Handwritten Notes

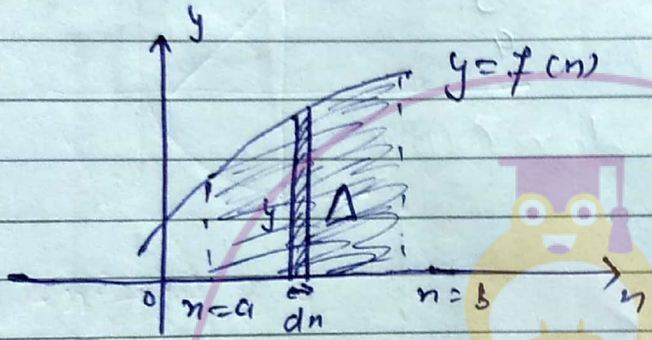
On

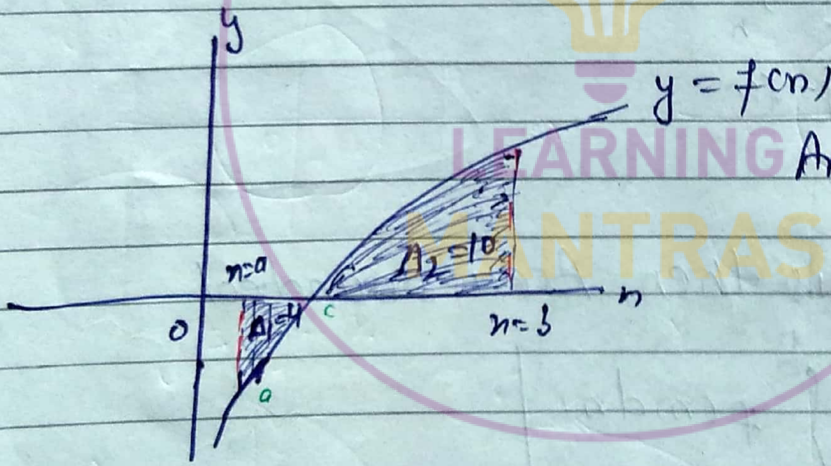
Area Under the Curve

Area Under the Curve

* $= \int_a^b y\,dx = \int_a^b f(x)\,dx = \Delta$

* algebric sum of Area bounded by curve $y = f(x)$ and ordinate $x = a$ and $x = b$





Actual Area

$$\text{Area} = \left|\int_a^c f(x)\,dx\right| + \int_c^b f(x)\,dx$$

$$= 4 + 10$$

$$= 14$$

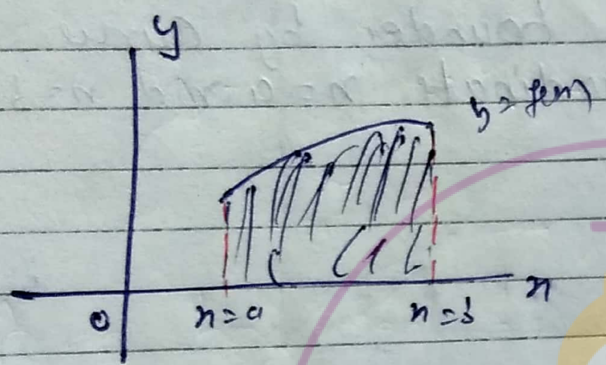
$$\int_a^b f(x)\,dx = 10 + (-4)$$

$$= 6.$$

* formula!

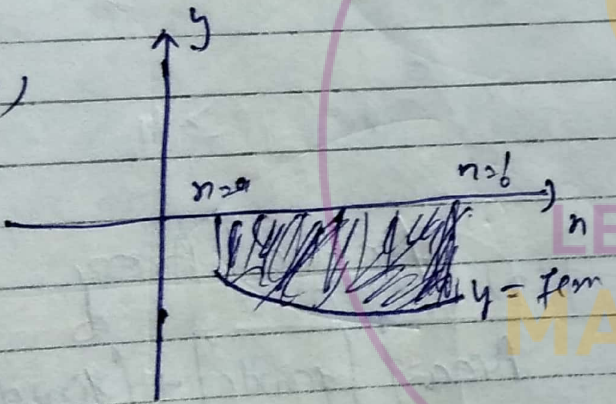
1. If curve $y = f(x)$ completely lie above x-axis then area bounded by curve, x-axis and lines $x = a$ and $x = b$.

(i)



$$= \int_a^b f(x)\,dx.$$

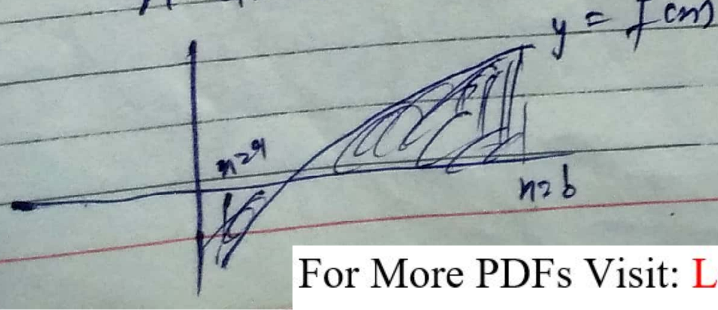
(ii)



If fig. is this then Area bounded

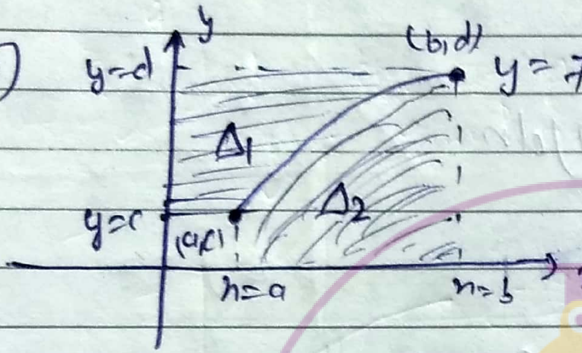
$$Ar.\ B = \left| \int_a^b f(x)\,dx \right|$$

(iii)
(Q) if part of curve lie above and below x axis



then area bounded =

$$A = \left|\int_a^c f(x)\,dx\right| + \int_c^b f(x)\,dx$$

(3) 

$y = f(x)$ or $x = g(y)$

$y = x^2 \Rightarrow x = \sqrt{y}$
$f(x)$ $\quad\quad$ $x = g(y)$

* Area bounded by curve $x = g(y)$, y-axis and the lines $y = c$ and $y = d$

$$\Delta_1 = \int_c^d g(y)\,dy$$

* $$\Delta_2 = \int_a^b f(x)\,dx.$$

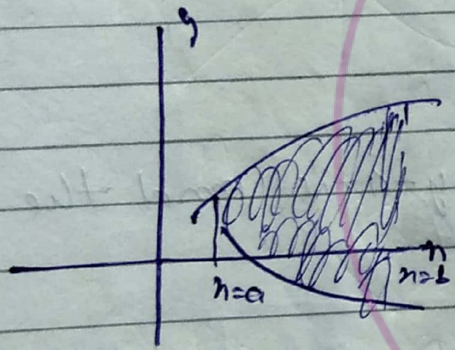
$$\boxed{\Delta_1 + \Delta_2 = bd - ac.}$$

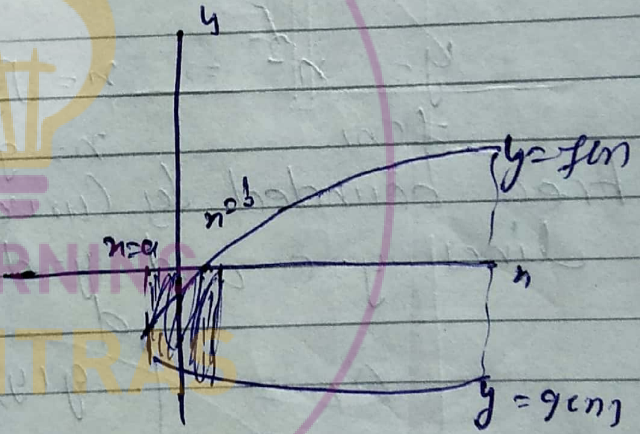
(4)

(4)

$$\Delta = \int_{x=a}^{x=b} (f(x) - g(x))\,dx$$

Area bounded b/w ~~two~~ curve

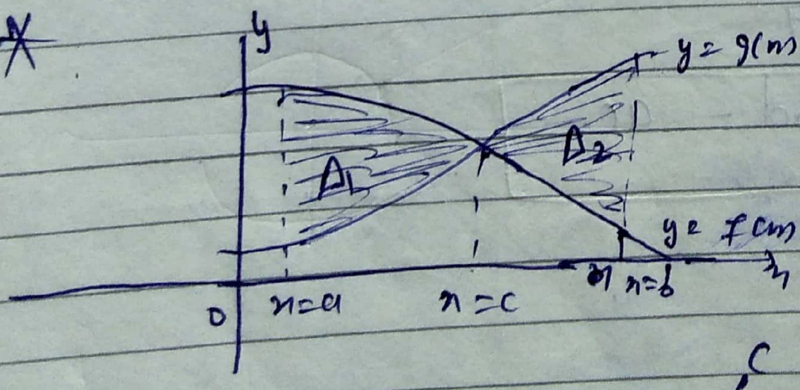




same

$$\Delta = \int_{x=a}^{x=b} (f(x) - g(x))\,dx.$$

*

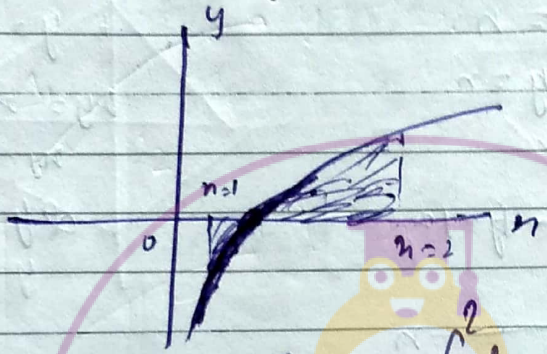


$$\text{Area bounded} = \Delta_1 + \Delta_2 = \int_a^c (f - g)\,dx + \int_c^b (g - f)\,dx$$

or

$$= \boxed{\int_a^b |f-g|\, dx}$$

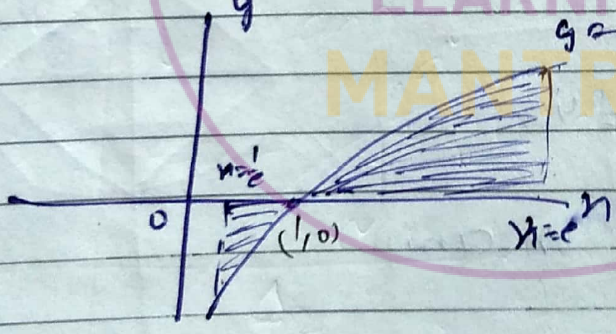
Que: find AB $y = \ln x$, $x=1$, $x=2$ and x axis.



