Handwritten Notes
On
Electrostatics

ELECTROSTATES

Properties of charges -

- Basic property of Matter
- charge Without mass can not exist Whereas mass Without charge can exist.

*- quantization of charge - charge on a body can only exist in the form of 'e'

$$\boxed{Q = ne} \quad (n = \text{Integer})$$

* charge is additive in nature

$q_1, -q_2$
$q_3, -q_4$

$$\boxed{Q_{net} = q_1 - q_2 + q_3 - q_4}$$

* conservation of charge - charge on an isolated system can neither be created nor, distroyed.

Total charge of a system = const.

NOTE → ←⊕ ⊕→ , ←⊖ ⊖→ , ⊕→ ←⊖

- Minimum possible charge $e = 1.6 \times 10^{-19}$ C (quanta)
- Exception of quantisation -

quark → up $\left(+\frac{2e}{3}\right)$, down $\left(-\frac{e}{3}\right)$

$\left[2U + 1D = 1 \text{ particle}\right]$
$\left[2\left(+\frac{2e}{3}\right) + 1\left(-\frac{e}{3}\right) = +e\right]$
$\left[1U + 2D = 1 \text{ neutron}\right]$
$\left[\left(\frac{2e}{3}\right) + 2\left(-\frac{e}{3}\right) = \text{zero}\right]$

- quark partide dont excist indepently, so quantisation is still correct.

*** - If quark particle would excist even then quantisation would be valid quanta will be (e/3).

- In a conductor charge is distributed at outer surface only while in non-conductor charge is distributed inside the surface.

METHOD OF CHARGING -

|i|→ Friction -
⊕ve ⇒ Glass Rod, Dry hair, cat skin, Wool.
⊖ve ⇒ Silk, comb, Ebonite, Plastic/Amber.

Ex→ cloud charging, charging of oil drop in Miliken oil drop experiment.

|ii|→ conduction -
* For ⊕ve charge will move [High → Low]
* ⊖ve charge will move [Low → High]

*NOTE ⇒ *In conduction total charge of system is re-distributed in the ratio of radius for making potential same.

* After conduction potential become same while charges will differ.

|iii|→ Induction - takes place in facing layer only.

$$\left[Q_{Induced} = Q_{Inducing}\left(1 - \frac{1}{\varepsilon_r}\right)\right] \quad (\varepsilon_r \to \text{dielectric const. of body})$$

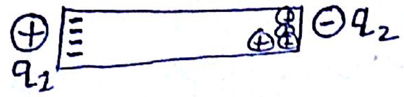
* For Metal $(\varepsilon_r = \infty) \to Q_{induced} = Q_{inducing}$
* For Non-Metal $(\varepsilon_r \neq \infty) \to Q_{induced} < Q_{inducing}$

Imp * Best method of charging.

NOTE ⇒ * Induction affect the distribution of charge not the magnitude of charge.
* There will be attraction b/w neutral charge body.
* There will be attraction b/w body having charge of same nature provided that magnitude of charges will be different.
* Sure test of charging is repulsion not attraction.

Ex → How will the force on q_1 will change if an insulated Rod is kept b/w them as shown?

Ans → Force will ↑



COLUMB'S LAW -

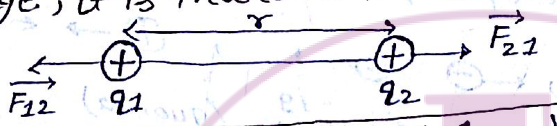
* It is not affected by pressure of any other charge
* It follow newton reaction.

$F \propto \frac{1}{r^2}$, $F \propto q_1 q_2$

$$\boxed{F = \frac{K q_1 q_2}{r^2}}$$

$K = 9 \times 10^9$ (MKS)
$K = 1$ (CGS)

* If the distance in discussion is very large as compared with the dimension of charge, it is treated as point charge.



$$\boxed{|\vec{F}_{21}| = |\vec{F}_{12}| = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_2}{r^2}}$$

ϵ_0 → permitivity of vaccum or, free space $= 8.85 \times 10^{-12} \frac{C^2}{N\text{-}m^2}$

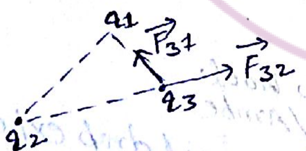
$$K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

IIT NOTE * If the charges are kept in some other medium, permitivity $= \epsilon_0\epsilon_r$ or, dielectric const. of medium will ↓

ϵ_r = Relative permitivity

$$\boxed{|\vec{F}_{net}| = \left(\frac{1}{4\pi\epsilon_0\epsilon_r}\right)\frac{q_1 q_2}{r^2}}$$

* coulomb's law follow principle of superposition.



$$\boxed{\vec{F}_{3net} = \vec{F}_{31} + \vec{F}_{32}}$$

AIIMS * PERMITIVITY [E] → E_0, E, E_r

- Permitivity of vaccume (ϵ_0) ⇒ $\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N \times m^2$
- Permitity of medium
  Absolute permitivity of medium (ϵ)
  unit → $C^2/N \times m^2$
- $\epsilon_r = \frac{\epsilon}{\epsilon_0}$

$\boxed{1 \le \epsilon_r < \infty}$

$\boxed{(\epsilon_r)_{air} = 1 \quad (\epsilon_r)_{metal} = \infty}$

$$\boxed{F_{vacume} = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}} \qquad \boxed{F_{medium} = \frac{1}{4\pi\epsilon_0\epsilon_r} \times \frac{q_1 q_2}{r^2}}$$

$\therefore \boxed{F_{medium} = \frac{F_{vacume}}{\epsilon_r}}$ $\because \boxed{\epsilon_r > 1}$

$\Rightarrow \boxed{F_{net} < F_{vacume}}$

|1| → case

If $F_{air} = F_{medium}$

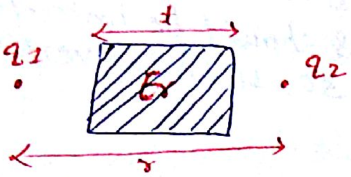
$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{air}^2} = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \cdot \frac{q_1 q_2}{r_{med}^2}$$

$$r_{air}^2 = \varepsilon_r r_{med}^2$$

*

$$\boxed{r_{air} = \sqrt{\varepsilon_r} \cdot r_{med}}$$

|2| → case

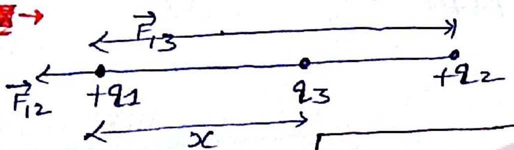


*

$$\boxed{F_{partial\ medium} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{\{(r-t) + t\sqrt{\varepsilon_r}\}^2}}$$

Imp concept III

→ There should be third charge q_3 placed in b/w, so that net force on all the charges become zero, Also find value of q_3.



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$$\boxed{x = \frac{d\sqrt{q_1}}{\sqrt{q_1} + \sqrt{q_2}} = \frac{d}{\left(1 + \sqrt{\frac{q_2}{q_1}}\right)}}$$

$$\boxed{q_3 = \frac{-q_1 q_2}{(\sqrt{q_1} + \sqrt{q_2})^2}}$$

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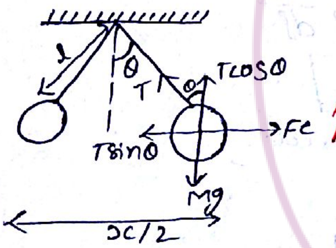
concept #

Two identical charge simple pendulum are in equilibrium as shown in fig. (Given θ = small)



|i| → calculate Repulsion b/w ball *

$$\boxed{\tan\theta = \frac{kq^2}{x^2 mg}} \qquad \boxed{x = \left\{\frac{2kq^2 l}{mg}\right\}^{1/3}}$$

2016 AIPMT AIEEE 2012

|ii| → If charge of ball start to leave at const. Rate & ball are moving towards each other with velocity (v) then Relation b/w x & v ?

$$q^2 \propto x^3 \quad \Big| \quad \frac{dq}{dt} \propto \frac{3}{2} x^{1/2} \frac{dx}{dt}$$
$$q \propto x^{3/2}$$

$$\boxed{v \propto x^{-1/2}}$$ *AIPMT 2016

|iii| → If $q_1 < q_2 \Rightarrow \theta_1 = \theta_2$ (due to Action Reaction pair.)

|iv| → If $M_1 > M_2 \Rightarrow \theta_1 < \theta_2$ (M1 is heavy so it will displace less so angle less) * $\boxed{\tan\theta \propto \frac{1}{M}}$

**|v| → If this system is carried in space or, Artificial satellite: [g = 0]

T → Fe   90° 90° $\tan\theta = \infty$, $\boxed{\theta = 90}$

***

- * Sepration b/w ball = l + l = 2l
- * Angle b/w string = 90° + 90° = 180°
- * Electric force b/w both = $Fe = \frac{kq^2}{(2l)^2} = \frac{kq^2}{4l^2}$
- * Tension in each string = $Fe = \frac{kq^2}{4l^2}$

JIPMER 2016

If we want to give charge on ball in due to Repulsion ~~ball~~ String become horizontal or, 180 then the value of q = ?

$$\boxed{T = \frac{kq_1 q_2}{(2l)^2}}, \quad \boxed{q = \sqrt{\frac{4l^2 mg}{k}}}$$

$$\boxed{q = \sqrt{\frac{4l^2 T}{k}}} \quad \boxed{T = mg}$$

2012 AIEEE

|vi| → If this system is dipped into the liq & angle of string with verticle remain unchanged find dielectric cost of liq. (density = 1.6 g/cm³, density liq = 0.8 g/cl)

*

$$\boxed{\varepsilon_r = \frac{1}{1 - \frac{d_l}{d_b}}} = \frac{1}{1 - \frac{0.8}{1.6}} = \frac{1}{\frac{1.6 - 0.8}{1.6}} = \frac{1}{\frac{0.8}{1.6}} = \frac{1}{\frac{1}{2}}$$

$$= \boxed{2}$$

concept
Two identical ball each with density 'ρ' are suspended with a common point by two insulating string of equal length. Both the ball have equal mass & charge. in equlibrium the string makes an angle 'θ' with vertical, now the whole system is emerged in liquid with density 'σ', If angle 'θ' does not change, what is dielectric const of liquid.

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$$\boxed{\epsilon_r = \frac{\rho}{\rho - \sigma}}$$ *

**
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concept
Three identical small balls each with mass 'm' are suspended at one point by threads of length 'l', what charges should be imparted to the ball for each thread to form an angle 30°, with the vertical.

$$\boxed{q = l\sqrt{\pi\epsilon_0 mg}}$$ *

**
***
concept
A small charge '+q' is distributed uniformally on an insulating Ring of Radius 'R'. If an additional charge '+Q' is kept at centre, find increament in tension of in Ring.

$$\boxed{T = \frac{KQq}{2\pi R^2}}$$ *

continuous charge distribution :→

|i|→ Linear charge density (λ) → charge per unit length.

* If distribution is uniform → $\boxed{\lambda = \frac{q}{l}}$
* If distribution is non uniform ⇒ $\lambda = \frac{dq}{dx}$ or, $\frac{dq}{dl}$

*$\boxed{dq = \lambda dx}$

|ii|→ surface charge density (σ) → charge per unit area.

* If distribution is uniform → $\boxed{\sigma = \frac{q}{A}}$
* If the distribution is non-uniform → $\boxed{\sigma = \frac{dq}{dA}}$

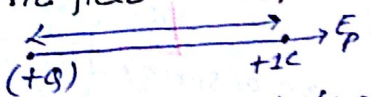
$\boxed{dq = \sigma dA}$

|iii|→ volume charge density (ρ) → charge per unit volume.

