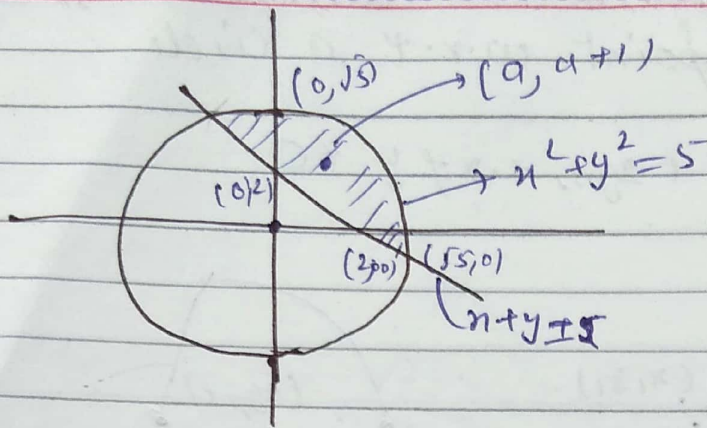




Handwritten Notes
On
Circle



For what value of a , the point $(a, a+1)$ lie inside the smaller region bounded by circle $x^2 + y^2 = 5$ and line $x + y = 2$

$$s_1 < 0$$

$$s = x^2 + y^2 - 5 = 0$$

$$a^2 + (a+1)^2 - 5 < 0$$

$$a^2 + a^2 + 2a + 1 - 5 < 0$$

$$2a^2 + 2a - 4 < 0$$

$$a^2 + a - 2 < 0$$

$$(a+2)(a-1) < 0$$

$$a \in (-2, 1)$$

$$x + y - 2 = 0$$

$$0 + 0 - 2 < 0$$

$$a + a + 1 - 2 > 0$$

$$2a > 1$$

$$a > \frac{1}{2}$$

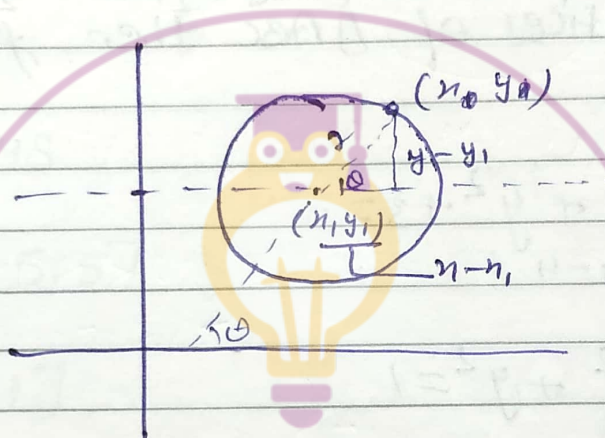
$$a \in \left(\frac{1}{2}, 1\right)$$

* Parametric eqⁿ of Circle :-

$$\frac{x-x_1}{r \cos \theta} = \frac{y-y_1}{r \sin \theta} = r$$

$$x = r \cos \theta + x_1$$

$$y = r \sin \theta + y_1$$



$$r \sin \theta = \frac{y - y_1}{r}$$

$$r \cos \theta = \frac{x - x_1}{r}$$

$$\frac{x-x_1}{r \cos \theta} = \frac{y-y_1}{r \sin \theta} = r$$

r is constant
 θ is parameter.

$$x = r \cos \theta + x_1$$

$$y = r \sin \theta + y_1$$

Q. $x^2 + y^2 - 4x + 3 = 0$
find range of

- (i) $x^2 + y^2$
(2) $x^2 + y^2 + 2y$
(3) $3x + 4y$

Q. If $A(\cos \theta_1, \sin \theta_1)$, $B(\cos \theta_2, \sin \theta_2)$, $C(\cos \theta_3, \sin \theta_3)$ are the vertices of $\triangle ABC$ then find Orthocentre.

Ans:

$$x^2 - 4x + y^2 + 3 = 0$$

$$(x-2)^2 + y^2 = 1$$

$$(x-x_1)^2 + (y-y_1)^2 = r^2$$

$$x - x_1 = r \cos \alpha$$

$$y - y_1 = r \sin \alpha$$

$$x = x_1 + r \cos \alpha$$

$$y = y_1 + r \sin \alpha$$

$$x = 2 + \cos \alpha$$

$$y = 0 + \sin \alpha$$

$$(1) \quad x^2 + y^2$$

$$4 + c^2 + 4c + s^2$$

$$5 + 4 \cos \alpha$$

$$[1 \quad 4]$$

$$(1, 1)$$

$$c \rightarrow \cos \alpha$$

$$(2) \quad x^2 + y^2 + 2y$$

$$5 + 4c + 2s$$

$$5 + [-2\sqrt{5}, 2\sqrt{5}]$$

$$[5 - 2\sqrt{5}, 5 + 2\sqrt{5}]$$

$$a \cos \theta + b \sin \theta$$

$$\in [\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$$

$$(3) \quad 3x + 4y$$

$$6 + 3c + 4s$$

$$6 + [-5, 5]$$

$$[1, 11]$$

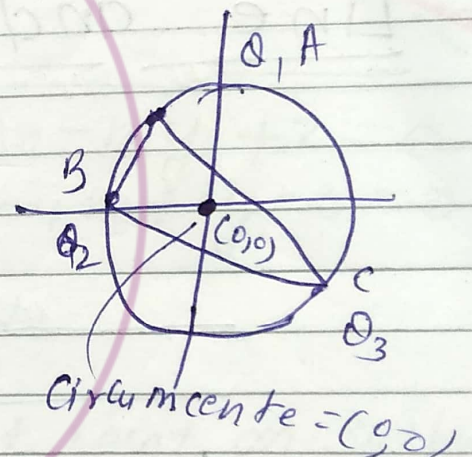
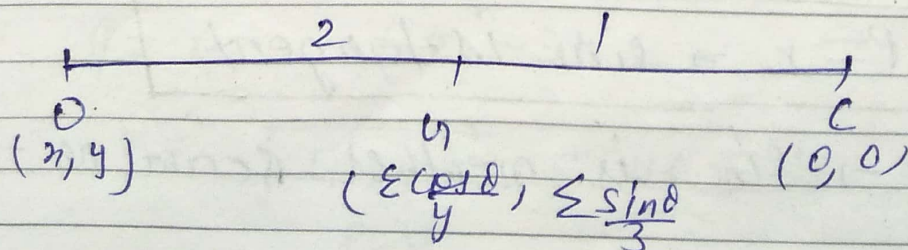
$$\text{Ans } x^2 + y^2 + 1$$

$$x = \cos \theta$$

$$y = \sin \theta$$

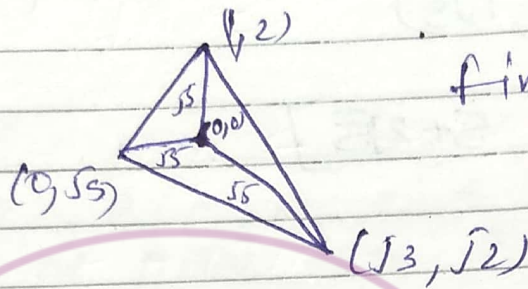
$$\text{Centroid} = G = \left[\frac{\cos \theta_1 + \cos \theta_2 + \cos \theta_3}{3} \right]$$

$$\frac{\sin \theta_1 + \sin \theta_2 + \sin \theta_3}{3}$$



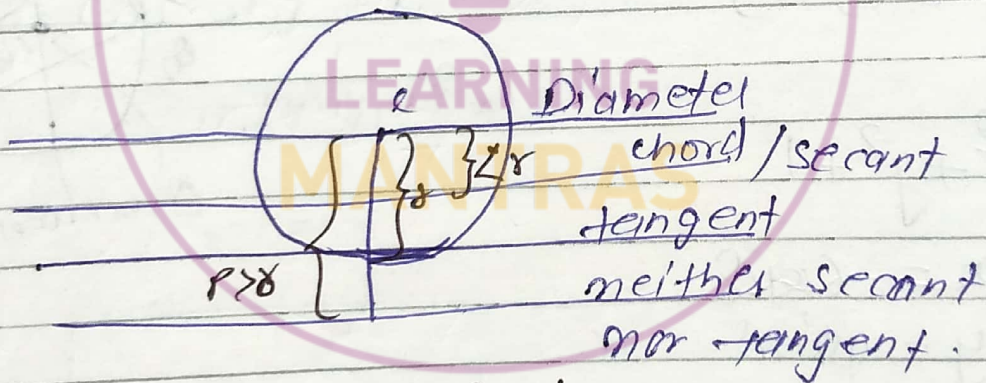
$$\frac{x}{3} = \frac{\sum \cos \theta}{3}$$

$$\frac{y}{3} = \frac{\sum \sin \theta}{3} \quad (x, y) = (\sum \cos \theta, \sum \sin \theta)$$



find orthocentre.

Line and Circle



$p \rightarrow$ dist. from centre

$p = 0 \Rightarrow$ line is diameter

$p < r \Rightarrow$ line is secant

$p = r \Rightarrow$ line is tangent

$p > r \Rightarrow$ line is neither secant nor tangent

$$x + y = 0$$

$$x^2 + y^2 = 1$$

$$x^2 + (1-x)^2 = 1$$

$$D = 0 \rightarrow \text{tangent}$$

$$D > 0 \rightarrow \text{secant}$$

$$D < 0 \rightarrow \text{neither secant nor tangent}$$

Q. Two Coincident Point

Tangent

Q.1 For what value of m , $3x - my + z = 0$ intersect circle $x^2 + y^2 - 4x + 4y - 3 = 0$

at two coincident points.

Q.2 If there are 3 distinct Point on $3x + by + 7$ which form a right angle at with two Point $A(0,0)$ and $B(0,4)$ then find b .

Q.3) Line $3x + 4y + \lambda = 0$ neither touches nor cuts circle $(x-1)^2 + (y-2)^2 = 9$ then find λ .

