{2n^2}$$

$$= \tan^{-1} \frac{1}{1+(2n^2-1)} = \tan^{-1} \frac{1}{1+(2n-1)(\sqrt{2n+1})}$$

$$= \tan^{-1} \left(\frac{(\sqrt{2n+1}) - (\sqrt{2n-1})}{1 + (\sqrt{2n-1})(\sqrt{2n+1})} \right) \times$$

$$\times = \tan^{-1}(\sqrt{2n+1}) - \tan^{-1}(\sqrt{2n-1})$$

$$\left[(n^4 + n^2 + 1) = (n^2 + n + 1)(n^2 - n + 1) \right]$$

$$= \tan^{-1} \frac{2}{4n^2}$$

$$= \tan^{-1} \frac{2}{1 + (4n^2 - 1)} = \tan^{-1} \frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)}$$

$$T_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

Ques:

$$(i) \quad S = \sum_{n=1}^{\infty} \tan^{-1} \left(\frac{2n}{2 + n^2 + n^4} \right)$$

$$(ii) \quad S = \sum_{n=1}^{\infty} \tan^{-1} \left(\frac{4n}{n^4 - 2n^2 + 2} \right)$$

$$(1) \quad T_n = \tan^{-1} \frac{(n^2 + n + 1) - (n^2 - n + 1)}{1 + (n^4 + n^2 + 1)(n^2 - n + 1)}$$

$$= \tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)$$

$$(2) \quad T_n = \tan^{-1} \frac{4n}{1 + (n^2 - 1)^2}$$

$$= \tan^{-1} \frac{(n+1)^2 - (n-1)^2}{1 + (n-1)^2 (n+1)^2}$$

$$= \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$$



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