* If 'ρ' is uniform → $\boxed{\rho = \frac{q}{V}}$
* If 'ρ' is non-uniform → $\boxed{\rho = \frac{dq}{dv}}$ or, $\boxed{dq = \rho dv}$

ELECTRIC FIELD INTENSITY (E) ⇒ It Represent strength of effect of charge at given point.

* Electric field due to point charge :–



$$|\vec{F_p}| = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{(+q)(1C)}{r^2} = |\vec{E}|$$

*$$\boxed{|\vec{E}| = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2}} \quad \frac{N}{C} \text{ or, } \frac{V}{M}$$

$$\boxed{E = \frac{kq}{r^2}}$$

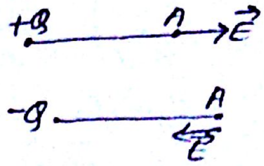
NOTE ⇒ * When Electric field is measured due to a point charge (Q), the test charge taken very small. & another defination can be given as

$$\boxed{|\vec{E}| = \lim_{q_0 \to 0} \frac{|\vec{F}|}{q_0}}$$

* Direction of Electric field :-

$\oplus$ve charge = Away from the charge $\left(E = \frac{KQ}{r^2}\right)$

$\ominus$ve charge = Towards the charge $\left(E = \frac{KQ}{r^2}\right)$



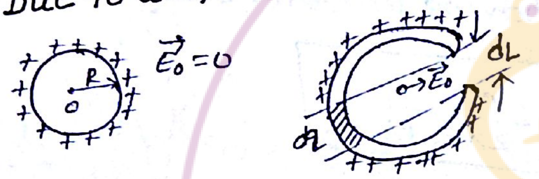
* If there are several no of charges in the system then, net Electric field is vector sum of all Electric field due to charges. (principle of superposition)

* If a charge 'q' is kept in a External Electric field $\vec{E}$, then net force acting on q is $\boxed{\vec{F} = q\vec{E}}$ $\left[\begin{array}{l} \text{Along } \vec{E}, \text{ If } q \to \oplus\text{ve} \\ \text{opp } \vec{E}, \text{ If } q \to \ominus\text{ve} \end{array}\right]$

ELECTRIC FIELD DUE TO CONTINUOUS CHARGE DISTRIBUTION :-

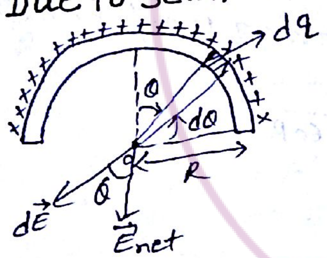
CASE I ⇒ Due to uniformaly charge Ring of Radius 'R' at the centre :-

$\lambda = \frac{Q}{2\pi R}$, $dq = \lambda dL = \frac{QdL}{2\pi R}$



$$\boxed{|\vec{E}_0| = \frac{1}{4\pi\epsilon_0 R}\frac{dQ}{R^2}}$$

CASE II ⇒ Due to semi-circular Ring at the centre :-



$$\boxed{\vec{E}_{net} = \frac{Q}{2\pi^2\epsilon_0 R^2} = \frac{\lambda}{2\pi\epsilon_0 R}}$$

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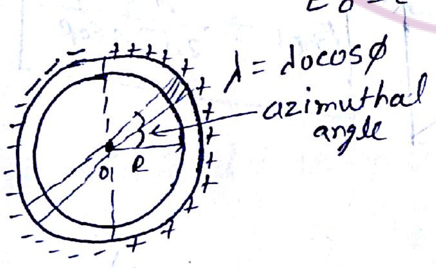
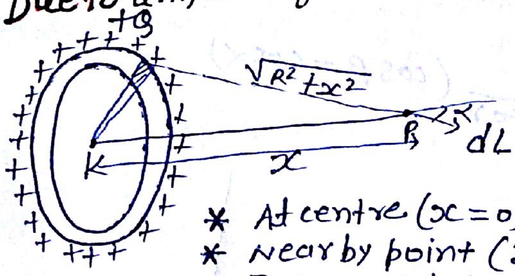
$\vec{E}_0 = ?$



$$\boxed{E = \frac{\lambda_0}{2\epsilon_0 R}}$$

$$\boxed{E_{net} = 2E = \frac{\lambda_0}{4\epsilon_0 R}}$$

CASE III ⇒ Due to uniformaly charged circular Ring along Its axis :-



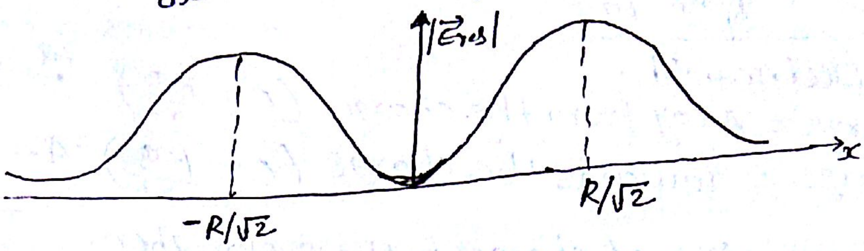
$$\boxed{E_{res} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}}$$

* At centre ($x = 0$) $E = 0$
* Near by point ($x <<< R$) $E = \frac{KQx}{(R^2)^{3/2}} = \frac{KQx}{R^3} \propto x$
* Far away point ($x >>> R$) $E = KQ/x^3 \propto 1/x^3$
* For $E_{max} = \frac{dE}{dx} = 0 \Rightarrow x = \pm R/\sqrt{2}$

NOTE →

* Max^m electric field along axis

$$\frac{dE}{dx} = 0 \Rightarrow x = \frac{R}{\sqrt{2}}$$



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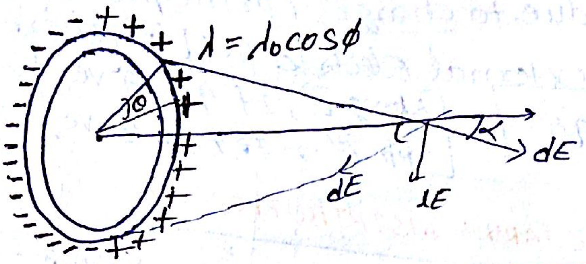
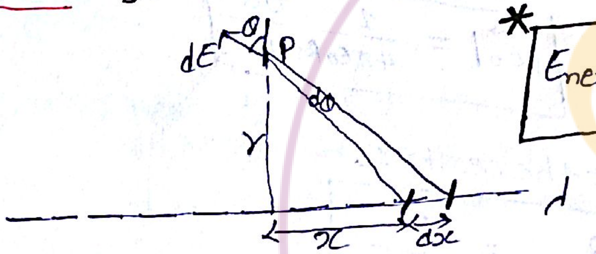


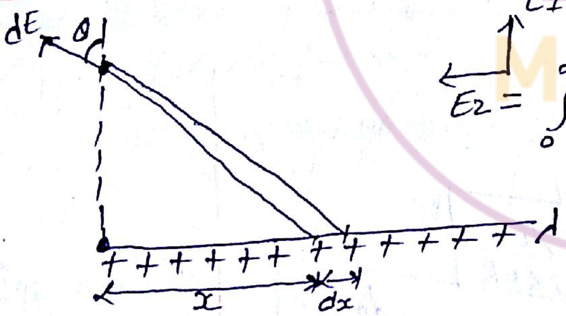
*
$$\boxed{2E = \frac{\lambda_0 R^2}{\pi \epsilon_0 (R^2 + x^2)^{3/2}}}$$

case IV ⇒ Due to infinite line charge having uniform charge density (λ) :-



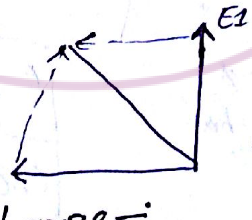
*
$$\boxed{E_{net} = \frac{\lambda}{2\pi\epsilon_0 R}}$$

case V → Due to semi-infinite line charge having uniform density 'λ'



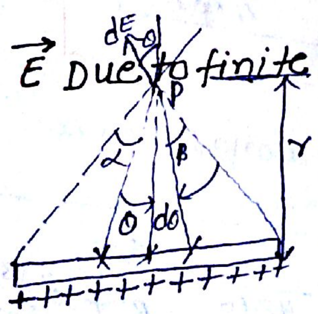
$$E_1 = \int_0^\infty dE\cos\theta = \frac{\lambda}{4\pi\epsilon_0 R}$$

$$E_2 = \int_0^\infty dE\sin\theta = \frac{\lambda}{4\pi\epsilon_0 R}$$



*
$$\boxed{E_{net} = \frac{\lambda}{2\sqrt{2}\pi\epsilon_0 R}}$$

case VI ⇒ $\vec{E}$ Due to finite line charge :-



$E_P = ?$

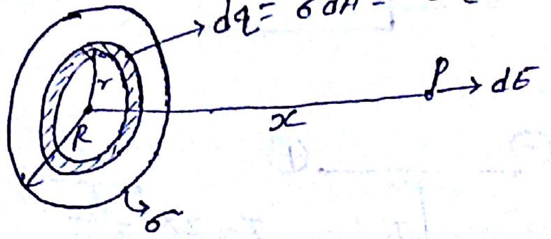
$$E_y = \frac{\lambda}{4\pi\epsilon_0 x}(\sin\beta + \sin\alpha)$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0 r}(\cos\beta - \cos\alpha)$$

Case VII ⇒ $\vec{E}$ due to circular Disc uniformally charge with (σ) ÷

$\vec{E}_{centre} = 0$

$dq = \sigma dA = \sigma(2\pi r)dr$



$$\boxed{\vec{E} = \frac{\sigma x}{2\epsilon_0}\left[\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right]}$$

NOTE →

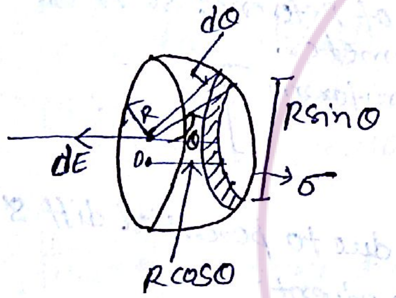
$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{1+\frac{R^2}{x^2}}}\right]$$

If $R \to \infty$

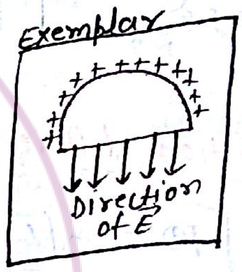
$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}}$$ For infinitely distributed plane sheet.

* After folding the Rings, net electric field will ↑.

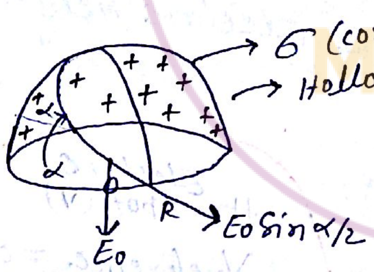
Case VIII ⇒ $\vec{E}$ at centre due to uniformally charged hemi-sphericall shell ÷

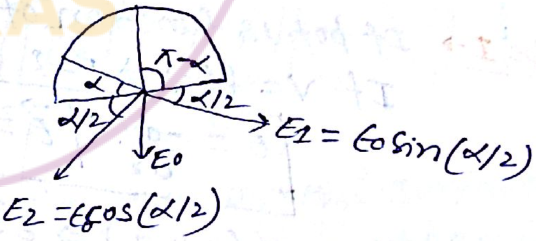


$$\boxed{E = \frac{\sigma}{4\epsilon_0}}$$



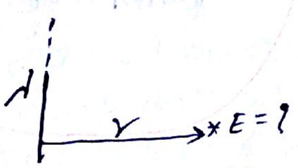
EX → Find $\vec{E}$ due to part which is cut at an angle α.





