Handwritten Notes

On

Indefinite Integrals

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Chapter : Indefinite Integrals

Que: $f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$ Find $f'(x)$

$$f'(x) = \begin{vmatrix} 1 & 2x & 3x^2 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 0 & 2 & 6x \\ 0 & 2 & 6x \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 0 & 6 \end{vmatrix}$$

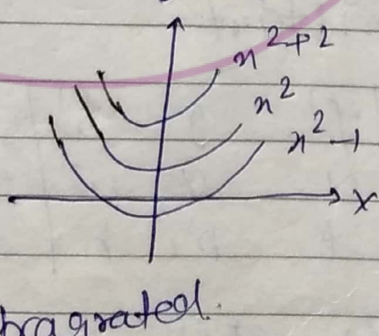
* Indefinate integral :

It is a ~~rivrse~~ reverse process of diff.

$$\frac{d}{dx} x^2 = 2x$$

$$\int 2x\,dx = x^2 + c$$ Constant

Family of Curve



$\because \frac{d}{dx} x^2 = 2x$

Intragrated

$$\int (2x)\,dx = \underbrace{x^2 + c}$$ anti darivate of 2x

Here x^2 is called Anti Deravity of $2x$.

$$\frac{d}{dx}(f(x)+c) = f(x)$$

$$\int f(x)\,dx = f(x)+C.$$

* Every cont. f^n is integravable but some of them are not Expressable.

$$\left\{ \int \sin\frac{1}{x}\,dx = \text{Integrable but not expressable.} \right\}$$

$$\int f(x)\,dx = f(x)+C$$

diff atn $\qquad f(\text{cont } f^n) =$

$$f(x) = f'(x)$$

* if $f(x)$ is cont then $F(x)$ is differenciable.

* $\int x^{-1/3}\,dx = \dfrac{x^{-1/3+1}}{-\frac{1}{3}+1}$

$f(x) = \dfrac{1}{x^{1/3}}$

$$= \cancel{\frac{3x^{2/3}}{2}} \quad \frac{3}{8}x^{2/3}$$

* $f(x)=0$ discon. at $x=0$

but its integration $F(x) = \frac{3}{8}x^{2/3}$ is cont. at $x=0$

~~Ttts~~

H.W = Trimetr formula Learn

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* Anti derivative of Periodic f^n need not be Periodic

Ex: $\int \cos x \, dx = \sin x + C$

Periodic with Periodic

Ex. $\int \underbrace{(\cos x + 1)\, dx}_{\text{Periodic}} = \underbrace{\sin x + x + d}_{\text{non-periodic}}$

Basic formula

1) $\int x^n dx = \dfrac{x^{n+1}}{n+1} + d$

2) $\int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{(n+1)a} + d$

3) $\log_a a^N = N$

4) $a^n \cdot b^n = (a \cdot b)^n$

5) $(a^x)^y = a^{xy}$.

6) $\log_a c = \log_c a$

7) $\int \frac{1}{\sqrt{x}} = \int x^{-1/2} = \dfrac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$
$= 2\sqrt{x}$.

8) $\int x^2 dx = \dfrac{x^{2+1}}{2+1}$

9)

Que:- 1 $\int e^{\ln\sqrt{x}}$ (2) $\int e^{-\ln x^2} dx$

Ans. $\int e^{\ln\sqrt{x}}$

$\int \sqrt{x}\, dx = \int x^{1/2} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

An : $\int e^{\ln(x^2)^{-1}} dx$.

$= \int (x^2)^{-1} dx$

$= \int x^{-2} dx = \int \frac{x^{-2+1}}{-2+1} + c$

(3) $\int \ln\left(\frac{1}{e^x}\right) dx$

Ans: $\int \ln\left(\frac{1}{e^x}\right) dx = \int \ln e^{-x} = \int -x(\ln e)\, dx =$

$= \int -x\, dx = -\frac{x^2}{2} + c$

Que: $\int \frac{(1+x)^3 dx}{\sqrt{x}}$

An $\frac{1+x^3+3x^2+3x}{\sqrt{x}} = (x^{-1/2} + x^{5/2} + 3x^{3/2} + 3x^{1/2}) dx$

$= \frac{x^{1/2}}{\frac{1}{2}} + \frac{x^{7/2}}{\frac{7}{2}} + 3 \cdot \frac{x^{3/2}}{3/2} + \frac{3x^{3/2}}{3/2} + c$

2) $\int e^{\ln 2 + \ln x} dx$

$\int e^{\ln 2x} \cdot dx = \int 2x\,dx = x^2 + c$

3) $\int e^{3\ln x} dx$

$\int e^{3\ln x} dx = e^{\ln x^3} = \int x^3 dx = \frac{x^4}{4} + c$

Second formula:

1. $\int \frac{dx}{x} = \ln x + c$ $\boxed{\text{or } \int \frac{dx}{x} = \ln|x| + c}$ $\boxed{x > 0}$

2. $\int \frac{dx}{ax+b} = \frac{\ln(ax+b)}{a} + c$

Que: $\int \frac{dx}{2x-1}$ (2) $\int \frac{dx}{3-4x}$

$= \frac{\ln(2x-1)}{2} + c$ $= \frac{\ln(3-4x)}{-4} + c$

(3) $\int \frac{\sqrt{x^4+x^{-4}+2}}{x^3}\,dx$

$= \cancel{\frac{\ln(x^3)}{\sqrt{x^4+x^{-4}+2}}}$

$\sqrt{x^4+\frac{1}{x^4}+2}$

$= \sqrt{\left(x^2+\frac{1}{x^2}\right)^2} = \int \frac{x^2+\frac{1}{x^2}}{x^3}\,dx$

$= \int \left(\frac{1}{x}+\frac{1}{x^5}\right)dx = \ln x + \frac{x^{-5+1}}{-5+1} + c$

Que: $\int \frac{2x+3}{x^2+3x-10}\,dx$

$\int \frac{(x+5)+(x-2)}{(x+5)(x-2)}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{dx}{x+5}$

$= \ln(x-2) + \ln(x+5) + c$ Ans

Que: $\int \frac{x}{x^2+2x+1}\,dx$ (2) $\int \frac{dx}{x^2-5x+6}$

$\int \frac{x}{x^2+x+x+1}$

$\frac{x}{x(x+1)(x+1)} = \int \frac{x}{(x+1)(x+1)}$

$\int \frac{(x+1)-(x+1)}{(x+1)(x+1)} = \int \frac{1}{(x+1)}\,dx + \int \frac{1}{(x+1)}\,dx = \cancel{\ln(x+1)}\ (x+1)^{-2+1} + c$

Ans:-1 $\Rightarrow \int \frac{(x+1)-1}{(x+1)^2} = \int \frac{dx}{x+1} - \int \frac{dx}{(x+1)^2} = \ln(x+1)$

$$= \frac{(x+1)^{-2+1}}{-2+1} + c$$

(2) $\int \frac{1}{(x-2)(x-3)} = \int \frac{(x-2)-(x-3)}{(x-2)(x-3)} = \int \frac{1}{x-3}\, dx -$

$$- \int \frac{dx}{x-2} = \ln(x-3) - \ln(x-2) + c$$

Que. $\int \frac{dx}{\sqrt{x+1} + \sqrt{x-1}}$

$=$

$$\int \frac{dx}{\sqrt{x+1} + \sqrt{x-1}} \times \left(\frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} \right)$$

$$\frac{1}{2} \int \left(\sqrt{x+1} - \sqrt{x-1} \right) dx = \frac{1}{2} \left[\frac{(x+1)^{3/2}}{\frac{3}{2}} + \frac{(x-1)^{3/2}}{\frac{3}{2}} \right] + c$$

* Third formula :

1. $\int e^x\, dx = e^x + c$

2. $\int e^{(ax+b)}\, dx = \frac{e^{(ax+b)}}{a} + c$

3) $\int a^x dx = \frac{a^x}{\ln a}$ $\boxed{a>0}$

Que: $\int e^{2x-3} dx$

$= \frac{e^{2x-3}}{2} + c$

(2) $\int 2^{3x+4} dx$

$= \boxed{\frac{2^{3x+4}}{3\ln 2} + c}$

$\frac{2^{3x+4}}{3\ln 2}$ Ans

(3) $\int 3^{-x} dx = \frac{3^{-x}}{-\ln 3} + c$

Que: $\int \frac{2^{x+1} - 5^{x-1}}{10^x}$ $10^x = (2 \cdot 5)^x = 2^x \cdot 5^x$

$= \frac{2^x \cdot 2 - \frac{5^x}{5}}{2^x \cdot 5^x} = 2\int 5^{-x} dx - \frac{1}{5}\int 2^{-x} = 2\frac{5^{-x}}{-\ln 5} - \frac{1}{5}\frac{2^{-x}}{-\ln 2} + c$

2) $\int 2^x \cdot e^x dx$

$a^x \cdot b^x = (ab)^x$ $= \int 2^x \cdot e^x dx = \int (2e)^x dx = \frac{(2e)^x}{\ln(2e)} + c$

3) $\int (2^x + 3^x)^2 dx$ $= (2^x)^2 + (3^x)^2 + 2 \cdot 2^x \cdot 3^x = \int (2^{2x} + 3^{2x} + 2 \cdot 6^x) dx$

$$= \frac{2^{2x}}{2\ln 2} + \frac{3^{2x}}{2\ln 3} + \frac{2\cdot 6^{x}}{\ln 6} + \lambda$$

Que: 1) $\displaystyle\int \frac{e^{3x} + e^{5x}}{e^{x} + e^{-x}}\, dx$

$$= \frac{e^{3x}(1+e^{2x})}{\frac{e^{2x}+1}{e^{x}}} = \int e^{4x}\, dx = \frac{e^{4x}}{4} + \lambda$$

2) $\displaystyle\int a^{mx}\cdot b^{nx}\, dx$

$$= \int (a^{m}\cdot b^{n})^{x} = \frac{(a^{m}\cdot b^{n})^{x}}{\ln(a^{m}\cdot b^{n})} + \lambda$$

3) $\displaystyle\int 2^{\ln x}\, dx$

$$\boxed{\log_b c,\ a^{c}}$$

$$= \int x^{\ln 2}\, dx = \frac{x^{(\ln 2+1)}}{(\ln 2+1)} + \lambda$$

* Fourth formula :

1. $\int \sin x \, dx = -\cos x$

2. $\int \sin(ax+b) dx = \frac{-\cos(ax+b)}{a}$

3. $\int \cos x \, dx = \sin x$

4. $\int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + \lambda$

Que

1. $\int \sin(2x-3) dx = -\frac{\cos(2x-3)}{2}$

2. $\int \cos(3x+4) dx = \frac{\sin(3x+4)}{3}$

3. $\int \cos^2 x \, dx = \sin^2 x + \lambda$ ✗

This is wrong

$= \cos$

$\int \cos x . d(\cos x) = \frac{(\cos x)^2}{2} + \lambda$ ✓

$\to \int \left(\frac{1+\cos 2x}{2}\right) dx = \frac{x}{2} + \frac{1}{2} \frac{\sin 2x}{2} + \lambda$ ✓

$$\cos 2x = 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$\left[8\int \sin^4 x \, dx \right]$ H.W

Q. $\int \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + \lambda$

Que: $\int 4\cos^4 x \, dx$

$$(2\cos^2 x)^2 = (1 + \cos 2x)^2 = 1 + \cos^2 2x + 2\cos 2x$$

$$= 1 + 2\cos 2x + \frac{1 + \cos 4x}{2}$$

$$= \frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x .$$

$$\int 4\cos^4 x = \frac{3}{2}x + \frac{2 \cdot \sin 2x}{2} + \frac{\sin 4x}{8} + \lambda .$$

Que: $\int \sin^3 x \, dx$ $\quad [\cos 3x = 4\cos^3 x - 3\cos x]$

$\int \frac{3\sin x - \sin 3x}{4} dx = \frac{3}{4}(-\cos x)$ $\quad [\sin 3x = 3\sin x - 4\sin^3 x]$

$$+ \frac{\cos 3x}{12} + \lambda$$

2) $\int \cos^3 x \, dx$

$$= \frac{-3}{4}\sin 3x - \frac{3}{4}\sin x + \lambda$$

Ques: $\int \sqrt{1+\sin x}\, dx$ $\quad x \in (0, \pi/2)$

Ans $\int \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}} = \left(\sqrt{\sin\frac{x}{2} + \cos\frac{x}{2}}\right)^2 = \cos\frac{x}{2} + \sin\frac{x}{2}$

$= \int \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) dx = 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + c$

2) $\int \cos 2x \cos 3x\, dx$

$\frac{1}{2}\int (2\cos 5x + \cos x)\, dx = \frac{1}{2}\left[\frac{\sin 5x}{5} + \sin x\right] + c$

3) $\int \cos x^\circ\, dx = \sin x + c$ $\quad$ (x is in Radian)

$180^\circ = \pi$

$1^\circ = \frac{\pi}{180} \Rightarrow x^\circ \frac{\pi x}{180}$

$\int \cos x^\circ\, dx =$

$= \int \cos\frac{\pi x}{180}\, dx$

$= \dfrac{\sin\frac{\pi x}{180}}{(\pi/180)} + c$

Que: $\int \frac{\cos x - \cos 2x}{1-\cos x}\, dx$

$[2\cos x^2 - \cos x - 1)$

$2\cos^2 x - 2\cos x + \cos x - 1$

$|2(\cos+1)(\cos x - 1)|$

$= \frac{\cos x - 2\cos^2 x + 1}{1-\cos x}$

$= \int (2\cos x + 1)\, dx = 2\sin x + x + c$

$= \int \frac{(\cos x + \sin x)(\cos x^2 + \sin x^2 - \sin x \cos x)}{(\cos x + \sin x)}$

Ans

$= \int \left(1 - \frac{1}{2}\sin 2x\right) dx = x + \frac{\cos 2x}{4} + c$

Que: $\int \frac{\cos^3 x + \sin^3 x}{\cos x + \sin x}\, dx$

$= \int \frac{(\cos x + \sin x)(\cos x^2 + \sin x^2 - \sin x(\cos x)}{(\cos x + \sin x)}$

$= \int \left(1 - \frac{1}{2}\sin 2x\right) dx = x + \frac{\cos 2x}{4} + c$

3) $\int 4\sin 2x \cdot \cos^2 x\, dx$

$2\sin 2x \cdot [\cos 2x + 1] = \int (\sin 4x + 2\sin 2x)\, dx$

$\int$

$= \frac{\cos 4x}{4} - \cos 2x + c$

I ... anionic defination of

$$\int \phi = t$$
$$\text{cosec} \Rightarrow -\cot$$

* V) formula!

