Learning Mantras

LEARNING MANTRAS

Handwritten Notes

On

Circular Motion

Circular Motion

① **Angular displacement (θ)**

2π rad $= 360°$

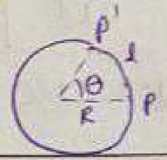
$\theta \rightarrow$ Axial Vector.

Axial Vector $\rightarrow \theta, \omega, \alpha, \tau$

dirn of $\theta \rightarrow$ by Right hand thumb rule.

$\theta \rightarrow$ dimensionless

$$\boxed{\theta = \frac{l}{R}}$$



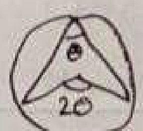
② **Angular Velocity (ω)**

Avg. Angular Vel. | Instantenous angular vel.

$$\boxed{\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}} \qquad \boxed{\omega_{inst} = \frac{d\theta}{dt}}$$

$$\int d\theta = \int \omega dt$$

③ Angle at Centre is twice angle at Circumference



④ $\omega \rightarrow$ Axial Vector

$\omega \rightarrow$ Rad/sec.

$$\boxed{\omega = \frac{d\theta}{dt}}$$

$\theta = 2\pi$

$$\omega = \frac{\theta}{T} = \frac{2\pi}{T}$$

$$\boxed{\omega = 2\pi\nu}$$

$\nu \rightarrow$ frequency

$$\boxed{\nu = \frac{\text{Revolution}}{\text{sec}} \quad \nu = \text{rpm.}}$$

⑤ **Angular acc^n (α)**

Avg. angular acc^n | Instantenous angular acc^n

$$\boxed{\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}} \qquad \boxed{\alpha_{inst} = \frac{d\omega}{dt}}$$

$$\omega = \int \alpha\, dt$$

⑥

Linear		Angular
(i) S	$S = \theta R$	θ
(ii) $V = \frac{ds}{dt}$	$V = \omega R$	$\omega = \frac{d\theta}{dt}$
(iii) $a = \frac{dv}{dt}$	$a = \alpha R$	$\alpha = \frac{d\omega}{dt}$

⑦ **For Constant / uniform angular acc^n (α)**

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

⑧

Angular	Linear
$\boxed{\theta = \left(\frac{\omega + \omega_0}{2}\right) \times t}$	$S = \left(\frac{V + U}{2}\right) \times t$
$\boxed{\theta^{th} = \omega_0 + \frac{\alpha}{2}(2t-1)}$	$S_{nth} = U + \frac{a}{2}(2n-1)$

9

$$\boxed{\text{No. of Revolution (N)} = \frac{\text{Angular disp.}}{2\pi}}$$

$$\boxed{N = \frac{\theta}{2\pi}}$$ 1 Rev. = 2π

Centripetal acc^n (a_c)

(i) When particle is uniform circular motion, it Speed remains Constant
⇓
U.C.M

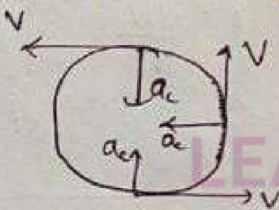
(ii) The Vel. of particle changes due to change in dir^n.

(iii) The acc^n due to change in dir^n of Vel. is Centripetal acc^n

(iv) Centripetal acc^n is directed towards Centre of circle.

$$\boxed{a_c = \frac{V^2}{R}}$$

Radial acc^n



$$\boxed{a_c = \omega^2 R}$$

$V = \omega R$

$$\boxed{a_c = V\omega}$$

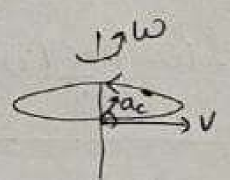
only change in dir^n

Centripetal acc^n also Called Radial acc^n

a_R or a_c

Centripetal acc^n is Constant for U.C.M
⇓
No. as its dir^n changes

$$\boxed{\vec{a}_c = \vec{\omega} \times \vec{V}}$$



$a_c = \omega V \sin 90$

$a_c = \omega V$ $V = \omega R$

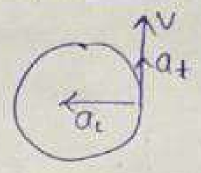
$$\boxed{a_c = \omega^2 R}$$

Tangential acc^n (a_t)