$$= \int_{x=1}^{x=2} f(x) = \int_{x=1}^{2} \ln x$$

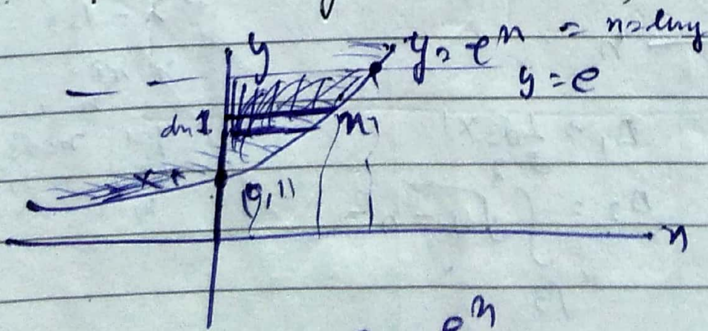
$$A = \int_1^2 \ln x\, dx$$

2. fAB $y = \ln x$ $x = \frac{1}{e}$, $x = e$, x-axis



$$A = \left|\int_{1/e}^{1} \ln x\, dx\right| + \int_1^e \ln x\, dx$$

Que: fAB $y = e^x$, $x=0$, $y=e$, $x>0$



$$= A = \int e^x dx =$$

$$= A = \int_0^1 (e - e^x) dx$$

$e = e^x$

$x = 1$

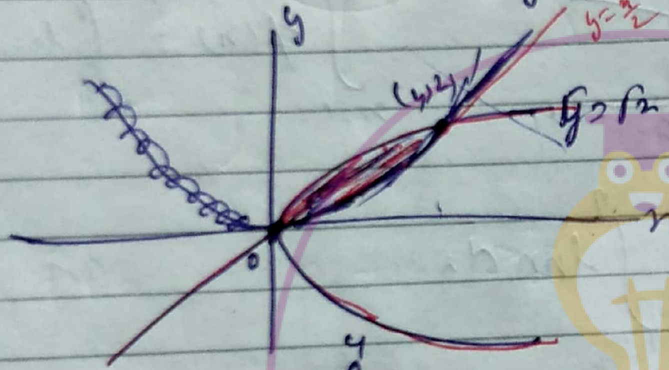
(M-2) $A = \int_1^e \ln y \, dn$

$A = 1 \cdot e - \int_0^1 e^n dn$

Que! fAB = $y^2 = n$
$2y = n$

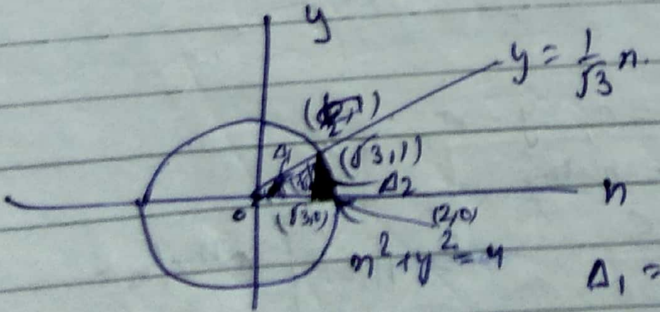
$y = \sqrt{n}$
$2y = n$
$ny = n$
$2y = n$
$y = \frac{n}{2}$

$n=1$ $y=\frac{1}{2}$
$n=2$ $y=1$
$n=4$ $y=3$



$$A = \int_0^4 \left(\sqrt{n} - \frac{n}{2}\right) dn$$

Q. fAB = $n^2 + y^2 = 4$
$n = \sqrt{3} y$
first quad. n axis



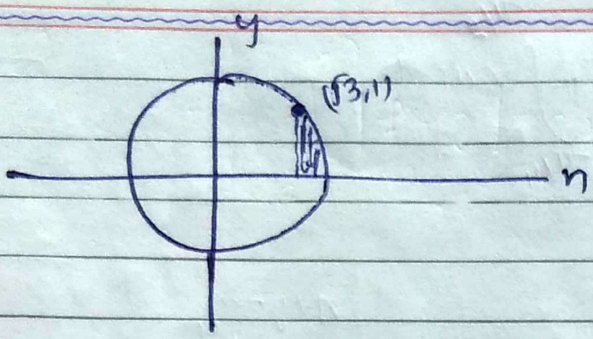
$n = \sqrt{3} y$.
$\Rightarrow \lambda = \sqrt{3} y$.
$n=1$
$y=1$ $n=\sqrt{3}$
$y=2$ $=2\sqrt{3}$
$y=3$ $n=3\sqrt{3}$

$\Delta_1 = \frac{1}{2} \sqrt{3} \times 1$

$\Delta_2 = \int_{\sqrt{3}}^{2} \sqrt{4 - n^2}$

$A = \int$

H.W. JA.



$$A = \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx$$

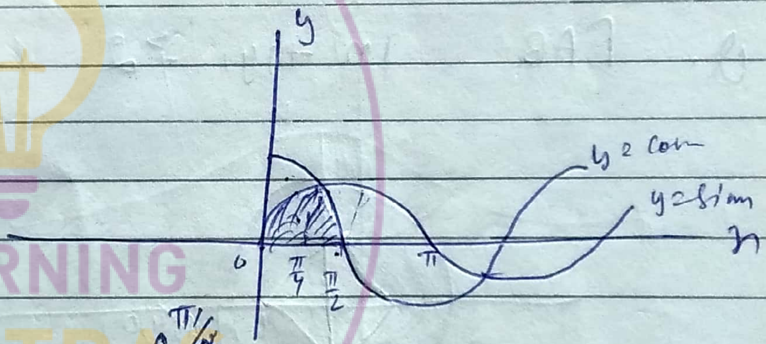
M-2

$$2\pi = \pi(2)^2$$

$$\because \quad 1^c = \frac{4\pi}{2\pi}$$

$$\therefore \quad \frac{\pi}{6} = \frac{\pi}{6} \times \frac{4\pi}{2\pi} = \frac{\pi}{3} \quad \text{Ans.}$$

* Q. FAB $y = \sin x$, $y = \cos x$, $x = 0$, $x = \frac{\pi}{2}$

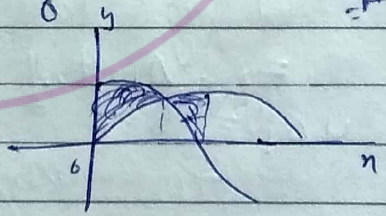


$$A = \int_0^{\pi/2} \cancel{\sin x}\,dx$$

$$A = \int_0^{\pi/2} \cancel{\sin x}\,dx$$

$$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)$$

$$A = \frac{1-0}{} = 1 \text{ sq}$$



$$[\cancel{\cos 90 - \cos 0}]_0^1$$

$$= \cancel{0 - 1} = -1$$

Q. FAB

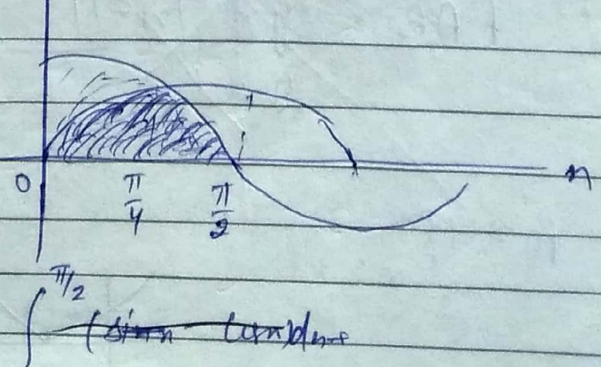
$y = \sin x$

$y = \cos x$

$x = 0$

$x = \frac{\pi}{2}$

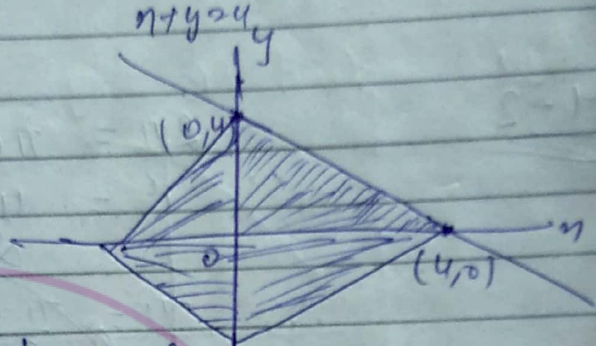
x-axis.