* $\int \sec^2 x \, dx = \tan x.$

* $\int \sec^2(ax+b) \, dx = \frac{\tan(ax+b)}{a} + c$

$\int \text{cosec}^2 x \, dx = -\cot x + c$

Que. $\int \cot^2 2x \, dx$

$$\int (\text{cosec}^2 2x - 1) \, dx = -\frac{\cot 2x}{2} - x + c$$

(2) $\int \frac{1}{1+\cos 2x} \, dx$

$$= \int \frac{dx}{1+2\cos^2 x + 1} = \frac{1}{2} \int \sec^2 x \, dx$$

$$= \frac{1}{2} \tan x + c$$

3) $\int \frac{dx}{1-\cos x}$

$$= \frac{1}{\cancel{1} - \cancel{1} + 2\sin^2 \frac{x}{2}} = \frac{1}{2} \int \text{cosec}^2 \frac{x}{2} = \frac{1}{2} \left(\frac{-\cot \frac{x}{2}}{1/2} \right) + c$$

Q.1 $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

$= \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x} = \int \sec^2 x = \tan x + c$

(2) $\int \tan^2 \frac{x}{4} dx$

$\int \left(\sec^2 \frac{x}{4} - 1\right) dx = \frac{\tan \frac{x}{4}}{1/4} - x + c$

(3) $\frac{1+\cos x}{1-\cos x} dx$

$= \int \frac{1 + 2\cos^2 \frac{x}{2} - 1}{1 - 1 + 2\sin^2 \frac{x}{2}} dx = \int \cot^2 \frac{x}{2} dx$

$= \int \cot^2 \frac{x}{2} dx = \int \left(\operatorname{cosec}^2 \frac{x}{2} - 1\right) dx$

$= \frac{-\cot x}{1/2} - x + c.$

VIth formula:

* $\int \sec x \tan x \, dx = \sec x + c$

* $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

Q.1 $\int \frac{a \sin^3 x + b \cos^3 x}{\sin^2 x \cos^2 x} dx$

$a \int \frac{\sin^3}{\sin^2 x \cos^2 x} + b \int \frac{\cos^3 x}{\sin^2 x \cos^2 x} = a\int \cancel{\tan x}$

$a \int \tan x \cdot \sec x \, dx + b \int \cot x \operatorname{cosec} x \, dx.$

$= a(\sec x - b(\operatorname{cosec} x + 1))$

Q.2 $\int \frac{\operatorname{cosec} x + \tan^2 x + \sin^2 x}{\sin x} dx$

$\int \operatorname{cosec}^2 x \, dx + \int \sin x \, dx + \int \tan \sec x \, dx$

VII formula

* $\int \frac{dx}{1+x^2} = \tan^{-1}x$

* $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

* $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x \quad \left(\text{or } \frac{\pi}{2} - \cos^{-1}x\right)$

* $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$

* $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x$, $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}$

Que:

1) $\int \frac{dx}{4+x^2}$

$= \frac{1}{2}\tan^{-1}\frac{x}{2}$

(2) $\int \frac{dx}{\sqrt{1-9x^2}}$

$= \frac{\sin^{-1}3x}{3}$

(3) $\int \frac{dx}{1+16x^2}$

$= \frac{\tan^{-1}4x}{4}$

(4) $\int \frac{dx}{x\sqrt{4x^2-1}}$

$= 2.\frac{\sec^{-1}2x}{2}$

5) $\int \frac{dx}{\sqrt{16-x^2}} = \sin^{-1}\frac{x}{4}$

Que: 1) $\int \frac{x^2+\cos^2 x}{x^2+1} \cdot \operatorname{cosec}^2 x \, dx$

$= \cancel{\frac{x^2+\cos^2 x}{x^2+1}} \cdot \cancel{\operatorname{cosec}^2 x} = \frac{x^2+1-\sin^2 x}{x^2+1} \cdot \operatorname{cosec}^2 x \, dx.$

$= \int \operatorname{cosec}^2 dx - \int \frac{1}{1+x^2} dx = -\cot x - \tan^{-1} x + c$

(2) $\int \frac{x^2}{1+x^2} dx = \cancel{\frac{\tan^{-1} x}{x}}$

deg of $N^r \geq$ deg of D^r

$\int \left(1 - \frac{1}{1+x^2}\right) dx = x - \tan^{-1} x + c$

(3) $\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{(3)^2-(2x)^2}} = \frac{1}{2} \sin^{-1} \frac{2x}{3} + c$

(4) $\int \frac{dx}{16+25x^2} = \int \frac{dx}{(4)^2+(5x)^2} = \left(\frac{1}{4} \tan^{-1}\left(\frac{5x}{4}\right) \frac{1}{5}\right) + c$

* $\int \frac{dx}{(x+1)(x+2)} = \int \left(\frac{1}{x+1} - \frac{1}{x+2}\right) dx =$

$= \ln(x+1) - \ln(x+2)$

Que:1) $\int \frac{x^4}{1+x^2}\, dx = \frac{\tan^{-1} x}{4x^4}$

$\frac{(x^4-1)+1}{1+x^2} = \frac{(x^2+1)(x^2-1)+1}{(x^2+1)}$

$= \int \left(x^2 - 1 + \frac{1}{1+x^2}\right) = \frac{x^3}{3} - x + \tan^{-1} x$

(2) $\int \frac{dx}{(x^2+1)(x^2+2)} = \int \frac{1}{(x^2+1)} - \frac{1}{(x^2+2)}\, dx$

~~$\tan^{-1} x (\ln(x^2+1) - \ln(x^2+2)$~~

$= \tan^{-1} x - \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + c$

(3) $\int \frac{dx}{(x-1)\sqrt{x^2-2x-3}} = \int \frac{dx}{(x-1)\sqrt{x^2-3x+1x-3}}$

~~$\frac{dx}{(x+1)(\sqrt{x^2 - x(x-3)+1(x-3)}}$~~ $= \frac{dx}{(x-1)(\sqrt{(x+1)(x-3)}}$

$\int \frac{dx}{(x-1)\sqrt{(x-1)^2-2^2}} = \frac{1}{2} \sec^{-1}\left(\frac{x-1}{2}\right)$

Que: $\int \frac{dx}{(x^2-4x+4)(x^2-4x+5)}$

$$\int\left(\frac{1}{x^2-4x+4} - \frac{1}{x^2-4x+5}\right) = \int \frac{dx}{(x-2)^2} - \int \frac{dx}{(x-2)^2+1}$$

$$= \frac{1}{x-2} - \tan^{-1}(x-2) + c$$

(2) $\int \frac{dx}{(2x-7)\sqrt{(x-3)(x-4)}}$

$x^2-7x+12$

$x^2 - 2\cdot\frac{7}{2}x + \left(\frac{7}{2}\right)^2 + 12 - \left(\frac{7}{2}\right)^2$

$\left(x-\frac{7}{2}\right)^2 - \frac{1}{4}$

$$\int \frac{dx}{(2x-7)\sqrt{x^2-7x+12}} = \frac{1}{2}\int \frac{dx}{\left(x-\frac{7}{2}\right)\cdot\sqrt{\left(x-\frac{7}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$= \frac{1}{2}\cdot\frac{1}{1/2}\sec^{-1}\frac{\left(x-\frac{7}{2}\right)}{1/2}$$

Que: $\int (x^2+1)^3\, dx$

$3\cancel{(x^2+1)}^2$ $= (x^6+1+3x^4+3x^2)\,dx$

$= \cancel{\frac{(x^2+1)^4}{4}}$ $= \frac{x^7}{7} + x + \frac{3x^5}{5} + \frac{3x^3}{3} + c$

(2) $\int \frac{dx}{\sin^2x\cos^2x} = \frac{\sin^2+\cos^2}{\sin^2x\cos^2x}$

$\int \sec^2x\,dx + \int \operatorname{cosec}^2 dx$

$= \tan - \cot x\ dx$

($\underline{M-2}$) $\frac{4}{\sin^2 2x}\,dx = 4\int \operatorname{cosec}^2 x\,dx$

$= 4(-\cot 2x) + c.$

(3) $\int \frac{\sin^6x + \cos^6x}{\sin^2x\cos^2x}$

(3) Ans $\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x$

$\sin^4 + \cos^4 x = 1 - 2\sin^2 x \cos^2 x$

$$= \int \frac{1 - 3\sin^2 x \cos^2 x}{\sin^2 x \cos^2} dx = \int \frac{dx}{\sin^2 \cos^2 x} - \int 3\, dx$$

Que:1) $\int \frac{(x^3+8)(x-1)}{x^2-2x+4} = \int \frac{(x^3+8)(x-1)}{}$

$a^3+b^3 = (a+b)(a^2+b^2-ab)$

$$\int \frac{(x+2)(x^2 \cancel{+x} - 2x)(x-1)}{(x^2-2x+4)} = \int (x^2+x-2)\, dx$$

(2) $\frac{x^3}{(x+1)} dx = \int \frac{(x^3+1)-1}{x+1}$

$$\frac{(x+1)(x^2-x+1)-1}{x+1} = \int (x^2-x+1) - \frac{1}{x+1})$$

(3) $\frac{dx}{\sin^2 x \cos x}$

$$\int \frac{\sec^2 + \cos^2}{\sec^2 \cos^2 x} = \int (\sec x + \cot x \operatorname{cosec} x \cdot \cot) dx$$

21/06/17

Integration

Virus. sub. — By Part — Partial — mix.

(1)

Integration Viras Subtitution :-

if $\int f(x)\,dx = \phi(x)$

then $J = \int f(\psi(z)) \cdot \psi'(z)\,dz$

$\psi'(z)\,dz = dx$

$\int f(k)\,dk$

$= \phi(k) + c$

$= \phi\,\psi(z)$

$\frac{d\,\psi(z)}{dz} = \frac{dk}{dz}$

$\psi'(z) = \frac{dx}{dz}$

$\psi'(z)\,dz = dx.$

Ex: $J = \int x \cos x^2\,dx$

$2x\,dx = dk$

$x^2 = k$

$2x = \frac{dk}{dx}$

$2x\,dx = dk$

diff w.r.t. x

$2x = \frac{dk}{dx}$

$2x\,dx = dk$

$= \frac{1}{2}\int \cos k \cdot dk$

$= \frac{1}{2} \sin k + c$

$= \frac{1}{2} \sin x^2 + c$

formula:

1. $\int \tan x\,dx = \ln(\sec x) + c$
2. $\int \cot x\,dx = \ln(\sin x)$
3. $\int \sec x\,dx = \ln(\sec x + \tan x)$ or $\ln \tan\left(\frac{\pi}{2} + \frac{x}{4}\right)$
4. $\int \operatorname{cosec} x\,dx \Rightarrow \ln(\operatorname{cosec} x - \cot x)$ or $\ln\left(\tan\frac{x}{2}\right) + c$

Que:-1 $\int \frac{\tan(\ln x)}{x}\,dx$ $\quad \ln x = t$

$\frac{1}{x}\,dx = dt$

Ans: $= \int \tan t\,dt = \ln(\sec t) + c$

$= \ln(\sec(\ln x)) + c$

2) $\frac{\tan(\sin^{-1}x)\,dx}{\sqrt{1-x^2}}$ $\quad \sin^{-1}x = k$

$\frac{1}{\sqrt{1-x^2}}\,dx = dk$

$= \int \tan k\,dx -$

$\tan k\,dk$

(3) $\int e^x \sin e^x\,dx$ $\quad e^x = k$

$e^x\,dx = dk$

$= \int \sin P\,dP = -\cancel{\cos P + c}$ $\quad \sin k\,dk$

$= -\cos e^x$ $\quad -\cos k + c$

Que:- 4 $\int \sec^2 x\,\sqrt{5+\cancel{t}\tan x}\;dx$ $\quad \left(\begin{array}{l} 5+\tan x = k \\ \sec^2 x\,dx = dk \end{array} \right)$

