Learning Mantras

Handwritten Notes

On

Circular Motion

'CIRCULAR MOTION'

Basic term Related to Angular motion

|1| → Angular displacement (θ) → Body Rotate w.r.t fixed point · Angle b/w Initial & Final position vector.

* When Magnitude of Initial & Final position vector is same.
* unit → Radian (R), Degree (θ)
* Dimensionless.
* Small Angular displacement is a Axial vector & its direction is define from Right hand thumb Rule.
* Large Angular displacement is scal quantity.

NOTE → * Axial vector → Vector Which is along the Rotational axis.

C·W (Inward)   A·C·W (outward)

$|\vec{r_f}|$, $|\vec{r_i}|$, θ

$$\boxed{|\vec{r_i}| = |\vec{r_f}|}$$

* ⟹ Angular displacement in one Rotation is → 2π.
* ⟹ 'N' Rotation ⟹ $2\pi N = \theta$

$$\boxed{\text{Angle} = \frac{\text{Arc}}{\text{Radius}}}$$

$$\boxed{\theta = \frac{\text{Arc}}{r}}$$

$$\boxed{\text{Arc} = \theta r}$$

|2| → Angular velocity (ω) → * Rate of change In Angular disp. represent Angular velocity.

* It is also Axial vector & direction ∥ to the angular disp.

$$\boxed{\vec{\omega} = \frac{\Delta\theta}{\Delta t}} \qquad \boxed{\text{* unit} \to R/sec} \qquad \boxed{\vec{\omega} \parallel \vec{\theta}}$$

$$\boxed{1 \text{ Rotation} \Rightarrow \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n}$$

Angular freq. or, velocity (ω); Time period (T); Frequency (n) [Hz, c.p.s, p.M, R.p.M]

Types of Angular velocity

|i| → uniform Angular velo
Direction + Magnitude same.

|ii| → Non-uniform Angular velocity
Direction change / Magnitude change / Both change.

|iii| → Inst. Angular velocity

$$\boxed{\omega_t = \lim_{t\to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

→ Ist derivative of Angular disp.

|iv| → Avg. Angular velocity

$$\boxed{\vec{\omega}_{avg} = \frac{\text{Total Angular disp.}}{\text{Total time}}}$$

$$\boxed{\vec{\omega}_{avg} = \frac{\vec{\theta}_1 + \vec{\theta}_2 + \vec{\theta}_3 + \cdots + \vec{\theta}_N}{t_1 + t_2 + t_3 + \cdots + t_N}}$$

Standard Result

|i| → Same time Interval.

$$\boxed{\omega_{Avg} = \frac{\omega_1 + \omega_2 + \omega_3 + \cdots + \omega_N}{N}}$$

|ii| → Same Angular displacement.
$(\theta_1 = \theta_2 = \cdots = \theta_N)$

$$\boxed{\omega_{avg} = \frac{N}{\frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3} + \cdots + \frac{1}{\omega_N}}}$$

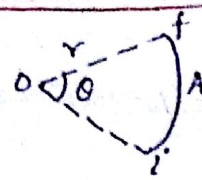
|iii| → $\theta = f(\text{time})$ ⟹ function of time ⟹ $\omega = \frac{d\theta}{dt} = f(t)$

$$\boxed{\langle\omega\rangle = \omega_{avg} = \frac{\int_{T_1}^{T_2} \omega(t)\,dt}{\int_{T_1}^{T_2} dt}}$$

|iv| → $\omega = f(\theta)$

$$\boxed{\langle\omega\rangle = \frac{\int_{\theta_1}^{\theta_2} \omega(\theta)\,d\theta}{\int_{\theta_1}^{\theta_2} d\theta}}$$

Relation b/w Linear velocity & Angular velocity

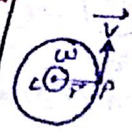


$\boxed{\theta = \frac{Arc}{r}} \Rightarrow \boxed{Arc = \theta r}$

$\boxed{V = r\omega}$ $\boxed{\vec{V} = \vec{\omega} \times \vec{r}}$ *

Linear velocity, Angular velocity, Tangential vector, Angular vector, Radial vector.

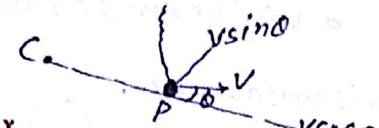
*



$\vec{r} \perp \vec{V}$
$\vec{\omega} \perp \vec{V}$
$\vec{\omega} \perp \vec{r}$

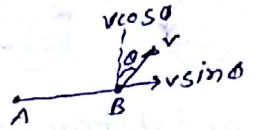
|V| → Relative Angular velocity (ω_R)

$$\boxed{\omega_R = \frac{V_T}{r} = \frac{V\perp}{r}}$$



Relative Angular velocity at point 'P' w.r.t point 'C'

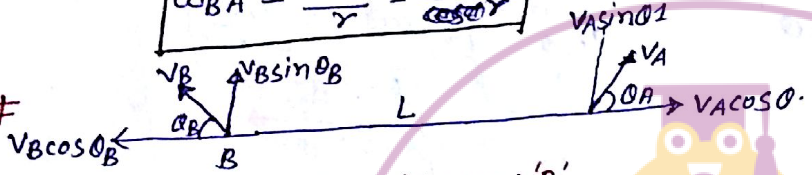
$$\boxed{\omega_R = \frac{V\perp}{r} = \frac{V\sin\theta}{r}}$$



