Handwritten Notes
On
Capacitor

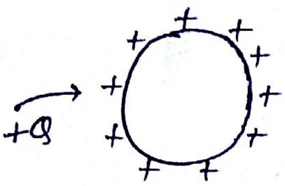
CAPACITOR

* c does not depend on Q & V.

The capacity of conductor to store charge on its Shape & size.

* Dielectric strength of medium → It is max electric field after which insulation of medium get punchered. for air = 3×10^6 V/m

solid conducting sphere:→



$$V_{sphere} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{R}$$

$$V_{sphere} - V_{infinite} = \Delta V = \frac{Q}{(4\pi\epsilon_0)R}$$

$$Q = (4\pi\epsilon_0 R)\Delta V$$

$$Q = C\Delta V$$

$$C = 4\pi\epsilon_0 R$$

* In conductor

$Q(\uparrow) = V(\uparrow)$

Means, charge (Q) ∝ pot. (V)

unit → MKS → C/volt, Farad (F)

CGS → stat farad

(1F = 9×10^{11} st. F)

Imp

* C_{earth} = ?

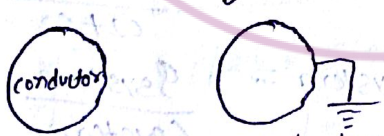
* Theoritical value → By considering It conductor of 6370 (≈6400) km

*$C = 711 \, \mu F$

* Practically → earth can accept unlimited charge so its capacitance is infinite (∞)

* capacitance of any earth conductor is infinite.

EX →



capacitance = c, take limited charge

capacitance = ∞, take unlimited charge { b/c $c = \frac{Q}{V} = \frac{Q}{0} = \infty$ }

potential energy stored in charge conductor →

which is some external work is done during charging of conductor which is stored in form of pot. energy.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

* P.E ⊕nt only in charge conductor but capacitance ⊕nt in charge or uncharge.

Imp * Energy spend by Battery

$\because$ Emf $= V$

1 charge $\longrightarrow$ Work $= V$

$q \longrightarrow$ Work $= qV$

Work by battree $= \Delta U +$ Heat

$$qV = \left(\frac{1}{2}qV - 0\right) + \text{Heat}$$

$$\text{Heat} = \frac{1}{2}qV$$

NOTE → During charging half of energy spend for battery is stored in the form of P.E of conductor & remaning half wasted in form of Heat in connected wire.

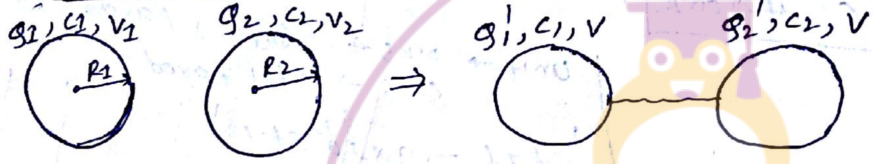
EX → For a conductor

If charge $= q = PE = U$

then charge $= q/2 = PE = ?$

$C = \text{const.}, \; U = \frac{q^2}{2C} \propto q^2, \; U' = \frac{U}{4}$

** # Re-distribution of charge :→



* $C_{system} = C_1 + C_2 + C$ ← It is capacitance of conductor wire it is negligible.

$C_{system} = C_1 + C_2$

* Total charge of system is re-distributed is ratio of capacitance.

$q = CV$

$q \propto C \propto R$

* $V_{common} = ?$

$q_1 + q_2 = q_1' + q_2'$

$C_1V_1 + C_2V_2 = C_1V + C_2V$

$$V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$V_{common} = \frac{q_1 + q_2}{C_1 + C_2}$$

$$V_{system} = \frac{q_{system}}{C_{system}}$$

* Energy loss (ΔU)

$U_{Initial} = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

$U_{Final} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$

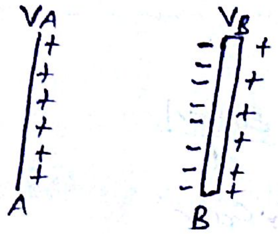
$U_{Initial} - U_{Final}$

$\Delta U = U_{final} - U_{Initial}$

$$\Delta U = \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$$

Concept of capacitor :→ A/R Generally, capacitor are termed by using 2 conductor having equal & opposite charge on them. The advantage of using opp. charge is to contive all the critical energy in certain volume so that it can be early extraxted when required.

Case : I



pot. of plate A

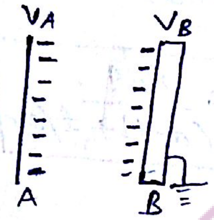
$V_A' = V_A - V_B^- + V_B^+$

$\because |V_B^-| > |V_B^+|$ (Here B^- more due to more impact of plate A)

$\Rightarrow V_A' < V_A$

$\because C = \frac{Q \leftrightarrow}{V(\downarrow)} = C(\uparrow)$

Case II



$V_A'' = V_A - V_B^-$

$V_A'' << V_A$

$\because C = \frac{Q \leftrightarrow}{V \downarrow\downarrow} = C(\uparrow\uparrow)$

A/R

*→ When a neutral or, opp. charge conductor is placed near a charge conductor then charge of Ist conductor remain uncharge while pot. change significantly so capacitance ↑

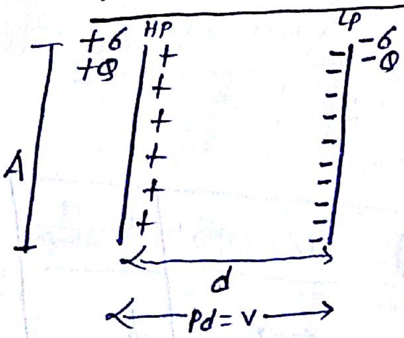
Type of capacitor →

|i|→ Parallel Plate capacitor (PPC) → $C = \frac{A\epsilon_0}{d}$

|ii|→ Spherical capacitor → $C = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$

|iii|→ cylindrical capacitor → $C = \frac{2\pi\epsilon_0 L}{\ln\frac{R_2}{R_1}}$

|i|→ Parallel plate capacitor :→



|a|→ Electric field (E)→

$$\left[\begin{aligned} E_{inside} &= \frac{\sigma}{\epsilon_0\epsilon_r} = V/d \\ E_{outside} &= 0 \end{aligned}\right] \text{(uniform)}$$

|b|→ Pot. energy (U) :→ $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$

|c|→ Energy density/Electrostatic pressure (u) :→ energy store in unit vol.

