Handwritten Notes
On
Error Analysis

* Metrology or Error Analysis:

Metrology

It is the science of measurement and finding error in measurement.

*** Measurement:**

(1) it is the process of determination of a value of a Physical Quantity experimently with the help of measuring instruments.

(2) The result of measurement must be a no. with Proper unit The purpose of measurement is to represent the property of object by a number and to find true value of the quantity.

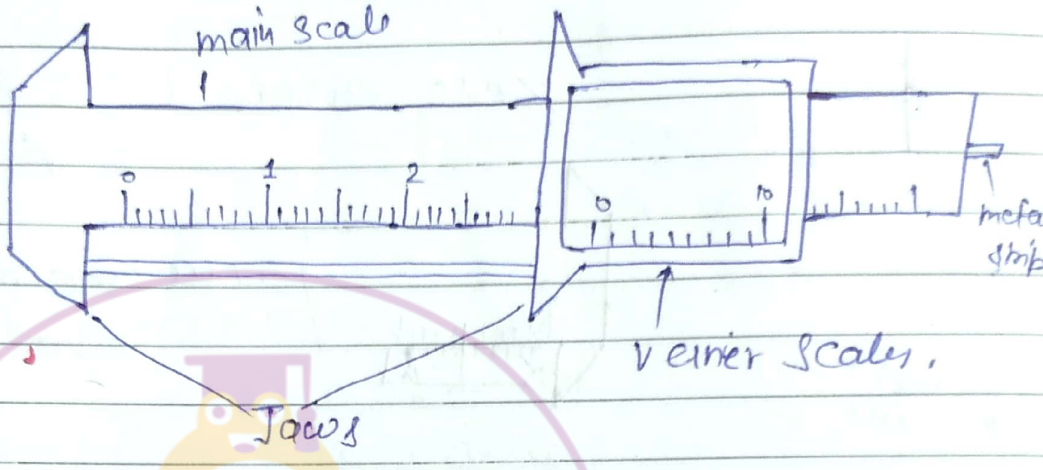
(i) True value of all the Quantities exist
(ii) True value is constant.
(iii)* True value can never be found.

*** Least Count:** It is the minimum value that can be measured accurately by an instrument. for a normal scale it is 1mm

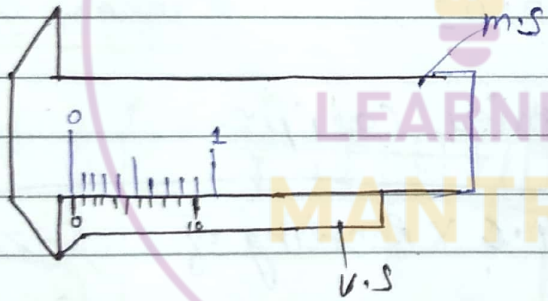
Scale 1	Scale 2	Instrument
10mm	20 mm	10 mm
30 mm	100 mm	10 mm
1 mm	10 mm	1 mm
4 mm	10 mm	2mm
0·9 mm	1 m	0·1 mm
0·8 mm	7 mm	0·2 mm.

* Measurement of Length :

① Vernier Calipers :



To find the value of 1 division on vernier scale jaws are made in contact.



1 division on m.s = 1mm
L.C of m.s = 1mm.

if 10th div. on v.s Considered to 9th div. on m.s

10 div. on v.s = 9 div on m.s
10 div on v.s = 9 mm
1 div on v.s = 0.9 mm

L.C of v.s = 0.9 mm
" " m.s = 1mm

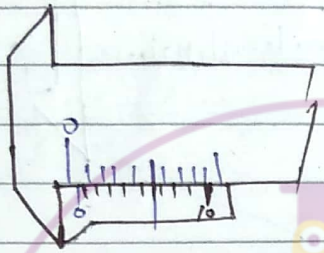
L.C of vernier calipers : 0.1 mm
= 0.01 cm

* zero error :

When jaws are made in contact and zero of both the scale do not coinside the Calipers is said to have zero error.

zero error = L.C × Reading of V.S

= 0.6 cm.



