

Handwritten Notes  
On  
Alternating Current

LEARNING  
MANTRAS



LEARNING  
MANTRAS



LearningMantrasOfficial

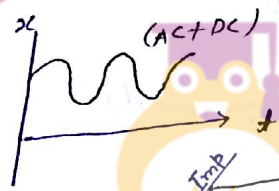
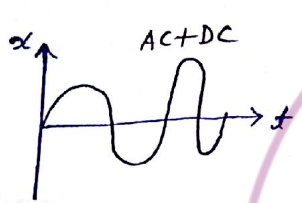
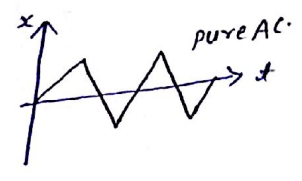
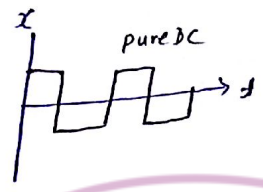
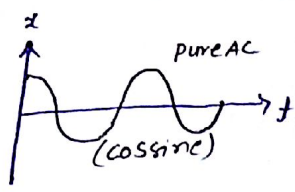
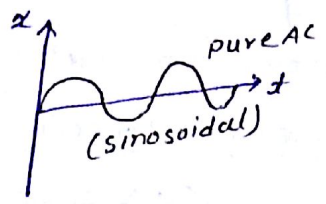
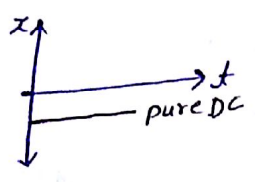
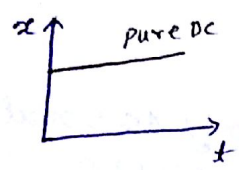


LearningMantras

# ALTERNATING CURRENT

## Electrical signal

→ Direct (unidirectional,  $\phi$  const)  
 → Alternating (Bidirectional, periodic, positive peak equal to negative peak position.)



$I_{imp}$   
 \* AC measured by Hot wire instrument  
 ↳ Heating effect of current prove AC.

$I_{imp}$   
 \* DC measured by moving coil voltmeter.  
 ↳ proved by magnetic effect of current.

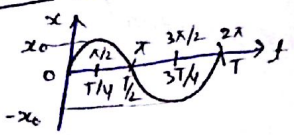
## Fundamental Alternating signal:

$$x = x_0 \sin(\omega t \pm \phi)$$

$x$  → Instantaneous value  
 $x_0$  → peak value  
 $2x_0$  → peak to peak value  
 Phase Angle →  $\omega t + \phi$   
 $\omega$  → Angular frequency.

$\phi$  = phase difference b/w current & voltage.

Case-I  $x = x_0 \cos \omega t$

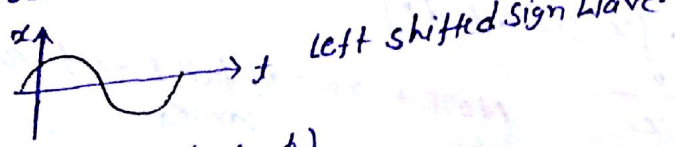


$$2\pi = T$$

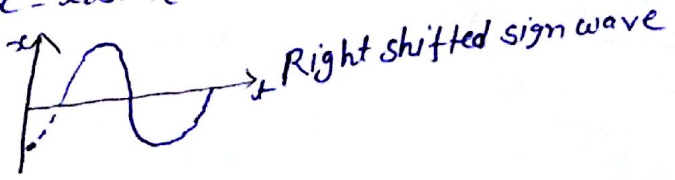
Eg →  $\frac{\pi}{3} = \frac{T}{6}$   
 $\frac{\pi}{4} = \frac{T}{8}$   
 $\frac{\pi}{6} = \frac{T}{12}$

→ In one cycle, direction change twice  
 →  $0 - T/2$  ⊕ve Half cycle.  
 →  $T/2 - T$  ⊖ve half cycle.