$= \int \sqrt{k}\,dk = \frac{k^{3/2}}{3/2} + c$ $\Big|$ $5+\tan x = t^2$

$\sec^2 x\,dx = 2t\,dt$

$I = \int t$

(5) $\int \frac{x^5}{1+x^{12}}\,dx$ $\quad x^6 = \lambda$

$6x^5\,dx = d\lambda$

$= \frac{1}{6}\int \frac{d\lambda}{1+\lambda^2} = \frac{1}{6}\tan^{-1}\lambda$ $\quad x^5\,dx = \frac{1}{6}\,d\lambda$

(6) $\int \sec x \cdot \ln(\sec x + \tan x)\, dx$

$\frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)\, dx = dP$

$\sec x\, dx = dP$

$= \int$

Q.7) $\frac{x^2 \tan^{-1} x^3}{1+x^6}\, dx$ $= x^3 = k$

$3x^2 dx = dk$

$x^2 dx = \frac{1}{3} dk$

$= \tan^{-1} x^3 = \lambda$

$\frac{1}{1+x^6} \times 3 \cdot x^2 dx = d\lambda$

$I = \frac{1}{3} \int \lambda\, d\lambda$

$= \frac{1}{3} \frac{\lambda^2}{2}$

(8) $\int \frac{\sqrt{\tan x}}{\sin 2x}\, dx = \int \frac{t}{2 \frac{\sin x}{\cos x} \cos^2 x} \cdot \frac{2t\,dt}{1+t^4}$

$\int \frac{t}{2t^2} \cdot \sec^2 x \cdot \frac{2t\,dt}{1+t^4} =$

$= \int \frac{t}{2t^2} \cdot \frac{2t\,dt}{1+t^4} (1+t^4)$

$= \int dt = t = \sqrt{\tan x} + \lambda$

Q. $\dfrac{dx}{\cos(x-1)\cos(x-2)}$

$$\frac{1}{\sin 1}\int \frac{\sin 1}{\cos(x-1)(\cos(x-2)}\,dx$$

$$= \frac{1}{\sin 1}\int\left[\frac{\sin[(x-1)-(x-2)]}{\cos(x-1)\cos(x-2)}\right]$$

$$= \frac{1}{\sin 1}\int \frac{\sin(x-1)\cos(x-2) - \cos(x-1)\sin(x-2)}{\cos(x-1)\cos(x-2)}\,dx$$

$$= \frac{1}{\sin 1}\int\left[\tan(x-1)\,dx - \int \tan(x-2)\,dx\right]$$

Q. 1) $\displaystyle\int \frac{\sec^4 x}{\sqrt{\tan x}}\,dx$

$$= \int \frac{1+\tan^2 x}{\sqrt{\tan x}} \cdot 2k\,dk = \int \frac{1+k^4}{k}\cdot 2k\,dk =$$

$$= 2\int (1+k^4)\,dk$$

$\tan x = k^2$

$\sec^2 x\,dx = 2k\,dk$

$$\int 2 + 2t^4 \cdot dt = 2t + \frac{2t^5}{5} + c$$

$dx = \dfrac{2t\,dt}{\sec^2 x}$

$$= 2\sqrt{\tan x} + \frac{2}{5}\sqrt[5]{\tan x} + c$$

$dx = \dfrac{2t\,dt}{1+\tan^2 x}$

$dx = \dfrac{2\tan t}{1-\cancel{\tan}\,t^4}$

$dx = \dfrac{2t\,dt}{1-t^4}$

$\frac{1}{\sec x} \cdot \sec x \cdot \tan x \cdot dx = dP$

$\rightarrow$

(1) $\frac{\ln^2(\sec x)}{\cot x} dx = \int P^2 dP$

$\int \frac{\ln(\sec x) \cdot \ln(\sec x) \cdot dx}{\cot x}$ $\qquad \ln(\sec x) = P$

$\int \frac{dt}{2} \cdot \sqrt{t}$

Que: $\int \frac{x \cos x}{(x \sin x + \cos x)^2} dx.$ $\qquad x \sin x + \cos x = P$

$= (x \cos x + \sin x - \sin x) dx = dP$

$= \int \frac{dP}{P^2}$

$\int \frac{dt}{t^2} = \int t^{-2} \cdot dt$

$\Rightarrow$

$= \frac{t^{-1}}{-1} + d = \frac{-1}{t} + d$

(2) $\int \frac{\sin 2x}{\sin 5x \cdot \sin 3x} dx = \frac{\sin(5x - 3x)}{\sin 5x \sin 3x}$

$= \frac{\sin 5x \cos 3x - \cos 5x \times \sin 3x}{\sin 5x \sin 3x}$

$= \int \cot 3x \, dx - \frac{\int \cot 5x \, dx \, \sin 3x}{\sin 5x \cdot \sin 3x}$

$= \int \cot 3x \cdot dx - \int \cot 5x \cdot dx$

$= \frac{\ln(\sin 3x)}{3} - \frac{\ln(\sin 3x)}{5} + d.$

Que: $\int \frac{\sin 2x}{(a\sin^2 x + b\cos^2 x)^2}\,dx$ $\qquad a\sin^2 x + b\cos^2 x = k$

$= (a-b)\, 2\sin 2x\, dx = dk$

$= \frac{1}{(a-b)} \cdot \int \frac{dk}{k^2}$

(2) $\int \frac{\ln^2\left(\frac{x}{x+1}\right)}{x(x+1)}\,dx$ $= \qquad \log^2 x = (\log x)^2$

$\int t^2\,dt$

$= \cancel{dx}$

$\frac{1}{\left(\frac{x}{x+1}\right)} \cdot \frac{d}{dx}\left(\frac{x}{x+1}\right) = dx \cdot dt$

$\Rightarrow \int t^2\,dt$

$\int \frac{t^3}{3} + \lambda$

$= \frac{x+1}{x} \cdot \frac{(x+1)-x}{(x+1)^2}\,dx = dt$

$= \frac{\ln^3\left(\frac{x}{x+1}\right)}{3} + \lambda$

$\frac{1}{x(x+1)}\,dx = dt$

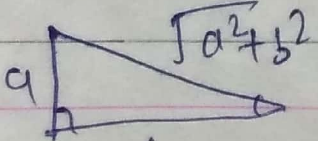
$\Rightarrow \int \frac{dx}{a\sin x + b\cos x}$ $\qquad \Rightarrow \sqrt{a^2+b^2}$

$a\sin x + b\cos x = \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x\right]$

$= \frac{1}{\sqrt{a^2+b^2}} \cdot \int \frac{dx}{\cos(x-\phi)}$

$= \sqrt{a^2+b^2}\,[\sin x \cdot \sin\phi + \cos x\cos\phi]$

$= \sqrt{a^2+b^2}\,[\cos(x-\phi)]$



Que
(1) $\int \frac{dx}{\sqrt{3}\cos x - \sin x}$

$\sqrt{3}\cos x - \sin x \quad \sqrt{(\sqrt{3})^2 + (-1)^2}$

$= 2\cdot\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right)$ $\left(2\sin\frac{\pi}{3}\cos x - \sin x(x\frac{\pi}{3})\right)$

$\left[2(\cos x(x\frac{\pi}{3}) - \sin\sin\frac{\pi}{3}\right)$

(2) $\int \frac{dx}{\sin x + \cos x}$

$\frac{1}{\sqrt{2}} \cdot \int \frac{dx}{\sin(x + \frac{\pi}{4})}$ $\quad \frac{1}{\sqrt{2}}\int \frac{dx}{\cos(x - \frac{\pi}{4})}$

Que. $\int \frac{2\,dx}{\cancel{\sec x} +} \int \frac{2\,dx}{\sec x + \operatorname{cosec} x}$.

Ans: $\frac{2\,dx}{\frac{1}{\cos x} + \frac{1}{\sin x}} =$

$\frac{2\sin x\cos x}{\sin x + \cos x}\,dx$

$\int \frac{(1 + 2\sin\cos x) - 1}{\sin x + \cos x} = \int \frac{(\sin^2 x + \cos^2 x + 2\sin\cos x)}{\sin x + \cos x}$

$= \int \frac{(\sin x + \cos x)^2 - 1}{\sin x + \cos x} = \int (\sin x + \cos x)\,dx - \int$

$\int \frac{1}{\sin x\cos x}\,dx$

O-I ⇒ 4, 6, 7, 8, 13

Do first → S-I ⇒ 1, 2, 3, 9,

Que: $\int (\sin x - \cos x)(\sin x + \cos x)^5 dx$

$\sin x + \cot x = k$

$(\cos x - \sin x) dx = dk$

$\int -k^5 dk$

(2) $\int \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1} dx$

$\int \frac{\tan x + (\sec x - 1)}{\tan x + (\sec x - 1)} \times \frac{\tan - (\sec x - 1)}{\tan x - (\sec x - 1)}$

$\int \frac{\tan x + \sec x - 1}{\tan x - \sec x + (\sec^2 x - \tan^2 x)}$

$\int \frac{\sec x + \tan x - 1}{(\sec x - \tan x)[\sec x + \tan x - 1)}$

$= \int (\sec x + \tan x) d$

(3) $\int \frac{\cos 2x}{\sin x} dx$

$\int \frac{1 - 2\sin^2 x}{\sin x} = \int \operatorname{cosec} dx - 2\int \sin x d$

Que. $\int \frac{dx}{e^x+1}$

$e^x = t$
$e^x dx = dt$
$dx = \frac{dt}{e^x}$

Ans: $\int \frac{dx}{e^x+1} = \int \frac{dt}{t e^x} =$

$= \int \frac{e^{-x}}{1+e^{-x}} = \int \frac{-dk}{k}$ $\quad e^{-x} dx = -dk.$

$= -\ln k$

Que: $\int \frac{e^x-1}{e^x+1} dx$

$e^x = t$
$e^x dx = dt$
$dx = \frac{dt}{e^x}$

$\int \frac{(e^x+1)-2}{e^x+1}$

$= \int dx - \int \frac{2}{e^x+1} dx$

$\int dx - \int \frac{2e^{-x}}{1+e^{-x}} dx \quad [1+e^{-x} = k]$

* Integration of the form

* $\int \sin^m x \cos^n x \, dx.$

1. if $m =$ odd then put $\cos x = t$
2. if $n =$ odd put $\sin x = t$
3. if n, m both are odd then put ~~which~~ either of them t.

QST = t =

* If both are even then use trigonometry Identity to solve

* Otherwise, Put $\tan x = t$ or $\cot x = t$

Ex.

Ex) $\int \sin^3 x \cos^2 x \, dx$ $\qquad$ $\cos x = t$

$= \int \sin^2 x \cdot t^2 (-dt)$

$= \int (1 - \cos^2 x)(-t^2) \, dx$

$= \int (1 - t^2)(-t^2) \, dx$

$= \int (t^4 - t^2) \, dt$

Q. $\int \sin^5 x \cos x \, dx$

$\sin^5 x = t$

$\cos^5 x \, dx = t$

$dx = \frac{t}{\cos^5 x}$

$\int k^5 \, dk.$

Q. $\int \sin^4 x \, dx$

$\sin x = k$

$\cos dx = dk$

Q. $\int \sin^4 x \cos^2 dx$

Q. $\int \cos^4 x \, dx$

use trigonometry Id.