Relative Angular velocity at point 'B' w.r.t. point 'A'

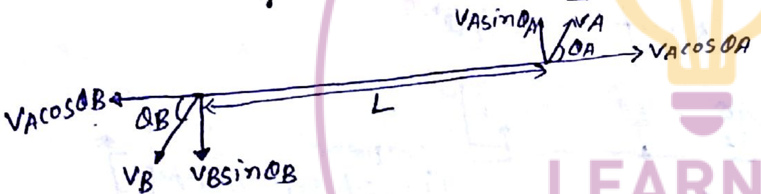
$$\boxed{\omega_{BA} = \frac{V\perp}{r} = \frac{V\cos\theta}{r}}$$

#



Relative Angular velo of 'A' w.r.t 'B'

$$\omega_{AB} = \frac{(V\perp)_{AB}}{r} = \frac{V_A\sin\theta_A - V_B\sin\theta_B}{L}$$

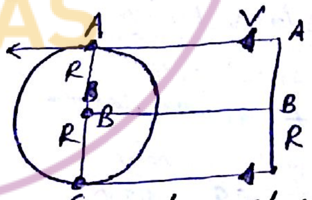


particle Rotate in a circular path of Radius 'R' with velocity 'V'. than Relative Angular velocity of particle 'A' point 'A' w.r.t point 'B' & 'C'.

$$\boxed{\omega_{AB} = \frac{V\perp}{r} = \frac{V}{R}}$$

$$\boxed{\omega_{AC} = \frac{V\perp}{r} = \frac{V}{2R}}$$

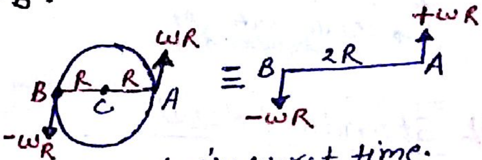
$$\boxed{\frac{\omega_{AB}}{\omega_{AC}} = \frac{V/R}{V/2R} = 2:1}$$



Two particle rotate in circular path of Radius 'R' with angular velocity 'ω' at point 'A' & 'B'. Relative angular velocity of 'A' w.r.t 'B'.

$$\omega_{AB} = \frac{(V\perp)_{AB}}{r} = \frac{\omega R + \omega R}{2R}$$

$$\boxed{\omega_{AB} = \omega}$$ *



|3| → Angular Acceleration (α) → Rate of change in Angular velocity w.r.t time.

$$\boxed{\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}}$$

* unit → R/sec^2
* Angular Accln is axial vector. & It is ∥ to the angular velocity & Angular disp. $\boxed{\vec{\alpha} \parallel \vec{\omega} \parallel \vec{\theta}}$ *

**

|a| → Uniform Accln. → Magnitude + Direction ⇒ same.

|b| → Non-uniform Accln → Magnitude/Direction/both change.

|c| → Inst. Angular Accln → $\boxed{\vec{\alpha}_t = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}}$ *

$\alpha_t = \frac{d\omega}{dt}$ = Ist derivative of Angular velo.

$\alpha_\omega = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$ = 2nd derivative of Angular disp.

$\boxed{\theta = f(t)} \underset{\int \omega\, dt}{\overset{d\theta/dt}{\rightleftarrows}} \boxed{\omega = f(t)} \underset{\int \alpha\, dt}{\overset{d\omega/dt}{\rightleftarrows}} \boxed{\alpha = f(t)}$

$\boxed{\omega = f(\theta)}$

$\rightarrow a = \frac{d\omega}{dt}\left(\frac{d\theta}{d\theta}\right) = \boxed{\omega \frac{d\omega}{d\theta}}$ *

$\rightarrow \omega = \frac{d\theta}{dt} = f(\theta) \Rightarrow \int \frac{d\theta}{f(\theta)} = \int dt \Rightarrow \boxed{\theta = f'(t)}$ *

$\boxed{\theta = f'(t)} \xrightarrow{d\theta/dt} \boxed{\omega = f'(t)}$ *

$\boxed{\omega = f'(t)} \xrightarrow{d\omega/dt} \boxed{\alpha = f'(t)}$ *

(d) → Avg. Angular Acceleration

$$\boxed{\vec{\alpha}_{Avg} = \frac{\vec{\omega}_f - \vec{\omega}_i}{t_f - t_i}}$$

NOTE → Direction of Inst. Acceleration in the direction of torque but direction of Avg Angular Accⁿ in the direction of change in the Angular velocity.

$\alpha = f(t)$ *

$$\boxed{\langle \alpha \rangle = \alpha_{Avg} = \frac{\int_{t_1}^{t_2} f(t)\,dt}{\int_{t_1}^{t_2} dt}}$$

$\alpha = f(\theta)$ *

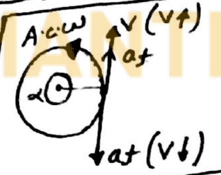
$$\boxed{\langle \alpha \rangle = \frac{\int_{\theta_1}^{\theta_2} f(\theta)\,d\theta}{\int_{\theta_1}^{\theta_2} d\theta}}$$

Relation b/w Linear Accⁿ & Angular Accⁿ

$\boxed{V = \omega r}$ * $\boxed{a = r\alpha}$ *

$\boxed{\frac{d}{dt}(v) = \frac{d}{dt}(\omega r)}$

* $\boxed{\vec{a} = \vec{\alpha} \times \vec{r}}$



$\boxed{\begin{matrix} \vec{r} \perp \vec{a} \\ \vec{\alpha} \perp \vec{r} \\ \vec{\alpha} \perp \vec{a} \end{matrix}}$ *