+ve zero error

-ve zero error = -L.C zero order error

= -0.01 × 2

= -0.02

Measure length by vernier calipers

$$L = \text{Reading of M.S} + (\text{L.C} \times \text{Reading of V.S}) \mp \text{zero error}$$

(- zero add, + zero sub)

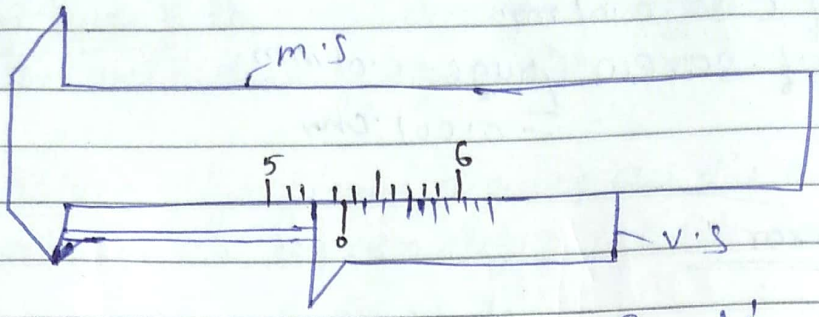
Q. Smallest division on main scale of a V. calipers is 1 mm. and 10 div. on vernier scale = 8 division on main scale when jaws are made in contact zero of vernie scale lies to the right of zero of main scale and 7th division on V.S considered to a division on main scale.

L.C = 0.02

when a rod is placed b/w the jaws zero of vernier scale lies b/w 5.3 and 5.4 on main scale and 3rd division on V.S considered to a division on M.S. find length of the rod.

Vernier Caliper

Sol:



5.3 = Reading of M.S

3 — Reading of V.S

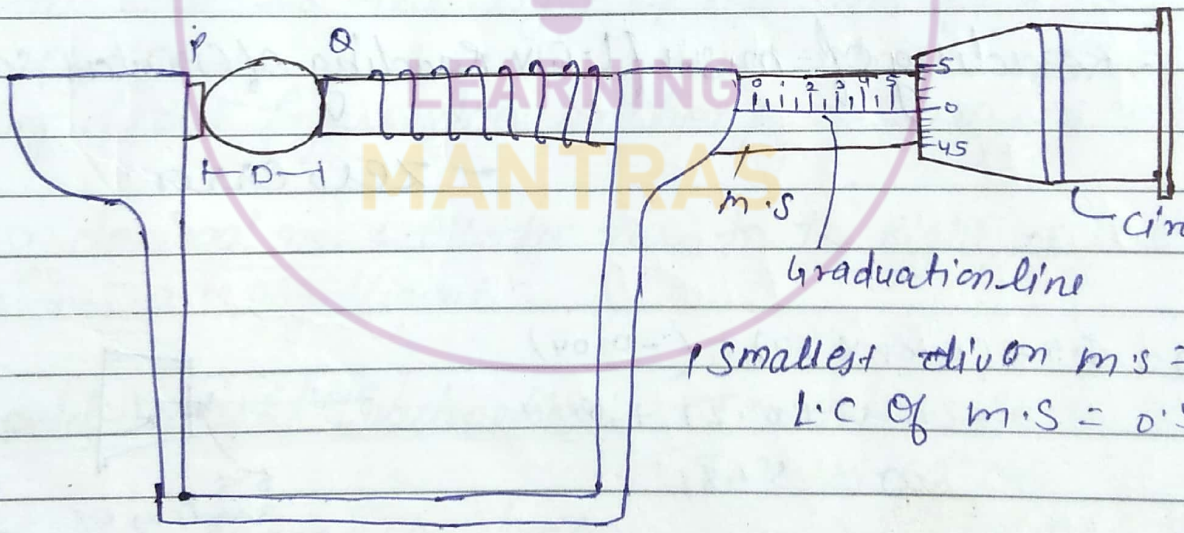
$= 5.3 + (0.02 \times 3) - 0.14$

$= 5.3 + 0.06 - .14$

$= 5.36 - 0.14$

$= 5.22$ cm

* Screw Gauge :



Smallest division M.S = 0.5 m

L.C of M.S = 0.5 mm

Pitch. It is the distance moved by Circular scale on main scale in one Rotation

$d = 0.5$ mm

50 division on C.S = 0.5 mm

1 division on C.S $= \frac{0.5}{50} = 0.001$ mm

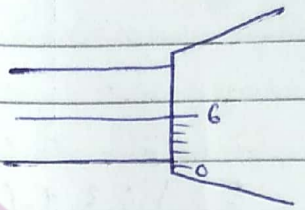
L.C of C.S = 0.01 mm

L.C of screw Gauge = 0.01 mm

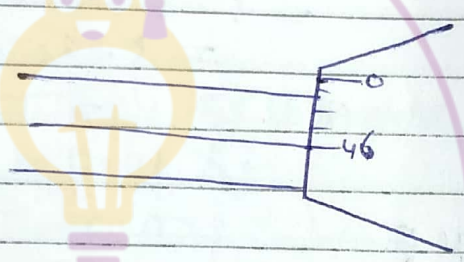
= 0.001 cm.

Zero Error :

+ve zero error = L.C × 6.