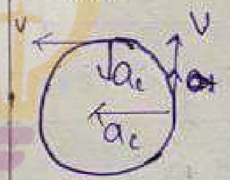
(i) If Speed of particle is also changing in circular motion i.e Vel. also changes in Magnitude as well as dir^n
⇓
we have tangential acc^n

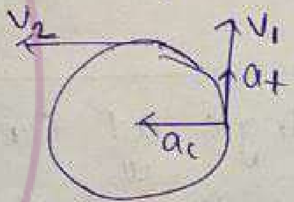
$$\boxed{a_t = \frac{d|V|}{dt}}$$

↳ act along tangent



UCM	NUCM
⇓	⇓
only dir of $\vec{V}$ changes	dir of $\vec{V}$ change → a_c & Magnitude also changes → a_t
⇓ a_c	





Qn.) In U.C.M

a) $a_c \neq 0$ $a_t = 0$

(10) Rel^n b/w a_t & angular acc^n (α)

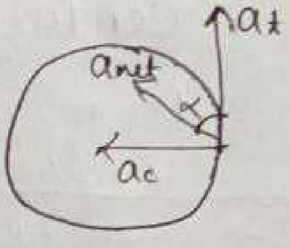
$$a_t = \frac{d|V|}{dt} = \frac{d[R\omega]}{dt} = \frac{R\,d[\omega]}{dt}$$

$$\boxed{a_t = R\alpha}$$

α tab hoga jab a_t hoga

$$\boxed{\begin{aligned} a_c &= \frac{V^2}{R} = \omega^2 R = \omega V \\ a_t &= \frac{d|V|}{dt} = \alpha R \end{aligned}}$$

Net acc^n



$$\boxed{a_{net} = \sqrt{a_c^2 + a_t^2}}$$

$$\boxed{\tan\alpha = \frac{a_c}{a_t}}$$

For UCM $\rightarrow$ $\alpha = 0$

$a_t = 0$

$|v| \rightarrow$ Constant

$|\omega| \rightarrow$ Constant $\quad \vec{v}$ = Constant ✗

$\vec{\omega}$ = Constant $\quad a_c \neq 0$

$|a_c| \rightarrow$ Constant

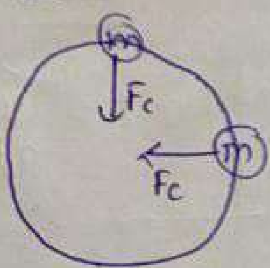
Dynamics of Circular Motion

Centripetal Force (F_c)

1) When particle moves in Circular path $\rightarrow$ Centripetal acc^n is present always.
2) This Centripetal acc^n $\underline{a_c}$ is directed along the radius of Circle towards Centre
3) As there is $\underline{\text{no } acc^n}$ without force
$\Downarrow$
A Centripetal force acts on particle along radius towards Centre.

$F = ma$

$$\boxed{F_c = ma_c}$$



$$\boxed{F_c = \frac{mv^2}{r}}$$

$$\boxed{F_c = m\omega^2 r}$$

$$\boxed{F_c = mv\omega}$$

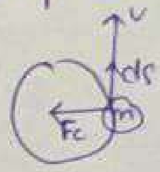
$a_c = v\omega$

(II) Work done by Centripetal Force (F_c) & Change in K.E

Work done in Round trip = 0

Work done in half trip = 0

$W = F \cdot ds$
$= Fs\cos\theta$
$W = 0$



Work Energy theorem

$$\boxed{\text{Work done} = \text{Change in K.E}}$$

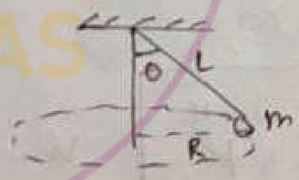
$\underset{\downarrow 0}{W} = K.E_f - K.E_i$

