Handwritten Notes
On
Function

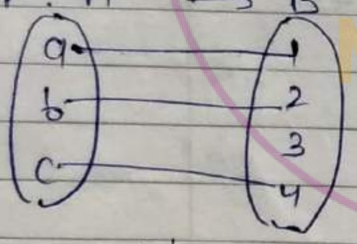
function

Set :

Let A, B be two sets and their exist a rule of mapling f which associate **each** element of Set A to unike element insectry. then f is called maping or function from A to B then it is called maping or function of Sat A to B
and dinoted by $A \xrightarrow{f} B$

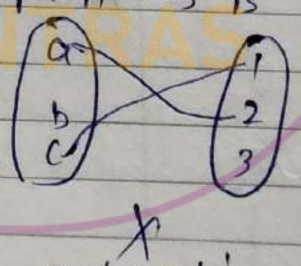
$$f: A \longrightarrow B$$

$f: A \longrightarrow B$



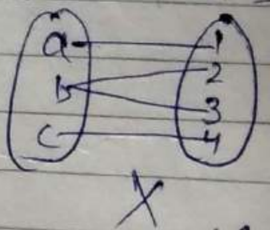
maping

$f: A \rightarrow B$



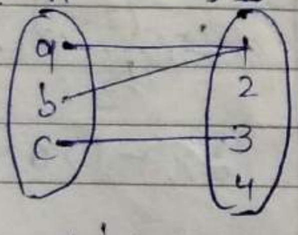
not maping

$f: A \rightarrow B$



non-maping

$f: A \longrightarrow B$



maping

1 is image of a ⇒ a is proamage of 1
3 " " of b ⇒
4 " " of c ⇒

Domain of f = Set A
= {a, b, c}

Rang = {1, 3, 4}

co-ordine = {1, 2, 3, 4}

Rang ≠ co-domain

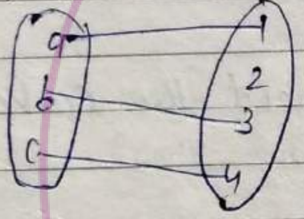
Domain of f = {a, b, c}

Range = {1, 3}

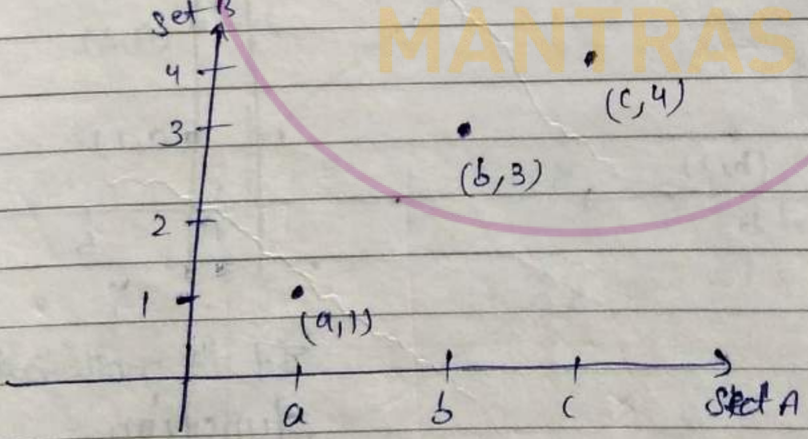
co-domain = {1, 3)

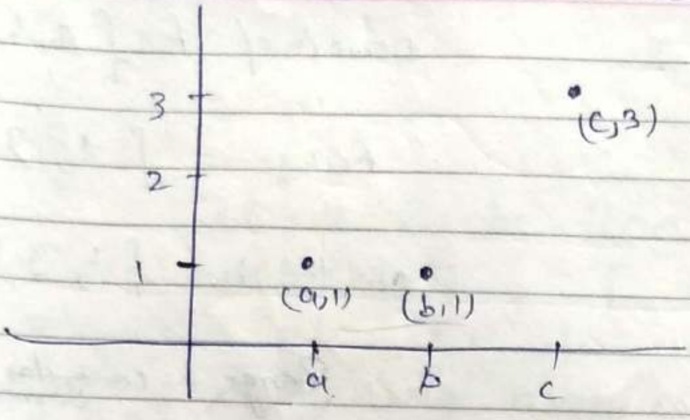
Renge = co-~~ordian~~ Domain

$f: A \longrightarrow B$

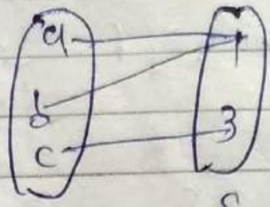


f(a) = 1 , f(b) = 3 , f(c) = 4





f : A → B



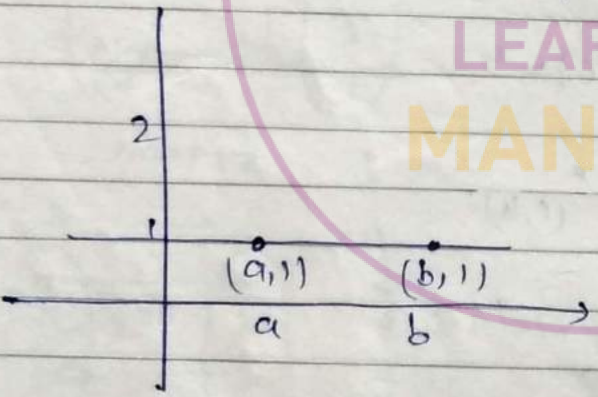
Domain = {a, b, c}

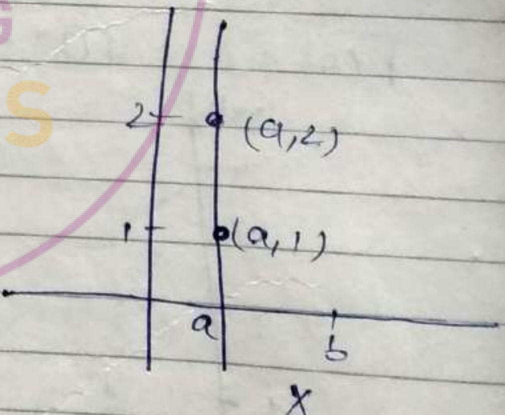
Range = {1, 3}

co-domain = {1, 3}

* If ==line Parallel== to ==x-axis== meet the curved then it is called function.

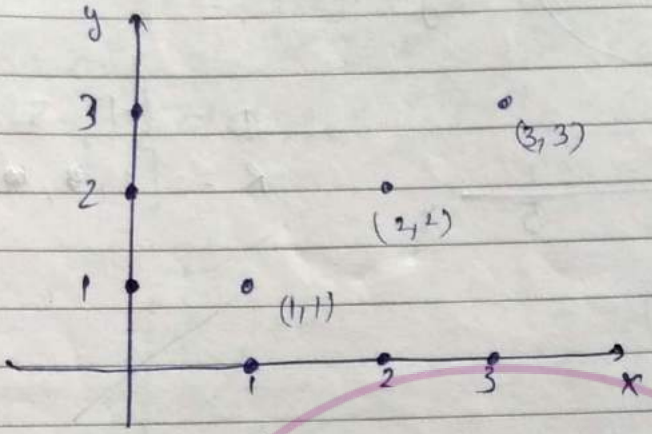
* If line Parallel to y axis meet the curved then it is not a function





It is not ~~a~~ function.
because Y axis Curve is Cut ~~2 time~~ of 2 Point.

$f: \{1, 2, 3\} \rightarrow \{1, 2, 3\}$

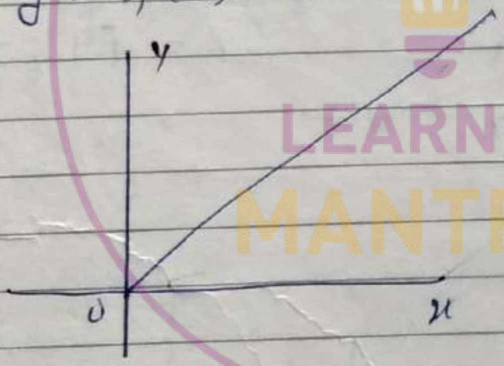


$R = (-\infty, \infty)$

$y = f(x) = x$

$f: (0, \infty) \rightarrow R$

$y = f(x) = x$

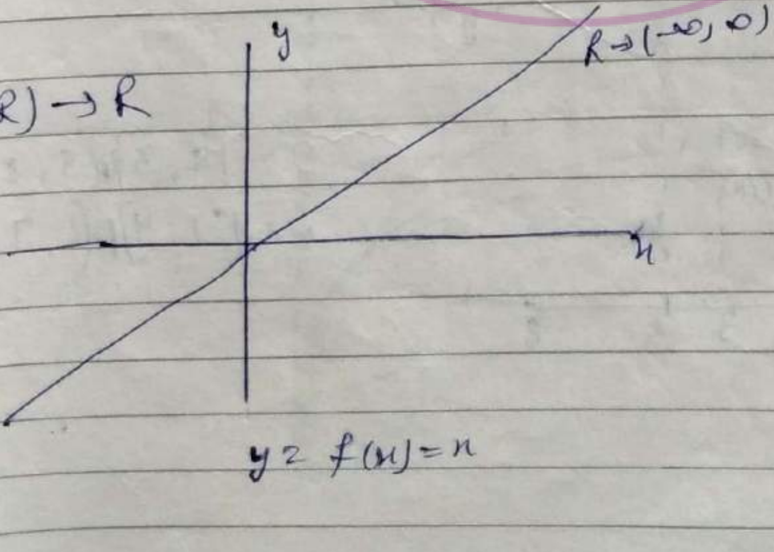


Domain = $[0, \infty)$

Range = $[0, \infty)$

co-domain = R

$f(R) \rightarrow R$



$y = f(x) = x$

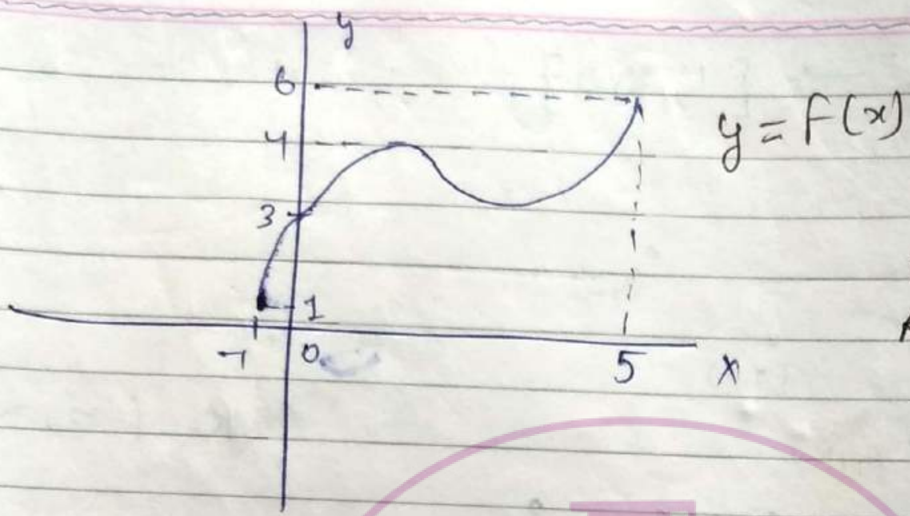
$y = f(x) = x$

Domain = R

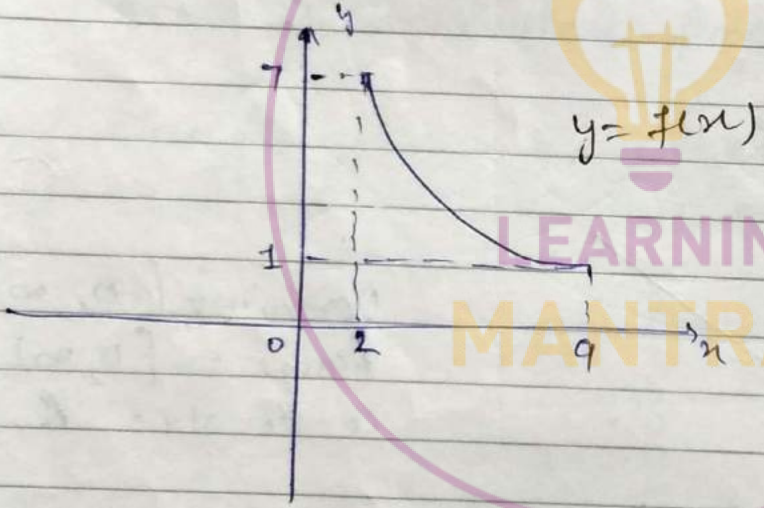
Co-domain = R

Range = R

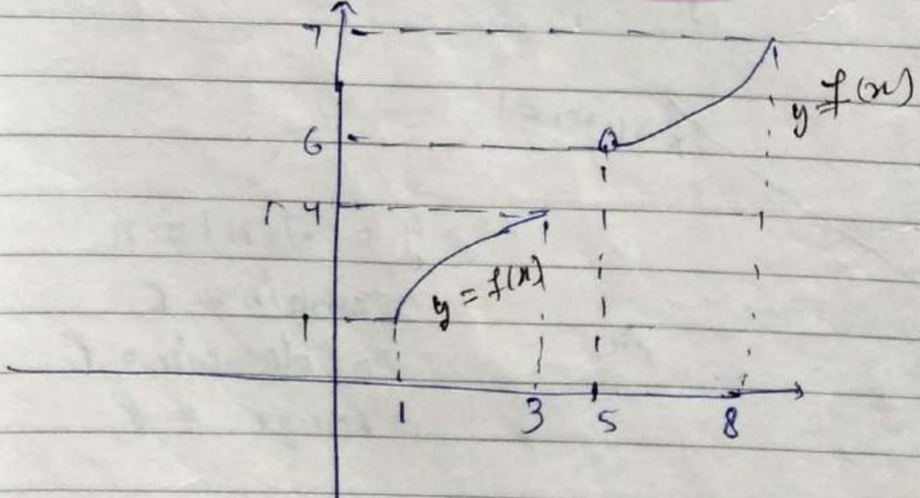
90 3-6



D = [-1, 5]
R = [1, [illegible], [illegible], 6]



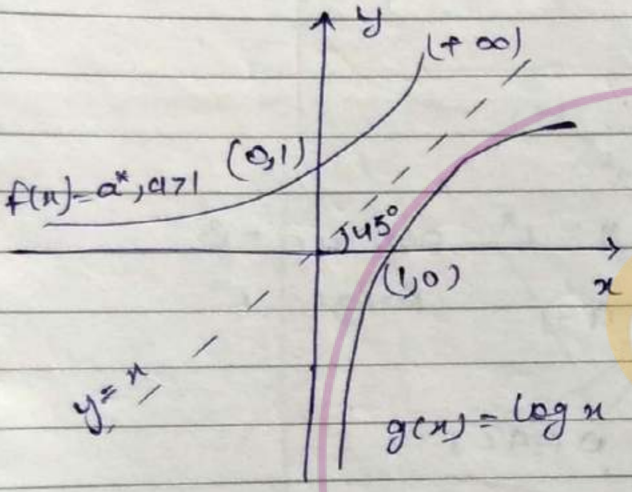
D = [2, 9)
R = [1, 7)

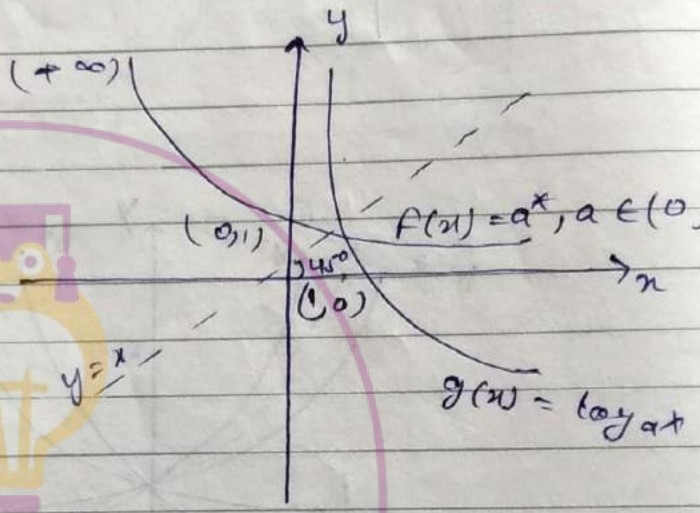


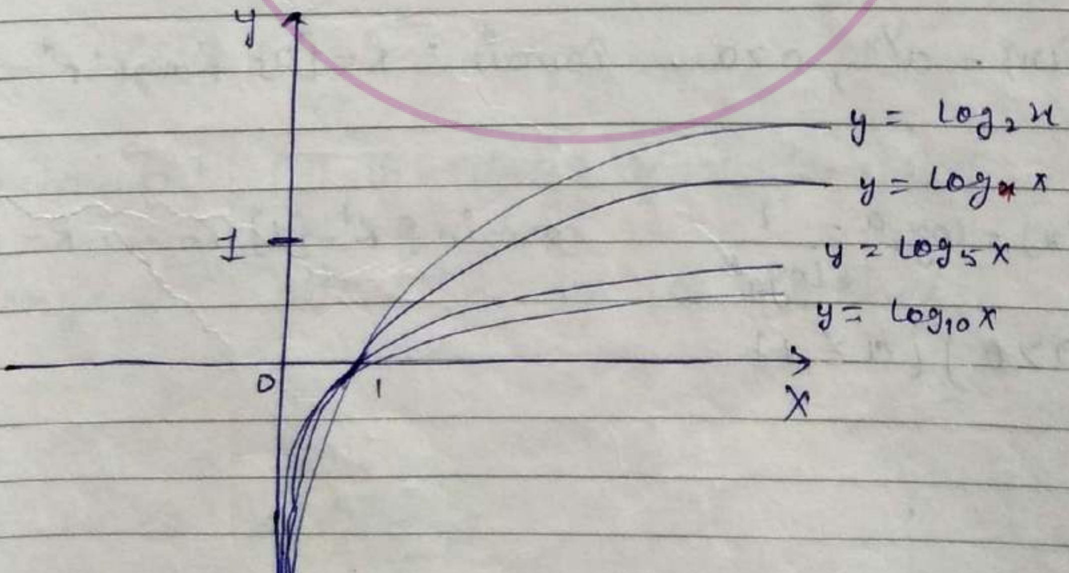
D = [1, 3]∪(5, 8]
R = [1, 4]∪(6, 7]

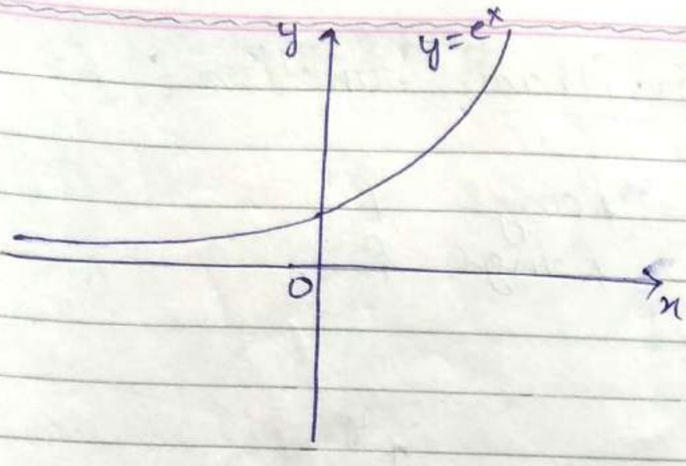
* Exponential and logarithmic function :

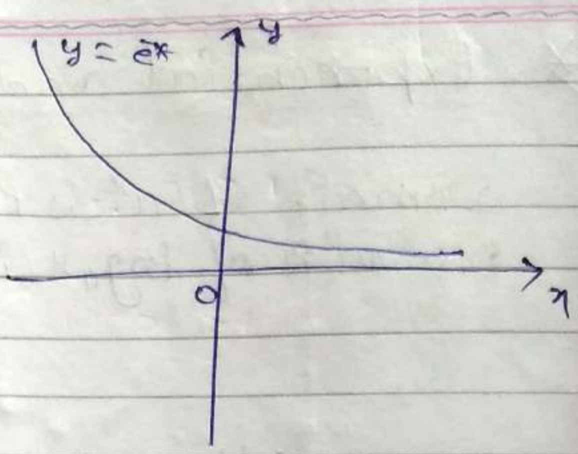
Domain of a^x is R   Range R^+
Domain of $\log_a x$ is R^+ Range R

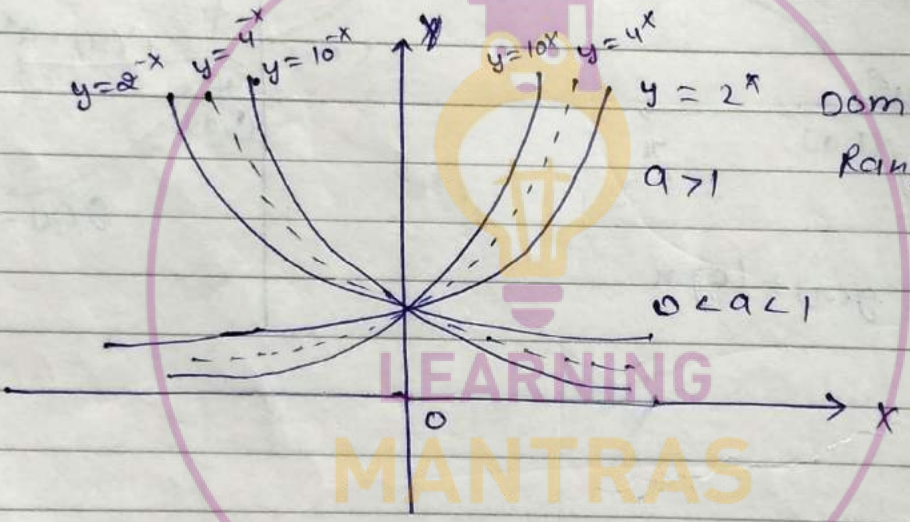












Domain : R
Range : R^+

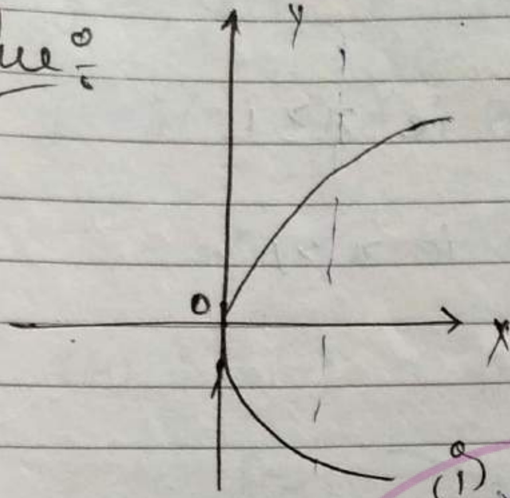
Note :- 1 $f(x) = a^{1/x}$, $a>0$ Domain :- $R-\{0\}$, Range :- $R^+-\{1\}$

Note :- 2 $f(x) = \log_x a = \frac{1}{\log_a x}$ Domain :- $R^+-\{1\}$, Range : $R-\{0\}$

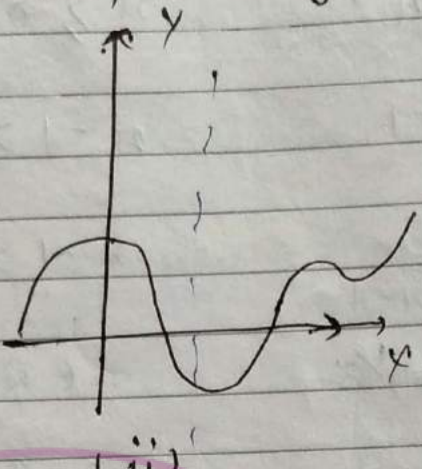
$(a>0)(a\neq 1)$

Date : 04/05/17 which is following function.

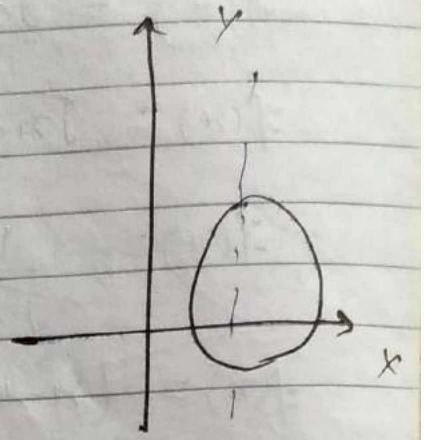
Que :



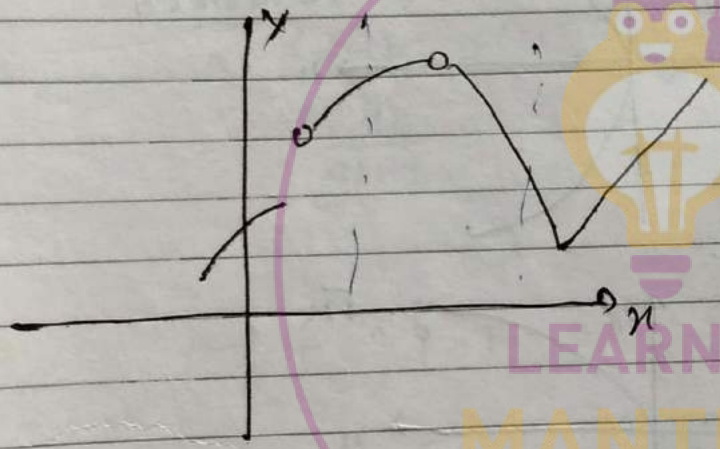
(i) ✗



(ii) ✓



(iii) ✗



(iv) ✓

Que : $f(x) = \sqrt{x}$
Do

Domain : All the value of x for which function $y = f(x)$

Range : Collection of the Outputs.

$\sqrt{4} = \pm 2$

$(4)^{1/2} = \sqrt{4} = 2$ ✓

$(-3)^{1/2} = \sqrt{-3} = N.D$

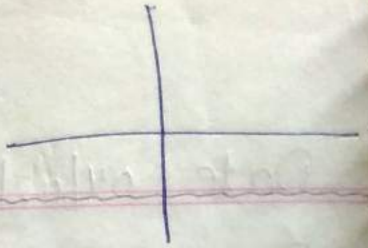
$(-27)^{1/3} = -3$

$(27)^{1/3} = 3$

* 24

Line eqn

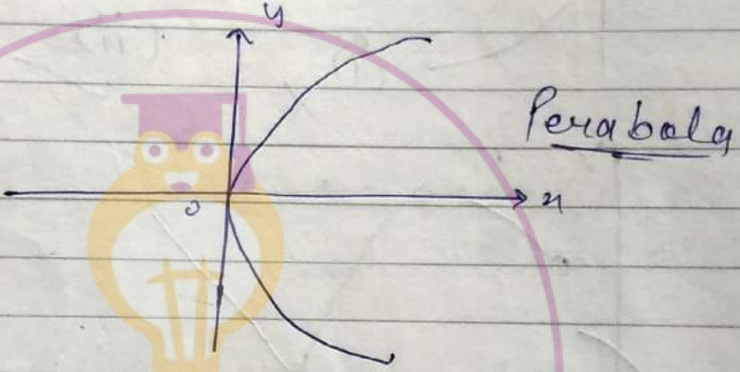
$y^2 = x$



	Domain
$f(x) = \sqrt{x}$	$x \geq 0$
$f(x) = \sqrt{x-1}$	$x-1 \geq 0$ ie $x \geq 1$
$f(x) = \dfrac{1}{\sqrt{x-1}}$	$x-1 > 0$ ie $x > 1$
$f(x) = (x-1)^{1/3}$	$x \in R$

$y^2 = 4ax$

Perabola

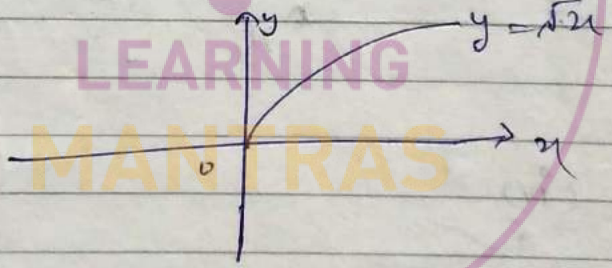


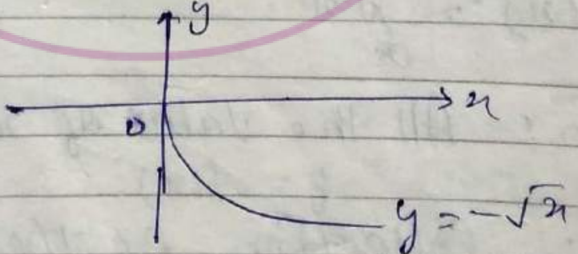
$y^2 = x$

$\sqrt{y^2} = \sqrt{x}$

$|y| = \sqrt{x}$

$y = \pm \sqrt{x}$





* $y = f(x)$ D_1

$y = g(x)$ D_2

Therefore domain of new function $f(x)+g(x)$ function.

Domain

$f(x) = f(x) + g(x)$ — $D_1 \cap D_2$

$= f(x) - g(x)$ $D_1 \cap D_2$

$= f(x) \cdot g(x$ $D_1 \cap D_2$

$= \frac{f(x)}{g(x)}$ $D_1 \cap D_2$ $\{x : g(x) = 0\}$

$(f+g)(x)$

Ex: $f(x) = \sqrt{x-1}$ $g(x) = \sqrt{4-x}$

Domain $x - 1 \geq 0$ Domain $4 - x \geq 0$

$x \geq 1$ — D_1 $x - 4 \leq 0$

$x \leq 4$ — D_2

→ $f(x) + g(x)$ — $[1, 4] - \{1\}$

$f(x) - g(x)$ — $[1, 4]$

$f(x) \cdot g(x)$ — $[1, 4]$

→ $\frac{f(x)}{g(x)}$ — $[1, 4] - \{4\}$

→ $\frac{g(x)}{f(x)}$ — $[1, 4] - \{1\}$

* Types of function :

1. Polynomial function :

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + - - . a_n x^n.$

Called polynomial function of degree where $a_0, a_1 - - - a_n$ are constent and n is +ve integer (I^+)

- $f(x) = 3x^2 - 4x + 7$ - ~~Linear~~ Polynomial Polynomial of ° 2
- $f(x) = x^3 - 72x^2 + 7x - 9$ - Quadratic 3° Polynomial

$f(x) = x^4 - 3x^3 + 7x - 1$ - Cubic 4° Polynomial

$f(x) = x^{17} - 3x + 5$ - by Quadratic 17° Polynomial

$f(x) = x^{16} - 4\sqrt{x} + 7$ - X not Polynomial.

$f(x) = ax + b \rightarrow$ linear polynomial.

$f(x) = ax$ odd ~~pol~~ linear polynomial.

$f(x) = x^4 - 3x^3 + 7x - 1$
$f(x) = x^{17} - 3x + 5$
} monic

* If leding of cofficient 1 is monic polynomial.

* If leading of Coff. 2,3,4,5 is non - monic

- Domain = R
- Range of odd degree polynomial will be real no.
- ~~For~~ Range of even degree polynomial will be be ($\neq R$) subset ~~[illegible]~~ of real no

$$f(x+y) = f(x) \cancel{+} f(y)$$ Called functional equation.

Particular types of polynomial function satisyfy functional equation.

$$f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$$

where $f(x) = \cancel{f(x)} \pm x^n + 1$

Qu:- A functional equation satisfy.

$$f(x)\ f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$$

if $f(4)$ is 65

Ans :-

$$f(x) = x^n + 1$$
$$65 = 4^n + 1$$
$$64 = 4^n$$
$$4^n = 4^3$$
$$n = 3$$
$$\boxed{f(x) = x^3 + 1}$$

$$\left[\begin{array}{l} \log_a^n = N \\ \therefore\ x = a^n \end{array}\right]$$

2b

35 Sheet Questions

* Alzabric function:- If function is constricted using Alzabric opration such as

$+ , - , \times , \div , \sqrt{x}$ . . .

$f(x) = \frac{\sqrt{x} + 3x}{4}$ - Alzabric.

$f(x) = x^2 - 3x + 4$ Alzabric function Polynomial

* ~~Relational~~ Rational function:-

$f(x) = \frac{P(x)}{Q(x)}$

$f(x) = \frac{x-1}{x+2}$

Domain:- All the values of x for which function $y = f(x)$

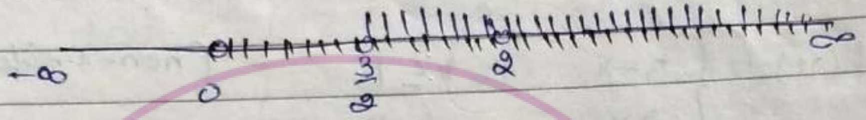
Range:- Callection of all the Outputs

Exercise : O-1

05/05/17

10) $y = \sqrt{x} + \frac{1}{(x-2)^3} - \log_{10}(2x-3)$

$\downarrow$ $x \geq 0$ $\qquad$ $\downarrow$ $x \in R - \{2\}$ $\qquad$ $\downarrow$ $2x-3 > 0$, $x > \frac{3}{2}$



$x \in (\frac{3}{2}, \infty) - \{2\}$

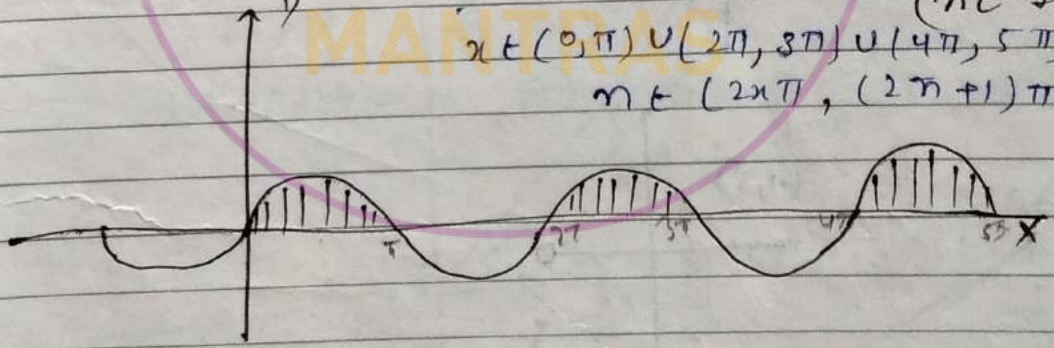
(12) $f(x) = \frac{1}{\sqrt{\sin}} + (\sin x)^{1/3}$

Ans: $\downarrow$ $\sin x > 0$ $\qquad$ $\downarrow$ $x \in R$ (Sin take all value)

($n \in$ Integer)

$x \in (0,\pi) \cup (2\pi, 3\pi) \cup (4\pi, 5\pi)$.

$x \in (2n\pi, (2n+1)\pi)$



(11) $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$

$x^2 - 4 \neq 0$

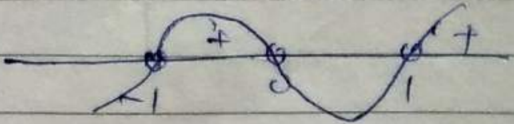
$x = \pm 2$

$R - \{2, -2\}$

$\rightarrow x^3 - x > 0$.