Case-II  $x = x_0 \sin(\omega t + \phi)$

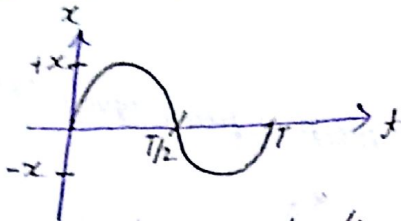


Case-III  $x = x_0 \sin(\omega t - \phi)$



# Fundamental Alternating signal

$$x = x_0 \sin \omega t$$



III → Average/Mean value/DC voltage

Case-I → For complete cycle

$$\begin{aligned} \langle x \rangle &= \frac{1}{T} \int_0^T x dt \\ &= \frac{1}{T} \int_0^T x_0 \sin \omega t dt \end{aligned}$$

$$\boxed{\langle x \rangle_T = 0}$$

NOTE → For any AC signal Average for one cycle is always zero.

Case-II → For Half cycle

$$\begin{aligned} \langle x \rangle_{+1/2} &= \frac{1}{T/2} \int_0^{T/2} x \cdot dt \\ &= \frac{2}{T} \int_0^{T/2} x_0 \sin \omega t dt \end{aligned}$$

$$\langle x \rangle_{+1/2} = \frac{2x_0}{\pi}$$

$$\begin{aligned} \langle x \rangle_{+1/2} &= +\frac{2x_0}{\pi} \\ \langle x \rangle_{-1/2} &= -\frac{2x_0}{\pi} \end{aligned}$$

NOTE → For Half cycle Average value for sinusoidal voltage is may be ⊕ve, ⊖ve, zero.

\* Average voltage/current

$$\langle V \rangle_{\text{cycle}} = 0$$

$$\langle i \rangle_{\text{cycle}} = 0$$

IV → Root Mean Square velocity (rms, Apparent, Effective, virtual)

$$x_{\text{rms}} = \left\{ \frac{1}{T} \int_0^T x^2 dt \right\}^{1/2}$$

$$x_{\text{rms}} = \left\{ \frac{1}{2} \cdot \int_0^T x_0^2 \sin^2 \omega t dt \right\}^{1/2}$$

$$\boxed{x_{\text{rms}} = \frac{x_0}{\sqrt{2}}}$$

NOTE → rms of full & Half cycle is same.

# POWER LOSS

$$P_{inst} = VI = V_0 \sin(\omega t) \cdot I_0 \sin(\omega t + \phi)$$

$$= \frac{V_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)]$$

$$\langle P_{avg} \rangle_{cycle} = \frac{V_0 I_0}{2} \cos \phi = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$$

$$= V_{rms} \cdot I_{rms} \cdot \cos \phi$$

$$P = I^2 R$$

$$H = I^2 R t$$

**NOTE** → \* If nothing is mentioned then given AC voltage consider rms only.  
 \* For Heat & power calculation only RMS value is used.  
 \* 220 volt AC is more dangerous than 220 volt DC b/c peak value of AC is 311. (अगर 2 व 311 के स्तर के मंजुरी)

## Form Factor (FF)

$$FF = \frac{x_{rms}}{x_{average}} = \frac{x_0}{\frac{2x_0}{\pi}} = \frac{\pi}{2\sqrt{2}}$$

**NOTE** → \*  $\langle \sin \omega t \rangle_T = 0$       \*  $\langle \sin^2 \omega t \rangle_T = 1/2$   
 \*  $\langle \cos \omega t \rangle_T = 0$       \*  $\langle \cos^2 \omega t \rangle_T = 1/2$   
 \*  $\langle \sin^2 \omega t \rangle_T = 0$   
 \*  $\langle \cos^2 \omega t \rangle_T = 0$

EX →  $I = I_1 + I_2 \sin \omega t$

Short trick Average for one cycle:  
 ①  $\langle I \rangle_T = \langle I_1 + I_2 \sin \omega t \rangle_T$   
 $I_1 + I_2 \times 0 = I_1$

②  $I_{rms}$  for one cycle  
 $I_{rms} = \left\{ \langle I^2 \rangle_T \right\}^{1/2}$   
 $= \left\{ \langle I_1^2 + I_2^2 \sin^2 \omega t + 2I_1 I_2 \sin \omega t \rangle_T \right\}^{1/2}$   
 $I_{rms} = \left\{ I_1^2 + \frac{I_2^2}{2} + 0 \right\}^{1/2}$   
 $= \sqrt{I_1^2 + \frac{I_2^2}{2}}$