Ans: $r = \sqrt{4 + 4 + 3} = \sqrt{11} = 4$
 $p = 4$

2560

$$\left| \frac{6+3m+2=0}{\sqrt{m^2-4}} \right| = 4$$

$$8+3m = 4\sqrt{m^2+4}$$

$$64+9m^2+48m = 16m^2+144$$

$$7m^2-48m+80=0$$

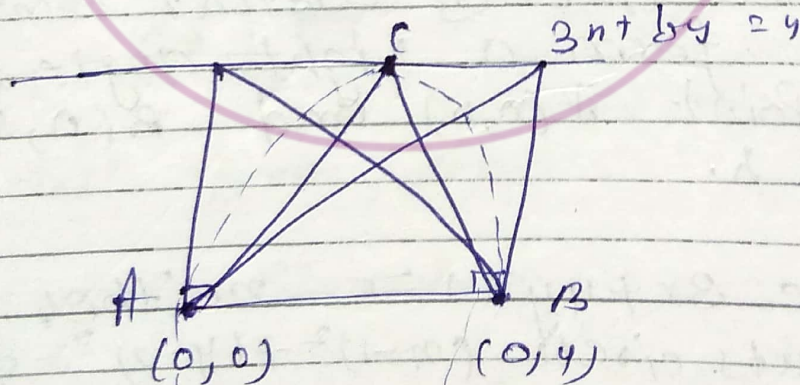
$$7m^2-28m-20m+80=0$$

$$7m(m-4)-20(m-4)=0$$

$$(m-4)(7m-20)=0$$

$$m=4, \frac{20}{7}$$

Ans: 2



$$(x-0)(x-0) + (y-0)(y-4) = 0$$

$$x^2 + y^2 - 4y = 0$$

4-secant . tangen - 3, ① . ②

$$P = x \quad (0, 2)$$

$$r = \sqrt{0 + 4} = 0$$

$$r = 2$$

$$\left| \frac{0 + 2b - 4}{\sqrt{a + b^2}} \right| = 2$$

$$a + b^2 = b^2 + 4 - 4b$$

$$4b = -5$$

$$b = \frac{-5}{4} \text{ Ans.}$$

Ans: -3 (1, 2), r = 3

$$3x + 4y + \lambda = 0$$

$$P > r$$

$$P > 3$$

$$\left| \frac{3 + 8 + \lambda}{5} \right| > 3$$

$$|n| > a$$

$$n \in (-\infty, -a) \cup (a, \infty)$$

$$|11 + \lambda| > 15$$

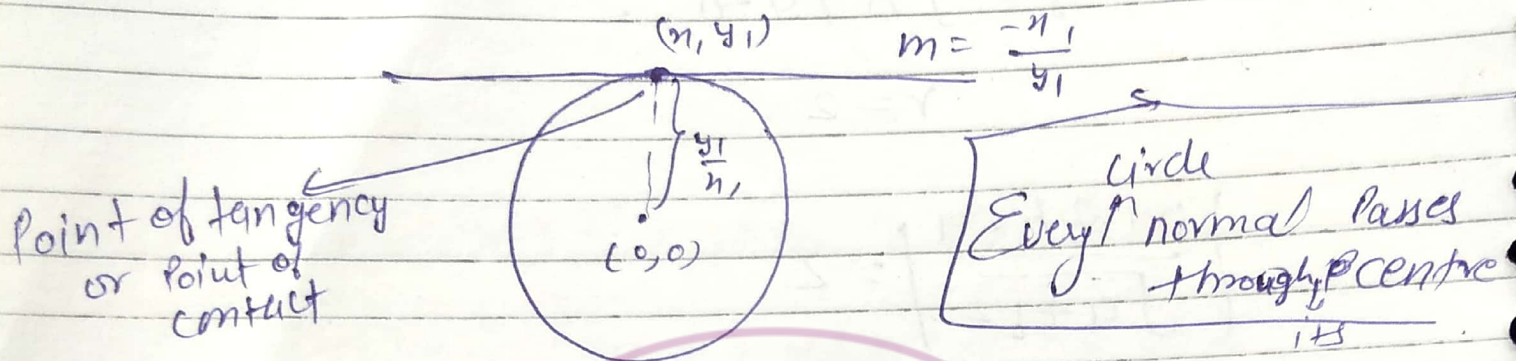
$$|n| < a$$

$$a \in (-a, a)$$

$$|11 + \lambda| \in (-\infty, -15] \cup (15, \infty)$$

$$\lambda \in (-\infty, -26] \cup (4, \infty)$$

Tangent of a circle



$$x^2 + y^2 = a^2$$

$$x_1^2 + y_1^2 = a^2$$

$$y - y_1 = -\frac{x_1}{y_1} (x - x_1)$$

$$y y_1 - y_1^2 = (-x_1 x_1 + x_1^2)$$

$$x x_1 + y y_1 = x_1^2 + y_1^2$$

$$\boxed{x x_1 + y y_1 = a^2} \quad \boxed{T=0}$$

Replace

$$x^2 = x x_1$$

$$y^2 = y y_1$$

$$2x = x + x_1$$

$$2y = y + y_1$$

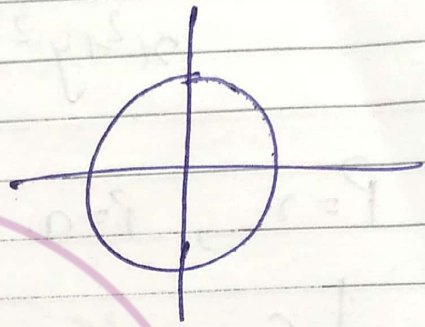
$$2xy = xy_1 + yx_1$$

It is valid for every second degree curve.

Every normal of a circle passes
~~thru~~ through its centre

$$* \quad x^2 + y^2 = r^2$$

$$(r \cos \theta, r \sin \theta)$$



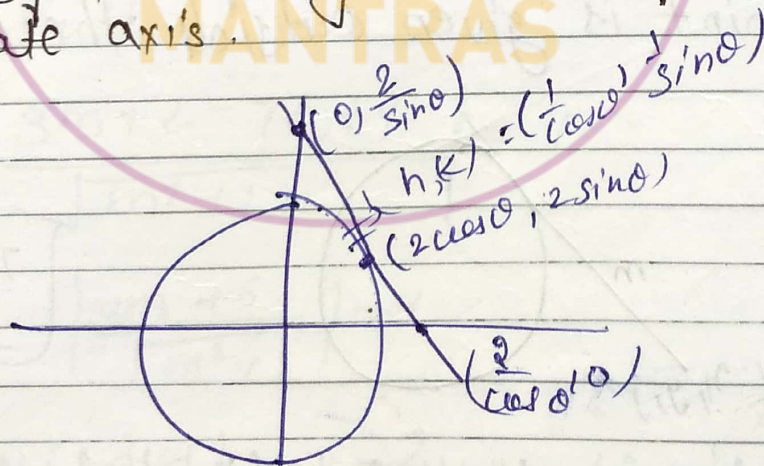
$$T = 0$$

$$x(r \cos \theta) + y(r \sin \theta) = r^2$$

$$x \cos \theta + y \sin \theta = r$$

Que Let there a circle $x^2 + y^2 = 4$ It has a
 tangent in first quadrant.

\Rightarrow Find the locus of mid point of the
 portion of the tangent intercept b/w
 coordinate axis.



$$T = 0$$

$$x \cos \theta + y \sin \theta = 2$$

$$h = \frac{1}{\cos \theta}$$

$$k = \frac{1}{\sin \theta}$$

$$\frac{1}{h^2} + \frac{1}{k^2} = 1$$

$$\frac{1}{x^2} + \frac{1}{y^2} = 1$$

Slope form of tangent.

if $y = mx + c$ is tangent of circle

$$x^2 + y^2 = a^2$$

$$(0,0) \rightarrow r = a$$

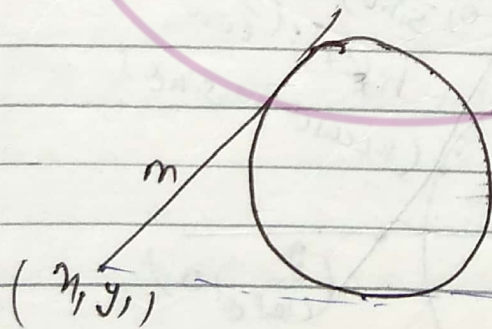
$$P = r, \quad P = a.$$

$$\left| \frac{c}{\sqrt{m^2 + 1}} \right| = a \quad |c| = a\sqrt{1+m^2}$$

$$c = \pm a\sqrt{1+m^2}$$

$$y = mx \pm a\sqrt{1+m^2}$$

If a point is given outside the circle.