$$A = \int_0^{\pi/4} \cancel{(\cos x - \sin x)}\,dx + \int_{\pi/4}^{\pi/2} \cancel{(\sin x - \cos x)}\,dx$$

$$A = \int_0^{\pi/4} \sin x \, dx + \int_{\pi/4}^{\pi/2} \cos x \, dx$$

Que: $|x| + |y| \le 4$

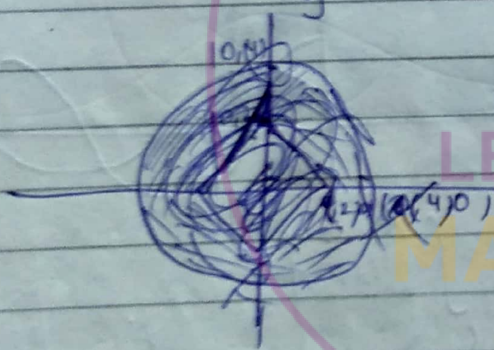


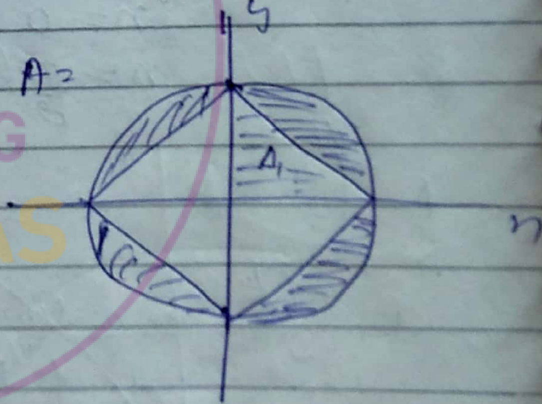
$A = \int_0$

$A = \frac{1}{2} \times 4 \times 4 = 8 \times 4 = 32$ Ans

far all

Q. FAB = $|x| + |y| \ge 2$ & $x^2 + y^2 \le 4$





A =

$= \Delta_1 = \frac{1}{2} \cdot 2 \cdot 2 = 2$

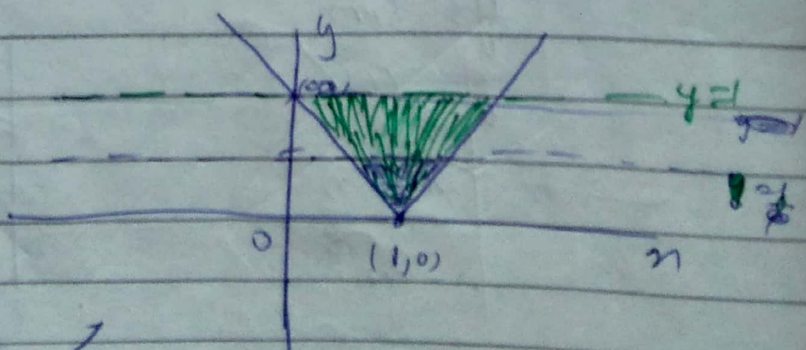
$4\Delta_1 = 8.$

$= \pi(2)^2 - 4\Delta_1$

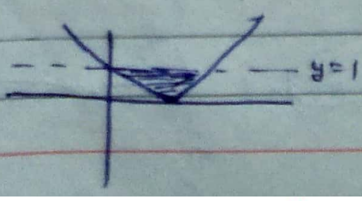
$= 4\pi - 8$

Que: FAB: $y = |x-1|$

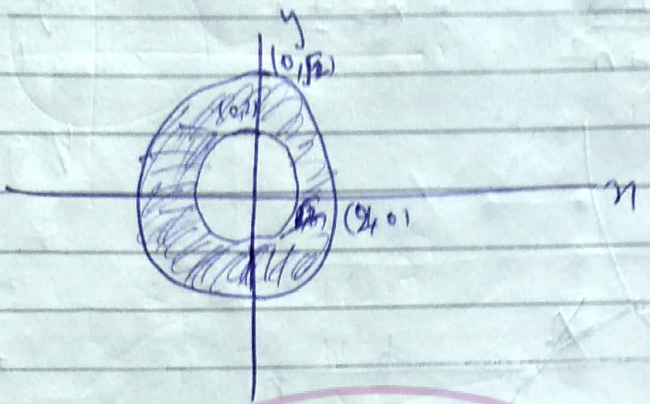
$y = 1$



$A = \frac{1}{2} \times 2 \times 1 = 1$



Q. FAB . $2 \le x^2 + y^2 \le 4$ — $x^2+y^2 \ge 2$, $x^2+y^2 \le 4$.



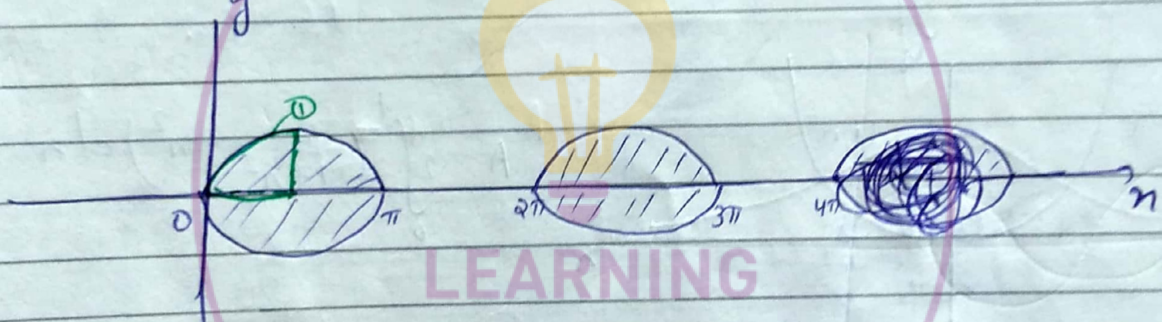
$A = \frac{1}{2} \times 2 \times 2 = 2$

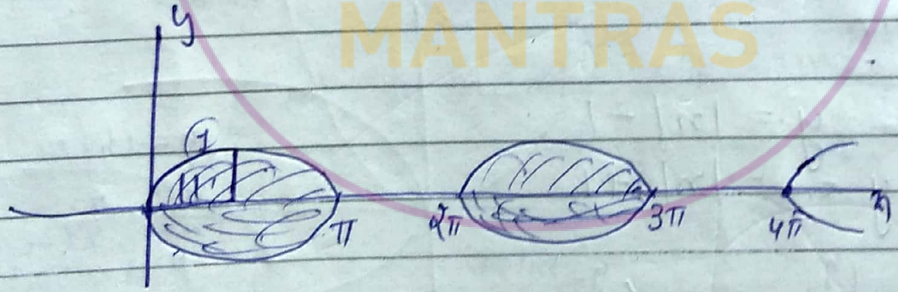
$A = \pi(2)^2 = 2\pi$

$A = \pi(2)^2 - \pi(\sqrt{2})^2$

$4\pi - 2\pi = 2\pi$

Q. FAB: $|y| = \sin x$ $\quad x \in (0, 4\pi)$





$A = \int_0^{\pi} \sin x\,dx + \int_{2\pi}^{3\pi} \sin x\,dx = \cos 3\pi - \cos 0$ $\quad \cos 2\pi - \cos 2\pi$

$= 8A$

$\int_0^{\pi/2} \sin x = 1$

$So = 1 \times 8 = 8$

Q. FAB = $y = x^2$

$y = \dfrac{2}{1+x^2}$ $= x = \pm 1$

$y = \dfrac{2}{1+(x)^2}$

$y = \dfrac{2}{1+x^2}$

$= x = 1,\ y = 1$

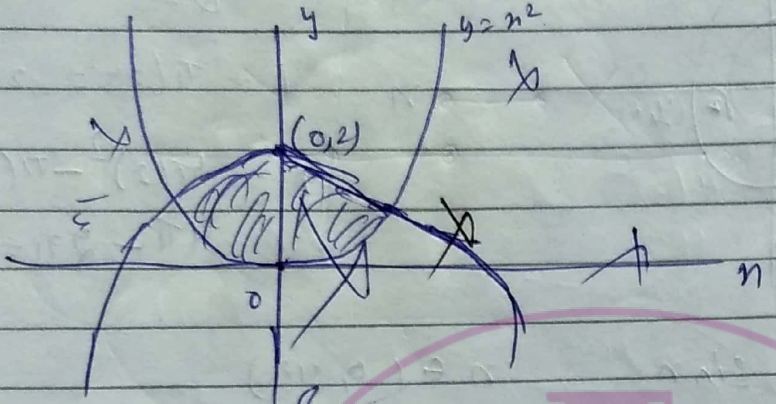
$x = 2,\ y = \frac{2}{5} = 0.4$

$x = 0,\ y = 2$

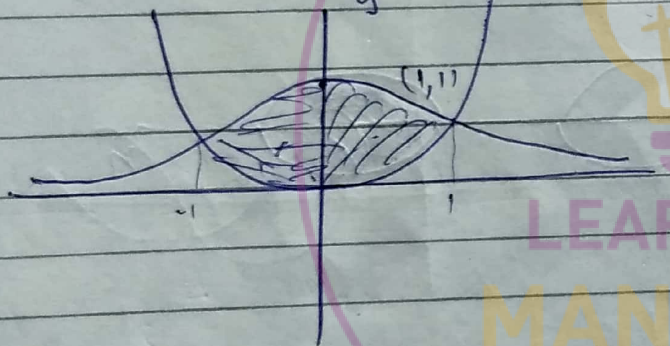
$x = 3,\ y = \frac{2}{10}$

$x = 5 = 4$

$x = -1,\ y = 1$



$A = \int_0^1$



$$A = 2\int_0^1 \left(\frac{2}{1+x^2} - x^2\right) dx$$

Q. FAB = $y = |x| - 1$

$y = -|x| + 1$

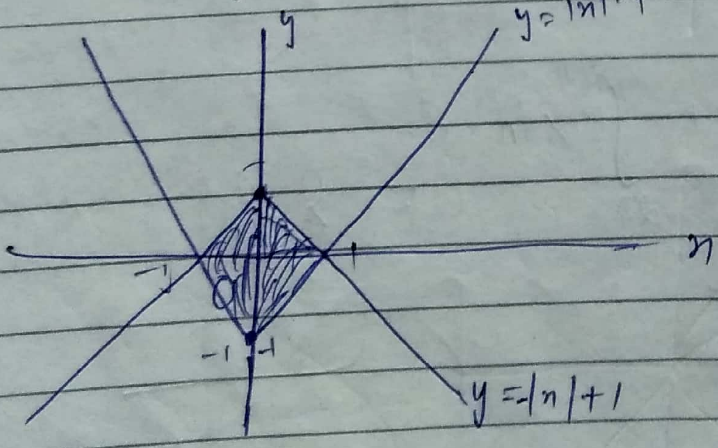
$y = -|x| + 1$

$x = 1,\ y = 0$

$x = 2,\ y = -1$

$x = 3,\ y = -2$

$x = 0,\ y = 1$



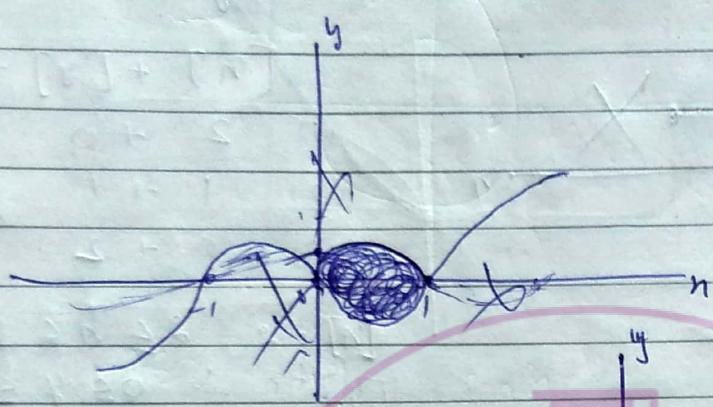
$= \int_{-1}^{1} \cancel{\left((-|x|+1) - (|x|-1)\right)} dx$