1) $\int \sin^3 x \cos^{15} x\, dx$

$\int \cancel{\sin^3 x}\ k^{15}$

$\int (1-k^3) \cdot k^3 \times \frac{k^{15}}{-\sin x}\, dk$

$\cos x = k$

$-\sin x\, dx = k$

$dx = \frac{k}{-\sin x}$

(2) $\int \cos^3 x \sqrt{\sin x}\, dx$

$\int dt^3 \cdot \frac{dt}{\cos x}$

$\sin x = t$

$\cos x\, dx = dt$

$dx = \frac{dt}{\cos x}$

(3) $\int \cos^3 x =$

(1) $\int (1-\cos^2 x)\, k^{15}\, dk$

$\int (k^2-1)\, k^{15}\, dx$

$\cos x = k$

$\sin x\, dx = -dk$

(2) $\int (1-\sin^2 x) = dt = \int (1-t^2)\sqrt{t}\, dt$ $\left(\begin{array}{l} \sin x = t \\ \cos x\, dx = dt \end{array} \right.$

(3) $\int (1-\sin^2 x)\, dy$

$= \int (1-u^2)\, du$

$\sin x = u$

$\cos x\, dx = du$

Que: $\int (\sin x)^{-11/3} \cdot (\cos x)^{-1/3}\, dx$

$\frac{-11}{3} - \frac{1}{3} = \frac{-12}{3} = (-4)$

$\sin 2x = 2\sin x \cos x$

$= \frac{(\sin x)^{-11/3}}{\cos x^{1/3}}\, dx = \int \tan x^{-4}\, dx$ $\quad (\tan x = t)$

$= \frac{t^{-4} \cdot t}{\sec^2 x}$ $\quad \sec^2 x\, dx = t$

$\int \cancel{\frac{1}{\sin x^{+11/3}} \cdot \frac{1}{\cos x^{1/3}}}\, dx$ $\quad = \int \frac{t^{-3}}{\sec^2 x}$ $\quad \cancel{dx = \frac{t}{\sec^2 x}}$

Ans: $\int \frac{(\sin x)^{-11/3}}{(\cos x)^{-11/3}} \cdot (\cos x)^{-1/3} \cdot (\cos x)^{-11/3}\, dx$

$\int (\tan x)^{-11/3} \cdot (\cos x)^{-4}\, dx =$ $\boxed{\begin{matrix} \tan x = \lambda \\ \sec^2 x\, dx = d\lambda \end{matrix}}$

$= \int (\tan x)^{-11/3} \cdot \frac{1}{\cos^4 x}\, dx = \int (\tan x)^{-11/3} \cdot \sec^2 x \cdot \sec^2 x\, dx$

$= \int \lambda^{-11/3} (1 + \lambda^2)\, d\lambda$

Expression	Sub
$a^2 - x^2$	Put $x = a\sin\theta$ or $a\cos\theta$
$a^2 + x^2$	$x = a\tan\theta$, $x = a\cot\theta$
$x^2 - a^2$	$x = a\sec x$, $\csc\theta$
$\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos\theta$ or $x = a\cos\theta$
$\sqrt{\frac{x}{a+x}}$ or $\sqrt{\frac{a+x}{x}}$	$x^3 = a\tan^2\theta$ or $x = a\cot^2\theta$

$-1/3^{+1}$

$\sqrt{\frac{a-x}{x-b}}$ & $\sqrt{\frac{x-b}{a-x}}$ or $\sqrt{(a-x)(x-b)}$ | $x = a\cos^2\theta + b\sin^2\theta$.

Que. $\int \sqrt{\frac{1+x}{1-x}}\, dx$

$\int \frac{\cos\theta/2}{\sin\theta/2} \cdot \left(-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) d\theta$

$x = \cos\theta$
$dx = -\sin\theta\, d\theta$

$\frac{1+x}{1-x} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+2\cos^2\theta/2-1}{1-1+2\sin^2\frac{\theta}{2}} = \cot\frac{\theta}{2}$

$\int -2\cos^2\frac{\theta}{2}\, d\theta = \int -(\cos\theta+1)\, d\theta.$

Que: $\int \sqrt{\frac{1-\cos x}{\sin x - 2}}\, dx$

$1\cos^2 x + 2\sin^2 x = k$
or $2\cos^2 x + \sin^2 x = P$

Ans:
$(-\sin 2x + 2\sin 2x)$

Que: $\int \sqrt{\frac{1-x}{x-2}} =$

x $2\cos^2 x + 1\sin^2 x = t$
$2(-\sin^2 x)$
$\cos^2 x \cdot 2(-\sin^2 x) + \sin^2 x \cdot \cos^2 x\, dx$
$= dt.$

Answer
on Next Page.

Ans $\int \sqrt{\frac{1-x}{x-2}}\, dx$

$x = 1\cos^2\theta + 2\sin^2\theta$

$dx = (-\sin 2\theta + 2\sin 2\theta)\, d\theta$

$dx = \sin 2\theta\, d\theta$

$1-x = 1-(\cos^2\theta + 2\sin^2\theta)$

$= 1-\cos^2\theta - 2\sin^2\theta$

$= \sin^2\theta - 2\sin^2\theta$

$= -\sin^2\theta$

$x-2 = \cos^2\theta + 2\sin^2\theta - 2$

$= \cos^2\theta - 2(1-\sin^2\theta) = \cos^2\theta - 2\cos^2\theta$

$= -\cos^2\theta$

$= \int \sqrt{\frac{-\sin^2\theta}{-\cos^2\theta}} \times \sin 2\theta\, d\theta$

$= \int \frac{\sin\theta}{\cos\theta} \times 2\sin\theta\cos\theta\, d\theta = \int (1-\cos 2\theta)\, d\theta.$

Ques. $\int \sqrt{\frac{x}{4+x}}\, dx$

Ans

$x = 4\tan^2\theta$

$4\tan^2\theta =$

$\cancel{dx = 4\sec^2 x + \tan^2\theta}$

$dx = 4\cdot 2\tan\theta \cdot \sec^2\theta\, d\theta$

$\frac{x}{4+x} = \frac{4\tan^2\theta}{4(1+\tan^2\theta)} = \frac{\tan^2\theta}{\sec^2\theta}$

$= \sin^2\theta$

$$\int \sin\theta \times 8\tan\theta \cdot \sec^2\theta d\theta$$

$$\sin\theta \int \frac{\sin^2\theta}{\cos^3\theta} d\theta$$

Q. $\int \sqrt{\frac{x}{1-x}}\, dx = \int \sqrt{\frac{x}{1-x}}\, dx.$ ~~$x = a\tan^2\theta$~~

$$\int \sqrt{\frac{x}{1-x}}\, dx = \frac{\cancel{\tan^2\theta}}{\cancel{1-\tan^2\theta}} = \frac{\cancel{\tan^2\theta}}{\sec^2\theta} =$$

Ans: $\int \sqrt{\frac{\sin^2\theta}{\cos^2\theta}}\, 2\sin 2\theta d\theta$ $\quad x = \sin^2\theta$ $\quad dx = \sin 2\theta$

$$\int \frac{\sin\theta}{\cos\theta} \cdot 2\sin\theta\cos\theta d\theta$$

$$\int 2\sin^2\theta d\theta = \int (1-\cos 2\theta) d\theta.$$

Que: $\int \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right) dx$

$x = a\cos\theta.$
$\cos\theta = x$
$\sin\theta$
$\sin\theta = x$
$-\cos^2\theta d\theta = dx$
$\cos^2\theta d\theta = -dx$

$$\int 2\cot^{-1}\sqrt{\frac{\cancel{1-\cos\theta}}{\cancel{1+\cos}}}\ \frac{1-\sin\theta}{1+\sin\theta} \times -dx$$

$\int$

Ans: $x = \cos 2\theta$

$dx = -2\sin 2\theta\, d\theta$

$$\sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \sqrt{\frac{\sin^2\theta}{\cos^2\theta}} = \tan\theta$$

$$\int \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right) dx$$

$$= \int \left(\cos 2 \cdot \cot^{-1} \cot\left(\frac{\pi}{2}-\theta\right)\right) dx$$

$$= \int \cos 2\left(\frac{\pi}{2}-\theta\right) dx$$

$$= \int \cos(\pi - 2\theta)\, dx$$

$$= \int -\cos 2x \cdot (-2\sin 2\theta\, d\theta) = \int \sin\theta\, d\theta$$

Que! $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$

Ans: $\sqrt{x} = \cos\theta$

$x = \cos^2\theta$

$dx = -\sin 2\theta\, d\theta$

$$\frac{1-\sqrt{x}}{1+\sqrt{x}} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{\sin^2\theta/2}{\cos^2\theta/2}$$

$$I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$$

$$= \int \frac{\sin\theta/2}{\cos\theta/2} \times -2\sin a \cos a \, da$$

$$- \int \frac{\sin\theta/2}{\cos\theta/2} \cdot 2 \cdot 2 \sin\frac{\theta}{2} \cos\frac{\theta}{2} \cos a \, d\theta$$

$$\cos a = 1 - 2\sin^2\frac{a}{2}$$

$$2\sin^2\frac{x}{2} = 1 - \cos a$$

$$= -2\int \left(2\sin^2\frac{a}{2}\right) \cos a \, da$$

Que: $\displaystyle\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} \, dx$ $\quad x^{3/2} = a^{3/2}\sin\theta$

$$\frac{3}{2} x^{1/2} dx = a^{3/2}\cos\theta \, d\theta$$

$$= \frac{2}{3} \cdot a^{3/2} \int \frac{\cos\theta}{\cos\theta} \, d\theta.$$

$$\boxed{x^{3/2} = a^{3/2}\sec\theta}$$

Que: $\displaystyle\int \frac{x^2}{64 - x^6} \, dx. = \int \frac{x^2}{\sqrt{2^6 - x^6}}$ $\quad x^3 = 2^3\sin\theta$

$3x^2 dx = 8\cos\theta \, d\theta$ $\quad = \displaystyle\int \frac{\frac{8}{3}\cos\theta \, d\theta}{2^3 \cdot \cos\theta}$

Que: $\dfrac{\sin 2x \, dx}{\sqrt{9-\sin^4 x}}$ $\quad$ $\sin^2 x = k$

$\sin 2x \, dx = dk.$

$$\int \frac{dk}{\sqrt{9-k^2}}$$

$$= \sin^{-1}\left(\frac{k}{3}\right) + c$$

Que: $\displaystyle\int \frac{e^x \, dx}{\sqrt{e^{2x}-1}}$ $\quad$ $e^x = k$

$e^x dx = dk$

$$\int \frac{dk}{\sqrt{k^2-1}} = \ln(k + \sqrt{k^2-1}) + c$$

Que) $\displaystyle\int \frac{e^x}{4+e^{2x}} \, dx$ $\quad$ $e^x = t$

$e^x dx = dt$

$$\int \frac{dt}{2^2+t^2} = \frac{1}{2} \tan^{-1} \frac{e^x}{2} + c$$

Que: $\displaystyle\int \frac{dx}{\sqrt{2x-x^2-1)+1}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \sin^{-1}(x-1)$

1. ~~x~~ take common x cofficient
2. Roor make +1 on tree

Integral of Rational functions
(without partial fraction)

(1) $\int \frac{1}{Q} dx$, $\int Q dx$, $\int \frac{1}{\sqrt{Q}} dx$. $Q = ax^2 + bx + c$

Q. $\int \frac{dx}{2x^2 - 3x + 4} =$

$2x^2 - 3x + 4$

$= 2 \cdot \left[x^2 - \frac{3}{2} x + 2 \right]$

$= \frac{1}{2} \int \frac{dx}{\left(x - \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2}$

$= 2 \cdot \left[x^2 - 2\left(\frac{3}{4}\right)x + \left(\frac{3}{4}\right)^2 + 2 - \left(\frac{3}{4}\right)^2 \right]$

$= 2\left[\left(x - \frac{3}{4}\right)^2 + \frac{23}{16} \right]$

$= \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{23}}{4}} \cancel{\tan^{-1}} ($

$\frac{1}{2} \cdot \frac{1}{\sqrt{\frac{23}{4}}}$

$= \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{23}}{4}} \cdot \tan^{-1}\left(\frac{\left(x - \frac{3}{4}\right)}{\sqrt{23}/4} \right) + c$

Q. $\frac{dx}{\sqrt{2x^2 - 3x + 4}} = \frac{1}{2} \int \frac{dx}{\left(x - \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2}$

Formula No - 18

$\frac{A}{B} = 1 \cdot A = \lambda$

Que: $\int \sqrt{3x^2 - 6x + 10}\, dx$

$= \int \sqrt{3x^2 - 6x + 10}\, dx$

$3x^2 - 6x + 10$

$3\left[\cancel{x^2 - 2x + 1 - 1} + \frac{10}{3}\right]$

$= \sqrt{3} \int \sqrt{(x-1)^2 + \left(\sqrt{\frac{7}{3}}\right)^2}\, dx$

$3 \cdot \left[x^2 - 2x + \frac{10}{3}\right]$

$3 \cdot \left[x^2 - 2x + 1 - 1 + \frac{10}{3}\right]$

$= \sqrt{3}\left(\frac{x-1}{2}\sqrt{(x-1)^2 + \frac{7}{3}}\right.$

$= 3\left[(x-1)^2 + \frac{7}{3}\right]$

Type (9) $\int \frac{L}{Q} dx,\ \int \frac{L}{\sqrt{Q}} dx,\ \int L\sqrt{Q}\, dx.$

$Q = ax^2 + bx + c$

$L = px + q$

Que: $L = \lambda \frac{d}{dx} Q + \mu$ Here λ = Constant

$px + q = \lambda(2ax + b) + \mu$

$p = 2a\lambda$ -

$q = \lambda b + \mu$ -

then move to type 1.