$x(x^2-1) > 0$

$x(x-1)(x+1) > 0$



$(-1,0) \cup (1,\infty) - \{2\}$

3°

Que:1 (i) $y = \sqrt{-Px}$

Ans:- $-Px \geq 0$

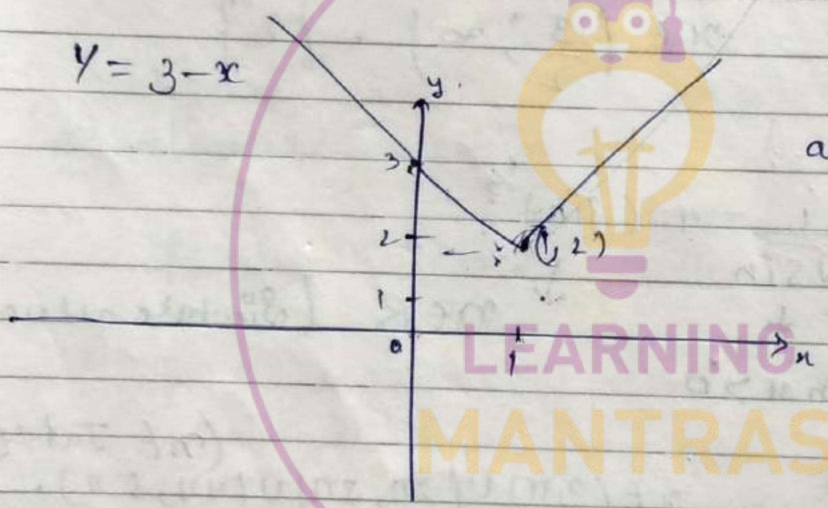
↑ +ve

$-4x \geq 0$

$x \leq 0$ Ans

Que: 4

Ans:- $f(x) = \begin{cases} 3-x & x \leq 1 \\ 2x & x > 1 \end{cases}$ (non-uniform function)

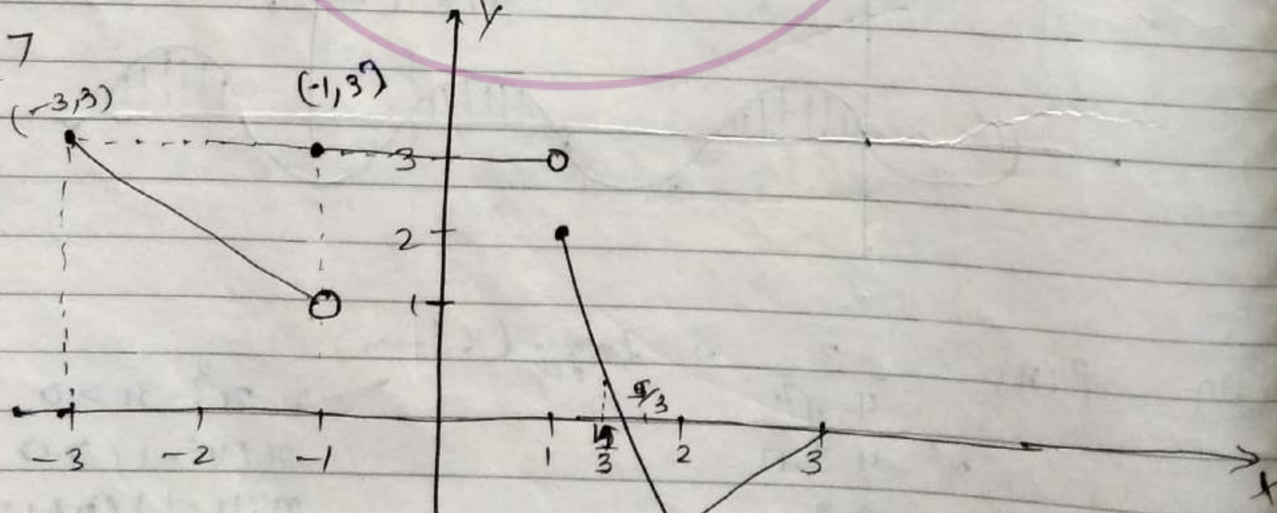


y = 3-x

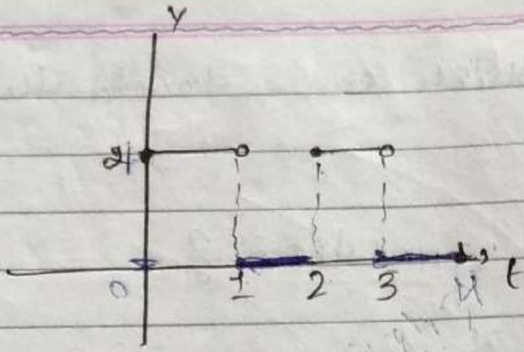
at x-axis y = 0
x = 3

at y axis x = 0
y = 3

Que: 7



Que: 5 (b):



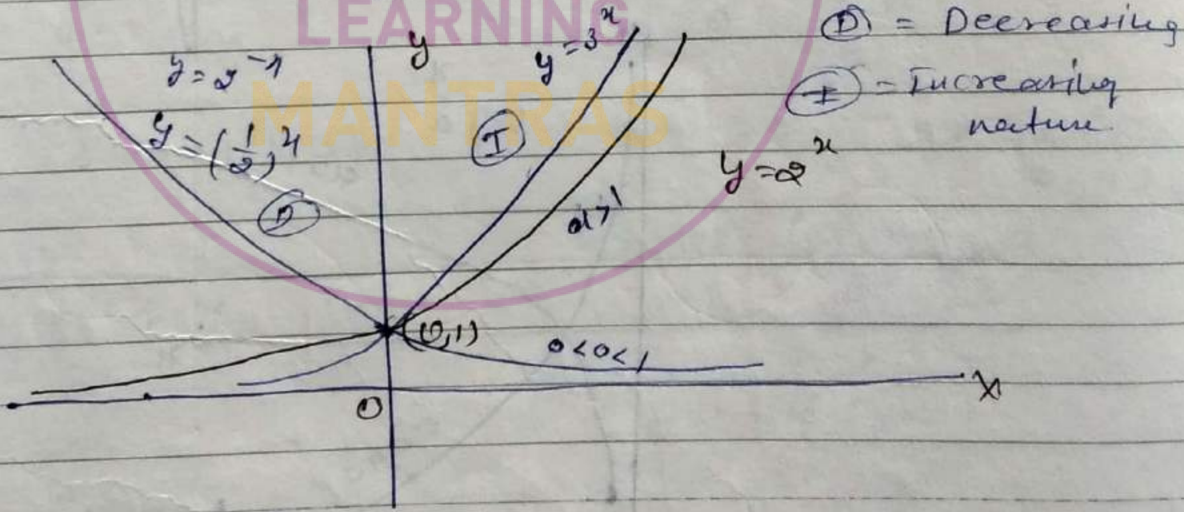
$$f(x) = \begin{cases} 2 & 2I \le x < 2I+1 \\ 0 & 2I+1 < x < 2I+2 \end{cases}$$

* Expontenial function:

$y = a^x$

$a > 0$ & $a \neq 1$

(y = 3x)



Domain = R
Range = $(0, \infty)$

Que: Solve $2^x = -8$
Ans = ϕ

asymptote

* asympote has which curve touch the y axi's on infinity

* Logtharmic function :

$y = \log_a^{x}$

↓

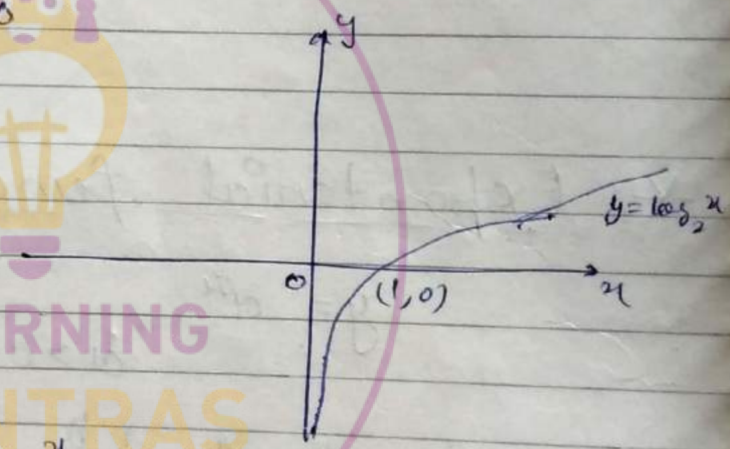
base

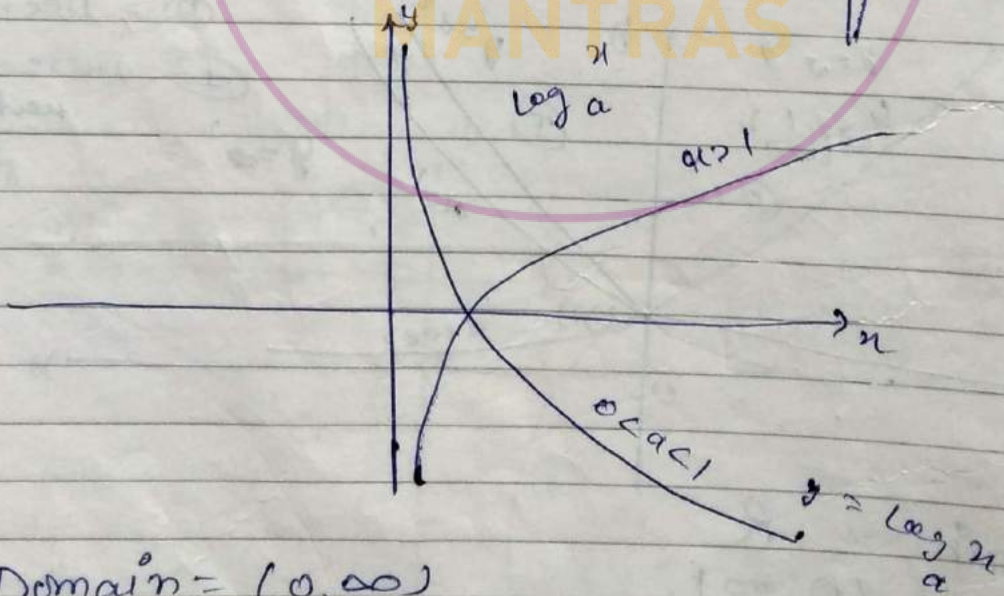
always (+) $a>0$

$a \neq 1$ $x>0$

$\log_n^{n}=n$ $n\neq 1$

x	$y=\log_2^{x}$
2	$y = \log_2^{2} = 1$
2^2	$y = l_2\ 2^2 = 2$
2^3	$= 3$





Domain $= (0, \infty)$

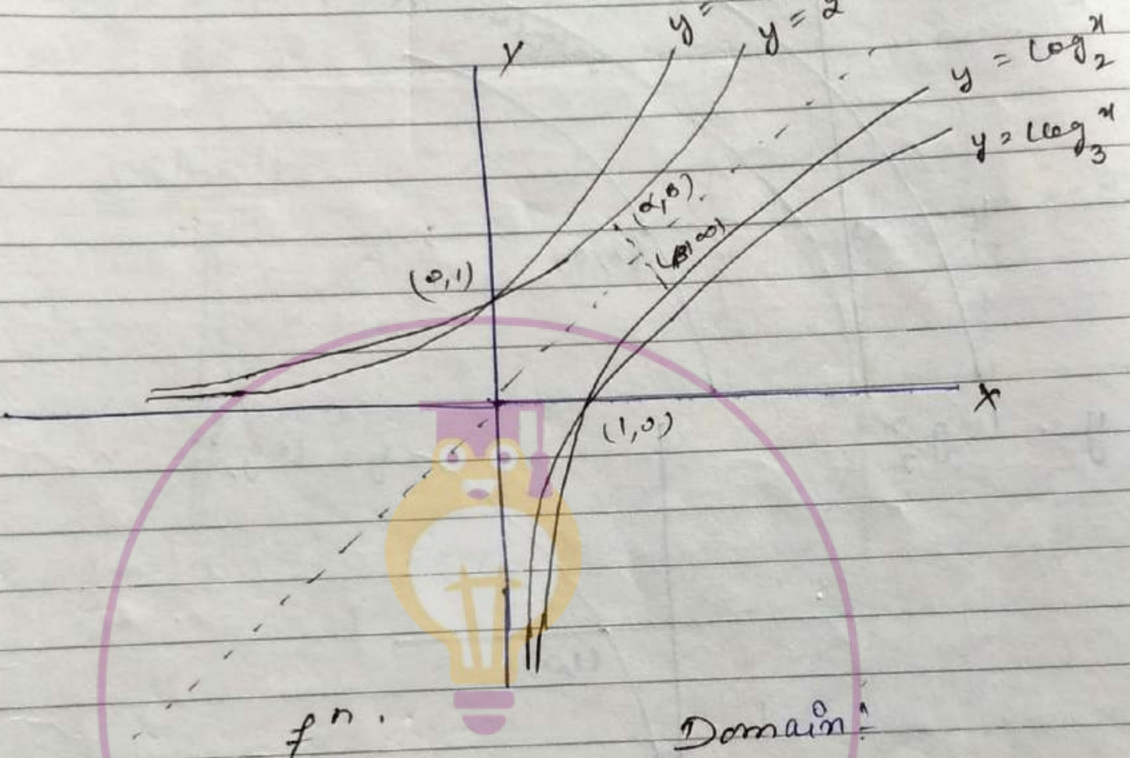
Range $=$ R

~~which~~

Y axi's is axcimpate

$\log_a^x = N \iff x = a^N$

$y = \log_a^x$



f^n.	Domain:
$y = \log_{10}^x$	$x > 0$
$y = \log(4-x)$	$4 - x > 0 \Rightarrow x < 4 < 0$ $x < 4$
$y = \dfrac{1}{\log(x-1)}$	$x > 0$ & $\log_2^x > 0$ $x > 2^0 = 1$ Ans = $(1, \infty)$.

$y = \log_{10} \log_{1/3}^x$

$\downarrow$ $x > 0$ & $\log_{\frac{1}{3}} x > 0$

$x < \left(\frac{1}{3}\right)^0$

$x < 1$

$x \in (0, 1)$

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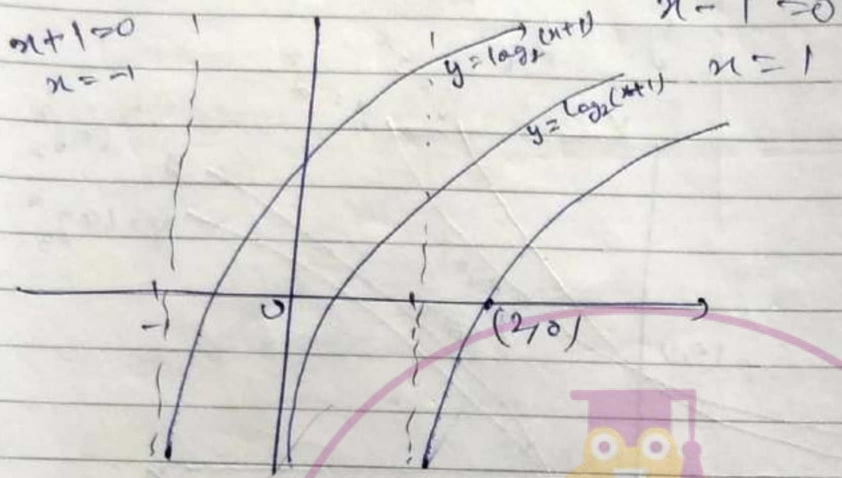
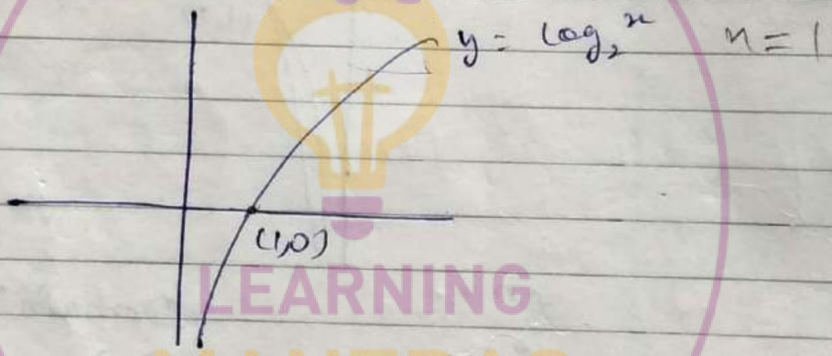
base > 1 = increasing.

* $y = \log_2(x-1)$



$y = \log_2 x$



* $y = \log_2(1+x^2)$

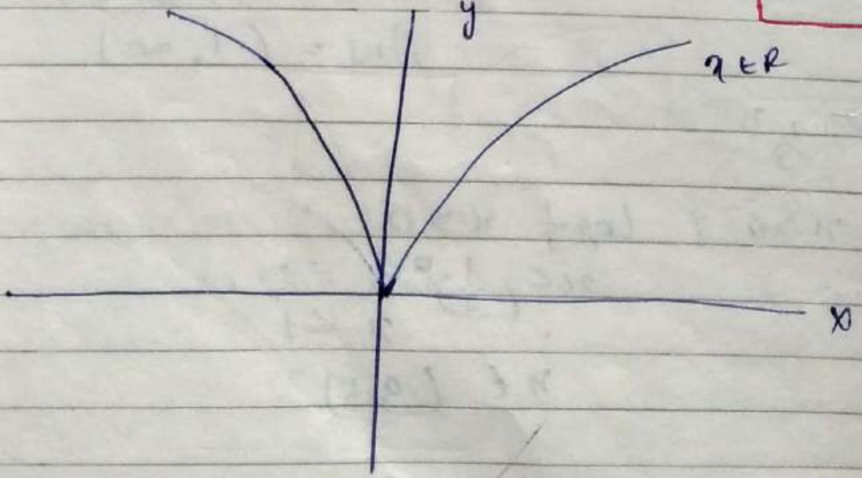
$1+x^2 > 0$ (Input +)

Domain = $x \in R$.

No asimtote.

$\left[\begin{matrix} 1+x^2 \\ x^2 = -1 \\ x \not> -1 \end{matrix}\right]$

$\boxed{x^{(\text{even})} \quad |x|^{\text{even}} \Rightarrow \text{fig symetrical of y axis}}$

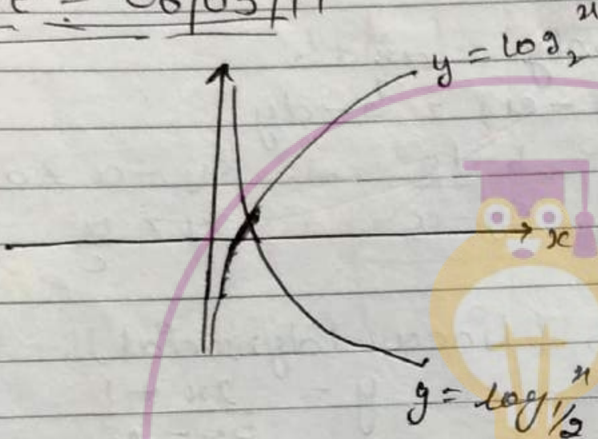


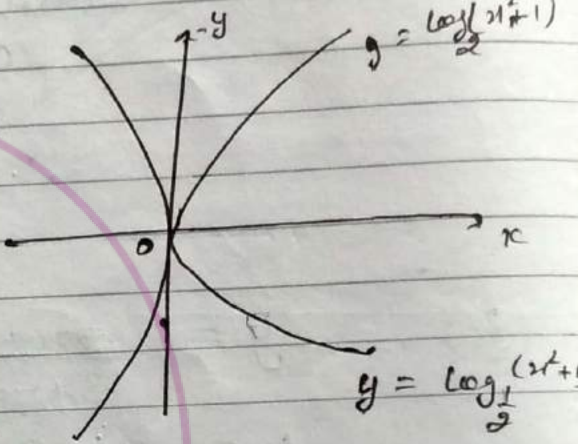
$y = \log_2 (1+x)^2$

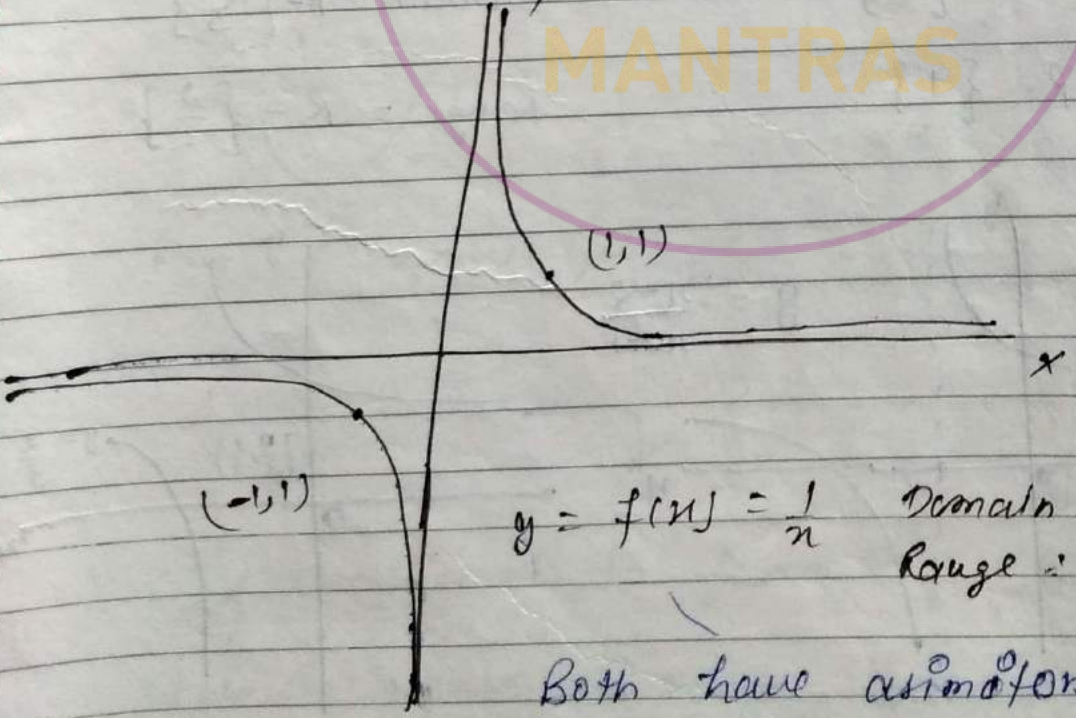
$1+x^2 > 0$ $\qquad\qquad$ $1+x^2 = 0$

$x \in R$

Date :- 06/05/17







$y = f(x) = \frac{1}{x}$ Domain = $R - \{0\}$

Range : R ~~o~~ $R - \{0\}$

Both have asimotone.

{ $a+b \Rightarrow$ Linear Polynomial
$ax+0 \Rightarrow$ odd linear polynomial. }

Que :- $y = \frac{ax+b}{cx+d}$

Domain $= R - \{-\frac{d}{c}\}$
Range $= R - \{\frac{a}{c}\}$

Solve for x

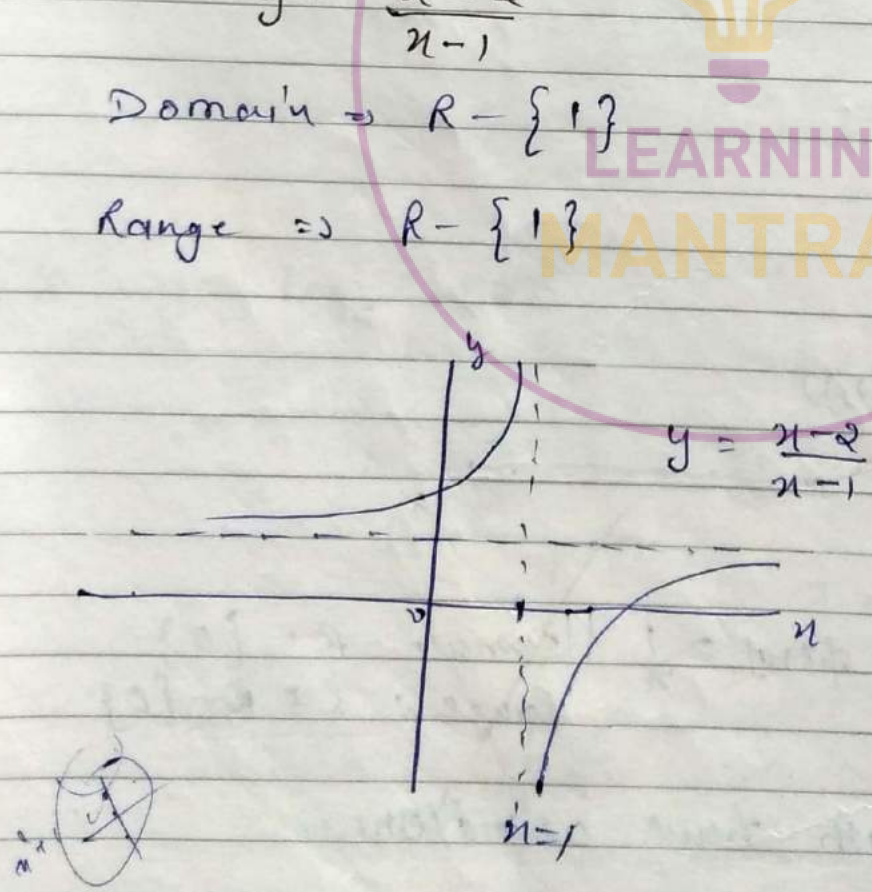
$$cxy + dy = ax + b$$
$$x(cy - a) = b - dy$$
$$\therefore x = \frac{b-dy}{cy-a} \rightarrow cy - a \neq 0$$
$$y \neq \frac{a}{c}$$

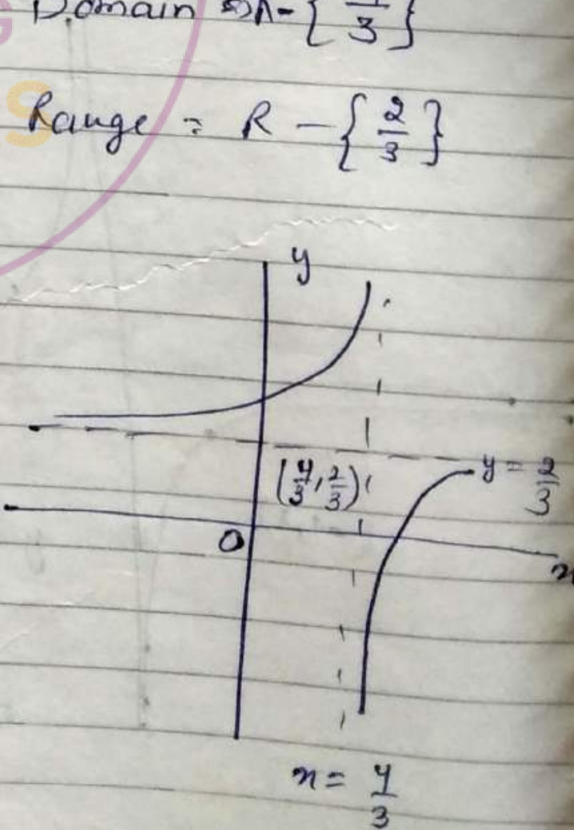
$(\frac{2}{2})$ $y = \frac{2x+1}{3x-2}$, (Linear Polynomial).

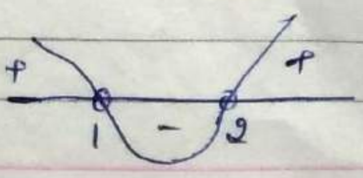
$y = \frac{x^{*}-2}{x-1}$ $\qquad$ $y = \frac{2x+1}{3x-4}$

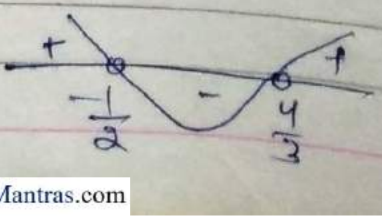
Domain $\Rightarrow R - \{1\}$ $\qquad$ Domain $\Rightarrow R - \{\frac{4}{3}\}$

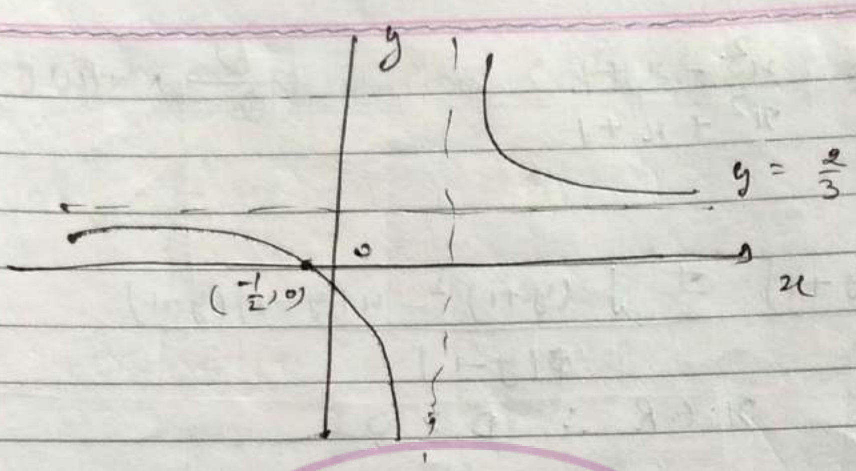
Range $\Rightarrow R - \{1\}$ $\qquad$ Range $= R - \{\frac{2}{3}\}$











→ ②

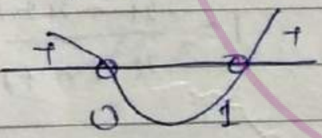
Que: $\frac{1}{x} < 1$ → $1 < x$ X

$x > 1$ Ans

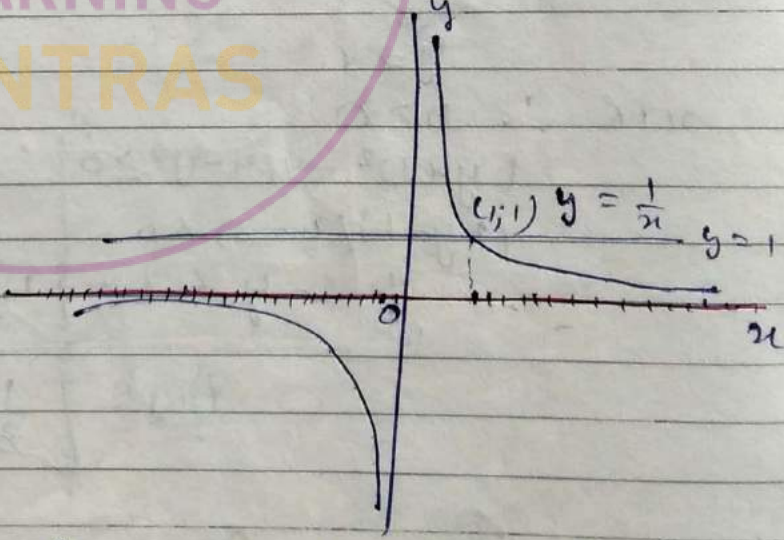
$\frac{1}{x} - 1 < 0$

$\frac{1-x}{x} < 0$

$\frac{x-1}{x-0} > 0$



Ans = $(-\infty,0] \cup (1,\infty)$



* If rational to form $Y = \frac{L}{L}, \frac{L}{Q}, \frac{Q}{L}, \frac{Q}{Q}$ then to obtained Range, solve for x.