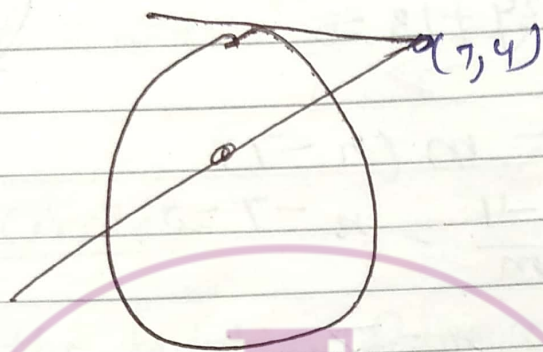
To find Value of m

$$\text{line} = y - y_1 = m(x - x_1)$$

if line is tangent $P = r$

Q. find the eqⁿ of tangent from (7, 4) to circle

$$x^2 + y^2 - 6x + 4y - 3 = 0$$



$$y - 4 = m(x - 7)$$

$$mx - y = 7m + 4 = 0$$

$$(3, -2)$$

$$r = \sqrt{9 + 4 + 3} = 4$$

$$r = 4$$

$$\left| \frac{3m + 2 - 7m + 4}{\sqrt{m^2 + 1}} \right| = 4$$

$$\left| \frac{4m + 6}{\sqrt{m^2 + 1}} \right| = 4$$

$$4(m^2 + 1) = 4m^2 + 9 - 12m$$

$$4m^2 + 4 = 4m^2 + 9 - 12m$$

$$\frac{12m = 5}{m = \frac{5}{12}}$$

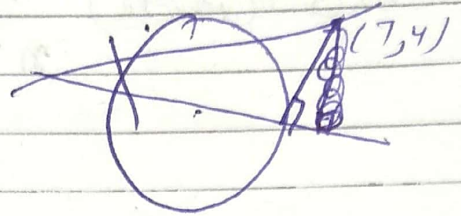
$$m = \frac{5}{12}$$

second value

$$y - 4 = \frac{5}{12}(x - 7)$$

$$12y - 48 = 5x - 35$$

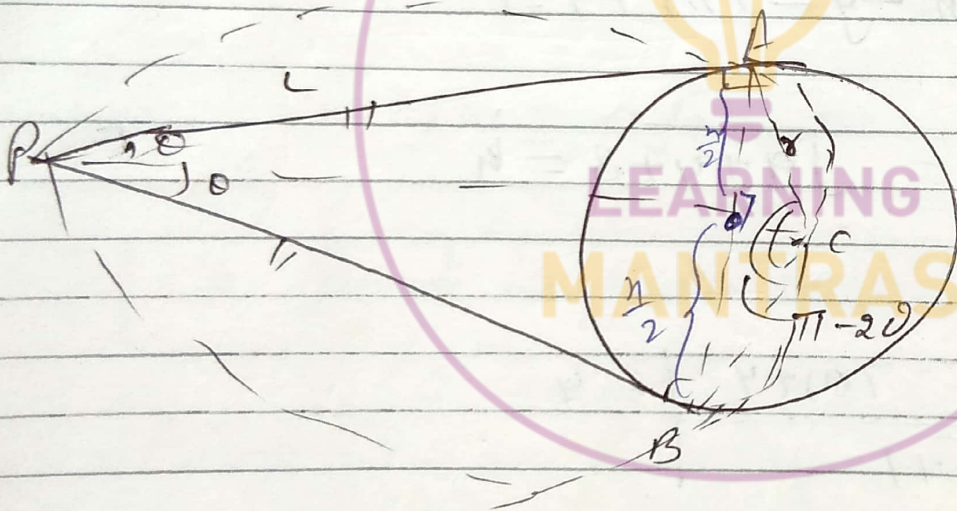
$$5x - 3y + 13 = 0$$



$$y - 4 = m(x - 7)$$

$$\frac{y - 4}{m} = x - 7 = 0$$

$$x = 7$$



$PACB \rightarrow$ cyclic quadrilateral with diameter PC

$$\text{Area of } \triangle APC = \frac{1}{2} \times L \times r$$

$$\text{Area of } PACB = L \times r$$

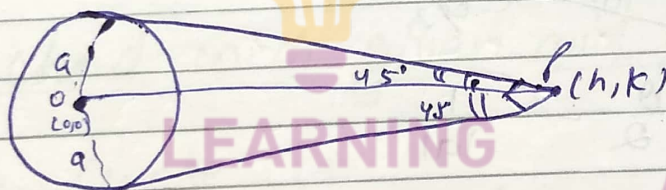
$$\tan \theta = \frac{r}{L}$$

$$\theta = \tan^{-1} \frac{r}{L}$$

$$2\theta = 2 \tan^{-1} \frac{r}{L}$$

* Director Circle :-

Locus of the point of Intersection of two mutually perpendicular tangent to a given curve is called director circle



$$\sin 45 \frac{a}{OP} = \frac{1}{\sqrt{2}} \quad OP^2 = 2a^2$$

$$OP = \sqrt{2}a \quad h^2 + k^2 = 2a^2$$

$$\boxed{x^2 + y^2 = 2a^2} = (\sqrt{2}a)^2$$

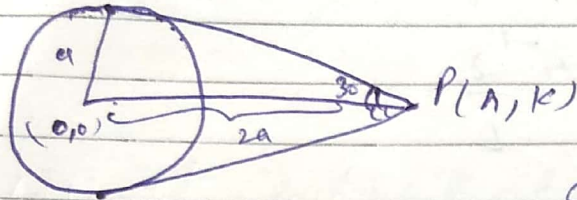
concentric circle.

Q. Locus of point P which moves such that angle made by tangent to circle $x^2 + y^2 = a^2$ is 60° .

Q. find range of a such that angle θ b/w tangents from $(a, 0)$ to circle $x^2 + y^2 = 1$ lying in $(\frac{\pi}{2}, \pi)$

Q. Find locus of centre of circle touching two mutually \perp lines from origin and $x+y=1$

Ans:
(1)



$$\sin 30^\circ = \frac{a}{OP} = \frac{1}{2}$$

$$h^2 + k^2 = 4a^2$$

$$OP = 2a$$

(2)



$$\sin \frac{\theta}{2} = \frac{r}{|a|}$$

$$|a| = r \operatorname{cosec} \frac{\theta}{2}$$

$$\theta \in \left(\frac{\pi}{2}, \pi \right)$$

$$|a| = r \operatorname{cosec} \frac{\theta}{2}$$

$$|a| \in (1, \sqrt{2})$$

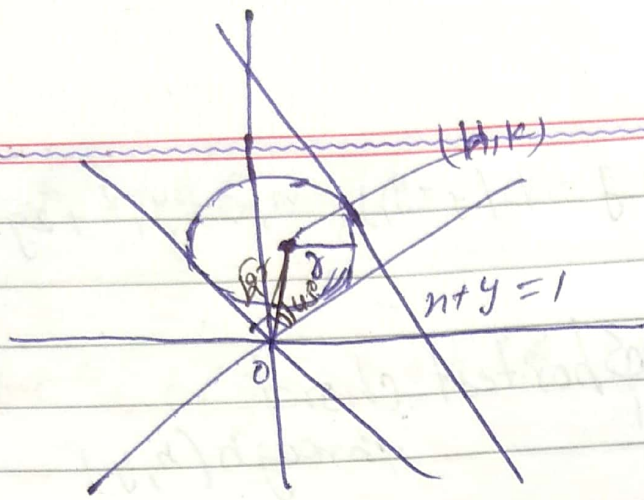
$$\sqrt{2} > |a| > 1$$

$$|a| < \sqrt{2} \Rightarrow a \in (-\sqrt{2}, \sqrt{2})$$

$$|a| > 1 \Rightarrow a \in (-\infty, -1) \cup (1, \infty)$$

$$a \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$$

3



$$OP = \sqrt{2}r$$

$$OP = \sqrt{2}r$$

$$OP^2 = 2r^2$$

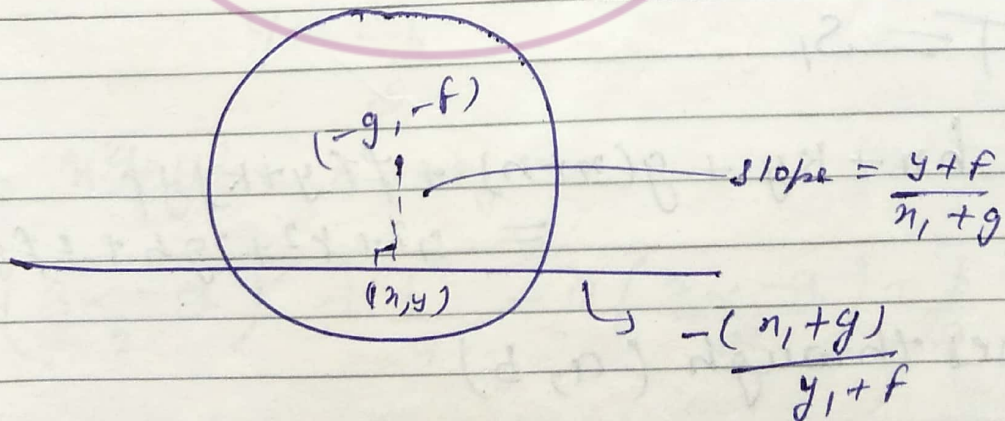
$$h^2 + k^2 = 2 \left(\frac{h+k-1}{\sqrt{2}} \right)^2 = h^2 + k^2 + 1 + 2hk - h - k$$

$$h + k - 2hk = 1$$

* eqⁿ of chord with given mid point:

let a circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$



$$y - y_1 = \left(\frac{y_1 + f}{-(x_1 + g)} \right) (x - x_1)$$

$$+ yy_1 + yf - y_1^2 - y_1f = -xx_1 + x^2 - gx_1 + gn_1$$

$$xx_1 + yy_1 + (x+x_1)g + f(y+y_1) = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

$$T = S_1$$

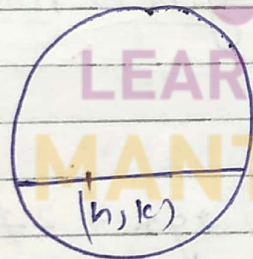
→ Shortest chord through (x, y)

Q. find locus of mid-point of chords of

circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which passes through (a, b) lying outside the circle.

(2) If two distinct chords are drawn from (a, b) to circle $x^2 + y^2 + ax + by = 0$ are divided by x -axis in ratio 2:1 then P.T. $a^2 > 3b^2$

Ans: (1)



$$T = S_1$$

$$hx + ky + g(x+h) + f(y+k) + f = h^2 + k^2 + 2gh + 2fy + f$$

Passes through (a, b)

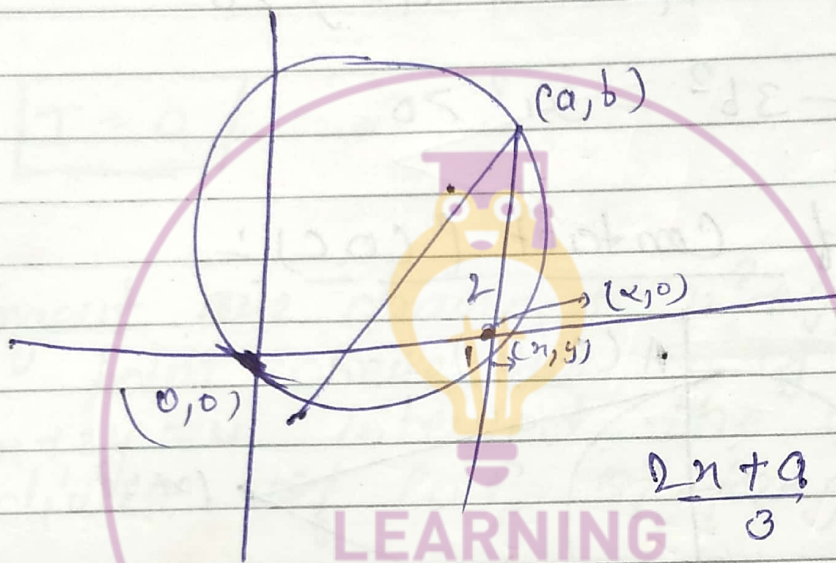
$$ah + bk + g(a+h) + f(b+k) = h^2 + k^2 + 2gh + 2fk$$