$A = \frac{1}{2} \times \cancel{2 \times 2 = 1}$

$A = \frac{1}{2} \times 4 = 2$

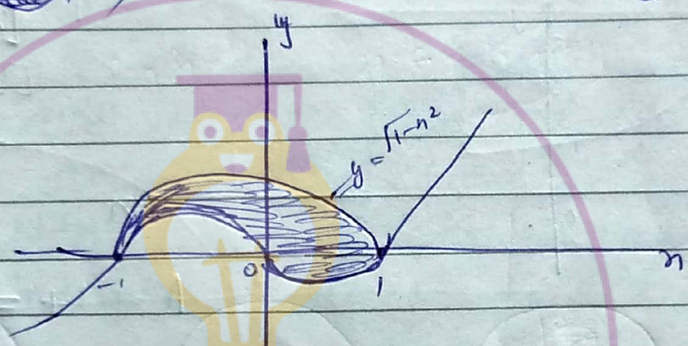
$y^2 = 1 - x^2$
$y^2 + x^2 = 1$

Que: FAB $y = \sqrt{1 - x^2}$
$y = x^3 - x \quad = x(x^2 - 1) \quad = x(x-1)(x+1)$



$y = \sqrt{1 - x^2}$ $\quad x=0, y=\pi$
$x=1 \;=\; y = 0$
$x = 2, \; y = \sqrt{3}$

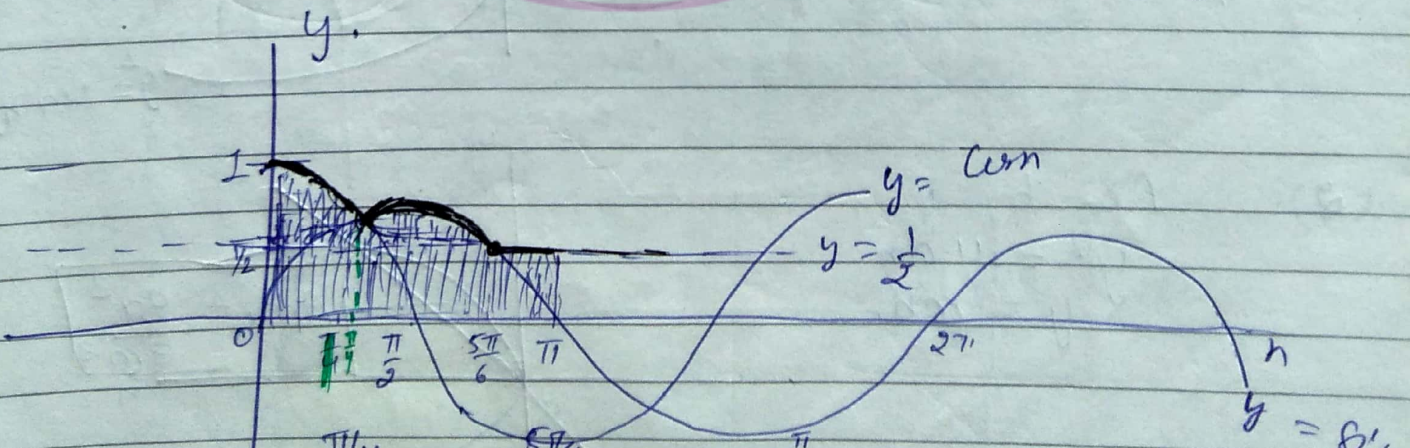
$= A = \cancel{d}$



$$A = \frac{\pi}{2} \quad = \frac{\pi(1)^2}{2} = \frac{\pi}{2}$$

Que: $f(x) = \max\left\{\sin x, \cos x, \frac{1}{2}\right\}$

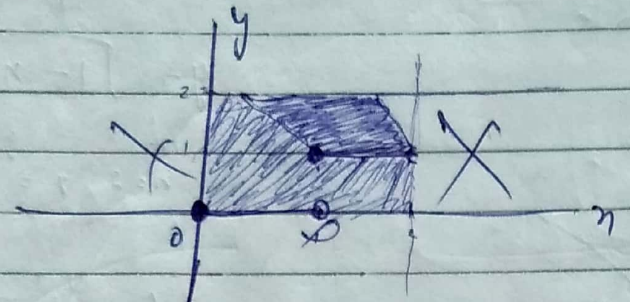
$x = 0, \; y = 0, \; x \in (0, \pi)$



$$A = \int_0^{\pi/4} \cos x + \int_{\pi/4}^{5\pi/6} \sin x + \int_{5\pi/6}^{\pi} \frac{1}{2}\, dx.$$

Que: $[x] + [y] = 2 \quad x, y \geq 0$

AAB =



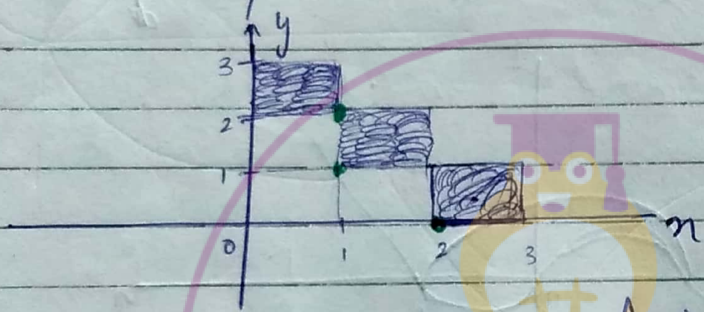
$x + y = 2$

$[x] + [y] = 2$

(1) $2 + 0$

(2) $1 + 1$

(3) $0 + 2$



$[x] = 2 \quad 2 \leq x < 3$

$[y] = 0 \therefore 0 \leq y < 1$

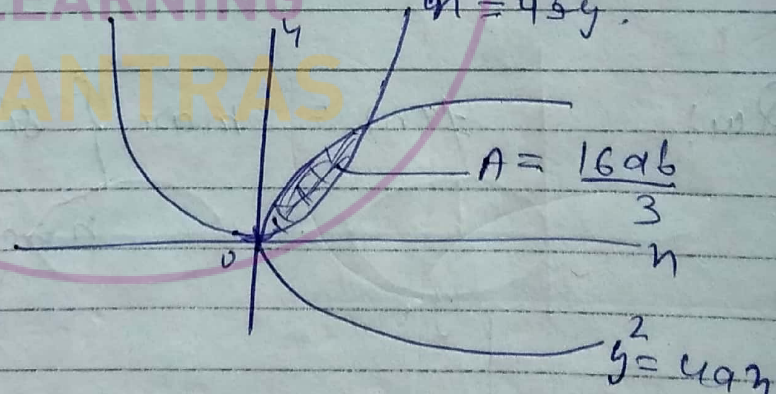
$1 \leq x < 2, \quad 1 \leq y < 2$

$A = 1 + 1 + 1 = \underline{\underline{3}}$

Note! Standard Area to be Remembered.

(1) $y^2 = 4ax$

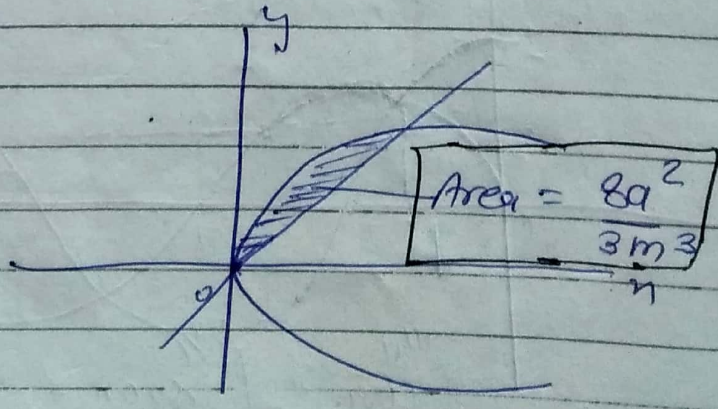
$x^2 = 4by$



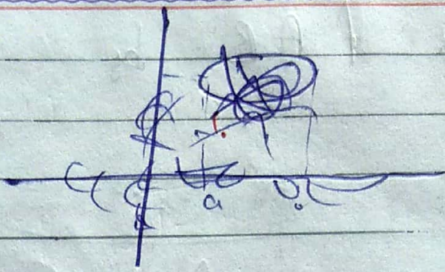
(2) AB by

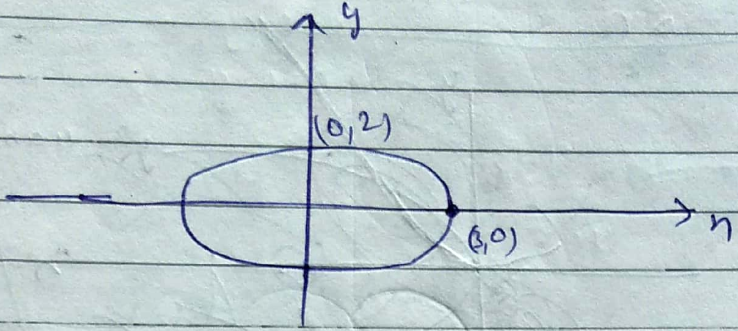
$y^2 = 4ax$

$y = mx$.