*

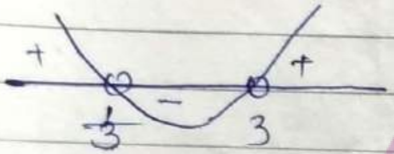
$$y = \frac{x^2 - x + 1}{x^2 + x + 1} \qquad \left(\frac{Q}{Q} \text{ Nature}\right)$$

$$x = \frac{(y+1) \pm \sqrt{(y+1)^2 - 4(y-1)(y-1)}}{2(y-1)}$$

$x \in R \therefore D \geq 0$

$(x+1)^2 - 4(y-1)^2 \geq 0$

$(3y-1)(y-3) \leq 0$



$\frac{1}{3} \leq y \leq 3$

$yx^2 + yx + y = x^2 - x + 1$

$x^2(y-1) + x(y+1) + (y-1) = 0$

$y \neq 1$

$x \in R \therefore D \geq 0$

$(y+1)^2 - 4(y-1)^2 \geq 0$

$(3y-1)(y-3) \leq 0$

$\frac{1}{3} \leq y \leq 3 - \{1\}$

$y = 1$

$1 = \frac{x^2 - x + 1}{x^2 + x + 1}$

$x = 0$

$\therefore x \in R \therefore y = 1$ accepted

Ans: $\left[\frac{1}{3}, 3\right]$

Que: $\frac{x^2 - 3x + 2}{x^2 - x}$ find D/R

(Hence a facter is common in neumator or dinometer)

$\frac{(x-1)(x-2)}{x(x-1)} = \frac{x-2}{x-0}$

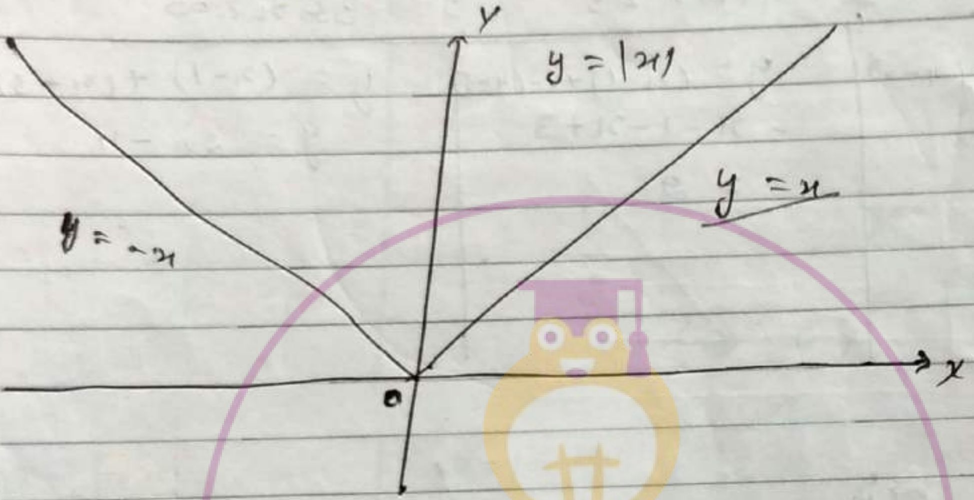
$y = \frac{x-2}{x-0}$

at $x = 1$, $y = \frac{1-2}{1-0} = -1$

Range $= R - \{1, -1\}$

* Modulus function (absolute value function):

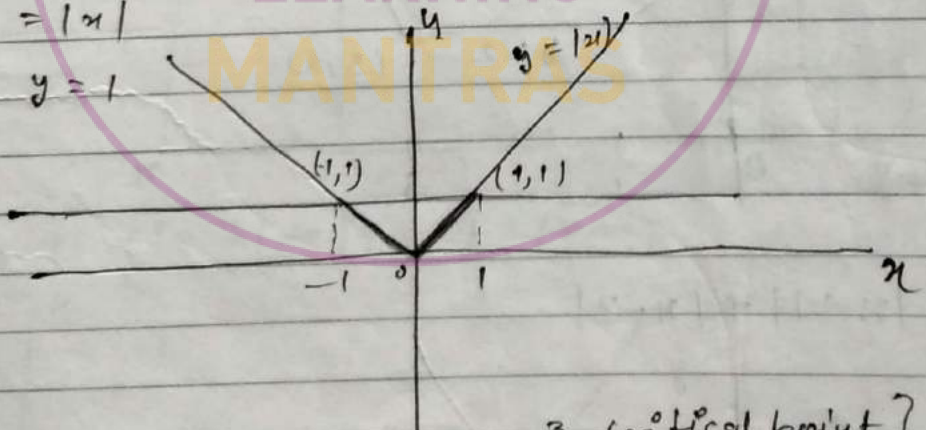
$$\sqrt{x^2} = |x| = \begin{cases} x , & x \geq 0 \\ -x & x < 0 \end{cases}$$



If $|x| \leq 1 \Rightarrow -1 \leq x \leq 1$

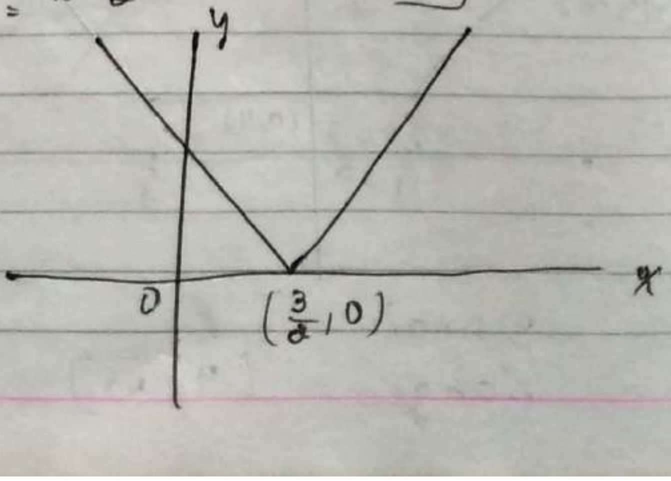
$\downarrow$

$y = |x|$



* $y = |2x-3|$ $\rightarrow \left[2x-3 = x = \frac{3}{2}\right.$ (critical point$\left.\right]$

$$\begin{cases} 2x-3 & x \geq \frac{3}{2} \\ -(2x-3) & x < \frac{3}{2} \end{cases}$$

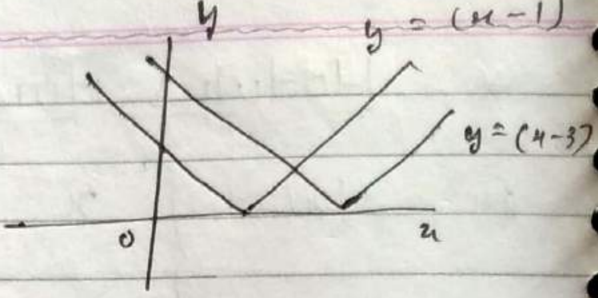


4°

$y = mx + c$

$|x-1| + |x-3| = 4$

$y = |x-1| + |x-3|$

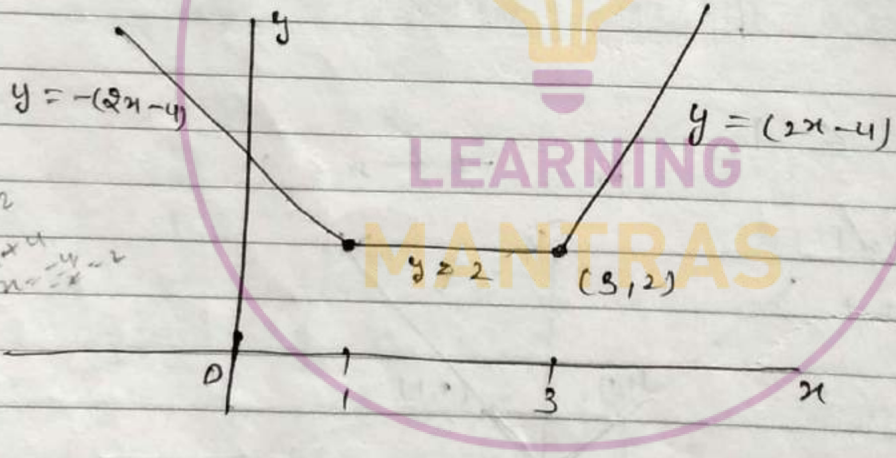


$-\infty < x < 1$ | 1 | $1 \le x < 3$ | 3 | $3 \le x < \infty$

$y = -(x-1) + -(x-3)$
$y = -(2x-4)$

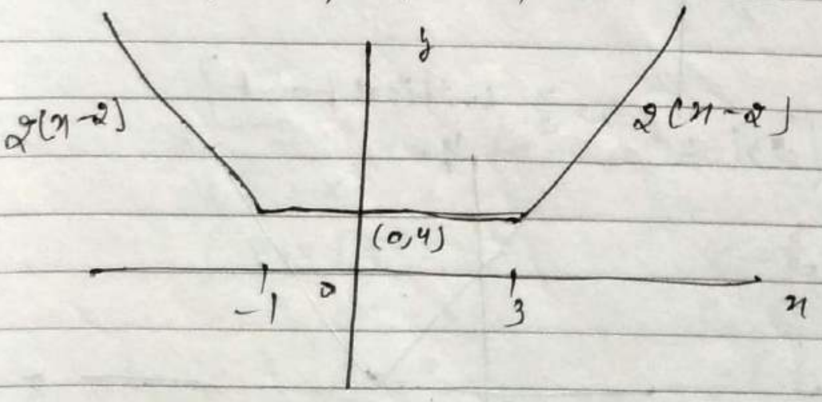
$y = (x-1) + -(x-3)$
$= x - 1 - x + 3$
$= 2$

$y = (x-1) + (x-3)$
$y = 2x - 4$



$2x-4$
$x = \frac{4}{2} = 2$
$-2x+4$
$x = \frac{-4}{-2} = 2$

Note: $|x+1| + |x-3|$

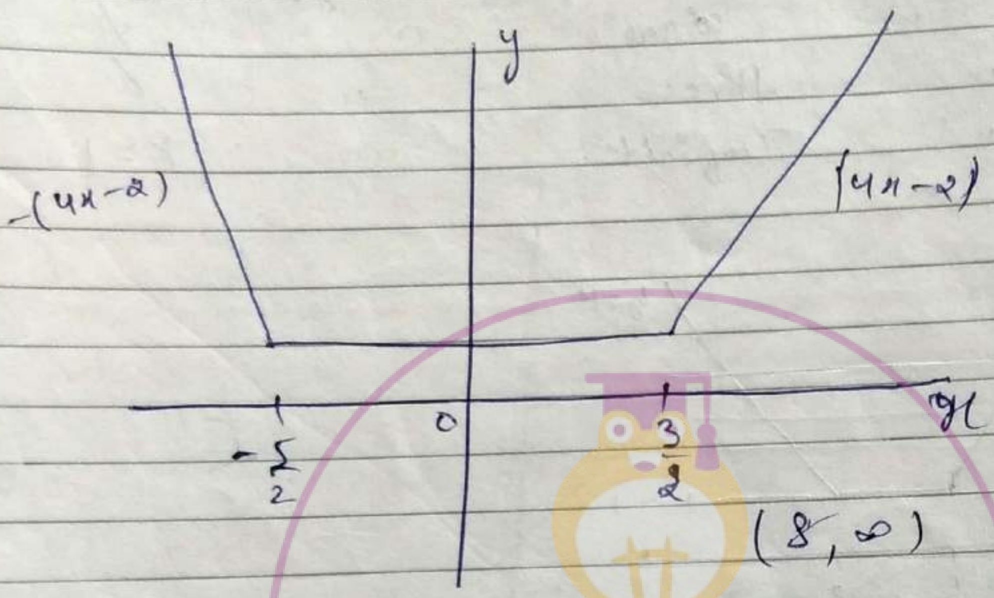


Domain = R
Range = [4, ∞]

no. |x| odd = mid ✓ sharp corne

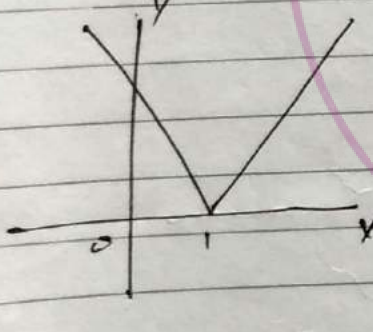
no |x| even = ⌴ hole shape

✗ $y = |2x-3| + |2x+5|$

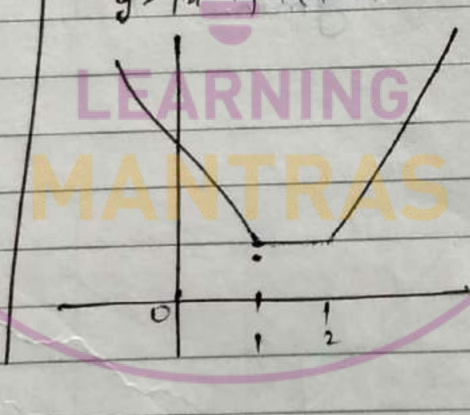


$(8, \infty)$

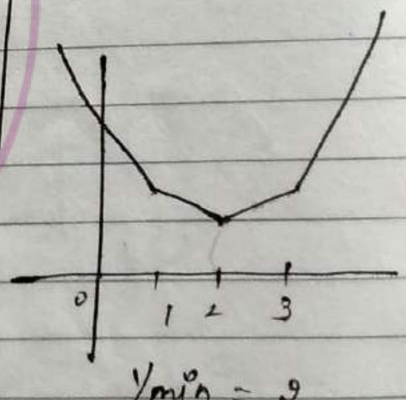
$y = |x-1|$



$y = |x-1| + |x-2|$



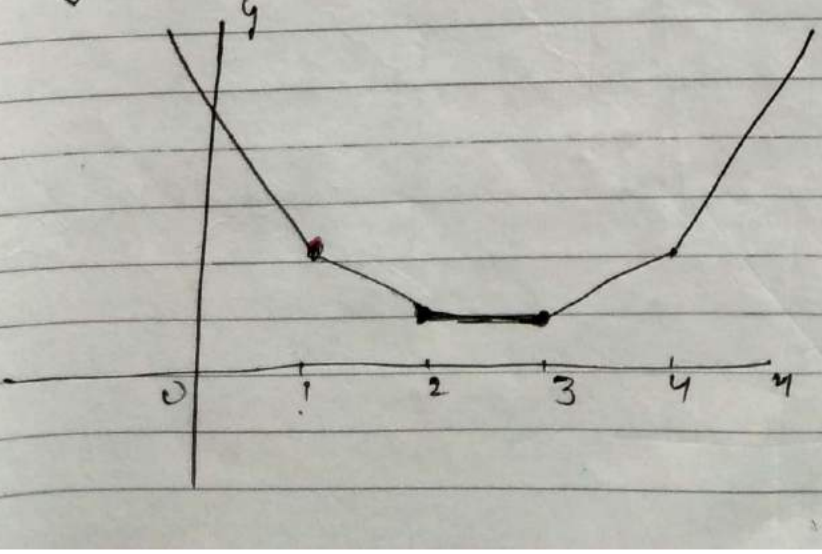
$y = |x-1| + |x-2| + |x-3|$



$y_{min} = 2$

range $= [2, \infty)$

$y = |x-1| + |x-2| + |x-3| + |x-4|$



$y_{min} = 4$

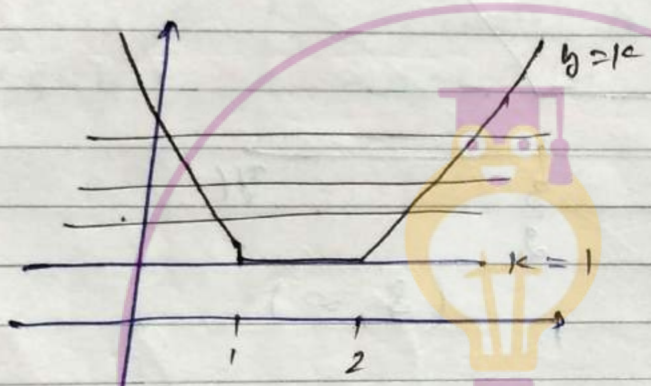
range $= [4, \infty]$

H.w O-1 ⇒ Q, 2, 13, 14,
S-1 ⇒ (1) (i) (ii) (iii) (iv) (vi)
(vii) (viii), (ix)

Que:

* $|x-1| + |x-2| = K$

(i) If no. of Solution is two then find $k = k > 1$
(ii) " " " " one " $= \phi$
(iii) " " three " $= \phi$
(iv) " " Infinite. " $k = 1$



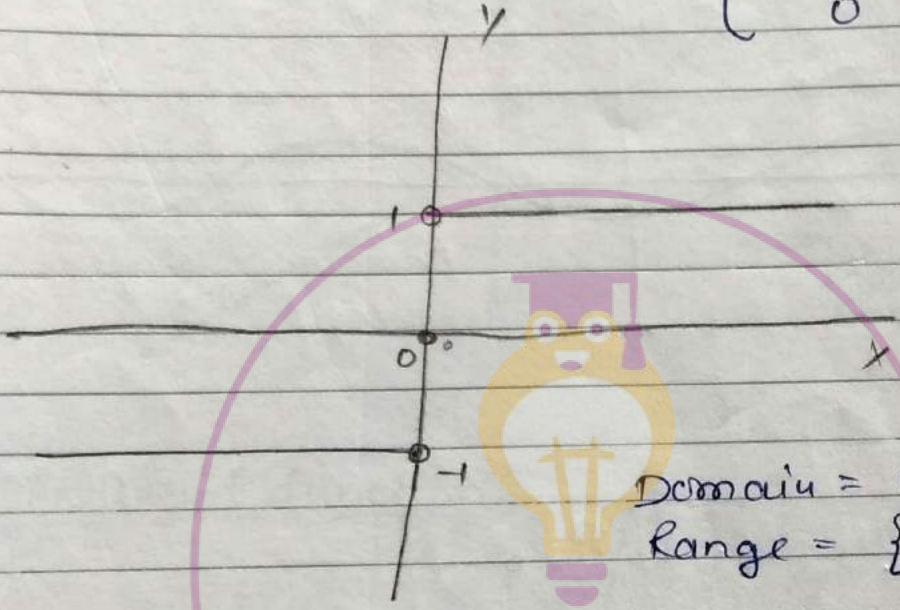
$y_{min.} = 1$

Date: 08/05/17

[Signum of x]

* Signum function:

$$\operatorname{Sgn} x = \begin{cases} \frac{|x|}{x} \text{ or } \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$$



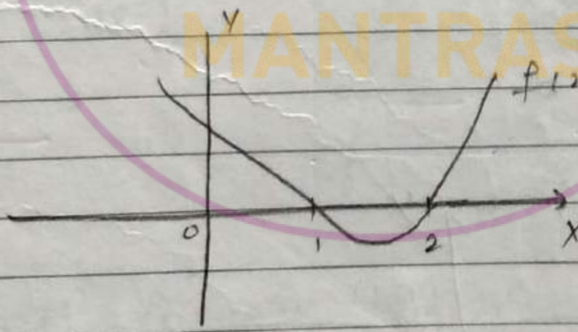
$\frac{|x|}{x}$

$x > 0$ $\quad$ $x < 0$

$\frac{x}{x} = 1$, $\frac{-x}{x} = -1$

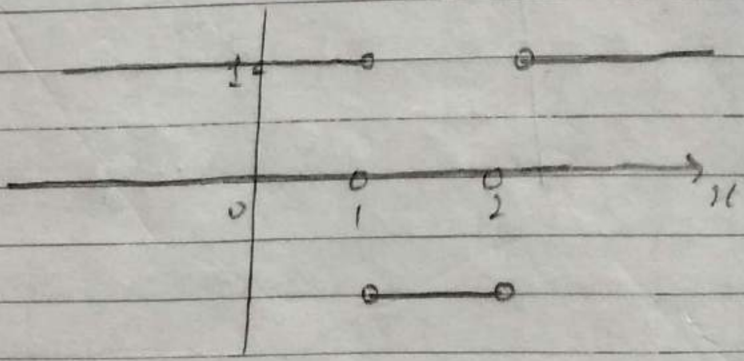
Domain = R
Range = {-1, 0, 1}

$\begin{Bmatrix} + = 1 \\ - = -1 \\ 0 = 0 \end{Bmatrix}$



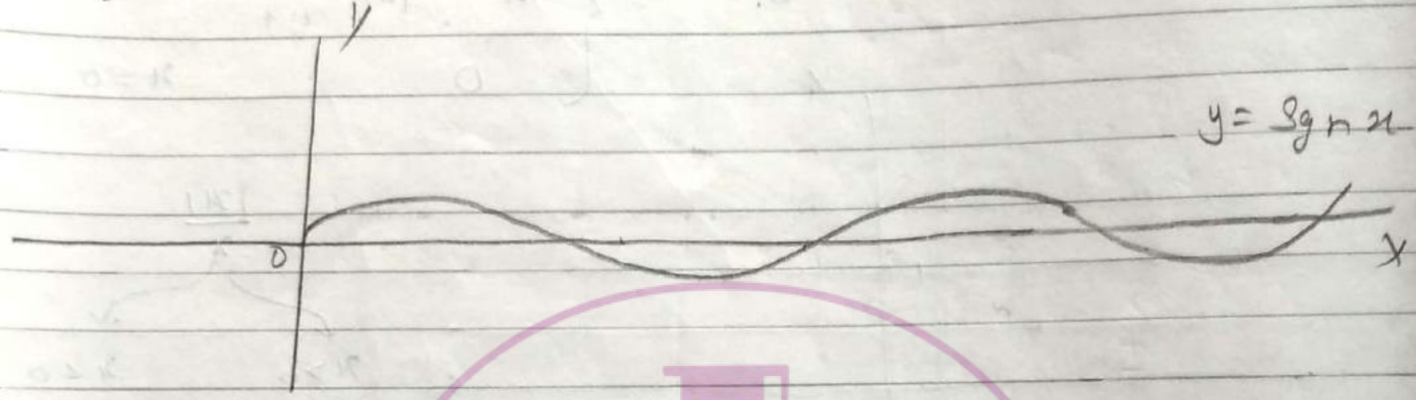
$f(x) = (x-1)(x-2)$

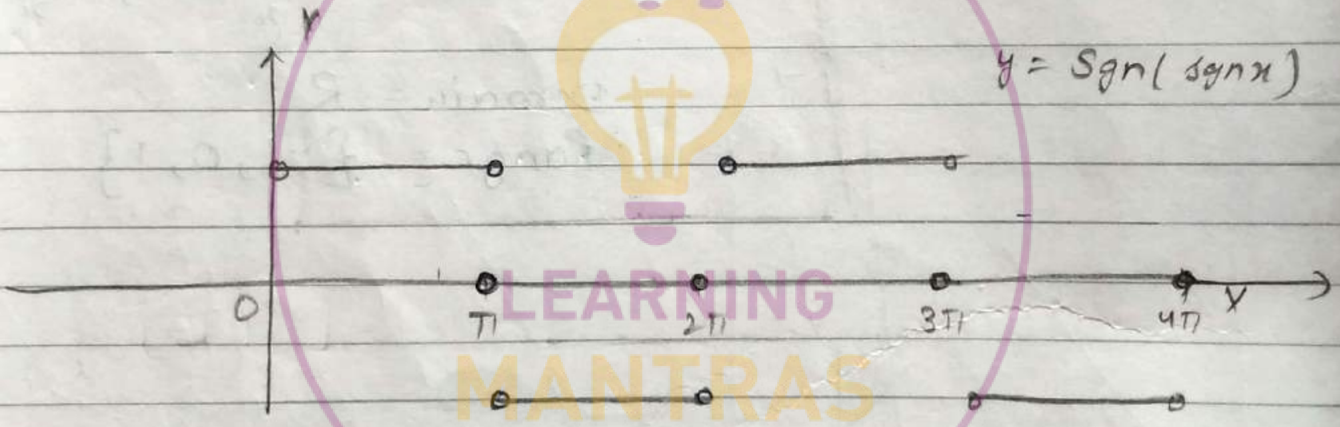
$y = \operatorname{Sgn}(x-1)(x-2)$



Note : $sgn(sgn(sgn)(sgn)) = sgn\,x$

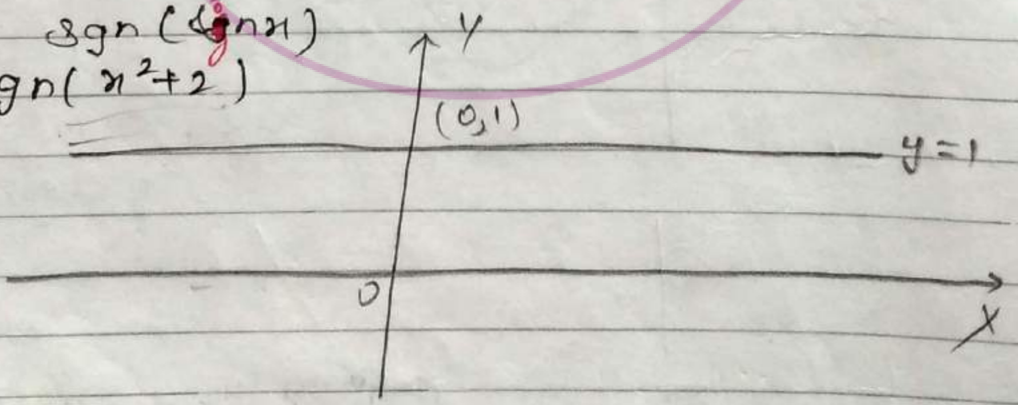
Que :





$y = sgn(sgn x)$

$sgn(x^2+2)$



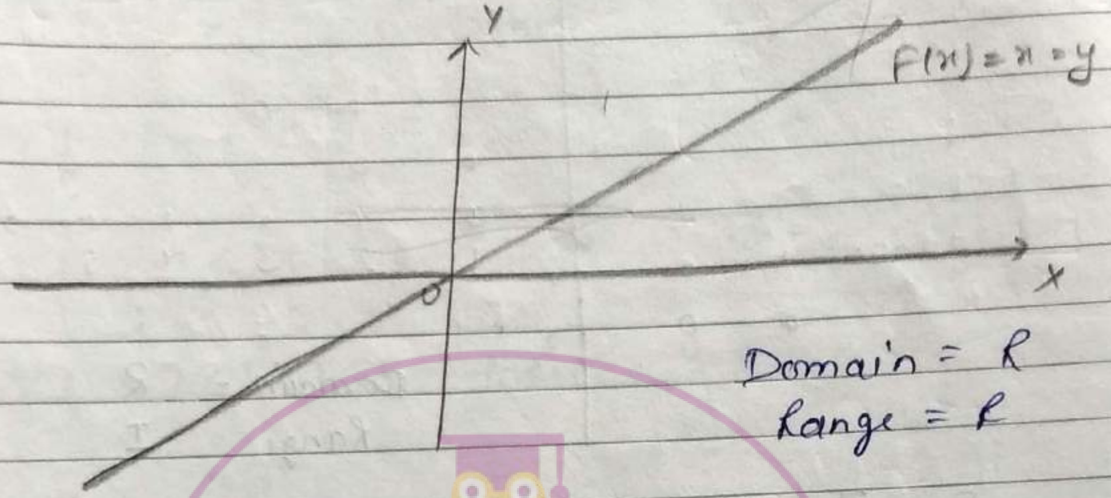
$[1.0007] = [1]$

$[-1.0001]$

~~move left~~

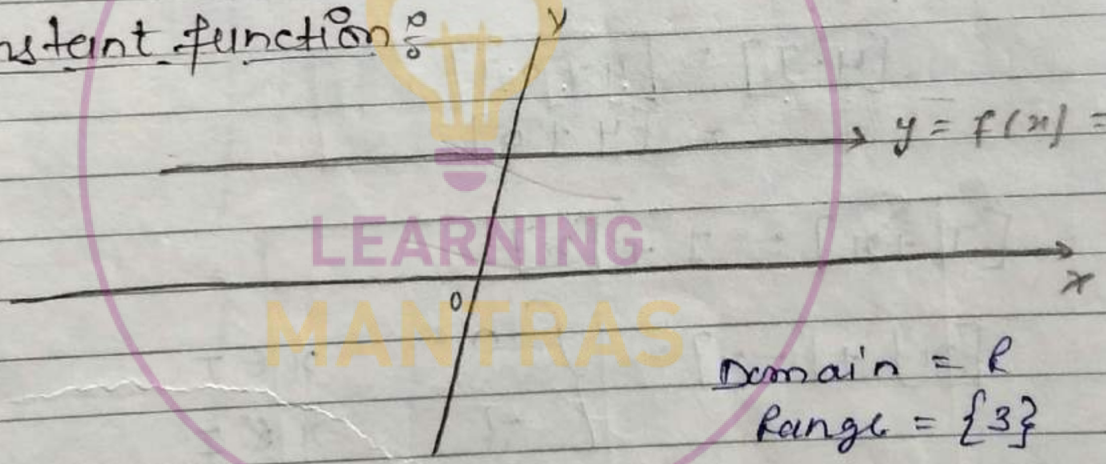
* Identity function :-

$f(x) = x$



Domain = R
Range = R

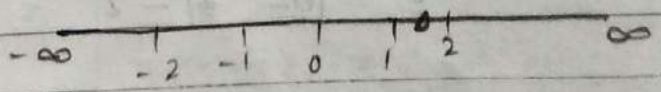
* Constant function :-



Domain = R
Range = {3}

* Greatest integer function :- (G.I.F) :-

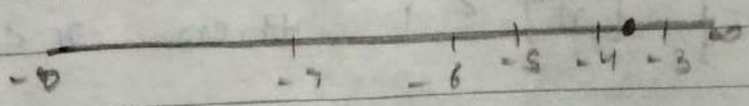
$[Sgn] =$



$[1.6] = 1$ (1, 0, -1, -2)

Greatest integer.

$[-3.2] = -4$

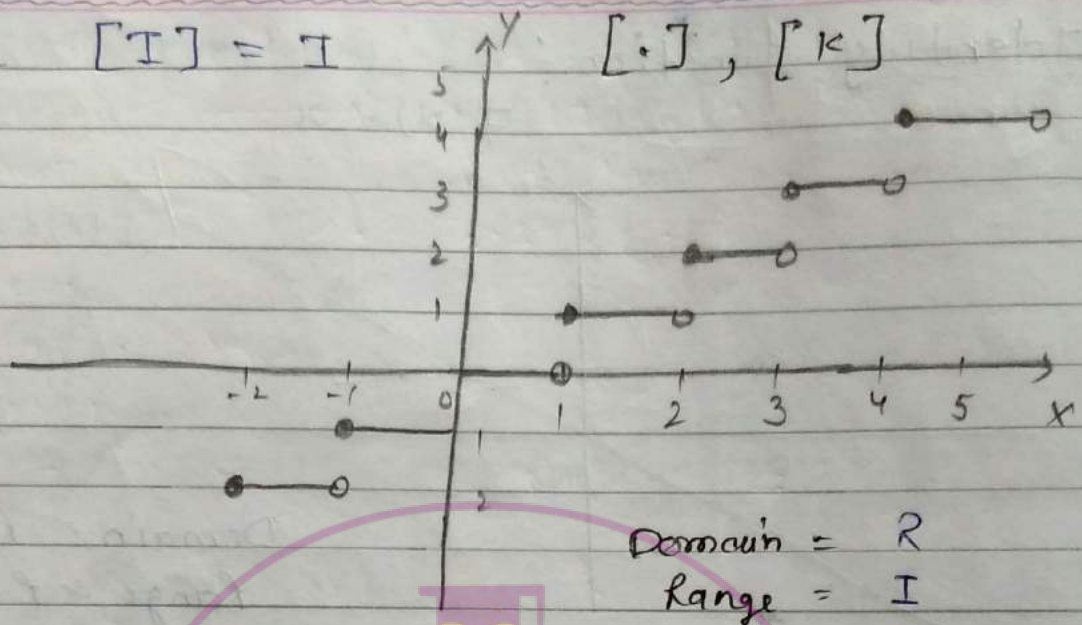


$[3] = I = 3$ Ans

W → Fout
f → inbuil

Fract
Integer ✓

Note: $[I] = I$ $[\cdot]$, $[K]$



Domain = R
Range = I

Note: $[[x]] = [x]$

$$[4.7] = [4 + 0.7]$$
$$= [4 + 0]$$

* $[I + x] = I + [x]$

$$[x] + [-x] = \begin{cases} 0 & x \in I \\ -1 & x \notin I \end{cases}$$

L.H.S

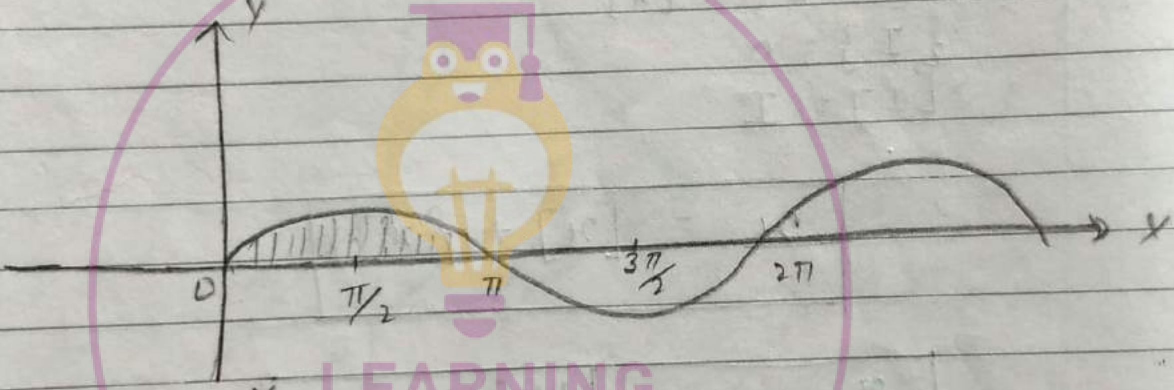
$$[x] + [-x]$$
$$= [1.7] + [-1.7]$$
$$= \cancel{[0-}\ \cancel{0}1 - 2 = -1$$

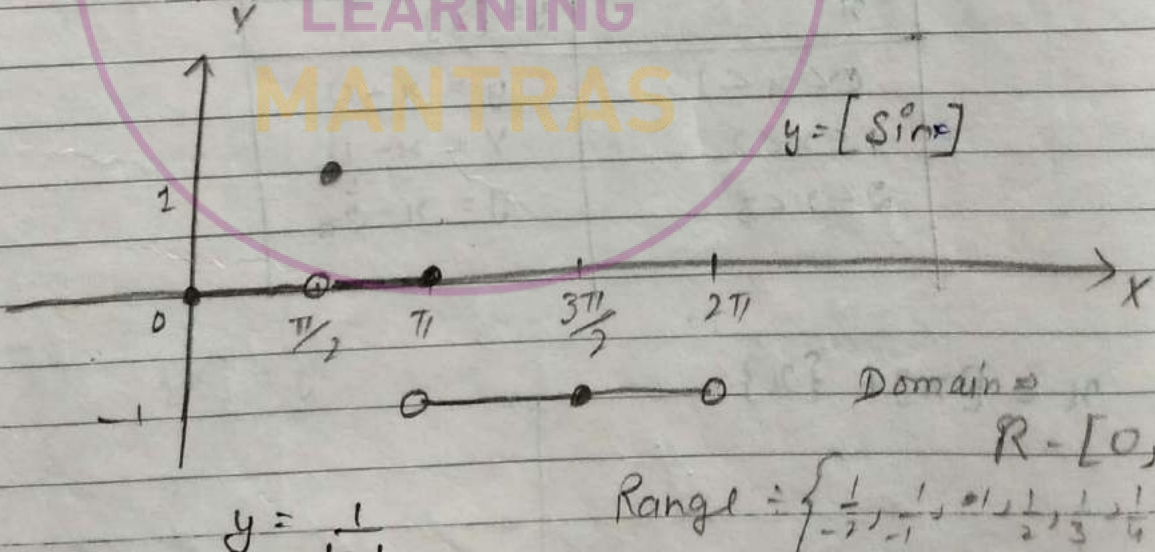
* If $[x] = 1$, then $x \in [1, 2)$ Ans.

* If $[x] \leq 1$ then $x < 2$ Ans

Que: $[x] = \frac{1}{2}$
Ans: ϕ

- * If $[x] > 1$ thom $x \geq 2$
- * If $[x] \geq 0$ then $x \geq 0$
- * If $[x] < 2$ than $x < 2$
- * If $1 < [x] < 2$ then ϕ
- * If $2 < [x] < 5$ then $3 \leq x < 5$





$y = \frac{1}{|x|}$

Domain ⇒ $R - [0, 1)$

Range ÷ $\left\{\frac{1}{-2}, \frac{1}{-1}, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

Que: $Y = \frac{\tan[\pi(x+1)]}{x^2+x+1}$ =

Domain = R

Range = $\{0\}$

(fractional Part always +ve)

* fractional function : $y = \{x\}$

$x = [x] + \{x\} \longrightarrow 1.7 = [1.7] + \{1.7\}$

$= 1 + 0.7$

$1.7 = 1 + 0.7 \longrightarrow$

$3 = 3 + 0$

$S.2 = [5.2] + \{5.2\}$

$5.2 = 5 + 0.2$

$= 5 + 0.2$

$-1.2 = -1 + (-0.2)$ ✗

$= -1 - 1 + 1 - 0.2$ ✓

$= -2 + 0.8$ ✓

Range = $0 \le \{x\} < 1$

Note : $\{x + I\} = \{x\}$

$\{I\} = 0$

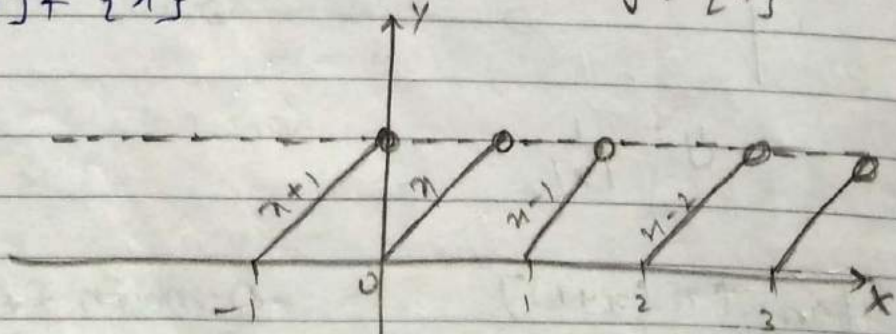
$[I] = I$

$$\boxed{x = [x] + \{x\}}$$

x	$y = \{x\}$
$0 \le x < 1$	$y = x - 0$
$-1 \le x < 2$	$y = x - 1$
$2 \le x < 3$	$y = x - 2$

$x = [x] + \{x\}$

$y = [x]$



$\{2.7\} = 2.7 - 2$

$= 0.7$

$\{2.7\} = \{2 + 0.7\}$

$= \{0.7\}$

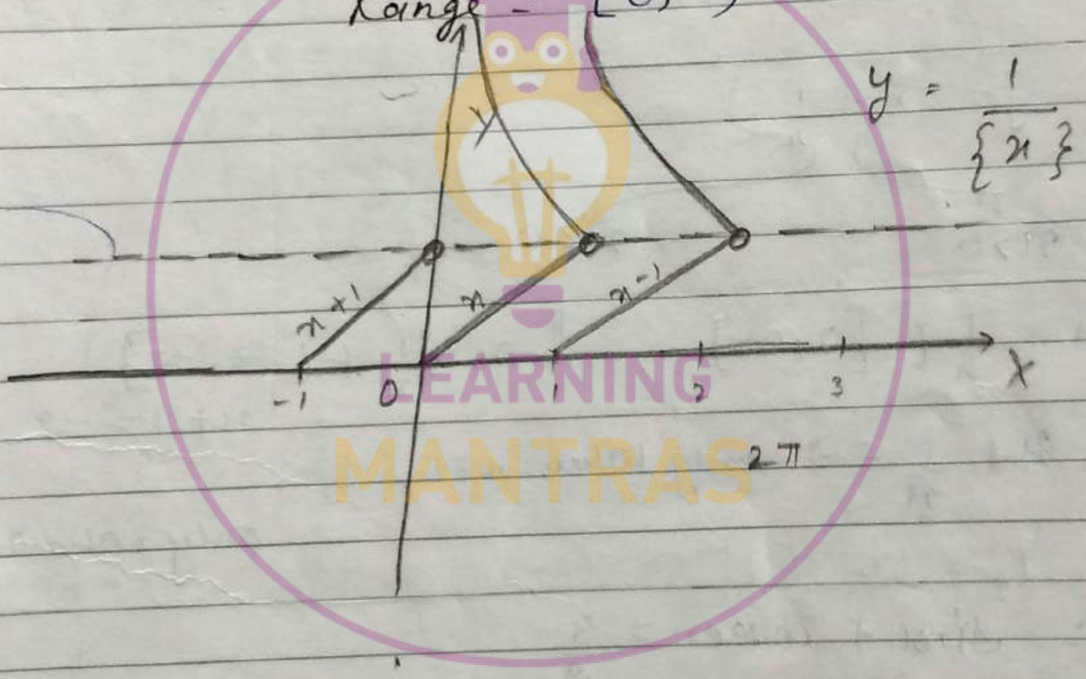
$$[x]+[-x] = \begin{cases} 0 & x \in I \\ -1 & x \notin I \end{cases}$$

* $$\{x\}+\{-x\} = \begin{cases} 0 & x \in I \\ 1 & x \notin I \end{cases}$$

* $[\{x\}] = 0$

Domain = R

Range = [0, 1)



5^0 $\boxed{6}$

Date : 09/05/17 Exercise - O-1

Que : 2 (viii) $\sin^2 x - 2\sin x + 1 + 3$

$= 3 + (\sin x - 1)^2 \quad (-1 \le \sin x \le 1)$

y min $= 3$

y maximum $= 3 + (-1-1)^2$

$= 3 + 4 = 7 \quad \sin x = 1$

$[3, 7]$

$y = x + \frac{1}{x}$

$x > 0$ $\qquad$ $x < 0$

$x + \frac{1}{x} \in [2, \infty]$ $\qquad$ $x + \frac{1}{x} \in (-\infty, -2]$

$x + \frac{1}{x} = 2$ only when $x = 1$ $\qquad$ $x + \frac{1}{x} = -2$ only when $x = -1$

Que : $\sin x + \operatorname{cosec} x = \frac{3}{2}$

Ans : $\sin x + \frac{1}{\sin x} = \frac{3}{2}$

$= \phi$

(xi) $y = \frac{(e^{2x} + e^x + 1) - 2e^x}{e^{2x} + e^x + 1} = 1 - \frac{2e^x}{e^{2x} + e^x + 1}$

$= 1 - \frac{2}{\left(e^x + \frac{1}{e^x}\right) + 1}$

$y = 1 - \frac{2}{3}$

$= \frac{1}{3}$

$y = 1 - \frac{2}{\infty}$

$= 1 - 0$

1

$\left[\frac{1}{3}, 1\right]$

(iii) $y = \frac{1}{x^2 - x + 1}$

$y = \frac{1}{(x - \frac{1}{2})^2 + \frac{3}{4}} = \left(0, \frac{4}{3}\right]$ Ans

(13) $f : N \rightarrow I$

$f(x) = (-1)^{x-1}$

$x = 1$; $f(1) = (-1)^{1-1} = (-1)^0 = 1$

$x = 2$; $f(2) = (-1) = -1$

$x = 3$; $f(3) = (-1)^{3-1} = (-1)^2 = 1$

$\{1, -1\}$

Ex : 2-1.