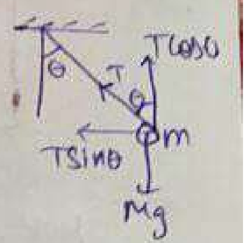
$$\boxed{K.E_i = K.E_f}$$

$$\boxed{V_i = V_f}$$

Speed can change if F tangential is present

Conical pendulum





$T\cos\theta = mg$ ①

$T\sin\theta = F_c = \frac{mv^2}{R}$ ⑪

⑪ ÷ ①

$\tan\theta = \frac{v^2}{gr}$

$$\boxed{V = \sqrt{gr\tan\theta}}$$

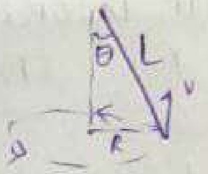
Time period $\quad T = \frac{2\pi}{\omega}$

$\omega = \frac{v}{r}$ $\quad T = \frac{2\pi}{\sqrt{\frac{g\tan\theta}{r}}}$

$$\boxed{T = 2\pi\sqrt{\frac{r}{g\tan\theta}}}$$

$\sin\theta = \frac{r}{L}$

$r = L\sin\theta$

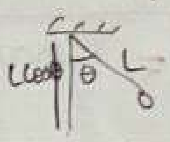


$T = 2\pi\sqrt{\frac{L\sin\theta}{g\tan\theta}} \Rightarrow \frac{\sin\theta}{\cos\theta}$

$$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

Shortcut for conical pendulum

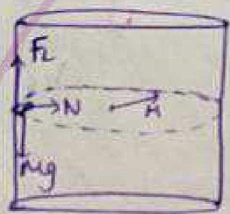
$T = 2\pi\sqrt{\frac{L}{g}}$



$$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

Death Well or Rotor

to save biker



$F_L \geq mg$

$\mu N \geq mg$

Here N act as centripetal force.

$N = F_c = \frac{mv^2}{r}$

$\mu \frac{\cancel{m}v^2}{r} \geq \cancel{m}g$

$$\boxed{v = \sqrt{\frac{gr}{\mu}}}$$

$v_{min} = \sqrt{\frac{gr}{\mu}}$

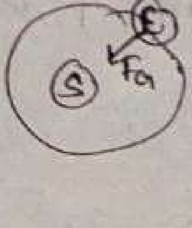
or

$\mu \cancel{m}\omega^2 r \geq \cancel{m}g$

Q→ Neet 2019

Note: Centripetal force is $\boxed{\text{NOT}}$ a new kind of force. Some or other force act as centripetal force.

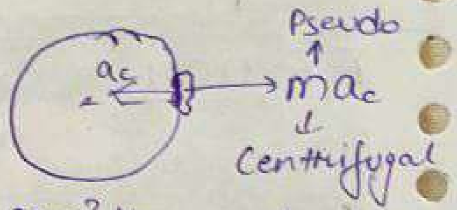
$F_G = F_c = \frac{Gm_1m_2}{r^2}$



Centrifugal Force

→ It is pseudo force i.e (Imaginary force)

→ when observer is in rotating frame (accelerated frame) then we apply centrifugal force on the system