$u = \frac{Energy}{vol.}$ $\quad U = \frac{1}{2}\frac{\epsilon_0}{a}\left(\frac{V}{d}\right)^2 = \frac{1}{2}\epsilon_0 E^2$

$$* \; U = \left\{\frac{1}{2}\epsilon_0\left(\frac{\sigma}{\epsilon_0}\right)^2 = \frac{\sigma^2}{2\epsilon_0}\right\}$$

($\frac{\sigma}{\epsilon_0}$ ↑ E^2)

(d) → Attraction force b/w // plate :→

$= \frac{\sigma^2}{2\epsilon_0}$

$= \left(\frac{\sigma^2}{2\epsilon_0}\right)A$

$$F = \left(\frac{\sigma^2}{2\epsilon_0}\right)A$$ *

(e) → capacitance (c) :→

|Air| $C_{air} = \frac{\epsilon_0 A}{d}$

|ϵ_r| $C_{Med} = \frac{\epsilon_0 \epsilon_r A}{d}$

$\because C_{Med} = \epsilon_r C_{air}$

$\because \epsilon_r > 1$

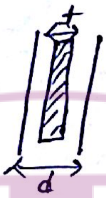
$\Rightarrow C_{Med} > C_{air}$.

If $C_{air} = C_{Med}$

$\frac{\epsilon_0 A}{d_{air}} = \frac{\epsilon_0 \epsilon_r A}{d_{Med}}$

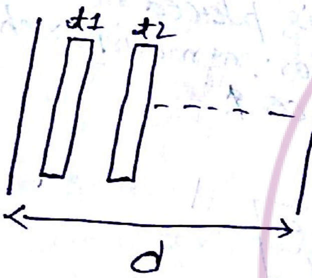
$d_{air} = \frac{d_{med}}{\epsilon_r}$

#



$$C_{partial} = \frac{\epsilon_0 A}{(d-t) + t/\epsilon_r}$$

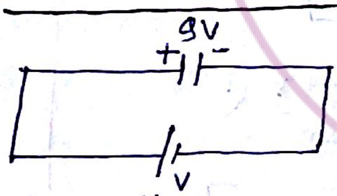
#



$$C = \frac{\epsilon_0 A}{(d - t_1 - t_2 - \cdots) + \left(\frac{t_1}{\epsilon_{r1}} + \frac{t_2}{\epsilon_{r2}} \cdots\right)}$$

* capacitance is independent of position of slab with respect to plate.

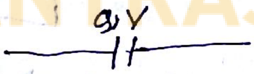
Variation in charge PPC :→



⇓

V_c = const

(battre connected)



⇓

q_c = const.

↓

battree disconected

case-I → Battery dis connected (Q_c = const) :→

Variation	$C = \frac{\epsilon_0 \epsilon_r A}{d}$	$V_c = \frac{q}{V}$		q_c = const	$E = \frac{q}{A\epsilon_0\epsilon_r}$	$U_e = \frac{q^2}{2C}$
ϵ_r (↑)	↑	↓		⟷	↓	↓
A (↑)	↑	↓		⟷	↓	↓
d (↑)	↓	↑		⟷	⟷	↑

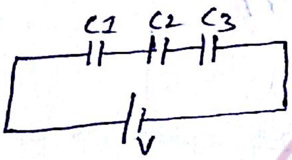
case II ⇒ Battery connected (V_c = const)

Variation	$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$	V_c = const	$q \propto C$, $Q_c = CV$	$E = \frac{V}{d}$	$U_c = \frac{1}{2}CV^2$
ε_r ↑	↑	⟷	↑	⟷	↑
A ↑	↑	⟷	↑	⟷	↑
d ↑	↓	⟷	↓	↓	↑

* If nothing is given in question then consider battree disconnected.

Grouping of capacitor :→

Series



→ Q = same

→ $\frac{1}{C_{net}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

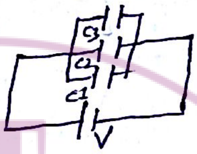
→ $\boxed{C_{net} = \frac{C_1 \times C_2}{C_1 + C_2}}$

→ voltage distribution

$V \propto \frac{1}{C}$

$\boxed{V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}}$

parallel



→ V = same

→ $\boxed{C_{net} = C_1 + C_2 + C_3}$

→ charge distribution

$Q = CV$

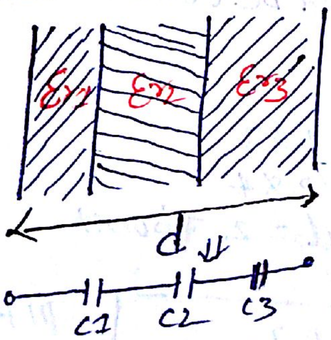
$Q \propto C$

$\boxed{Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3}$

Distribution in PPC by dielectric medium :→

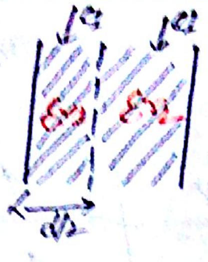
Air PPC $\boxed{C = \frac{\varepsilon_0 A}{d}}$

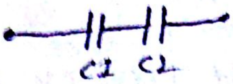
(a) → Division in distance →



[Each section is capacitor & all capacitor are in series]

EX →





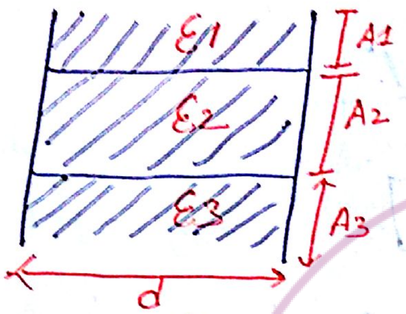
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = \frac{2\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}}$$

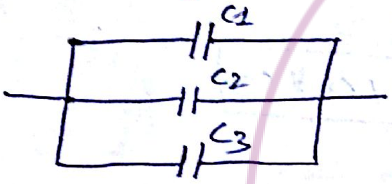
$$C_{eq} = \frac{\varepsilon_0 A}{(d - t_1 + t_2 - -) + \left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}}\right)}$$

* Equivalent dielectric const $(\varepsilon_r)_{eq} = \dfrac{2\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}}$

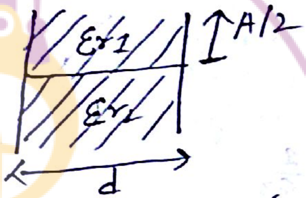
(b) → Division in Area :→



[Each section is a capacitor and all capacitor are in ||.]



EX →



$$C_{eq} = C_1 + C_2 = \left(\frac{\varepsilon_{r1} + \varepsilon_{r2}}{2}\right) C$$

$$(\varepsilon_r)_{eq} = \frac{\varepsilon_{r1} + \varepsilon_r}{2}$$

*

$x \,\&\, y$ A·M ⇒ $\dfrac{x+y}{2}$, G·M $= \sqrt{xy}$, H·M $= \dfrac{2xy}{x+y}$

Multiple capacitor →

→* Given plate arrangement converted into equivalent ckt using point potential Method.