Que 1

(iii) $f(x) = \ln\left(\sqrt{x^2 - 5x - 24} - (x+2)\right)$

$\sqrt{x^2 - 5x - 24} - (x+2) > 0$

$\sqrt{x^2 - 5x - 24} > (x+2)$

$x^2 - 5x - 24 > x^2 + 4 + 4x$

$-28 > 4x$

$9x < -28$

$x < -\frac{28}{9} = -3.11.$

$x^2 - 5x - 24 \geq 0$

$x^2 - 8x + 3x - 24 \geq 0$

$(x-8)(x+3) \geq 0$

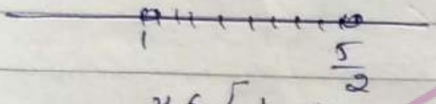
* $\sqrt{(5x-6-x^2)\cdot[\{3\}]} + \sqrt{7x-5-2x^2} + \frac{1}{\log(\frac{7}{2}-x)}$

$7x-5-2x^2 \geq 0$

$2x^2-7x+5 \leq 0$

$2x^2-5x-2x+5 \leq 0$

$(2x-5)(x-1) \leq 0$



$x \in \left[1, \frac{5}{2}\right]$

$\frac{7}{2} - x > 0$

$x - \frac{7}{2} < 0$

$x < \frac{7}{2}$

&

$\log\left(\frac{7}{2}-x\right) \neq 0$

$\frac{7}{2} - x \neq e^0 = 1$

$\frac{7}{2} - x \neq 1$

$x = \frac{5}{2}$

* $f(x)$ $l_7 l_5 l_3 l_2(\lambda)$

Ans: $l_5(l_3 l_2 \lambda) > 0$

$l_3 l_2 \lambda > 5^0 = 1$

$l_3(l_2 \lambda) > 1$

$(l_2 \lambda) > 3$

$\lambda > 2^3 > 8$

$2x^3+5x^2-14x > 27$

$2x^3+5x^2-14x-8 > 0$

* $f(x) = {}^{16-x}C_{2x} \Rightarrow$

$x, r \in I^+$
$x \geq r \geq 0$

$^{n}C_{r} \Rightarrow$

~~$16-x \geq 2x$~~

$16 - x > 0, \quad 2x \geq 0$

$x < 16 \quad x \geq 0$

&

$16 - x \geq 2x$

$16 \geq 3x$

$3x \leq 16$

$x \leq \frac{16}{3} = 5 \cdots$

$x = \{0, 1, 2, 3, 4, 5\}$

$x + \frac{1}{x} \Rightarrow [2,\infty)$ always.

* $f(x) = \log^{|x^2-x-6|} + {}^{16-x}C_{2x} +$

$\boxed{x>0} \longrightarrow [x+\frac{1}{x}]$

$\downarrow$

$x+\frac{1}{x} \in (-\infty,-2] \cup (2,\infty)$

$x^2-x-6 \neq 0$

$x^2-3x+2x-6 \neq 0$

$(x-3)(x+2) \neq 0$

$x \neq 3, x \neq -2$

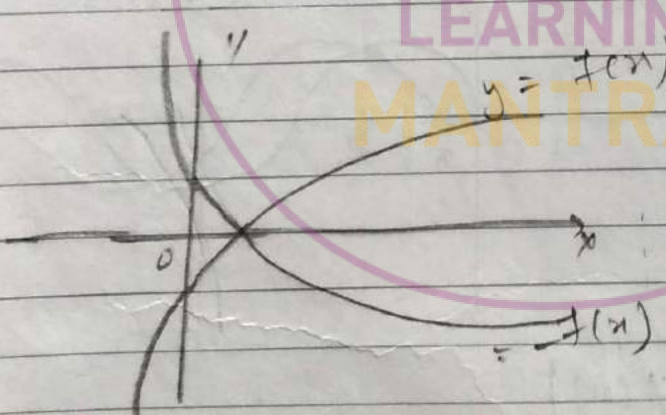
$(0,\infty) - \{3\}$.

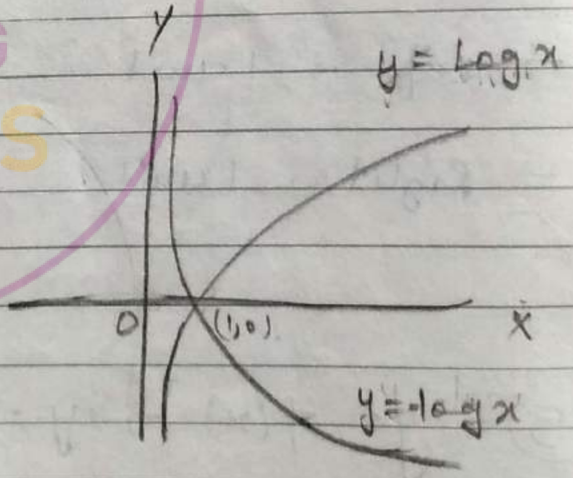
⊛

---

* <u>Transformation of function</u> : Take M.I of whole curve about x-axis

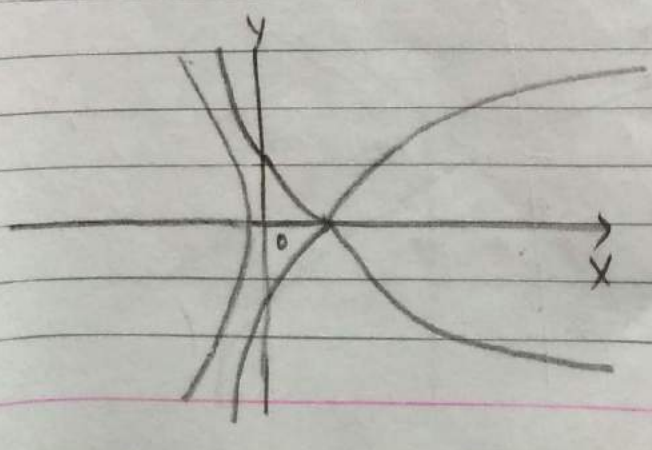
$y = f(x) \longrightarrow y = -f(x)$

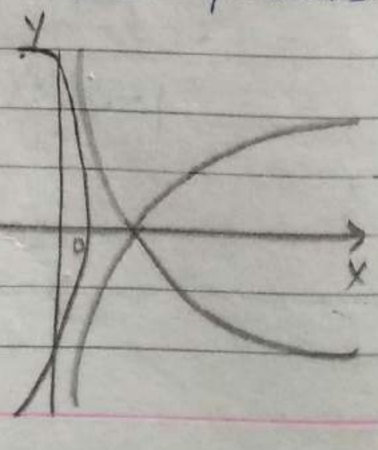




1) ~~y~~ $y = f(x)$

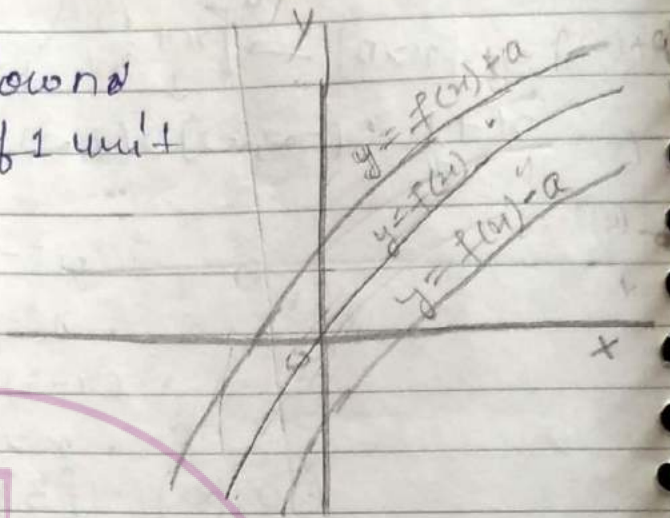
Take M.I of curve about y axis





3) $y = f(x) \rightarrow y = f(x) \pm a$

~~action~~ Shift upward / downward whole curve of 1 unit

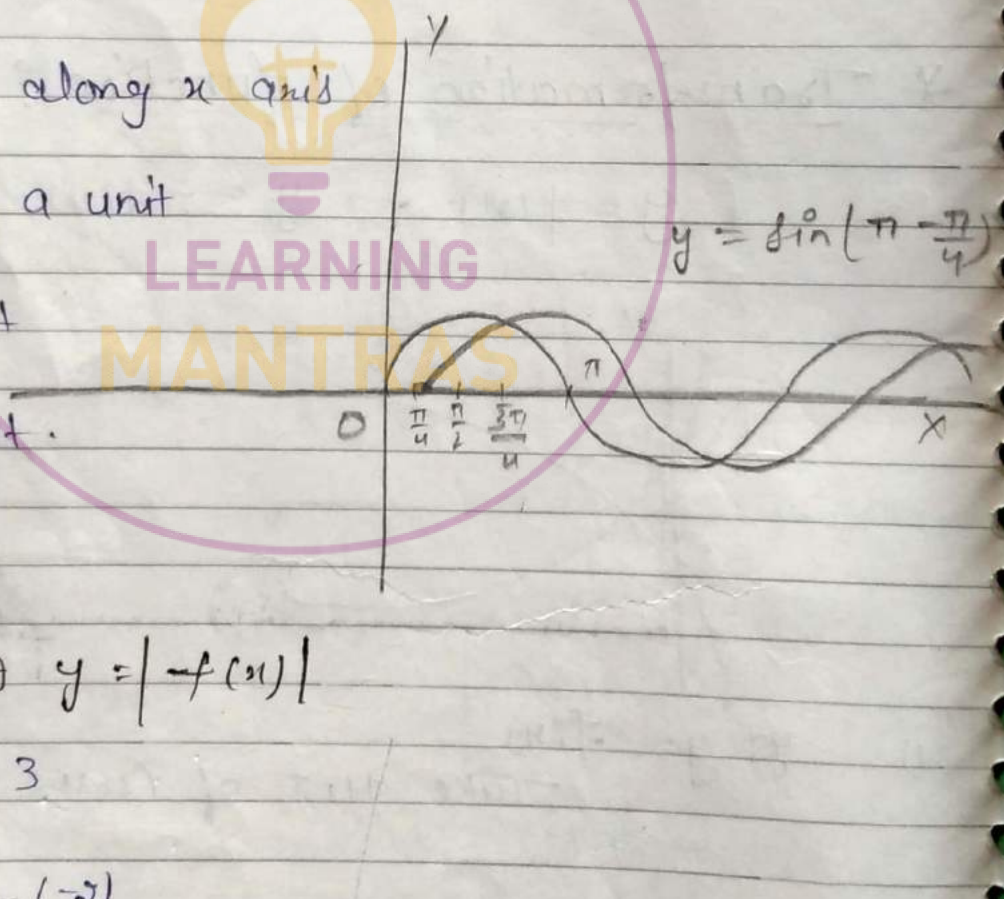


(4) $y = f(x) \rightarrow y = f(x \pm a)$

- + Left / Right along x axis
- Shift curve by a unit

+ Left → 1 unit

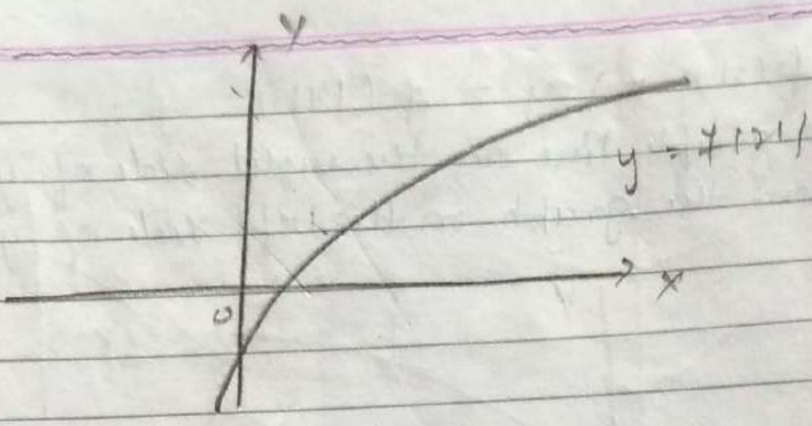
− Right → 1 unit.

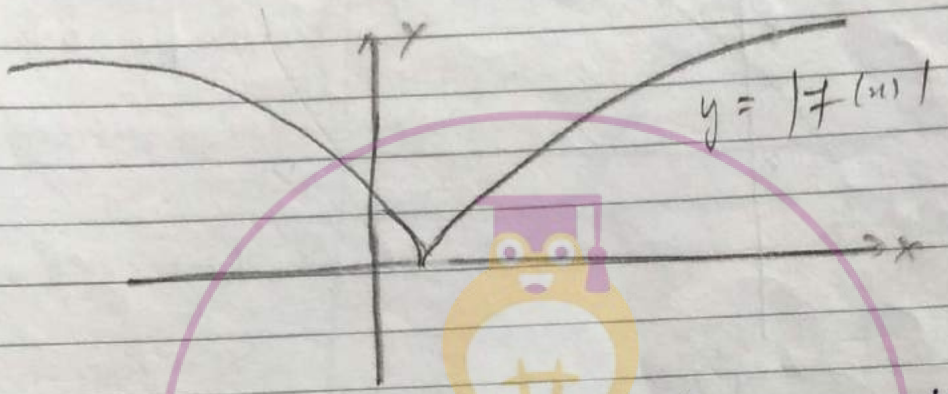


(5) $y = f(x) \rightarrow y = |f(x)|$

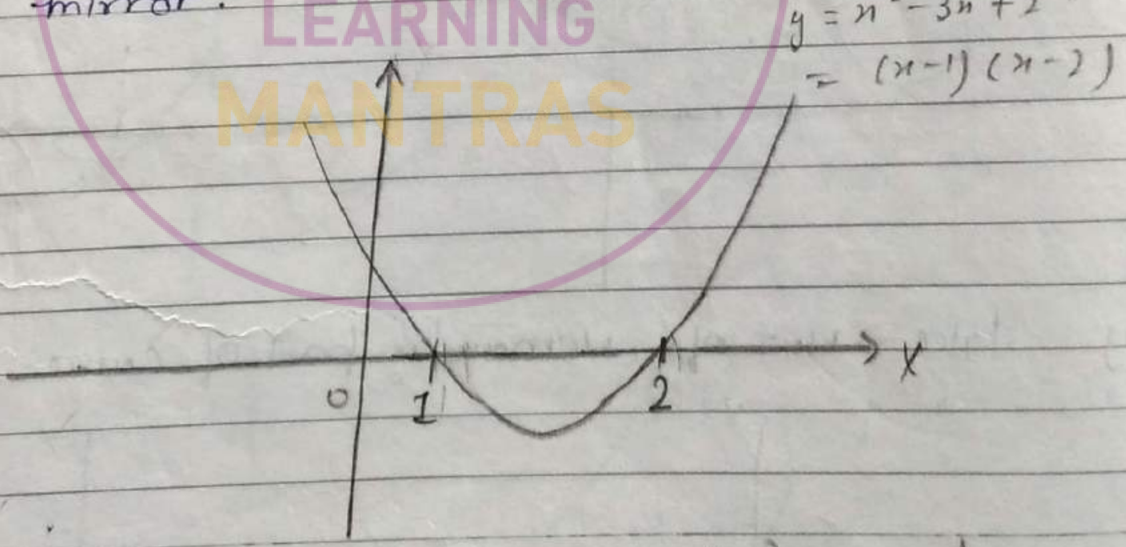
$|3| = 3$

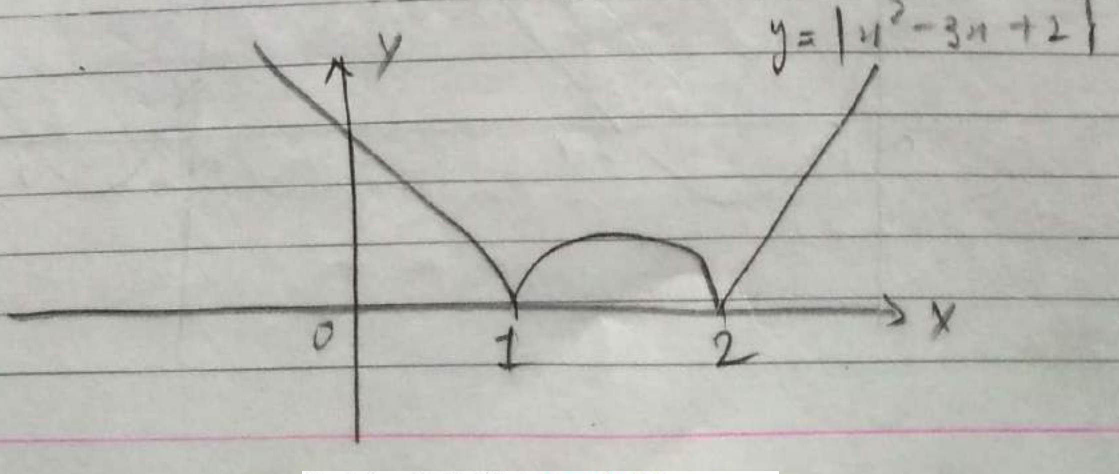
$|-2| = -(-2)$





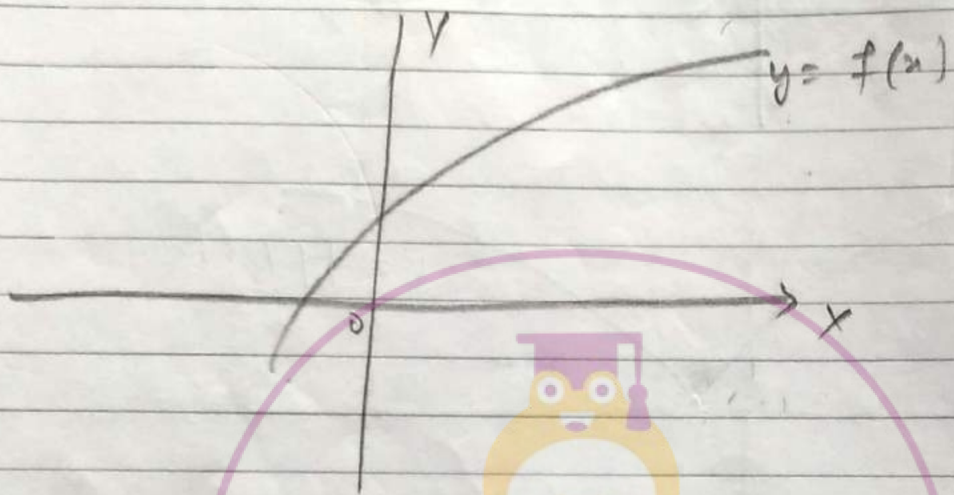
→ leave the +ve part of f(x) exactly.
take M.I of -ve part of f(x) about x-axis as a mirror.

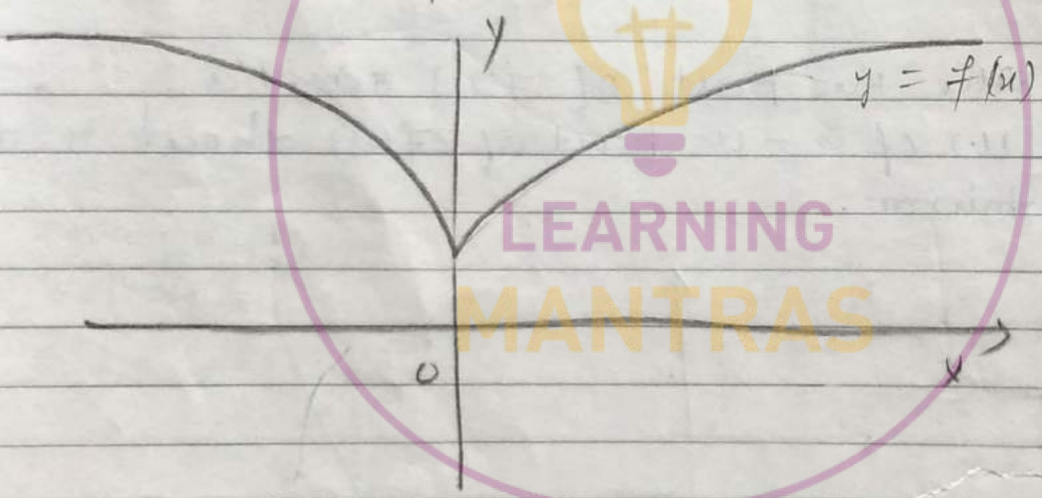




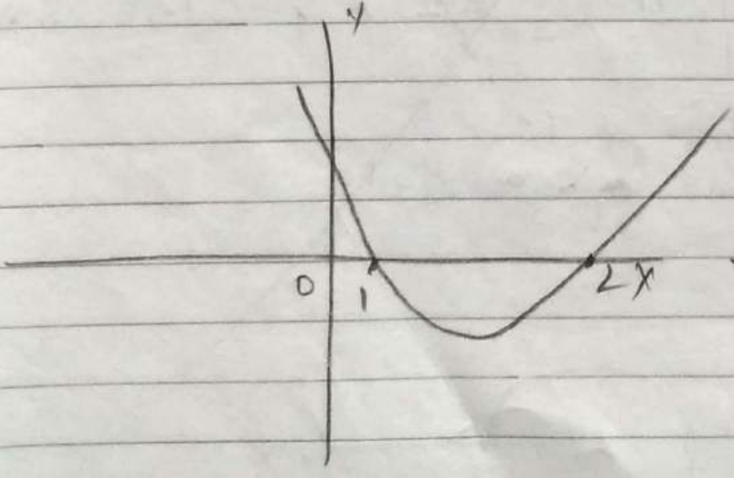
⑥ $y = f(x) \rightarrow y = f(|x|)$.

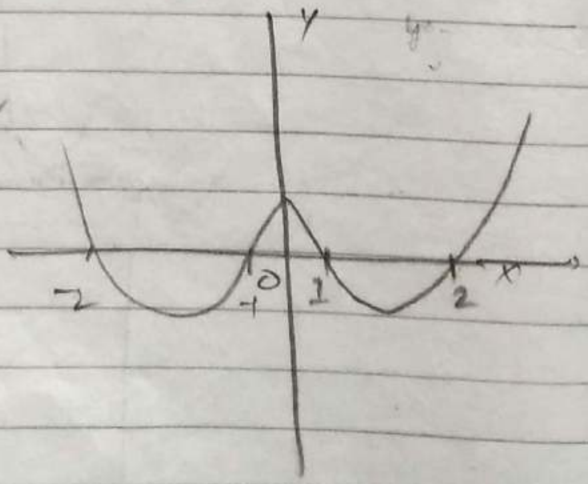
1) Leave the graph line on the right side of y axis existices and remove the graph on the left side of y axis.





2) take M.I of remaining part of curve about y axis





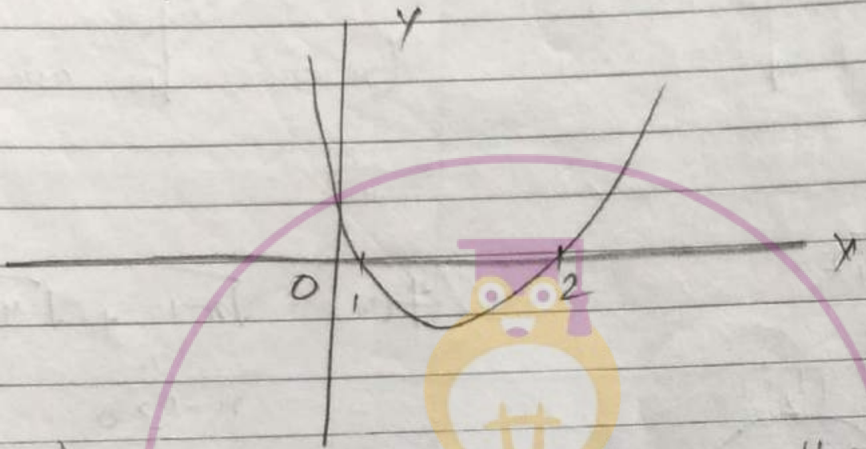
$|x|^2 = |x^2| = x^2 > 0$

*(7) $y = f(x) \longrightarrow y = |f(|x|)|$

$\boxed{|x|^2 = |x|^2 = x^2 > 0}$

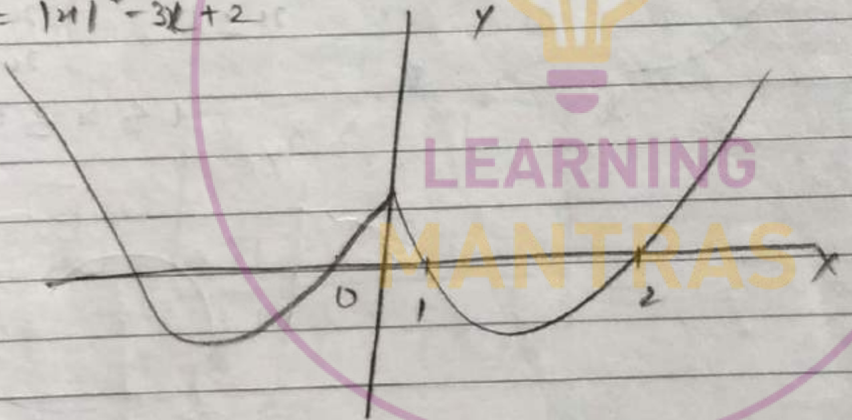
take 5 and 6.

$y = \left| |x|^2 - 3|x| + 2 \right|$

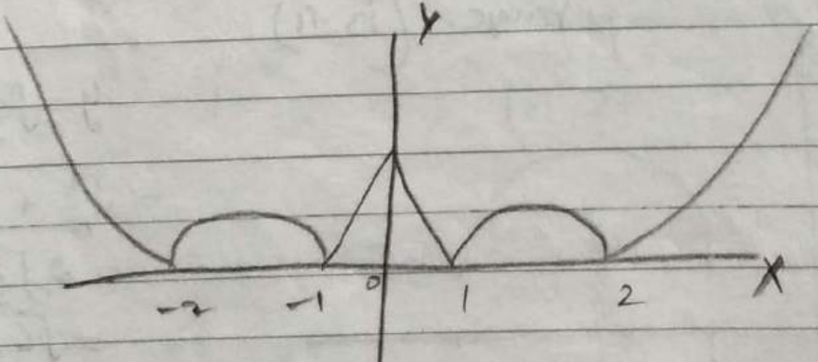


$y = |x|^2 - 3x + 2$

$y = |x|^2 - 3|x| + 2$

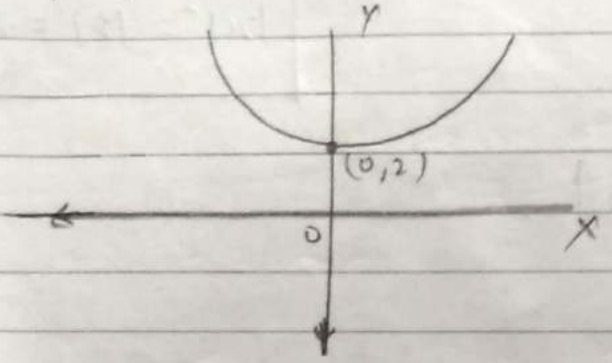


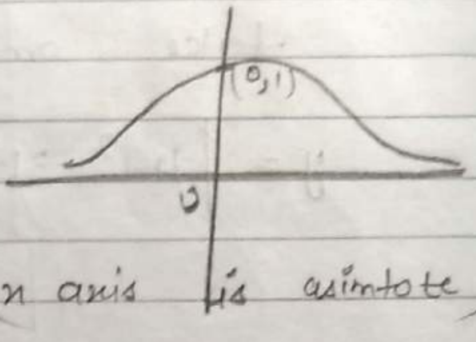
$y = \left| |x|^2 - 3|x| + 2 \right|$



I

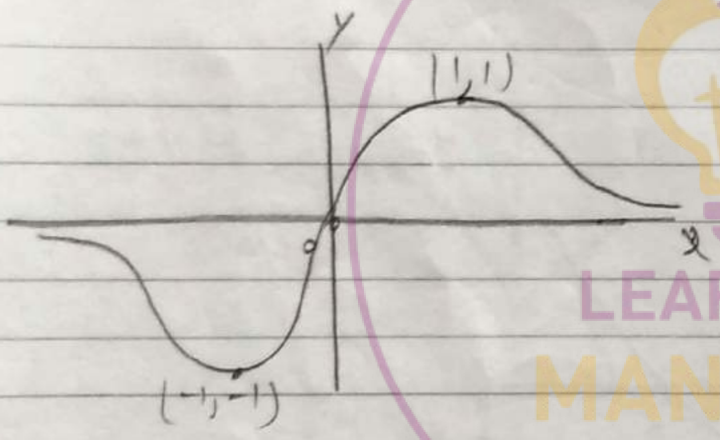
* $f(x) = e^{x} + e^{-x}$ $\qquad$ $y = \frac{1}{1+x^2}$





(x axis is asimtote)

* $y = \frac{2x}{1+x^2}$



$f(x) = \sqrt{x-1} + \sqrt{4-x}$

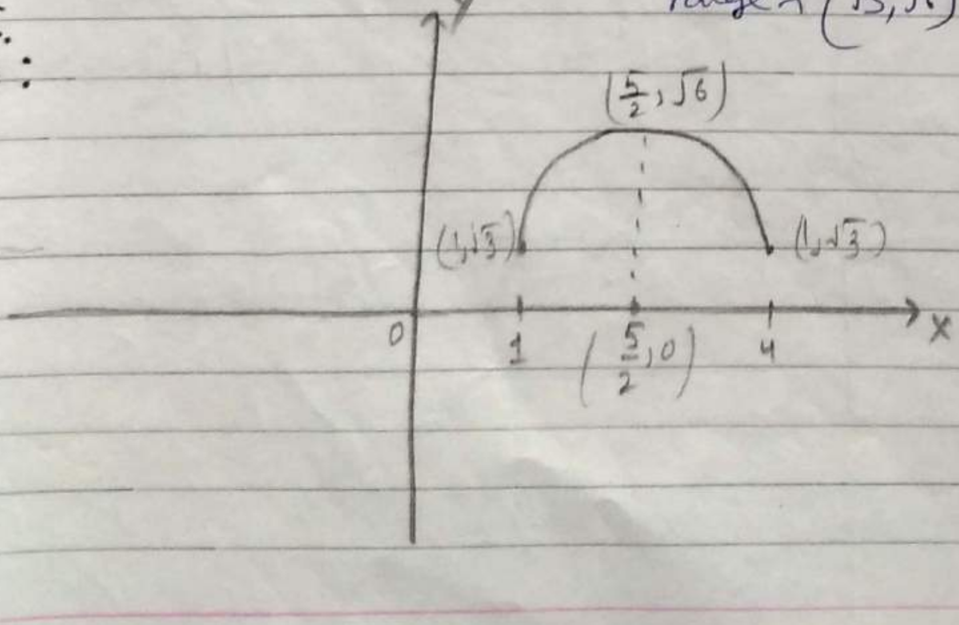
$x - 1 \geq 0$ $\qquad$ $4 - x \geq 0$

$x \geq 1$ $\qquad$ $x - 4 \leq 0$

$x \leq 4$

$1 \leq x \leq 4$

range = $(\sqrt{3}, \sqrt{6})$



$y = \sqrt{\frac{5}{2} - 1} + \sqrt{4 - \frac{5}{2}}$

$= \sqrt{\frac{3}{2}} + \sqrt{\frac{3}{2}}$

$= 2\sqrt{\frac{3}{2}}$

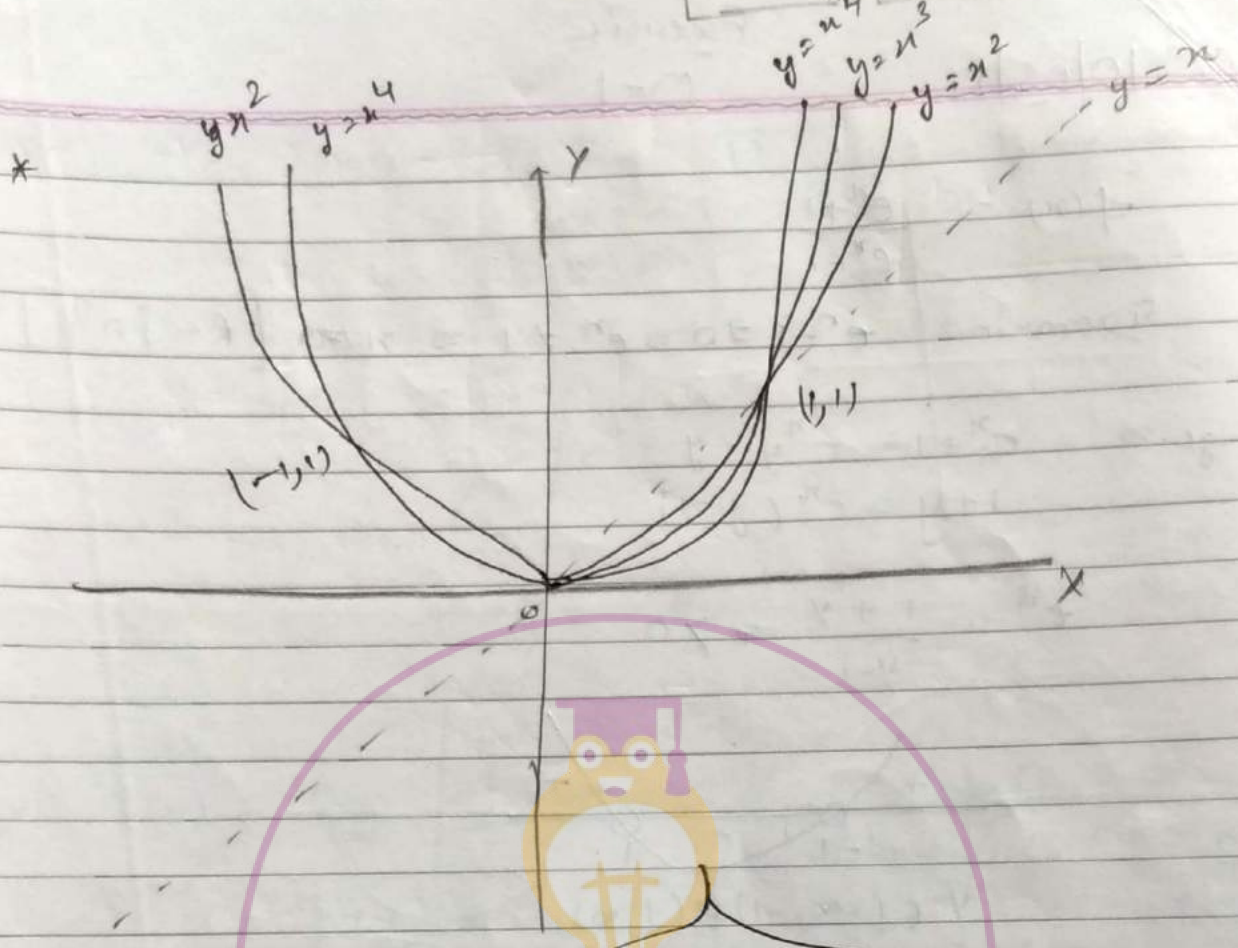
$= \sqrt{6}.$

① 02/08/7

O-1 : 8, 10, 12, 18, 23, 29, 28, 27, 31, 32
35

E-1 : 2(i), ii, iii, (v), vi, 3(a)

*



$0 < x < 1$

$x = \frac{1}{2}$ $x > x^2 > x^3 > x^4$

$\frac{1}{2} > \frac{1}{4} > \frac{1}{8} > \frac{1}{16}$

$x > 1$

$x = 2$ $x < x^2 < x^3 < x^4$

$2 < 4 < 8 < 16.$

Date:- 10/05/17 Exercise 0-1

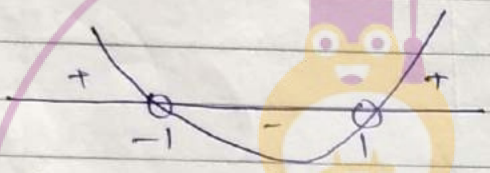
(18) $f(x) = \frac{e^x+1}{e^x-1}$

Domain: $e^x - 1 \neq 0 \Rightarrow e^x \neq 1 \Rightarrow x \neq 0$ $[R - \{0\}]$

Range ⇒ $e^x + 1 = e^x y - y$

$1 + y = e^x(y-1)$

$e^x = \frac{1+y}{y-1} \Rightarrow > 0$



$y \in (-\infty, -1) \cup (1, \infty)$.

