Handwritten Notes

On

Ellipse

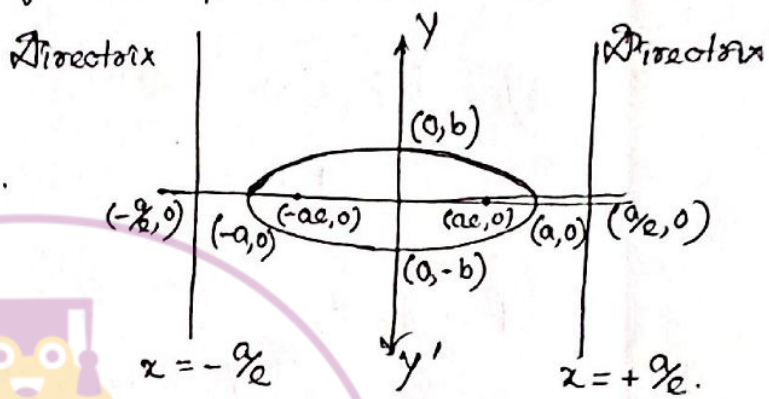
Ellipse

* Ellipse: Locus of a point which moves on a plane such that its distance from a fixed point is a constant ratio from a fixed line & this ratio, $e < 1$.

* Standard Equation of Ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

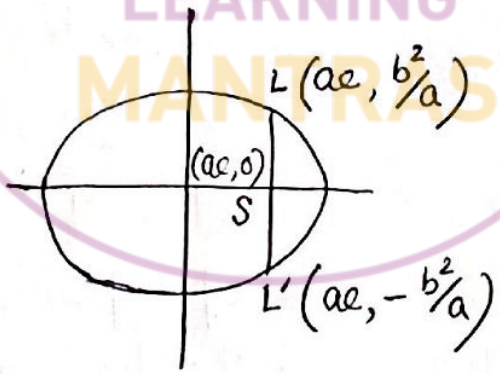
where $b^2 = a^2(1 - e^2)$.



* Length of Latus rectum:

i) for ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $LR = \frac{2b^2}{a}$.

ii) for ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \left(0 < a < b\right)$, $LR = \frac{2a^2}{b}$.



* If P is a point on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci S & S' then $PS + PS' = 2a = \text{Constant}$.

* Let $P(x_1, y_1)$ be a point & $S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ be an ellipse, then i) P lies on ellipse $\Leftrightarrow S_1 = 0$

ii) P lies inside the ellipse $\Leftrightarrow S_1 < 0$,

iii) P lies outside ellipse $\Leftrightarrow S_1 > 0$.

* The equation of tangent to the ellipse $S = 0$ at $P(x_1, y_1)$ is $S_1 = 0$. ie $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.

* The equation of normal to the ellipse $S = 0$ at $P(x_1, y_1)$ is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$.

* The condition that the line $y = mx + c$ may be a tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $c^2 = a^2m^2 + b^2$.

• Eqn of tangent — m form —

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

point of contact $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$.

* Two tangents can be drawn to an ellipse from an external point.

* Director Circle: Points of intersection of perpendicular tangents to an ellipse $S = 0$ lies on a circle, concentric with the ellipse, called director circle.

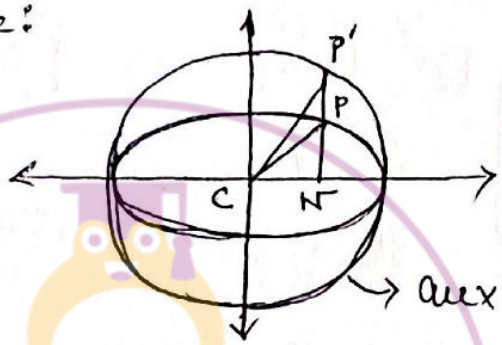
Eqn — $x^2 + y^2 = a^2 + b^2$.

Ellipse

* **Auxiliary Circle:** The feet of the perpendiculars drawn from either of the foci to any tangent to the ellipse $S=0$ lies on a circle, called auxiliary circle.

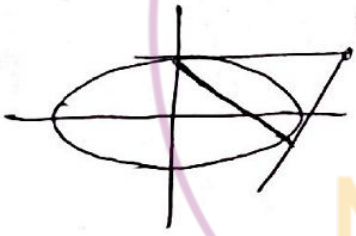
$$\text{Equ}^n - \quad x^2+y^2=a^2$$

* **Eccentric angle:**



$\angle NCP' \rightarrow$ eccentric angle of P.

* **Chord of contact:** Of ellipse $S=0$, $S_1=0$ of $P(x_1,y_1)$.



* **Parametric equ^n:** If $P(x,y)$ is a point on the ellipse then

$x=a\cos\theta$, $y=b\sin\theta$ where θ is eccentric angle of P.

* **Equ^n** of chord joining the points with eccentric angles α & β is

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}.$$

* Equⁿ of tangent at P(θ) on ellipse $S=0$ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$.

* Equⁿ of normal at P(θ) on ellipse $S=0$ is $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$.

* Four normals can be drawn from any point to the ellipse & sum of the eccentric angles of their feet is an odd multiple of π.