Eqⁿ of Motion ****

Linear Motion	Angular Motion
* → $V = u + at$	* $\omega = \omega_0 + \alpha t$
* → $S = ut + \frac{1}{2}at^2$	* $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
* → $V^2 = u^2 + 2aS$	* $\omega^2 = \omega_0^2 + 2\alpha\theta$
* $S_{nth} = u + \frac{1}{2}a(2n-1)$	* $\theta_n^{th} = \omega_0 + \frac{1}{2}\alpha(2n-1)$

u = Initial velocity
V = Final velocity
a = Linear Acceleration
S = Linear displacement

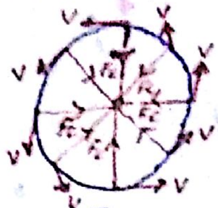
ω_0 = Initial Angular velocity
ω = Final Angular velocity.
α = Angular Acceleration
θ = Angular displacement.

NOTE → Eqⁿ of motion is used only when Angular Accⁿ Remain same.

$\boxed{\begin{matrix} S \to \theta \\ u \to \omega_0 \end{matrix}}\boxed{\begin{matrix} V \to \omega \\ a \to \alpha \end{matrix}}$

Centripetal force & Accelration

When particle Rotate in a circular path direction of particle continuously change that's why force Require to change the direction that's why Require force in circular Motion is called centripetal force. & Its direction is always towards centre of circular path.



\# $F_c = \frac{mv^2}{r} = \frac{m(\omega r)^2}{r} = m\omega^2 r = m(\omega r)$

$$\boxed{\vec{F_c} = \frac{mv^2}{r}(-\hat{r}) = m\omega^2 r(-\hat{r}) = m(\vec{\omega}\times\vec{v}(-\hat{r}))}$$ *

$\hat{r}$ = unit vector along the Radial vector
V = Linear (tangential) velocity [Tangential vector]
ω = Angular velocity [Axial vector]

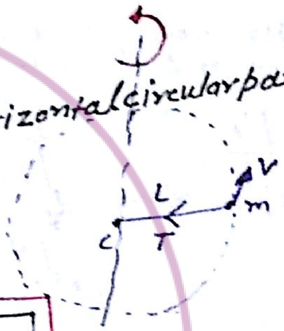
Centripetal Accln ($\vec{a_c}$)

$$\boxed{\vec{a_c} = \frac{\vec{F_c}}{m} = \frac{v^2}{r}(\hat{r}) = \omega^2 r(-\hat{r}) = \vec{\omega}\times\vec{v}(-\hat{r})}$$ *

NOTE → * centripetal force is necessory for circular motion but it can't change speed of particle.
* centripetal force is provided by natural forces like Gravitational, Electrostatic, Friction force etc.

**Ex → of centripetal Force

(i) → When particle attach with string & Rotate in horizontal circular path & Tenson of string is provided by necessory centripetal force.



$$\boxed{F_c = T}$$

$$\boxed{T = \frac{mv^2}{L} = m\omega^2 L = m(\omega v)}$$

**

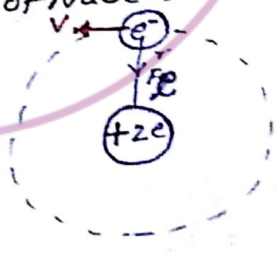
$$\boxed{T_{Max} = \frac{mv^2_{max}}{L} = m\omega^2_{max}L}$$

$$\boxed{V_{Max} = \sqrt{\frac{T_{max}L}{m}}}$$

$$\boxed{\omega_{Max} = \frac{T_{max}}{mL}}$$

T_{Max} = Max possible Tenson in String

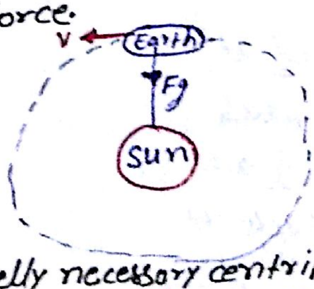
(ii) → Electron Rotate around the Nucleus then necessory centripetal force is provided by Electrostatic attractive force of Nucleus.



$$\left[F_c = F_e = \frac{K(ze)(e)}{r^2}\right]$$

$$\boxed{\frac{mv^2}{r} = \frac{K(ze^2)}{r^2}}$$ *

(iii) → When Earth Rotate around the sun then necessory centripetal force is provided by Gravitational Attractive force.



$$\left[F_c = F_g = \frac{G(M_s)(M_e)}{r^2}\right]$$

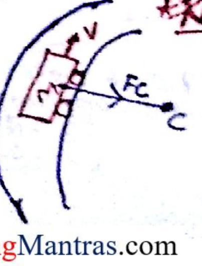
$$\boxed{\frac{m_e v_e^2}{r} = \frac{G(M_s)(M_e)}{r^2}}$$ *

(iv) → In horizontal circular track to turn safelly necessory centripetal force is provided by Friction force.

$\mu(mg) \geq \frac{mv^2}{r}$

$V \leq \sqrt{\mu r g}$

$$\boxed{V_{Max\ safe\ velo} = \sqrt{\mu r g}} \propto m^0$$



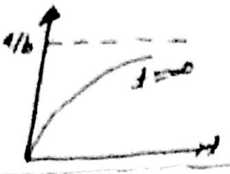
*NOTE → Max safe velo depend on friction coefficient, Radius of circular path, Gravitation Accln & It is Independent from Mass of vechle.