(i) → No. of all plate is done

(ii) → For making capacitor plate should be consecutive there pot. should be diff.

EX →

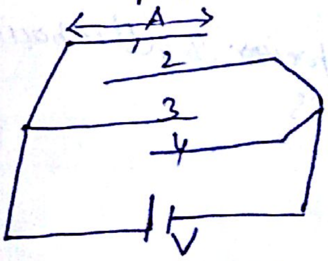
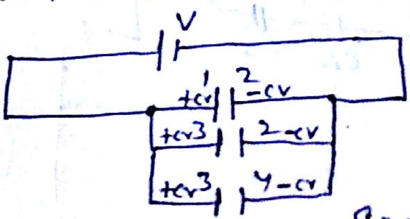


plate = 4
conductor = 2 < 1 & 3 , 2 & 4
seprate potential = 2 = point



(i) → $C_{eq} = 3C = 3\left(\frac{\varepsilon_0 A}{d}\right)$

(ii) → charge on each plate

$q_1 = +CV$

$q_2 = -CV - CV = -2CV$

$q_3 = +CV + CV = +2CV$

$q_4 = -C$

* 3e ⇒ In case of n-parallel plate, if alternate plate are connected with each other then max (n-1) capacitor are there in parallel groping so, $C_{Max} = (n-1)C$

Spherical capacitor ⇒

Case I → If Inner sphere is charged & outer is earthed →

$$V_{small} = \frac{Kq}{r} - \frac{Kq}{R}$$

$$V_{big} = \frac{Kq}{R} - \frac{Kq}{R} = 0$$

pot. difference

$$V = Kq\left(\frac{1}{r} - \frac{1}{R}\right) \quad , \quad V = \frac{q}{4\pi\epsilon_0}\left(\frac{R-r}{Rr}\right)$$

$$\frac{q}{V} = \frac{4\pi\epsilon_0 Rr}{R-r} \qquad \boxed{C_2' = \frac{4\pi\epsilon_0 Rr}{R-r}}$$

* If r = fixed R = ↑ then capacitance = ?

$$C = \frac{4\pi\epsilon_0 r}{1 - \frac{r}{R}} \quad \text{(divided by R)}$$

$C = ↓$ bcoz $\frac{r}{R}$ ↓ so, $1 - \frac{r}{R}$ ↑ something by घटाया (↓)

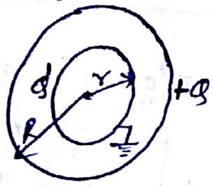
* If r = Fixed, R ↓ then capacitance → ↑

* If R = ∞ then capacitance ?

$$C = \frac{4\pi\epsilon_0\epsilon_r r}{1 - \frac{r}{R}} \qquad (C = 4\pi\epsilon_0\epsilon_r r)$$

Each indivisual conductor is also a capacitor considering that its other conductor is at infinite.

Case II → If outer sphere is charged & inner is earthed.



As inner sphere is earthed the some charge q' is shifted from earth to this sphere for making its pot zero.

$$V_{big} = \frac{Kq}{R} + \frac{Kq'}{R}$$

$$V_{small} = 0$$

∴ P.D

$$V = \frac{Kq}{R} + \frac{K}{R}\left(\frac{-r}{R}q\right)$$

$$V = \frac{Kq}{R}\left(1 - \frac{r}{R}\right)$$

$$V = \frac{q}{4\pi\epsilon_0\epsilon_r R}\left(\frac{R-r}{R}\right)$$

$$C_{II} = 4\pi\epsilon_0\epsilon_r R\left(\frac{R}{R-r}\right)$$

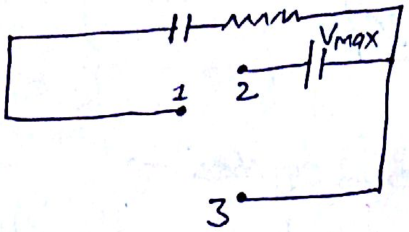
$$C_{II} = \frac{4\pi\epsilon_0\epsilon_r Rr}{R-r} + 4\pi\epsilon_0\epsilon_r R$$

$$\boxed{C_{II} = C_I + 4\pi\epsilon_0\epsilon_r R}$$

$$\boxed{C_{II} > C_I}$$

* If nothing is given in question then we use case I

Charging & discharging of capacitor →



case I ⇒ Charging ⇒

(1-2)

$t = 0$	$t = \uparrow$	$t = \infty$ (full charging)
$V_c = 0$	$V_c = \uparrow$	$V_c = V_{max}$ (max)
$Q_c = 0$	$Q_c = \uparrow$	$Q_c = CV_{max}$ (max)
$U_c = 0$	$U_c = (\uparrow)$	$U_c = \frac{1}{2}CV_{max}^2$ (max)
$I_{ckt} = \frac{V_{max}}{R}$ (max)	$I_{ckt} \downarrow$	$I_{ckt} = 0$

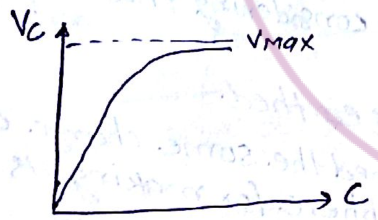
* Here, R is used to safe the capacitor from starting excess current.

By KVL

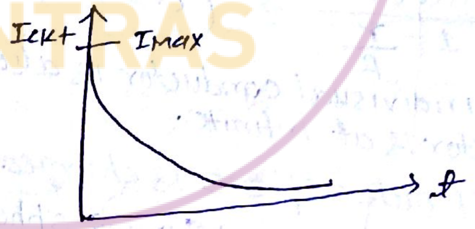
$$V_c = V_{max}\left(1 - e^{-t/RC}\right)$$

$$I_{ckt} = I_{max}\, e^{-t/RC}$$

Graph b/w V_c & t



Graph b/w I_{ckt} & t



$e = 2.73$	$\log_{10} 2 = 0.301$	$\log_e 2 = 2.303 \log_{10} 2 = 2.303 \times 0.301 = 0.693$
$\frac{1}{e} = 0.37$	$\log_{10} 3 = 0.473$	
$e^0 = 1$	$\log_{10} 5 = 0.699$	$\log_x x = 1$, $\log_{10} 10 = 1$
$e^{\infty} = \infty$	$\log_e x = 2.303 \log$	
$e^{-\infty} = 0$		

Time cost (τ) → It is time in wich approx 63% working take place.