-zero error = -(L.C × 4)



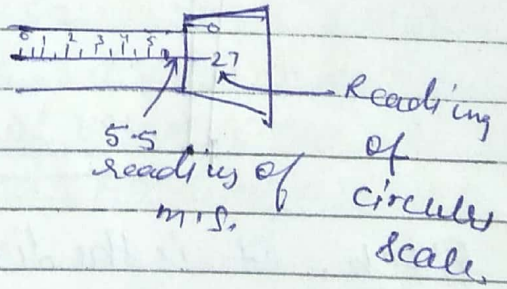
D = Reading of m.s + (L.C × Reading of circular scale)

- zero error.

D = 5.5 + (0.01 × 27) - (-0.04)

= 5.5 + 0.27 + 0.04

D = 5.81



* Significent figures :- In a no. significant figures are the no. of digits which are statistaly significant.

* for measured Value :- Significant figures are the no. of digits which are accurate + one more digit which have uncertainity

Vaunier & Callipey (L.C) = 0.01 cm.

Accurate, uncertain.

$d = \underbrace{8.23}_{\text{S.f}} 7.62$

***

x Rule for S.f in Calculated or computed value :-

1) all the non-zero no. are significant figure.
2) all the zero b/w two non-zero no. are significant.
3) all the zero to the right of last non-zero no. are not significant.
4) S.f does not change on changing unit.

x for a decimal no. :- All the zero to the Right of Last non-zero no. are significant

* for addition and Substraction :- $T_i = 20.1234°C$

$T_f = 46.65°C$

66.65

find change in temp.

$T_f - T_i = 26.5266°C.$

$= 26.53°C$

* division & Product :- $s = 3.6\,m$, $t = 1.2000\,sec$ } least S.f

$v = \frac{s}{t} = \frac{3.6}{1.2000} = 3\,m/s$ ✗

$= 3.0\,m/s$

* **Accuracy:** If experimental result are very close to the true value of Quantity. Results are called accurate

* **Precision:** if experimental result are very close to each other but may not be close to the true value. results are called precisies.

* **Error:** It is the slight different b/w measured value and true value.

① **Systematic error:** It has definate dirn and magnitude
2) It has an assignable cause. (like zero error).
3) It can be eliminated.
4) It can not be found by repeating the experiment with same instrument.
5) It occur due to faulty instrument, wrong procesion or taking wrong standard values

2) **Random errors:** These error occurs randomly do not have fixed sign and size can not be eliminated because their cause is not known. Can can be minimised by repeating experiment several times and taking average value. It occurs due to external factors as well as human mistake (like Rxn time)

• more the accuracy less is the systemic error and more the Random errors less is the Precision

error is always added

* **Least count error:** maximum error by an instrument = its least count.

* **absolute error:** It is the difference b/w measured value and true value X_0 = true value X = measured value

$$\Delta X = |X_0 - X| \quad , \Rightarrow X_0 \sim X$$

* **mean or average absolute error:**

$x_1, x_2, x_3 \dots x_n$

$$X_0 = \frac{x_1 + x_2 + x_3 \dots x_n}{n}$$

$$\Delta x_1 = X_0 - x_1$$
$$\Delta x_2 = X_0 - x_2$$
$$\vdots$$
$$\Delta x_n = X_0 - x_n$$

$$\overline{\Delta x} = \frac{\Delta x_1 + \Delta x_2 + \dots \Delta x_n}{n}$$

* **fractional or Relative Error:**

$$= \frac{\text{absolute error}}{\text{true value}} = \frac{\Delta x}{x_0}$$

$$\frac{\Delta x}{x_0} \times 100 = \%\ \text{Error}$$

* **Propagation of Errors:**

(1) $R = x \pm y \quad \Delta R = \Delta x + \Delta y$

(2) $R = xy \quad \frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$

(3) $R = \frac{x}{y} \quad \frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$

(4) $R = x^q \quad \frac{\Delta R}{R} = a\frac{\Delta x}{x}$

$R = x + y$

$R = xy$

$dR = dxy + xdy$

$\frac{dR}{R} = \frac{dx}{x} + \frac{dy}{y}$

$\frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$

(iv) $R = \frac{x^a y^b}{z^c}$ $\quad \frac{\Delta R}{R} = a\frac{\Delta x}{x} + b\frac{\Delta y}{y} + c\frac{\Delta z}{z}$

$R = x^a$

$\ln R = a \ln x$

$\frac{dR}{R} = a\frac{dx}{x}$

$R = x^3$

(v) $\frac{1}{R} = \frac{1}{x} + \frac{1}{y}$

$-\frac{dR}{R^2} = -\frac{dx}{x^2} - \frac{dy}{y^2}$

$$\boxed{\frac{\Delta R}{R^2} = \frac{\Delta x}{x^2} + \frac{\Delta y}{y^2}}$$

Qu: $T_i = (42.36 \pm 0.34)^\circ C$

$T_f = (78.6 \pm 0.7)^\circ C$ — find change in temp.