Angular Accel^n of particle is change with Angular velocity as Relation $\alpha = a - b\omega$. where a & b is const. & particle start from Rest. than Angular velocity of particle at time 't'.

$\alpha = \frac{d\omega}{dt} = a - b\omega$

previous Yr of

Ans $\boxed{\omega = \frac{a}{b}\left(1 - e^{-bt}\right)}$



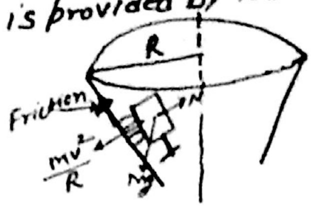
$\boxed{t\uparrow \Rightarrow e^{bt}\uparrow \Rightarrow e^{-bt} = \frac{1}{e^{bt}}\downarrow \Rightarrow (1 - e^{-bt})\uparrow}$ *

~~Centripetal force & Acceleration~~

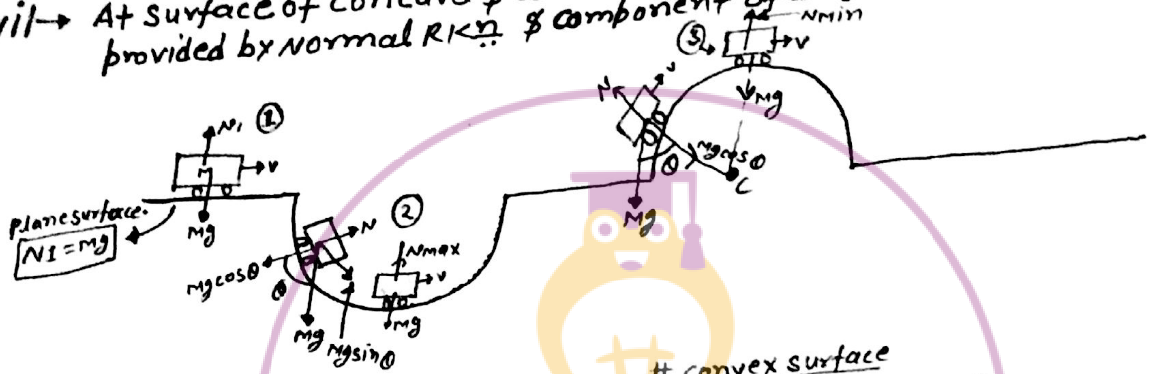
|VI| → In a death well necessory centripetal force is provided by Normal Reaction.

$\boxed{V \geq \sqrt{\frac{Rg}{\mu}}}$ *

$\boxed{V_{\text{Minimum safe velo}} = \sqrt{Rg/\mu}}$ * $\propto m^0$ *



|VII| → At surface of concave & convex briedge necessory centripetal force is provided by Normal R^n & component of weight.



concave surface

$N - Mg\cos\theta = F_c = \frac{Mv^2}{r}$

$\boxed{N = Mg\cos\theta + \frac{Mv^2}{r}}$

$\boxed{(N_{Max})_2 = Mg + \frac{Mv^2}{r}}$ *

Bottom point

convex surface

$Mg\cos\theta - N = F_c = \frac{Mv^2}{r}$

$\boxed{N = Mg\cos\theta - \frac{Mv^2}{r}}$

$\boxed{N_{min} = Mg - \frac{Mv^2}{r}}$ *

Top point

**

# $V\uparrow \Rightarrow N1 = $ same $N_2\uparrow$ $N_3\downarrow$	# $\boxed{N_3 \geq} \Rightarrow$ Safe cond^n. $\boxed{N_3 < 0} \Rightarrow$ unsafe cond^n.

**

Max possible velocity at top point for safe condition.

$\boxed{V_{max} = \sqrt{rg}}$ *

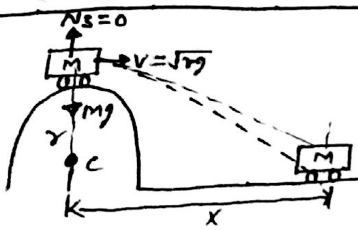
$N_3 \geq 0$

$Mg - \frac{Mv^2}{r} \geq 0$

$\frac{Mv^2}{r} \leq Mg \Rightarrow \boxed{v \leq \sqrt{rg}}$ *

***

For Speed breaker



$\boxed{x = u\sqrt{\frac{2h}{g}}}$

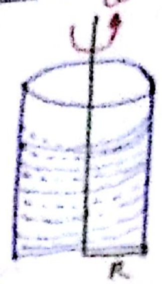
$= (\sqrt{rg})\sqrt{\frac{2r}{g}}$

$\boxed{x = \sqrt{2}r = 1.414r}$ *

S

|viii| → By Rotation of container, all liq. particle moves in circular path & necessary 'centripetal force is provided by pressure difference along radial direction', hence there is a pressure difference in horizontal direction due to this Rotation. *

$$\boxed{\frac{dP}{dr} = \rho r \omega^2}$$



Centrifugal Force (Pseudoforce)

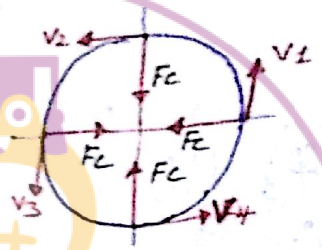
When particle moves in a circular path then from Non inertial frame of Reference force is appeared outward from centre of circular path wich is called pseudoforce. & Its direction Radially outward.