$V_c = max\left(1 - e^{-t/RC}\right)$

If ⇒ $t = RC$

* $V_c = V_{max}\left(1 - e^{-1}\right)$

* $V_c = 0.63\, V_{max}$

$\tau = RC$ *

** Imp +

$t = 5\tau = 99.3\%$

$t = \tau = 63\%$

* Heat produced $= \frac{1}{2}CV^2$ [Not depend on Resistance]

* If $R\uparrow \Rightarrow$ Heat $\leftrightarrow$
  timing $\uparrow$
  $I_{ckt}\downarrow$

* If $R\downarrow \Rightarrow$ Heat $\leftrightarrow$
  timing $\downarrow$
  $I_{ckt}\uparrow$

[* Time for 99% charging $\rightarrow$ 0.921 msec.
Trick
* $1\tau = 200\,\mu sec$
  $5\tau = 1000\,\mu sec$
  $t = 5\tau = 99.9\%$ work done so $t =$ less than $1000\,\mu sec$.]

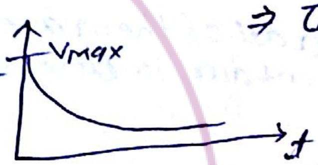
* time for 50% charging $\rightarrow t = 200\times10^{-6}\times0.693 = t = 138.6\,\mu sec$.

case II $\Rightarrow$ **Discharging** :$\rightarrow$

$t=0$	$t=\uparrow$	$t=\infty$ (Full discharge)
$V_c = V_{max}$	$V_c=\downarrow$	$V_c=0$
$q_c = q_{max}$	$q_c=\downarrow$	$q_c=0$
$U_c = \frac{1}{2}CV_{max}^2$	$U_c=\downarrow$	$U_c=0$
$I_{ckt} = \frac{V_{max}}{R}$	$I_{ckt}=\downarrow$	$I_{ckt}=0$

$\therefore V_c = V_{max}e^{-t/RC}$

$I_{ckt} = -I_{max}e^{-t/RC}$

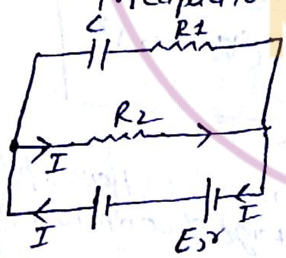


$\Rightarrow \tau = RC$
63% discharge
$\Rightarrow t = 5\tau$
$= 99.3\%$ discharge.

Trick
* $\tau = 1\,sec$ में 63% discharge ho jayega, so 50% discharge less than 1 sec.

RC ckt $\rightarrow$ In steady state condn capacitor are fully charge so current in capacitor branched is zero.

EX$\rightarrow$



In stable condn find charge on capacitor

$I = \frac{E}{R_2+r}$

$\therefore V_{R_2} = I\times R_2 = \frac{R_2}{R_2+r}E$

$\therefore V_{R1} = const\times R1 = 0\times R1 = 0$

$V_{c1} = V_{R_2} = \frac{R_2}{R_2+r}E$

$Q = CV_c = \frac{R_2}{R_2+r}CE$

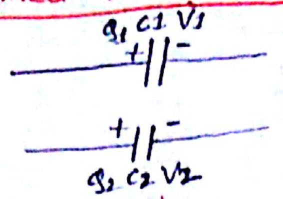
* If $R_2(\uparrow) \Rightarrow Q = ?$

$Q = \frac{R_2}{1+\frac{r}{R_2}}$

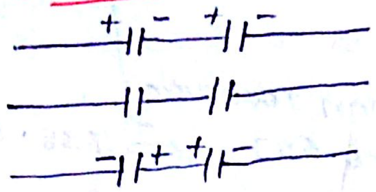
Trick
[$\frac{r}{R_2}$ ratio कम $+1 =$ कम, something upon कम उछाला]

NOTE$\rightarrow$ [* At $(t=0)$, $I = I_{max}$ 'C' is like short ckt.
* At $(t=\infty)$, $I = 0$ 'C' is like open ckt.]

To connect two charge PPC →

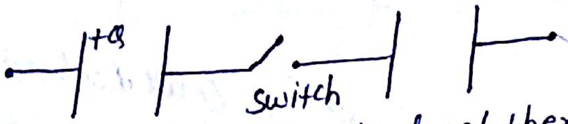


case-I → series –



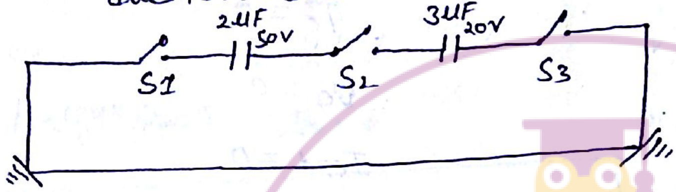
!! * No close loop so no charge flow so no charge in detail of capacitor.

IIT ** Ex → After switch on after long ting what will charge on uncharged capacitor.



Ans → When switch is closed then no charge flow in uncharged capacitor due to no close loop.

IIT Ex →

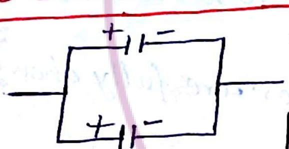


* If any one or two switch are made on then there is no close loop no charge, no current flow so, the details of charge & voltaage remain same.

* When all of them are switch are made then there is charge in charge current due to close loop is formed.

case II → parallel –

(a) → To connect same polarity ends →

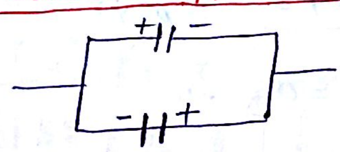


* If voltage are different then charge sharing takeplace to obtain same voltage.

$$V_{common} = \frac{V_1C_1 + V_2C_2}{C_1 + C_2}$$

$$\Delta U = \frac{C_1C_2}{2(C_1C_2)}(V_1 - V_2)^2$$

(b) → To connect opposite polarity plate →



* charge sharing does take place to obtain common voltage & common polarity.

$V_2 \rightarrow -V_2$

$$V_{common} = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

$$\Delta U = \frac{C_1C_2}{2(C_1+C_2)}(V_1 + V_2)^2$$

(plus)

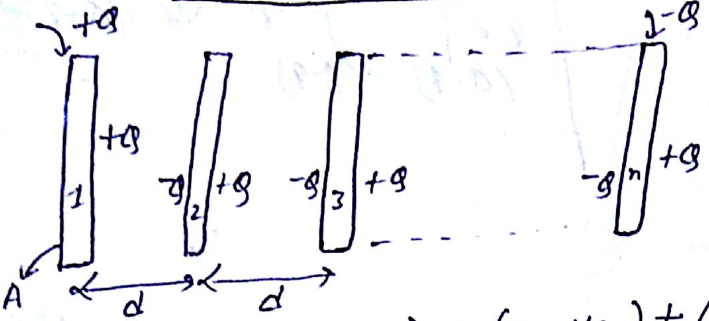
* Nothing is said in q. then we assume same polarity.