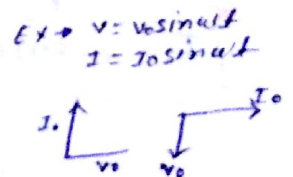
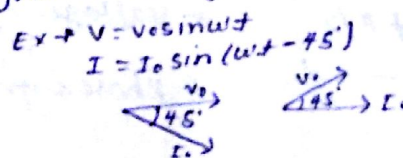
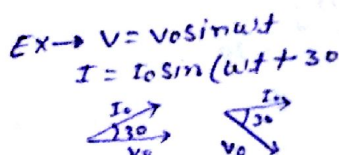
\*\*\*

## Phase

When a physical parameter changes sinusoidally with sign then it can be represented by an arrowhead straight line & this representation is called phase. Where length of line represent peak value.

## Phasor diagram

It is diagram having phase current & voltage both.

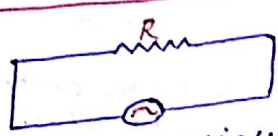


Phase difference ( $\phi$ )  $\rightarrow$  It is difference b/w phase of  $V$  &  $I$ .

Power factor =  $\cos \phi$

## AC CKTS

1.1  $\rightarrow$  pure Resistive ckt (R)

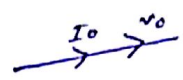


$V = V_0 \sin \omega t$

By KVL at any instant.

\*  $I = \frac{V}{R} = \frac{V_0}{R} \sin \omega t$

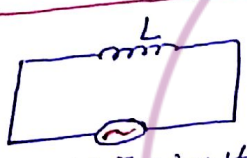
$I = I_0 \sin \omega t$

ii)  $\rightarrow$  phase diagram 

iii)  $\rightarrow$  Phase difference = 0

iii)  $\rightarrow$  power factor  $\cos \phi = 1$  (max)  
(R consume the total power of ckt).

1.2  $\rightarrow$  pure Inductive ckt (L)



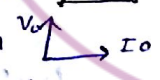
$V = V_0 \sin \omega t$

By KVL

\*  $I = I_0 \sin(\omega t - \pi/2)$

\* voltage leading the current by  $\pi/2$

\* phase difference =  $\pi/2$


\* phasor diagram 

\* power factor =  $\cos \phi = 0$

\* power =  $V_{rms} I_{rms} \cos \phi = 0$

Inductive Reactance ( $X_L$ )

\*  $X_L = \omega L = 2\pi f L$

\*  $X_L \propto f$  

\* unit  $\rightarrow$  ohm ( $R, X_L, X_C$ )

R  $\rightarrow$  Resistance = R

L ] Reactance (X)  $\leftarrow$   $X_L$

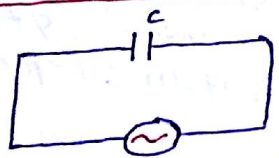
C ]  $\leftarrow$   $X_C$

$\Downarrow$

Z = Impedance

\* [General term that include both resistance & reactance.]

1.3  $\rightarrow$  pure capacitive ckt (C)



$V = V_0 \sin \omega t$

$V = \frac{q}{C} = 0$


$q = CV$

\*  $I = I_0 \sin \omega t + \pi/2$

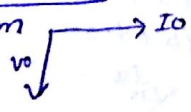
capacitive Reactance ( $X_C$ )

\*  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

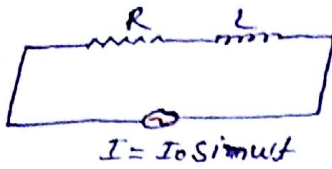
\* unit  $\rightarrow$  ohm

\*  $X_C \propto \frac{1}{f}$  

\* voltage lagging by current by  $\pi/2$  angle  
so  $\phi = \pi/2$

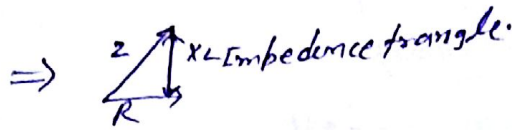
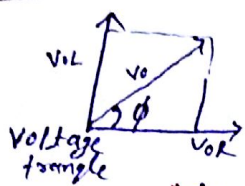
\* phase diagram 