$\boxed{F = ma}$ * * $\boxed{F = \frac{mv^2}{r}(\hat{r}) = m\omega^2 r(\hat{r}) = m(\omega v)\hat{r}}$

NOTE → magnitude of centrifugal force is equal to centripetal force.

Types of circular motion

|1| → Uniform circular Motion



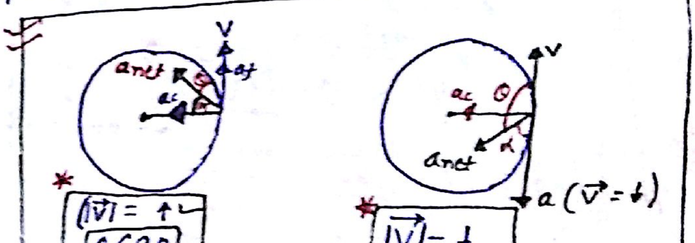
* $|\vec{V_1}| = |\vec{V_2}| = |\vec{V_3}| = |\vec{V_4}|$ ⇒ same
* $\boxed{\vec{\omega} = \text{same}}$ *
* $\boxed{\vec{\alpha} = 0}$ $\boxed{\vec{a_t} = 0}$ *
* $\boxed{|\vec{a_c}| = \frac{v^2}{r}}$ *
* $\boxed{a_{net} = \vec{a_c} = \frac{v^2}{r}(-\hat{r})}$ *
* $\boxed{F_{net} = \vec{F_c} = \frac{mv^2}{r}(-\hat{r})}$ *
* $\vec{V}, \vec{P}$ ⇒ change (magnitude same, direction change) *
* change in velocity $|\Delta\vec{V}| = \boxed{2V\sin(\theta/2)}$
* change in Linear Momentum
  $$\boxed{\begin{aligned}\Delta\vec{P} &= 2mv\sin(\theta/2)\\ &= 2p_i\sin(\theta/2)\end{aligned}}$$ *
* Avg. velocity $|\vec{V}_{avg}| = \boxed{V\frac{\sin(\theta/2)}{(\theta/2)}}$

* Avg. Acceleration *
  $$\boxed{|\vec{A}_{avg}| = \frac{V^2}{R} \cdot \frac{\sin(\theta/2)}{(\theta/2)}}$$
* K.E = same ⇒ $\boxed{\Delta K.E = 0}$ *
* Work done by centripetal Force.
  $\boxed{(\vec{F} \perp \vec{V}) \parallel \vec{S}}$ *
* $\boxed{P = \vec{F_c} \cdot \vec{V} = F_c V\cos 90° = 0}$

|2| → Non-uniform circular Motion

* $|\vec{V_1}| \neq |\vec{V_2}| \neq |\vec{V_3}| \neq |\vec{V_4}|$ ⇒ speed change.
* $\boxed{\vec{\omega} = \text{change}}$
* $\boxed{\vec{\alpha} \neq 0, \vec{a_t} = \frac{dv}{dt} \neq 0}$
* $\boxed{\vec{a_c} = \frac{v^2}{r}, F_c = \frac{mv^2}{r} \neq 0}$
* $\boxed{|\vec{a}_{net}| = \sqrt{a_t^2 + a_c^2}}$ *
* angle of $|\vec{a}_{net}|$ from $|\vec{a_c}|$
  $$\boxed{\alpha = \tan^{-1}\left(\frac{a_t}{a_c}\right)}$$ *
* $\boxed{|F_{net}| = \sqrt{F_c^2 + F_t^2}}$ *
* $\left.\begin{matrix}\vec{V}, \vec{P}, K.E\\ |\vec{V}|, |\vec{P}|\end{matrix}\right]$ = change *

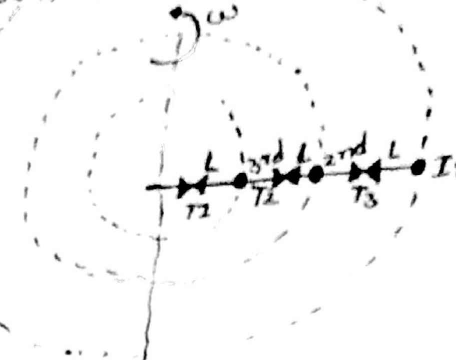
***



* $\boxed{|\vec{V}| = \uparrow}$ $\boxed{\theta < 90}$
* $\boxed{|\vec{V}| = \downarrow}$ $\boxed{\theta > 90°}$

* $\boxed{W = \underset{\substack{\neq\\0}}{\vec{F_t} \cdot d\vec{S}} + \underset{\substack{\parallel\\0}}{\vec{F_c} \cdot d\vec{S}} \neq 0}$ *
* $\boxed{P = \vec{F_c}\,\vec{V} + \vec{F_t} \cdot \vec{V} \neq 0}$

3-particle of mass 'm' is attach with string as shown & Rotate in a Horizontal circular path with uniform Angular velocity. then Ratio of tension in different path of string.