*** NOTE →** If $C_1 = C$, $C_2 = xC$, the $V_1 = C$, $V_2 = V/x$ b/c $q = \text{constant}$ $q = CV$

$V \propto \frac{1}{C}$

Different cases of capacitor

case-I ⇒ 'n' no. of plates are placed: —



$$V_1 - V_n = (V_1 - V_2) + (V_2 - V_3) + (V_3 - V_4) + \dots + (V_{n-1} - V_n)$$

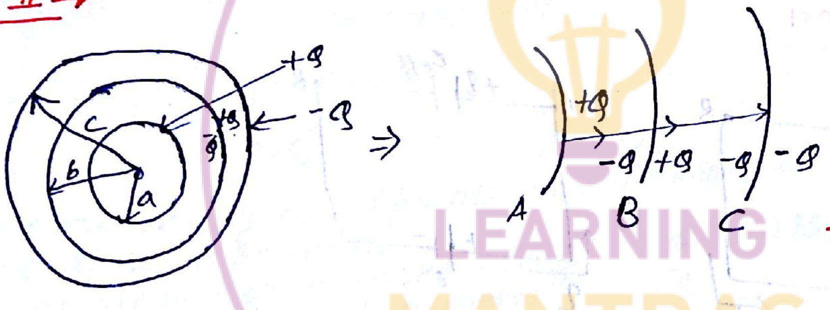
$$\Delta V = \frac{Qd}{A\epsilon_0} + \frac{Qd}{A\epsilon_0} + \dots + \frac{Qd}{A\epsilon_0}$$

$$\Delta V = \frac{Qd(n-1)}{A\epsilon_0}$$

$$Q = \left[\frac{A\epsilon_0}{(n-1)d}\right]$$

* $$C = \frac{A\epsilon_0}{(n-1)d}$$

case-II ⇒



$$V_A = \frac{kq}{a} - \frac{kq}{c}$$

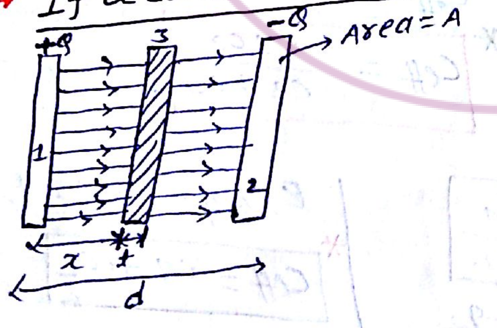
$$V_c = \frac{kq}{c} - \frac{kq}{c} = 0$$

$$\Delta V = V_A - V_c = kq\left[\frac{c-a}{ac}\right]$$

$$q = \frac{(4\pi\epsilon_0)ac}{(c-a)}\Delta V$$

* $$C = \frac{(4\pi\epsilon_0)ac}{c-a}$$

case-III ⇒ If a conducting slab with thickness 't' is introduced. →



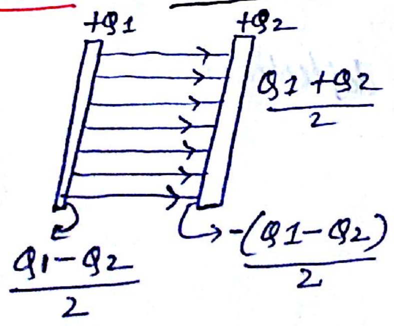
$$V_1 - V_2 = (V_1 - V_3) + (V_3 - V_2)$$

$$= \frac{qx}{A\epsilon_0} + \frac{q(d - t - x)}{A\epsilon_0}$$

$$\Delta V = \frac{Q(d-t)}{A\epsilon_0}$$

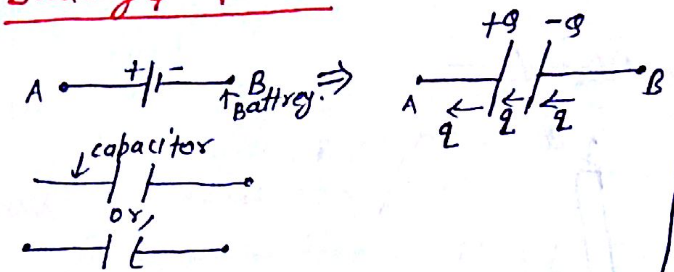
* $$C = \frac{A\epsilon_0}{d-t}$$

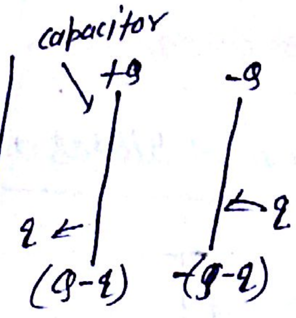
case-IV ⇒ Actual charge on capacitor →



* $$\text{charge on capacitor} = \frac{q_1 - q_2}{2}$$

Battery & capacitor →

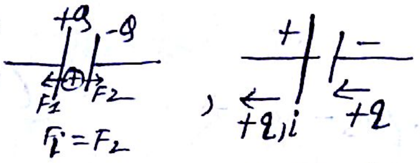




$V_i = \frac{Qd}{A\epsilon_0}$

$V_f = \frac{(Q-q)d}{A\epsilon_0}$

Battery →

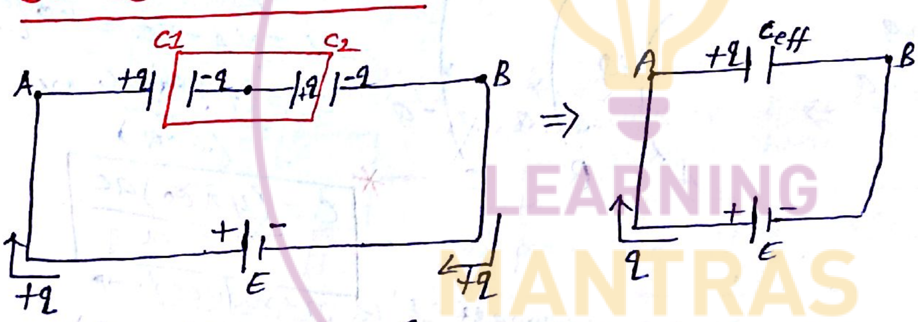


$F_1 = F_2$

* ⊕ve charge will leave from ⊕ve terminal & must entre the ⊖ve terminal of the battery.
* Inside the battery charge is taken from lower pot. plate to higher pot. plate with the help of battery mechanism.
- Work done by battery = charge flow (q) × pot difference b/w terminal (Emf of battery)

Series & parallel combination

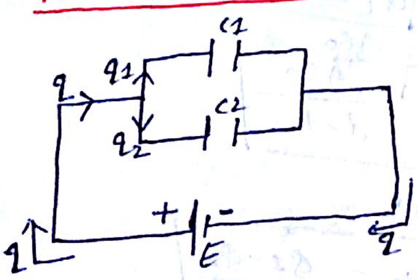
Series combination



$V_A - V_B = \frac{q}{c_1} + \frac{q}{c_2} = \frac{q}{C_{eff}} = E$, $\frac{1}{C_{eff}} = \frac{1}{c_1} + \frac{1}{c_2}$