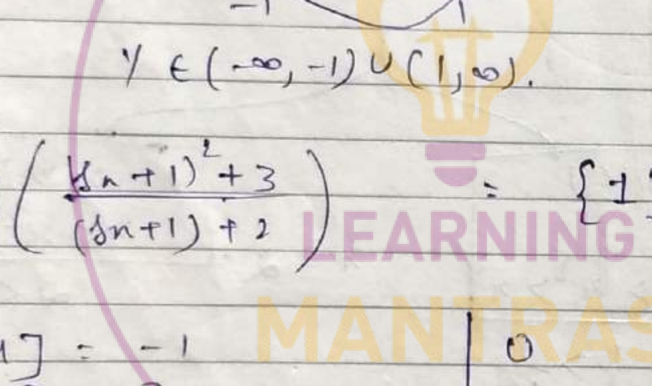
(10) $Sg\left(\frac{(\{x\}+1)^2+3}{(\{x\}+1)+2}\right) = \{1\}$.

(8) $[x] = -1$
$[y] = 0$
$[z] = 1$

$$\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 2 \end{vmatrix}$$

(32) $f(x) = x^2 + 3x + 2$
$(x+1)(x+2)$

ⓐ



Exercise

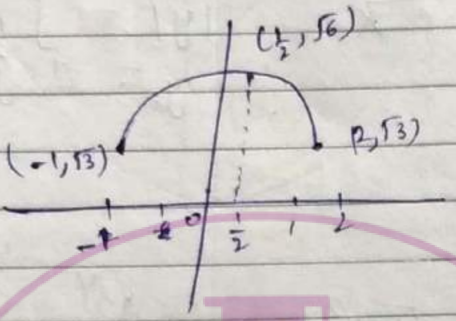
Q - 1

② (v) $y = f(x) = \sqrt{2-x} + \sqrt{1+x}$

$2 - x \geq 0 \qquad x + 1 \geq 0$

$x \leq 2 \qquad x \geq -1$

$-1 \leq x \leq 2$



(vi) $f(x) = ~~\log~~ \frac{\sqrt{x+4} - 3}{x-5}$

~~Domain = R - {5}~~

Domain, $x + 4 \geq 0$

$x > -4$ & $x \neq 5$

$[-4, \infty) - \{5\}$

$\frac{\sqrt{x+4} - 3}{x-5} \times \frac{\sqrt{x+4} + 3}{(\sqrt{x+4} + 3)}$

$\frac{x+4-9}{(x-5)\sqrt{x+4}+3}$

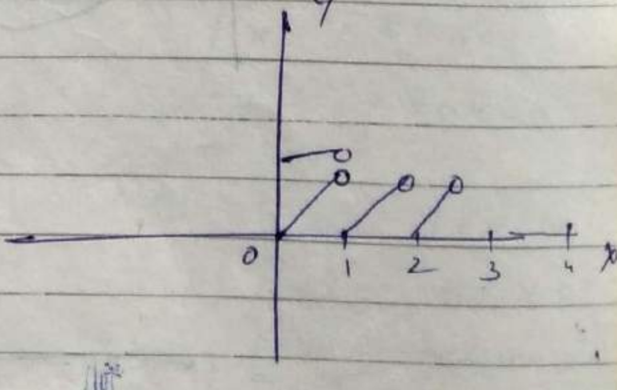
$f(x) = \frac{1}{\sqrt{x+4} + 3}$

$\left(0, \frac{1}{3}\right] - \frac{1}{6}$ Ans

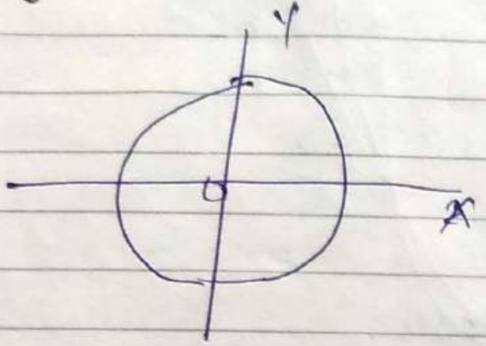
③ (ii) $x = [x] + \{x\}$

$y = \{x\}^{[x]}$

$0 < x < 1$	$x = 1$
$x = 1$	0
$1 < x < 2$	$(x-1)$
$x = 2$	0
$2 < x < 3$	$(x-2)^2$



* $x^2+y^2=1$

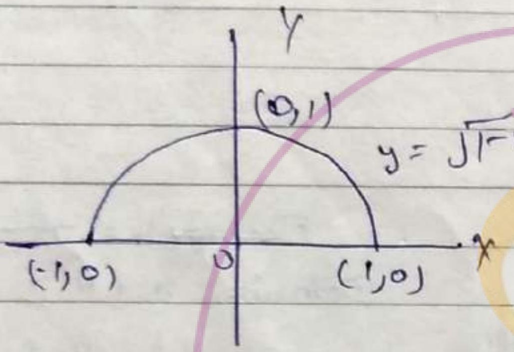


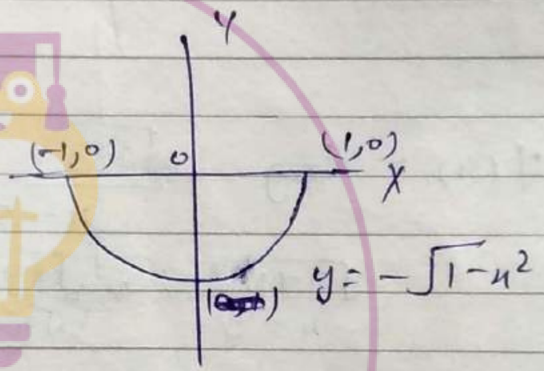
$y^2 = 1-x^2$

$\sqrt{y^2} = \sqrt{1-x^2}$

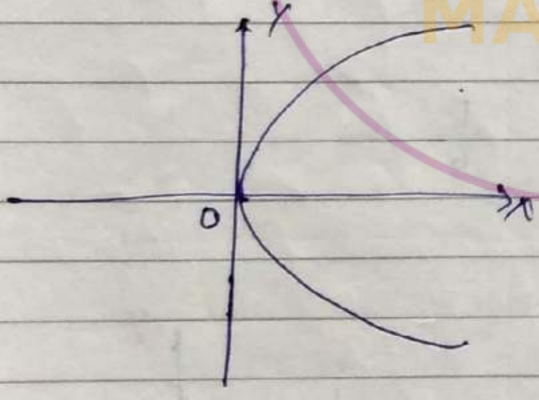
$|y| = \sqrt{1-x^2}$

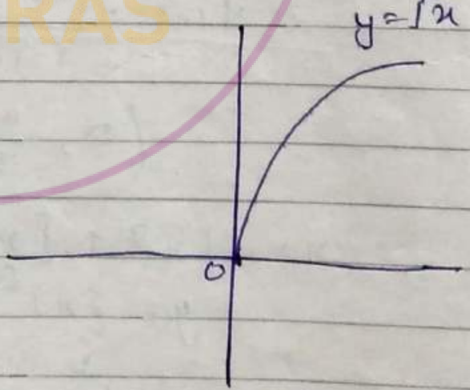
$y = \pm\sqrt{1-x^2}$





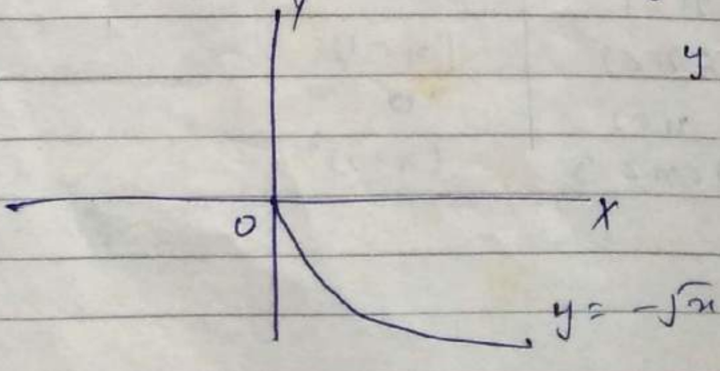
* $y^2 = x$ Perabala.





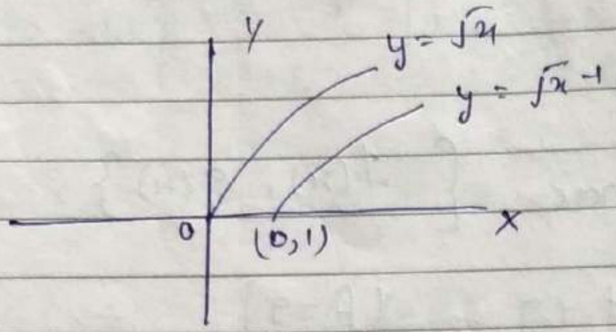
$\sqrt{y^2} = \sqrt{x}$

$y = \pm\sqrt{x}$



* $y = \sqrt{x-1}$

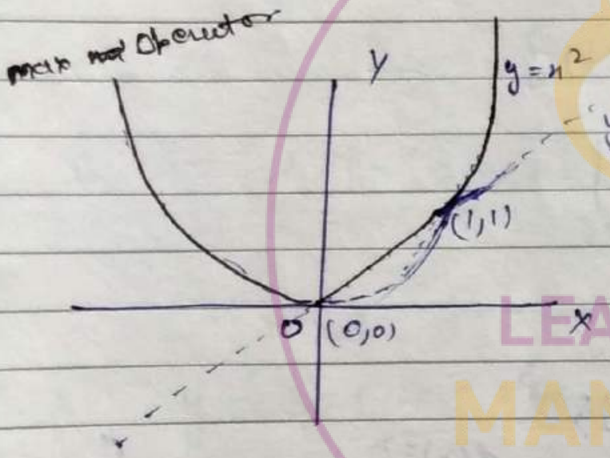
Domain ⇒ $x-1 > 0$
$x \geq 1$

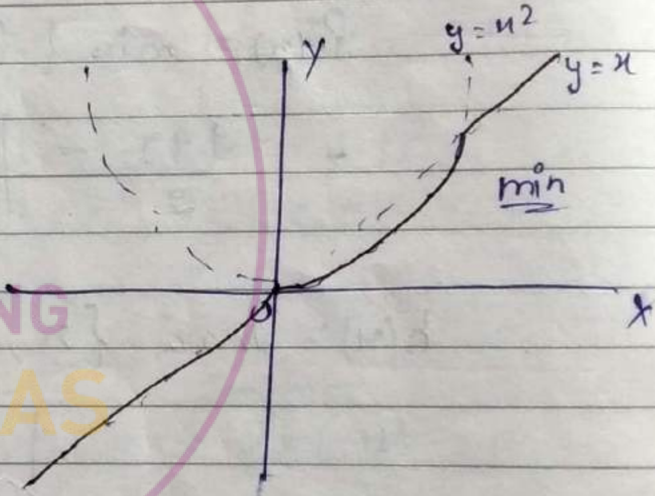


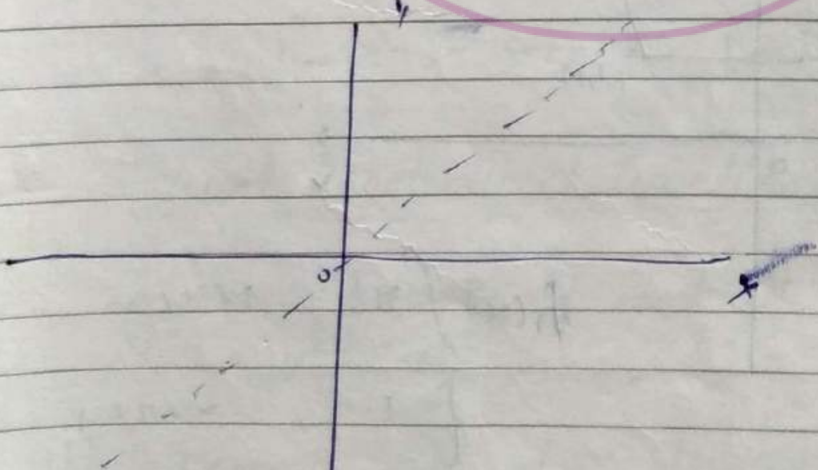
* Max min operator:

$f(x) = \max\{x, x^2\}$

$f(x) = \min\{x, x^3\}$







max

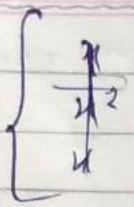
$$f(x) = \begin{cases} x^2, & 1 \leq x < \infty \\ x, & 0 \leq x < 1 \\ x^2, & -\infty < x < 0 \end{cases}$$

Min:

$$f(x) \begin{cases} x, & 1 < x < \infty \\ x^2, & 0 \leq x < 1 \\ x, & -\infty < x < 0 \end{cases}$$

Note:-

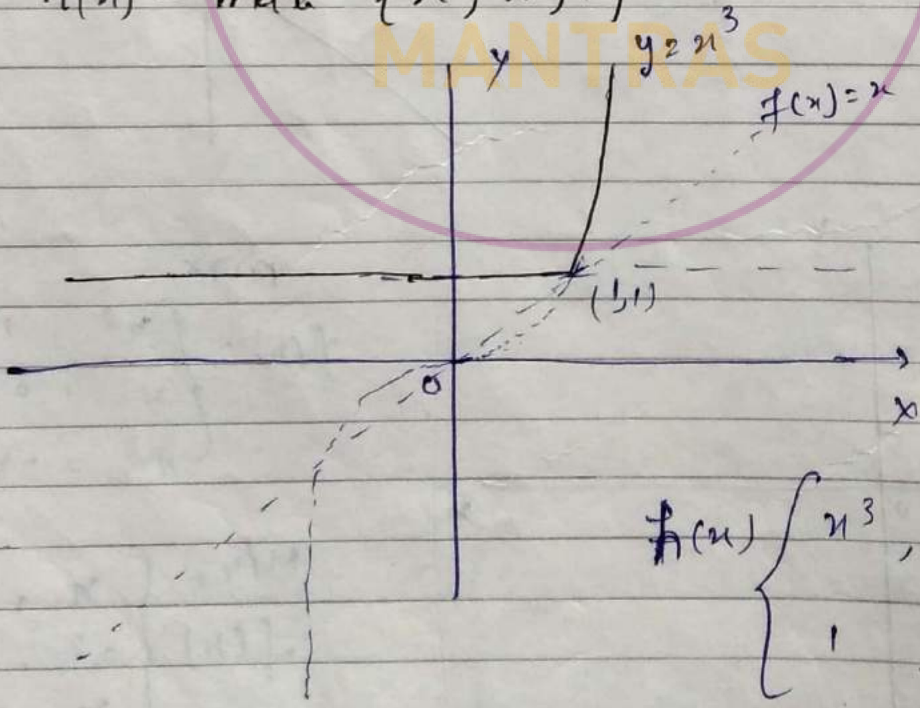
→ $g(x) = \min\{\cancel{x}, x^2\}$



$$h(x) = \max\{f(x), g(x)\}$$
$$= \frac{f+g}{2} + \left|\frac{f-g}{2}\right|$$

$$P(x) = \min\{f(x), g(x)\}$$
$$= \frac{f+g}{2} - \left|\frac{f-g}{2}\right|$$

$h(x) = \max\{x, x^3, 1\}$



$$h(x)\begin{cases} x^3, & 1<x<\infty \\ 1, & -\infty<x<1 \end{cases}$$

angle same apply.

(*) $a \sin x + b \cos x$ $\qquad \sqrt{(a)^2+(b)^2}$

$$= \sqrt{a^2+b^2}\left\{\frac{a}{\sqrt{a^2+b^2}} \sin + \frac{b}{\sqrt{a^2+b^2}} \cos x\right]$$

$$\sqrt{a^2+b^2}\left\{\sin \sin\phi + \cos \cos\phi\right)$$

$$\sqrt{a^2+b^2} \quad \underset{y}{\cos(x-\phi)}$$

$$\sqrt{a^2+b^2} \times [-1, 1] \Rightarrow \text{range of } y \in$$

$$\Rightarrow \boxed{\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right)}$$

* $y = \sin x - 2\cos x$: $\sqrt{(1)^2+(-2)^2} = \sqrt{5}$

range $= [-\sqrt{5}, \sqrt{5}]$ Ans

* $y = 3\cos 4x - 4\sin 4x + 7$ $\qquad \sqrt{(3)^2+(-4)^2} = 5$

range $= [-5, 5] + 7$

$[-5+7, 5+7] = [2, 12]$ Ans

* Equal (Identical Function): Two function f and g ~~are equal if and if the~~ are said to be equal iff.

(1) Domain of f = domain of g
(2) range of f = range of g
(3) f = g ∀ x ∈ dom.

[19, 22]

HW: S-1 ⇒ 3(b), 16, 18, 20, 22, [illegible]

* $\log x^n = n\log x$

$f(x) = \log x^2$ $\qquad$ $g(x) = 2\log x$

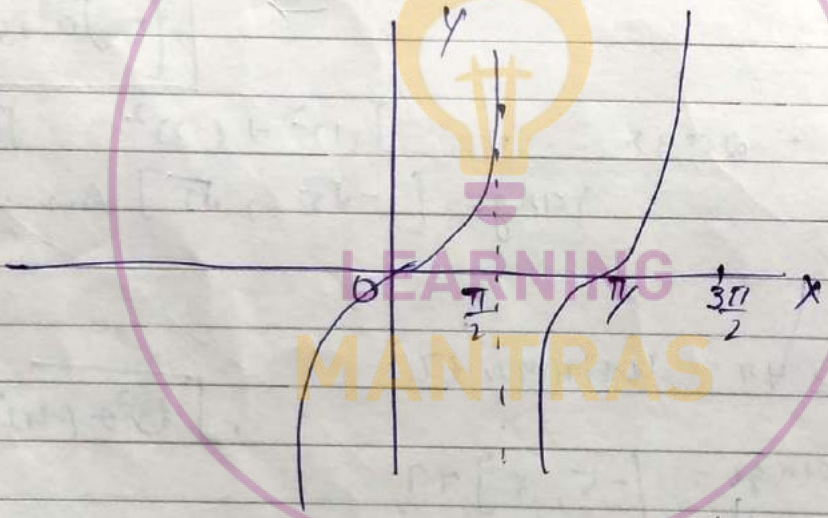
Domain − $x^2 > 0$ $\qquad$ $x > 0$

$x \in R - \{0\}$

* $f(x) = \sec^2 x - \tan^2 x$ $\qquad$ $g(x) = \sin^2 x + \cos^2 x$

$x \in R - (2n+1)\frac{\pi}{2}$ $\qquad$ ~~$y = \tan x$~~

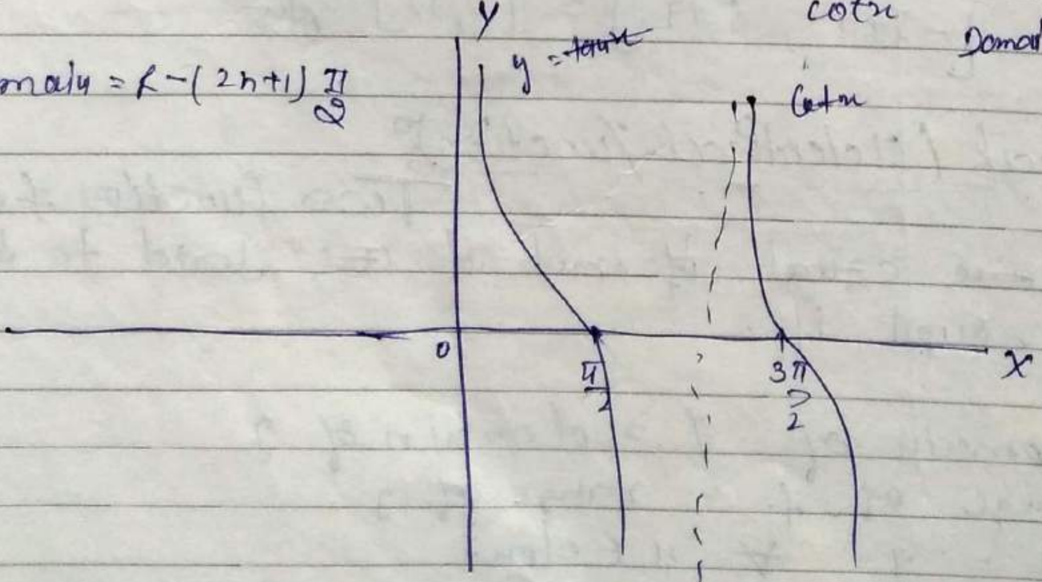
$x \in R$



* $y = \tan x$ $\qquad$ $y = \frac{1}{\cot x}$

Domain $= R - (2n+1)\frac{\pi}{2}$

Domain - $R - n\pi - (2n+1)\frac{\pi}{2}$



$\sin x^4 + \cos^4 x = 1 - 2\sin^2 x \cos^2 x$

$\sin x^6 + \cos^6 x = 1 - 3\sin^2 x \cos^2 x$.

$f(x) = \sin x$ | $g(x) = \cos x$

Domain : R | Domain = R -

Range : $[-1, 1)$ | Range :- $[-1, 1)$

Sin ≠ Cos

	f	g
1)	$\{[x]\}$	$[\{x\}]$
2)	$\operatorname{Sgn}(x^2+1)$	$\sin^2 x + \cos^2 x$
(3)	$\log_x e$	$\frac{1}{\log_e x}$
(4)	$\sqrt{\frac{1-\cos 2x}{2}}$	$\sin x$
⑤	$\sqrt{x^2-1}$	$\sqrt{x-1}\,\sqrt{x+1}$
⑥	$\operatorname{cosec} x$	$\frac{1}{\sin x}$.

$\{x\} \Rightarrow$

× Classification of function:

f | g

~~Ans~~:-1 $\{[x]\}$ | $[\{x\}]$

Domain $= \{0\}$ | Domain $= \{0\}$

Range $\{0\}$ | Range $= \{0\}$

$f = g$.

(3) $\log_x e$ | $\frac{1}{\log_e x}$

$f(x) \neq f(g)$

(8) $\operatorname{Cosec}(x)$ $\qquad$ $g(x) = \frac{1}{\sin(x)}$

$x \in R - \{n\pi\}$ $\qquad$ $x \in R$

$f(x) \neq g(x)$.

(4) $\sqrt{\frac{1-\cos 2x}{2}}$ $\qquad$ $\sin x$

$f(x) = g(x)$

⑨ $f(x) = \operatorname{sgn}(x^2+1)$ $\qquad$ $g(x) = \sin^2 x + \cos^2 x$

$D(f) = x \in R$ $\qquad$ Domain of $(f) = x \in R$

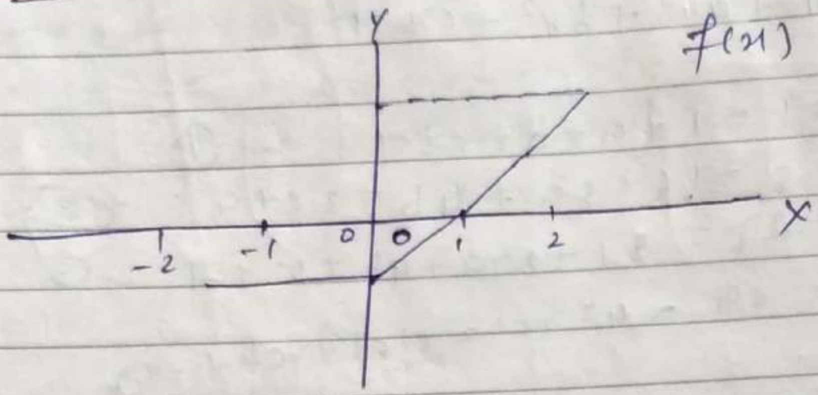
$f(x) = g(x)$

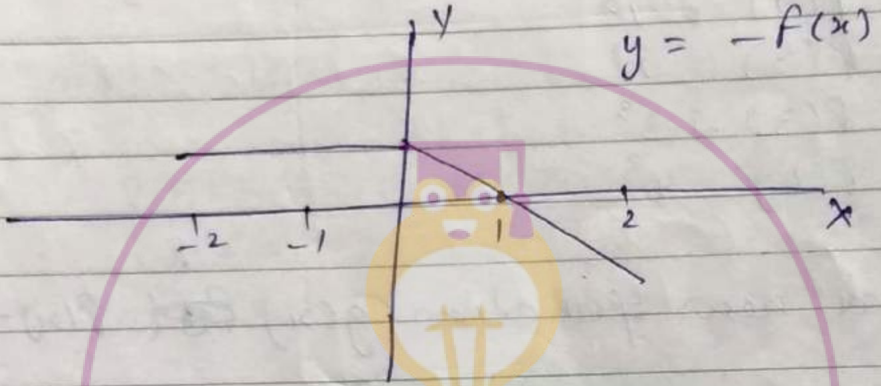
Exercise : B-1

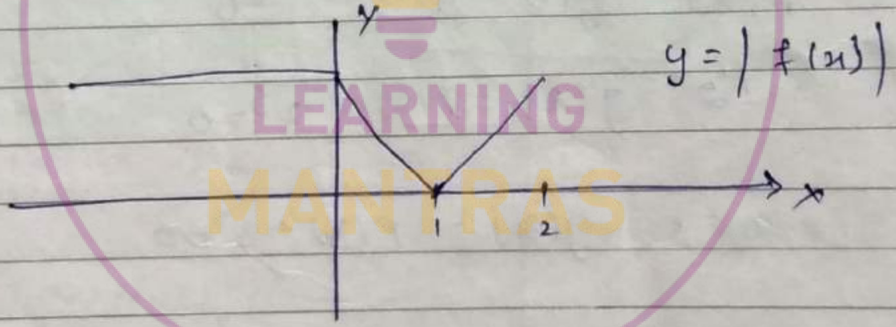
Date: 11/05/17

Que: 16

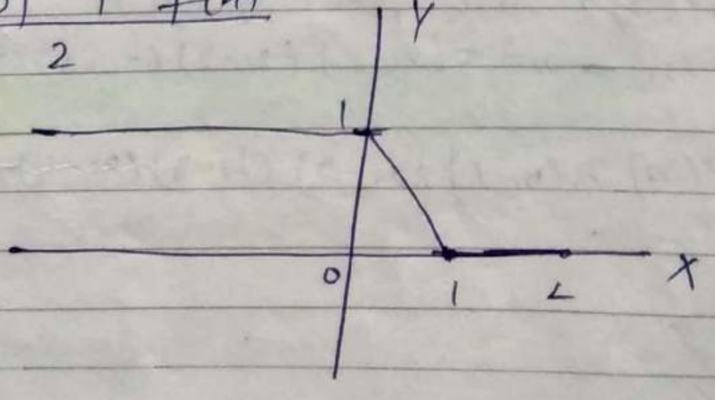
(d)




$y = -f(x)$


$y = |f(x)|$

$\frac{|f(x)| + f(x)}{2}$



1) $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$P(1) = 1 = 1 + a + b + c + d$ — ①

$P(2) = 8 = 16 + 8a + 4b + 2c + d$ — ②

$P(3) = 27 = 81 + 27a + 9b + 3c + d$ — ③

$P(4) = 64 = 4^4 + 64a + 16b + 4c + d$ — ④

$P(1) = 1^3$

$P(2) = 2^3$

$P(3) = 3^3$

$P(4) = 4^3$

Define a new function $g(x) = P(x) - x^3$.

$x = 1 \quad g(1) = P(1) - 1^3 = 0$

$x = 2 \quad g(2) = P(2) - 2^3 = 0$

$x = 3 \quad g(3) = P(3) - 3^3 = 0$

$x = 4 \quad g(4) = P(4) - 4^3 = 0$

$x = 1, 2, 3, 4$ are roots of $g(x)$

$P(x) - x^3 = (x-1)(x-2)(x-3)(x-4)$

$P(x) = (x-1)(x-2)(x-3)(x-4) + x^3$ Ans

* $f(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$

$P(1) = 2 = 1 + a + b + c + d + e$

$P(2) = 4 = 2 + 4a + 8b + 16b$

$P(3) = 6 = 3 +$

$P(4) = 8 =$

$P(5) = 10 =$

$\therefore g(x) = P(x) - 2x$

then find P(6)

$x = 1 = g(1) =$

$g(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$

$P(x) - 2x =$

$f(x) = (x-1)(x-2)(x-3)(x-4)(x-5) + 2x.$

* Classification of function!

$f : A \to B$

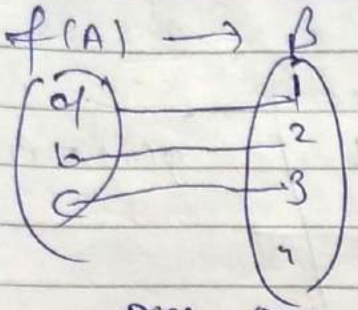
one one (Injective)

many one

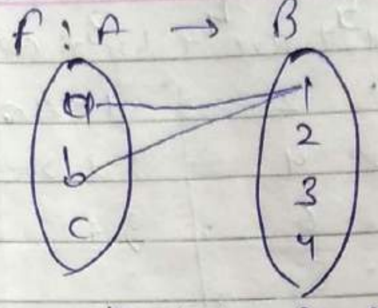
onto (Subjective)

into

* one one onto (Bijective).
* one one into
* many one onto
* many one into



one - one

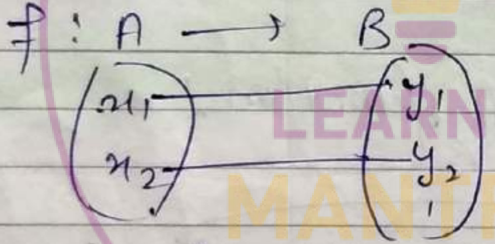


many - one.

* **One one:** If distinck element of set A associate distinct element in set B other wise function is many one.

Thus $x_1, x_2 \in A$

& $f(x_1), f(x_2) \in R$



$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$

Or

$f(x_1) \neq f(x_2) \Leftrightarrow x_1 \neq x_2$

then function is one one.

only increasing $\}$ one one fn.
or only Decreasing

$\left[\frac{dy}{dx}\right]$

* Method of Identify one one and Many one

1) If function is not one one than it is many one.

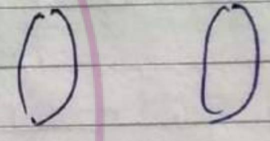
2) Total no. of function = no. of one one function + no. of many one function

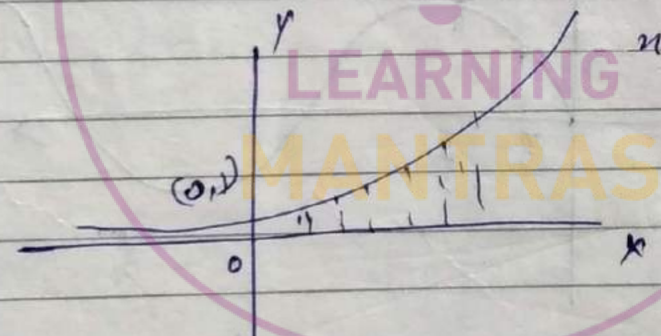
8

3) A continuous function always increase (or discrease) than it is one one function).