(3) $\frac{x^2}{a} + \frac{y^2}{b} = 1$





★ $A = \pi a b$

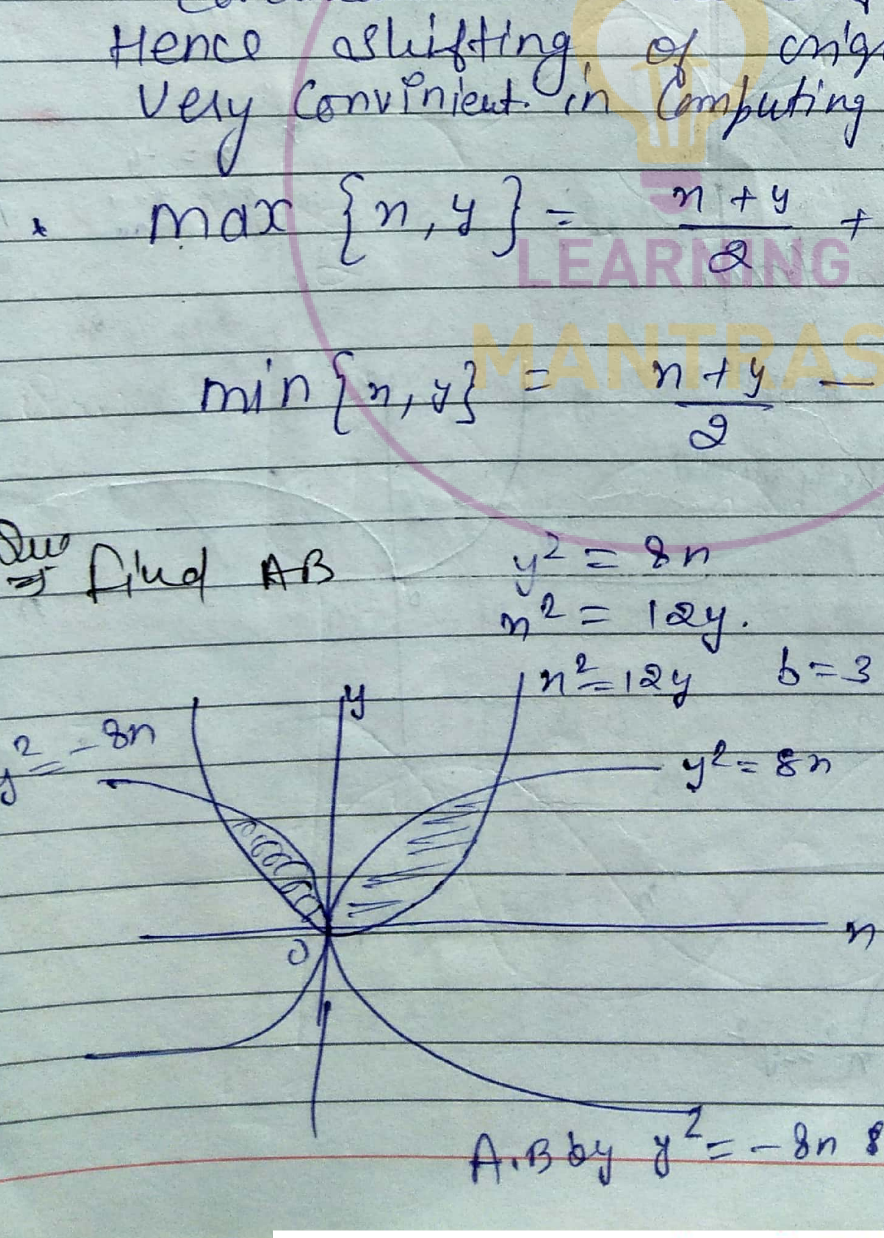
★ **Note:**

Area Remains invarient even if the cordinate axis are shifted.
Hence shifting of origin in many case proves to very convinient in computing Areas

$$\max\{x, y\} = \frac{x+y}{2} + \left|\frac{x-y}{2}\right|$$

$$\min\{x, y\} = \frac{x+y}{2} - \left|\frac{x-y}{2}\right|$$

Que ⇒ find AB $y^2 = 8x$
$x^2 = 12y$.



$b = 3$

$a = 2$

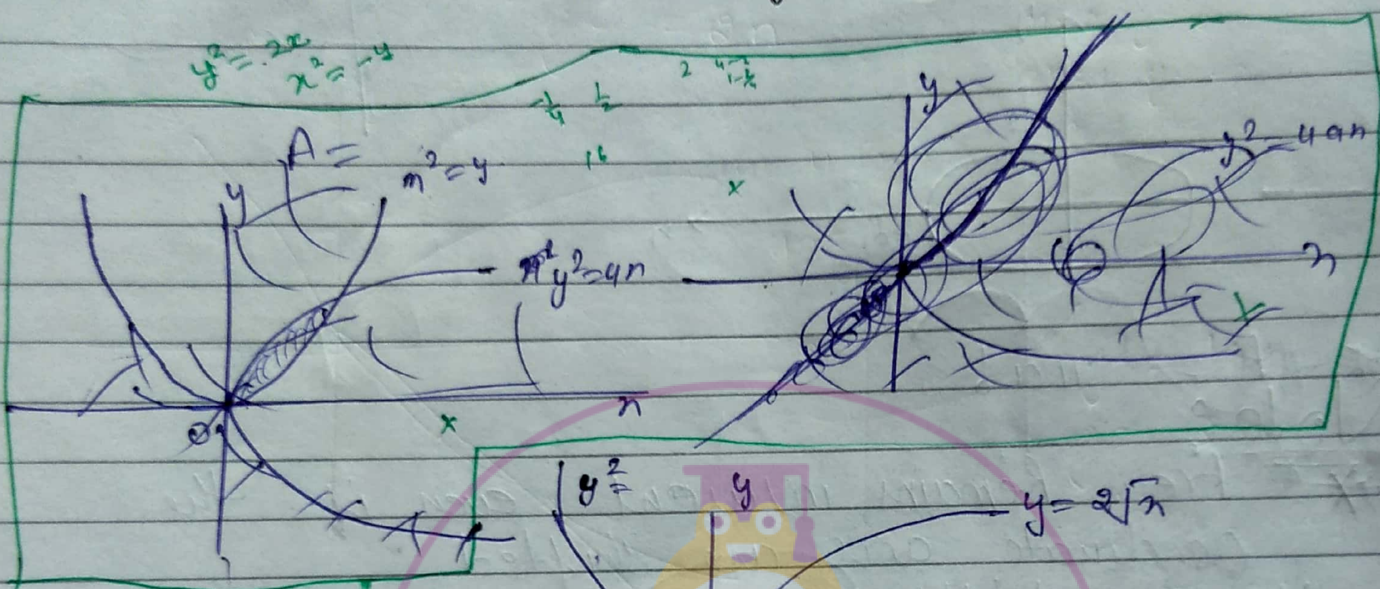
$$\frac{16ab}{3} = \frac{16 \cdot 2 \cdot 3}{3} = 32$$

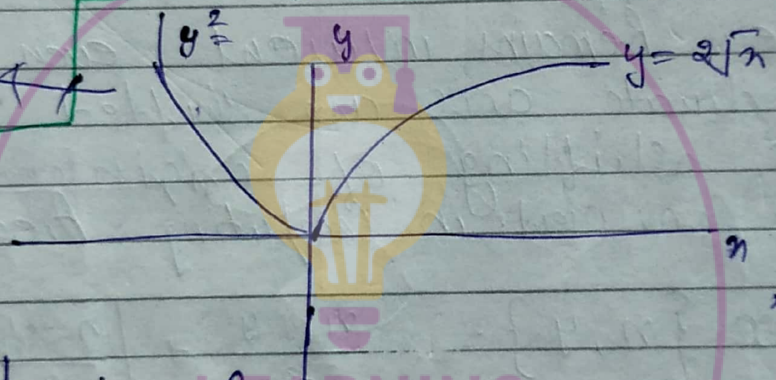
A.B by $y^2 = -8x$ & $x^2 = 12y$ ⇒ $A = 32$

$x=1,\ y=1$
$x=2,\ y=4$ $x=-1$
$x=3,\ y=9$ $y=$
$x=0,\ y=0$

Q. f AB by $y = 2\sqrt{x}$
$x = -\sqrt{y}$

$= y^2 = 4x$
$x^2 = y$

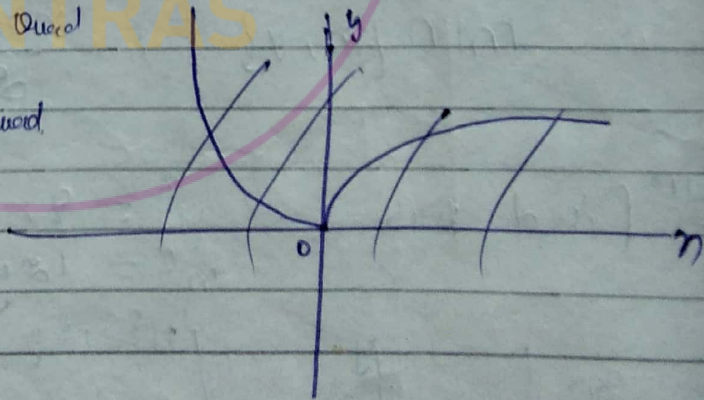




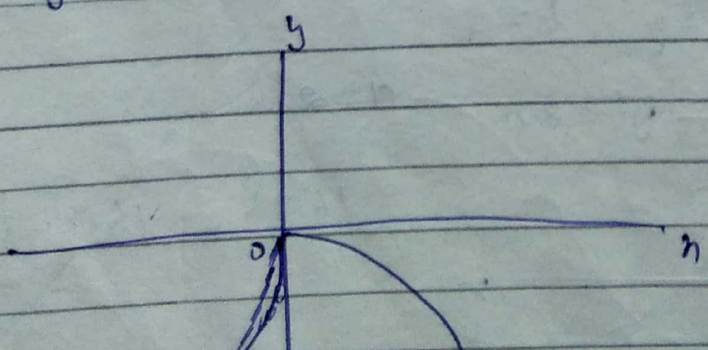
$x = -y$
$x=1 = y=-1$

No Bounded reigon form.