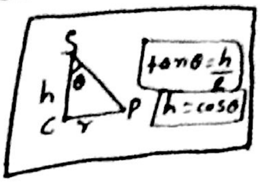
* Ist → $T_3 = m\omega^2(3L)$ — (I)
* 2nd → $T_2 - T_3 = m\omega^2(2L)$ — (II)
* 3rd → $T_1 - T_2 = m\omega^2(L)$ — (III)

After Finding $\boxed{T_2 = 5m\omega^2 L}$, $\boxed{T_1 = 6m\omega^2 L}$

**

$$\boxed{T_1 : T_2 : T_3 = 6m\omega^2 L : 5m\omega^2 L : 3m\omega^2 L = 6 : 5 : 3}$$

Particle of mass 'm' attached with string of Length 'L' & Rotate in a horizontal circular path of Radius 'r'. If centre of circular path 'h' below the point of suspension than time period of ossicilation.



$\boxed{\tan\theta = \frac{\omega^2 r}{g}}$

$\boxed{\omega = \sqrt{\frac{g}{r}\tan\theta}}$

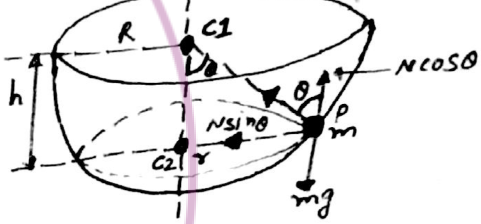
$\boxed{\omega = \sqrt{g/h}}$

**

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

Particle of mass 'm' Rotate in circular path of Radius 'r' in a bowl of Radius 'R'. If centre of circular path 'h' height below from centre of bowl than time period of ossicilation.

$\boxed{\tan\theta = \frac{r}{h}}$



*

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{R\cos\theta}{g}}}$$

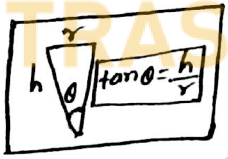
$\tan\theta = \frac{\omega^2 r}{g}$

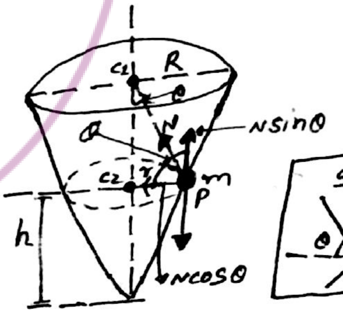
$\boxed{\omega = \sqrt{g\tan\theta}}$

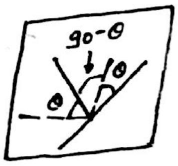
$\omega = \sqrt{\frac{g}{r}\left(\frac{r}{h}\right)}$

$\boxed{\omega = \sqrt{g/h}}$

Same as Previous Q.







$\tan\theta = \frac{g}{\omega^2 R}$

$\boxed{\omega = \sqrt{\frac{g}{r\tan\theta}}}$

$\omega = \sqrt{gh/r^2}$

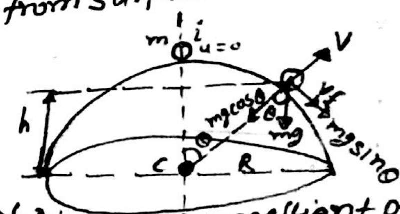
*

$$\boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{gh}}r}$$

Particle of mass 'm' drop from top point having Radius 'R'. Surface is frictionless. than height from the bottom where it leave the contact from surface.

*

$$\boxed{h = \frac{2R}{3}}$$



$N = mg\cos\theta = \frac{mv^2}{R}$ — (1)

$\cos\theta = \frac{2(R-h)}{R}$ — (2)

Particle of mass 'm' placed at distance 'r' from centre of Disc, friction coefficient of surface is 'μ' than max. angular velocity of disc so that particle Ⓒnt at Rest.

*

$$\boxed{\omega_{max} = \sqrt{\frac{\mu g}{r}}}$$

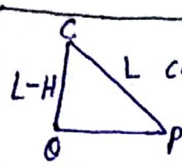
$F_{fr} \geq m\omega^2 r$

$\mu mg \geq m\omega^2 r$

$\boxed{\omega \leq \sqrt{\mu g/r}}$

Vertical circular Motion

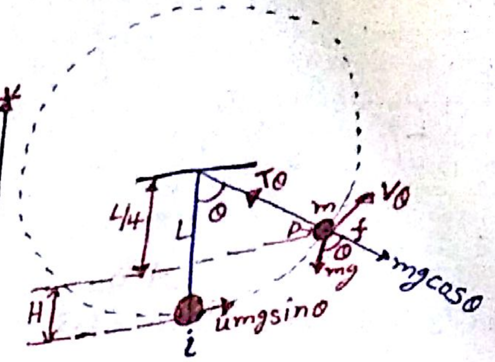
(i) → Velocity at 'θ' angular position



$\cos\theta = \frac{L-h}{L}$

$h = L(1-\cos\theta)$

$V_\theta = \sqrt{u^2 - 2gh}$
$= u^2 - 2gL(1-\cos\theta)$ *



(ii) → Tenson at 'θ' angular position

$$\boxed{T_\theta = mg\cos\theta + \frac{mV_\theta^2}{L}}$$

$$\boxed{T_\theta = mg\cos\theta + \frac{m}{L}\left[u^2 - 2gL(1-\cos\theta)\right]}$$

$$\boxed{T_\theta = \frac{mu^2}{L} + mg(3\cos\theta - 2)}$$

* u = velocity at bottom.
* θ = Angular position
* L = Length of string/ Radius of circular path.