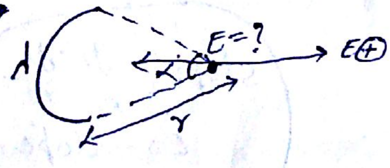
* Electric field due to uniformally charge wire –

$$\boxed{E = \frac{2K\lambda}{r}}$$



* $\vec{E}$ centre for uniformally charged arc ÷

$$\boxed{E = \frac{2K\lambda}{r}\sin\left(\frac{\alpha}{2}\right)}$$

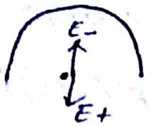


|ii|→

$\alpha = 90^\circ$

$$\boxed{E = \frac{\sqrt{2}K\lambda}{r}}$$

|iii|→



$\alpha = 180^\circ$

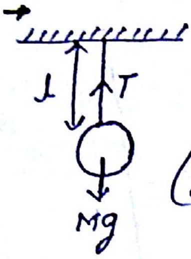
$$\boxed{E = \frac{2K\lambda}{r}}$$

|iv|→

$\alpha = 360^\circ$
$E = 0$

* Pendulum concept:-

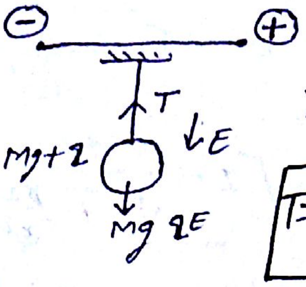
+→



$$\boxed{T = 2\pi\sqrt{\frac{l}{g_{eff}}}}$$

(g_{eff} = acceln for net force except tension)

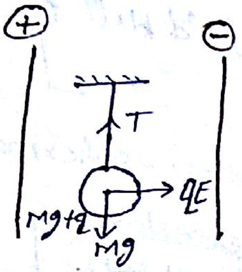
+→



$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g + \frac{qE}{M}}}}$$

+→



$$\boxed{F_{net} = \sqrt{(Mg)^2 + (qE)^2}}$$

$$\boxed{g_{eff} = \sqrt{g^2 + \left(\frac{qE}{M}\right)^2}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{qE}{M}\right)^2}}}}$$

Potential difference (PD):→ It is independent of Reference so it is absalute parameter.

$$\left[\begin{array}{l} PD = -\int \vec{E}\cdot d\vec{r} \text{ (If E is uniform /Non-uniform)} \\ PD = -\vec{E}\cdot d\vec{r} \text{ (If E is Uniform)} \end{array}\right]$$

Relation b/w E & V :- Electric field is due to potential diff & it is equal to ⊖ve of potential gradient

Type I→ If pot. is function of 'r' then find Electric field ÷

If $V = f(r) \Rightarrow E = ?$

$$\boxed{E = \frac{-dV}{dr} \;\Big|\; \vec{E} = -\frac{dV}{dr}\hat{r}}$$

Type II→ If $V = f(x,y,z) \Rightarrow E = ?$

$\vec{E} = -\nabla V$

$\vec{\nabla}$ → Del operator (Gradient)

$$\nabla = \frac{i\partial}{\partial x} + \frac{j\partial}{\partial y} + \frac{K\partial}{\partial z}$$

$$\boxed{\vec{E} = -\left[i\frac{\partial V}{\partial x} + j\frac{\partial V}{\partial y} + K\frac{\partial V}{\partial z}\right]}$$

Type III→ If V-r Graph is given at E = ?

$$E = \frac{-dV}{dr} \quad \boxed{E = -\text{slope}}$$

Electric pot (V)

$V_{reference} = 0$

$V_{\infty\,location} = 0$

$V_{earth} = -0.12\,V$

$V_{earth} = 0$ (consider zero)

slope $= \frac{y_2 - y_1}{x_2 - x_1}$

$E = -$slope

Type IV → If $E = f(r) = PD = ?$

$$\boxed{PD = -\int \vec{E}\cdot d\vec{r}}$$

Type V → If $E = f(x, y, z) \Rightarrow PD = ?$

$$PD = -\int \vec{E}\cdot d\vec{r}$$

$$= -\int (E_x\hat{i} + E_y\hat{j} + E_z\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\boxed{= -\int E_x dx - \int E_y dy - \int E_z dz}$$

Type V → If E-r Graph is given

$$PD = \left|-\int \vec{E}\cdot dr\right|$$

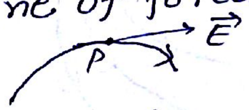
= Area under the curve.

Electric line of forces or, Electric field lines :-

We know that electric field is invisible to generate the picture of Electric field, we draw Electric line of forces.

properties of ELF →

|i| → Tangent drawn at a point on the line of forces gives the direction of field at that point.



|ii| → Two electric field lines can never intersect as there will be two direction of Electric field at that point.

NOT POSSIBLE

|iii| → conservative Electric field lines can never form close loop.

|iv| → The density of electric field lines in a certain region gives us a qualitative idea about the stregth of field in that Region.

$$\boxed{|\vec{E_A}| > |\vec{E_B}|}$$

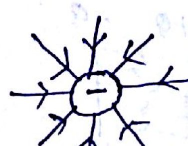
|v| → They are always perpendicular to equipotential surface.

|vi| → If the positive charge at Rest is free to move then it may or, may not follow the line of forces.

|vii| → Electric lines are differentiable at all point they can't have Sharp turnings.

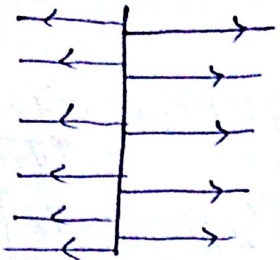
|viii| → Electric field lines can be discontinuous.

Isolated ⊕ve charge orginates at charge $ end at infinity.

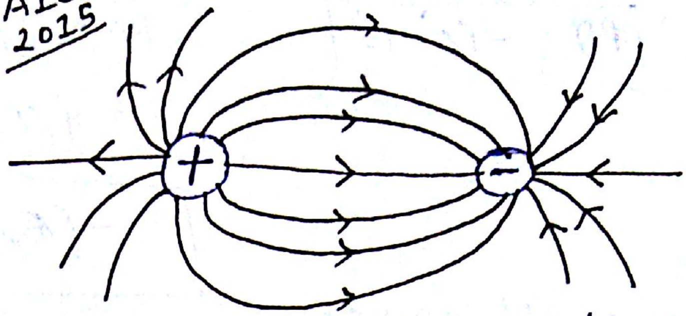


Isolated ⊖ve charge orginates at ∞ $ End at ⊖ve charge.

AIEEE 2015

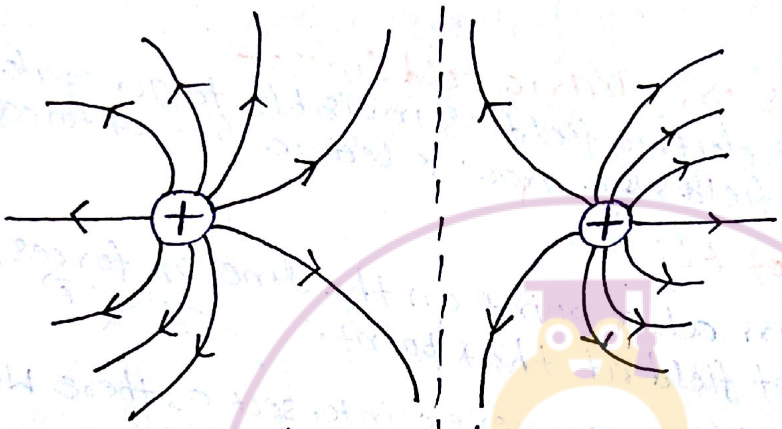


Infinite plane Sheet of charge



Equal ⊕ve & ⊖ve charge.

*



Equal ⊕ve charges

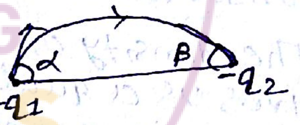
* Equidistance ∥ line represent uniform Electric field.

$$E \propto \frac{1}{\text{sepration b/w lines}}$$

IIT *****

Imp property :—

$$\boxed{\sqrt{\frac{q_1}{q_2}}\,\sin\left(\frac{\alpha}{2}\right) = \sin\left(\frac{\beta}{2}\right)}$$



EX → If a field line originate at $+2Q$ at angle $30°$. Find the angle at wich it enter $-Q$ charge.

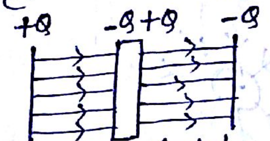
$$\sqrt{\frac{2Q}{Q}}\sin 30° = \sin\left(\frac{\beta}{2}\right)$$

$$\sqrt{2}\times\frac{1}{2} = \sin(\beta/2)$$

$$\frac{\beta}{2} = 45°$$

$$\boxed{\beta = 90°}$$

|IX| → In a conductor there is no line of force while in non-conductor few line are there.



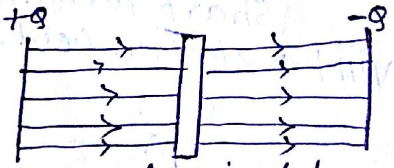
metal slab

$E_{applied}$ → Right

$E_{induced}$ → left

$E_{ind} = E_{App}$

$E_{net} = 0$



Dielectric slab

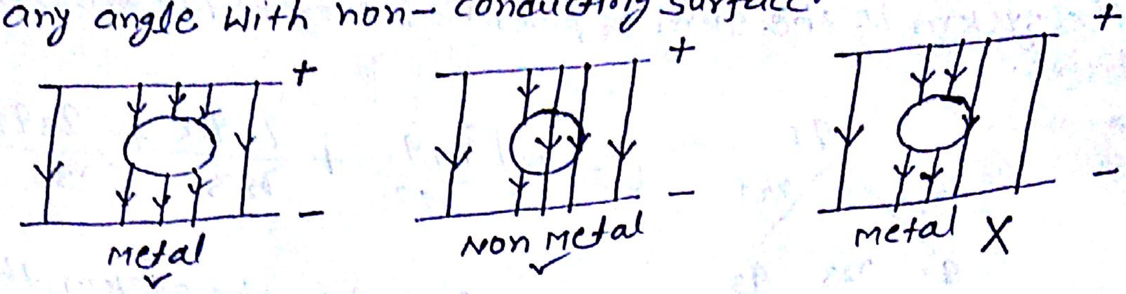
$E_{applied}$ → Right

$E_{induced}$ → Left

$E_{ind} < E_{App}$

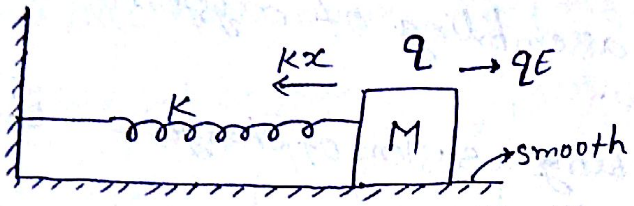
$E_{net} \neq 0 \ (< E_{app})$

(X) → Electric at ELF are always ⊥ to conducting surface this can make any angle with non-conducting surface.



concept Ex →

An Electric field is switched on at $t=0$ as shown.



|a| → amplitude of oscillation *

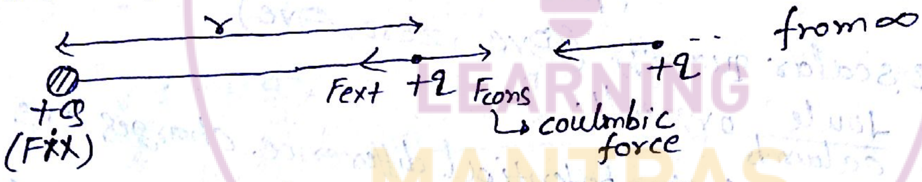
$Kx = qE$ $\boxed{x = \frac{qE}{K}}$

|b| → Time period of oscillation.

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

ELECTRIC POTENTIAL ENERGY (U):-

Defined for conservative field only.