Que: $x = \sqrt{-y}$ → lie in 4th Quad
$y = -\sqrt{-x}$ → lie in 3rd Quad



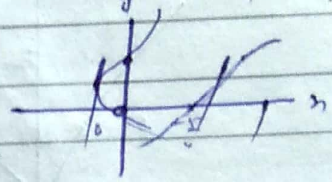
$x^2 = -y^2$
$y^2 = -x$



Q.No. 25 → 1 Page / 32 − 1
28 — 1 Page / 39 − 1/2
30 — 1 Page
34 — 1 Page

$(y-2)^2$
$= y^2+4-4y^2+2y$
$-2y$

$y^2 = 2$
$y(y-2) + 4x + 5$
$x = -\frac{5}{4}$



Que. : FAB $\quad y^2 - 2y + 4x + 5 = 0$

$$x^2 + 2x - y + 2 = 0$$

$$y^2 - 2y + 4x + 4 + 1 = 0$$

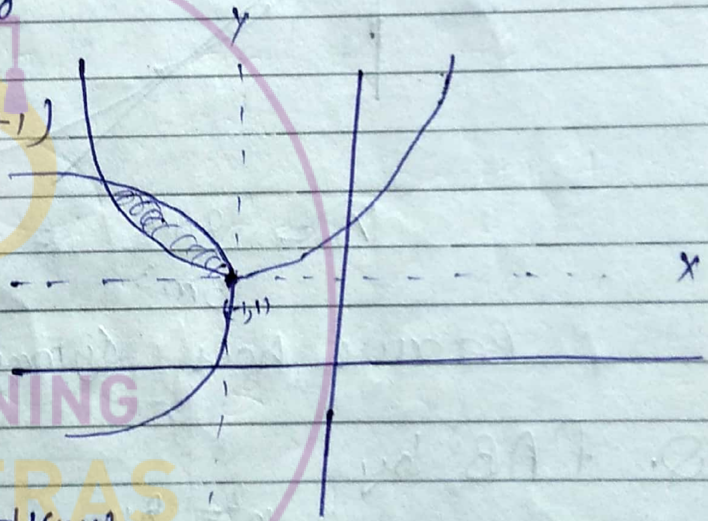
$$(y-1)^2 + 4(x+1) = 0$$

$$x^2 + 2x - y + 1 + 1 = 0$$

$$(x+1)^2 - (y-1) = 0$$

$$(x+1)^2 = (y-1)$$

$$(y-1)^2 = -4(x+1)$$



$y - 1 = Y \Rightarrow 0 \quad y = 1$

$x + 1 = X \Rightarrow 0 \quad x = -1$

$X^2 = Y \Rightarrow (x+1)^2 = (y-1)$

$Y^2 = -4X \Rightarrow (y-1)^2 = -4(x+1)$

$$a = \frac{1}{4} \qquad b = 1$$

$$A = \frac{16ab}{3} \Rightarrow \frac{4}{3} \text{ A}$$

Q = $|x-1| + |y+3| \leq 4$ $\quad$ FAB

$|X| + |Y| \leq 4$

$$x - 1 = X$$

$$y + 3 = Y$$

Area = 32.

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$\frac{8\times16}{3(\cancel{128})} = \frac{1}{}$

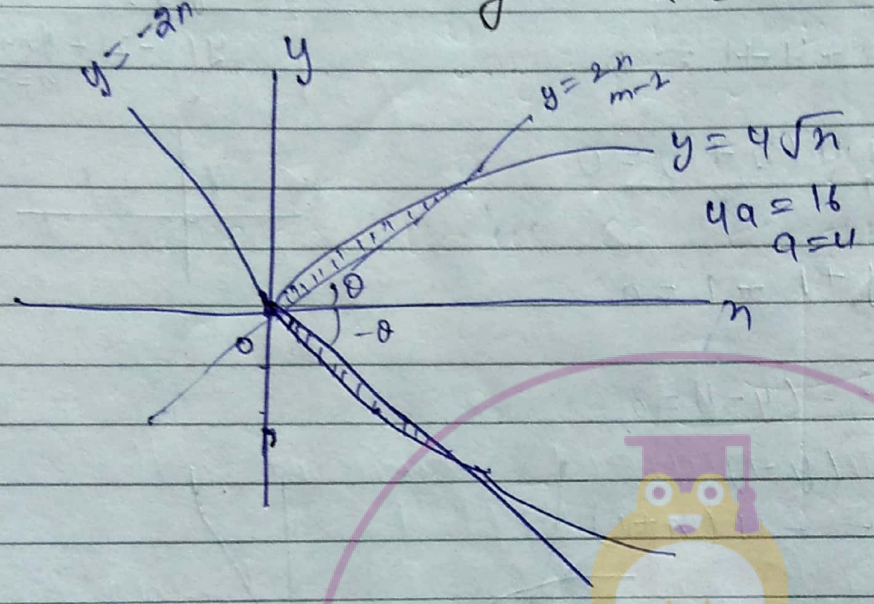
Q. fAB by $y^2 = 16x$ $\quad$ $y = 4\sqrt{x}$

$y = -2x$ $\quad$ $y = -2x$

$x=0,\ y=0$

$x=1,\ y=-2$

$x=2,\ y=-4$



$4a = 16$

$a = 4$

$y^2 = 16x$

$y = 2x$

Same as that Area.

$A = \frac{8a^2}{3m^3} = \frac{8\cdot16}{3\cdot8} = \frac{16}{3}$

Because has symmetical about x-axis.

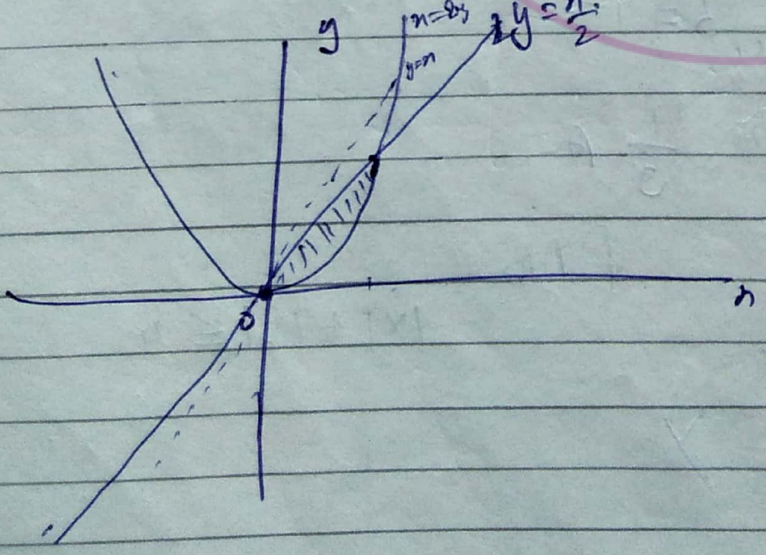
Q. fAB by $x^2 = 8y$ $\quad$ $x^2 = 8y$

$2y = x$ $\quad$ $2y = x$ $\quad$ $y = \frac{x^2}{8}$

$x=1,\ y=2$ $\quad$ $x=1 = y = \frac{1}{8} \cdot 0.1$

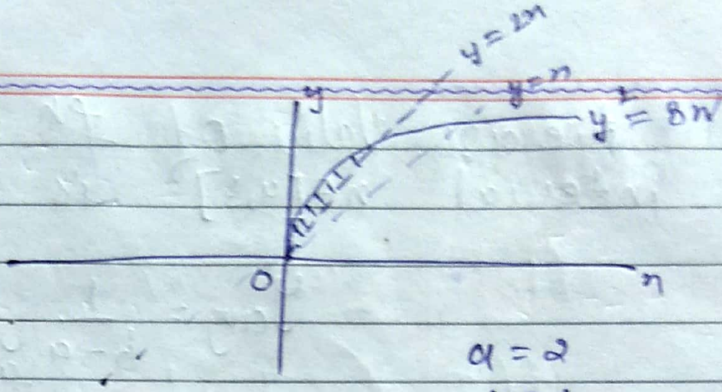
$x=2,\ y=4$ $\quad$ $x=0 = y=0$

$x=$



(14.2)

Ans $y^2 = 8x$
$y = 2x$



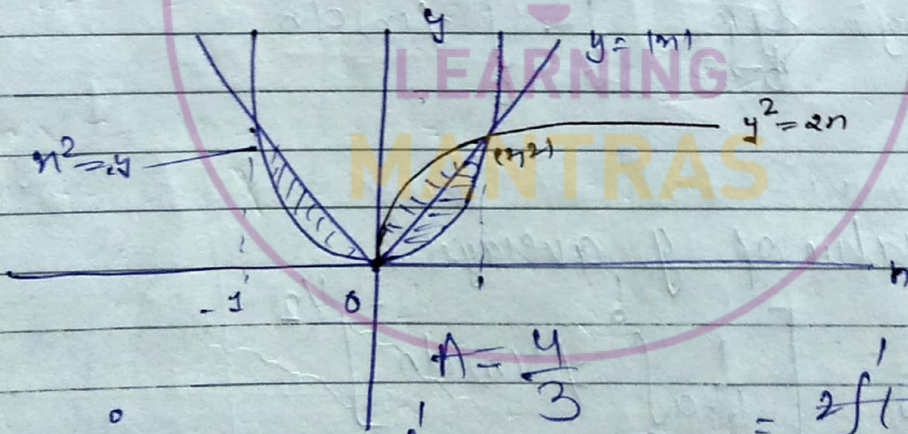
$a = 2$
$m = 2$

$A = \frac{8 \cdot 4}{3 \cdot 8} = \frac{4}{3}$

Note: Area bounded by $x^2 = 8y$ & $2y = x$ and AB by curve $y^2 = 8x$ & $y = 2x$ will be same.