Ques! $\int \cancel{3x} + \frac{5x+7}{2x^2-3x+4}\,dx$

$$5x+7 = \lambda \frac{d}{dx}(2x^2-3x+4)\,dx$$

$$5x+7 = \lambda(4x-3) + \mu.$$

$5x = 4x\lambda$

$\lambda = \frac{5x}{4x} = \frac{5}{4}$

$7 = -3\lambda + \mu.$

$\mu = 7 + 3 \cdot \frac{5}{4} = \frac{43}{4}$

=

$$= \int \frac{\frac{5}{4}(4x-3) + \frac{43}{4}}{2x^2-3x+4}\,dx$$

$$= \frac{5}{4}\int \frac{4x-3}{2x^2-3x+4}\,dx + \frac{49}{4}\int \frac{dx}{2x^2-3x+4}$$

$\downarrow$

$2x^2-3x+4 = p$

$(4x-3)dx = dp$

$= \frac{5}{4}\int \frac{dp}{p} +$ —

$= \frac{5}{4}\ln(2x^2-3x+4) +$ -

Que: $\int \frac{3\cos x + 5}{\sqrt{\sin^2 x - 2\sin x + 5}}$

Que: $\int \frac{\frac{3}{2}\sin 2x + 5\cos x}{\sqrt{\sin^2 x - 2\sin x + 5}}\, dx$

$\sin x = t$

$\cos dx = dt$

$= \int \frac{(3\sin x + 5)\cos x}{\sqrt{\sin^2 x - 2\sin x + 5}}\, dx$

$\int \frac{3t+5}{\sqrt{t^2-2t+5}}\, dt =$

$\frac{3}{2}\int \frac{2t-2}{\sqrt{t^2-2t+5}}\, dt + 8\int \frac{dt}{\sqrt{(t^2-2t+1)+4}}$

$\frac{3}{2}\cdot\int \frac{2k\,dk}{k} + 8\int \frac{dt}{2^2+(t-1)^2}$

$3t+5 = \lambda \frac{d}{dt}(t^2-2t+5) + \mu$

$= \lambda(2t-2)+\mu$

$3 = 2\lambda \Rightarrow \lambda = \frac{3}{2}$

$= \frac{3k+8\,dt}{\sqrt{t^2-2t+5}}$

$\mu = 5+2\lambda$

$= 5 + 2\times\frac{3}{2} = 8$

$= \int \frac{2t-2}{\sqrt{t^2-2t+5}}\, dt + 8\int \frac{dt}{\sqrt{(t^2)-2t+1)+4}}$

$= \frac{3}{2}\int \frac{2k\,dk}{k} + 8\int \frac{dt}{\sqrt{2^2+(t-1)^2}}$

$= 3k + 8$

Type :- 3

$\int \frac{Q_1}{Q_2}\,dx , \int \frac{Q_1}{\sqrt{Q_2}}\,dx$ $Q_1 = ax^2 + bx + c$

$Q_2 = px^2 + qx + r$

$Q_1 = \lambda \cdot Q_2 + \mu \frac{d}{dx}(Q_2) + \gamma .$

{Alternately Question goes to type 1 and type 2}

get the value of λ, μ and γ

λ, μ, γ are constant and balance.

Que: $\int \frac{3x^2 - 2x + 5}{x^2 - 2x + 10}\,dx$ $\frac{d}{d}$

$3x^2 - 2x + 5 = \lambda(x^2 - 2x + 10) + \mu(2x - 2) + \gamma .$

$\lambda = 3, \quad -2 = -2\lambda + 2\mu$

$\mu = 2, \quad \gamma = -21$

$\int \frac{3x^2 - 2x + 5}{x^2 - 2x + 10}\,dx$

$3\int \frac{\cancel{x^2 - 2x + 10}}{\cancel{x^2 - 2x + 10}}\,dx + 2\int \frac{2x-2}{x^2 + 2x + 10}\,dx - 21\int \frac{dx}{(x^2 - 2x + 1) + 9}$

$= 3x + 2\ln(x^2 - 2x + 10) - 21 \cdot \frac{1}{3} \cdot \tan^{-1}\left(\frac{x-1}{3}\right)$ Ans

Integration of Trigonometric function:

: Type 1:

$$\int \frac{dx}{a+b\sin^2 x}, \quad \int \frac{dx}{a+b\cos^2 x} \quad \int \frac{dx}{a\sin^2 x + b\cos^2 x + c}$$

$$, \int \frac{dx}{a\sin^2 x + b\cos^2 x + c\sin x\cos x}$$

Tech: Const. is replaced by

(1) $a \to a(\sin^2 x + \cos^2 x)$

(2) take $\cos^2 x$ common from dinomenator and put $\tan x = t$.

Que: $\int \frac{dx}{3+4\sin^2 x} = \int \frac{}{3(\sin^2 x + \cos^2 x) + 4\sin^2 x} =$

$$\int \frac{dx}{7\sin^2 x + 3\cos^2 x} = \int \frac{\sec^2 x \cdot dx}{7\tan^2 x + 3}$$

$$= \frac{1}{7}\int \frac{\sec^2 x \cdot dx}{\tan^2 x + \left(\sqrt{\frac{3}{7}}\right)^2} = \frac{1}{7}\int \frac{dk}{k^2 + \left(\sqrt{\frac{3}{7}}\right)^2}$$

Que: $\int \frac{dx}{3+\cos^2 x} = \int \frac{dx}{3\sin^2 x + 4\cos^2 x}$

$= \int \frac{\sec^2 x\,dx}{3\tan^2 x + 4} = \int \frac{dt}{3t^2+4}$ $\quad \tan x = t$

Que: $\int \frac{dx}{3\sin^2 x + 4\cos^2 x - \sin x\cos x}$

$= \frac{dx}{\cancel{3\sin}\cos x} \quad \int \frac{dt}{3t^2 - t + 4}$

Type: 2 $\int \frac{dx}{a+b\sin x}, \int \frac{dx}{a+b\cos x}, \int \frac{dx}{a\sin x + b\cos x + c}$

$a \to a\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right)$

$\cos x \to \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$\sin x \to 2\sin\frac{x}{2}\cos\frac{x}{2}$

take $\cos^2\frac{x}{2}$ common and put $\tan\frac{x}{2} = t$

J.A → 2, 5

Que! $\int \frac{dx}{5+4\sin x}$

$$5+4\sin x$$
$$5\sin^2\frac{x}{2}+5\cos^2\frac{x}{2}+8\sin\frac{x}{2}\cos\frac{x}{2}$$

Ans

$$\int \frac{\sec^2\frac{x}{2}dx}{5\tan^2\frac{x}{2}+8\tan\frac{x}{2}+5}$$

$\tan\frac{x}{2}=P$

$\sin^2\frac{x}{2}dx = 2dP$

$$= \int \frac{2dP}{5P^2+8P+5}$$

Que! $\int \frac{dx}{3+2\sin x+\cos x}$

$~~3+2\sin x+\cos x~~$

$= 3\cancel{\sin^2\frac{x}{2}+}$

$$= \int \frac{\cancel{dx}}{3(\cancel{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}})+}$$

Ans!

$$\int \frac{}{3(\sin^2\frac{x}{2}+\cos^2\frac{x}{2})+2\cdot 2\sin\frac{x}{2}\cos\frac{x}{2}+(\cos^2\frac{x}{2}-\sin^2\frac{x}{2})}$$

$$= \frac{d}{4\cos^2x+2\sin^2x+4\sin x\cos x}$$

$$= \frac{1}{2}\int \frac{\cancel{\sec}\ \sec^2\frac{x}{2}dx}{\tan^2\frac{x}{2}+2\tan\frac{x}{2}+2}$$

$\tan\frac{x}{2}=k$ $$= \frac{1}{\cancel{2}}\int \frac{\cancel{2}\cdot dk}{k^2+2k+2}$$

24/06/17

Que: $\int \frac{dx}{3\cos^2 x + 4\sin^2 x - 2}$ as

$\frac{d}{3\cos^2 x + 4\sin^2 x - 2\sin^2 x - 2\cos^2 x}$ $\tan x = t$

$\int \frac{d}{\cos^2 x + 2\sin^2 x} = \int \frac{\sec^2 x\,dx}{2\tan^2 x + 1} = \int \frac{dt}{2t^2+1}$

Q. $\int \frac{dx}{1+2\sin x + 3\cos x}$

$= \int \frac{}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 4\sin\frac{x}{2}\cos\frac{x}{2} + 4\cos^2\frac{x}{2} - 4\sin^2\frac{x}{2}}$

$= \int \frac{}{5\cos^2\frac{x}{2} \cdot 2}$

Type : 3

$$\int \frac{a\sin x + b\cos x + c}{p\sin x + q\cos x + r}\,dx.$$

Tech:

$a\sin x + b\cos x + c = \lambda(p\sin x + q\cos x + r) + \mu \frac{d}{dx}((p\sin x + q\cos x + r) + \nu.$

compare Cofficient of $\sin x$, $\cos x$ and get the value of λ, μ and ν.

Qu: $\int \frac{3\sin x + 2\cos x + 1}{\cos x + 2\sin x - 3}\,dx$

An.

$3\sin x + 2\cos x + 1 = \lambda(\cos x + 2\sin x - 3) + \mu(-\sin x + 2\cos x) + \nu$

$= (3\sin x + 2\cos x + 1) = \lambda(2\sin x + \cos x - 3) + \mu(-\sin x + 2\cos x) + \nu$

$3 = 2\lambda - \lambda \quad -(1) \qquad \lambda = 8/5$

$2 = \lambda + 2\mu \quad -(2) \qquad \mu = 1/5$

$1 = -3\lambda + \nu \quad -(3) \qquad \nu = 29/5$

$$\int \frac{3\sin x + 2\cos x + 1}{\cos x + 2\sin x - 3}\,dx =$$

$$\frac{8}{5}\int dx + \frac{1}{5}\int \frac{-\sin x + 2\cos x}{\cos x + 2\sin x - 3}\,dx + \frac{29}{5}\int \frac{dx}{\cos x + 2\sin x - 3}$$

$$= \frac{8}{5}x + \frac{1}{5}\ln(\cos x + 2\sin x - 3) + \text{---}$$

and goes type 2.

Qu. $\displaystyle\int \frac{6 + 3\sin x + 14\cos x}{3 + 4\sin x + 5\cos x}\,dx$

$6 + 3\sin x + 14\cos x = \lambda(3 + 4\sin x + 5\cos x) + \mu(4\cos x - 5\sin x) + \nu$

$6 = \lambda 3$

$\lambda = \frac{6}{3} = 2.$

$3 = \cancel{3 - 4\lambda}$

$3 = \cancel{4\mu - 5\mu}$

$-\mu = 3$, $\mu = -3$.

$\cancel{\mu = 1}$

$14 = (4\mu - 5\mu) + \nu$

$\cancel{14 = -\mu + \nu}$ $= \nu = \cancel{14 + \mu = 14 - 3 = 11}$.

$\nu = 0$

$\lambda = 2, \quad B = 1, \quad C = 0$

$$= 2\int \frac{3 + 4\sin x + 5\cos x}{3 + 4\sin x + 5\cos x}\,dx + \int \frac{4\cos x - 5\sin x}{3 + 4\sin x + 5\cos x}\,dx$$

$$= 2x + \ln(3 + 4\sin x + 5\cos x) + c$$

Que:- $\int \frac{3e^{x}+5e^{-x}}{4e^{x}-5e^{-x}}\,dx$

$= \int \frac{3e^{x}}{}$

$3e^{x}+5e^{-x} = \lambda(4e^{x}-5e^{-x}) + \mu(4e^{x}-5e^{-x}) + \nu$

$3 = 4A + B \quad -(i)$
$5 = -5A + 5B \quad -(ii)$

$A = -\frac{1}{8}, \quad B = \frac{7}{8}$

$= A\int dx + \frac{7}{8}\int \frac{4e^{x}+5e^{-x}}{4e^{x}-5e^{-x}}\,dx$

$= -\frac{1}{8}x + \frac{7}{8}\ln(4e^{x}-5e^{-x}) + \lambda$

Q. => $\int \frac{\sin x}{e^{x}-\sin x-\cos x}\,dx$

$= \int \frac{e^{-x}\sin x\,dx}{1-(\sin x+\cos x)e^{-x}}$

$= -\frac{1}{2}\int \frac{dt}{1-t}$

$(\sin x+\cos x)e^{-x} = t$

$(\cos x-\sin x)e^{-x} - e^{-x}(\sin x+\cos x)$
$\times dx = dt$

$-2\sin x\,e^{-x}dx = dt$
$\sin x\,e^{-x}dx = -\frac{1}{2}dt$

* Integration using Partial fraction:

$$\int \frac{P(x)}{Q(x)}\, dx$$

Where P and Q both are polynomial f^n.
and degree of $P(x)$ < degree of $Q(x)$

In Partial fraction $Q(x)$ can be
(i) Linear and non Repeated.
(ii) Linear and repeated
(iii) Quadratic and non-Repeated
(iv) Quadratic and Repeated.

