Handwritten Notes
On
Elasticity

ELASTICITY

If We apply any force on any body then the shape of body will change & after Removing the force body again come back to its original shape. then that property of body is called Elasticity. & the force applied on body is called deforming force.

\# Stress = $\frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$ * SI unit = N/m^2 * $[ML^{-1}T^{-2}]$

\# Strain = $\frac{\text{change in configuration (length or, volume)}}{\text{origenal configuration (length or, volume)}}$

\# Hook's Law → $\boxed{\text{Stress} \propto \text{Strain}}$

$\boxed{\text{Stress} = \epsilon \times \text{Strain}}$ ϵ → cofficient of Elasticity.

It is of three types–

|i| → young Modulus of Elasticity (Y) →

$$Y = \frac{\text{Longitudnal Stress}}{\text{Longitunal Strain}}$$

$$\boxed{Y = \frac{F/A}{\Delta L/l}}$$

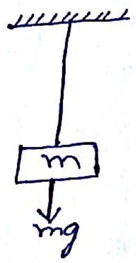
$$\boxed{Y = \frac{FL}{A\Delta L}}$$

A → Area of cross section

l → orginal length

ΔL → change in length.

|a| → change in length of a Massless wire when 'M' Mass is hanged at Lower end →



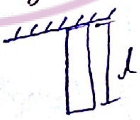
$$\boxed{Y = \frac{FL}{A\Delta L}}$$

r → Radius of Wire

$$\boxed{\Delta L = \frac{mgl}{YA}}$$

$$\boxed{\Delta L = \frac{mgl}{4\pi r^2}}$$

|b| → change in length due to its own weight of a uniform of mass 'm'



$$* \boxed{\Delta L = \frac{mg(l/2)}{4\pi r^2}}$$

$$\boxed{\Delta l = \frac{mgl}{2Y\pi r^2}}$$

|c| → potential energy stored in a streached wire–

$F = K\Delta L$ (like spring)

so, ~~because~~ behave like spring of force const.

$\boxed{K = \frac{YA}{l}}$

pot. energy stored ⇒ $\boxed{U = \frac{1}{2}K\Delta L^2}$ $\boxed{AL = U(\text{vol.})}$

$$\boxed{U = \frac{1}{2}\left(\frac{YA}{L}\right)(\Delta L)^2}$$

$$\boxed{\frac{U}{V} = \frac{1}{2} \times \text{Stress} \times \text{strain}}$$

$$\boxed{\frac{U}{V} = \frac{1}{2}\frac{(\text{stress})^2}{Y}}$$

(ii) → Bulk Modulus (B) or, K (Maxwell) →

$$B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\boxed{B = \frac{-(\Delta P)V}{\Delta V}}$$

$$\boxed{\text{compressibility } (c) = \frac{1}{B}}$$

(a) → For same $\Delta P, V$

$\boxed{B \propto \frac{1}{\Delta V}}$ $\boxed{B_{solid} > B_{Liq} > B_{gas}}$

$\because$ $\boxed{(\Delta V)_{solid} < (\Delta V)_{liq} < (\Delta V)_{gas}}$

(b) → There are two bulk Modulus in gases →

* Isothermal bulk modulus $= P \Rightarrow P \to gas$
* Adiabatic " " γP $\gamma \to$ gama/γ

Isothermal

$PV = \text{const}$

$P\Delta V + V\Delta P = 0$

$P\Delta V = V\Delta P$

$-\frac{V\Delta P}{\Delta V} = P$

$\boxed{B_{isothermal} = P}$

Adiabatic

$PV^{\gamma} = \text{const}$

$\gamma P = -\frac{(\Delta P)V}{\Delta V}$

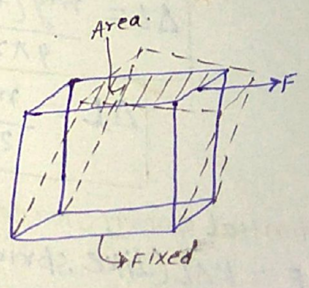
$\boxed{B_{adiabatic} = \gamma P}$

(iii) → Rigidity Modulus / shear modulus (η)

$$\eta = \frac{\text{Shear Stress}}{\text{Shear strain}}$$

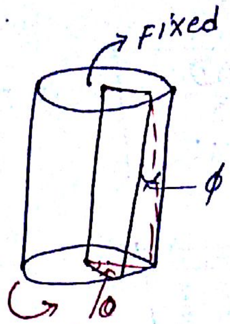
$\boxed{\eta = \frac{F/A}{\theta}}$ ($\theta \to$ shear angle)

$\boxed{\eta = \frac{F}{A\theta}}$



NOTE → * Twisting of solid cylinder.
* Angle of shear always take in Radian.