Ques! fAB

$x^2 = y$ & $y = |x|$



$m = 1, a = \frac{1}{2}$

$A = \frac{8a^3}{3m^3}$

$= \frac{8 \cdot \frac{1}{4}}{3} = \frac{2}{3}$

$A = \frac{4}{3}$

$\int_{-1}^{0} \cancel{(|x| - x^2)\,dx} + \int_0^1 \cancel{x - x^2}$

$= 2\int_0^1 (x - x^2)$

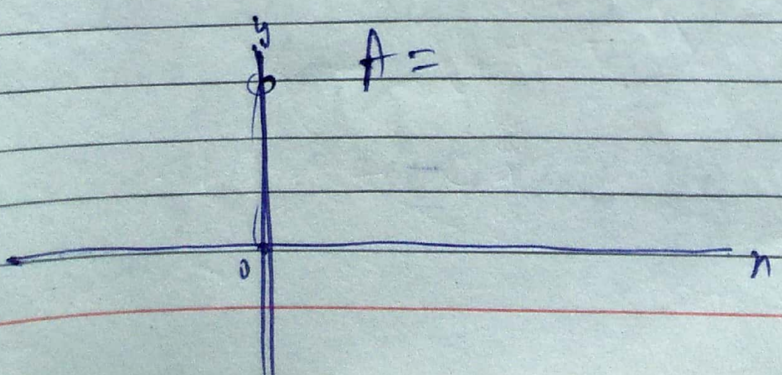
$= 2 \times \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1$

$2 \times \frac{1}{2} - \frac{1}{3}$

$2 \times \frac{3-2}{6} = 2 \times \frac{1}{6} = \frac{1}{3} A$

$A = \frac{1}{3} A_1$

fAB = $x^2 = 2y$ & $y = |x|$



A =

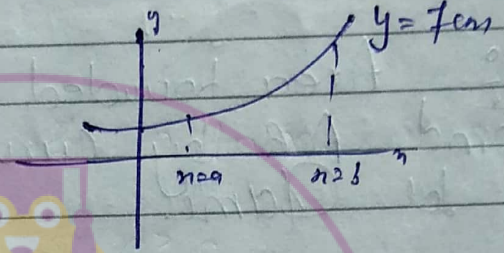
H.W [J.A and J.H.O] [0–1]

* (i) Average Value of f^n $y = f(x)$ in the interval $x \in [a,b]$ is define as

$$y_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

- average value can be +ve, −ve, or zero.

(ii)



(ii) if f^n is define in the int. $x \in (0, \infty)$

$$y_{avg} = \lim_{b \to \infty} \frac{1}{b}\int_0^b f(x)\,dx$$

Root means square

(iii) Rms value of y average

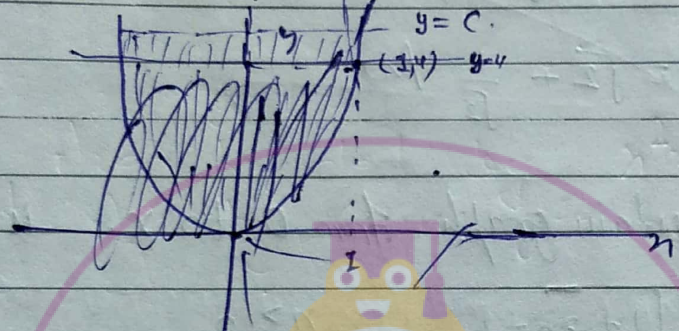
$$y_{avg} = \left[\frac{1}{b-a}\int_a^b f^2(x)\,dx\right]^{1/2}$$

* Determination of Parameter :-

Que find value of c for which AB by curve

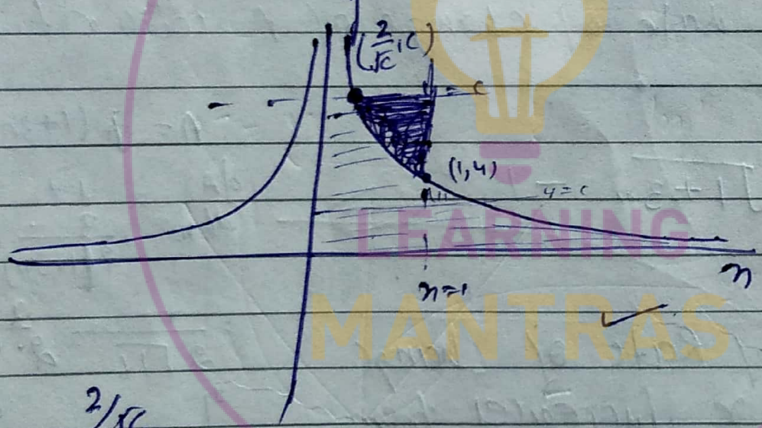
$y = \frac{4}{x^2}$, $x=1$, & $y=c$ is equal to $\frac{9}{4}$

An



$y = \frac{4}{x^2}$
$x=0$ $y=0$
$x=1$ $y=4$

A = $x=1$, $y=$



$$\int_{\frac{2}{\sqrt{c}}}^{1} \left(c - \frac{4}{x^2}\right) dx = \frac{9}{4}$$

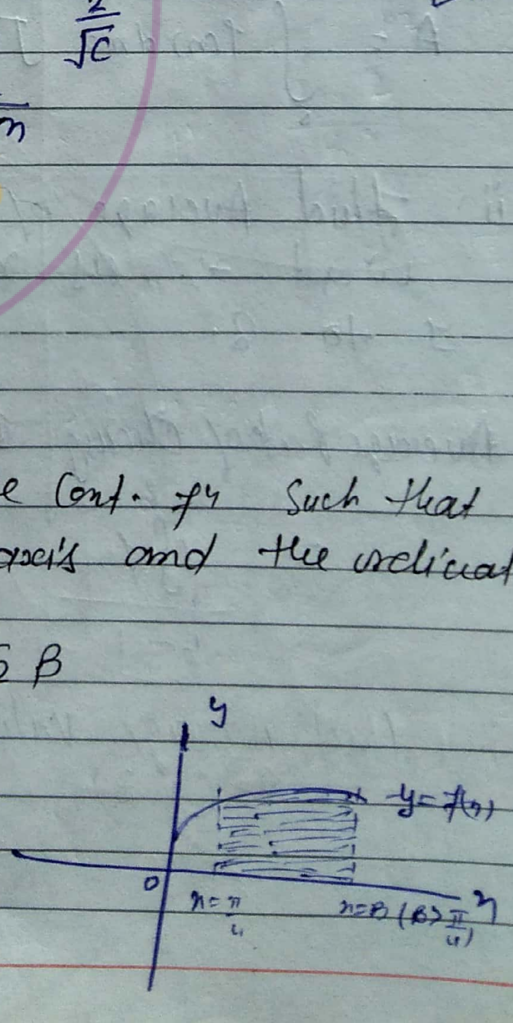
or

$$\frac{9}{4} = \int_{1}^{2/\sqrt{c}} \left(\frac{4}{x^2} - c\right) dx$$

Que :- 2 let f(x) is non -ve cont. fn such that AB by curve $y = f(x)$, x axis and the vertical $x = \frac{\pi}{4}$ and $x = \beta$ ($\beta > \frac{\pi}{4}$) is equal to $\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\,\beta$

then find $f(\frac{\pi}{4})$

$$= \int_{\pi/4}^{\beta} f(x) = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\,\beta$$



diff w.r.t β.

$$f(\beta) = \beta\cos\beta + \sin\beta - \frac{\pi}{4}\sin\beta - \cancel{\frac{\pi}{4}\sin\beta} + \sqrt{2}$$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{\pi}{4}\frac{1}{\sqrt{2}} + \sqrt{2}$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}} \text{ Ans}$$

Q. The AB by certain graph from 0 to x. is given by

$$A = \sqrt{1+3x} - 1, \quad x \geq 0$$

find instantaneous rate of change of A w.r.t x at $x = 5$

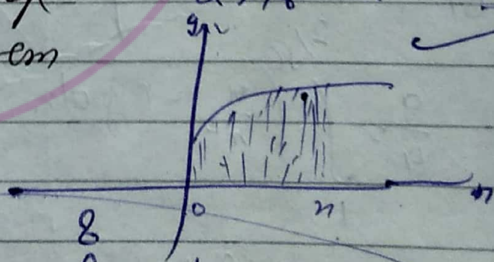
$$A = \int_0^x f(x)\,dx = \sqrt{1+3x} - 1 \Longrightarrow$$

$A = (1+3x)^{1/2} - 1$

$$\left(\frac{dA}{dx}\right)_{x=5} = \frac{3}{2(1+3x)^{1/2}}$$

$$= \frac{1}{2\sqrt{16}} = \frac{3}{2\cdot 4} = \frac{3}{8}.\text{ A}$$

(ii) Find Average of Rate of change w.r.t x as x increases from 1 to 8.



Average Rate of change 1 to 8.

$$= \frac{\int_1^8 f(x)}{8-1}$$

$$x = \int_1^8 \frac{3}{8}\,dx$$

$$= \frac{3}{8}[x]_1^8$$

$$\frac{3}{8}(8-1)$$

$$= \frac{3}{8}\times 7 = \frac{21}{8}$$

(iii) find Average value of fn in $x \in (1, 3)$

$$x^2 - 2x + 2$$

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M. Sir.

H.W J.A.

$x^2 - 2x + 2$

$x^2 - x - x + 2$

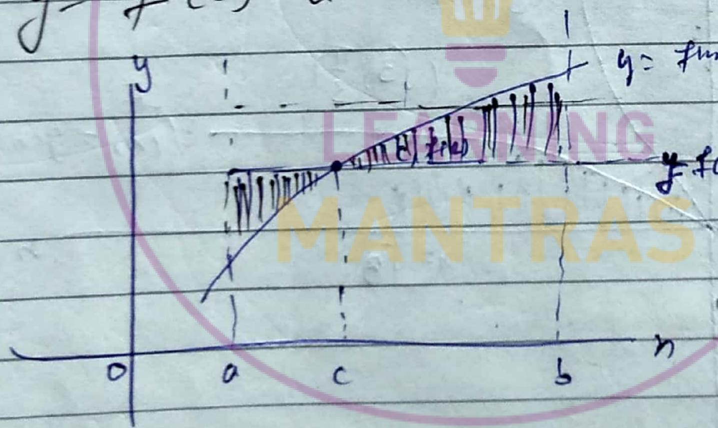
$x(x-1) - 1(x$

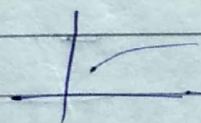
$$y_{avg} = \frac{\int_1^3 (x^2 - 2x + 2)\,dx}{3-1}$$

Note!