$f: R \rightarrow R$ or $f: R — R$

$f(x) = e^x$

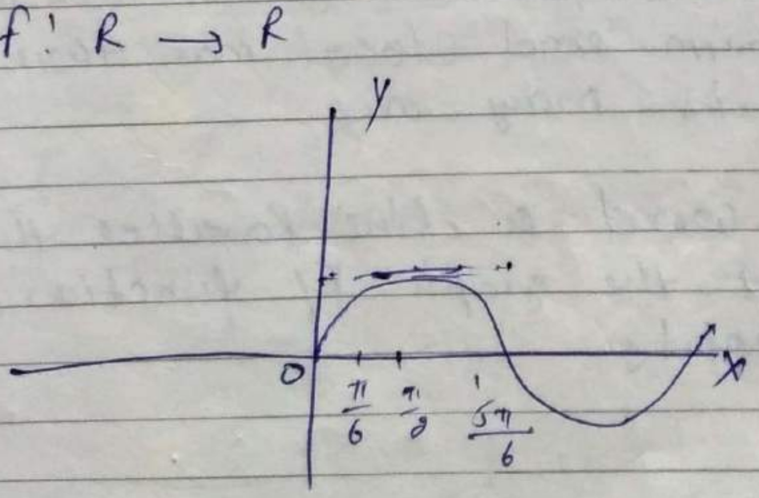




$x = e^x$

one one

$f: R \rightarrow R$



Many one

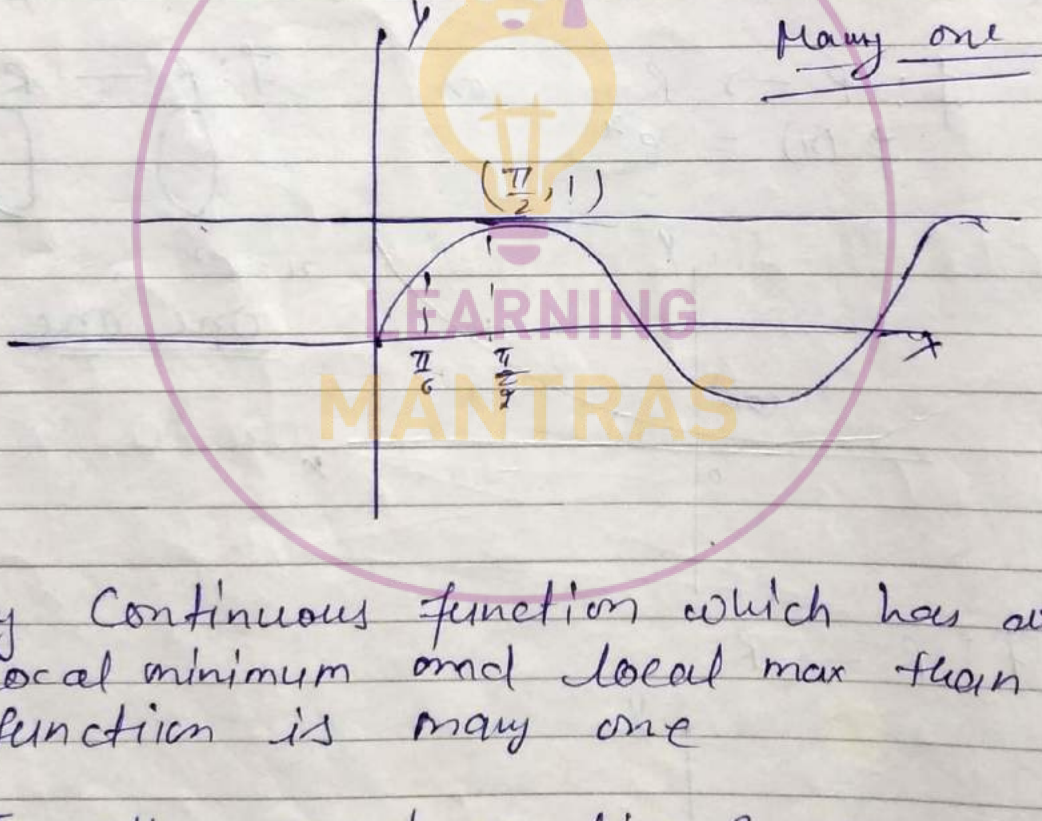
Cubic increasing ≥ 0 ↑ ↑ $\frac{dy}{dx} > 0$

for increasing :-
$\frac{dy}{dx} > 0$

for decreasing :- $\frac{dy}{dx} < 0$

for cubic increasing $= \geq 0$

for cubic decreasing $= \leq 0$



Many one

⇒ Any Continuous function which has atleast local minimum and local max than function is many one

⇒ In other word a line Parallel the x axis cut the graph of function atleast at two points.

R=CD

* **Onto**[redacted] :
If Range = cordomain then function is onto[redacted]

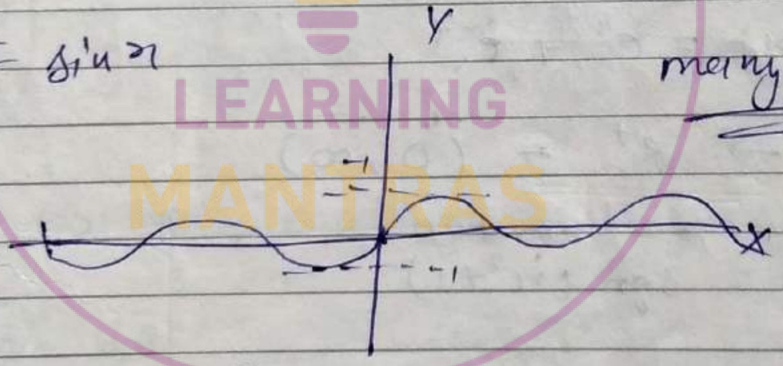
Into[redacted] :
If Range is not equal to codomain then function is In t[redacted]

* No. of onto[redacted] function + no of in to[redacted] function = Total no. of function.

* $f : R \rightarrow R$ odd degree polynomial is always onto[redacted]. ✓

$f(x) = \sin x$

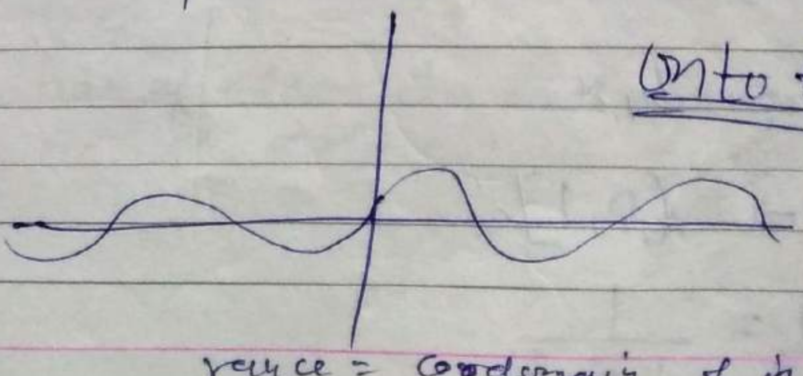
many one



rang $[-1, 1]$
range $\neq$ cordomain $= (R)$

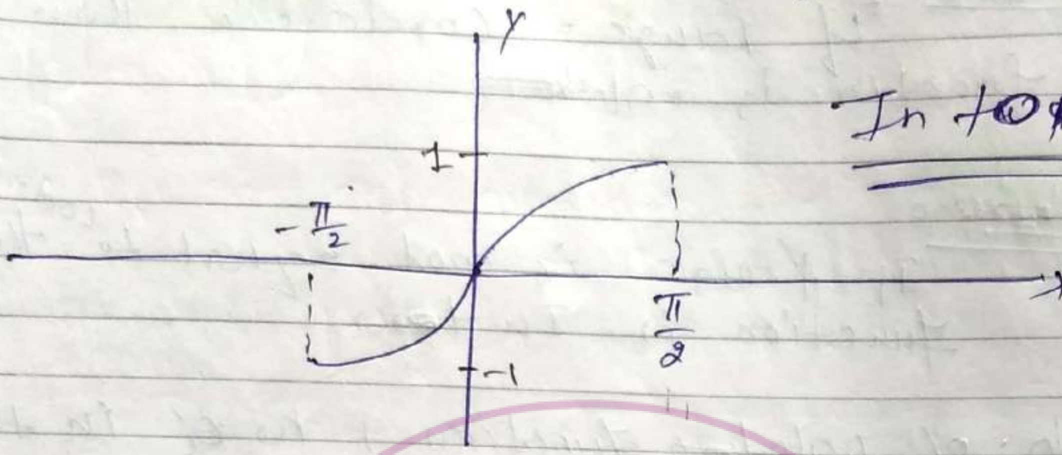
$f : R \rightarrow (-1, 1)$
$f(x) = \sin x$

Onto[redacted]



rance = cordomain f is many one

* $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow [-1, 1\}$ $f(x) = \sin$



Into

f is ~~many~~ one-one function

* Identify

1) $f : R \rightarrow R$
$f(x) = e^x + e^{-x}$ One one.

2) $f : R \rightarrow R^+ \rightarrow (0, \infty)$
$f(x) = \text{sgn}(x^2+1)$ One one

③ $f : R \rightarrow R$
$f(x) = x^3+1$ One one, onto

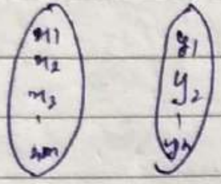
4) $f : (0, \infty) \rightarrow R$
$f(x) = \ln x$

(5) $f : R \rightarrow [0, 1]$
$f(x) = \frac{1}{1+x^2}$

(6) $f : R \to [1, \infty)$ one one

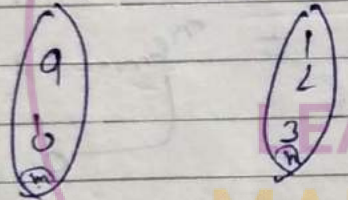
$f(x) = \sqrt{x^2+1}$

* $f : A \to B$



No. of Different types of function :

$f : A \to B$



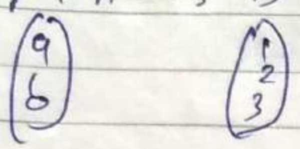
Total no. of maping.

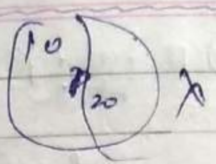
x_1 can take n images
x_2 can take n "
x_n can take n "

$\therefore$ total no. of map = $\underbrace{n \times n \times \cdots n}_{m \text{ times}} = n^m$

n = no. of elements in Codomain
m = no. of element in domain.

f : A → B





* No. of one one mapping

* x_1 can take n images.

x_2 " " (n-1) images

x_3 " " (n-2) images.

.

.

x^{m} " " (n-(m-1) images

total mapping = $n(n-1) \times (n-2) \times$ -- -- $(n-m+1)$

$= {}^{n}P_m \rightarrow n \geq m$

0 $m < m$

$^{n}P_r = \frac{|\underline{n}}{|\underline{n-r}}$

total map = n^m

$= {}^{n}P_m$.

$= {}^{3}P_2 = \frac{|\underline{3}}{|\underline{3-2}}$

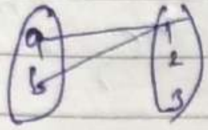
$= |\underline{3}$

$= 6$

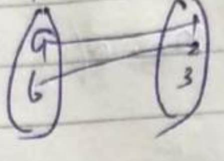
∴ many ~~one~~ map = total map − one one map.

f : A →

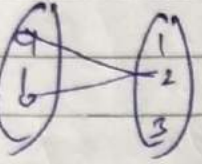
f : A → B



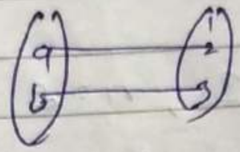
f : A → B



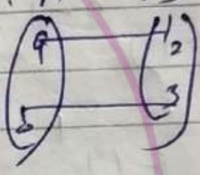
f : A → B



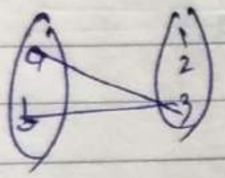
f : A → B



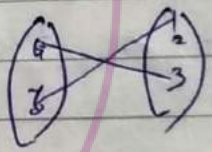
f : A → B



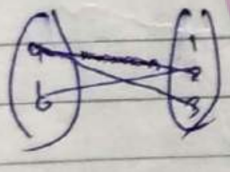
f : A → B



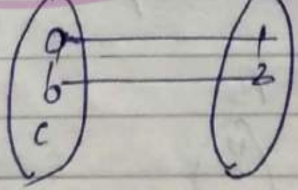
f : A → B



f : A → B



f : A → B

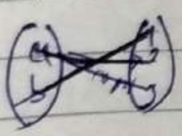


not one one mapping.

one one mapping = 0

m = 3 , n = 2 m > n .

f : A → B



* **No. of ONTO :**

~~no. of~~ $\sum_{r=1}^{n} (-1)^{n-r} \; {}^{n}C_{r} \; r^{m}.$

no. of into map = Total no. of map −
− no of onto map.

$m = 3 \; , \; n = 2$

$\sum_{r=1}^{n} (-1)^{2-r} \; {}^{2}C_{r} \; r^{3} + (-1)^{2\cdot 2} \; {}^{2}C_{2} \; 2^{3}$

$(-1)^{2-1} \; {}^{2}C_{1} \; r^{3} + (-1)^{0} \cdot 1 \cdot 8$

$= (-1)^{1} \; 2 \times 1 + 8$

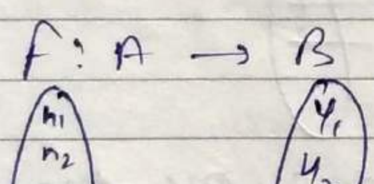
$= -1 \times 2$

$= -2 + 8 = 6$ Ans.

Total map = ~~2~~ $n^{m} = 2^{3} = 8$

Into map $= 8 - 6 = 2$

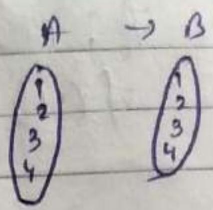
* $f : A \rightarrow B$



No. of one one and onto map.
no. of Bijective map = $\lfloor n$.

Date :- 12|05|17

Let A = {1, 2, 3, 4} then
If f : A → A then find



I] no. of bijective maping.

$$\lfloor 4 = 4 \times 3 \times 2 \times 1 = 24$$

ii) Total no of maping ⇒
Ans ⇒ 4^4

(iii) No. of one-one maping :
Ans! 1 can take 4 images.
2 can take 3
3 ——— 2
4 ——— 1
$4 \times 3 \times 2 \times 1 = 24$.

iv) No. of one one maping such that $f(1) = 1$
Ans: 1 can take = 1 images
2 can take = 3 images
3 can take = 2 images
4 ——— = 1 images
$1 \times 3 \times 2 \times 1 = 6$ images

(v) No. of one-one maping
when $f(1) = 1$
$f(2) \neq 2$
1 can take — 1 image
2 can take — 3 image
3 can take — 2 image
4 can take — 1 image
$1 \times 3 \times 2 \times 1 = 6$

x^{odd} - onto

* $f: R \rightarrow R$

$f(x) = x^3 + ax^2 + bx + c$

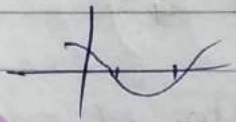
$f'(x) = 3x^2 + 2ax + b$

$D \leq 0$

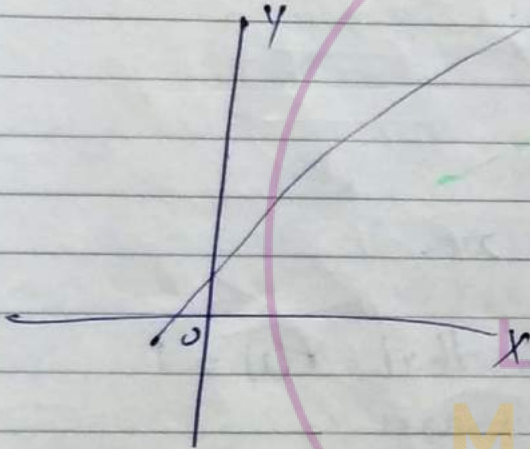
f is one one

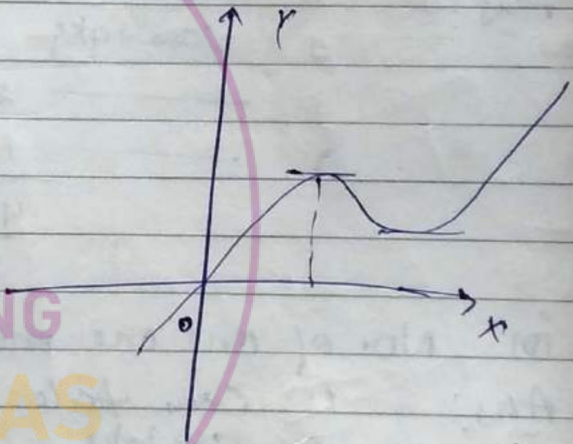
$D > 0$

f is many one.



f is I function.





Que: $f: R — R$

(i) $f(x) = x^3 - 2x^2 + 5x + 3$

(ii) $f(x) = 2x^3 - 6x^2 - 18x + 17$

Ans: 1) $3x - 4x + 5x + 3$

f is one-one

$D \leq 0$

f(x) is many one

~~4x+6~~

② $f'(x) = 6x^2 - 12x - 18$

$\downarrow$

$D > 0$

f many one onto

① $3x^2$ $3x^2 - 4x + 5$

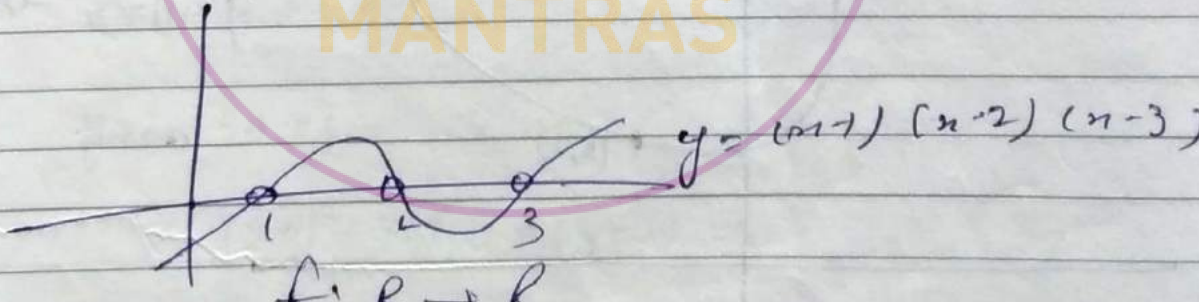
$D = 16 - 4\cdot3\cdot5 < 0$

$= -$

~~f(x) is one one function~~

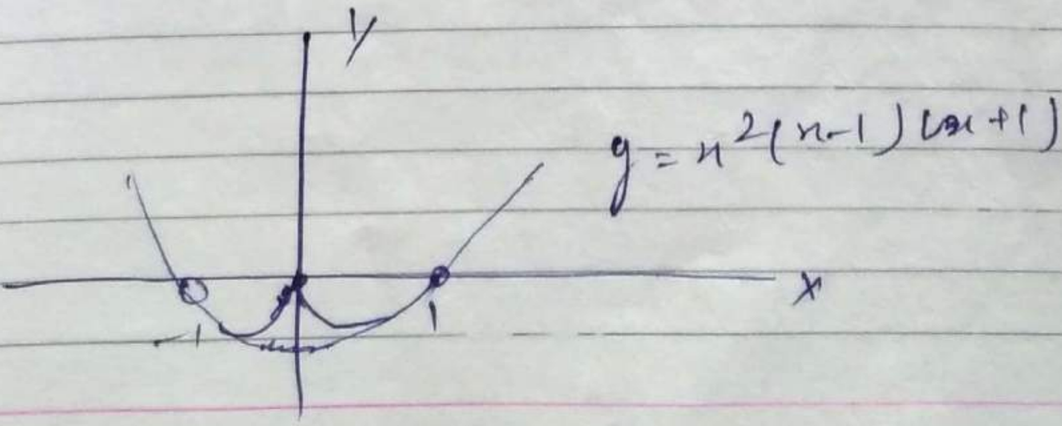
* $f : R \to R$ onto

~~f~~ $y = f(x) = (x-1)(x-2)(x-3)$



$f : R \to R$

⑨ $f(x) = x^2(x-1)(x+1)$



(3) $f : [2, \infty) \to A$

let f defined by $f(x) = x^2 - 4x + 5$ is both one one and onto.

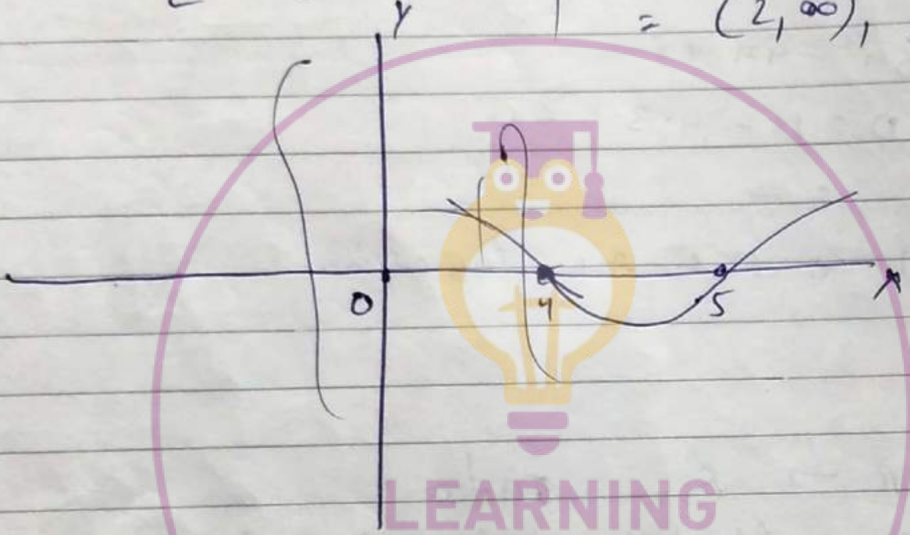
A → (i) R

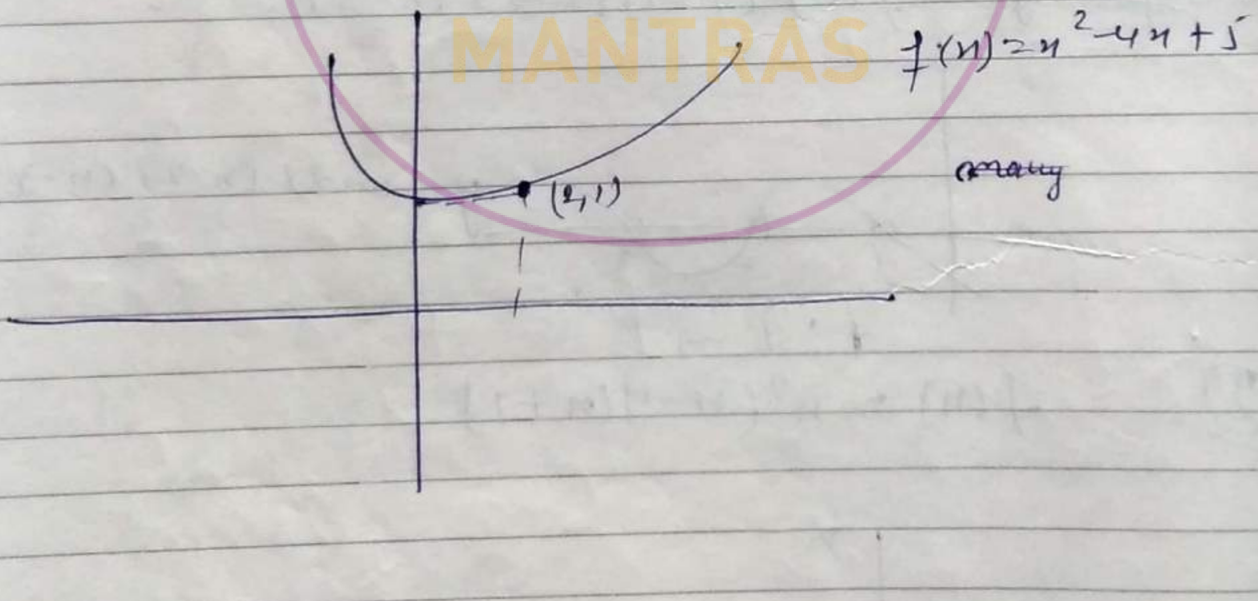
B - ~~(ii)~~ $(1, \infty)$

(iii) $[4, \infty]$

(iv) $[5, \infty)$

$$\left[\begin{array}{l} x^2 - 3 4x + 5 > 0 \\ x(x-4) + 5 \times \\ (x-4)(x+5) = 0 \\ = (2, \infty), \end{array}\right]$$





(4) $f : R \to R$

$$f(x) = \frac{x^2+x+2}{x^2+x+1}$$ — $a>0$, $D<0$

$\hookrightarrow a>0, D<0$

$$f(x) = \frac{x^2+x+1+1}{x^2+x+1}$$

$$= 1 + \frac{1}{x^2+x+1}$$

$$f(x) = 1 + \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}$$

$x=0$, $f = 1+\frac{1}{\frac{1}{4}+\frac{3}{4}}$, $x=-1$, $f = 1+\frac{1}{\frac{1}{4}+\frac{3}{4}}$.

$$y = \frac{x^2+x+2}{x^2+x+1}$$

$$yx^2+yx+y = x^2+x+2$$

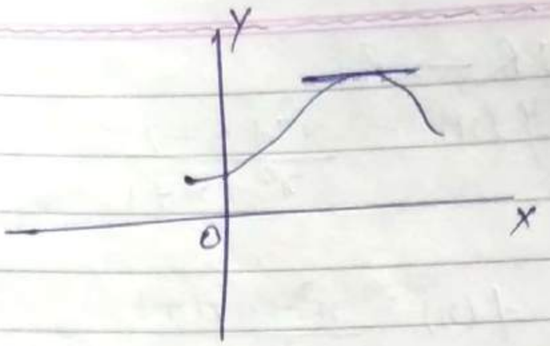
$$x^2(y-1)+x(y-1)+(y-2) = 0$$

$$x = \frac{-(y-1) \pm \sqrt{(y-1)^2-4(y-1)(y-2)}}{2(y-1)}.$$

Cubic max and min ki
Reason.

8) $f(x) = \frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow \frac{d(f)x}{dx} = 0 \Rightarrow$



fu

* functional equation:

Que: if $f(x) + 2f(\frac{1}{x}) = x$ — ①

& $f(\frac{1}{x}) + f(x) = 3x-1$ — ①

Ans: From i and 2 remove $f(\frac{1}{x})$.

$f(\frac{1}{x}) = 3x-1-f(x)$

$f(x)+2(3x-1-f(x)] = x$

$f(x)+6x-2-2f(x) = x$

$\downarrow$ $f(x) = 5x-2$.

Que-2 $f(f(x) \cdot [1+f(x)] = -f(x)$

& ~~1+f(x)~~
Real value function satisfy

Real valued function
Domain - R

② $f(x) = \lambda$

$$f(\lambda) \cdot [1+\lambda] = -\lambda$$

$$f(\lambda) = -\frac{\lambda}{1+\lambda}$$

$$\boxed{f(x) = -\frac{x}{1+x}}$$

③ Real value function f satisfy

$f(x + f(x)) = 4f(x)$ & $f(1) = 4$

$f(21) = ?$

21
4
84

$f(1) = 4$

$f(x + 4) = 4 \times 4$

$f(4 + x) = 16$

$f(5) = 16$

$f(21) =$ $f(x+21) = 4\times 21$

$f(x+21) = 84$

$f(5 + f(5)) = 4(5)$

$f(21) = 4 \times 16$

$= 64.$

range finite
elm

* bounded function:

A function whose range lies b/w two finite ~~function~~ is called bounded function.

$0 < e^x < \infty$ $\sin x =$ ✓

[whose Range only finite.]

* Implicit/Explict function:

Explicit: If y has been expressed in terms of x along then it is called an explict function otherwise Implicit function.

$x + y = 3 \rightarrow$ Implict.

$y = 3 - x \rightarrow$ Ex

$x^2 + y^2 - 3xy = 0$ — I

$y =$ Always Implicit.

* Homogenious equation:

function is said to be homogenious with respect to variable (x, y) if Each of its term of the same degree with respect to those variable.

H.W : QS-1 → 4, 5(a, d), 7, 8(b)
(a), 12 (i) a, b, 21,
(O-1,) 11, 15, 16, 21, 22, 23, 33, 36

Note: If.

$f(tx, ty) = t^n(f(x,y)$ then function is Homogeneous, with respect to variable (x,y), of degree n

$$f(x,y) = x^2 - 3y^2 + 4xy$$

Hom. of degree ②

$$f(tx, ty) = t^n(f(x,y)$$

$$f(tx, ty) = t^2x^2 - 3t^2y^2 + 4tx \cdot ty$$

$$= t^2 \ (x^2 - 3y^2 + 4xy)$$

$$= t^2 f(x,y).$$

Date: 13|05|17 Ex- 0.1

(22) $F : R \to R$

$$F(x) = \frac{\sin([x])\pi}{x^2+2x+3} + 2x-1+\sqrt{x^2-x+\frac{1}{4}}$$

$$\frac{N}{D} \neq 0$$

$$x^2-x+\frac{1}{4}$$

$$= x^2-2\cdot\frac{1}{2}x+\left(\frac{1}{2}\right)^2$$

$$= \sqrt{\left(x-\frac{1}{2}\right)^2}$$

$$= 0+2x-1+\left|x-\frac{1}{2}\right|$$

$$f(x) = \begin{cases} 2x-1+x-\frac{1}{2} & x \geq \frac{1}{2} \\ 2x-1+-\left(x-\frac{1}{2}\right) & x < \frac{1}{2} \end{cases}$$

$$\begin{cases} 3x \cdot \frac{3}{2} & x \geq \frac{1}{2} \\ x-\frac{1}{2} & x < \frac{1}{2} \end{cases}$$

$$\boxed{\begin{array}{l} \sqrt{x^2} = |x| \\ \sqrt{(x-1)^2} = |x-1| = |1-x| \end{array}}$$

(15)

$$f(x) = \frac{4^x}{4^x+2}$$

$$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2} = \frac{4^1\cdot4^{-x}}{4^1 4^{-x}+2}$$

$$= \frac{\frac{4}{4^x}}{\frac{4}{4^x}+2} = \frac{4}{4+2\cdot4^x} = \frac{2}{2+4^x}$$

$$f(x)+f(1-x) = \frac{\cancel{4^x+2}}{\cancel{4^x+2}} = 1 \quad A$$

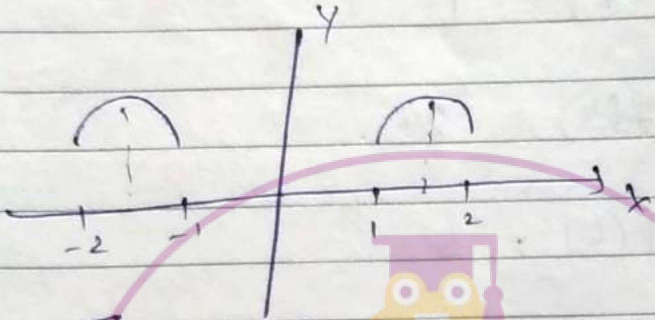
even -
Recall ⇒ Subset

(16) $f(x) = \sqrt{4-x^2} + \sqrt{x^2-1}$

$4-x^2 \geq 0 \qquad x^2-1 \geq 0$

$x^2 \leq 4 \qquad x^2 \geq 1$

$1 \leq x^2 \leq 4 \Rightarrow x \in [-2,-1] \cup [1,2]$



$$\left[\frac{dy}{dx} = 0\right]$$

(36) $f(x) = 1 - 3\sin^2 x \cos^2 x$

$= 1 - \frac{3}{4}(2\sin\cos x)^2$

$= 1 - \frac{3}{4}\sin^2 2x$

min = 1 $\qquad = 1 - \frac{3}{4}$

$= \frac{1}{4} \qquad \left[\frac{1}{4}, 1\right]$

(23)
(A) $f(x) = x^4 + 2x^3 + x^2 + 1$

(33) (C) ~~$f: R \to R^+$~~ $f(x)$

☆ [Method of difference]

Q-R

⑤ ⓓ

$f(3) = 1$

$f(3x) - f(3x-3) = x$

$x = 1 \quad f(3) - f(0) = 1$

$x = 2 \quad f(6) - f(3) = 2$

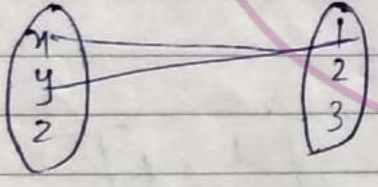
$x = 30 \quad f(9) - f(6) = 3$

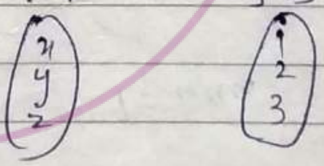
$x = 100 \quad f(300) - f(297) = 100$

$f(300) - f(0) = 1 + 2 + 3 + \text{---} 100$

$\frac{100(100+1)}{2} = 5050$

(9) $f : A \longrightarrow B \qquad f : A \longrightarrow B$





$f(x) = 1$

$f(y) \neq 1$

$f(z) \neq 2$

$f(x) = 1 \quad \times$

$\not\exists \; f : A \to B$

(सभी बनाते हैं
है गलत)

(21) $f(x) + f(-x) = ?$

* Composite function :-

Let $f: A \to B$ $g: B \to C$ two function then function gof $A \to C$ will be define by

$gof(x) = g(f(x))$ $\forall x \in A$ is called composite of the function $f(x)$ & $f(g)$

where Range of f must be subset domain of g.

$$\text{range of } f \underset{\text{subset}}{\subseteq} \text{domain}$$

$gof(x) = g f(x)$

$C = ~~g~~ f(A) = g(B)$

Ex1 $f(x) = \sin x$

$g(x) = x^2.$

$fog(x) = f g(x) = \sin g(x) = \sin x^2$

$f(x^2) = \sin^2 x.$

$gof = g(f(x) = f(x))^2 = \sin^2$

$fof(x) = f f(x) = \sin f(x) = \sin \sin x$

$gog(x) =$

1) $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

Write Domain of $fg(x)$ $(-2, \infty) \cup (2, \infty)$

~~$fg(x) = f(\sqrt{2-x})$ $(-1, \infty) \cup (1, \infty)$~~

~~$gf(x) =$ $(-0, 1) \cup (1, \infty)$~~

~~$f(f(x)) = f(\sqrt{x}) \to f(x) = \sqrt{R^+}$~~

~~$gg(x) = g(\sqrt{2-x})$~~

$fg(x) = \sqrt{g(x} = \sqrt{\sqrt{2-x}} = 2-x \geq 0$

$x \leq 2$

$fg(x) = \sqrt{2-f(x))} = \sqrt{2-\sqrt{x}} \rightarrow$

$$\left.\begin{array}{l} x \geq 0, -① \\ 2-\sqrt{x} \geq 0 \\ \sqrt{x}-2 < 0 \\ x \leq 4 -② \end{array}\right\} 0 \leq x \leq 4$$

⑬ If $f(x) = \dfrac{x}{\sqrt{1+x^2}} \longrightarrow$

then find $\underbrace{f o f o f \text{ -- } o f(x)}_{100 \text{ times}} = ?$

$$f o f(x) = f\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$= 1+x^2$$

$$x^2 = -1$$

~~$fofof(x) = \frac{x}{\sqrt{1+x^2}} f(x)$~~

~~$fof = f(f(x)) = f\left(\frac{x}{\sqrt{1+x^2}}\right)$~~

~~$(x) = \frac{f(x)}{\sqrt{1+x^2}}$ $1+x^2$~~

~~$x^2+1 =$~~

~~$x^2 = -1$~~

~~x~~

~~$1+x^2 \geq 2$~~ $\boxed{2}$

$$\Rightarrow \quad ff(x) = \frac{f}{\sqrt{1+f^2}} = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^2}} = \left(\frac{x}{\sqrt{1+2x^2}}\right)$$

$$\left[= \frac{x}{1+100x^2}\right]$$

Que! If $f(x) = \frac{2x-7}{x+3}$ & $g(f(x)) = x$

$$g\left(\frac{2x-7}{x+3}\right) = x$$

$$g(\quad g(\lambda) = \frac{3\lambda+7}{2-\lambda}$$

$$\frac{2x-7}{x+3} = \lambda$$

$$2x-7 = \lambda x + 3\lambda$$

$$x(2-\lambda) = 7+3\lambda$$

$$x = \frac{7+3\lambda}{2-\lambda}$$

$$\boxed{g(x) = \frac{3x+7}{2-x}}$$ Ans

Que: if $\overset{y_2}{f(x)}$ — $\overset{\text{Domain}}{[0,1)}$ and range is then find domain of $y = f(2x-1)$
(2) $y = f(\tan x)$,

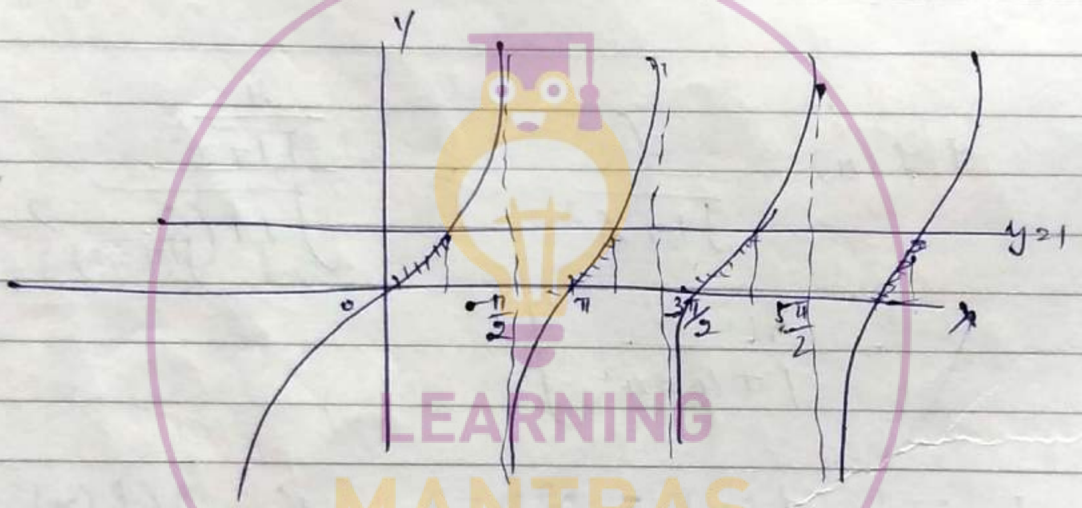
Ans 1, i) $0 \le 2x - 1 \le 1$

$1 \le 2x \le 2$

$\frac{1}{2} \le x \le 1$

Domain $= \left[\frac{1}{2}, 1\right]$

(ii)



$y = f(2x-1)$

$y = f(\tan x)$

$0 \le \tan x \le 1$

$\downarrow \qquad\qquad \uparrow$

$y=0 \qquad y=1$

$x \in \left(0, \frac{\pi}{4}\right), \cup \left[\pi, \pi + \frac{\pi}{4}\right]$ ——

$x \in \left[n\pi, n\pi + \frac{\pi}{4}\right]$

Domain Range

Que: $y = f(x)$ — $[0, 1]$ $[-1, 4]$

then find D/R of

(i) $y = f(2x-1)$

(ii) $y = 2f(x-1)+4$

Ans: (i) range $= [-1, 4]$

(ii) $0 \le x-1 \le 1$

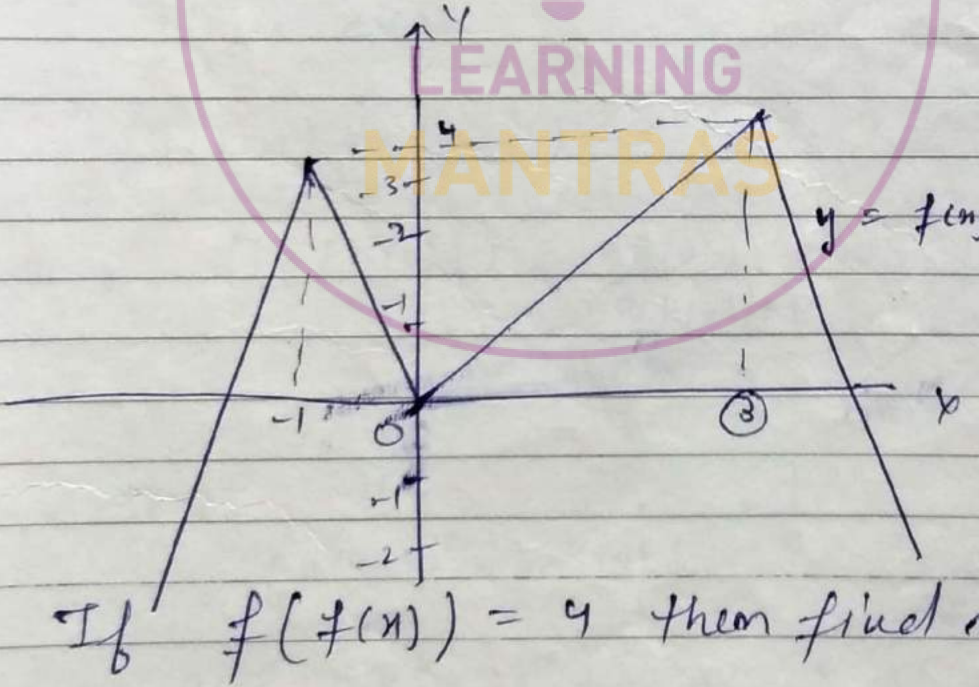
$1 \le x \le 2$ — Domain

$-1 \le f(x-1) \le 4$

$-2 \le 2f(x-1) < 8$

$-2+4 < 2f(x-1)+4 \le 8+4$ $[2, 12]$

(3)



If $f(f(x)) = 4$ then find no. of solution

$f(f(x) = 4$

H.W: O-1, 38, 34,
S-1, 6, 7, 8, 12, 17,

* Property :

(1) generally composition is not communicatided

$f \circ g \neq g \circ f$

2) Composition of 3 function, If defined, then associated $f \circ (g \circ h) = (f \circ g) \circ h$

3) $f : A \to B$ $g : B \to C$
Both one one the $g \circ f : A - C$ also one one.