$\theta \rightarrow$ angle of twist
$r \rightarrow$ radius
$L \rightarrow$ Length
$\phi \rightarrow$ Shearangle



$AB \Rightarrow r\theta = L\phi$

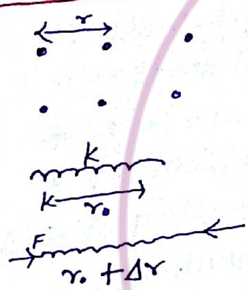
** $\boxed{\text{Shearangle } \phi = \frac{r\theta}{L}}$

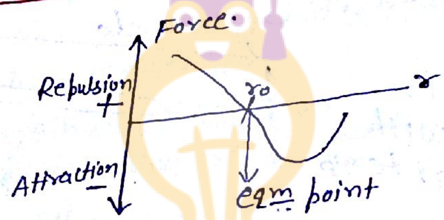
*** # A Wire has length L_1 When tension is T_1 & length is found to be L_2 When tension is T_2. Find its natural length.

$$\boxed{L = \frac{L_1 T_2 - L_2 T_1}{T_2 - T_1}} \; *$$

A Massless wire of length 'L' Radius 'r' is suspended vertically & pulled by force 'F' then change in length is 'l' another wire made of of same material having length '2L' & radius '2r' & pulled by force 2F then change in Length will be $\rightarrow$ $\boxed{l' = l}$ *

Explansion of Elasticity by Interatomic force $\rightarrow$

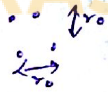




If $K \rightarrow$ Interatomic force const.
then, $F = K\Delta r$ — (I)

Strain $= \frac{\Delta r}{r_0}$ — (II)

corss secction area 'A'



no. of atom $n = \frac{A}{r_0^2}$

Total Force $F_T = nF$
$= \frac{A}{r_0^2} K\Delta r$

$\frac{F_T}{A} = \frac{K\Delta r}{r_0^2}$

Stress $= \frac{K\Delta r}{r_0^2}$ — (III)

* $\boxed{\text{Stress} = Y \times \text{strain}}$

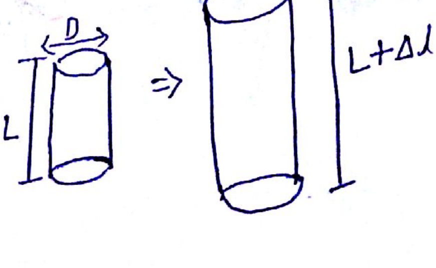
$\frac{K\Delta r}{r_0^2} = Y \times \frac{\Delta r}{r_0}$

* $\boxed{K = Y r_0}$

$r_0 \rightarrow$ Interatomic distance.

*** # Poisson's Ratio (σ) :$\rightarrow$

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudnal strain}}$$



$$\boxed{\sigma = \frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}}}$$

* If volume is const on streching

Then, $V = \text{const}$

$$\frac{\pi D^2}{4} L = \text{const.}$$

$$D^2 L = \text{const.}$$

$$\boxed{0 < \sigma < \frac{1}{2}}$$

$$\boxed{\sigma = \frac{1}{2}}$$

MPPMT

Relation b/w Y, β, η, σ

young → Y, Bulk → β, Shear → η, poisson → σ

(i) → $\boxed{Y = 3\beta(1-2\sigma)}$

(ii) → $\boxed{Y = 2\eta(1+\sigma)}$

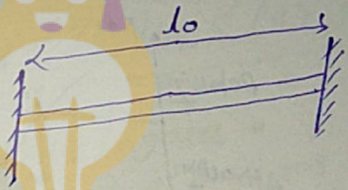
(iii) → $\boxed{\frac{9}{Y} = \frac{1}{\beta} + \frac{3}{\eta}}$

BHU

$\sigma = -1 < \sigma < 0.5$ (Theoritical limit)

$\sigma = 0.2$ to 0.4 (Experimental limit)

Thermal stress →



Let, a Rod of length l_0 is clamped at its end with rigid support & then temp. is increased by $\Delta\theta$