Que: $\displaystyle\int \frac{dx}{(x-1)(2x-3)}$

$$\frac{1}{(x-1)(2x-3)} = \frac{A}{x-1} + \frac{B}{2x-3}$$

constant

Linear

जितनी हो ऊपर आधार की & Degree कम रखनी है।

$$\frac{1}{(i)}$$

$$1 = A(2x-3) + B(x-1) \quad \text{(LCM)} \quad (\text{An Identity})$$

$$0 = 2A + B \quad \text{—(i)}$$
$$1 = -3A - B \quad \text{—(ii)}$$

$x = \frac{3}{2}$, $1 = B\cdot\left(\frac{3}{2} - 1\right) = B = 2$

$A = -1$

$$= -\int \frac{dx}{x-1} + 2\int \frac{dx}{2x-3}$$

$$= -\ln(x-1) + 2\,\frac{\ln(x-3)}{2} + \lambda$$

Que.: $\displaystyle\int \frac{dx}{(x-1)^2\,(x+2)}$

$$= \frac{px+q}{(x-1)^2} + \frac{r}{(x+2)}$$

$$\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)} \Rightarrow (\text{lcm})$$

$$= \frac{A(x-1)+B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$= \frac{Ax+(B-x)}{(x-1)^2}$$

$$\frac{A}{(x-1)} = \cancel{A(x-1)+B}$$

$$\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$1 = \cancel{A(x+} \; A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

$x=1$ $\quad B = \frac{1}{3}$

$\rightarrow x=-2$ $\quad C = \frac{1}{9}$

$x=0$ $\quad 1 = -2A + 2B + C$

$$\cancel{9A} \quad 9A = 2B + C - 1$$

$$= \frac{2}{3} + \frac{1}{9} - 1$$

$$\frac{6+1-9}{9} = \frac{2}{9} \quad \left[A = -\frac{1}{9} \right]$$

$$\int \cancel{dx} \quad \frac{1}{9}\int \frac{dx}{x-1} + \frac{1}{3}\int \frac{dx}{(x-1)^2} - \frac{1}{9}\int \frac{dx}{(x+2)}$$

$$= -\frac{1}{9}\ln(x-1) + \frac{1}{3}\cdot\left(-\frac{1}{x-1}\right) + \frac{1}{9}\ln(x+2)$$

Que: $\dfrac{1}{(x-1)(x+2)(2x+3)}$

$$= \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(2x+3)}$$

$$1 = A(x+2)(2x+3) + B(x-1)(2x+3) + C(x+1)(x+2)$$

$\cancel{A \quad 0 = A+2+B+2+C}$

$1 = \cancel{2A+3A+-B+3B} + C + 2C.$

Ans:

$x = 1 \qquad A = \frac{1}{15}$

$x = -2 \qquad B = \frac{1}{3}$

$x = -\frac{3}{2} \qquad C = \frac{4}{5}$

$= \frac{1}{15} \ln(x-1) + \frac{1}{3} \ln(x+2) - \frac{4}{5} \ln\left(\frac{2x+3}{2}\right) + c$

Q. $\int \frac{dx}{(x-1)(x+2)} = \frac{1}{3}\left[\frac{dx}{x-1} - \frac{dx}{x+2}\right]$

$= \frac{1}{3}\left(\ln(x-1) - \ln(x+2)\right)$

Que! $\int \frac{x+5}{(x-2)^2} dx$

$= \frac{x+5}{(x-2)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2}$

$x+5 = A(x-2) + B$

$x = 2, \quad B = 7$

$x = 0 \quad A = 1$

$\int \frac{dx}{x-2} + 7\int \frac{dx}{(x-2)^2}$

$= \ln(x-2) - 7\frac{1}{(x-2)} + c$

Que! $\int \frac{dx}{(x^2+1)(x^2+2)}$

$\int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+2}$

$= \tan^{-1}x - \frac{1}{\sqrt{2}} \tan^{-1}\frac{x}{\sqrt{2}} + c$

Que! $\int \frac{dx}{(\sin x - \sin 2x)}$

$$\int \frac{dx}{\sin x(1-2\cos x)} = \int \frac{\sin x\, dx}{\sin^2 x\,(1-2\cos x)}$$

$$\int \frac{\sin x\, dx}{(1-\cos^2 x)(1-2\cos x)}$$

$$= \int \frac{\sin x\, dx}{(2\cos x-1)(\cos x-1)(\cos x+1)}$$

$$= -\int \frac{dt}{(2t-1)(t-1)(t+1)}$$

Que) $\int \frac{2x+7}{(x+1)(x^2+4)}$

An. $\frac{A}{x+1} + \frac{Bx+C}{x^2+4}$

$2x+7 = A(x^2+4) + (Bx+1)(x+1)$

$x=-1$ $\quad A = 1 \qquad 2 = A+B$

$B = -1$

$C = 3$

$5 = A(1+4) + (Bx+1)\overset{0}{(-1+1)}$

$A = \frac{5}{5} = 1$

* Integration by Parts:

$$\int f(x)\cdot g(x)\,dx = f(x)\cdot\int g(x)\,dx - \int\left(\left(\frac{d}{dx}f(x)\right)\int g(x)\,dx\right)dx$$

1. Ist f^n $f(x)$ is diff. easily.
   IInd easily integrable

Ex: $\int \underset{I}{x}\ \underset{II}{\sin x}\,dx = x\int \sin x\,dx - \int\left(\left(\frac{d}{dx}x\right)\cdot\int \sin x\,dx\right)dx$

$= -x\cos x - \int 1\cdot(-\cos x)\,dx$

$= -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + c.$

Some times ILATE Rule is helpfull to Choose in decending order

decending order of Ist f^n or 2nd Accending or.

I L A T E

Inverse, Log, Alg, Trig, exponential

$$\int \underset{I}{f}\,\underset{II}{g}\,dx = f\int g(x)\,dx - \int\left(f'\int g(x)\,dx\right)dx.$$

$$\int \underset{\substack{\downarrow\\A}}{\overset{I}{x}}\ \underset{\substack{\downarrow\\T}}{\overset{II}{\sin x}}\,dx \qquad \int \underset{II}{1}\cdot\underset{I}{\sin^{-1}x}\,dx$$

$$\int \underset{\substack{|\\Exp}}{\overset{II}{e^{x}}}\ \underset{\substack{\downarrow\\Ag.}}{\overset{I}{(x^2-3x+4)}} \qquad \int \underset{I}{(\ln x)}\ \underset{II}{\cos x}\,dx$$

Que: $\int \underset{II}{x}\,\underset{I}{\tan^{-1}x}\,dx$ (2) $\int \underset{II}{x}\,\underset{I}{e^{x}}\,dx$

$= x\int \tan^{-1}x - \int\left(\left(\frac{d}{dx}x\right)\tan^{-1}x\,dx\right)dx$

$= (\tan^{-1}x)\frac{x^2}{2} - \int \frac{x^2}{1+x^2}\,dx$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{(x^2+1)-1}{1+x^2}\,dx$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx = \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\left[x-\tan^{-1}x\right] + c$

(2) $\int \underset{I}{x}\,\underset{II}{e^{x}}\,dx$

$= x\int e^{x}dx - \int\left(\left(\frac{d}{dx}x\right)\cdot\int e^{x}dx\right)dx$

$= xe^{x} - \int 1\cdot e^{x}dx$

$= xe^{x} - e^{x}$

Qu. $\int \underset{II}{\operatorname{cosec}^2 x} \cdot \underset{I}{\ln(\sec x)}\, dx$

$\operatorname{cosec}^2 x \int \frac{1}{\sec x} + \int \left(\operatorname{cosec} x \sec x - \int \sec x \, dx\right)$

$\operatorname{cosec}^2 x$

$= \ln(\sec x)(-\cot x) - \int \frac{1}{\sec x} \sec x \tan x \times (-\cot x) dx$

$= -\cot x \ln(\sec x) + x + c$

Qu. 11) $\int \underset{II}{\sin x} \underset{I}{\ln(\sec x + \tan x)} dx$

$\ln(\sec x + \tan x) \cdot \cos x + \int \cos x \cdot \int \frac{1(\sec x \tan x + \sec^2 x)}{\sec x + \tan x} dx$

$\ln(\sec x + \tan x) - \cos x - \sin x \frac{1}{\sec x + \tan x} \int dx$

Imp.

(2) $\frac{\cos^{-1} x}{x^3} dx$

$\cos^{-1} x = \theta$
$x = \cos\theta$
$dx = -\sin\theta\, d\theta$

$\frac{\cos^{-1}\cos\theta}{\cos^3\theta} =$

$$\int \frac{\theta \times -\sin\theta\, d\theta}{\cos^3\theta} = \int \underset{I}{\theta} (\underset{II}{\tan\theta \cdot \sec^2\theta}) dx$$

$= -\theta \cdot \frac{1}{2}\tan^2\theta + \frac{1}{2}\int 1 \cdot \tan^2\theta\, d\theta$

$= -\frac{\theta}{2}\tan^2\theta + \frac{1}{2}\int(\sec^2\theta - 1)d\theta$

$= -\frac{\theta}{2}\tan^2\theta + \frac{1}{2}[\tan\theta - \theta] + c$

$\int \tan \sec^2\theta\, dx$
$= \int k\, dk = \frac{k^2}{2}$

$\tan\theta = k$
$\sec^2\theta\, d\theta = dk$

Que: $\int x(\underset{II}{\sin x} \underset{II}{\cos^2 x}) dx.$

$\Rightarrow$

$-\frac{x\cos^3 x}{3} + \frac{1}{3}\int \cos^3 x\, dx.$

$\int \sin\cos^2 x\, dx$
$= -\int t^2 dt$
$= -\frac{t^3}{3}$
$= -\frac{1}{3}\cos^3 x$

$\cos x = t$
$\sec^2 dx = dt$

Q.) $\int \underset{II}{1} \cdot \underset{I}{\ln(x + \sqrt{x^2+1})}\, dx$

Ans

$$x \ln(x+\sqrt{x^2+1}) - \int \frac{x}{\sqrt{x^2+1}}dx$$

$$\frac{1+\frac{1\cdot 2x}{2\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}$$

$$= \frac{\sqrt{x^2+1}+x}{x+\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}}$$

$$= \frac{1}{\sqrt{x^2+1}}$$

$$= -\int \frac{p\,dp}{p}$$

$$-p + \lambda$$

$$-\sqrt{x^2+1} + \lambda$$

$$x^2+1 = p^2$$

$$x\,dx = p\,dp.$$

Q. $\int \underset{I}{x^2} \underset{II}{e^x} dx$

$$x^2 e^x dx - \int (2x \cdot \int e^x dx)\,dx$$

$$x^2 e^x - 2\int \underset{I}{x}\underset{II}{e^x} dx = x^2 e^x - 2(x-1)e^x + \lambda \text{ Ans}$$

$$\downarrow$$

$$J$$

$$J = \int \underset{I}{x} \underset{II}{e^x}$$

$$= xe^x - \int 1 \cdot e^x dx$$

$$= xe^x - e^x$$

$$= (x-1)e^x$$

Imp.

* Two classic Integration:

$$\int e^x [f(x) + f'(x)]\,dx = e^x f(x) + c$$

$$\int [f(x) + x f'(x)] = x f(x) + c.$$

Q. $\int e^x f(x)\,dx + \int \underset{I}{e^x}\, \underset{II}{f'(x)}\,dx$

$$= \int e^x f(x)\,dx + e^x f(x) - \int e^x f(x)\,dx.$$

Que. $\int e^x (\underset{f}{\sin x} + \underset{f'}{\cos x})\,d = e^x \sin x + c$

Que. $\int e^x [\underset{f'}{\sec^2 x} + \underset{f}{\tan x}]\,dx = e^x \cancel{\sec^2} x.$
$= e^x \tan x$

Q. $\int e^x [\cot x + \operatorname{cosec}^2 x]\,dx = e^x \cot x.$

Q. $\int (\underset{f}{\sin x} + \underset{x}{x}\,\underset{f'}{\cos x})\,dx = x \sin x + c$

J. M => 5 Class

H.W: Q.1, 3 ~~~~ both side.
14, Q.9 Classic but.