(4) $f : A \to B$ $g : B \to C$ onto
then $g : A \to C$

(5) $g \circ f = A \to C$ one one
onto

(8)

(b) $f(x) = \text{map}\{x, \frac{1}{x}\}$

$f(\frac{1}{x}) = \max\{\frac{1}{x}, x\}$ $\quad g(x) = f(x) + (\frac{1}{x})$

$g(x) = f(x) + (\frac{1}{x})$

$$= \begin{cases} \frac{1}{x^2}, & 0 \le x \le 1 \\ x^2 & x > 1 \end{cases}$$

(a) $f(x) = \frac{x}{x+1}$

$g(x) = x^{10}$

$h(x) = x+3$

$(g(h(x))) = (h(x))^{10} = (x+3)^{10}$

$f(g(h(x))) = \frac{g(h(x))}{g(h(x))+1} = \frac{(x+3)^{10}}{(x+3)^{10}+1}$

(17)

$f(x) = \frac{1}{(x^6+2x^4+3x^2+1)} > \frac{7}{3}$

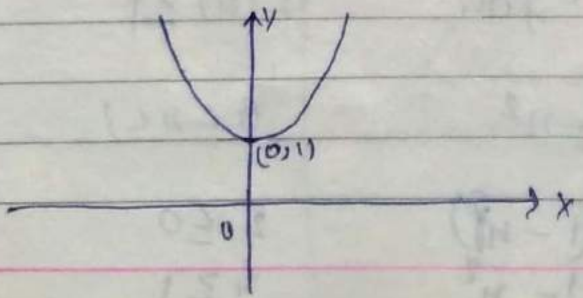
③

$g(x) = x^6+2x^4+3x^2+1$

$f\text{max} = \frac{1}{1} = 1$

$f\text{min} = \frac{1}{\infty} = 0$

$(0, \frac{7}{3})$ Ans



✲ Composition of non-uniform function

$$f(x) = \begin{cases} 1-x, & x \le 0 \\ x^2 & x > 0 \end{cases}$$

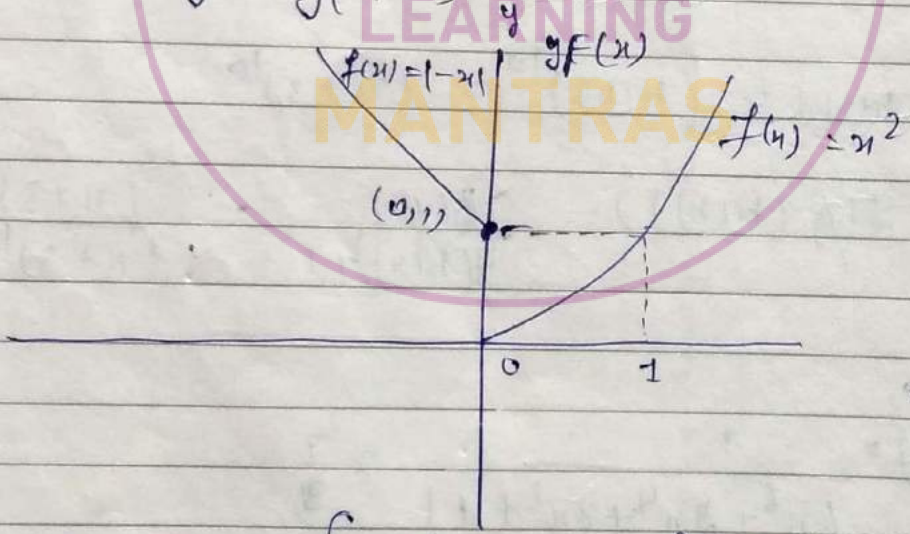
$$g(x) = \begin{cases} -x & x < 1 \\ 1-x & x \ge 1 \end{cases}$$

find gof(x) and fog(x)

$y = mx + c \quad m < 0$

$y = -x+1$

$y = g(f(x))$



$$g(f(x)) = \begin{cases} -f(x) & f(x) < 1 \\ 1-f(x) & f(x) \ge 1 \end{cases}$$

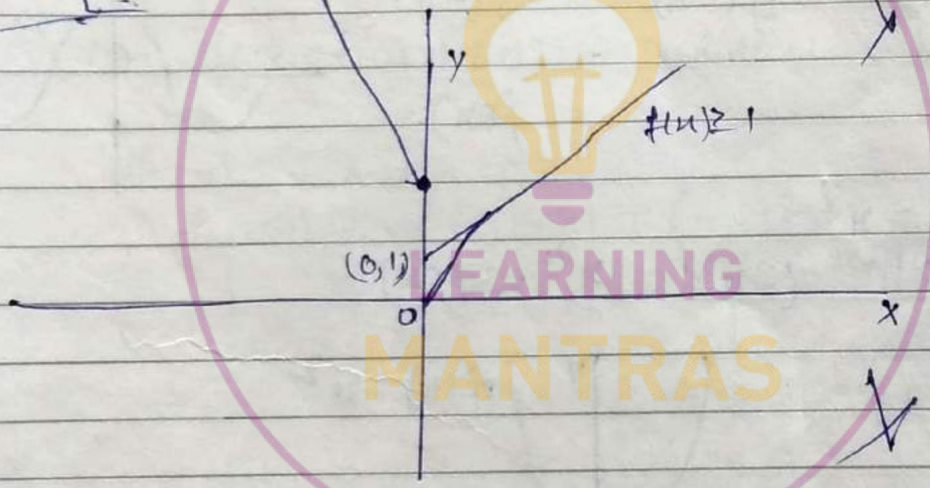
$$\begin{cases} -x^2 & 0 < x < 1 \\ 1-(1-x)^2 & 2 \le 0 \\ 1-x^2 & x \ge 1 \end{cases}$$

$y = gf(x)$

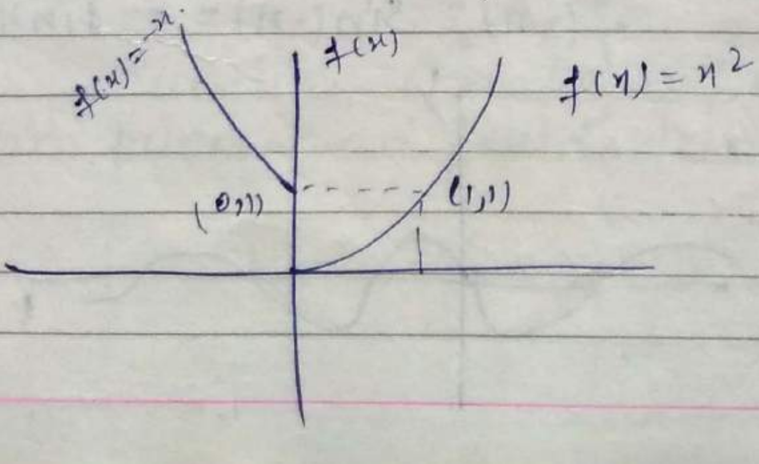
$$\begin{cases} -x^2 , & 0 < x < 1 \\ x & -\infty < x \leq 0 \\ 1 - x^2 & x \geq 1 \end{cases}$$

② $f(fx) = \begin{cases} 1 - f(x) & f(x) \leq 0 \\ (f(x))^2 & f(x) > 0 \end{cases}$

$g(x) \begin{cases} -f(x) & f(x) < 1 \\ 1 - f(x) & f(x) \geq 1 \end{cases}$



$g(f(x) \begin{cases} (x^2)^2 & x > 0 \\ (1-x)^2 , & x \leq 0 \end{cases}$



* Even/odd function :-

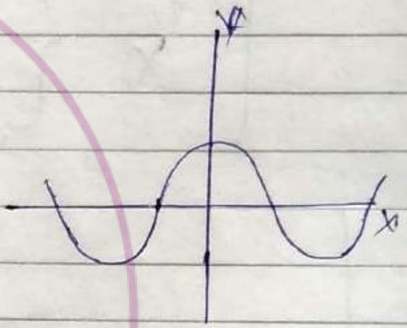
A function $y = f(x)$ defined in $(-a, a)$ symmetrical interval are said to be an even function if $f(-x) = f(x)$

Even. $f(-x) = f(x)$

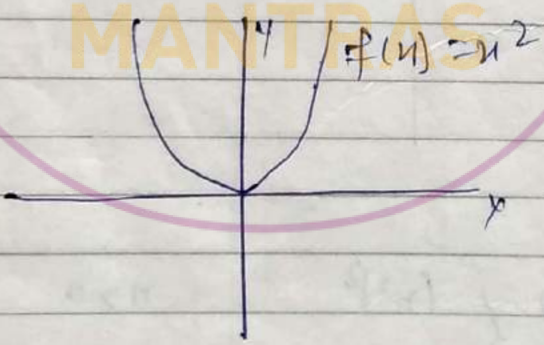
odd $f(-x) = -f(x)$

Example!

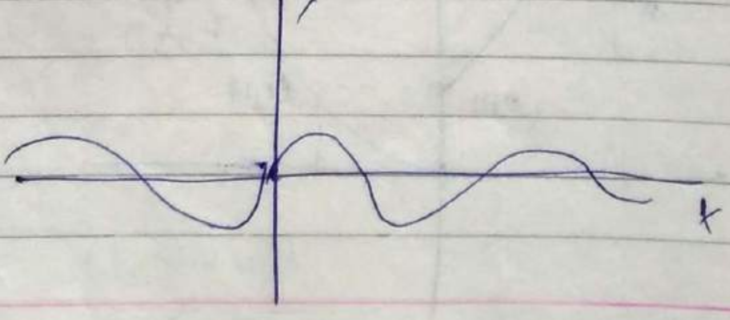
Even: $f(x) = \cos x$
$f(-x) = \cos(-x)$
$= \cos x$
$= f(x)$.



Ex! $f(x) = x^2$
$f(-x) = (-x)^2 = x^2 = f(x)$

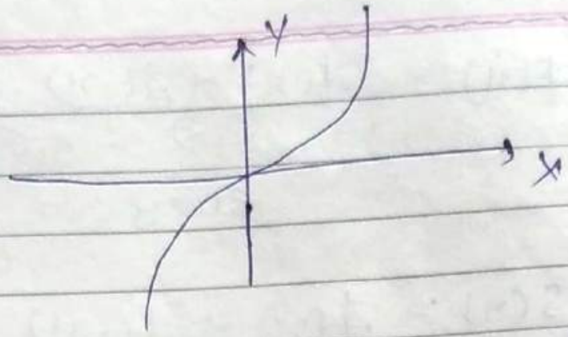


Odd: Example! $f(x) = \sin x$
$f(-x) = \sin(-x) = -\sin x = -f(x)$



$x = -x$.   even. symetry - y
c=even

$$f(-x) = (-x)^3$$
$$= -x^3$$
$$= -f(x)$$



Properties/Information: of even/odd function:

1) even function is symetrical about y axis.

2) odd function is symetrical about opposite cordi quadrates.

3) Some function ~~is~~ are neither even or odd

$$f(x) = \sin x + x^2$$
$$f(-x) = \sin(-x) + (-x)^2$$
$$= -\sin x + x^2 \quad f(x).$$
$$\ne -f(x)$$

4) A function which is both even aswell as odd then $f(x) = 0$.

5) constant function is always even ~~and odd~~ function.

6) every function can be expressed as sum of an even and odd function.

$$f(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{\text{Even}} + \underbrace{\frac{f(x)-f(-x)}{2}}_{\text{odd}}$$

$$S(x) = \frac{f(x)+f(-x)}{2}$$

$$S(-x) = \frac{f(-x)+f(x)}{2}$$

$$= S(x)$$

Even

$$P(-x) = \frac{f(-x)-f(x)}{2}$$

$$= -\frac{f(x)-f(-x)}{2}$$

$$= -P(x)$$

odd

$$2^x = \underbrace{\frac{2^x+2^{-x}}{2}}_{\text{Even}} + \underbrace{\frac{2^x-2^{-x}}{2}}_{\text{odd.}}$$

f	g	$f+g$	$f \cdot g$	$f \cdot g$	$\frac{f}{g}$	f^2	$fg(x)$	$gf(x)$
E	E	E	E	E	E	E	E	E
E	O	N	~~O~~N	O	O	E	E	E
O	E	N	N	O	O	N	E	E
O	O	O	O	E	E	E	O	O

N = Neither
E = Even
O = odd

$h(x) = f(x) - g(x)$

$h(-x) = f(-x) - g(-x) = f(x) - g(x) = h x$

$h_1(x) = f(x) / g(x)$

$h(-x) = f(-x) / g(-h)$

$= -f(x) / g(x)$

$= -h(x)$

$f(x) = g f(x)$

$h(-x) = g f(-x) \longrightarrow g(f(x))$

$= -g f(x)$

$= -h(x)$

$g f(x)$

$h(x)$

$\downarrow$

f(odd

$f(f(x))$.

$f(x) = \sin x - x^3 + \tan x$

$\uparrow \qquad \uparrow \qquad \uparrow$

$O \qquad O \qquad O$

$= \text{Odd}$

$\Delta \quad f(x) = x^2 - \cos x + x$ ~~$f(x)$~~ $\sin$

$\uparrow$

$E \qquad E \qquad O \times \text{Ⓞ}$

$E \times E \times E$

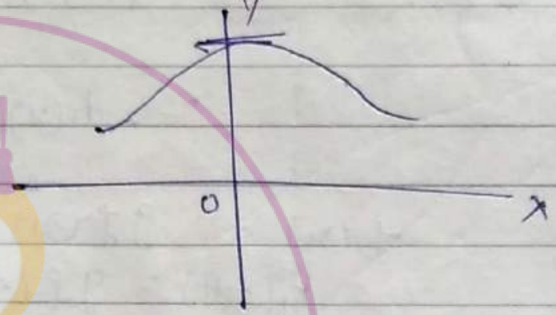
$= E$

* If $x=0$ lie in the domain of the odd function then $f(0)=0$

* $f(x) = f(-x)$

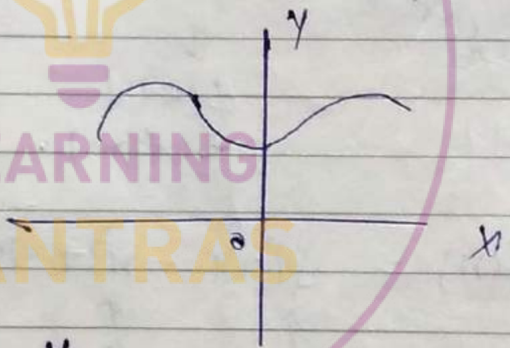
even f ⇒ 0 difference

* If even function is differencable at 0. the $\boxed{f'(0)=0}$

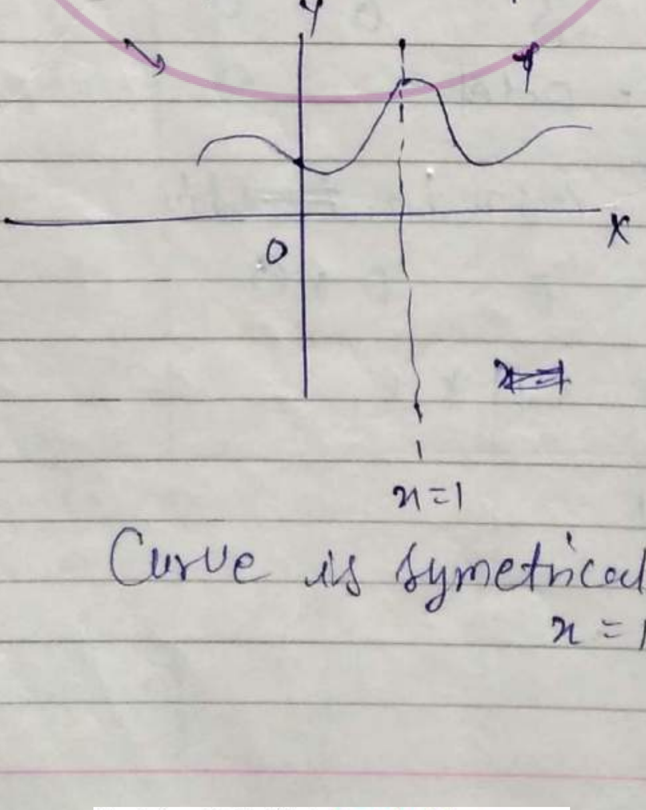


* $f(x) = f(-x)$

$f(0+x) = f(0-x)$



* $f(1-x) = f(1+x)$



Curve is symetrical about $x = 1$

$f(a-x) = f(a+x)$
then curve is symetrical about line $x=a$

M.

* Inverse function!

Let $F: A \rightarrow B$ be a one $\overset{one}{\text{and}}$ onto function.

then there exest unike function $g: B \rightarrow A$
such that $f(x) = y$
$$g(y) = 1 \quad \forall\ x \in (A, y \in B.$$
the g is called inverse of F.

$$F^{-1} = g: B \rightarrow A$$

* Method to find inverse:

Even function $y = f(x)$
(i) Solve for x.
(ii) Interchange x and y.
Hence we will get inverse of function.

Note! If f and g are inverse of each other then they will be mirror image of each other about line $y=x$ has mirror

x find

Find inverse.

i) $f(x) = 2x-1$

solve for x. as $y = 2x - 1 \longrightarrow$ $y = 2x - 1$

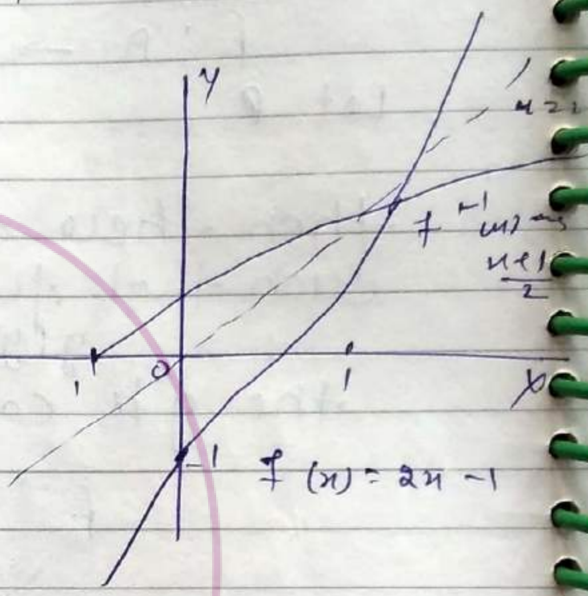
$2x = y + 1$ $\qquad$ $2x - y = 1$

$x = \frac{y+1}{2}$ $\qquad$ $\frac{x}{1/2} + \frac{y}{-1} = 1$

$y = \frac{x+1}{2} = 2y = x+1$

$-x + 2y = 1$

$\frac{x}{-1} + \frac{y}{1/2} = 1$

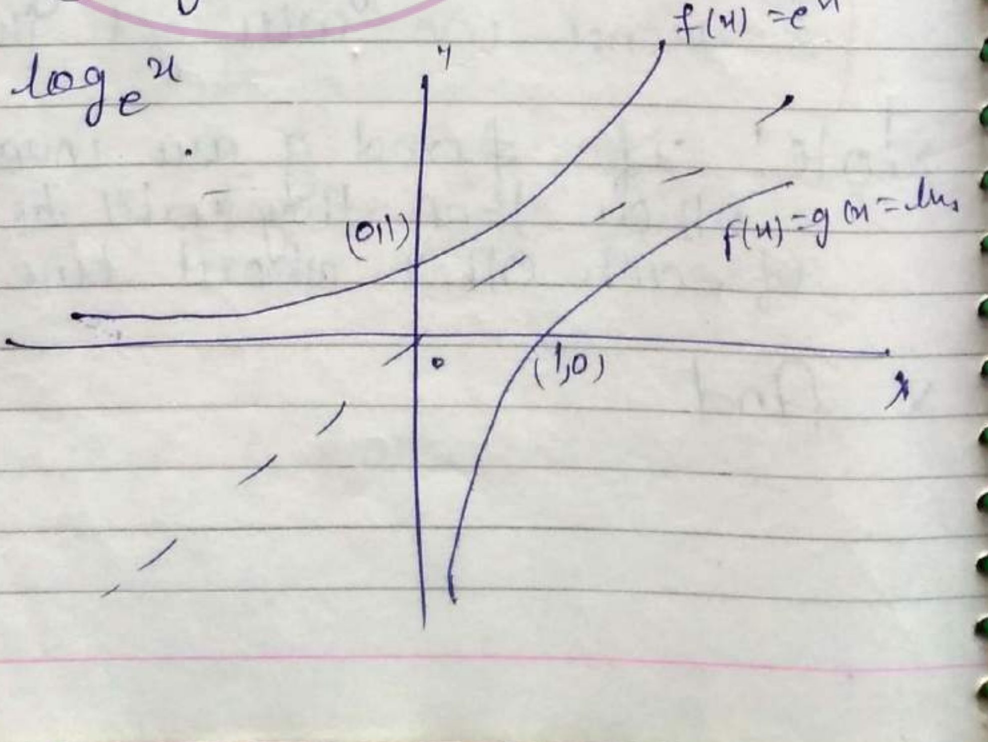


(ii) $f : R \to (0, \infty)$

$f(x) = e^x$ $\qquad$ $y = e^x$

① $\log y = \log_e e^x = x \log_e e = x$

② $y = \log_e x$



Sheet : S-1 : 11, 10, 13, 23, 25

$f : R \rightarrow (0, \infty)$

$g(x) = f(x) : (0, \infty) \rightarrow R$

$f : R \rightarrow R$

$g : R \rightarrow R$

$g(x) = f^{-1}(x) = \frac{x+1}{2}$

$f(x) = y \Rightarrow g(y) = x \quad \forall\ x \in A, y \in B$

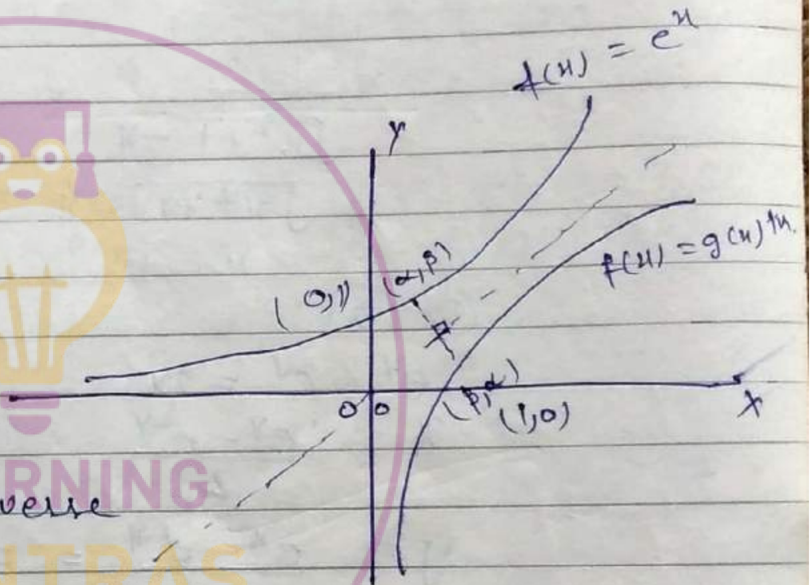
$f(0) = e^0 = 1$

$g(1) = \log_e 1$

$= 0$ Ans.

$f(\alpha) = \beta$

$g(\beta) = \alpha$



If ~~because~~ f(x) & are inverse each other.

If f and g are invers each other and $f(o)g(x) = x$

Date : 16/08/17   Ex : 5-1

Que: 13(i)

Sol: $y = \ln(\sqrt{x^2+1} + x)$

$\sqrt{x^2+1} + x = e^y$ ①

$= \frac{(\sqrt{x^2+1} + x)(\sqrt{x^2+1} - x)}{\sqrt{x^2+1} - x}$

$= \frac{1}{\sqrt{x^2+1} - x} = e^y$

$\therefore \sqrt{x^2+1} - x = e^{-y}$ ②

① − ②

$e^y - e^y = 2x$

$\therefore x = \frac{e^y - e^{-y}}{2}$

$y = \frac{e^x - e^{-y}}{2} = f^{-1}(x)$

23) $fff(x) = \begin{cases} 0 & x < \frac{7}{4} \\ 8x-14 & x \geq \frac{7}{4} \end{cases}$

$y =$

$y = x.$

(ii) (f) $f(x) = \frac{(1+2^x)^2}{2^x}$

$= \frac{1 + (2^x)^2 + 2 \cdot 2^x}{2^x}$

$= \frac{1}{2^x} + (2^x)^2 + 2$

$f(x) = 2^{-x} + 2^x + 2$ $f(-x) = f(x).$

⇒

* Differendation.

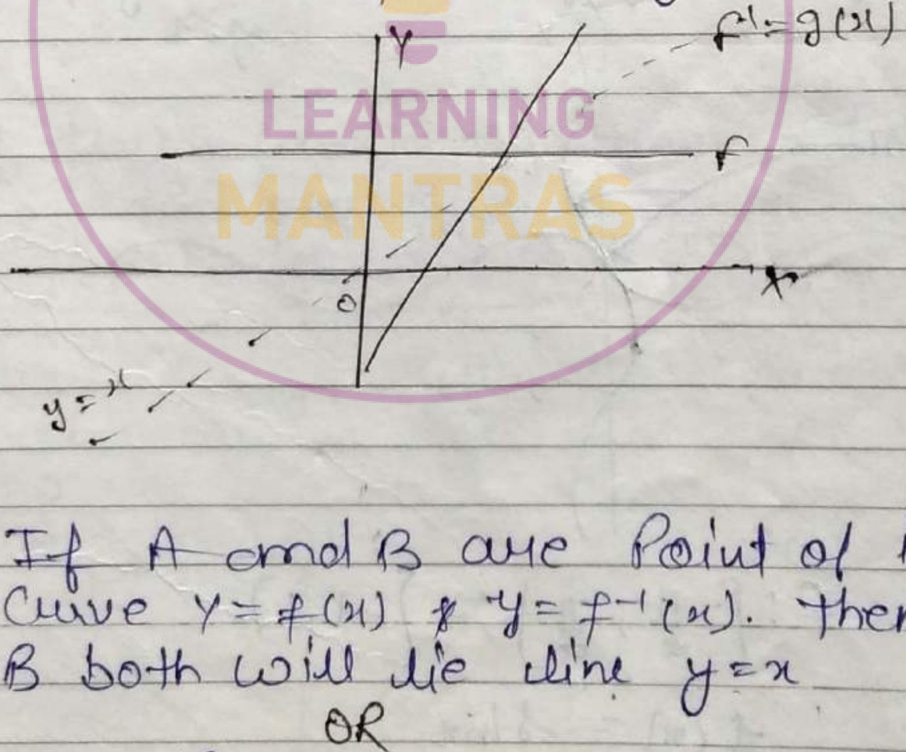
$f(g(x)) = x$
$f'(g(x)) \cdot g'(x) = 1$

If $g\,f(x) = x$
$g'(f(x)) \cdot f'(x) = 1$

* If f and g are inverse of each other then their composition is always an idendity function

* If f and g are inverse of each other then solution of $f(x) = g(x)$, or $f(x) = f(x)$ are solution of $f(x) = x$, $g(x) = x$.

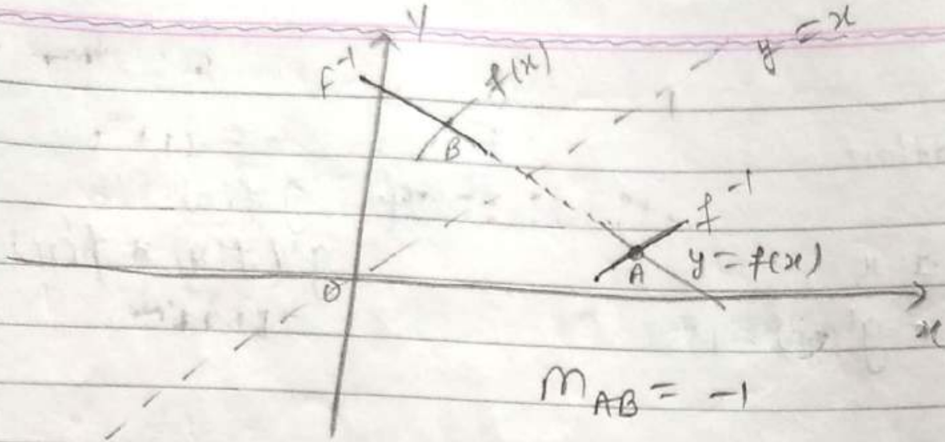
*



Note: If A and B are Point of intersectio of curve $y = f(x)$ & $y = f^{-1}(x)$. then A and B both will lie line $y = x$

OR

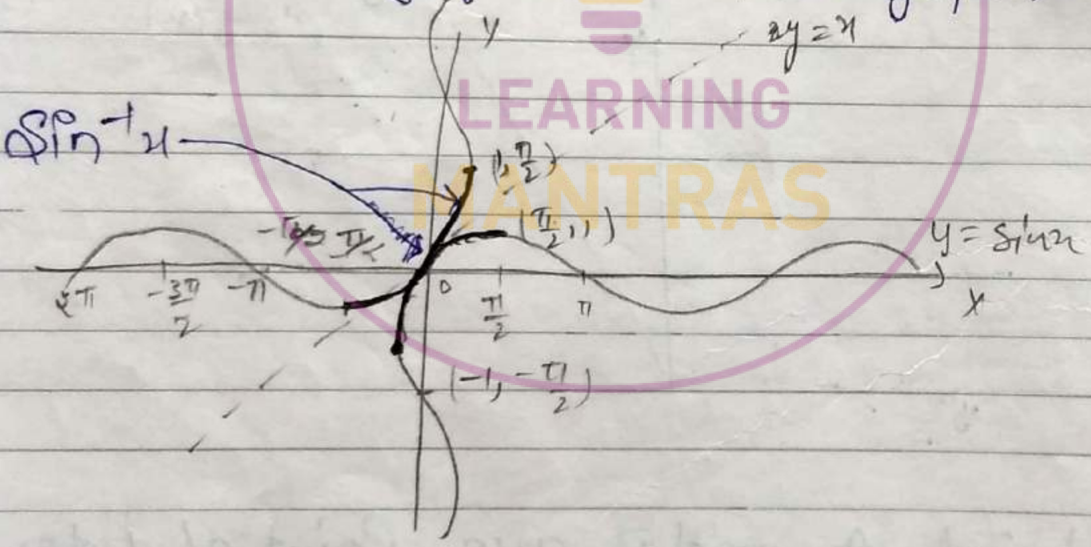
Slope of line AB $= -1$



$m_{AB} = -1$

* If x is Bijective function ~~if~~ the their inverse will also ~~to~~ be bijective.