* $C_{eff} = \frac{c_1 c_2}{c_1 + c_2}$

Parallel combination



* $\frac{q_1}{q_2} = \frac{c_1}{c_2}$

$q = q_1 + q_2$

$E \times C_{eff} = E \times c_1 + E \times c_2$

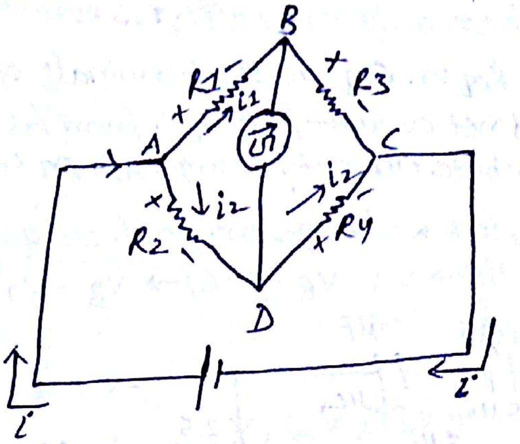
* $C_{eff} = c_1 + c_2$

NOTE Wheatstone Bridge

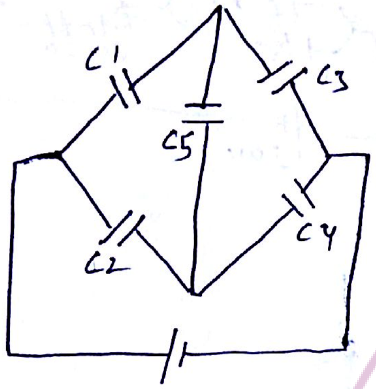
When galvanometer should null (zero) deflection

$V_A - V_B = +i_1 R_1$
$V_A - V_D = +i_2 R_2$] $V_B - V_D$

$i_1 R_1 = R_2 i_2$ —① , $i_3 R_3 = i_2 R_4$ —②



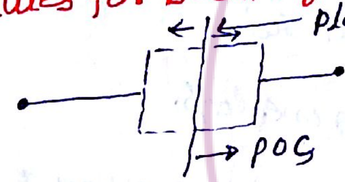
* $$\boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$



* $$\boxed{\frac{C_1}{C_2} = \frac{C_3}{C_4}}$$

Balanced Wheat stone Bridge

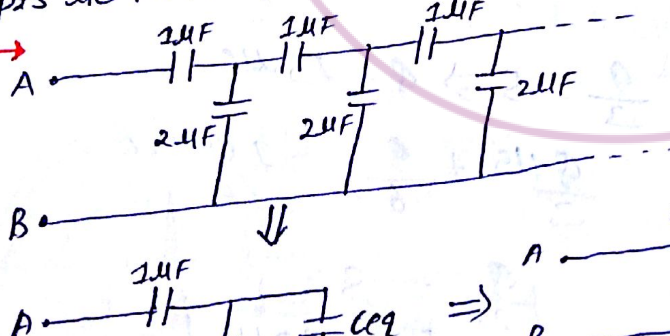
Rules for Breaking or, folding of ckt.



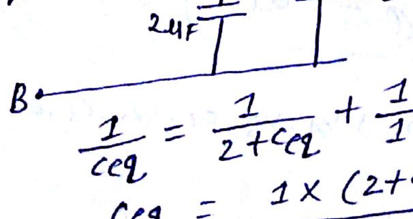
* We can join any no. of points wich are laying on plane of symmetry. They all are at same potential.

* we can break a point in ckt in any no. of points if after breacking all the pts lie in the plane of symmetry.

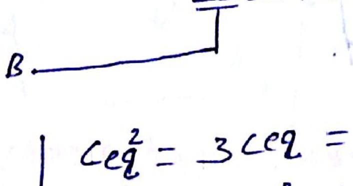
EX→



⇓



⇒



$$\frac{1}{c_{eq}} = \frac{1}{2+c_{eq}} + \frac{1}{1}$$

$$c_{eq} = \frac{1 \times (2+c_{eq})}{1+2+c_{eq}}$$

$$c_{eq}^2 = 3c_{eq} = c_{eq}+2$$

$$c_{eq}^2 + 2c_{eq} - 2 = 0$$

$$c_{eq} = \frac{-2 \pm \sqrt{4+8}}{2}$$

$$= (-1+\sqrt{3})\,\mu F$$

General ckt solving -

Case-I ⇒ When only one battery is connected -

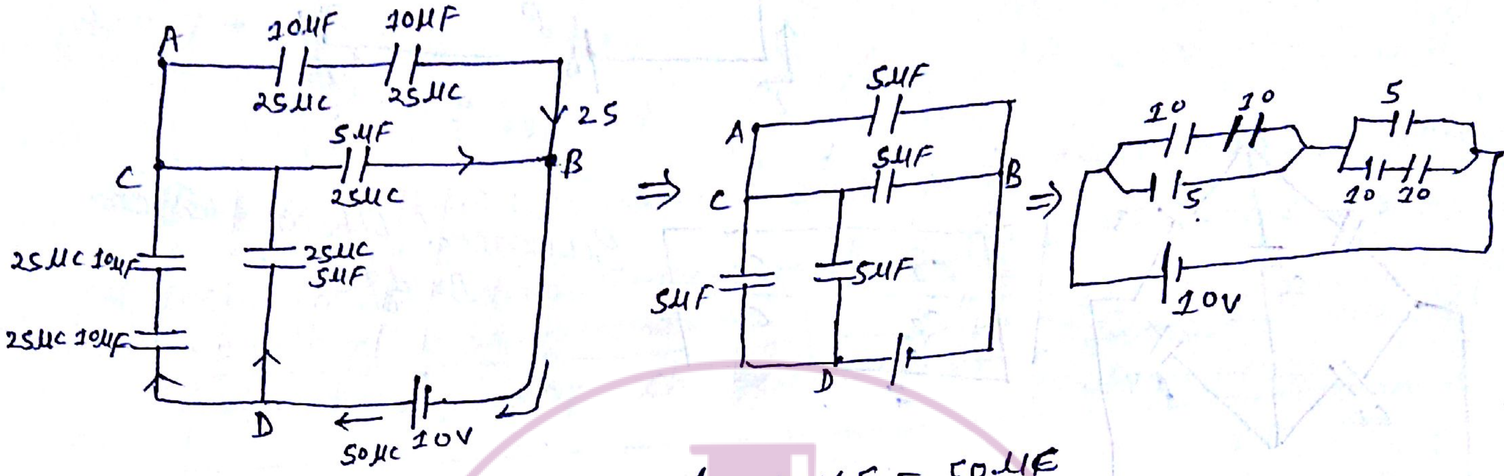
(i) → Find R_{eq} or, C_{eq} across terminals of battery.