S-1: Q.6, 7, 8, 10, 11, 17,

Que: $\int \frac{x e^x}{(1+x)^2} dx$

$= \int e^x \left[\frac{x+1-1}{(x+1)^2}\right] dx$

$= \int e^x \cdot \left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right] dx$

$= \int e^x \cdot \left[\underbrace{\frac{1}{1+x}}_{f} + \underbrace{\frac{-1}{(1+x)^2}}_{f'}\right] dx$

$= \frac{e^x}{1+x} + \cot x$

* Two Classic Integrals

$$\int e^x[f(x) + f'(x)]dx = e^x(f(x)) + c$$

$\int e^x[f($

* $$\int e^x[f(x) - f''(x)]dx = \int e^x[f(x) + f'(x) - f'(x) - f''(x)]dx$$

$$\int e^x[f(x) + f'(x)]dx - \int e^x[f'(x) + f''(x)]dx$$

$$= e^x f(x) - e^x \cancel{[f'(x) + f''(x)]dx}$$

$$= e^x f(x) - e^x f'(x)$$

$$= e^x(f(x) - f'(x)) + c$$

Que $\int \frac{x + \sin x}{1 + \cos x} dx$

$$= \int \frac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{1 + 2\cos^2\frac{x}{2} - 1} = \int \Big(\underbrace{\tan\frac{x}{2}}_{f} + \underbrace{x}_{x} \cdot \underbrace{\frac{1}{2}\sec^2\frac{x}{2}}_{f'}\Big) dx$$

$$= x\tan\frac{x}{2} + c$$

(2) $\int \frac{e^x (x-1)}{(x+1)^3}$

$\int \frac{e^x (x+1-2)}{(x+1)^3} = \int e^x \left[\frac{1}{(1+x)^2} - \frac{2}{(1+x)^3} \right) d$

$= \int e^x \left[\frac{1}{1+x^2} + \frac{-2}{(1+x)^3} \right) dx$

$\quad f \qquad f'$

$= e^x \cdot \frac{1}{(1+x)^2} + c$

Que: $\int e^x [\ln(\sec x + \tan x) + \sec x] \, dx$

$= e^x \ln(\sec x + \tan x) + c$

(2) $\int \frac{e^x (x^2+1)}{(x+1)^2} dx$

$= \frac{e^x (x^2-1+2)}{(x+1)^2} = e^x \left[\frac{(x-1)(x+1)}{(x+1)^2} + \frac{2}{(x+1)^2} \right)$

$$\int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] = \left(\frac{x-1}{x+1}\right) e^x$$

$\uparrow f \qquad \uparrow f'$

Que. $\int (\sin(\ln x) + \cos(\ln x))\, dx$ $\qquad \ln x = t$, $x = e^t$, $dx = e^t dt$

$$I = \int (\sin t + \cos t) e^t dt$$

$$= e^t \sin t + c$$

M-2 $I = \int \left(\sin(\ln x) + x \frac{\cos(\ln x)}{x}\right)$

$\uparrow f' \quad \uparrow x \quad \uparrow f'$

Q) $e^{\tan^{-1}} \frac{(1+x+x^2)}{1+x^2}$ $\qquad \tan^{-1} x = \theta$, $x = \tan\theta$, $dx = \sec^2\theta d\theta$

$= \cancel{f(x)} = \cancel{\cos}(\cancel{\ln x}) \cdot \frac{1}{\cancel{x}}$

$$\int e^\theta \left(\frac{1+\tan\theta+\tan^2\theta}{(1+\tan^2\theta)}\right) \cdot \sec^2\theta\, d\theta$$

$$= \int e^\theta (\sec^2\theta + \tan\theta)\, d\theta = e^\theta \tan\theta + c$$

$f' \qquad f$

$\ln^2 x = (\ln x)^2$

Que: $\dfrac{\ln x + 1 - 1}{(1+\ln x)^2} dx = \dfrac{1}{1+\ln x} - \dfrac{1}{(1+\ln x)^2} dx$

$f(x) = \dfrac{1}{1+\ln x}$

$f'(x) = \dfrac{-1}{(1+\ln x)^2} \times \dfrac{1}{x}$

$= \int \left(\dfrac{1}{1+\ln x} + x \cdot \dfrac{-1}{x(1+\ln x)^2} \right)$

$= x \cdot \dfrac{1}{1+\ln x} + c$

(2) $\int \left(\ln(\ln x) + \dfrac{1}{\ln^2 x} \right) dx$

$\ln x = y$

$x = e^y$

$dx = e^y\, dy$

$\ln^2 x = (\ln x)^2$

$f(y) = \ln y$

$f'(y) = \dfrac{1}{y}$; $f''(y) = -\dfrac{1}{y^2}$

Ans $\Rightarrow \int \left(\ln y + \dfrac{1}{y^2} \right) e^y \cdot dy = \int \left(\ln y + \dfrac{1}{y} - \dfrac{1}{y} + \dfrac{1}{y^2} \right) e^y$

$= \int \left(\ln y + \dfrac{1}{y} \right) e^y dy - \int \left(\dfrac{1}{y} + \left(-\dfrac{1}{y^2} \right) \right) e^y dy$

$= e^y \cdot \ln y - e^y \cdot \dfrac{1}{y} + c$

$= e^y \left(\ln y - \dfrac{1}{y} \right) + c = x \left(\ln(\ln x) - \dfrac{1}{\ln x} \right) + c$

Imp.

Put $x = \frac{1}{t}$ always moveculy

स्कूटर स्कूटर (सास बहू के मांडे)

* Meniculating / Culating Integration :

$$\int \frac{dx}{x(x^6+1)} = \int \frac{1}{x^7\left(1+\frac{1}{x^6}\right)} dx$$

$$\frac{1}{x^6} = k$$

$$x^{-6} = k$$

$$-6\frac{1}{x^7}dx = dk$$

$$= -\frac{1}{6}\int \frac{dk}{1+k} = -\frac{1}{6}\ln(1+k)$$

Method - 2.

Put $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}$

$$\int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\left(\frac{1}{t^6}+1\right)} = \int \frac{1}{t}\frac{t^6}{(1+t^6)}dt$$

$$= -\int \frac{t^5}{1+t^6}dt$$

$$= 1+t^6 = \lambda$$

Que: $\int \frac{x^7}{(1-x^2)^5}dx = \int \frac{x^7}{x^{10}\left(\frac{1}{x^2}-1\right)}$

$$k = \frac{1}{x^2}$$

$$= \frac{\frac{1}{x^3}}{\left(\frac{1}{x^2}-1\right)^5}dx$$

$$\frac{1}{x^2} - 1 = t$$

$$-\frac{2}{x^3}dx = dt$$

Que:- $\int \frac{x\,dx}{(1-x^4)^{3/2}}$

$\int \frac{x\,dx}{x^6\left(\frac{1}{x^4}+1\right)^{3/2}}$ $\qquad \frac{1}{x^4}=t$

$\qquad \frac{-4}{x^5}dx = dt$

$= \int \frac{\frac{1}{x^5}\,dx}{\left(\frac{1}{x^4}-1\right)^{3/2}}$ $\qquad \frac{1}{x^4}-1=t$

Que:- 1) $\int \frac{x-1}{x^2\sqrt{2x^2-2x+1}} = \int \frac{\frac{1}{x^2}-\frac{1}{x^3}}{\sqrt{2-\frac{2}{x}+\frac{1}{x^2}}}$

$= 2-\frac{2}{x}+\frac{1}{x^3} = t^2$

2) $\int \frac{dx}{x^4(x^3+1)^2}$

$\int \frac{dx}{x^{10}\left(1+\frac{1}{x^3}\right)^2} =$ $\qquad 1+\frac{1}{x^3}=t$

25^{th} <u>H.W</u> S-1 → 25, 27 ~~assigning~~ left full comp. full sheet.

Q. $\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}\,dx = \int \frac{1-\frac{1}{x^4}}{\sqrt{1+\frac{1}{x^2}+\frac{1}{x^4}}}\,dx$ ✗

$= \int \frac{x^3\left(x-\frac{1}{x^3}\right)dx}{x^3\sqrt{x^2+1+\frac{1}{x^2}}}$ $\quad x^2+1+\frac{1}{x^2}=k^2$

$\left(2x-\frac{2}{x^3}\right)dx = 2k\,dk$

$\int \frac{k\,dk}{k}$

<u>Que:</u> $\int \frac{(ax^2-b)}{x\sqrt{c^2x^2-(ax^2+b)^2}}\,dx$

$= \int \frac{x^2\left(a-\frac{b}{x^2}\right)dx}{x^2\sqrt{c^2-\left(ax+\frac{b}{x}\right)^2}}$ $\quad ax+\frac{b}{x}=t$

$\left(a-\frac{b}{x^2}\right)dx = dt$

$= \int \frac{dt}{\sqrt{c^2-t^2}}$

29/06/17

(S-1)

(25) diff. both side

$$\frac{1-(\cot x)^{2008}}{\tan x + (\cot x)^{2009}} = \frac{1}{k}\cdot\left(\frac{1}{\sin^k x + \cos^k x}\left(k\sin^{k-1}x\cdot\cos x + k\cos^{k-1}x(-\sin x)\right)\right)$$

$$= \frac{\frac{1}{\sin x^{2008}}(\sin^{2008} - \cos^{2008})}{\frac{\sin x}{\cos x} + \left(\frac{\cos x}{\sin x}\right)^{2009}}$$

$$= \frac{\sin x\cos x(\sin x^{k-2} - \cos^{k-2}x)}{\sin^k x + \cos^k x}$$

$$\sin x\cos x\left(\frac{\sin^{2008} - \cos x^{2008}}{\sin^{2010} + \cos^{2010}}\right)$$

$$= \frac{\frac{1}{\sin^{2008}}(\sin^{2008} - \cos x^{2008})}{\frac{\sin x^{2010} + \cos x^{2010}}{\sin^{2009}\cdot\cos x}}$$

$k = 2010$

* algebric / trigonometric Functions :

$$\int \frac{1}{x^4+1} \qquad \int \sqrt{\tan x}\, dx$$

$$\int \frac{x^2}{x^4+1} \qquad \int \sqrt{\cot x}\, dx$$

$$\int \frac{1}{x^4+kx^2+1} \qquad \int \frac{1}{\sin^4 x+\cos^4 x} dx$$

$$\int \frac{x^2}{x^4+kx^2+1} dx \qquad \int \frac{1}{\sin^6 x+\cos^6 x} dx$$

$$\int \frac{\pm \sin x \pm \cos x}{1+\sin x \cos x} dx$$

Q. $I = \int \frac{1}{x^4+1} dx = \int \frac{(1+x^2)+(1-x^2)}{x^4+1}$

$$= \int \frac{1+x^2}{1+x^4} + \int \frac{1-x^2}{1+x^4} = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx - \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx$$

$x - \frac{1}{x} = t$

$\left(1+\frac{1}{x^2}\right) dx = dt$

$x^2 + \frac{1}{x^2} = t^2+2$

$$= \int \frac{dt}{t^2+2} - \int \frac{dp}{p^2-2}$$

② $\cancel{I} =$

$x + \frac{1}{x} = p$

$\left(1-\frac{1}{x^2}\right) dx = dp$

$x^2 + \frac{1}{x^2} + 2 = p^2$

(2) $I = \int \frac{x^2}{x^4+1} = \frac{1}{2}\int \frac{2x^2}{x^4+1} = \cancel{\frac{1}{2}}$

$\frac{1}{2}\int \frac{(x^2+1)+(x^2-1)}{x^4+1}$

$= \frac{1}{2}\int \frac{x^2+1}{x^4+1}\,dx + \frac{1}{2}\int \frac{x^2-1}{x^4+1}\,dx$

$1 + \frac{1}{x^2} = t$

-2

Q. $\int \frac{dx}{x^2+kx^2+1}$

Ans:

Que: $\int \frac{2x^2}{x^4+1} dx$

(2) $\int \frac{x^2+1}{x^4+3x^2+1} dx = \int \frac{1+\frac{1}{x^2}}{x^2+3+\frac{1}{x^2}}$

$= \int \frac{dt}{t^2+5}$

~~Q.~~ Trigonometric twins

1 – $\int \sqrt{\tan\theta}\, d\theta$

$\tan\theta = k^2$

$\sec^2\theta\, d\theta = 2k\, dk$

$d\theta = \frac{2k\,dk}{\sec^2\theta}$

$= \frac{2k\,dk}{1+\tan^2\theta}$

$= \frac{2k\,dk}{1+k^4}$

$\int k \cdot \frac{2k\,dk}{1+k^4}$

$= \int \frac{2k^2\,dk}{1+k^4}$

$= \int \frac{(k^2+1)+(k^2-1)}{k^4+1} dk$

Que: $\int \frac{d\theta}{\sin^4\theta+\cos^4\theta}$

$\tan\theta = k$

$\sec^2\theta\, d\theta = dk$

$\int \frac{d\theta}{\cos^4\theta(\tan^4\theta+1)} = \int \frac{\sec^4\theta\, d\theta}{\tan^4\theta+1}$

$= \int \frac{(1+\tan^2\theta)}{1+k^4} dk = \int \frac{1+k^2}{1+k^4} dx$

$k - \frac{1}{k} = u$

$= \int \frac{1+\frac{1}{k^2}}{k^2+\frac{1}{k^2}} dk$

Q. $I = \int \left(\sqrt{\tan\theta} + \sqrt{\cot\theta}\right) d\theta$

$$\Rightarrow \int \sqrt{\frac{\sin\theta}{\cos\theta}} + \sqrt{\frac{\cos}{s}}$$

$\sin\theta - \cos\theta = k$

$(\cos\theta + \sin\theta)\, d\theta = dk$

$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = k^2$

$\frac{1-k^2}{2} = \sin\cos$

$$= \int \frac{\sin\theta + \cos\theta\, d\theta}{\sqrt{\sin\theta\cos\theta}} = \sqrt{2}\int \frac{dk}{\sqrt{1-k^2}}$$