14) → Series RL →



Let,  $I = I_0 \sin \omega t$   
 $V_R = V_0 R \sin \omega t$   
 $V_L = V_0 L \sin(\omega t + \pi/2)$   
 When  $\begin{cases} V_0 R = I_0 R \\ V_0 L = X_0 L \end{cases}$

Phase Diagram



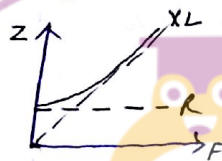
\*i)  $V_0 = \sqrt{V_0 R^2 + V_0 L^2}$

\*ii) Impedance (Z)

$Z = \sqrt{R^2 + X_L^2}$

\*iii) Unit = Ohm

liv) Z v/s f



\*  $f = 0 \Rightarrow X_L = 0 \Rightarrow Z = R$   
 $f(\uparrow) \Rightarrow X_L(\uparrow) \Rightarrow Z(\uparrow)$   
 $f(\infty) \Rightarrow X_L(\infty) \Rightarrow Z(\infty)$

Imp  $V$  Leading

\* phase difference (φ) = Acute Angle

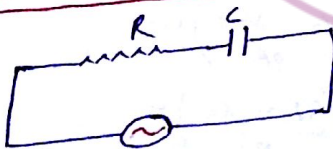
$\phi = \tan^{-1} \left( \frac{V_0 L}{V_0 R} \right) = \tan^{-1} \left( \frac{X_L}{R} \right)$

\* power factor =  $\cos \phi = \frac{V_0 R}{V_0} = \frac{R}{Z}$

Imp Equation of supply voltage

$V_{\text{supply}} = V_0 \sin(\omega t + \phi)$

15) → Series RC

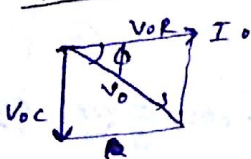


Let's ckt current  $I = I_0 \sin \omega t$

$V_R = V_0 R \sin \omega t$   
 $V_C = V_0 C \sin(\omega t - \pi/2)$

When  $\begin{cases} V_0 R = I_0 R \\ V_0 C = X_0 C \end{cases}$

Phase diagram



\*  $V_0 = \sqrt{V_0 R^2 + V_0 C^2}$

\* Impedance (Z)

$Z = \sqrt{R^2 + X_C^2}$

\* Z v/s f

\*  $f = 0 \Rightarrow X_C = \infty \Rightarrow Z = \sqrt{R^2 + X_C^2} = \infty$   
 \*  $f(\uparrow) \Rightarrow X_C = \downarrow \Rightarrow Z \downarrow$   
 \*  $f = \infty \Rightarrow X_C = 0 \Rightarrow Z = R$



\* voltage lagging.

\* Phase difference ( $\phi$ )  $\Rightarrow$  Acute Angle.

$$\phi = \tan^{-1} \left( \frac{V_{0C}}{V_{0R}} \right) = \tan^{-1} \left( \frac{X_C}{R} \right)$$

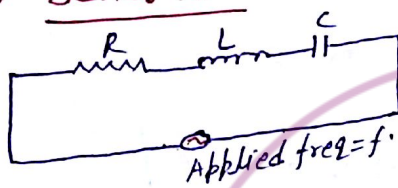
\* power factor

$$PF = \cos \phi = \frac{V_{0R}}{V_0} = \frac{R}{Z}$$

\* Equation of power supply

$$V_{\text{supply}} = V_0 \sin(\omega t - \phi)$$

161  $\rightarrow$  Series RLC

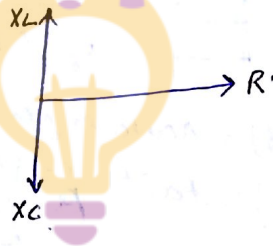
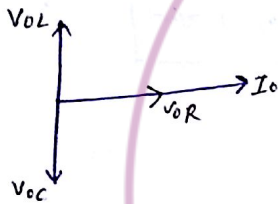


Let  $I_{\text{ext}} = I_0 \sin \omega t$

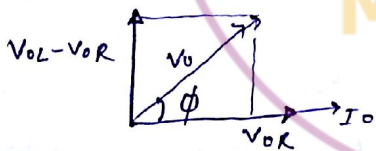
$$V_R = V_{0R} \sin \omega t$$

$$V_L = V_{0L} \sin(\omega t + \pi/2)$$

$$V_C = V_{0C} \sin(\omega t - \pi/2)$$



Case-I If  $V_{0L} > V_{0C}$



\* Series R-L behaviour.

$$* V_0 = \sqrt{V_{0R}^2 + (V_{0L} - V_{0C})^2}$$

$$* Z = \sqrt{R^2 + (X_L - X_C)^2}$$

\* voltage leading.