* point charge → PE = zero.
* If we consider displacement $\vec{dr}$ at position vector $\vec{r}$

$dW_{cons} = \vec{F} \cdot \vec{dr}$ $\boxed{\left(\vec{F} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Qq}{r^2}\cdot\hat{r}\right)}$

$$\boxed{U = \frac{Qq}{4\pi\epsilon_0 r}}$$

NOTE |i| → Q, q are kept along with sign as If Q, q both are of same sign

$W_{ext} = \oplus ve$, $U = \oplus ve$ (unstable

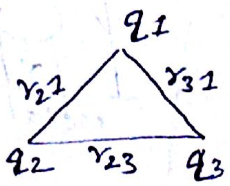
* As If Q, q are of opp. sign

$W_{ext} = \ominus ve$

$U = \ominus$ (stable)

|ii| → If there are 'n' no. of point charges then the total energy of the system is the sum of potential energies due to all these pairs.

Ex →



$$U = k\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]$$

|iii| → If there are 'n' no. of charges (point) in the system then no. of pair $= \boxed{nC_2}$

|iv| → External work done in assembling of a system of charges $= U_{system}$.

Work done in disassembling a system of charges $= -U_{system} =$ = Binding Energy of system.

** # ELECTRIC POTENTIAL (V) ÷ Work done on an unit ⊕ve charge in displacing it from A to B.

q_0 q_0

A → B

$$V_{AB} = V_B - V_A = \frac{W_{ext}}{q_0} = \frac{U_B - U_A}{q_0}$$

$$= (\text{pot of } B - \text{pot. of } A)$$

NOTE → * It is scalar quantity (⊕ve, zero, ⊖ve)

* unit $\frac{\text{Joule}}{\text{Coulomb}}$ or, volt.

* While calculating potential difference charges are used along with sign.

Absolute potential at a point ÷ If we bring charge 'q_0' from infinity to a point P. then work done per unit charge become potential at P.

P   ← q_0 ∞

$$V_P = V_P - V_\infty = \boxed{\frac{U_P - U_\infty}{q_0} = \frac{U_P}{q_0}}$$

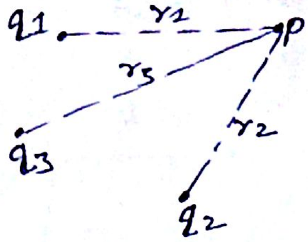
Electric potential due to a system of charges —

|i| → Due to point charge (Q) —

$+Q$ ←— r —→ A   ← at ∞ q_0

$$\boxed{V_A = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}}$$ (Q with sign)

|ii| → Due to system of several point charges –



$$\boxed{V_p = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]}$$

$q_1, q_2, q_3 \Rightarrow$ With sign

Due to continuous charge distribution –

|i| → Due to circular Ring (+Q) →

|a| → At centre

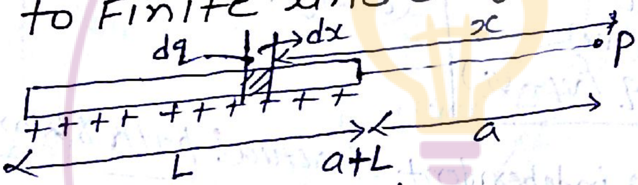
$$\boxed{V = \frac{KQ}{R}}$$

*(Wheather this dist. is uniform or, non-uniform)

|b| → At a point on axis

$$\boxed{V = \frac{KQ}{\sqrt{R^2+x^2}}}$$

** |ii| → Due to Finite line charge –



$$V = K\frac{dq}{x} = \int_a^{a+L} K\lambda\frac{dx}{x}$$

$$* \boxed{V = K\lambda \ln\left|\frac{a+L}{L}\right|}$$

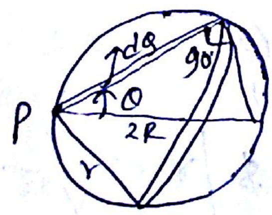
*** |iii| → uniformally charged disc (σ, R) → (sigma)

|A| → At centre $\boxed{V = \frac{\sigma R}{2\epsilon_0}}$

|B| → At a point on the axis 'x'

$$\boxed{V = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2+x^2} - x\right]}$$

|C| → At a point on the edge of the disc –



$$\boxed{V = \frac{\sigma R}{\pi\epsilon_0}}$$

pot. energy of 2 point charge.

q_1 ——— r ——— q_2 $\boxed{U = \frac{Kq_1q_2}{r}}$ $U_0 = \frac{Kq_1q_2}{\infty} = 0$

EX→ * ⊕ ⟶ ⟵ ⊕ | ⊕ ⟶ ⟵ ⊖

$r\downarrow$ | $r\downarrow$

U (⊕ve) ↑ | U = (⊖ve)

PE↑ | PE↓

* When two point charge are brought close to each other, then PE of system may ↑ or ↓.

Potential energy of system having 'n' point charges – Equal to algebric sum of PE of all possible pair of 2 charges.

$$\boxed{\text{no. of pair} = \frac{n(n-1)}{2}}$$

Work done by Electric field: → conservative field so it perform work then its PE ↓.

$$\boxed{W_{conser.} = -\Delta U}$$

$$\boxed{\begin{aligned} W_{electric\ field} &= -\Delta U \\ &= -q(\Delta V) \\ &= -q(V_f - V_i) \end{aligned}}$$

(charge must use c̄ sign)

* Work in electric field is independent of actual path & work for closed loop is zero.

Work Energy Relation

$W_{Total} = \Delta K.E$

$W_{external} + W_{cons} + W_{non\text{-}cons} = \Delta KE$

$W_{Ext} + W_{cons} = \Delta K.E$

* $\boxed{W_{Ext} = \Delta K + \Delta U}$

PMP Sir!! SMS * When an α-particle is projected towards a nucli it can reachup to a closest distance, Higher the K.E of Incident α-particle closer to it is to nucleus, success in life analogous to it more effort you put in closer to succes. COME

$K.E = const \Rightarrow \Delta K.E = 0$

$W_{Ext} = ?$

$W_{Ext} = \Delta U$

$$\boxed{\begin{aligned} &= q\Delta V \\ &= q(V_f - V_i) \end{aligned}}$$

If $K.E \neq const$

$W_{Ext} = 0$

$$\boxed{\begin{aligned} \Delta K.E + \Delta U = 0 \\ K + U = const \end{aligned}}$$

2016 AIPMT concept

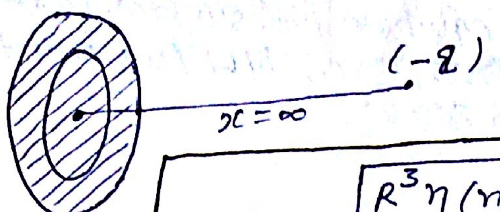
An α-particle is thrown from with velocity 'v' towards nucleus of atomic no. 'Z' calculate closest distance of approch?

(+Ze) ⟵ r ⟶ * $v = 0$ v ⟵ α α-particle (+2e)

* $\boxed{r = \frac{4KZe^2}{Mv^2}}$ Here $\boxed{r \propto \frac{1}{M}}$ & $\boxed{r \propto \frac{1}{v^2}}$

AIPMT 2016

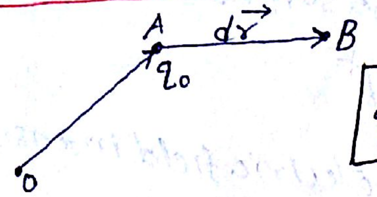
Potential at a distance 'x' from axis ÷



$$V_0 = \sqrt{\frac{Qq}{\pi\epsilon_0 m R(n+1)}}$$

$$T = 2\pi\sqrt{\frac{R^3\eta(n+1)m}{2KQq}}$$

Relation b/w $\vec{E}$ & V :→



"displaced very slowly by Ext. agent"

$$\boxed{dV_{AB} = -\vec{E}\cdot d\vec{r}}$$

NOTE ⇒

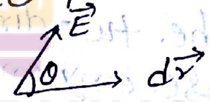
|i| → To find potential difference b/w point A & B

$$\int_A^B dV_{AB} = -\int \vec{E}\cdot d\vec{r}$$

$$\boxed{V_{AB} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot d\vec{r}}$$

|ii| → To find direction & magnitude of $\vec{E}$

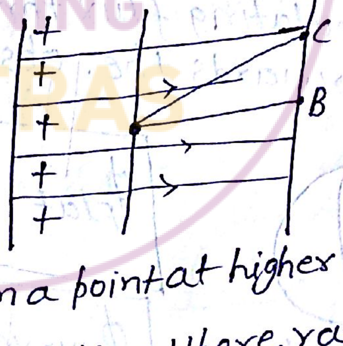
$$dV = -\vec{E}\cdot d\vec{r}$$



$$dV = -E\,dr\cos\theta$$

$$\boxed{-\frac{dV}{dr} = E\cos\theta}$$

$$\boxed{\left|\frac{-dV}{dr}\right|_{max} = E}$$



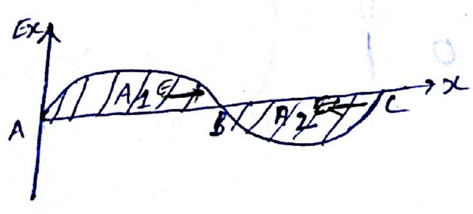
* 'Electric field is always directed from a point at higher potential to the point at lower potential'

* 'Direction of electric field is along the line where rate of ↓ of potential (i.e $-\frac{dV}{dr}$) is maxm.'

calculating potential from E-x curve

$$E_x = -\frac{dV}{dx}$$

$$\Delta V = -\int E_x \cdot dx \approx \text{Area under } E_x\text{-}x \text{ curve.}$$



$$V_{AB} = V_B - V_A = -A_1$$

$$V_{BC} = V_C - V_B = +A_2$$

Equipotential surface :→ It is the surface where pot at every point is const, It can be spherical, cylindrical or, plane surface.

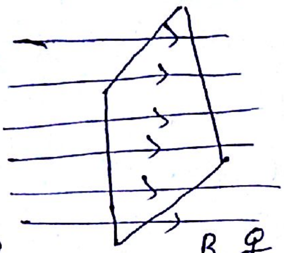
* Electric field is always ⊥r to the equipotential surface. ($E \perp dv$)
* Work done is moving a charge (⊕ve/⊖ve) b/w two point on equipotential surface is always zero.

$$W = q(V_f - V_i)$$

$$\because V_f = V_i$$

$$\boxed{W=0}$$

Electric Flux (φ) → Measure of no of lines passing through a surface.

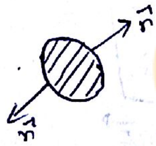


* Flux going into the surface is consider as ⊖ve & coming out of the surface is consider as ⊕ve.

$$\boxed{d\phi = \vec{E} \cdot \vec{ds}}$$ *

$\vec{E}$ = Electric field intensity.

Area vector ($\vec{ds}$) —

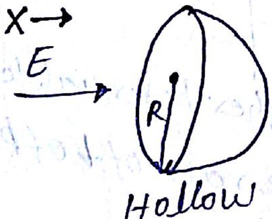


$\vec{ds} = (ds)\vec{n}$

$\vec{n}$ = a unit vector ⊥r to the small area ds

* There can be two directions of $\vec{n}$ & theoritically we can choose any one of them but while calculating flux through a surface $\hat{n}$ is taken towards the observer.

EX→



$$\boxed{\phi_{Flat} = -E\pi R^2}$$

$$\boxed{\phi_{Total} = 0}$$

$$\boxed{\phi_{curved} = E\pi R^2}$$

$$\boxed{\phi_{bowl} = \frac{q}{2\epsilon_0}}$$

NOTE →* To find electric flux either we can find area ⊥r to $\vec{E}$ or, we can find component of $\vec{E}$ ⊥r to the given Area. [Remember area in a case should be plane surface.]

* If a closed surface is kept in a uniform electric field or, If it is does not contain any charge, the total flux passing through it is always zero.

$\vec{E}$ ▭ $\phi_{Total} = 0$

The statement of Gauss law:-

"Net flux passing through any closed surface is equal to charge inclosed by it divided by ϵ_0"

$$\boxed{\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\epsilon_0}}$$

surface integral = Intigration of complete surface.