* If $f : A \to B$
$f : B \to C$ are two Bijective function and their gof exist then $gof^{-1}(x) = f^{-1} o\, g^{-1}$



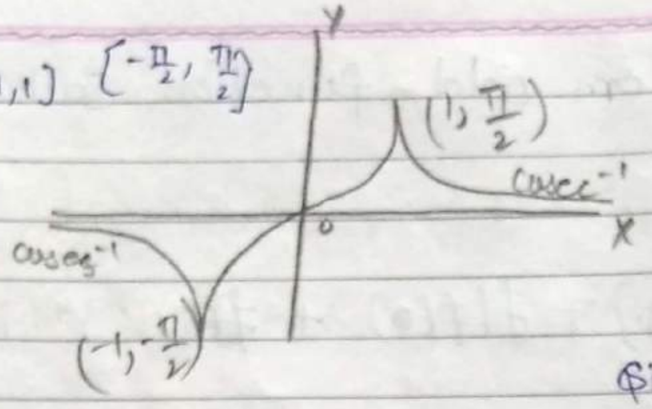
$$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$$

$$f(x) = \sin x$$

$$f^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$f^{-1}(x) = g(x) = \sin^{-1}$$

$y = \sin^{-1} [-1,1]$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$



D = ~~Range~~ R.

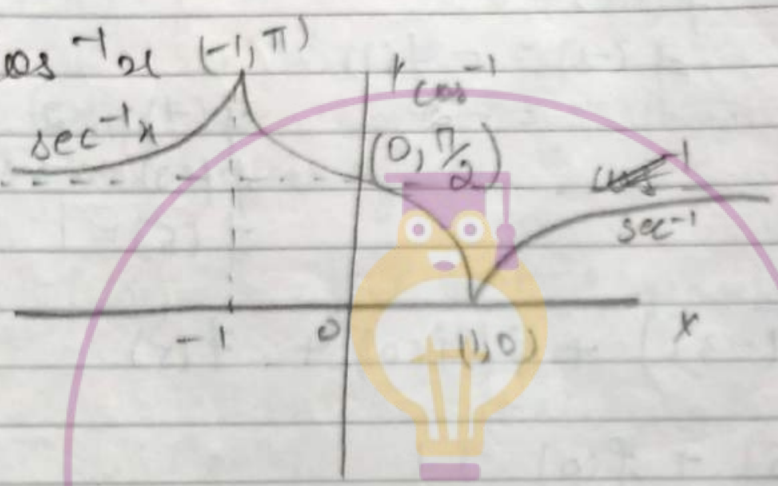
$y = \sin^{-1}x$

odd function.

bounded

$\sin^{-1}(-x) = -\sin^{-1}(x)$

$y = \cos^{-1}x$ $(-1, \pi)$



D = $[-1, 1]$

Range = $[0, \pi)$

Que: If $f(x) = (a-2)x + 3a - 4$ E/O

find Even odd.

$f(x) = (a-2)x + 3a - 4$

$a = 2$

$f(x) = 3a - 4$

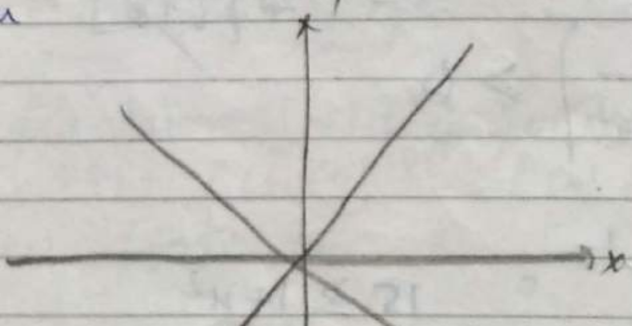
$= 2$

Even

$3a - 4 = 0$ $f(0) = 0$

$\Rightarrow a = \frac{4}{3}$.

$f(x) = (\frac{4}{3} - 2)x$



Ques!

② If f is an odd function such that

$f(1) = 2$

$f(3) = 5$

$f(5) = -1$

$\Rightarrow$ find $f(f(f(-3)] + f(f(0) + f(5)$

Ans! $-f(x) = f(-x)$

$f(1), \quad f(-1) = -f(1)$

$= -2$

$f(-1) = -2$

$f(-3) = -5$

$f(5) = 1$

$f(f(f(-3)) + f(f(0)) + f(5)$

$= f\, f(-5) + f(0) + 1$

$= F(-1) + f(0) + 1$

$= -2 + 0 + 1$

$= -1$

③ If $Sgn\left(\left[\frac{15}{1+x^2}\right]\right) = [1 + \{2x\}]$

$\Rightarrow$ find no. of Integral ~~no.~~ values of x satisfying above equation.

$= 1 + [\cancel{\{x\}}]^{0}$

$$\left[\frac{15}{1+x^2}\right] > 0$$

$\frac{15}{1+x^2} \geq 1 \quad = \quad 15 \geq 1+x^2$

$1 + x^2 \leq 15$

$x^2 \leq 14$

F odd = 0 $f(0) = 0$ even + odd = neither

$-\sqrt{14} \le x \le \sqrt{14}$

$x = \{-3, -2, -1, 0, 1, 2, 3\}$

Que! $f: [-4, 4] \to R$

$f(x) = \left[\frac{x^2}{a}\right] \sin x + \cos\theta$

If $f(x)$ is an even no. then find smallest integral value of a.

Ans! $f(-x) = \left[\frac{(-x)^2}{a}\right] \sin(-x) + \cos(-x)$

$f(x) = f(-x)$

$\left[\frac{(-x)^2}{a}\right] \sin + \cancel{\cos x} = -\left[\frac{(-x)^2}{a}\right] \sin + \cancel{\cos\theta}$

$2 \overset{0}{\cancel{\left[\frac{x^2}{a}\right]}} \sin x = 0$

$-4 \le x \le 4$

$0 \le x^2 \le 4$

$\left[\frac{\overset{16}{x^2}}{a}\right] = 0$

$a > 16$

Smallest integer = 17.

Que: A function satisfy functional equation

$f(x+y) + f(x-y) = 2f(x) \cdot f(y)$

when $f(0) = 0$

$x\, y \in R$.

then check h is even, odd, neither

~~odd~~ even $f(x) \neq 0$

even $f(-y) = f(y)$

Ans: $x = y = 0$ $\qquad$ $f(0) + f(0) = 2f(0)f(0)$

$f(0) + f(0) = 2f(0)f(0)$

$2f(0) = 2(f(0))^2$

$f(0) = 0$ or 1

$\therefore f(0) = 1$

$\searrow x = 0$

$f(y) + f(-y) = 2f(0)f(y)$

$= 2(f(y)$

$\underbrace{f(-y) = f(y)}_{\text{even function}}$

Que: $f : \left[\frac{3}{2}, \infty\right) \to \left[\frac{7}{4}, \infty\right)$

$f(x) = x^2 - 3x + 4$

then find its inverse.

$\frac{b \pm \sqrt{4ac}}{2}$

Ans. $x^2 - 3x + (4-y) = 0$

① $x = \frac{3 \pm \sqrt{9 - 4 \cdot 1(4-y)}}{2}$

$x = \frac{3 \pm \sqrt{4y - 7}}{2}$

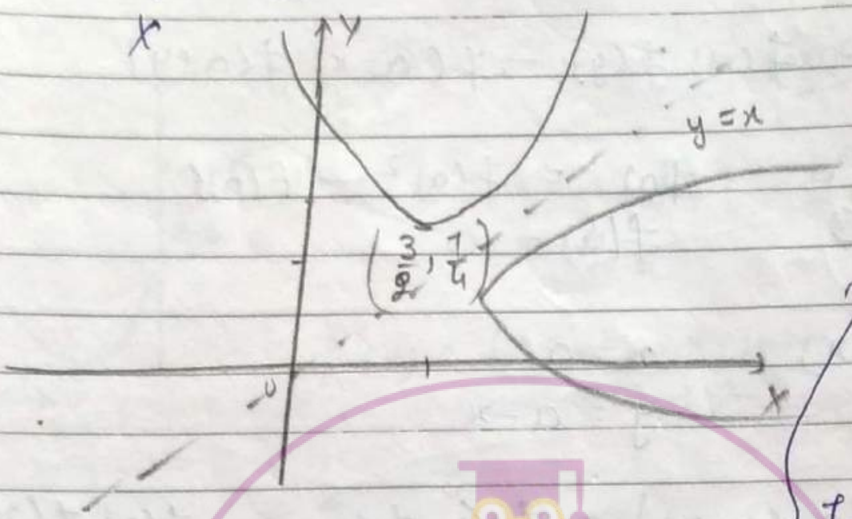
② Interchanging $y = \frac{3 \pm \sqrt{4x - 7}}{2}$

$f^{-1}(x) = \frac{3 - \sqrt{4x-7}}{2}$ $\qquad$ $\therefore f^{-1}(x) = \frac{3 + \sqrt{4x-7}}{2}$

$f^{-1}(x) : \left[\frac{7}{4}, \infty\right) \to \left[\frac{3}{2}, \infty\right)$ $\qquad$ $f^{-1}(x) : \left[\frac{7}{4}, \infty\right) \to \left[\frac{3}{2}, \infty\right)$

H.W S-1 : 14, 15 (a), 25
J.M : 1, 2, 3, 4, 5, 6, 7, 9, 10
J.A = 2, 3, 4,

$f^{-1}(x) = \frac{3}{2} - \frac{1}{2}\sqrt{4x-7}$ $= \frac{3}{2} + \frac{1}{2}\sqrt{4x-7}$ Ans



$f(x) = x^2 - 3x + 4$

$f'(x) = 2x - 3 = 0$

$x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} + 4$

$= \frac{9-18+16}{4} = \frac{7}{4}$

Que: Let g(x) be the inverse of f(x) and $f'(x) = \frac{1}{1+x^3}$

then Proof that $g'(x) = 1 + (g(x))^3$.

Ans!

$f(g(x)) = x$

$f'(g(x) \cdot g'(x) = 1$

$\frac{1}{1+g(x)^3} \cdot g'(x) = 1$

$g'(x) = 1 + g(f(x))^3$

Ans.

$g(f(x) = x$

$g'(f(x) \cdot f'(x) = 1$

$g'(f(x) \cdot \frac{1}{1+x^3} = 1$

$g'(f(x) = 1 + x^3$

$x \to g(x)$ $g'(f(g(x)) = 1 + (g(x))^3$

$g'(x) = 1 + g(x)^3$

Ans

Date:- 17/05/17   Ex ⇒ (JM)

Que:- 1

Ans: $f(x-y) = f(x)\,f(y) - f(a-x)\,f(a+y)$

$f(0) = 1$

$x = y = 0 \qquad f(a) = f(a)^2 - f(a)^2$

$f(a) = 0$

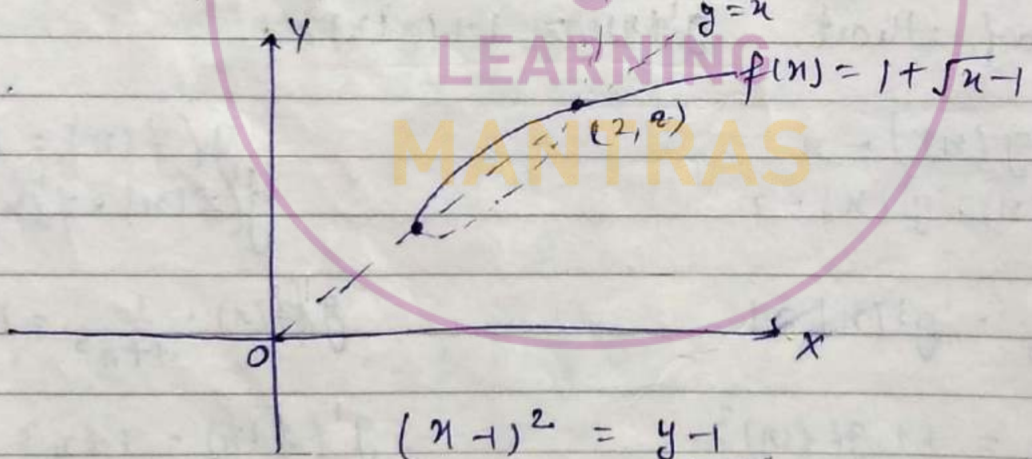
$f(2a-x) \qquad x = a$

$y = a-x$

$f(a-(a-x)) = \cancel{f(a)}^{0}\,f(a-x) - f(0)\,f(2a-x)$

$f(x) = \quad - f(2a-x)$

(7) $y = f(x) = (x-1)^2 + 1 \qquad x \geq 1$



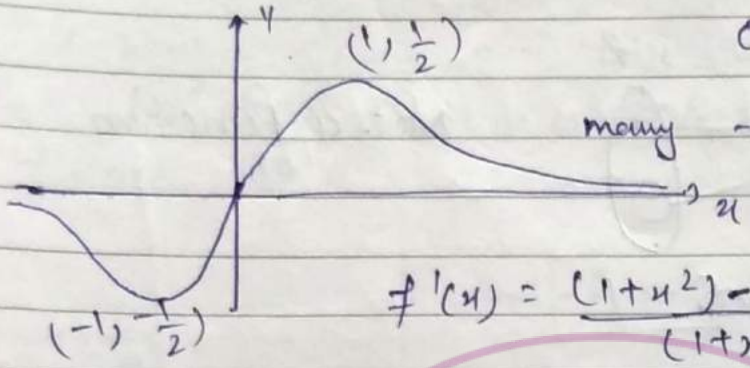
$(x-1)^2 = y-1$

$x-1 = \pm\sqrt{y-1}$

$x = 1 \pm \sqrt{y-1}$

$f^{-1} = 1 + \sqrt{y-1}$

10) $f(x) = \frac{x}{1+x^2}$



odd function

many -onto

$$f'(x) = \frac{(1+x^2) - x(2x)}{(1+x^2)^2} = 0$$

$$x = \pm 1$$

(8) $x - [x] = f$

$0 \le f < 1$

Ex = J. A

(4) $f(-1, 1) \longrightarrow R$

$$f(\cos 4\theta) = \frac{2}{2 - \frac{1}{\cos^2\theta}}$$

$$= \frac{2\cos^2\theta}{2\cos^2\theta - 1}$$

$$f(2\cos^2 - 1) = \frac{1 + \cos^2\theta}{\cos^2\theta}$$

$$2\cos^2\theta - 1 = \frac{1}{3}$$

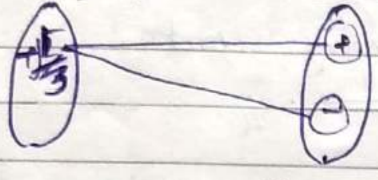
$$2\cos^2\theta = \frac{4}{3}$$

$$\cos^2\theta = \frac{2}{3}$$

$$\cos\theta = \pm\sqrt{\frac{2}{3}}$$

$$f\left(\frac{1}{3}\right) = \frac{1 \pm \sqrt{2/3}}{\pm \sqrt{2/3}} \begin{cases} + \\ - \end{cases}$$

$$f : (-1, 1) \longrightarrow R$$



Not a function.

Q−1

(19)

Ans: $f(x) = x(x+2)(x+4)(x+6) + 7, \; x \in (-4, 2)$

$$= (x^2 + 6x)(x^2 + 6x + 8) + 7$$

$x^2 + 6x = K$ $\quad = K(K+8) + 7$ $\quad$ K =

$$\boxed{f(K) = K^2 + 8K + 7}$$

$$K = x(x+6)$$

$$\frac{dy}{dx} = 2x + 6 = 0$$

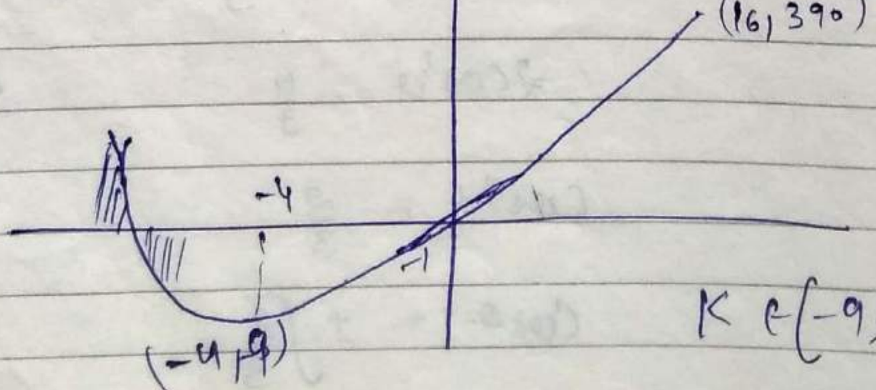
$$x = -3$$

$$f(K) = 2K + 8 = \quad K = -4$$

$$f(-4) = 16 - 32 + 7 = -9$$

$$K^2 + 7K + K + 7$$

$$= (K+7)(K+1)$$



$K \in (-9, 16)$

H.w ⇒ J.A ≠ 1
S.1 ⇒ } Fresh
0-1 ⇒

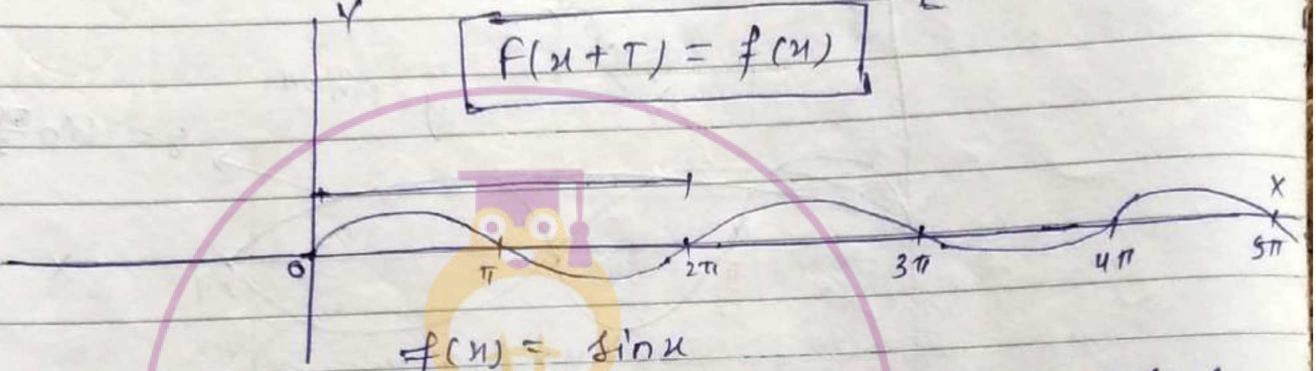
⇒

* Periodic function :-

function $y = f(x)$ is called Periodic function of Period T $(T>0)$, if functional satisfy $f(x+T) = f(x)$ ∀ $x \in$ in the domain of f where T is smallest Positive no. and independent from x.

[Period Repeat]

$$\boxed{f(x+T) = f(x)}$$



$f(x) = \sin x$

$f(2\pi + x) = \sin(2\pi + x) = \sin x = f(x)$

$f(4\pi + x) = f(x)$

Method - 2 :-

Suppose period of $\sin x = T$

$\therefore f(x+T) = f(x)$

$\sin(x+T) = \sin x$ → $n=0,\ x+T = 0 + x \Rightarrow T = 0$ ✗

$x + T = n\pi + (-1)^n x$ → $n=1,\ \cancel{x}+T = \pi - \cancel{x} = 2x + T = \pi$

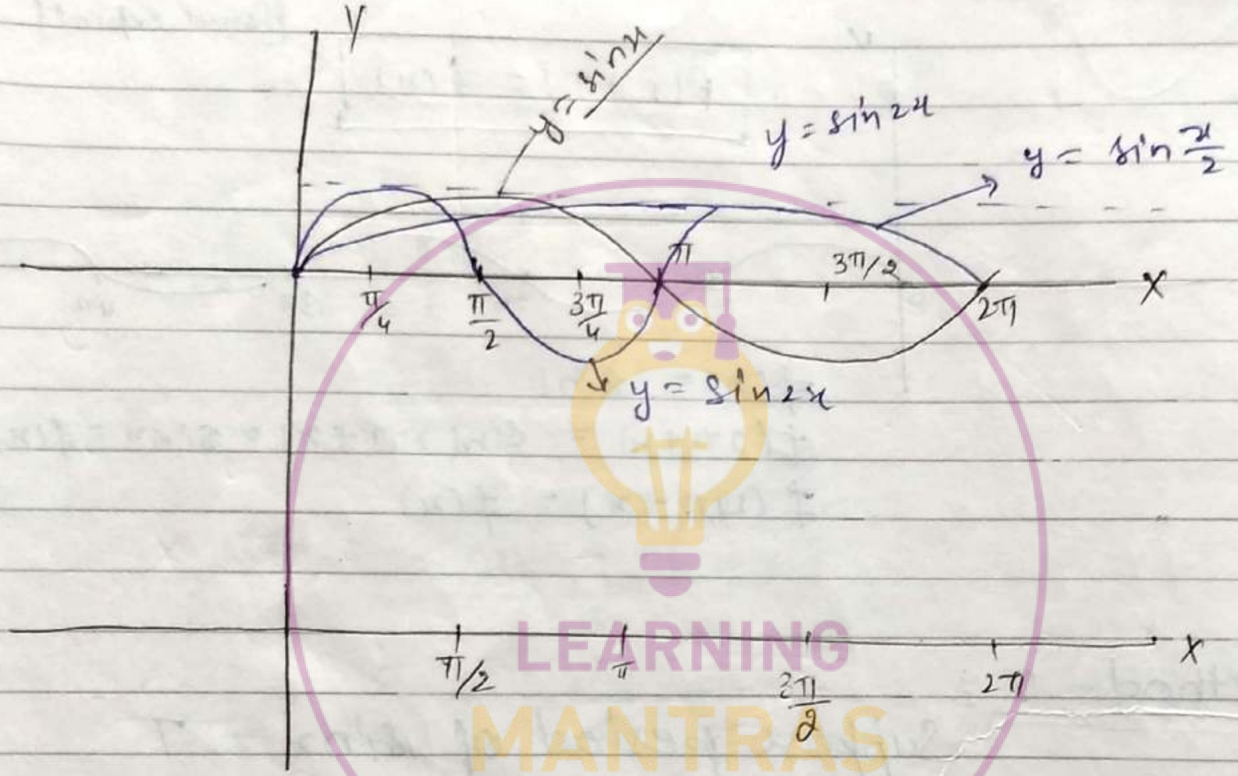
(∵ T is independent it 2π).

$n = 2,\ \cancel{x} + T = 2\pi + \cancel{x} \Rightarrow T = 2\pi$ Au

* Properties / Information :

1) if $y = f(x)$ is T

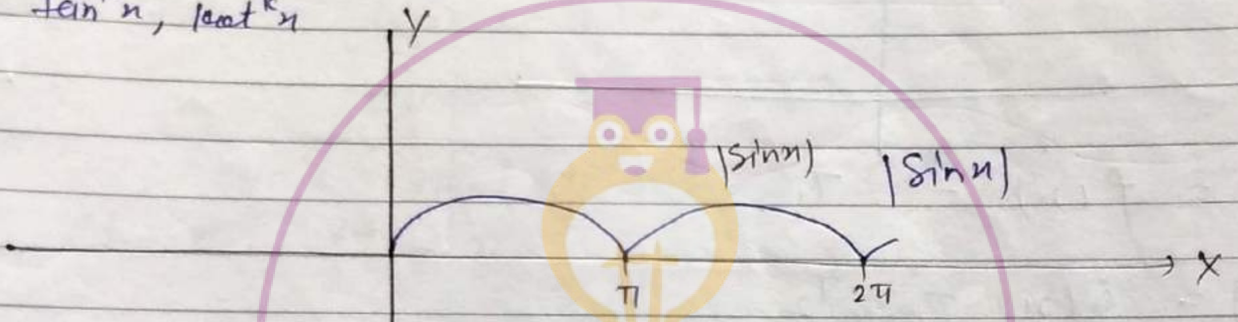
then $y = af(bx \pm c) \pm d$ is $\frac{T}{|b|}$

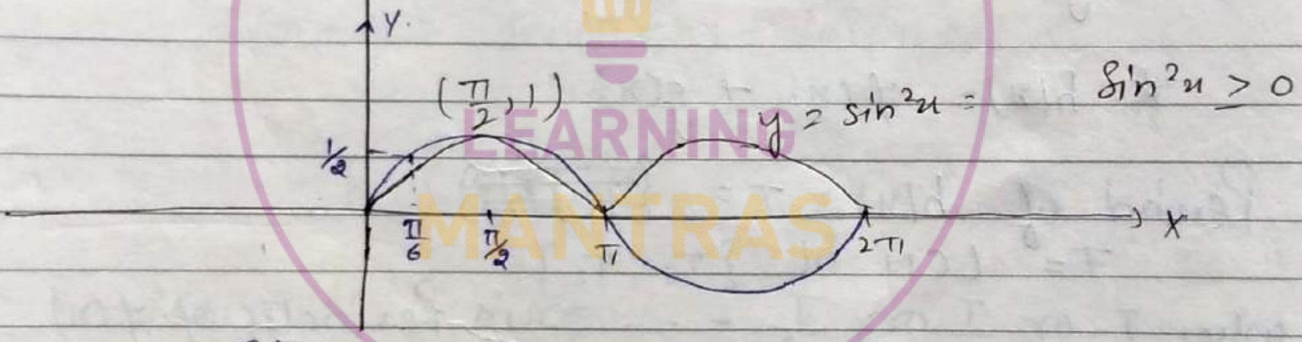


fn	Period
$\sin x$	2π
$3\sin x$	2π
$5\sin(x \pm 4) - 100$	2π
$\sin 2x$	$\frac{2\pi}{\|2\|} = \pi$
$\sin \frac{x}{2}$	$\frac{2\pi}{1/2} = 4\pi$

2)

f^n	Period
i) $\sin x$, $\cos x$, $\operatorname{cosec} x$, $\sec x$	2π
$\tan x$, $\cot x$	π
ii) $\lvert\sin x\rvert$, $\lvert\cos x\rvert$, $\lvert\tan x\rvert$, $\lvert\cot x\rvert$, $\lvert\sec x\rvert$, $\lvert\operatorname{cosec} x\rvert$,	$\xrightarrow{}$ π
iii) $\sin^k x$, $\operatorname{cosec}^k x$, $\sec^k x$, $\cos^k x$	$\to$ E = even = π $\to$ E = 0 Periodwise - 2π
$\tan^k x$, $\lvert\cot^k x$	





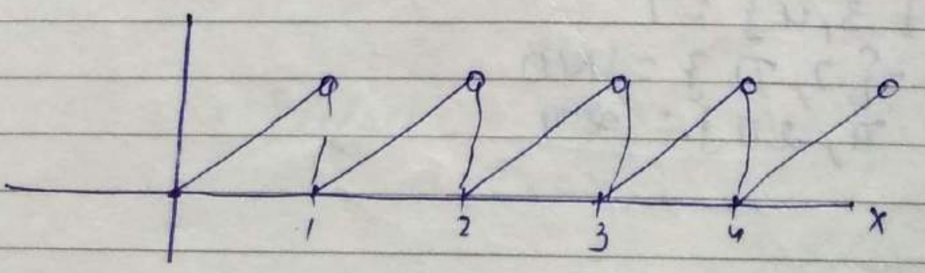
$\sin^2 x = \pi$

Si $\cos^4 x = \pi$

$\sin^3 x$ — 2π

$(\sin x)^{1/3} = 2\pi$

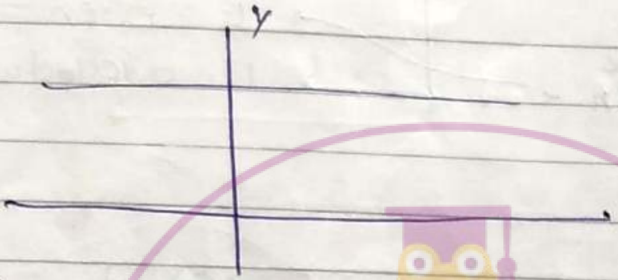
f^n	Period
(iv) $\{x\}$	1



(iv) Other alzebric function ——

Non - Periodic.

Constant function is Periodic function with no fundamental Periodic.



$y = f(x)$ —— T_1

$y = g(x)$ ——→ T_2

* $h(x) = f(x) + g(x)$

Period of $h(x) = T =$ ~~$\{T_1, T_2\}$~~

$T =$ LCM of $\{T_1, T_2\}$

when T or $\frac{T}{2}$ or $\frac{T}{3}$ - - = is Periodic of $f(x)$.

$LCM = \{2, 4\} = 4$

$HCF = \{2, 4\} = 2$

$LCM\ \{2, 4, 12\} = 12$

$HCF\ \{3, 4\} = 1$

$LCM - \{2, \pi\} = ND$

$LCM\ \{\pi, 2\pi\} = 2\pi$

$LCM = \left\{\frac{a}{b}, \frac{c}{d}, \frac{e}{f} - -\right\}$

$= \frac{LCM = \{a\ \ c\ \ e\}}{HCF = \{b, d, f\}}$

$$\text{LCM} = \left\{ \frac{2}{5}, \frac{6}{7} \right\}$$

$$= \frac{\text{LCM}\{2, 6\}}{\text{HCF}\{5, 7\}}$$

$$= \frac{6}{1}$$

Ex: $f(x) = \underset{\overset{\downarrow}{2\pi}}{\sin x} + \underset{\overset{\downarrow}{2\pi}}{\cos x}$

$$T = \text{LCM}\{2\pi, 2\pi\} = 2\pi$$

Check, $\frac{2\pi}{2} = \pi$

$$f(\pi + x) = \sin(\pi + x) + \cos(\pi + x)$$
$$= -\sin x - \cos x$$
$$\neq f(x)$$

Period $= 2\pi$

$\Rightarrow$ $f(x) = \underset{\overset{\downarrow}{\pi}}{|\sin x|} + \underset{\overset{\downarrow}{\pi}}{|\cos x|}$

$$T = \text{LCM}(\pi, \pi) = \pi$$

Check, $\pi/2$ $\quad f\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2} + x\right)| + \left|\cos\left(\frac{\pi}{2} + x\right)\right|$

$$= |\cos x| + |\sin x|$$

$$= f(x)$$

Period $= \pi/2$

(iii) $f(x) = |\sin x| - |\cos x|$

$T = LCM \{\pi, \pi\} = \pi$

Check $\frac{\pi}{2}$

$$f(\tfrac{\pi}{2}+x) = |\sin(\tfrac{\pi}{2}+x)| - |\cos(\tfrac{\pi}{2}+x)|$$

$$= |\cos x| - |\sin x|$$

$$= -(f(x)$$

$$\neq f(x)$$

Period $= \pi$.

(iv) $$h(x) = \frac{|\sin x| + |\cos x| \longleftarrow \pi/2}{|\sin| - |\cos x| \longleftarrow \pi}$$

$$T = LCM \left\{\frac{\pi}{2}, \pi\right\}$$

$$= \frac{LCM\{\pi, \pi\}}{HCF\{2, 1\}} = \frac{\pi}{1}$$

check $\frac{\pi}{2} \longrightarrow$ failed. $\therefore T = \pi$

(v) $f(x) = \sin \frac{3\pi}{2} + \cos \frac{5\pi}{3}$

$$T = \frac{2\pi}{3/2} = \frac{4\pi}{3}, \quad T_2 = \frac{2\pi}{5/3} = \frac{6\pi}{5}$$

$$T = LCM \left\{\frac{4\pi}{3}, \frac{6\pi}{5}\right\}$$

$$= \frac{LCM(4\pi, 6\pi)}{HCF\{3, 5\}} = \frac{12\pi}{1}$$

Cheack not

(vi) $f(x) = \sin x + \{x\}$

$\downarrow \quad\quad \downarrow$

$2\pi \quad\quad 1$

$T = \text{LCM}\{2\pi, 1\}$

$= \text{ND}.$

Non-periodic → aperiodic

* $f(x) = \{x\} + \{x + \frac{1}{2}\}$

$T_1 = 1 \quad T_2 = 1$

$T = \text{LCM} = \{1, 1\} = 1$

$f(x + \frac{1}{2}) = \{x + \frac{1}{2}\} + \{x + \frac{1}{2} + \frac{1}{2}\}$

$= \{x + \frac{1}{2}\} + \{x + 1\}$

$= \{x\} + \{x + \frac{1}{2}\}$

$= f(x)$

$T = \frac{1}{2}$

* $f(x) = \{x\} + \{x + \frac{1}{3}\} + \{x + \frac{2}{3}\}$

$T = \frac{1}{3}$

$f(x + \frac{1}{3}) = f(x)$

$T = \frac{1}{3}$

(vi) $f(x) = \underset{\underset{2\pi}{\downarrow}}{\sin x} + \underset{\underset{1}{\downarrow}}{\{x\}}$

$T = LCM\ \{2\pi, 1\}$

$= NO.$

Non-periodic → aperiodic

* $f(x) = \{x\} + \{x + \frac{1}{2}\}$

$T_1 = 1 \quad T_2 = 1$

$T = LCM = \{1, 1\} = 1$

$f(x + \frac{1}{2}) = \{x + \frac{1}{2}) + \{x + \frac{1}{2} + \frac{1}{2}\}$

$= \{x + \frac{1}{2}\} + \{x + 1\}$

$= \{x\} + \{x + \frac{1}{2}\}$

$= f(x)$

$T = \frac{1}{2}$

* $f(x) = \{x\} + \{x + \frac{1}{3}\} + \{x + \frac{2}{3}\}$

$T = \frac{1}{3}$

$f(x + \frac{1}{3}) = f(x)$

$T = \frac{1}{3}$

Difference = 2X

Qu: $f(x) + f(x+a) = 0$ — ①

then Period of $y = f(x) = 2a$
$T = 2a$

Proof: $x \to x+a$

$f(x+a) + f(x+2a) = 0$ — ②

① − ② $f(x) = f(\underset{\substack{\downarrow \\ x}}{x} + \underset{\substack{\uparrow \\ T}}{2a})$

$$\boxed{T = 2a}$$

Que! ~~$f(x+4)$~~

$\overset{8}{\frown}$ difference = 2X.

* if $f(x-1) + f(x+7) =$

$T = 16$

$$f(x+4) + f(x-4) = f(x) \quad \text{— ①}$$

Que! $\overbrace{f(\underbrace{x+4) + f(x}_{8}\underbrace{-4) = f(x}_{4})}^{4}$ — ①

$x \to$ ~~f~~

minimum Jump = 4

$x \to x+4$

$f(x+8) + f(x) = f(x+4)$ — ②

①+② $\underbrace{f(x+8) + f(x-4)}_{12 \times 2} = 0$ — [illegible]

$T = 24$

Even: *

Que: Q-1, Que: {24, 26}
0-1

(30) $y = \min\{\sin, |x|\} + \frac{x}{\pi} - \left\{\frac{x}{\pi}\right\}$

$= \sin x + \left\{\frac{x}{\pi}\right\}$

$\downarrow \qquad \downarrow$

$T_1 = 2\pi \qquad T_2 = \frac{1}{1/\pi} = \pi$

LCM $\{2\pi, \pi\}$ $= 2\pi$ Ans

(24) $f(x) = 2^x - 1 \quad 0 \le x < 2$

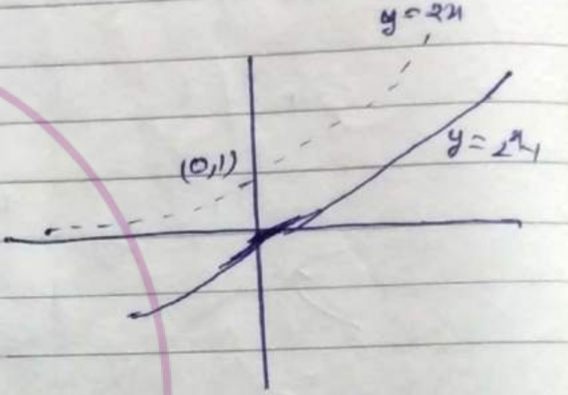
$f(x) = 1$

f Even, Periodic

$T = 4$

$f(x) = 2^x - 1 \quad 0 \le x < 2$

$(-2, 6)$



(26) $f(x) + f(x+3) + f(x+6) \;---\; f(x+42) = \lambda$

$f(x+3) + f(x+6) \;----\; f(x+45] = \lambda$

$f(x) - f(x+45] = 0$

$f(x+45] = 5(x)$

$T = 45$