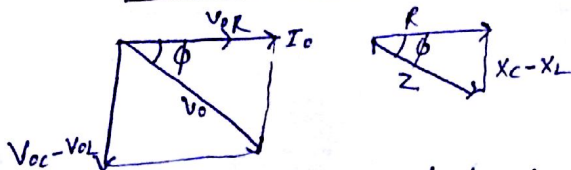
$$* \phi = \tan^{-1} \left( \frac{V_{0L} - V_{0C}}{V_{0R}} \right)$$

$$= \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$$

\* power factor

$$P.F = \cos \phi = \frac{V_{0R}}{V_0} = \frac{R}{Z}$$

Case-II  $\rightarrow$  If  $V_{0C} > V_{0L}$



\* Series R-C behaviour

$$* V_0 = \sqrt{V_{0R}^2 + (V_{0C} - V_{0L})^2}$$

$$* Z = \sqrt{R^2 + (X_C - X_L)^2}$$

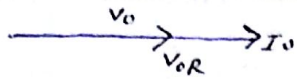
$$* \text{Power factor } \cos \phi = \frac{V_{0R}}{V_0} = R/Z$$

\* 'V' Lagging.

$$* \phi = \tan^{-1} \left( \frac{V_{0C} - V_{0L}}{V_{0R}} \right)$$

$$= \tan^{-1} \left( \frac{X_C - X_L}{R} \right)$$

Case - III → Voltage Resonance ( $V_{oL} = V_{oC}$ )



- \* pure 'R'
- \*  $V_o = V_{oR}$
- \*  $\phi = 0$
- \* power factor =  $\cos \phi = 1$  (Max)
- \*  $Z = R$  (min)
- \*  $I = \frac{V}{Z} = \frac{V}{R}$  (Max)

At Resonance

$$(X_L)_r = (X_C)_r$$

$$\rightarrow (V_{oL})_r = (V_{oC})_r$$

$$(I_o X_L)_r = (I_o X_C)_r$$

$$(\omega L)_r = \left(\frac{1}{\omega C}\right)_r$$

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ (Rad/sec)}$$

$$F_r = \frac{1}{2\pi\sqrt{LC}} (XL)$$

⇒ Z vs f

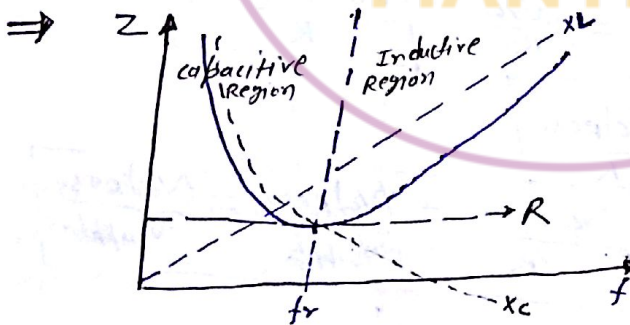
ii) →  $f = 0$   
 $R = R$   
 $X_L = 0$   
 $X_C = \infty$   
 $Z = \infty = X_C$

iii) →  $0 < f < f_r$   
 $R = R$   
 $X_L \uparrow$   
 $X_C \downarrow$   
 $X_C > X_L$  } Series R-C

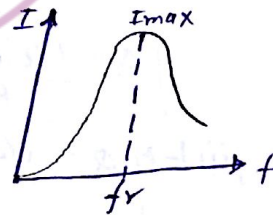
iii) →  $f = f_r$   
 $R = R$   
 $X_L = X_C$  } pure 'R'

iv) →  $f_r < f < \infty$   
 $R = R$   
 $X_L > X_C$  } Series R-L

v) →  $f = \infty$   
 $R = R$   
 $X_L = \infty$   
 $X_C = 0$  }  $Z = \infty = X_L$   
 pure 'L'