NOTE →

(a) → * Net flux is always calculated due to charges inside the surface by while integrating $\vec{E}\cdot d\vec{s}$ the $\vec{E}$ at the surface is due to all the charges in that system.

(b) → * With the help of Gauss law, we can find electric field due to the some charged system but they are very limited.

* Angle b/w $\vec{E}$ & $d\vec{s}$ at every point on Gaussian surface should be same.
* Magnitude of $\vec{E}$ should be same through out the surface.

* ϕ_{net} doesn't depepend on size & shape of body.

Application of Gauss law:→

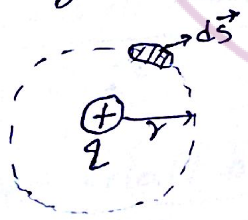
Electric field due to some symmatric charge distribution.

$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\epsilon_0}$$

$$\oint E\, ds \cos\theta = \frac{q_{in}}{\epsilon_0}$$

* Magnitude 'E' at every point on surface must be same.
* Angle b/w $\vec{E}$ & $d\vec{s}$ should same.

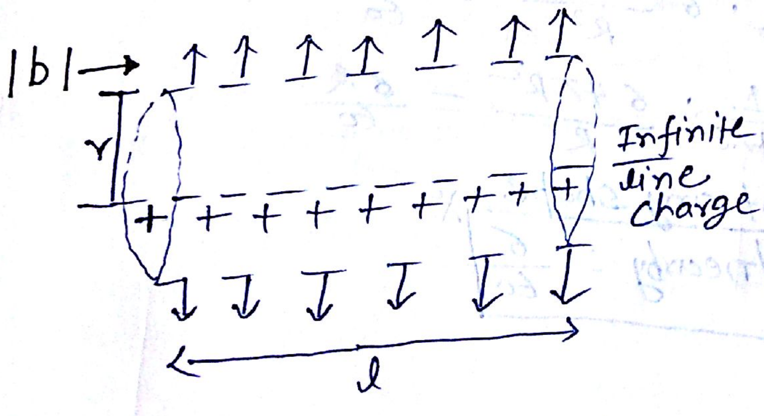
(a) →



$$\oint = \vec{E}\cdot d\vec{s} = \oint E\,ds = E\oint ds$$
$$= E \times 4\pi r^2$$
$$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$* \boxed{E = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2}}$$

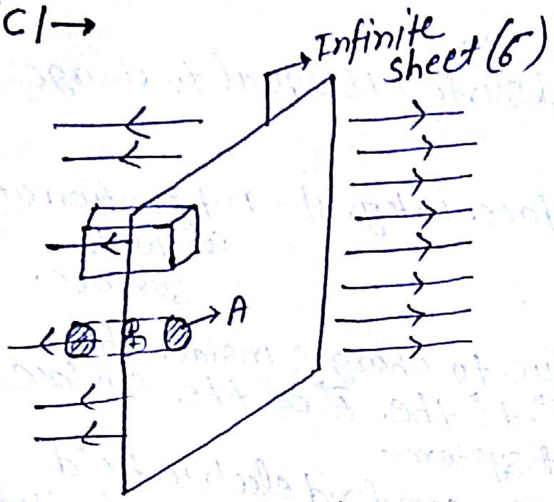
(b) →



$$\oint \vec{E}\cdot d\vec{s} = E\oint ds$$
$$= E\left[2\pi r l\right]$$
$$= \frac{q_{in}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

(c) →



$$\oint \vec{E}\cdot d\vec{S} = E\oint ds = E(2A)$$

$$\boxed{q_{in} = \sigma A}$$

* Infinite/long wire : → $E = \frac{2K\lambda}{r}$

* Non conducting wire → $E = \frac{\sigma}{2\epsilon_0} \propto r^0$
* conducting plate → $E_{outside} = \frac{\sigma}{\epsilon_0}$, $E_{inside} = 0$

All conducting [solid/Hollow] & Hollow non conducting sphere →

distance of observer point from the centre of sphere.

r → distance of observer point from the centre of sphere.

position	E	V
$r > R$	KQ/r^2	KQ/r
$r = R$	KQ/R^2	KQ/R
$r < R$	0	KQ/R
$r = 0$	0	KQ/R

NOTE →

EX → $\sigma = \frac{Q}{4\pi R^2}$

$Q = \sigma(4\pi R^2)$

* $E_{surface} = \frac{KQ}{R^2} = \frac{1}{4\pi\epsilon_0}\cdot\sigma\frac{4\pi R^2}{R^2} = \frac{\sigma}{\epsilon_0}$

* $V_{surface} = \frac{KQ}{R} = \frac{1}{4\pi\epsilon_0}\cdot\frac{\sigma 4\pi R^2}{R} = \frac{\sigma R}{\epsilon_0}$

For any conductor of any shape *

$$\boxed{E_{outside/nearby} = \frac{\sigma}{\epsilon_0}}$$

EX→ * $E_{surface} = \frac{Kq}{R^2} \propto \frac{1}{R^2}$ (If q = const)

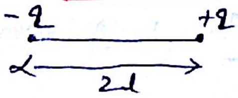
* $E_{surface} = \frac{\sigma}{\epsilon_0} \propto R^0$ (If σ = const)

* $E_{surface} = \frac{V}{R} \propto \frac{1}{R}$ (If V = const)

solid non-conductor sphere:-

Position	E	V
$r > R$	Kq/r^2	Kq/r
$r = R$	Kq/R^2	Kq/R
$r < R$	$\frac{Kq}{R^3} r$	$\frac{Kq(3R^2 - r^2)}{2R^3}$
$r = 0$	0	$\frac{3}{2}\left(\frac{Kq}{R}\right)$

Electric dipole :→ Two point of same magnitude & opposite nature at small sepration.



$2l$ → length vector
direction ($-q$ to $+q$)

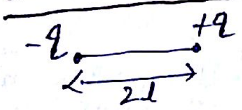
|I|→ Dipole Moment (P) :-

P = charge × length of Dipole

$$\boxed{\begin{array}{l} P = q(2l) \\ \vec{P} = q(2\vec{l}) \end{array}}$$

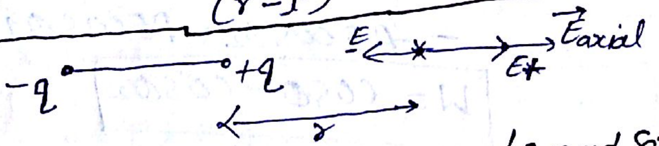
vector = ($-q$ to $+q$)
unit = C×m
1 Debye(D) = 303×10^{-30} C×m

|II|→ Electric field due to Dipole

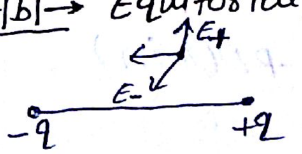


case-|a|→ Axial/Longitudinal/tanA/End on position

$$\boxed{E_{axial} = \frac{2KPr}{(r^2 - l^2)^2} = \frac{2KP}{r^3} \text{ (Along P direction)}}$$

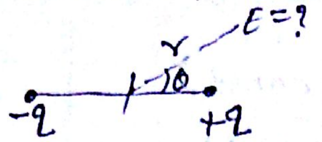


case-|b|→ Equitorial/Transverse/Broad side/Tan β →



$$\boxed{E_{equitorial} = \frac{KP}{(r^2 + l^2)^{3/2}} = \frac{KP}{r^3} \text{ (opp direction)}}$$

case-|E| → General point (r, θ) ÷



$$E = \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}$$

~~Angle~~ Angle b/w E & P

$$\theta + \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

From same distance 's' in case of dipole $\frac{E_{axis}}{E_{equitorial}} = ?$

$$\frac{2kP/r^3}{kP/r^3} = 2:1 \text{ approx}$$
$$= 2:1 \text{ approx}$$

NOTE

* In axial point dipole the electric field direction in the direction of net dipole.
* In equitorial dipole electric field direction in opposite to the direction of net dipole.

* $E_{dipole} \propto \frac{1}{r^3}$, $E_{point charge} \propto \frac{1}{r^2}$, $E_{long wire} \propto \frac{1}{r}$, $E_{sheet} \propto r^0$

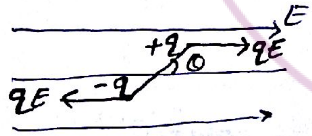
III → potential due to dipole :→

$$V = \frac{kP\cos\theta}{r^2 - l^2\cos^2\theta} = \frac{kP\cos\theta}{r^2}$$

|a| → $\theta = 0$, $V = \frac{kP}{r^2}$ |b| → $\theta = 90°$

$V_{eq} = 0$

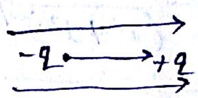
|IV| → Behaviour of dipole in Ext. uniform field :→



* $F_{net} = 0$ (no translational motion)
* $\boxed{\tau = PE\sin\theta \\ \vec{\tau} = \vec{P}\times\vec{E}}$
* $\boxed{U = -PE\cos\theta \\ U = -\vec{P}\cdot\vec{E}}$
* $W_{\theta_1 \to \theta_2} = \Delta U$
$= U_{\theta_2} - U_{\theta_1}$
$= -PE\cos\theta_2 - PE\cos\theta_1$

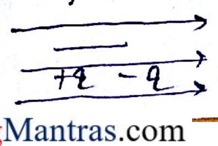
$$\boxed{W = \cos\theta - \cos\theta_2}$$

case -|a| → If $\theta = 0°$



$f = 0$ | $U = -PE$ (min)
$\tau = 0$

case-|b| → If $\theta = 180°$



$f = 0$
$\tau = 0$
$U = +PE$

NOTE → In a stable equilibrium dipole is given small angular disp. & it perform angular SHM with time period

$$\boxed{T = 2\pi\sqrt{\frac{I}{PE}}} \quad I = M.O.I$$

** BIG DROP ⇌ n SMALL DROPS

(i) → $R_{Big} = n^{1/3} r_{small}$	(iv) → $E_{Big} = n^{1/3} E_{small}$
(ii) → $Q_{Big} = n\, q_{small}$	(v) → $C_{Big} = n^{1/3} C_{small}$
(iii) → $\sigma_{Big} = n^{1/3} \sigma_{small}$	(vi) → $V_{Big} = n^{2/3} V_{small}$

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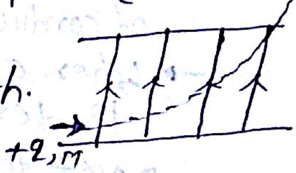
* path of charge particle in uniform ⊥r electric field (g = negligible)

$$\boxed{y = \left(\frac{qE}{2mv^2}\right)x^2} \rightarrow \text{Deviation.}$$

$$\boxed{y \propto x^2} \rightarrow \text{parabolic path.}$$



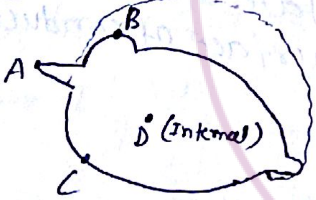
conductor of Irregular shape: →

(R → Radius of curvature)

* $Q \propto R$ * $\sigma \propto \frac{1}{R}$ * $E = \frac{\sigma}{\epsilon_0} \propto \frac{1}{R}$ * $V \propto R^0$ [conductor are equipotential surface]

AIIMS

A Metal Body: →

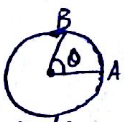


* R ⇒ $R_C > R_B > R_A$
* Q ⇒ $Q_C > R_B > R_A > q_D = 0$
* σ ⇒ $\sigma_A > \sigma_B > \sigma_C$
* E → $E_A > E_B > E_C > E_D = 0$
* V → $V_A = V_B = V_C = V_D$

*** Solid Angle (Ω)

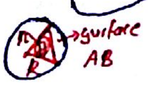
- plane Angle (θ), Radian measured in 2-D
- solid angle (Ω), Stradian measured in 3-D.