(iii) → A bottom point (θ = 0)

$\boxed{V_{bottom} = u}$ *

$\boxed{T_{bottom} = \frac{mu^2}{L} + mg}$ *

(iv) → At Horizontal point (θ = 90°)

* $\boxed{V_{Horizontal} = \sqrt{u^2 - 2gL}}$ *

* $\boxed{T_{Horizontal} = \frac{mu^2}{L} - 2mg}$ * $\Rightarrow \frac{mv_H^2}{L}$

(v) → Top point (θ = 180°)

$\boxed{V_{Top} = \sqrt{u^2 - 4gL}}$ *

$\boxed{T_{top} = \frac{mu^2}{L} - 5mg}$ *

***

* $T_{max} - T_{min} = T_B - T_T = 6mg$
* $T_B - T_H = 3mg$
* $T_H - T_H = 3mg$.

***

* $V_{bottom} = u$
* $V_{Horizontal} = \sqrt{u^2 - 2gh}$
* $V_{Top} = \sqrt{u^2 - 4gL}$
* $T_{bottom} = \frac{mu^2}{L} + mg = T_{max}$
* $T_H = \frac{mu^2}{L} - 2mg$
* $T_{Top} = \frac{mu^2}{L} - 5mg = T_{min}$.

condition to complete vertical circular path.

$T_{Min} = T_{Top} \geq 0$

$\frac{mu^2}{L} - 5mg \geq 0$

$\boxed{u \geq \sqrt{5mg}}$

* $\boxed{(u_{bottom})_{min} = \sqrt{5gL}}$ *
* $\boxed{T_{bottom} = 6mg}$ *
* $\boxed{V_H = \sqrt{3Lg}}$ *
* $\boxed{V_T = \sqrt{gL}}$ *
* $\boxed{T_{top} = 0}$

***

* $T_B \geq 6mg$
* $T_H \geq 3mg$
* $T_T = 0$
* $V_B \geq \sqrt{5Lg}$
* $V_H \geq \sqrt{3Lg}$
* $V_T \geq \sqrt{Lg} \Rightarrow$ ** V_{min} at top point.
↳ Critical velocity.

AIPMT 2016

***

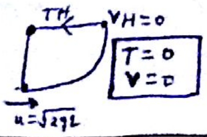
(a) → $\sqrt{2Lg} < V_{bottom} < \sqrt{5Lg}$

* $L < h < 2L$,
* $90° < \theta < 180°$

$T = 0$
$V \neq 0$

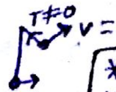
(b) → $V_{bottom} = \sqrt{2gL}$

* $L = h$, $\theta = 90°$



$T = 0$
$V = 0$

(c) → $0 < u < \sqrt{2Lg} \Rightarrow$ simple pendulum



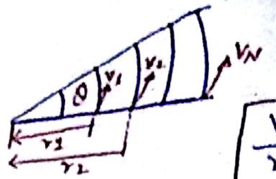
* $h < L$
* $\theta < 90°$
* $T \neq 0$
* $V = 0$

* $T_{bottom} > T_{Horizontal} > T_{Top}$
* $T_{bottom} - T_{Top} = 6mg$
* $T_{bottom} - T_{Horizontal} = 3mg$
* $T_{Horizontal} - T_{Top} = 3mg$

* $u_{bottom} = \sqrt{4lg} \Rightarrow$ Parabolic
* $u_{bottom} = \sqrt{2lg} \Rightarrow \theta = 90° \Rightarrow$ Horizontal
* $u_{bottom} = \sqrt{lg} \Rightarrow \theta < 90° \Rightarrow$ not horizontal or simple pendulum.

* Magnitude of change in velo. b/w Horizontal & bottom $\rightarrow \boxed{= \sqrt{2(u^2 - gl)}}$

When two or ... horizontal circular path-



$$\boxed{\frac{\theta}{t} = \omega = \text{same} = \frac{v}{r}}$$

$$\boxed{\frac{V1}{r1} = \frac{V_2}{r_2} = \cdots \frac{V_N}{r_N}}$$

Vechle of mass 'M' is drop from height 'H' as shown. than value of 'R' If it will complete V.t. circular motion as shown in fig.

$V = \sqrt{2g(H-h)} \geq \sqrt{5Rg}$

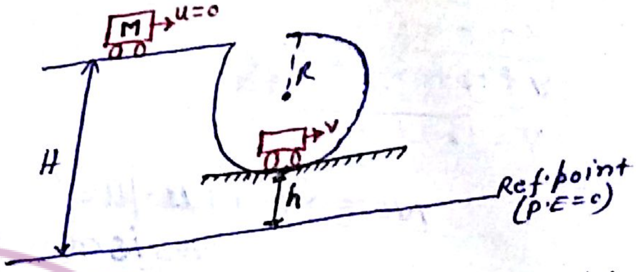
$\boxed{2g(H-h) \geq 5Rg}$ *

$R \leq \frac{2}{5}(H-h)$

$H - h \geq \frac{5}{2}R$

$\boxed{H \geq \frac{5}{2}R + h}$

$$\boxed{\begin{array}{c} h = 0 \Rightarrow H \geq \frac{5}{2}R \\ R \leq \frac{2}{5}H \end{array}}$$ *

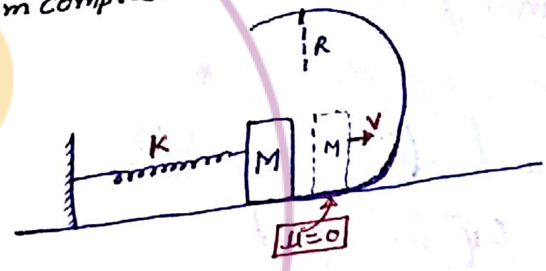


Mass 'M' is compresed with spring of spring coefficient 'K' & Release If it will complete vertical circular path as shown. then Minimum compression in spring.