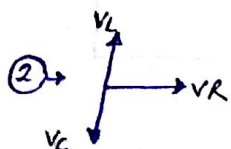
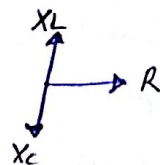
Graph  
 $I = V/Z$



General Knowledge

① → Resistance = R  
 Reactance = X

$$\left. \begin{matrix} X_L = \omega L \\ X_C = \frac{1}{\omega C} \end{matrix} \right\} Z \Rightarrow$$



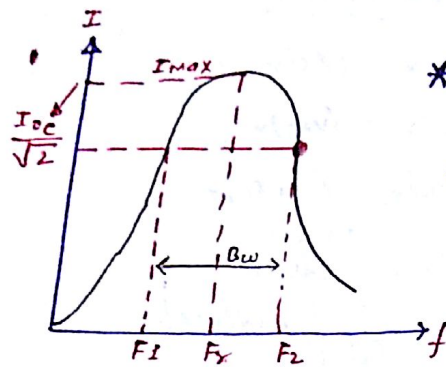
③ →  $\phi = \tan^{-1} \left( \frac{V}{R} \right)$

④ →  $\cos \phi = \left( \frac{R}{Z} \right) = \text{power factor}$



## # Half power frequency

At this freq. current become  $\frac{1}{\sqrt{2}}$  times of  $I_{max}$ . So power become MAX.



$$* f_1 = f_r - \frac{R}{4\pi L}$$

$$* f_2 = f_r + \frac{R}{4\pi L}$$

\* Bandwidth (Bw)

$$\Delta f = f_r - f_1$$

$$\Delta f = \frac{R}{2\pi L} \text{ (Hz)}$$

$$\Delta \omega = \frac{R}{L} \text{ (Rad/sec)}$$

\* Fractional Band width

$$\frac{\Delta f}{f_r} = \frac{\Delta \omega}{\omega_r}$$

$$\frac{R/L}{1/\sqrt{2}}$$

$$= R \sqrt{\frac{C}{L}}$$

\* Quality factor (Q)

→ Measure of sharpness of 'I' v/s 'f' curve.

→ If 'Q' ↑ ⇒ sharpness ↑

→ 'Q' ↓ ⇒ sharpness ↓

→ It is inverse of fractional Bandwidth.

$$\text{ii} \rightarrow Q = \frac{I}{f_r \cdot Bw} = \frac{I}{R \sqrt{\frac{L}{C}}}$$

$$\text{iii} \rightarrow Q = \frac{\omega_r}{\Delta \omega} = \frac{\omega_r}{R/L} = \frac{\omega_r L}{R} = \frac{(X_L)_{\text{resonance}}}{R}$$

$$Q = \frac{(X_L)_{\text{reso}}}{R} = \frac{(X_C)_{\text{reso}}}{R}$$

$$\text{iiii} \rightarrow Q = \frac{(V_L)_{\text{reso}}}{V_R} = \frac{(V_C)_{\text{reso}}}{V_R} = \frac{(V_L)_{\text{reso}}}{V_{\text{supply}}} = \frac{(V_C)_{\text{reso}}}{V_{\text{supply}}}$$

## Power in AC ckt

$$\text{ii} \rightarrow P_{\text{inst}} = V_{\text{inst}} \cdot I_{\text{inst}}$$

$$\text{iii} \rightarrow P_{\text{peak}} = V_{\text{peak}} \cdot I_{\text{peak}}$$

$$\text{iv} \rightarrow P_{\text{apparent}} = V_{\text{rms}} \cdot I_{\text{rms}}$$

\* Real/Average power of ckt.