$$ah + bk + ga + gh + fb + fk = h^2 + k^2 + 2gh + 2fk$$

$$h^2 + k^2 + gh + fk - ah - bk - ga - bf = 0$$

$$h^2 + k^2 + h(g-a) + k(f-b) - bk - bf = 0$$

(2)



$$\frac{2x+a}{3} = \alpha$$

$$x = \frac{3\alpha - a}{2}$$

$$\frac{2y+b}{3} = 0$$

$$y = -\frac{b}{2}$$

$$x^2 + y^2 - ax - by = 0$$

$$\left(\frac{3\alpha - a}{2}\right)^2 + \frac{b^2}{4} - a\left(\frac{3\alpha - a}{2}\right) + \frac{b^2}{2} = 0$$

$$a\alpha^2 + a^2 - 6a\alpha + b^2 - 2a(3\alpha - a) + 2b^2 = 0$$

~~eqn of chord with given~~

$$D = b^2 - 4ac$$

$$x^2 =$$

$$9x^2 + a^2 - 6ax + b^2 - 6ax + 2a^2 + 2b^2 = 0$$

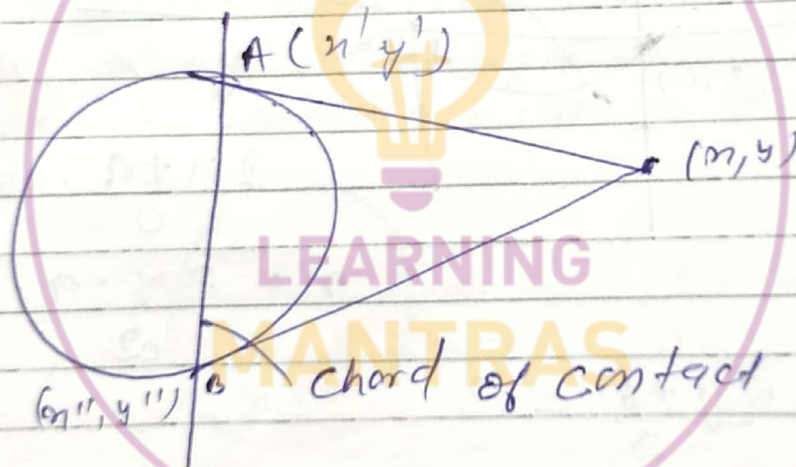
$$9x^2 - 12ax + 3b^2 + 3a^2 = 0$$

$$D > 0$$

$$4 \times 9a^2 - 4 \times 9(3b^2 + 3a^2) > 0$$

$$4a^2 - 3b^2 - 3a^2 > 0$$

* chord of contact (COC) :-



$$x^2 + y^2 = a^2$$

eqⁿ of tangent passing through A

$$T = 0 \Rightarrow xx' + yy' = a^2$$

Tangent from B

$$T = 0, \quad xx'' + yy'' = a^2$$

Too may be chord of contact to out of circle
 Point on curve \longrightarrow Tangent

Passes through (x_1, y_1)

$$x_1 x' + y_1 y' = a^2$$

$$x_1 x'' + y_1 y'' = a^2$$

\rightarrow $\boxed{x_1 x' + y_1 y' = a^2}$ \rightarrow this is the eqⁿ of chord of contact

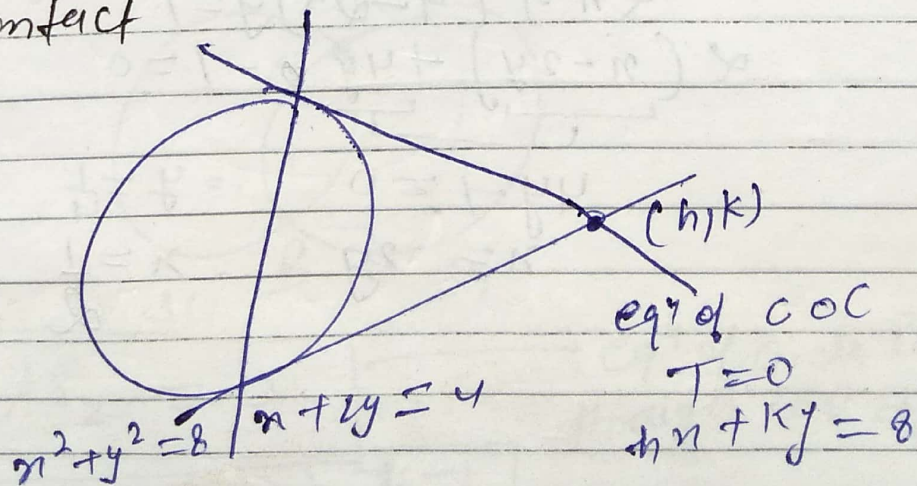
$$\boxed{T=0}$$

Q.1 Tangent are drawn to $x^2 + y^2 = 8$ from the point where the ~~$x^2 + y^2$~~ line $x + 2y = 4$ intersect the circle. find coordinate of POC of tangent.

Q.2 Tangent are drawn to unit circle centred at origin from every point on the line $x + y = 4$

- (i) chord of contact passes through a fix point, find that point
 (ii) Find the eqⁿ of locus of middle point of chord of contact

Ans (1)

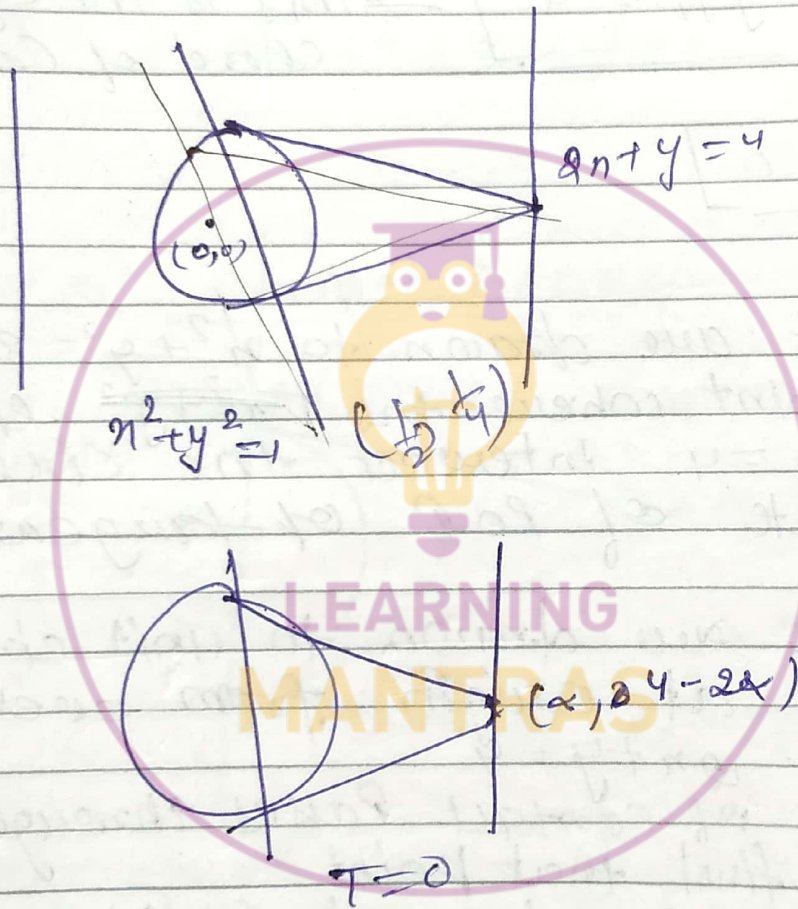


$$\frac{h}{1} = \frac{k}{2} = \frac{8}{2} \quad \text{compare}$$

$$\boxed{\begin{matrix} h=2 \\ k=4 \end{matrix}} \quad (2, 4)$$

②

(i)



$$\begin{aligned} \alpha x + (4 - 2\alpha)y &= 1 \\ \alpha(x - 2y) + 4y - 1 &= 0 \end{aligned}$$

$$4y - 1 = 0$$

$$x = 2y$$

$$y = \frac{1}{4}$$

$$x = \frac{1}{2}$$

Point P

(ii) mid-point (h, k)

$$T = S_1$$

$$hx + ky - 1 = h^2 + k^2 - 1$$

$$hx + ky = h^2 + k^2$$

- compare

$$\frac{h}{\alpha} = \frac{k}{4-2\alpha} = \frac{h^2+k^2}{1}$$

$$\alpha = \frac{h}{h^2+k^2} = \frac{k}{h^2+k^2} = 4-2\alpha$$

$$\frac{k}{h^2+k^2} = 4 - 2\frac{h}{h^2+k^2}$$

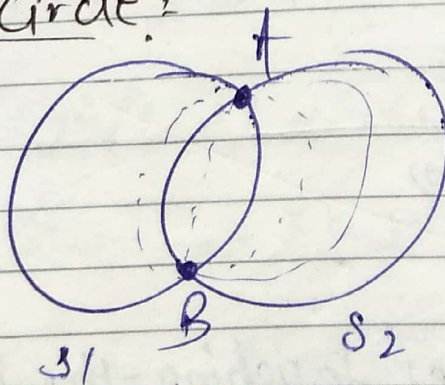
$$k = 4h^2 + 4k^2 - 2h$$

$$4h^2 + 4k^2 - 2h - k = 0$$

* Family of circle:

$$S_1 = 0$$

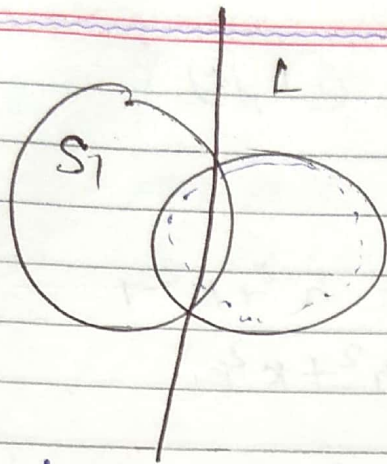
$$S_2 = 0$$



$$S_1 + \lambda S_2 = 0$$

$$\lambda \neq -1$$

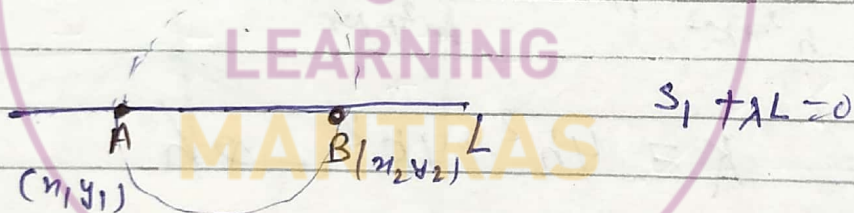
→ Eqⁿ of circle passing through IOD of S_1 & S_2



eqⁿ of circle passing through PoI of circle S_1 and line L is given by

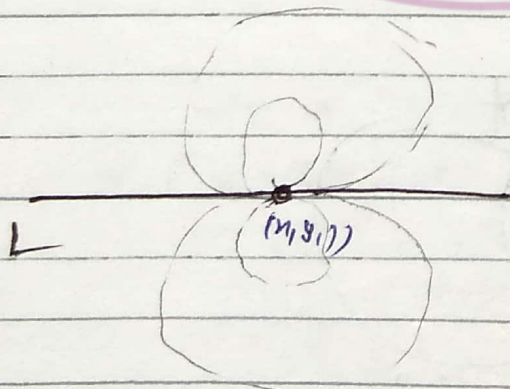
$$S_1 + \lambda L = 0$$

⇒ If only two points are given



$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda L_{AB} = 0$$

*



eqⁿ of circles touching the line L at (x_1, y_1) is given by

$$(x - x_1)^2 + (y - y_1)^2 + \lambda L = 0$$

Dmp:

Q. Find the eqⁿ of circle passing through P.O.I of $x^2 + y^2 - 4x + 6y - 3 = 0$ and $x^2 + y^2 + 4x - 6y + 12 = 0$ and

(1) Passes through $(0,0)$

(2) Centre lie on $y = x$

(3) $4x - 7y = 10$ is normal to Circle

Q. Find the eqⁿ of circle which touches $x - y = 4$ at $(1, -2)$ and also passes through $(3, 4)$

(3) Find the eqⁿ of which passes through origin and through the point of contact of the tangent from origin to circle

$$x^2 + y^2 + 11x + 3y + 17 = 0$$

(1) $S_1 + \lambda S_2 = 0$

$$x^2 + y^2 - 4x + 6y - 3 + \lambda(x^2 + y^2 + 4x - 6y + 12) = 0$$

$$x^2(1+\lambda) + y^2(1+\lambda) + x(4\lambda - 4) + y(-6\lambda + 6) - 12\lambda - 3 = 0$$

(1) $12\lambda - 3 = 0 \Rightarrow \lambda = \frac{1}{4}$

$$\frac{5}{4}x^2 + \frac{5}{4}y^2 - 3x - \frac{3}{2}y - \frac{9}{2} = 0$$

$$\frac{5}{4}x^2 + \frac{5}{4}y^2 - 3x - \frac{9}{2} = 0$$

$$(2) \text{ Centre} = \left(\frac{-(2\lambda - 2)}{\lambda + 1}, \frac{3\lambda - 3}{\lambda + 1} \right)$$

$$\frac{3\lambda - 3}{\lambda + 1} = -\frac{(2\lambda - 2)}{\lambda + 1}$$

$$3\lambda - 3 = -2\lambda + 2$$

$$5\lambda = 5$$

$$\lambda = 1$$

$$x^2 + y^2 + 9 = 0$$

(3)

Centre

$$2x - y = 4$$

$$4 \left(\frac{-(2\lambda - 2)}{\lambda + 1} \right) - 7 \left(\frac{3\lambda - 3}{\lambda + 1} \right) = 10$$

$$\frac{-8\lambda - 8}{\lambda + 1} - \frac{21\lambda + 21}{\lambda + 1} = 10$$

LEARNING MANTRAS

Ans

2

$$2x - y = 4$$

$$(1, -2)$$

$$(x-1)^2 + (y+2)^2 + \lambda(2x-y-4) = 0$$

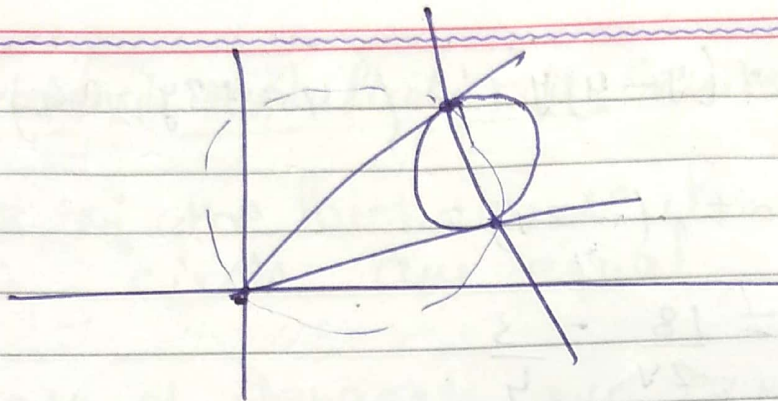
$$4 + 36 + \lambda(6 - 4 - 4)$$

$$\lambda = \frac{40}{2}$$

$$\lambda = 20$$

Ans 1

3



$$COC \Rightarrow T=0$$

$$0 \cdot 0 + -\frac{11}{2}(x+0) + \frac{13}{2}(y+0) + 17 = 0$$

$$-11x + 13y + 34 = 0$$

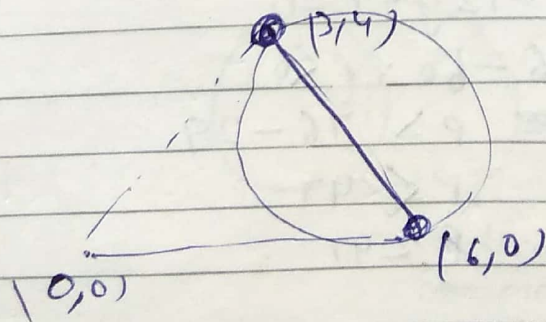
$$S + \lambda L = 0$$

$$x^2 + y^2 - 11x + 13y + 17 + \lambda(-11x + 13y + 34) = 0$$

$$17 + \lambda(34) = 0$$

$$\lambda = -\frac{1}{2}$$

Q. find the eqⁿ of circle which passes through (0,0) (3,4) (6,0)



$$(x-3)(x-6) + (y-4)(y-0) = 0$$

$$y-0 = \frac{4}{-3}(x-6)$$

$$3y = -4x + 24$$

$$4x + 3y - 24 = 0$$

$$(x-3)(x-6) + (y-4)y + \lambda(4x+3y-24) = 0$$

$$18 + 0 + \lambda(-24) = 0$$

$$\lambda = \frac{18}{24} = \frac{3}{4}$$

* Power of a point:

Square of the length of tangent from point P is called power of point P w.r.t given circle

$$\text{Length of tangent} = \sqrt{S_1}$$

$$\text{Power} = S_1$$

Que: find the value of P for which power of point P(2, 5) is negative w.r.t circle $x^2 + y^2 - 8x - 12y + P = 0$ and the circle neither touches nor intersects the y axis.

$$x^2 + y^2 - 8x - 12y + P = 0$$

$$4 + 25 - 16 - 60 + P < 0$$

$$P > 76 - 29$$

$$P < 47$$

$$P < 47$$

$$D \sqrt{f^2 - c}$$

$$f^2 - c < 0$$

$$6^2 - P < 0$$

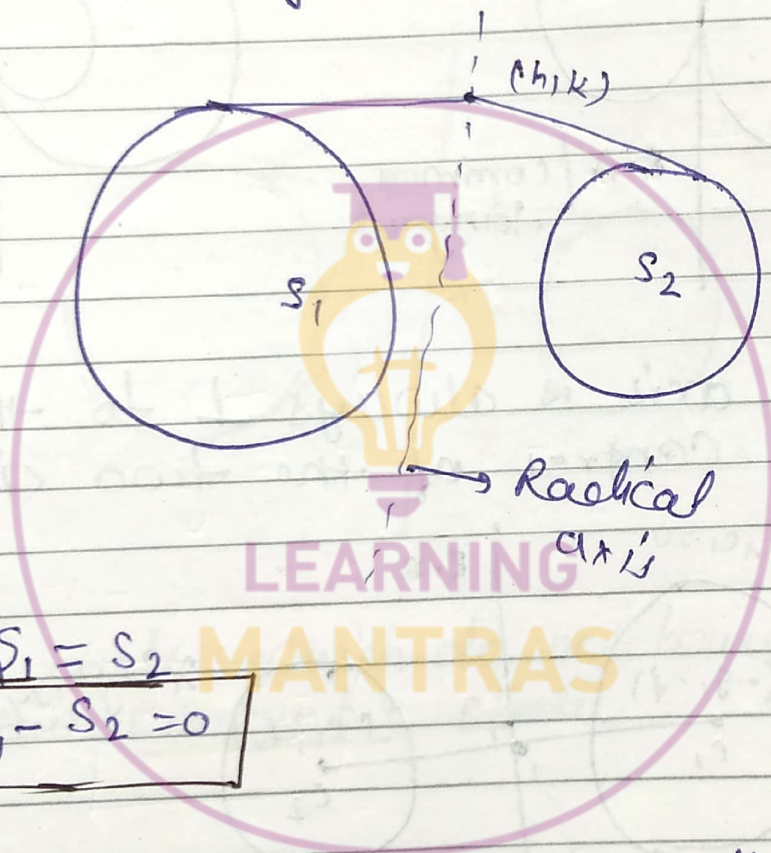
$$P > 36$$

$$P \in (36, 47)$$

* Radical axis and Radical Centre!

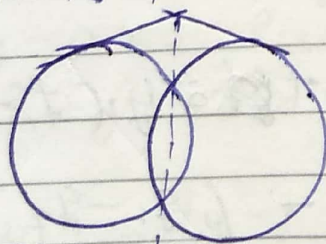
Locus of the point whose Power w.r.t the two circles are equal.

(Length of tangents are equal)



$$\begin{aligned} S_1 &= S_2 \\ S_1 - S_2 &= 0 \end{aligned}$$

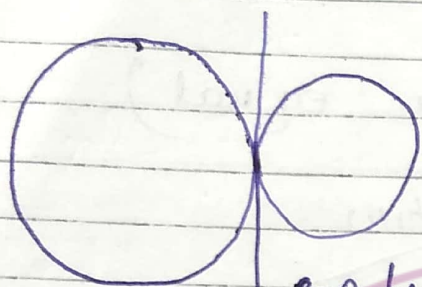
1) If two circles intersect then the radical axis is the common chord of two circles.