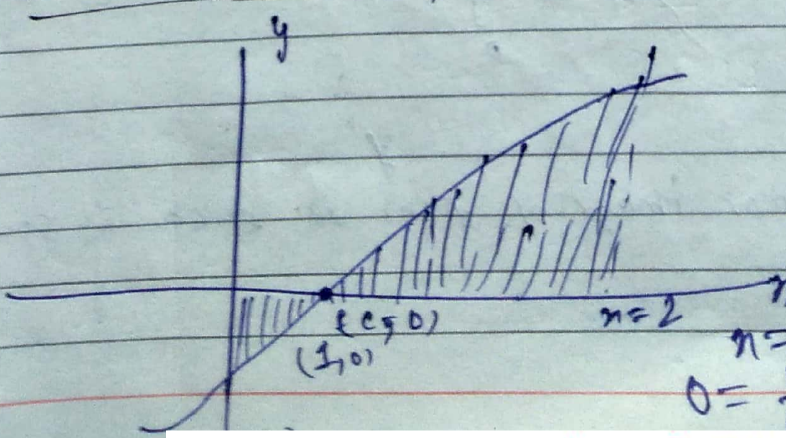
Que: $y = f(x)$ is monotonic f^n in (a, b) the AB by ordinates at $x = a$, $x = b$, $y = f(x)$ and $y = f(c)$ is minimum when $c = \frac{a+b}{2}$

$c \in (a, b)$





Que If AB by Curve $f(x) = \frac{x^3}{3} - x^2 + a$ and the lines $x=0$, $x=2$, and x axis is minimum then find A



$\int_0^2 \left(\frac{x^3}{3} - x^2 + a\right)$

$= \left(\frac{x^4}{12} - \frac{x^3}{3} + ax\right)_0^2$

$= \frac{16}{12} - \frac{8}{3} + 2a$

$= \frac{4}{3} - \frac{8}{3} + 2a$

$\frac{-4}{3} + 2a$

$2a = \frac{4}{3}$

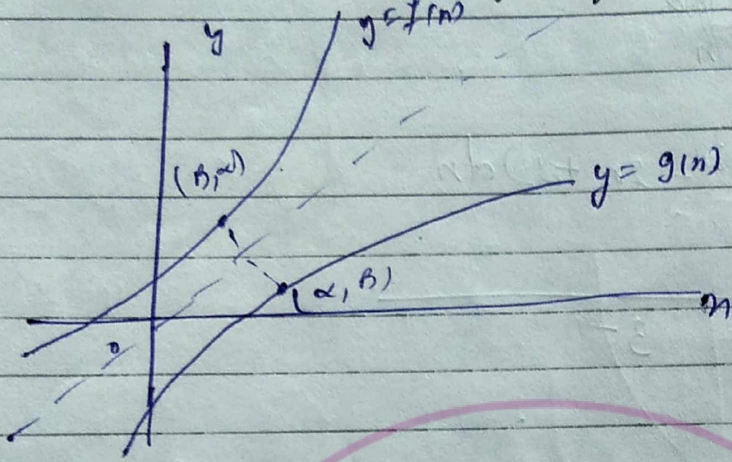
$a = \frac{4}{6}$

$x = 1$

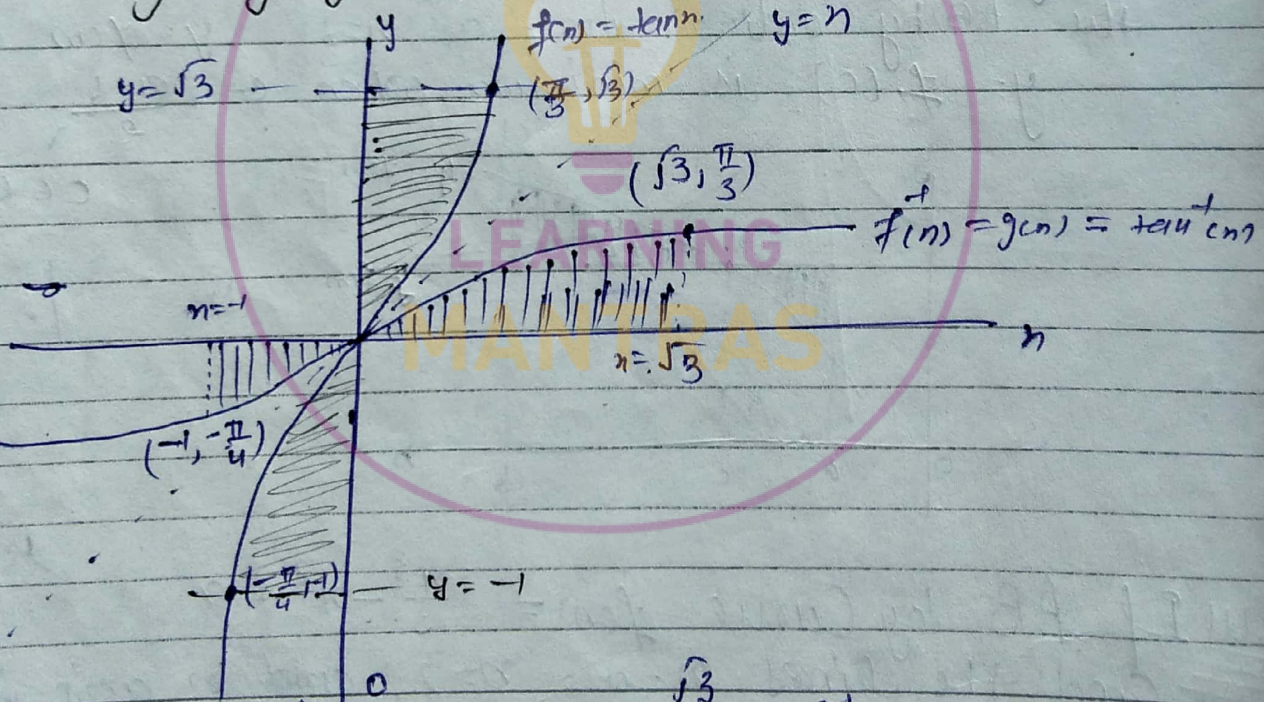
$0 = \frac{1}{3} - 1 + a$

$a = \frac{2}{3}$ Ans

Area Under the Image of f^n:



Que: Let $f(x)=\tan x$ and $g(x)$ is its inverse. Find AB by $y=g(x)$, x axis and ordinat $x=-1$, and $x=\sqrt{3}$.



$$A=\left|\int_{-1}^{0}\tan^{-1}dx\right|+\int_{0}^{\sqrt{3}}\tan^{-1}x\,dx = A_{n}$$

Method-2:

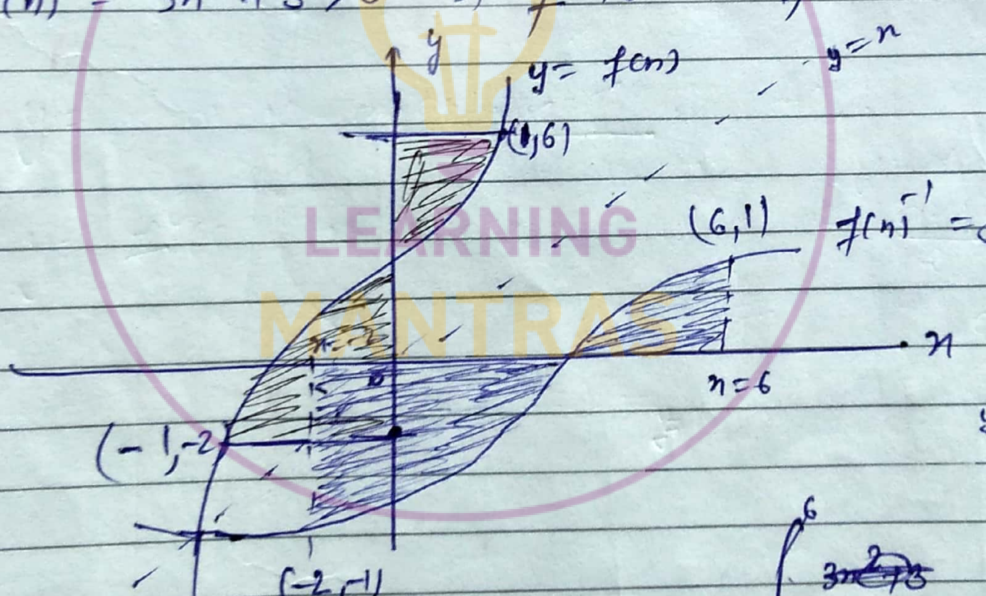
$f(x)$ mirror image of $f(x)$ is area is same of the $f^{-1}(x)$

H.W Adv. Advance.

$$\int_0^{\pi/3} (\sqrt{3} - \tan x)\,dx + \int_{-\pi/4}^{0} (\tan x - (-1))\,dy =$$

(ii) Let $f(x) = x^3 + 3x + 2$ and $g(x)$ is its inverse. Find ABbd $y = g(x)$, x axis and the ordinate at $x = -2$, at $x = 6$.

$f'(x) = 3x^2 + 3 > 0 \Rightarrow f$ is inc f^n.



$x = 6$
$y = 3x^2 +$
$\cancel{x^3 + 3x + 2}$
$8 + \cancel{6 + 2}$
-11

$$\int_{-2}^{6} \frac{\cancel{3x^2+3}}{x^3 + 3x + 2} =$$

$\int_{-1} \cancel{x^3+3x+2}$

$$A = \int_0^1 (6 - f(x))\,dx + \int_{-1}^{0} (f(x) - (-2))\,dx$$