$$\boxed{\Omega = \frac{\text{area}}{(\perp \text{distance})^2}}$$



* Angle = $\boxed{\frac{\text{arc}}{\text{radius}}}$

complete plane Angle $\frac{2\pi R}{R} = \boxed{2\pi}$



* solid Angle = $\boxed{\frac{\text{Area of surface}}{R^2}}$

* complete solid Angle = $\frac{4\pi R^2}{R^2} = \boxed{4\pi}$

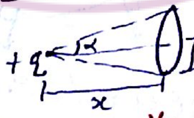
* Relation b/w plane Angle & solid Angle

$$* \boxed{\Omega = 2\pi(1 - \cos\alpha)}$$

Application of solid Angle (Ω)

$\phi_{total} = \frac{q}{\epsilon_0}$, 4π

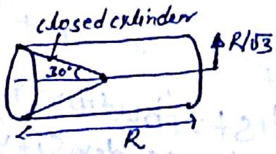
Flux through per unit solid Angle $= \frac{q}{4\pi\epsilon_0}$



$\cos\alpha = \frac{x}{\sqrt{R^2+x^2}}$, * $\boxed{\Omega = 2\pi\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]}$

$$* \boxed{\phi_{through\ disc} = \frac{q}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]}$$

Ex →



$\phi_{curved} = ?$

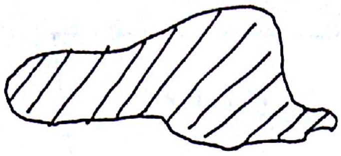
$\Omega_{flat} = 2\pi(1 - \cos 30°) = 2\pi\left(1 - \frac{\sqrt{3}}{2}\right)$

$\Omega_{curved} = 4\pi - 2\Omega_{flat} = 4\pi\left[\cancel{1} - \cancel{1} + \sqrt{3}/2\right] = 2\sqrt{3}\pi$

$\phi_{curved} = \frac{q}{4\pi\epsilon_0} \times 2\sqrt{3}\pi = \boxed{\frac{q\sqrt{3}}{2\epsilon_0}}$

CONDUCTOR

[METAL]



It has infinite no. of free e^- wich can move inside the volume or, on the vol. or, on the surface if External force is applied, They can not leave conductor.

concept of Electrostatic Equilibrium

suppose by some mecharenism an excess charge +q is given to a conductor.

→ To gain minm energy state, all the charge come on surface of conductor.

→ When all the charge come in state of Rest, this equilibrium is termed as electrostatic equilibrium.

→ net force on each charge along the surface become zero.

In state of Electrostatic equilibrium

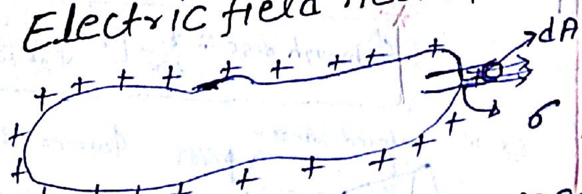
* Electric field along the surface is zero as $\vec{F}_{net\,surfce} = 0$.
* Electric potential of whole body becomes some.
* Electric field inside the body of conductor become zero.

$$\boxed{\vec{E}_{inside} = 0}$$

* It can be assumed as lowest energy state of conductor.
* Electric field lines start ⊥r from the surface of conductor at every point.

Electric field near the surface of conductor.

case I ⇒ Electric field near the surface of conductor.



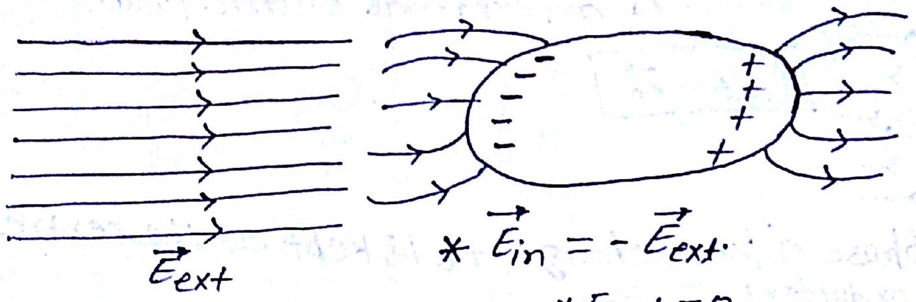
* Charge distribution may not be uniform.
* Surface charge density at diffent points will be different.

$$\oint \vec{E}\cdot d\vec{s} = E \cdot ds = \frac{q_{in}}{\epsilon_0}$$

$$E ds = \sigma \frac{ds}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}}$$ ⊥r to surface.

Case II ⟹ conductor left in an external electric field



* $\vec{E}_{in} = -\vec{E}_{ext}$

* $E_{net} = 0$

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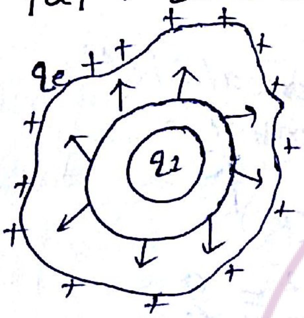
Case III ⟹ Conductor Having cavity :→

|a| → Excess charge given but no charge in cavity



* under electrostatic condition

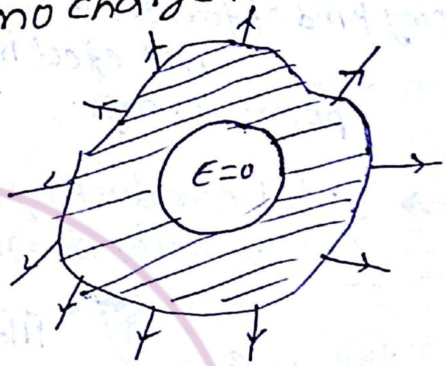
$V_{conductor} = const$

$E_{inside} = 0$

$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\epsilon_0} = \frac{q_1}{\epsilon_0}$$

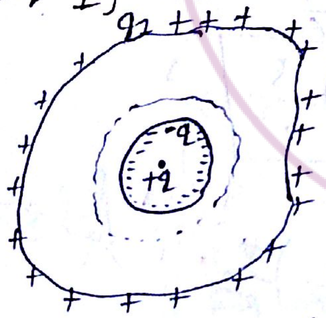
$\boxed{\vec{E} = 0}$

$\therefore \boxed{q_{in} = 0}$



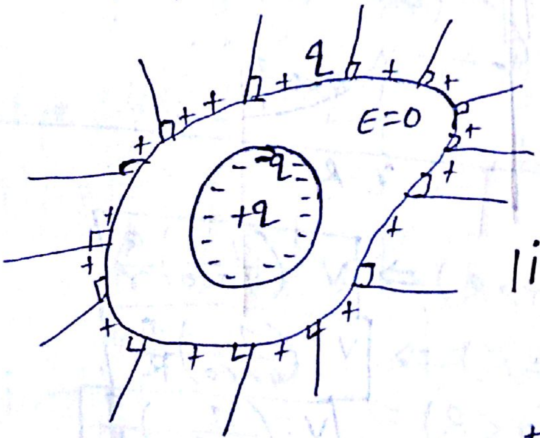
|b| → A point charge is kept inside the cavity :→

|i| → If no. charge given to conductor →



$$\oint \vec{E}\cdot d\vec{s} = 0$$

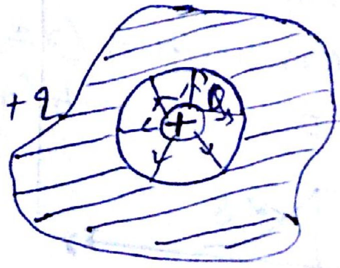
$$\frac{q - q_1}{\epsilon_0} = 0 \Rightarrow \boxed{q_1 = q}$$



Electric field inside cavity at a distance r from O

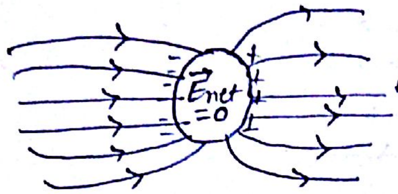
$\Rightarrow \boxed{E = \frac{Kq}{r^2}}$

|ii| → If additional charge is also given to the conductor.



$\boxed{Net = +Q + q}$

** **case IV** ⇒ **Electrostatic shielding** ⇒ suppose a conductor is kept in an external electric field.



$$\boxed{E_{out} = \vec{E_0}}$$

Now, suppose a point charge $+q$ is kept at the centre of conductor

$$\boxed{\vec{E}_{ext} = \vec{E_0}}$$

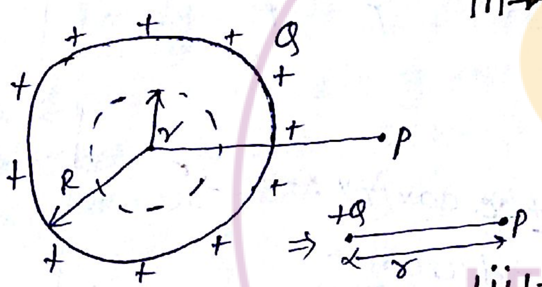
(ii)

AIIMS ** ***

Q → Will $+q$ experience any force ?

Ans → If charge $+q$ is placed inside conductor, it will not expirience any kind of force, it is known as Electrostatic shielding. Net effect inside conductor will be only due to point charge.

case V ⇒ solid conducting sphere or, Hollow spherical shell (uniformally charged) Electric field Intensity :→



|i| → For inside point $(r<R)$

$$\oint \vec{E}\cdot d\vec{s} = E\cdot 4\pi r^2 = \frac{0}{\epsilon_0}$$

$$\boxed{E_{inside} = 0}$$

|ii| → For outside point $(r>R)$

$$\oint \vec{E}\cdot d\vec{s} \quad E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\boxed{E_{outside} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2}}$$

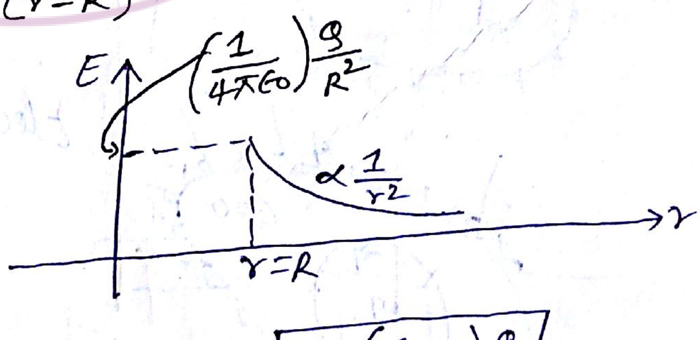
AIIMS

|iii| → on surface (just outside) $(r=R)$

$$\boxed{E = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{R^2}\right)}$$

Just inside ⇒ $(r=R)$

$$\boxed{E = 0}$$



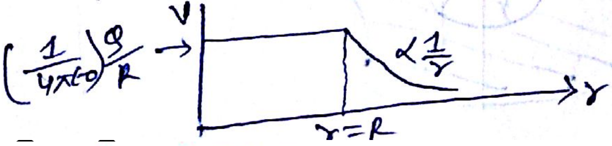
Electric potential

|a| → For outside point $(r>R)$ ⇒ $\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r}}$

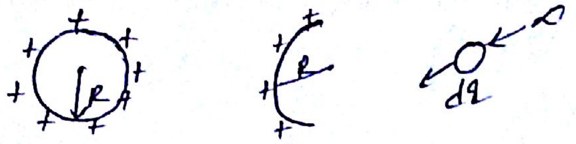
|b| → For on surface $(r=R)$ ⇒ $\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{R}}$

|c| → For Inside point $(r<R)$ ⇒ $\boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{R}}$

= const = $V_{surface}$.