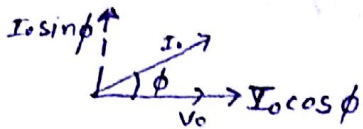
$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$P = I_{\text{rms}}^2 R = \left(\frac{V_R}{R}\right)^2 R$$

\* ckt voltage & current  
 $P = V_{\text{rms}} I_{\text{rms}} \cos \phi$   
 \*  $P = I_{\text{rms}}^2 R = \frac{V_R^2}{R}$   
 (Resistance voltage)

# # Wattless current / Workless current / Powerless current

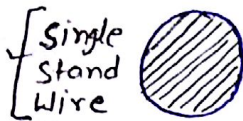
It is part of ckt current which is not responsible for any power consumption.



$$\begin{aligned}
 * I_{\text{Wattless}} &= I_{\text{rms}} \sin \phi \\
 * I_{\text{Wattfull}} &= I_{\text{rms}} \cos \phi \\
 * I_{\text{ckt}} &= \text{Pythagorous of both}
 \end{aligned}$$

## # SKIN EFFECT →

- \* High freq. AC does flow near the surface of wire.
- \* So, AC cable is made by multiple wire of smaller cross sectional area rather than thick cable.



$$R = \frac{\rho L}{A} \quad \therefore A (\downarrow\downarrow\downarrow) \Rightarrow \text{Res.} (\uparrow\uparrow\uparrow)$$

$$\therefore \text{Power Loss} = I^2 R \text{ Loss} (\uparrow\uparrow)$$



$$A_{\text{eff}} (\uparrow) \Rightarrow R_{\text{eff}} (\downarrow)$$

$$\text{Power Loss} \Rightarrow \downarrow$$

## # Hot Wire Ammeter & Voltmeter

- i) Moving coil galvanometer does not work on AC ckt. since it's reversed direction continuously.
- So, measure AC, a device is made based on heating effect of current called Hot wire Ammeter & Voltmeter.
- \* Its direction is direction independent.

$$\text{Deflection } \theta \propto H$$

$$\begin{aligned}
 \theta &\propto i_{\text{rms}}^2 \\
 \theta &\propto V_{\text{rms}}^2
 \end{aligned}
 \quad
 \begin{aligned}
 H &\propto i_{\text{rms}}^2 \\
 H &\propto V_{\text{rms}}^2
 \end{aligned}$$

- iii) It measure r.m.s value.
- iii) It has non-linear scale.
- iv) It can work in AC & DC Both.

## \* NOTE → i) In case of Resonance in R-L-C ckt

Amplitude of current is max.

For max Amplitude  $X_C = X_L$

Resonance Angular Frequency →  $\omega = \frac{1}{\sqrt{LC}}$

Linear Frequency →  $f = \frac{1}{2\pi\sqrt{LC}}$

iii) Avg power

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2 \cdot R}{|Z|^2}$$

$\cos \phi \rightarrow$  power factor.

\* pure 'R'  $\Rightarrow \phi = 0$   
 $\cos \phi = \max = 1$   
 \* pure 'L' or 'C'  $\Rightarrow \phi = \frac{\pi}{2}$  or  $-\frac{\pi}{2}$   
 $\cos \phi = 0$   
 (Wattless current found in ckt).

* In India 220V 50 Hz	* In USA 110V 60 Hz
-----------------------------	---------------------------

\* 1 unit = 1 kWh =  $1000 \times 60 \times 60 = 3.6 \times 10^6$  Jule.  
 \* Any scalar quantity come with phase diff. in physics then vector addition occur. not simple addition.

- \* R  $\rightarrow$  power खाता है।
- \* L  $\rightarrow$  power नहीं खाता है।
- \* C  $\rightarrow$  power नहीं खाता है।
- \* L-C  $\rightarrow$  खसकाता खाता है।

Phase & Amplitude Relation for Alternating current & voltage.

SYMBOL	Impedance	Phase of current In phase $\bar{v}_R$	Phase Angle ( $\phi$ )	Amplitude Relation
R	R	In phase $\bar{v}_R$	$0^\circ$	$V_R = IR$
C	$X_C$	Leads $V_C$ by $90^\circ$	$-90^\circ$	$V_C = IX_C$
L	$X_L$	Lags $V_L$ by $90^\circ$	$+90^\circ$	$V_L = IX_L$

Mnemonic  $\Rightarrow$  designed to assist the memory.  
 $\hookrightarrow$  (ELI the ICE man)

- \* L  $\Rightarrow$  Inductance
- \* C  $\Rightarrow$  capacitance
- \* E  $\Rightarrow$  voltage
- \* I  $\Rightarrow$  current.

- \* In Inductive ckt (ELI)  $\rightarrow$  the current (I) lags the voltage (E)
- \* In capacitive ckt (ICE)  $\rightarrow$  the current leads the voltage.