$V = x\sqrt{\frac{K}{m}} \geq \sqrt{5Rg}$

$X_{min} \geq \sqrt{\frac{m}{K}(5Rg)}$ *

$$\boxed{X_{min} = \sqrt{\frac{m}{K}(5Rg)}}$$



Circular Motion in Daily Life

|1| → Skidding condition

$\mu mg \geq \frac{mv^2}{r}$

$\boxed{V \leq \sqrt{\mu rg}}$ *

$$\boxed{V_{\substack{\text{maximum} \\ \text{safe velo.} \\ \text{without skiding}}} = \sqrt{\mu rg}}$$ *

r = Radius
μ = Friction b/w Road & Tyre. For safe driving without skidding.

** 2016 AIEEE

|2| → overturning condition (toppling condition)

$R1 + R2 = Mg$ —(i)

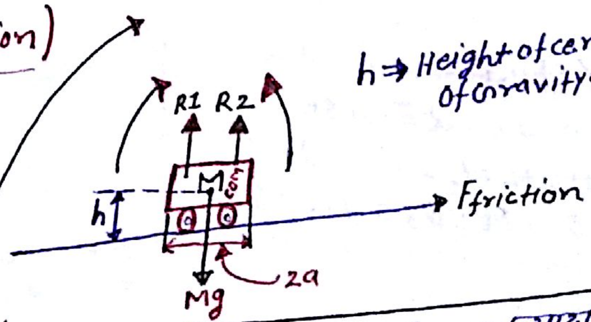
$R1 - R2 = \left(\frac{Mv^2}{r}\right)\frac{h}{a}$ —(ii)

Normal Reaction on outer wheel.

$$\boxed{R1 = \frac{Mg}{2}\left[1 + \left(\frac{v^2}{rg}\right)\left(\frac{h}{a}\right)\right]}$$ *

Normal R×n on Inner wheel.

$$\boxed{R_2 = \frac{Mg}{2}\left[1 - \left(\frac{v^2}{rg}\right)\left(\frac{h}{a}\right)\right]}$$ *



h ⇒ Height of centre of Gravity.

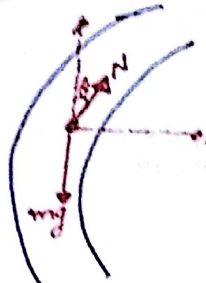
* बड़ी vechle पलटने का chance ज्यादा!
* जितना down होगा C.M पलटने का chance कम!

Safe condition

* $V\uparrow, R_1\uparrow, R_2\downarrow$ *
* $R_2 > 0 \rightarrow$ Normal Rkn on Inner Wheel.
* $V < \sqrt{rg\left(\frac{a}{h}\right)}$ *
* $V_{max} = \sqrt{rg\left(\frac{a}{h}\right)}$ * (Safe velocity)

NOTE → To avoid skidding & overturing in plane circular path safe velocity of vechle is less than $\sqrt{\mu rg}$ & $\sqrt{rg\left(\frac{a}{h}\right)}$

Bending of cyclist in a circular path



* $r \Rightarrow$ Radius of circular path.
* $\theta \Rightarrow$ Bending angle from verticle.

$N\sin\theta = \frac{mv^2}{rg}$ *

$\tan\theta \propto v^2$ *

$V\uparrow \Rightarrow \tan\theta\uparrow$ *

$V = \sqrt{rg\tan\theta}$ *

NOTE → * If $u = \sqrt{rg\tan\theta}$, then frictional force on vechle is zero. this is called Ideal speed.
* If $u < \sqrt{rg\tan\theta}$, then fr. force will be outward.

Banking of Track

Max safe velocity *

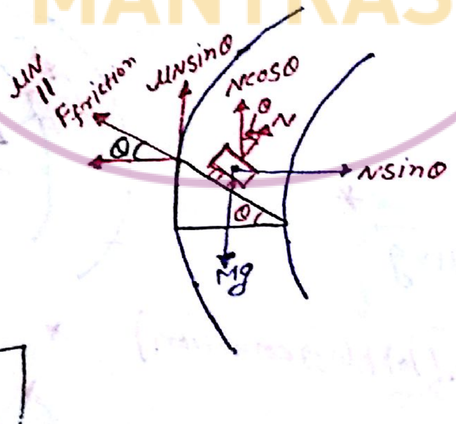
$$V = \sqrt{rg\left(\frac{\mu+\tan\theta}{1-\mu\tan\theta}\right)}$$ 2016 NEET

(If fr. ⊕nt.)

* $\mu = 0$
$V = \sqrt{rg\tan\theta}$
$\tan\theta = \frac{h}{W}$

Minimum safe velocity

$$V = \sqrt{rg\left(\frac{\tan\theta-\mu}{1+\mu\tan\theta}\right)}$$



* Standard
* $\omega_0 = 0, \alpha = const.$
$t_1 = t_2 = t_3 = \dots = t_N$
* $\theta_1 : \theta_2 : \theta_3 = 1:3:5:7\dots$