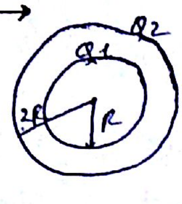


Self potential energy of conducting shell ⇒



$$\boxed{W_{ext} = \frac{Kq}{2R} = \frac{q^2}{8\pi\epsilon_0 R} = U_{self}}$$

Ex →



$U_{system} = ?$

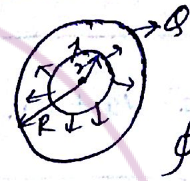
$$\boxed{U_{system} = U_{self} + U_{Interaction}}$$

$$U_{self} = \frac{q_1^2}{8\pi\epsilon_0 R} + \frac{q_2^2}{8\pi\epsilon_0 (2R)}$$

$$U_{Interaction} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{2R}\right)$$

Uniformally charged Non-conducting solid sphere :–

(a) → Electric field $(E(r))$



$$\rho = \frac{q}{\frac{4}{3}\pi R^3}$$

$$\oint \vec{E}\cdot d\vec{s} = E\cdot 4\pi r^2$$

$$q_{in} = \rho \times \frac{4}{3}\pi r^3$$

(i) → $r < R$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\cdot\frac{qr}{R^3}}$$

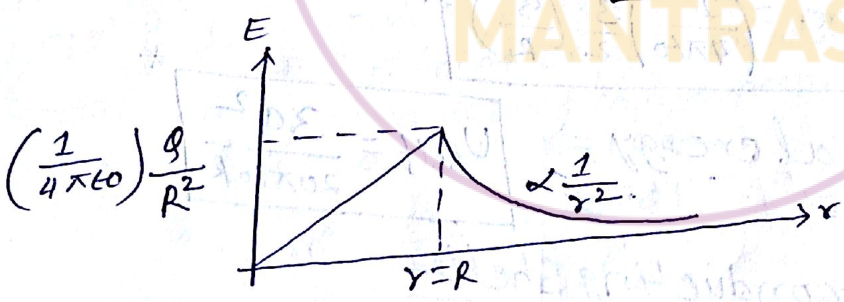
(ii) → $r = R$

$$\boxed{E_{surface} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{R^2}}$$

(iii) → For outside points whole charge of sphere can be assumed at centre. $(r > R)$

$$E\cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\boxed{E = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2}}$$



(b) → Electric potential $V(r)$ →

(i) → $r > R$ (For outside point)

$$\boxed{V_{(r)} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r}}$$

(ii) → $r = R$ (at surface)

$$\boxed{V_{surface} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{R}}$$

(iii) → $r < R$ (For Inside point)

$$-\frac{dV}{dr} = E$$

$$-\int_{V_{surface}}^{V_{inside}} dV = \int_{r=R}^{r=r} E \cdot dr$$

$V_{Inside} = V_1 + V_2$

V_1 ↳ due to point $r < r$

V_2 ↳ due to point $r < r < R$

$$\boxed{V_1 = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{qr^3}{R^3}}$$

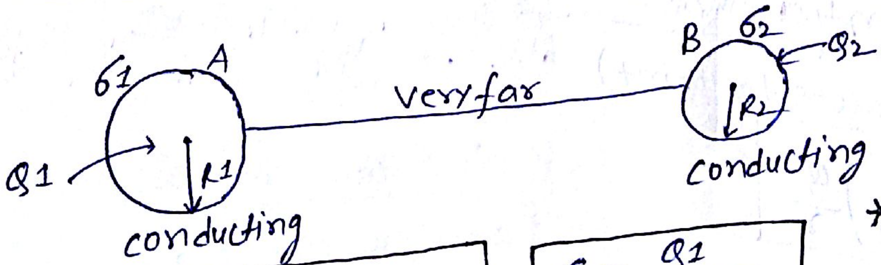
*

$$\boxed{V_{Inside} = V_1 + V_2 = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{2R^3}(3R^2 - r^2)}$$

$$\boxed{V_{centre} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{3}{2}\left(\frac{q}{R}\right)}$$

(C) → self potential energy ⇒ $\boxed{U_{self} = \frac{3q^2}{20\pi\epsilon_0 R}}$ *

connecting two conducting Shells



$$\boxed{V_A = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q_1}{R_1}}$$

$$\boxed{V_B = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q_2}{R_2}}$$

$$\boxed{\sigma_1 = \frac{Q_1}{4\pi R_1^2}}$$

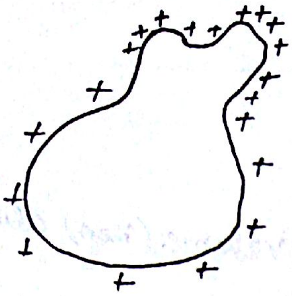
$$\boxed{\sigma_2 = \frac{Q_2}{R_2^2}}$$

*

$$\boxed{\sigma_1 R_1 = \sigma_2 R_2}$$

***

$$\boxed{\sigma \propto \frac{1}{\text{Radius of curvature}}}$$

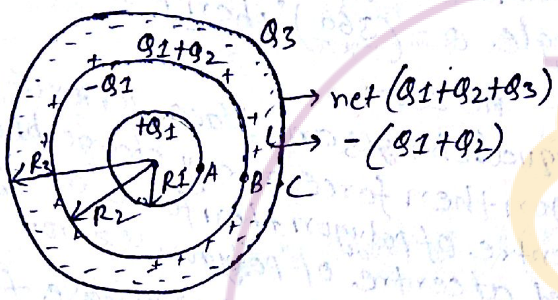


It is clear that at sharp edges surface charge density become too high.

$$\sigma = \frac{1}{\text{Radius of curvature}}$$

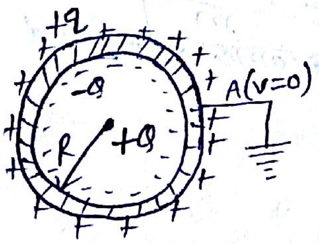
!! ** AIIMS* → If Electric field just outside the conductor become greater than 3×10^6 V/m, break down of air malecule near conductor start which is commonly known as 'Corona Discharge' (Ionisation of air Malecules)

Potential calculating in conducting shells



$$\boxed{\begin{aligned} V_A &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1}{R_1}+\frac{Q_2}{R_2}+\frac{Q_3}{R_3}\right) \\ V_B &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1}{R_2}+\frac{Q_2}{R_2}+\frac{Q_3}{R_3}\right) \\ V_C &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1+Q_2+Q_3}{R_3}\right)\end{aligned}}$$

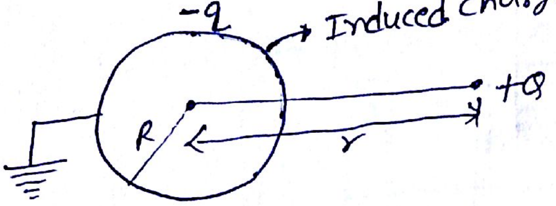
NOTE ⇒



* By convention potential of earth is assumed to be zero.
* Two points which are earthed can be connected by conducting wire. ($V = \text{const} = 0$)
* The point or body connected to earth can recive or send desired charge accordingly charge conservation law will not hold.

$$V_A = \frac{KQ}{R} + \frac{K(q-Q)}{R} = 0 \Rightarrow q = 0$$

*** Q → Induced charge on sphere = ?



$$V_{sphere} = V_{centre}$$

$$= \frac{KQ}{r} - \frac{Kq_1}{R} = 0$$

*

$$\boxed{q_1 = \frac{QR}{r}}$$

Point from question

AIIMS * $\boxed{F > Ab \cdot C > coulumb > stat \cdot coulumb}$

* Faraday > Ab·C > coulumb > stat·coulumb

* $\boxed{m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$ $m_0 \Rightarrow$ rest mass.

* Rest mass of photon zero.
* When a body is charge (either (+ve)ly or (−ve)ly) its volume (size) always ↑ & density always ↓
* In all type of Rkn̈ total charge of system remain const.
* Total no. of ions in a universe is const → wrong.
* $\boxed{1 \le \varepsilon_r \le \infty}$
* $K = [M^{-1}L^{-3}T^{4}A^{2}]$ → kappa particle
* When dielectric medium place b/w charge then electric force direction.
* When metal is placed b/w charge then electric force become zero.

#

θ	F_R
0°	2F
2016 AIPMT * 60°	$\sqrt{3}F$
90°	$\sqrt{2}F$
120°	F
180°	0

* If an equal forces are acting at an angle $\theta = \left(\frac{360}{N}\right)^\circ$ then Resultant will be zero.
* If equal charges are place vertex of Regular polygon then force on any charge placed at centre of polygon then force on any charge placed at centre of polygon is zero.
* A charge particle (q, m) is Release from rest in an uniform E·field then K·E after time 't' ⇒ $\boxed{K \cdot E = \frac{1}{2}\frac{q^2E^2t^2}{m}}$

* When temp. of dielectric medium ↑ then molecule of medium become disturbed so net induced electric field ↓ that's why ε_r also ↓.

temp ↑ ⇒ E_{ind} ↓ $\left(\frac{E_0}{\varepsilon_r} = E_0 - E_{ind}\right)$

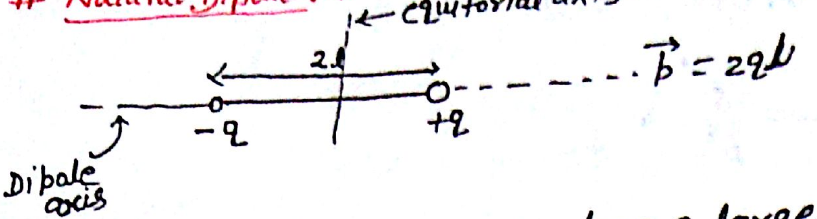
$$\boxed{\downarrow \varepsilon_r = \frac{E_0}{(E_0 - E_{ind})\uparrow}}$$

* Hollow sphere में centre to surface pot. same होता है। तथा centre to surface pot. calculate करने हेतु radius काम में लेते हैं। न कि दूरी !!
* Displacement current is same as is not a conduction current but caused by time ~~time~~ varying electric field.

*

AIIMS, IIT

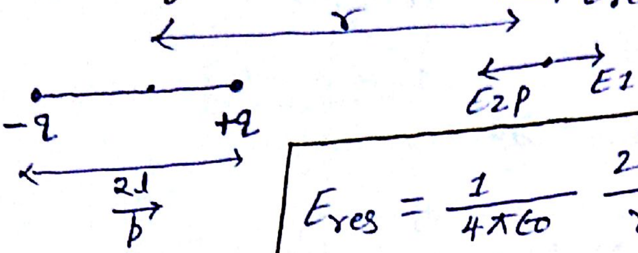
Natural Dipole :→



a large distance 'r' on dipole axis:→

Case I ⇒ $\vec{E}$ along a point at ~~[illegible]~~ a large distance 'r' on dipole axis

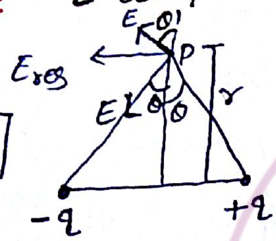
$\rightarrow E_{res} = E_1 - E_2$



$$\boxed{E_{res} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}} \quad (r >> l)$$

Case II ⇒ $\vec{E}$ at point on equitorial Axis →

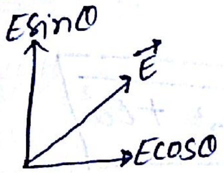
$\boxed{r >> l}$

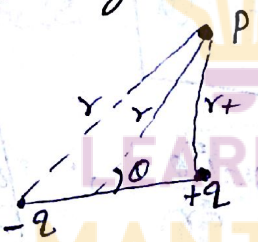


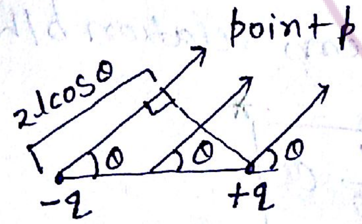
$$\boxed{E_{res} = 2E\sin\theta}$$

$$\boxed{\vec{E}_{res} = \frac{1}{4\pi\epsilon_0}\frac{(-\vec{p})}{r^3}} \; *$$

Case III ⇒ Electric pot. at a general point (r, θ) →







* If point P at very large distance.→

$$= \frac{q(2l\cos\theta)}{4\pi\epsilon_0 r^2}$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\left(\frac{p\cos\theta}{r^2}\right)} \; *$$