Que:- $\int \frac{d\theta}{\sin^6\theta + \cos^6\theta}$

$\left[a^3+b^3 = (a+b)(a^2+b^2-ab)\right]$

$$= \quad d \quad \frac{\sec^4\theta\,\sec^2\theta}{1+\tan^6\theta}$$

$$= \int \frac{(1+k^2)^2}{1+k^6}\, dk$$

$$= \int \frac{(1+k^2)^{\cancel{2}}}{(1+k^2)(1+k^4-k^2)}\, dk$$

$$= \int \frac{1+\frac{1}{k^2}}{k^2+\frac{1}{k}-1}\, dk. \quad \text{An}$$

* $\int \frac{\pm \sin x \pm \cos x}{1+\sin x\cos x}\, dx$

Q.u $\int \frac{(s+c)+(s-c)}{1+s\,c}$

$$= \int \frac{\sin x + \cos x}{1 + \sin x \cos x} dx + \int \frac{s - c}{1 + sc} dx$$

$$= \int \frac{dt}{1 + \frac{1-t^2}{2}} + \int \frac{-dd}{1 + \frac{d^2-1}{2}}$$

$\sin x + \cos x = d$

$(\cos x - \sin x) dx = dd$

$\sin x - \cos x = t$

$(\cos x + s) dx = dt$

$$= 2\int \frac{dt}{3-t^2} - 2\int \frac{dd}{1+d^2}$$

$\sin^2 + c^2 + 2sc = d^2$

$s^2 + c^2 - 2sc = t^2$

$sc = \frac{d^2-1}{2}$

$1 - t^2 = 2sc$

$sc = \frac{1-t^2}{2}$

$\int$

Que.) $\int \frac{2\cos x}{10 + \sin 2x} dx$

$$= \frac{\cos x + \sin x + \cos - \sin}{10 + 2\sin \cos}$$

$$I = \int \frac{c + s}{10 + 2\sin \cos x} dx + \int \frac{\cos - \sin}{10 + 2sc}$$

$$= \int \frac{dt}{10 + 1 - t^2} + \int \frac{dd}{10 + d^2 - 1}$$

V. Imp.

H.W → all full concept

$\rightarrow \left(x \pm \frac{4}{x}\right)^2 = x^2 + \frac{16}{x^2} \pm 8$

Q. $\int \frac{dx}{x^4+16} = \int \frac{dx}{x^2\left(x^2+\frac{16}{x^2}\right)}$

$\left(x \pm \frac{4}{x}\right)^2 =$

$\left(x \pm \frac{4}{x}\right)^2 = x^2 + \frac{16}{x^2} \pm 8$

$x + \frac{4}{x} = u$

$\left(1 - \frac{4}{x^2}\right)dx = du$

$x^2 + \frac{16}{x^2} = u^2 - 8$

$x - \frac{4}{x} = t$

$\left(1 + \frac{4}{x^2}\right)dx = dt$

$x^2 + \frac{16}{x^2} = t^2 + 8$

$\frac{1}{8}\int \frac{8\,dx}{x^2\left(x^2+\frac{16}{x^2}\right)}$

$= \frac{1}{8}\int \frac{\left(\frac{4}{x^2}+1\right) + \left(\frac{4}{x^2}-1\right)}{x^2+\frac{16}{x^2}}$

$= \frac{1}{8}\int \frac{1+\frac{4}{x^2}}{x^2+\frac{16}{x^2}}\,dx - \frac{1}{8}\int \frac{1-\frac{4}{x^2}}{x^2+\frac{16}{x^2}}\,dx$

$= \frac{1}{8}\int \frac{dt}{t^2+8} - \frac{1}{8}\int \frac{du}{u^2-8}$

Que: $\int \frac{x^2+3}{x^4+19x^2+19x}\,dx = \frac{1+\frac{3}{x^2}}{x^2+19+\frac{9}{x^2}}$

$= \int \frac{1+\frac{3}{x^2}}{x^2+19+\frac{9}{x^2}}$

$x - \frac{3}{x} = t$

$\int \left(1 + \frac{3}{x^2}\right)dx = dt$

$x^2 + \frac{9}{x^2} = t^2 + 6$

$= \int \frac{dt}{t^2+25}$

Imp

$\int \frac{1}{A\sqrt{B}}$ Integration of the form

Where A and B are linear or quadratic expression

L – Linear
Q – Quadratic

A	B	Substitution.
L	L	Put $B = t^2$
Q	L	Put $B = t^2$
L	Q	Put $A = \frac{1}{t}$
Q	Q	

goes to twin

$$\frac{dx}{(ax^2+bx+c)\sqrt{px^2+qx+r}}$$

Break into Linear

$(ax^2+bx+c) = a(x-\alpha)(x-\beta)$

(1) $\therefore \frac{1}{ax^2+bx+c} = \frac{A}{x-\alpha} + \frac{B}{x-\beta}$

$\therefore I = A\int \frac{dx}{(x-\alpha)\sqrt{px^2+qx+r}}$

(1) $+ B\int \frac{dx}{(x-\beta)\sqrt{px^2+qx+r}}$

Q $\int \frac{1}{(x-1)\sqrt{x-2}}$

$A = (x-1)$
$B = (x-2)$

Q $\int \frac{dx}{(x^2+x+1)\sqrt{x+1}}$

Q L

(2) if $ax^2+bx+c = (lx+m)^2$

$\therefore I = \int \frac{dx}{(lx+m)^2\sqrt{px^2+qx+r}}$

$lx+m = \frac{1}{t}$

— Q

$b = q = 0$ Imp

$I = \int \frac{dx}{(ax^2+c)\sqrt{px^2+r}}$

Put $x = \frac{1}{t}$

Ques $\int \frac{dx}{(x-1)\sqrt{x-2}}$

$= \quad A = (x-1)$

$B = (x-2)$

$\int \frac{d(x)}{(x-1)\,t^2} \qquad B = t^2$

$(x-2) = t^2$

$1dx = 2t\,dt$

$\frac{2t\,dt}{(x-1)\,t^2}$

$2\int \frac{dt}{t^2+1}$

Q.2 $\int \frac{dx}{(x-1)\sqrt{x^2+x+1}}$

$= \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\cdot\sqrt{\frac{3t^2+3t+1}{t}}}$

$x-1 = \frac{1}{t}$

$x-1 = \frac{1}{t}$

$= -\int \frac{dt}{\sqrt{3t^2+3t+1}} =$

$dx = -\frac{1}{t^2}dt$

$x = \frac{1}{t}+1 = \frac{t+1}{t}$

$x^2+x+1 = \left(\frac{t+1}{t}\right)^2 + \frac{t+1}{t} + 1$

$= \frac{(t+1)^2 + t(t+1) + t^2}{t^2}$

$= \frac{3t^2+3t+1}{t^2}$

Q $\int \frac{dx}{(x^2-2x+1)\sqrt{x^2+x+1}}$

$= \int \frac{dx}{(x-1)^2\sqrt{x^2+x+1}} = \int \frac{t}{\sqrt{3t^2+3t+1}}\,dt$

Q. $\int \frac{\cancel{16}\, dx}{(x^2+5x+2)\sqrt{x-2}}$ $\qquad x-2=t^2$, $dx = 2t\,dt$

$= \int \frac{8t\,dt}{x^2+5x+2 \times t}$

$= \int \frac{8\,dt}{t^2+at^2+16}$ $\qquad t \pm \frac{4}{t} =$

$= \int \frac{\frac{8}{t^2}\,dt}{t^2+\left(\frac{4}{t}\right)^2+9}$ $\qquad 1 \mp \frac{4}{t^2} =$

$= \int \frac{\left(1+\frac{4}{t^2}\right)-\left(1-\frac{4}{t^2}\right)dx}{t^2+\left(\frac{4}{t}\right)^2+9}$

$= \int \frac{1+\frac{4}{t^2}}{t^2+\left(\frac{4}{t}\right)^2+9}\,dt - \int \frac{\left(1-\frac{4}{t^2}\right)}{t^2+\left(\frac{4}{t}\right)^2+9}\,dt$

$t - \frac{4}{t} = \lambda$ $\qquad t+\frac{4}{t} = P$

Que: $\int \frac{dx}{(x^2-x)\sqrt{x^2+x+t}}$ $\qquad \frac{1}{x^2-1} = \frac{1}{x-1} - \frac{1}{x}$

$\int \frac{dx}{(x-1)\sqrt{x^2+x+1}} - \int \frac{dx}{x\sqrt{x^2+x-1}}$

$x = \frac{1}{t}$

Q. $\int \frac{dx}{(x^2+3x+2)\sqrt{x-1}}$

$= \int \frac{1}{x^2-3x+2} = \frac{1}{(x-1)(x-2)}$

$\sqrt{x-1}$
$x-1 = t^2$
$dx = 2t\,dt$

$= \frac{1}{x-2} - \frac{1}{x-1}$

$= \int \frac{1}{(x-2)(\sqrt{x+1})} dx - \int \frac{dx}{(x-1)\sqrt{x+1}}$

Step:

$x = t^2-1$
$x+1 = t^2$
$dx = 2t\,dt$

$\int \frac{dx}{t}$

$\int \frac{2t\,dt}{x\,t}$

$\int \frac{2dt}{}$

Que: $\int \frac{dx}{(x^2+1)\sqrt{x^2-2}}$

$x = \frac{1}{t}$

$dx = -\frac{1}{t^2}dt$

$= \int \frac{-\frac{1}{t^2}dt}{\frac{1+t^2}{t^2} \cdot \sqrt{\frac{1-2t^2}{t}}}$

$-t\,dt = \frac{1}{2}u\,du$

$1-2t^2 = u^2$
$-4t\,dt = 2u\,du$
$t\,dt = \frac{1}{2}u\,du$
$t^2 = \frac{1-u^2}{2}$
$1+t^2 = 1+\frac{1-u^2}{2}$

$= -\int \frac{t\,dt}{(1+t^2)(\sqrt{1-2t^2})}$

$= \frac{1}{2}\int \frac{u\,du}{u\left(3-\frac{u^2}{2}\right)} = \int \frac{du}{3-u^2}$ A

M-2 $t = \sin\theta$

Reduction formula :

$\sin^2 x + \cos^2 x = 1$

$$I_n = \int \sin^n x \, dx.$$

$$= \underset{II}{\sin x} \cdot \underset{I}{\sin^{n-1} x} \, dx$$

Break $\sin^{n-1}$ Imp. line

$$= -\cos x \sin^{n-1} x - (n-1) \int (-\cos x) \cdot \sin^{n-2} x \cdot \cos x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x - (n-1) \int (\sin^2 x - 1) \cdot \sin^{n-2} x \, dx$$

$$I_n = -\cos x (\sin x)^{n-1} - (n-1) \int \sin^n x \, dx + (n-1) \int \sin^{n-2} x \, dx$$

$$= -\cos x (\sin x)^{n-1} - (n-1) I_n + (n-1) I_{n-2}$$

$$((n-1)+1) I_n = -\cos x \sin^{n-1} x + (n-1) I_{n-2}$$

$$\boxed{I_n = -\frac{1}{n} \cdot \cos x \sin^{n-1} x + \frac{n-1}{n} I_{n-2}}$$

$$I_3 = -\frac{1}{3} \cos x \sin^2 x + \frac{2}{3} I_1$$

$$I_3 = \int \sin^3 x \, dx.$$

$$I_1 = \int \sin x \, dx = -\cos x.$$

Learn

fimb

Q $I_n = \int \tan^n x\, dx$

$\int \tan^2 x \cdot \tan^{n-2} x\, dx$

$\int (\sec^2 x - 1) \tan^{n-2} x\, dx$

$I_n = \int \sec^2 x \cdot \tan^{n-2} x\, dx - \int \tan^{n-2} x\, dx$

$I_n + I_{n-2} = \int \sec^2 x \cdot \tan^{n-2} x\, dx$

$= \int t^{n-2}\, dt = \frac{t^{n-1}}{n-1} = \frac{(\tan x)^{n-1}}{n-1}$

$I_3 = \int \sin^3 x\, dx$

$I_1 = \int \sin x\, dx = -\cos x.$

Q2 $I_n = \int \sec^n x\, dx$

$= \int \underset{\text{II}}{\sec^2 x}\ \underset{\text{I}}{\sec^{n-2} x}\, dx$

$= \sec^{n-2} x \tan x - (n-2) \int \sec^{n-3} x \cdot \sec x \tan x \cdot \tan x\, dx$

$= \sec^{n-2} x \cdot \tan x - (n-2) \int \sec^{n-2} x \cdot (\sec^2 x - 1)\, dx$

$= \_\_\_\_\_\_\_\_ - (n-2) \int \sec^n x\, dx + (n-2) \int \sec^{n-2} x\, dx$

$$\boxed{I_n = \sec^{n-2} x \tan x - (n-2) I_n + (n-2) I_{n-2}}$$

1/7/17

ELATE

→ full complete

Q. $I_n = \int \frac{dx}{(x^2+a^2)^n}$

$\int \underset{II}{1} \cdot \underset{I}{\frac{dx}{(x^2+a^2)^n}} =$

$x \frac{1}{(x^2+a^2)^n} - \int \frac{(-n)\cdot(2x)}{(x^2+a^2)^{n+1}} \, x \cdot dx$

$= \frac{x}{(x^2+a^2)^n} + 2n \int \frac{(x^2+a^2) - a^2}{(x^2+a^2)^{n+1}} \, dx$

$I_n = \frac{x}{(x^2+a^2)^n} + 2n \int \frac{dx}{(x^2+a^2)^n} - 2na^2 \int \frac{dx}{(x^2+a^2)^{n+1}}$

$I_n = \frac{x}{(x^2+a^2)^n} + 2n I_n - 2na^2 I_{n+1}$