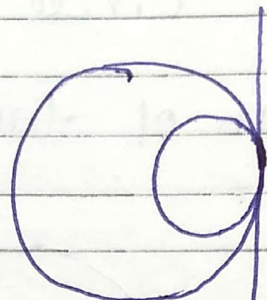
Radical axis
common chord.

equation of common chord = $S_1 - S_2 = 0$

(2) If two circles touch each other then the radical axis is the common tangent of the two circles at the point of contact.

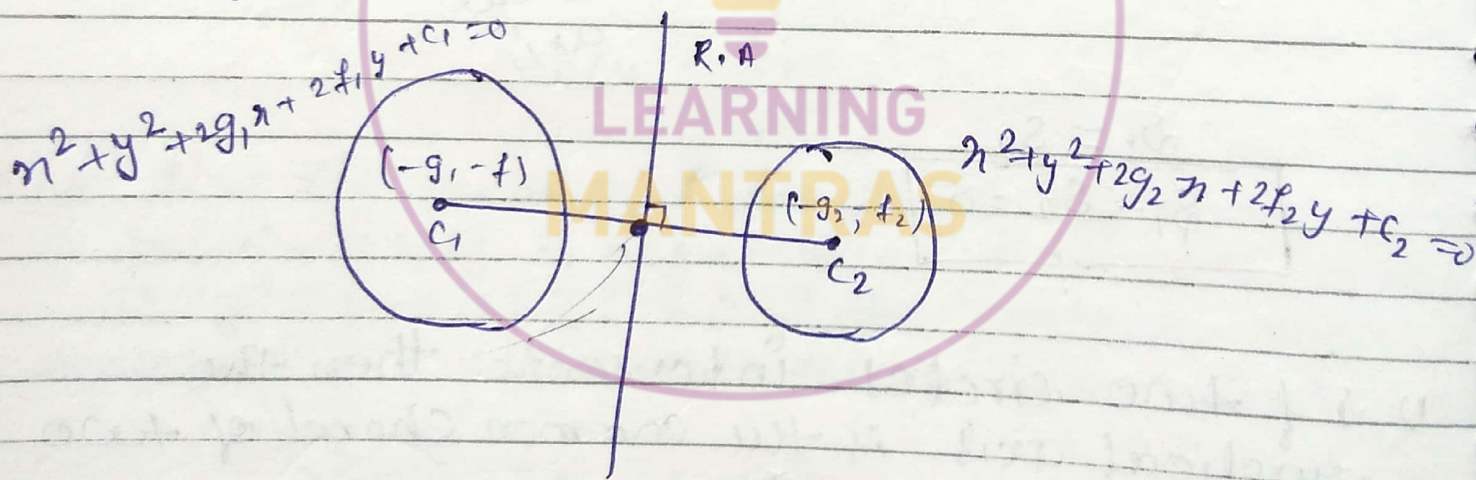


R.A / common tangent



R.A / common tangent

(3) Radical axis is always \perp to the line joining the centres of the two circles.



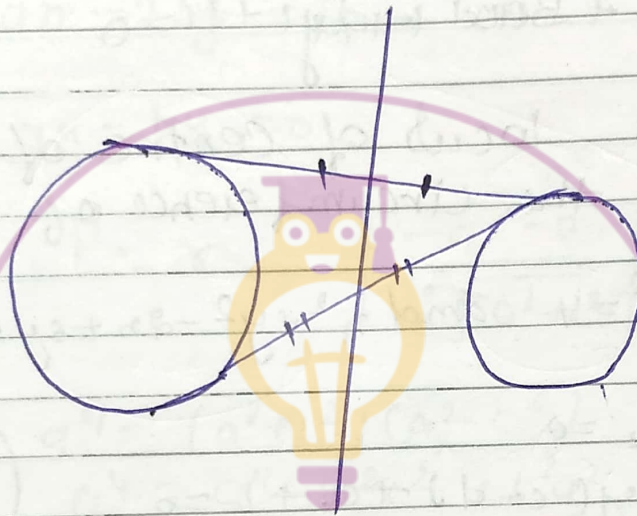
$$2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0$$

$$m_{R.A} = - \left(\frac{g_1 - g_2}{f_1 - f_2} \right) = - \left(\frac{g_2 - g_1}{f_2 - f_1} \right)$$

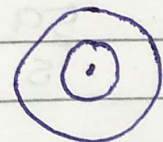
$$m_{C_1C_2} \cdot m_{R.A} = -1$$

* Radical axis need not always pass through the mid-point of the line joining of the centres of two circles.

* Radical axis bisect a common tangent b/w the two circles
Given



* Pair of circle which do not have radical axis are Concentric circles



Concentric.

Que: No. of real values of a for which line $5x + by - a = 0$ passes through POI of circles $x^2 + y^2 + 2ax + cy + a = 0$ and

$$x^2 + y^2 - 3ax - dy - 1 = 0$$

Q.2 Tangents are drawn to circle $x^2 + y^2 = 12$ at point where it meet $x^2 + y^2 - 5x + 3y = 0$
find the POI of tangents

Q. If the circle $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other. then

P.T $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

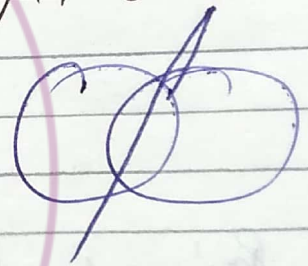
Ans: $x^2 + y^2 + 2ax + c^2 = 0$

Q. Find the locus of centre of circle which bisect the circumference of circle

$x^2 + y^2 = 4$ and $x^2 + y^2 - 2x + 6y + 1 = 0$

Ans:

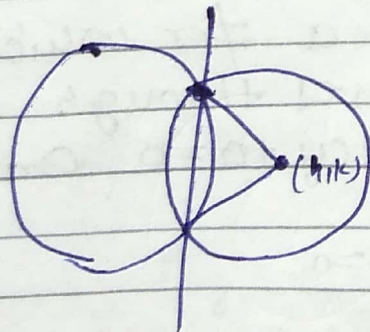
$S_1 - S_2 = 0$
 $5x + y(c-d) + a + 1 = 0$
 $5x + by - a = 0$



$\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$ $-a^2 = a+1$
 $a^2 + a + 1 = 0$

No real value is possible.

(2)



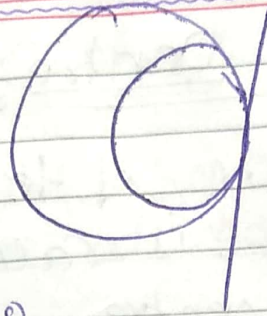
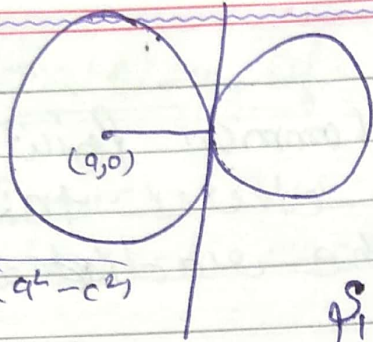
$S_1 - S_2 = 0$
 $5x - 3y = 10$

$T = 0$ $hx + ky = 12$

$\frac{h}{5} = \frac{k}{-3} = \frac{12}{10}$ $h = 6$

$k = -\frac{18}{5}$

(3)



$$r = \sqrt{a^2 - c^2}$$

$$d_1 - d_2 = 0$$

$$2ax - 2by = 0$$

$$ax - by = 0$$

$$P = r \left| \frac{-a^2 - 0}{\sqrt{a^2 + b^2}} \right| = \sqrt{a^2 - c^2}$$

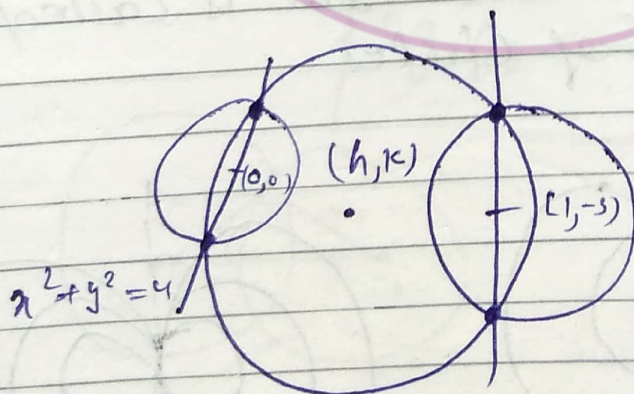
$$a^4 = (a^2 + b^2)(a^2 - c^2)$$

$$a^4 = a^4 - a^2c^2 + b^2a^2 - b^2c^2$$

$$a^2c^2 + b^2c^2 = a^2b^2$$

$$\frac{1}{b^2} + \frac{1}{a^2} = \frac{1}{c^2}$$

(4)