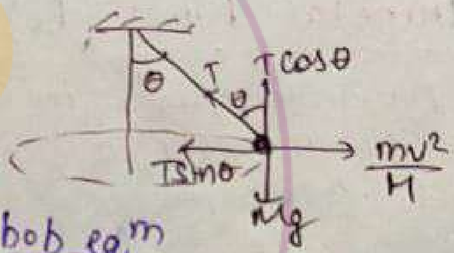


$F_{centrifugal} = m\omega^2 r$ = outward

System eq^n

When observer is in accelerated frame

Conical pendulum



bob eq^n

$T\cos\theta = mg$

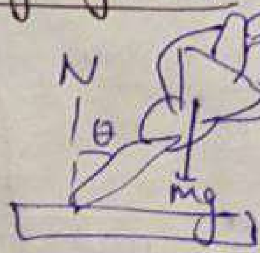
$T\sin\theta = \frac{mv^2}{r}$

$\tan\theta = \frac{v^2}{gR}$

$$\boxed{v = \sqrt{gR\tan\theta}}$$

Bending of Cyclist

$$\boxed{\tan\theta = \frac{v^2}{gR}}$$



(12) Turning of a Car :-

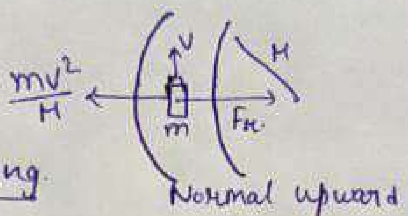
- In level Road (friction): $\boxed{V_{max} \leq \sqrt{\mu g r}}$
- Banking (Normal Rx^n): $\boxed{V = \sqrt{gR\tan\theta}}$
- Banking + friction:

$$\boxed{V_{max} = \sqrt{\frac{gr(\tan\theta + \mu)}{1-\mu\tan\theta}}}$$

$$\boxed{V_{min} = \sqrt{\frac{gr(\tan\theta - \mu)}{1+\mu\tan\theta}}}$$

(I) Turning of Car on level Road

N.I.F.O.R.



For no skiding

$$\frac{mv^2}{r} \leq F_{lim}$$

$$\frac{mv^2}{r} \leq \mu N$$

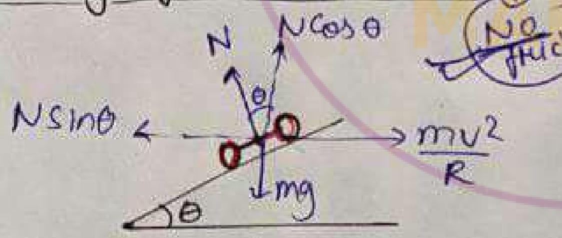
$$\boxed{V_{max} \leq \sqrt{\mu g r}}$$

In Raining $\mu \downarrow\downarrow \rightarrow F_{lim} \downarrow\downarrow$.

$V > V_{max} \rightarrow$ Skid.

(II) Turning of Car on banked Road :-

No fric^n



$N\cos\theta = mg$ | $N\sin\theta = \frac{mv^2}{r}$

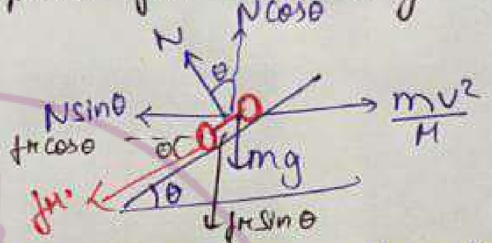
$$\boxed{\tan\theta = \frac{v^2}{gr}}$$

θ → angle of banking.

$$\boxed{V = \sqrt{gr\tan\theta}}$$

(III) Banking with Friction

Case I → If speed of vehicle is greater than the designed speed for banking.



$V\uparrow\uparrow \rightarrow \frac{mv^2}{r}\uparrow\uparrow$, then $N\sin\theta$ will can't able to balance $\frac{mv^2}{r}$ then frictional force will come in down ward dir^n.

$$N\cos\theta = mg + f_r\sin\theta$$

$$N\cos\theta = mg + \mu N\sin\theta$$

$$N\cos\theta - \mu N\sin\theta = mg \quad \text{(I)}$$

$$N\sin\theta + \mu N\cos\theta = \frac{mv^2}{r} \quad \text{(II)}$$

(II) ÷ (I)

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{V_{max}^2}{gr}$$

divide by $\cos\theta$.

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{V_{max}^2}{gr}$$

$$\boxed{V_{max} = \sqrt{\frac{gr(\tan\theta + \mu)}{1-\mu\tan\theta}}}$$

Case II :- $V < V_{max}$

fr. force upward dir^n mai lagega.