* At a point on dipole axis:→

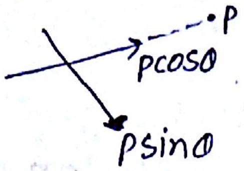
$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2}}$$

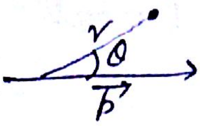
* At a point on equitorial axis →

$$\theta = 90^\circ$$

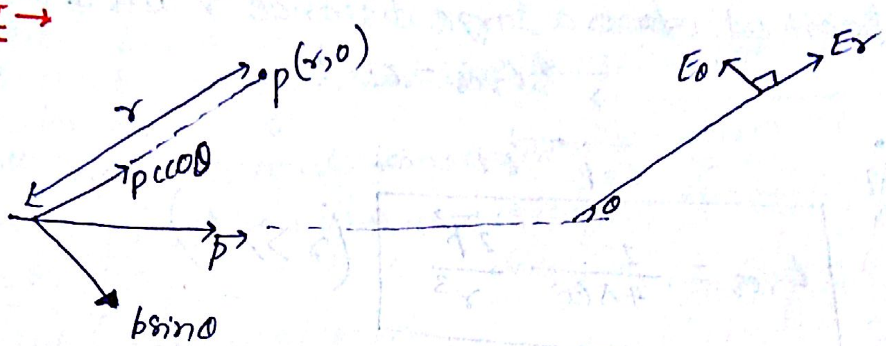
$$\boxed{V = 0} \; *$$

NOTE → Dipole as in form of components





Case IV →



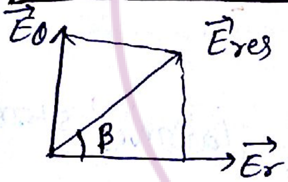
Electric field due to

|a| → pcosθ

$$\boxed{\vec{E_r} = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}\hat{r}}$$

|b| → psinθ

$$\boxed{E_\theta = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}\hat{\theta}}$$

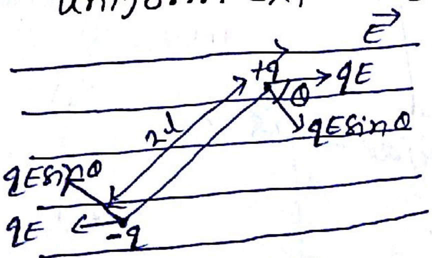


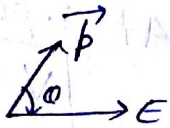
$\tan\beta = \frac{E_\theta}{E_r}$, $\boxed{E_{res} = \sqrt{E_r^2 + E_\theta^2}}$

NOTE ⇒ In case of polar co-ordinate system, relation b/w $\vec{E}$ & V can be given by

$$* \boxed{\vec{E} = -\frac{\partial V}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}} *$$

Case V ⇒ Net force & torque acting on dipole kept in uniform external $\vec{E}$.





$$\boxed{\vec{F}_{net} = 0}$$

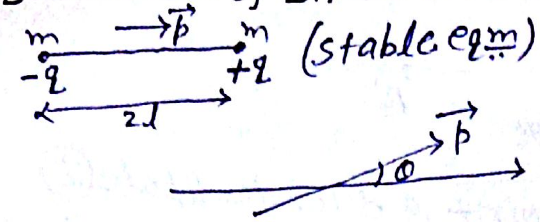
calculating $\vec{\tau}$ about centre of mass

$\tau = 2(qE\sin\theta)l$
$= 2qE\sin\theta l$

$\tau = pE\sin\theta$

$* \boxed{\vec{\tau} = \vec{p} \times \vec{E}} *$

Case VI ⇒ Oscillation of Dipole in uniform $\vec{E}$ →



Linear oscillation

$$\boxed{\omega = \sqrt{\frac{K}{m}}}$$

Angular oscillation

$$\boxed{\omega = \sqrt{\frac{K}{I}}}$$

$$\tau = -PE\sin\theta$$

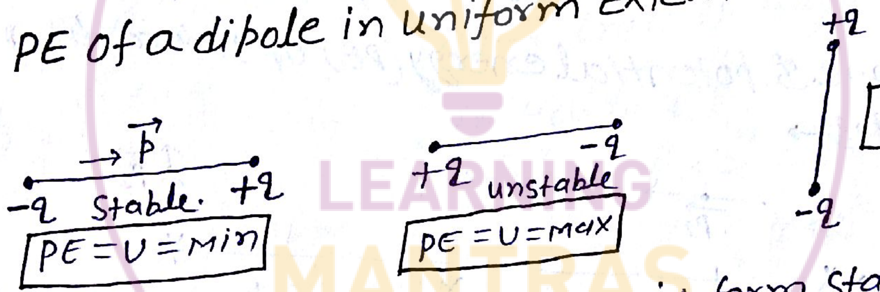
$\because$ θ is very small

$\sin\theta \approx \theta$ *

$$\boxed{\text{Time period} = T = 2\pi\sqrt{\frac{I}{PE}}}$$ *

$$\boxed{I = ml^2 + ml^2 = 2ml^2}$$ *

Case VII ⇒ PE of a dipole in uniform External field :→



$$\boxed{U_{min} < U < U_{max}}$$

" Work done on dipole in rotating it form stable position is stored in the form of PE of dipole."

suppose from general position $\theta = 0$; dipole is rotated very slowely through an angle '$d\theta$'

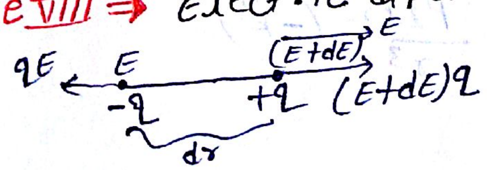
$$U = -PE\cos\theta$$ *

$$\boxed{U = -\vec{p}\cdot\vec{E}}$$ *

If dipole is rotated from $\theta = \theta_1$ to $\theta = \theta_2$ work done by ext agent $= W_{ext} = U_f - U_i$

$$\boxed{W_{ext} = PE(\cos\theta_1 - \cos\theta_2)}$$ **

Case VIII ⇒ Electric dipole in non-uniform electric field →



Net force $= F = q(dE)$

$= q(dr)\left(\frac{dE}{dr}\right)$ *

$$\boxed{F = P\left(\frac{dE}{dr}\right)}$$

Case IX ⇒ Force of interaction b/w two dipole →

r = very large.

Method ① Electric field at point A due to dipole ①

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{2P_1}{r^3} \qquad \frac{dE_1}{dr} = \frac{1}{4\pi\epsilon_0}\frac{(-6)P_1}{r^4}$$

* Force acting on dipole ②

$$\boxed{F = P_2\left(\frac{dE_1}{dr}\right) = -\frac{1}{4\pi\epsilon_0}\frac{6P_1P_2}{r^4}}$$

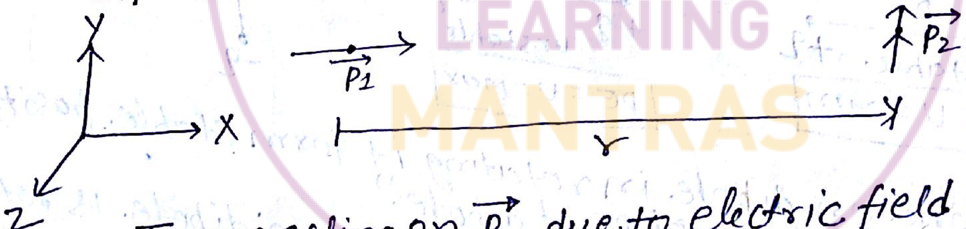
NOTE ⇒ $\frac{dE}{dr} \longrightarrow \ominus$ve ← sign indicates that force is attractive in nature.

Method ② - PE of Dipole ② kept in electric field of dipole ① is given by –

$$U = -P_2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{2P_1}{r^3}$$

✓ Force $= F = -\frac{dU}{dr} = -\frac{1}{4\pi\epsilon_0}\frac{6P_1P_2}{r^4}$

Case X ⇒ Torque & potential energy (PE) of interaction b/w two dipoles :→



Torque acting on $\vec{P_2}$ due to electric field of $\vec{P_1}$

$\vec{P_1} = P_1\hat{i}$, $\vec{E_1}$ at distance $r \Rightarrow \vec{E_1} = \frac{1}{4\pi\epsilon_0}\frac{2P_1}{r^3}\hat{i}$

$\vec{P_2} = P_2\hat{j} \qquad \vec{\tau_{P_2}} = \vec{P_2}\times\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2P_1P_2}{r^3}(-\hat{k})$

$\vec{E_1}$ at distance 'r' $\Rightarrow \vec{E_2} = \frac{1}{4\pi\epsilon_0}\frac{2P_2}{r^3}(-\hat{j})$

(due to $\vec{P_2}$ at $\vec{P_1}$)

$\vec{P_1} = P_1\hat{i}$

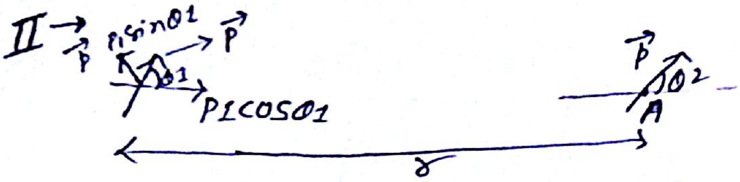
$$\vec{\tau_{P_1}} = \vec{P_1}\times\vec{E_2} = \frac{1}{4\pi\epsilon_0}\frac{P_1P_2}{r^3}(-\hat{k})$$

For General System of Dipoles interaction energy (U) ÷→

I →

$$\boxed{U = -\frac{1}{4\pi\epsilon_0}\frac{2P_1P_2}{r^3}}$$

II →



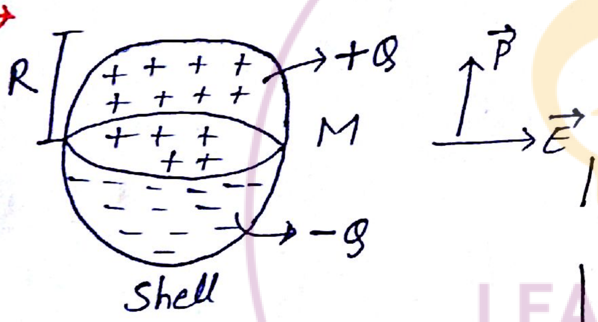
Electric field due to $\vec{P_1}$ at point A

$$\vec{E}_{P_1} = \frac{1}{4\pi\epsilon_0}\frac{2P_1\cos\theta_1}{r^3}\hat{i} - \frac{1}{4\pi\epsilon_0}\cdot\frac{P_1\sin\theta_1}{r^3}\hat{j}$$

$$\vec{P_2} = P_2\cos\theta_2\,\hat{i} + P_2\sin\theta_2\,\hat{j}$$

* $$\boxed{U = -\vec{P_2}\cdot\vec{E}_{P_1}}$$

***** Q →



If the shell is Released from the position shown, then Find -

(a) → Initial Angular Accln -

Initial torque

$$\vec{\tau_i} = \vec{P}\times\vec{E} = PE = I\alpha$$

$$I = \frac{2}{3}MR^2$$

$$\alpha = \frac{3PE}{2MR^2} = \frac{3EQR}{2MR^2} = \boxed{\frac{3EQ}{2MR}}$$

(b) → Angular Speed when it Rotated through $\theta = 90°$ *

$$\boxed{\omega = \sqrt{\frac{2EQR}{I}}}$$