$$x^2 + y^2 = 4$$

$$x^2 + y^2 - 2hx - 2ky + c = 0$$

$$x^2 + y^2 - 2hx - 2ky + c = 0$$

$$2hx - 2h + 2ky - 6k + c = 4$$

$$0 + 0 - c = 4$$

$$c = -4$$

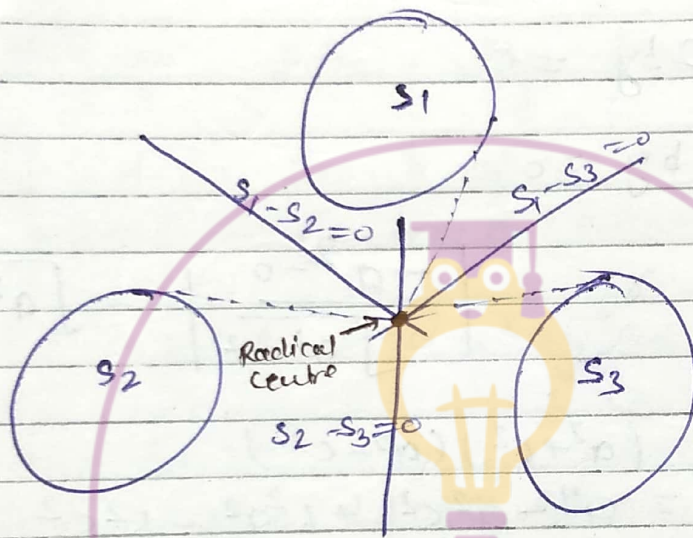
$$2hx - 2h + 2ky - 6k + c = 0$$

$$2h - 2 - 6k + 18 + 1 + 4 = 0$$

$$2h - 6k - 15 = 0$$

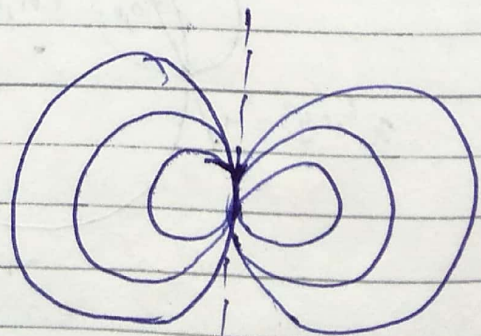
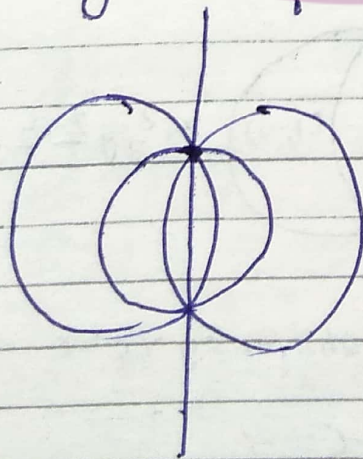
* Radical Centre:

The common point of intersection of the R.A of three circles taken at a time is called the radical centre of a centre.



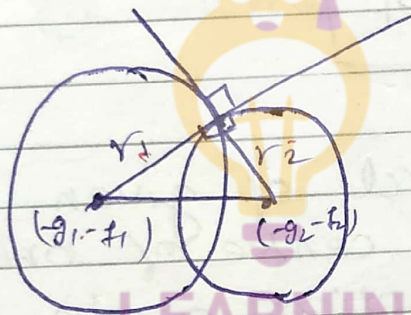
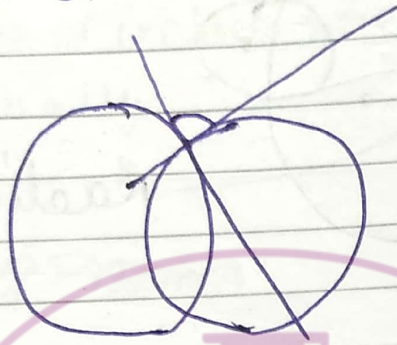
* Co-axial System of Circles:

A system of circle every two of which have the same radical axis ~~have~~ is called co-axial system of circles



* Orthogonality of two Circles:

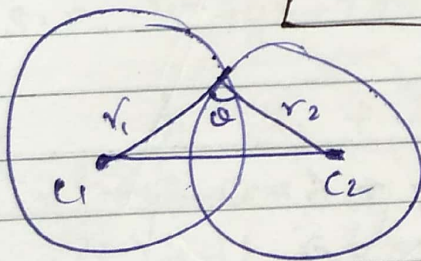
Two curves are said to be orthogonal if they intersect each other at 90° .



$$r_1^2 + r_2^2 = (g_1 - g_2)^2 + (f_1 - f_2)^2$$

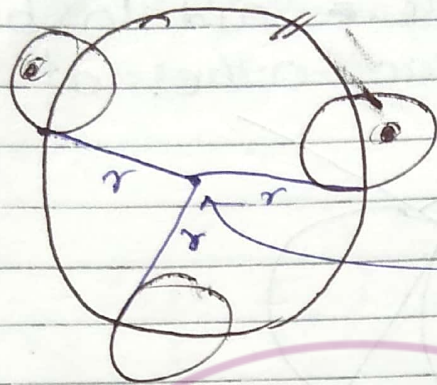
$$g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2 = g_1^2 + g_2^2 - 2g_1g_2 + f_1^2$$

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$



$$\cos \theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$$

Circle Orthogonal to 3 given Circle



Radical centre

Length of tangent

$$r = \sqrt{s_1}$$

Steps:

(1) * find Radical Circle of given circles.
It will be the centre of orthogonally circle
 \Rightarrow centre = radical centre

(2) then find length of tangent from radical axis to any of circle It will be radius of Orthogonally circle

$$r = \sqrt{s_1}$$

$$Q. \quad s_1 \equiv x^2 + y^2 + 2h + 2ky + 6 = 0$$

$$s_2 \equiv x^2 + y^2 + 2ky + k = 0$$

If s_1 & s_2 are orthogonal. then find k .

Q.2 find the eqn of circle which passes through the foci of $S_1 = x^2 + y^2 - 4x + 6y = 0$ and $x^2 + y^2 - 6x + 4y = 0$

cut the circle $x^2 + y^2 - 2x - 4y - 6 = 0$ orthogonally.

Ans: 1

$$= 2 \times (1) \times (0) + 2 \times k \times k = 6 + k$$

$$S_1 - S_2 = 2x - 2y - 6$$

$$2x - 2y - 6 = k(x - y - 3)$$

$$2k^2 = 6 + k$$

$$2k^2 - k - 6 = 0$$

$$(2k + 3)(k - 2) = 0$$

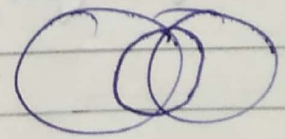
$$k = 2, -3/2$$

Q.3 find the eqn of circle orthogonally to given circle

$$S_1 = x^2 + y^2 - 2x + 3y - 7 = 0$$

$$S_2 = x^2 + y^2 + 5x - 5y + 9 = 0$$

$$S_3 = x^2 + y^2 + 7x - 9y + 29 = 0$$



Ans: 2

$$S_1 + \lambda S_2 = 0$$

$$x^2 + y^2 - 2x + 3y - 7 + \lambda(x^2 + y^2 + 5x - 5y + 9) = 0$$

$$x^2(1+\lambda) + y^2(1+\lambda) - x(2-5\lambda) + y(3-5\lambda) - 7 + 9\lambda = 0$$

$$k \times (-1) - \left(\frac{6\lambda + 4}{\lambda + 1} \right) + 2(-2) \left(\frac{4\lambda + 6}{1 + \lambda} \right) = -4 + 0$$

$$6\lambda + 4 - 8\lambda - 12 = -4 - 4\lambda$$

~~60~~

$$\phi_1 = 4$$

$$\lambda = 2$$

$$x^2 + y^2 - \frac{16x}{3} + \frac{14y}{3} = 0$$

Ans + 6

$$S_1 - S_2 = -7x + 2y - 16 = 0$$

$$S_2 - S_3 = -2x + 4y - 20 = 0$$

$$x - 2y + 10 = 0$$

$$\begin{cases} 7x - 8y + 16 = 0 \\ 4x - 8y + 40 = 0 \end{cases}$$

$$4x - 8y + 40 = 0$$

$$\frac{3x - 24 = 0}{3x - 24 = 0}$$

$$x = \frac{24}{3} = 8$$

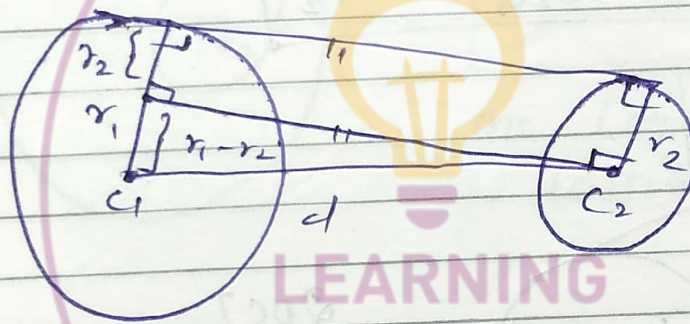
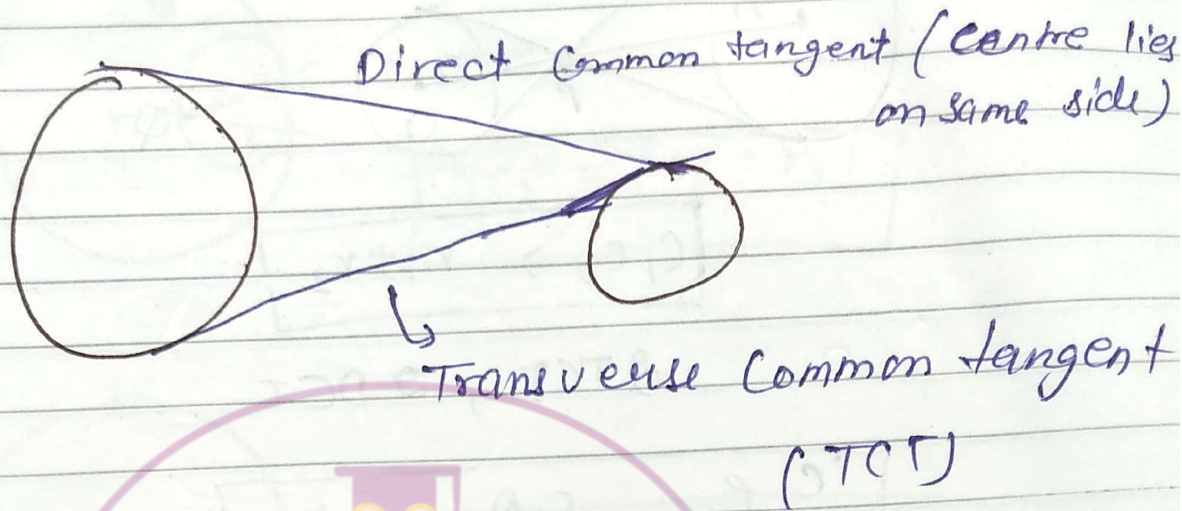
$x = 8$
 $y = 9$ } Radical centre

$$r = \sqrt{S_1} = \sqrt{8^2 + 9^2 - 16 + 27 - 7}$$

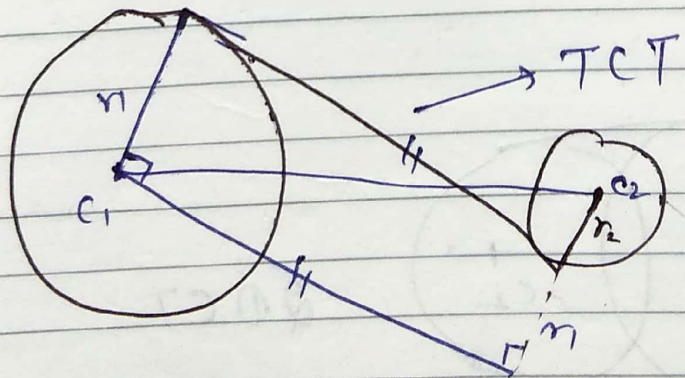
$$= \sqrt{85 + 64} = \sqrt{149}$$

$$(x - 8)^2 + (y - 9)^2 = 149$$

* Common Tangent of two given circles:



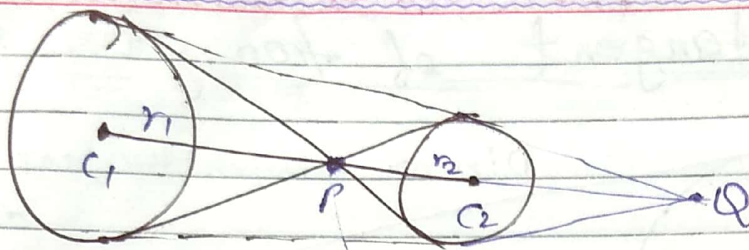
$$DCT = \sqrt{d^2 - (r_1 - r_2)^2}$$



$$DCT > TCT$$

$$TCT = \sqrt{d^2 - (r_1 + r_2)^2}$$

x

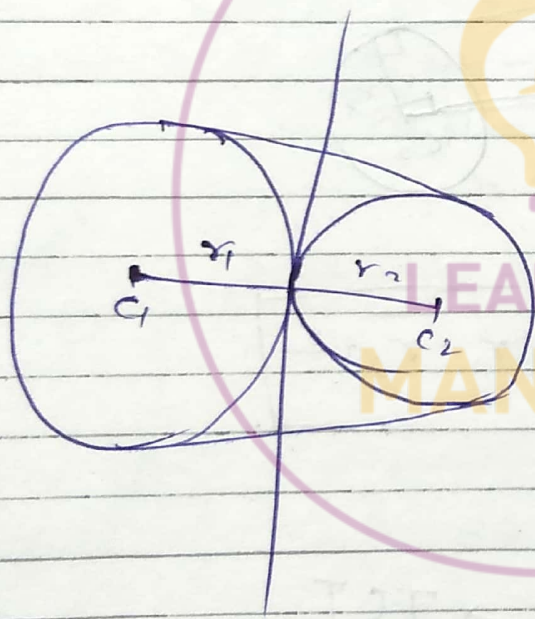


$$C_1 C_2 > r_1 + r_2$$

2 TCT, 2 DCT

$$\frac{C_1 P}{C_2 P} = \frac{C_1 Q}{C_2 Q} = \frac{r_1}{r_2}$$

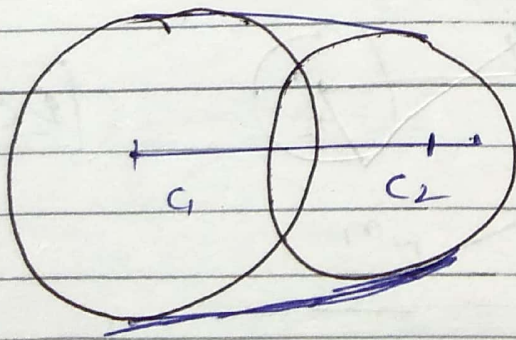
x



2 DCT
1 TCT

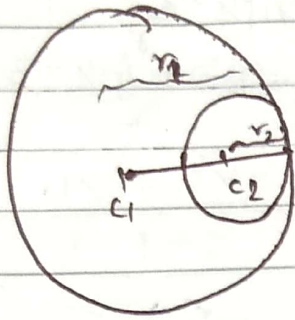
$$C_1 C_2 = r_1 + r_2$$

x



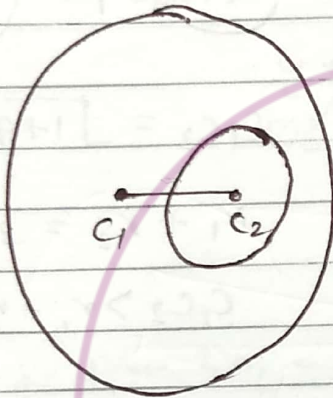
2 DCT

$$|r_1 - r_2| < C_1 C_2 < r_1 + r_2$$



1 DCT

$$C_1 C_2 = |r_1 - r_2|$$

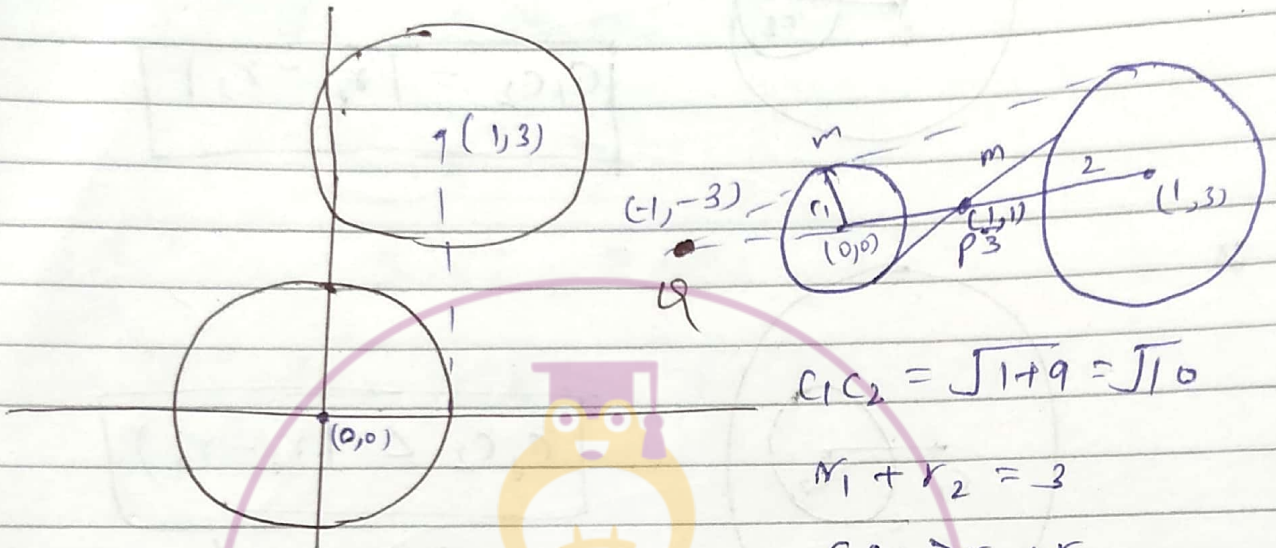


$$C_1 C_2 < |r_1 - r_2|$$

no common tangent is possible

LEARNING
MANTRAS

find eqⁿ of common tangents of
 $x^2 + y^2 = 1$ and $(x-1)^2 + (y-3)^2 = 4$



$$C_1C_2 = \sqrt{1+9} = \sqrt{10}$$

$$r_1 + r_2 = 3$$

$$C_1C_2 > r_1 + r_2$$

$$\frac{C_1P}{C_2P} = \frac{r_1}{r_2}$$

$$P = \frac{2 \times 0 + 1 \times 3}{1+2} = \frac{1}{3}$$

$$\frac{3+0}{1+2} = 1$$

$$y - 1 = m(x - \frac{1}{3})$$

$$mx - y - m + 1 = 0$$

$$P = 1$$

$$\left| \frac{-\frac{1}{3} + 1}{\sqrt{m^2 + 1}} \right| = 1$$

$$m^2 + 1 = (-\frac{1}{3} + 1)^2$$

$$m^2 + 1 = m^2 + 1 - 2m \quad m \rightarrow \infty$$

$$m \neq 0$$

$$T^2 = SS_1$$

$$T^2 = SS_1 \text{ } \left. \begin{array}{l} \text{Pair of} \\ \text{Tangent.} \end{array} \right\}$$

$$y = 1, \quad x-1=0 \quad x=1$$

Common Tangent (TCT)

$$\frac{1 \times 1 - 2 \times 0}{1-2}, \quad \frac{1 \times 3 - 2 \times 0}{1-2} = (-1, -3)$$

$$\frac{C_1}{C_2} = \frac{r_1}{r_2}$$

$$y + \beta = m(x + \alpha)$$
$$mx - y + m - 3 = 0$$

$$\left| \frac{m-1}{\sqrt{m^2+1}} \right| = 1$$

$$\Rightarrow \frac{m^2}{9} + 1 = \frac{2m}{3} = m^2 + 1$$

$$\frac{8m^2}{9} = \frac{-2m}{3} \quad m=0$$

$$\frac{8m}{9} = \frac{-2}{3} \quad m = -\frac{3}{4}$$

$$m=0 \quad y=1$$

$$\text{at } m = -\frac{3}{4}$$

Common Tangent (TCT)

$$y - 1 = -\frac{3}{4} \left(x - \frac{1}{3} \right)$$

$$m^2 + 9 - 6m = m^2 + 1$$

$$m \rightarrow \infty, \quad 6m = 8$$

$$x+1=0$$
$$x = -1$$

$$m = \frac{4}{3}$$

$$y + 3 = \frac{4}{3} (x + 1)$$

$$Q \quad (x-2)^2 + (y+3)^2 = 1$$

$$x^2 + 4 - 4x + y^2 + 9 + 6y = 1$$

$$x^2 + y^2 - 4x + 6y + 12 = 0$$

then find eqⁿ of pair of tangent from origin

$$T = SS_1, \quad (x_1, y_1) = (0, 0)$$

$$[0 + 0 - 2(x+0) + 3(y+0) + 12]^2 = [x^2 + y^2 - 4x + 6y + 12] \times (12)$$

$$[2x - 3y + 12]^2 = 12(x^2 + y^2 - 4x + 6y + 12)$$

$$4x^2 + 9y^2 + 144 - 12xy + 72y - 48x = 12x^2 +$$

$$12y - 48x + 72y + 144$$

$$8x^2 + 3y^2 + 12xy = 0$$



Learning Mantras

Our Guidance, Your Success