(ii) → Find net current / charge from battery as $i_{net} = \frac{V}{R_{eq}}$ or, $q_{total} = C_{eq} \times V$

(iii) → Distribute current / charge A/C to the ckt.

Ex → Find → (a) → charge on each capacitor

(b) → $V_A - V_B = ?$ (c) → $V_B - V_D = ?$



(a) → $C_{eq} = 5\mu F$, $q_{total} = 10 \times 5 = 50\mu C$

(b) → $V_A - V_B = \frac{+25\mu C}{5\mu F} = +5V$

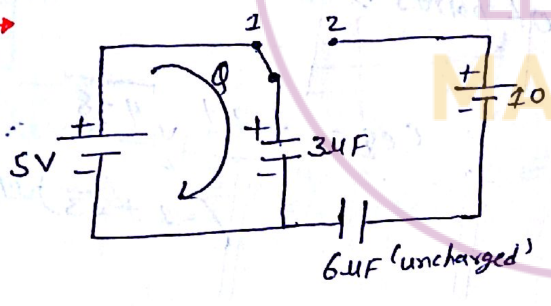
(c) → $V_B - V_D = -10V$

Case-II ⇒ When more than one battery are connected -

→ Apply loop rule (KVL) in each loop of ckt.

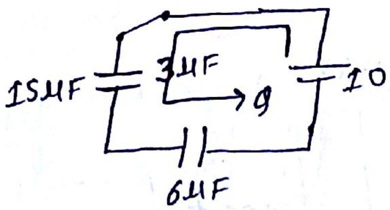
→ only one current / charge will exist along one loop.

Ex → Initially switch was at position 1. Find final charge on capacitor when switch is thrown to position 2.



$\frac{q}{3} = 5 \Rightarrow q = 15\mu C$

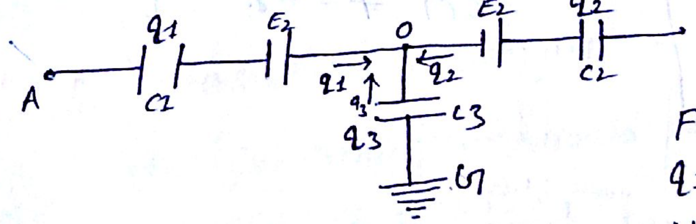
$\frac{q+15}{3} + \frac{q}{6} - 10 = 0$



$\frac{q}{2} = 5$

$\boxed{q = 10\mu F}$

Case-III ⇒ When given ckt is an open ckt.



Given ⇒ $V_A, V_B, C_1, C_2, C_3, E_1, E_2$

From KCL

$q_1 + q_2 + q_3 = 0$ — (1)

$V_A - V_0 = \frac{q_1}{a} - E_1$ — (2)

$V_A - V_0 = +\frac{q_2}{C_2} + E_2$ — (3)

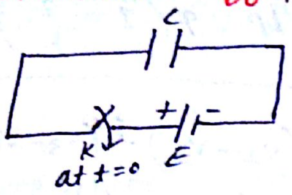
$V_{in} - V_0 = \frac{q_3}{C_3}$ — (4)

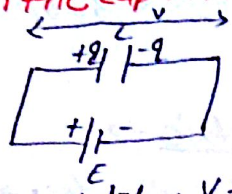
From (1) (2) (3) (4) $V = ?$

$q_1 = ?$, $q_2 = ?$, $q_3 = ?$

***

Potential energy stored in the capacitor :—





at $t=t$, $V = \frac{q}{C}$

In next 'dt' time battery delivers dq charge to the capacitor

dW on capacitor

$$dW_{cap} = dU_{cap} = dq \cdot V$$

$$\int dU_{cap} = \int_0^{Q} \frac{q}{C}\, dq$$

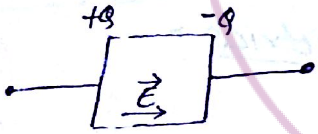
*
$$U_{cap} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} CE^2 = \frac{1}{2} QE$$

Total Work done by battery.

*
$$W_{Batt.} = QE = 2\,U_{cap}$$

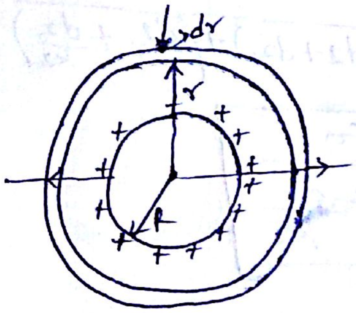
*
Heat generated = H = $W_{Battery}$ − ↑ in PE of capacitor.

A/R
****
NOTE → Energy density / Electrostatic pressure (u) →



$\vec{E} = \frac{Q}{A\epsilon_0}$, $U = \frac{1}{2}\frac{Q^2}{C}$, $U = \frac{Q^2 d}{2A\epsilon_0}$

*
$$u = \frac{U}{Ad} = \frac{Q^2}{2A^2\epsilon_0 d} = \frac{1}{2}\epsilon_0 E^2$$



$C = 4\pi\epsilon_0 R$

$U = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\epsilon_0 R}$

*
$$U_{self} = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\epsilon_0 R}$$

For linear Isotropic dielectrics :→

$\vec{E}_{net} = \vec{E}_{Ext} + \vec{E}_{in}$

$|\vec{E}_{net}| = |\vec{E}_{ext}| - |\vec{E}_{in}|$

*
$$|\vec{E}_{net}| = \frac{|\vec{E}_{ext}|}{K}$$

K → dielectric const.