$\therefore$ Its length will be

$$l = l_0(1 + \alpha\Delta\theta)$$

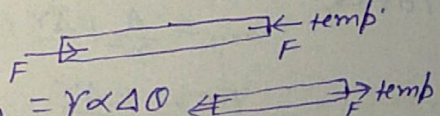
$$l - l_0 = l_0 \alpha \Delta\theta$$

$$\Delta l = l_0 \alpha \Delta\theta$$

$$\frac{\Delta l}{l_0} = \alpha\Delta\theta$$

$$\boxed{\text{Thermal stress} = Y\alpha\Delta\theta}$$

Let $A \to$ area of cross section



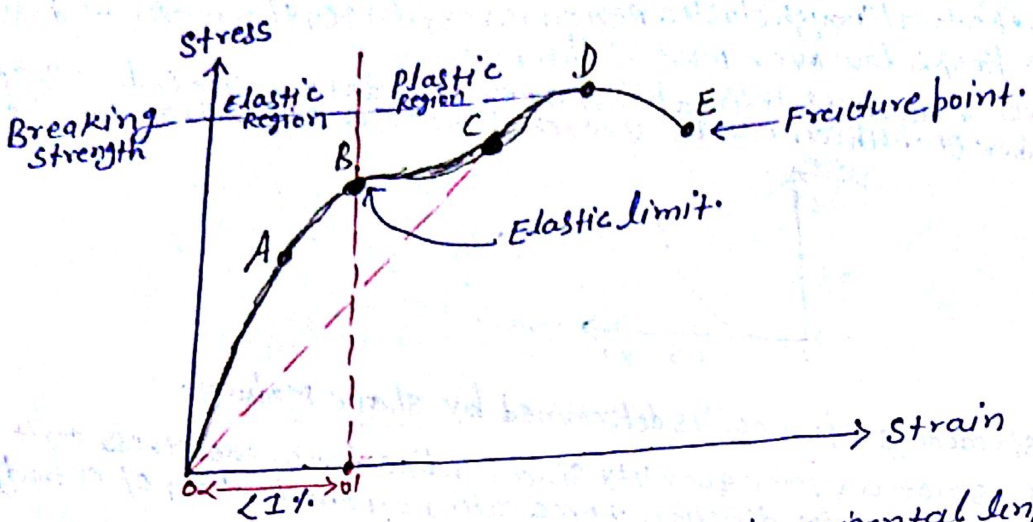
then, thermal stress $= F/A = Y\alpha\Delta\theta$

$$\boxed{F = YA\alpha\Delta\theta}$$

Energy stored/volume $= \frac{1}{2}$ stress $\times$ strain

$$\boxed{= \frac{1}{2} Y(\alpha\Delta\theta)^2}$$

Strain - stress curve →

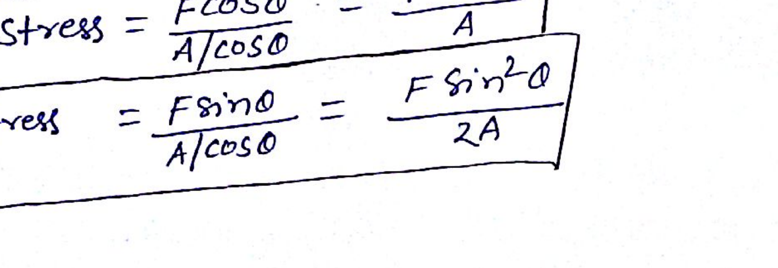


* OA → Fallows Hooks law & wire return in origental length when weight/Force is Removed.
* AB → Doesn't fallow Hooks law.
* BC → When weight removed, some permanent strain Remain.
* CD → Little extra stress cause large strain.
* D → Max stress without breaking.
* → If metal has very small plastic Region there called brittle material & having large plastic region called ductile material.

→ Elastic Fatigue → When weight on wire is applied & Removed continuosly then after some time it losses its elastic property called Elastic Fatigue.

→ Elastic After Affect → Time taken by material to regain its origenal shape when deforming force is Removed, there are some material like quartz**, Phosphor bronze** regain its origenal shape imadiately after deforming force is Removed. i.e these material has no elastic effect.

The force 'F' is applied on the face of Rectangular block as shown in fig. define the tensile & sheer stress at section PQRS.



$\cos\theta = A/S$

$S = A/\cos\theta$

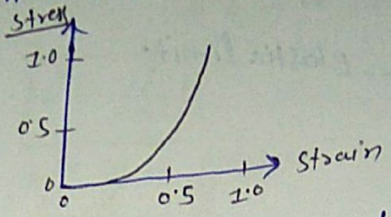
$$\boxed{\text{* Tensile Stress} = \frac{F\cos\theta}{A/\cos\theta} = \frac{F\cos^2\theta}{A}}$$

$$\boxed{\text{* Sheer Stress} = \frac{F\sin\theta}{A/\cos\theta} = \frac{F\sin^2\theta}{2A}}$$

*** → *Rubber can be pulled to several times ~~to~~ its origenal length & still Returns to origenal shape. ~~Stress~~

* Stress & strain curve for elastic tissue of Aurta, prescent in Heart. Note that, although elastic Region is very large, the material does not obey Hooks law over most of the Region.

AIIMS Imp
* There is no well defined plastic region. substance like tissue of Aorta, Rubber etc. Which can be stretched to cause large strains called Elastomer.



* The streaching of a coil is determined by <u>shear Modulus</u>.
* Stress is not a vector quantity since, unlike force, the stress can't be assignet a specific direction. Force acting on the portion of a body on a specific side of a section has definite direction.