$\Delta T = \Delta T_f + \Delta T_i$

$\Delta T = T_f - T_i = \quad = \Delta T = 1.04$

$= 78.6 - 42.36 = 36.24 = 36.2$

$T = (36.2 \pm 1.04)^\circ C$

Optimistic – $(36.2 \pm 1.0)^\circ C$

Pessimistic – $(36.2 \pm 1.1)^\circ C$

Q An object travel (40.0 ± 0.4) m in (5.0 ± 0.5) sec. find average speed of object

$\frac{40.0}{2.0} = 22.5$ $\quad 0.4 + 0.5 = 0.9$

(22.5 ± 0.9)

$V = SR$ $R =$

$V = \frac{S}{T} = \frac{40.0}{5.0} = 8.0$ ✓

$= 8 \times$

$\frac{\Delta V}{V} = \frac{\Delta S}{S} + \frac{\Delta t}{t}$

$= \frac{0.4}{40.0} + \frac{0.5}{5.0} = 0.01 + 0.1$

$\frac{\Delta V}{V} = 0.11$ $\Delta V = 0.11 \times 8.00$

$\Delta V = 0.88$

$V = (8.0 \pm 0.88)$

$= (8.0 \pm 0.9)$

% Error in speed = $0.11 \times 100 = 11\%$

Que: Potential difference across a Resistor is (12.0 ± 0.6) volt and Current (3.0 ± 0.3) A. Find R. of the coil

$V = IR$ $= R = \frac{V}{I}$

$R = \frac{V}{I} = \frac{12.0}{3.0} = 4.0$

$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

~~$\frac{\Delta R}{R} = 0.6 + 0.3$~~

$\frac{\Delta R}{R} = 0.9$

~~$R = (4.0 \pm 0.9)$~~

$R = (4.0 \pm 0.16)$

$\Delta R =$

$\frac{\Delta R}{4.0} = \frac{0.6}{12.0} + \frac{0.3}{3.0}$

$= 0.05 + 0.1$

$\frac{\Delta R}{R} = 0.15$

$\Delta R = 0.6$

Q. $R_1 = (6.0 \pm 0.6)\ \Omega$

$R_2 = (12.0 \pm 0.3)\ \Omega$

Find equivalent Resistance

(a) in Series Combination

$R = R_1 + R_2$

$= 6.0 + 12.0 = 18.0 = 18.0$

$= (18.0 \pm 0.9)\ \Omega$

(ii) in Parallel Comb.

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R} = \frac{1}{6.0} + \frac{1}{12.0} = \frac{2.0 + 1.0}{12.0} = \frac{3.0}{12.0} = \frac{1}{4.0}$

$R = 4.0$

$\frac{R}{\Delta R} + \frac{R_1}{\Delta R_1} + \frac{R_2}{\Delta R_2} = \frac{4.0}{\Delta R} = \frac{6.0}{0.6} + \frac{12.0}{0.3}$

$= \frac{4.0}{\Delta R} = 0.5$

$\Delta R = \frac{4.0}{0.5}$

$\Delta R = \frac{40}{5} = 8.0$

$\frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

$\frac{\Delta R}{(4.0)^2} = \frac{0.6}{(6.0)^2} + \frac{0.3}{(12.0)^2}$

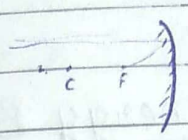
$\Delta R = 0.29$

(4.0 ± 0.29)

Q. A Convex lens form Real Image of a real object given. find focal length of lens

$u = (20.0 \pm 4.0)$ cm

$v = (60.0 \pm 3.6)$ cm



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{20.0} + \frac{1}{60.0} = \frac{3+1}{60.0} = \frac{4}{60.0} = \frac{1}{15.0}$$

$f = 15.0$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{\Delta u^2}$$

$$\frac{\Delta f}{225} = \frac{3.6}{3600} + \frac{4.0}{400}$$

$$\Delta f = 225\,(0.0001 + 0.001)$$

$$\Delta f = 225(.0011)$$

$$\Delta f = 0.2475$$

$$\Delta f = 0.25$$

(15.0 ± 0.2) or (15.0 ± 0.3).

Q. $y = \sqrt{x}$

if $x = 100 \pm 6$

Find y.

$y = \sqrt{100}$

$= 10$

$$\frac{\Delta y}{y} = \frac{1}{2}\frac{\Delta x}{x}$$

$$= \frac{1}{2} \times \frac{6}{100} \times 3$$

$y = (10 \pm 0.3)$ X

$= (10.0 \pm 0.3)$ ✓

$$\frac{\Delta y}{10} = 0.03$$

$\Delta y = 0.3$