$\vec{E}_{in} = |\vec{E}_{ext}| - |\vec{E}_{net}|$

*
$$|\vec{E}_{in}| = |\vec{E}_{ext}|\left(1 - \frac{1}{K}\right)$$

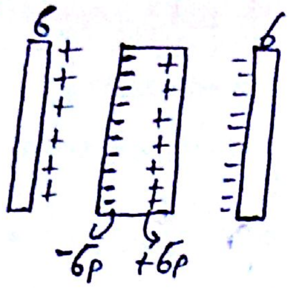
For conductor

* $K = \infty$

* $1 \leq K \leq \infty$

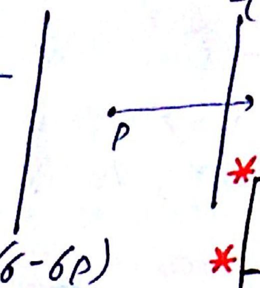
* $K = 1$ → For vaccum

NOTE→



$\vec{E}_{P\,initially} = \frac{\sigma}{\epsilon_0}$

$\vec{E}_{P\,initially} = \frac{\sigma}{K\epsilon_0}$

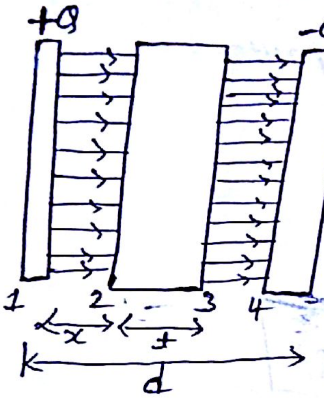


$\frac{\sigma - \sigma_p}{\epsilon_0} = \frac{\sigma}{K\epsilon_0}$

* $\sigma_p = \sigma\left(1 - \frac{1}{K}\right)$

* $q_p = q\left(1 - \frac{1}{K}\right)$

Case IV → capacitor with dielectric →



$V_1 - V_4 = (V_1 - V_2) + (V_2 - V_3) + (V_3 - V_4)$

$$\Delta V = \frac{Qx}{A\epsilon_0} + \frac{Qt}{KA\epsilon_0} + \frac{Q(d-x-t)}{A\epsilon_0}$$

* $$C = \frac{A\epsilon_0}{d - t + \frac{t}{K}}$$

NOTE→

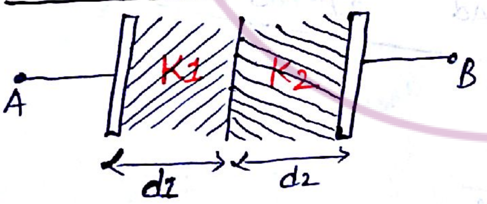
|i|→ If 'n' no. of plates with $(t_1, K_1), (t_2, K_2), (t_3, K_3) \ldots (t_n, K_n)$ are introduced,

* $$C = \frac{A\epsilon_0}{d - (t_1 + t_2 + \cdots + t_n) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \cdots + \frac{t_n}{K_n}\right)}$$

|ii|→ If a conducting slab with thickness 't' is introduced.

* $$C = \frac{A\epsilon_0}{d - t}$$

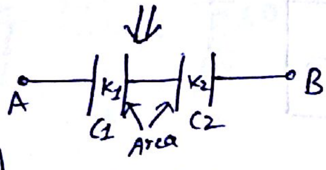
Case V → capacitor filled with different dielectrics →



$$C = \frac{A\epsilon_0}{(d_1 + d_2) - (d_1 + d_2) + \left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)}$$

* $$C = \frac{A\epsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

⇓



$C_1 = \frac{K_1 A\epsilon_0}{d_1}$

$C_2 = \frac{K_2 A\epsilon_0}{d_2}$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

Case-VI → Dielectric const is varying →

$K = K_0\left(1 + \frac{x}{L}\right)$

$dC = \frac{(L\,dx)\epsilon_0 K}{d}$

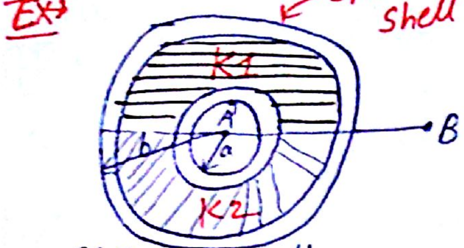
$\int dx = \int_0^L \frac{K_0 \epsilon_0 L}{d}\left(1 + \frac{x}{L}\right) \cdot dx$

$K = K_0\left(1 + \frac{x}{d}\right)$

$dC = \frac{LAK\epsilon_0\left(1 + \frac{x}{d}\right)}{dx}$

$\therefore \frac{1}{C_{eq}} = \int \frac{1}{dC} = \int_0^d \frac{dx}{\left(1 + \frac{x}{d}\right)} \times \frac{1}{LAK_0\epsilon_0}$

***EX→



C1 & C2 are in ||

$$C_1 = \frac{2\pi\epsilon_0 abK_1}{b-a}$$

$$C_2 = \frac{2\pi\epsilon_0 abK_2}{b-a}$$

$$C_{eq} = C_1 + C_2 = \frac{2\pi\epsilon_0 ab}{b-a}(K_1+K_2)$$

case-VII → When dielectric slab is inserted in charged capacitor →

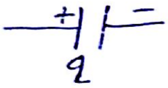
(a) → When battery is disconnected →

$\boxed{q = \text{const}}$*

$$C_i = \frac{A\epsilon_0}{d}$$

$V' = V/K$

$C' = KC$



(i) → capacitance (c) = KC

(ii) → potential $= \frac{V}{K}$

(iii) → $E = \frac{V}{d} \Rightarrow E' = \frac{E}{K}$

(iv) → $W_{ext} = U' = \frac{U}{K}$

$W_{ext} = \Delta U = \ominus ve.$

(b) → When battery remains connected →

$*\boxed{V = \text{const}}$

(i) → capacitance (c) ⇒ $c' = Kc$

(ii) → charge (q) = $q' = Kq$

(iii) → Electric field (E) ⇒ const.

(iv) → pot energy (U) ⇒ $U' = KU$

(v) → $W_{ext} = ?$

$W_{ext} + W_{battery} = \Delta V$

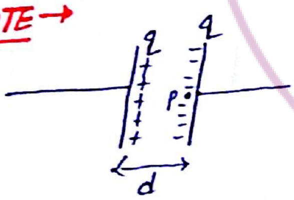
$W_{ext} = (K-1)\frac{CV^2}{2} - (K-1)CV^2$

$W_{ext} = (K-1)\frac{CV^2}{2} - (K-1)CV^2 < 0$

$* \; W_{ext} = \frac{-1}{2}CV^2(K-1)$

***

NOTE →



Electric field at 'P' due to +q plate

$$*E_P = \frac{q}{2A\epsilon_0}$$

Force on 2nd plate $\Rightarrow F = qE_P = \frac{q^2}{2A\epsilon_0}$*

case-VIII → Force acting on dielectric slab while insertion.

(a) → When battery is disconnected – (system have only capacitor)

$$F = -\frac{dU_{capacitor}}{dx}$$

$$*\,U_{capacitor} = \frac{1}{2}\frac{q^2}{C} = \frac{q^2}{2(Ax+b)}$$

$$*\boxed{F = -\frac{dU}{dx}}$$

(b) → When battery remain connected →

$dU = dU_{capacitor} + dU_{Battery}$.

$*\,dU_{Battery} = -dW_{Battery}$.

✓ charge Release from battery $(dq) = (dc)V$

$$*\,dq = \frac{(K-1)l\epsilon_0 V}{d}(dx)$$

$$F = -\frac{dU_{system}}{dx}$$

$$*\boxed{F = \frac{(K-1)l\epsilon_0 V^2}{2d} = \text{const.}}$$